Development of a pseudo-uniform structural quantity for the active control of structural radiation

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Active noise control has been a highly researched field over the past few decades but the active control of the radiating structures has recently excited interest. Multiple structural quantities and their relationships to acoustic radiation are investigated. This paper also looks at the control of a new structural quantity developed taking advantage of the principle of Rayleigh's integral and radiated power being strongly dependent on volume velocity. The benefit of this new quantity is that while most active control techniques are highly dependent on sensor location, this technique is not. The control of this quantity and its effect on radiated power and acoustic radiation modes is presented.

Introduction

Active noise control has been a topic of interest in the past few decades with heightened interest recently due to advancement in computer processing power and speed. Many different methods have been developed and tested. Active noise control uses sensors and separate control methods to interfere with unwanted noise and effectively cancel it out. Most ANC (active noise control) systems use speakers to cancel out the noise. This paper focuses on current control techniques, with their benefits and drawbacks, reasons for altering the structural vibrations to control radiation rather than cancel sound after being radiated into the enclosure. This paper also introduces a new structural quantity which can be used in active control of vibrating structures.

Background

ANC systems usually consist of a sensor, control filter and one or more control sources as shown in figure 1.

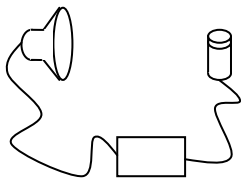
Figure 1: ANC Setup

Currently, many systems in the literature employ a filtered-x algorithm[1,2] (FXLMS) which is a feed-forward system. Because of the feed forward nature, system identification must be preformed prior to controlling. This is performed by computing a transfer function between the control source(s) and the error sensor. Most of the change in active control systems has been brought about by the error sensor, the algorithm performance function. Multiple performance functions have been investigated with a widely used and easy to implement function given by minimizing squared pressure. A limitation of current systems, mostly due to the error sensor, is the strong correlation between the amount of global control and the sensor location. If the sensor for minimizing squared pressure is placed at a pressure node, the sound field will not be attenuated and in most cases will be boosted by the control speaker(s). Most systems will only respond well when placed in specific locations, leading to the notion that the acoustic field or structure response must be known, a fact which could an expensive investment of time and equipment. Sommerfeldt[3] proposed a new performance function, acoustic energy density, which instead of solely minimizing squared pressure at a point minimizes pressure and particle velocity resulting in a more global energy control and which is less dependent on sensor location.

Structural ANC

In most scenarios, vibrating structures are the noise sources in enclosures. For this reason, this paper focuses on the control of structural vibrations to minimize the acoustic radiation. The same methods mentioned previously can be applied to structural active control. To control acoustic radiation, relationships must first be investigated. Two common structural quantities which were focused on strongly were power flow (structural intensity) and structural energy density (SED). Pavic[4] introduced a simple technique to measure power flow using an array of four accelerometers, but this requires processing in the frequency domain, which for active control techniques will not be sufficient but has been done by Pereira[5]. By altering the structural performance function and altering the vibrations of the structure, the acoustic radiation in theory could be minimized. Given in Figure 2 are the structural metrics as explained.
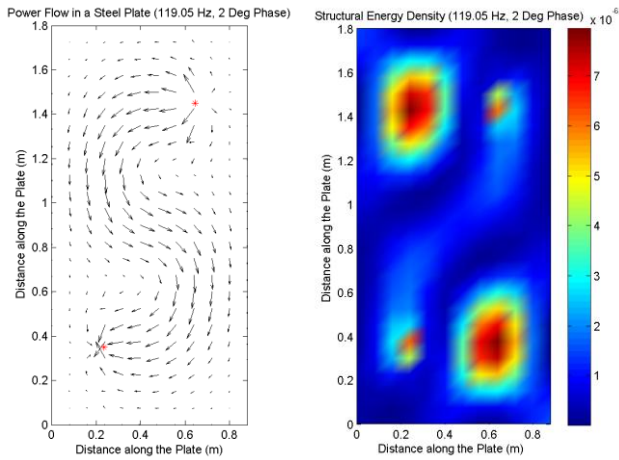

Figure 2: Power Flow and Structural Energy Density

Structural Acoustic Relationships

After a further investigation of power flow and structural energy density, it was determined that no practical relationships exist between power flow and the acoustic radiation. Structural energy density provides some relationships but for active noise control techniques are not practical because, for good control, the correct placement of the sensor on the structure requires pervious knowledge of the structural vibrations. A new quantity is under investigation which does not require prior knowledge of the structural vibrations, is independent of sensor location, and only requires a four accelerometer array. To show how this quantity was developed, it is important to understand how the structural and acoustic radiation relationships were determined.

When computing the pressure in a room, Rayleigh's integral is given by

$$P(r,t) = \frac{j\omega\rho_o}{2\pi} e^{j\omega t} \int_S \frac{\tilde{v}_n(r_s)e^{-jkR}}{R} dS \quad (1)$$

where R is the distance from a point on the source a point in space and $\tilde{v}_n$ is the normal velocity amplitude at a position given by the surface vector r_s. Also, as a structure vibrates, it emits a certain amount of power, which power is dependent on the driving frequency and the velocity response of the plate. The power radiated from a plate in terms of elementary radiators is given by

$$\bar{P}(\omega) = \{\tilde{v}_e\}^H [R]\{\tilde{v}_e\} \quad (2)$$

The plate is broken up into individual elements as given by Figure 2 and $\{\tilde{v}_e\}$ is a velocity vector containing the velocities of the individual radiators.
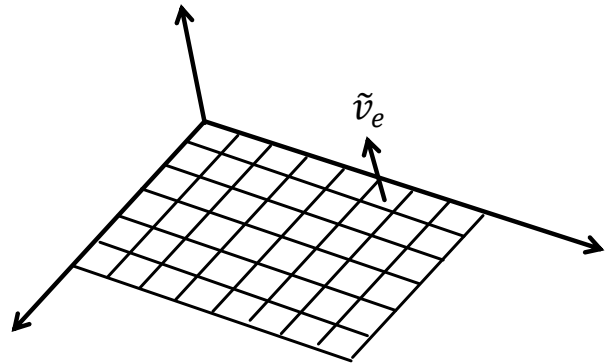

Figure 3: Panel broken up into elementary radiators

The [R] matrix is given by

$$[R] = \frac{\omega^2 \rho_0 A_e^2}{4\pi c} \begin{bmatrix} 1 & \frac{\sin(kR_{12})}{kR_{12}} & \cdots & \frac{\sin(kR_{1R})}{kR_{1R}} \\ \frac{\sin(kR_{21})}{kR_{21}} & 1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\sin(kR_{R1})}{kR_{R1}} & \cdots & \cdots & 1 \end{bmatrix}$$

Where R_{mn} is the distance between the m and nth element and $\{\tilde{v}_e\}$ is a velocity vector containing the velocities of the individual radiators.

Structural Radiation Energy – V_{comp}

As shown in equations (1) & (2), the far-field pressure and radiated power are highly dependent on the velocity response of the panel, given that ω, ρ_o and c are constants. Based on this, the development of a new structural quantity, V_{comp}, was created in order to obtain a constant velocity field over a plate.

In observing the (1,1) mode of a simply supported vibrating plate, it was determined that four velocity terms would provide a constant velocity over the plate, which terms are

$$\left(\frac{dw}{dt}\right)^2, \left(\frac{d^2w}{dxdt}\right)^2, \left(\frac{d^2w}{dydt}\right)^2, \left(\frac{d^3w}{dxdydt}\right)^2$$

These terms correspond to a transverse, bending in x and y directions and a twisting velocity, respectively. Scaling factors were added to each velocity term in order to minimizing the standard deviation over the plate. The new quantity with the scaling factors is shown in equation (3).

$$V_{comp} = \alpha\left(\frac{dw}{dt}\right)^2 + \beta\left(\frac{d^2w}{dxdt}\right)^2 + \gamma\left(\frac{d^2w}{dydt}\right)^2 + \delta\left(\frac{d^3w}{dxdydt}\right)^2 \quad (3)$$

Simulation

The structural quantities were investigated using a modal summation model of a simply supported plate with multiple forcing locations and includes structural damping. The plate displacement is given by

$$
\begin{aligned}
w(x,y) &= \frac{f_1}{\rho_s h} \sum_m^\infty \sum_n^\infty \frac{W_{mn}(x,y)W_{mn}(x_1,y_1)[\omega_{mn}^2 - \omega^2 - i\eta\omega_{mn}^2]}{[\omega_{mn}^2 - \omega^2]^2 + \eta^2\omega_{mn}^4} \\
&+ \frac{f_2}{\rho_s h} \sum_m^\infty \sum_n^\infty \frac{W_{mn}(x,y)W_{mn}(x_2,y_2)[\omega_{mn}^2 - \omega^2 - i\eta\omega_{mn}^2]}{[\omega_{mn}^2 - \omega^2]^2 + \eta^2\omega_{mn}^4} \quad (4)
\end{aligned}
$$

$$W_{mn}(x,y) = \frac{2}{\sqrt{L_x L_y}} sin\left(\frac{m\pi x}{L_x}\right) sin\left(\frac{n\pi y}{L_y}\right) \quad (5)$$

$$\omega_{mn} = \sqrt{\frac{D}{\rho_s h}\left(\frac{m^2\pi^2}{L_x^2} + \frac{n^2\pi^2}{L_y^2}\right)} \quad (6)$$

$$D = \frac{Eh^3}{12(1-v^2)} \quad (7)$$

The dimensions of the steel plate investigated were 19'' X 30'' X 0.0359''. The first fifteen modes of the plate are given in Table 1.

Table 1: Modal frequencies for the first fifteen modes of the simply supported plate

Mode	Modal frequency Hz
(1,1)	13.390
(2,1)	24.889
(1,2)	42.059
(3,1)	44.055
(2,2)	53.559
(4,1)	70.888
(3,2)	72.725
(1,3)	89.841
(4,2)	99.557
(2,3)	101.341
(5,1)	105.386
(3,3)	120.507
(2,5)	134.056
(4,3)	147.339
(6,1)	147.552

The scaling factors of V_{comp}, $\alpha,\beta,\gamma,\delta$, were solved at each of the fifteen modes and are set to the average values for the rest of the simulations. The location of the primary force f_1 is $(x_1,y_1) = (3.25'', 25.25'')$, the control force f_2 is $(x_2,y_2) = (5'',5'')$ with the error senor located at $(12'',12'')$. Figure 2 shows the force and sensor locations.
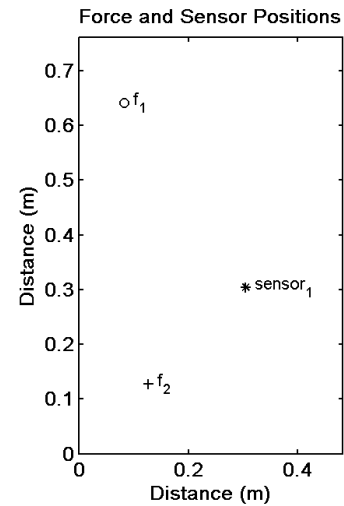

Figure 4: Force and sensor locations

To control V_{comp} at the desired location, the magnitude and phase of the control force f_2 is optimized.

Radiated Power

The power radiated by the plate was computed at frequencies ranging from 1 to 150 Hz. A comparison between the powers is shown in Figure 3.
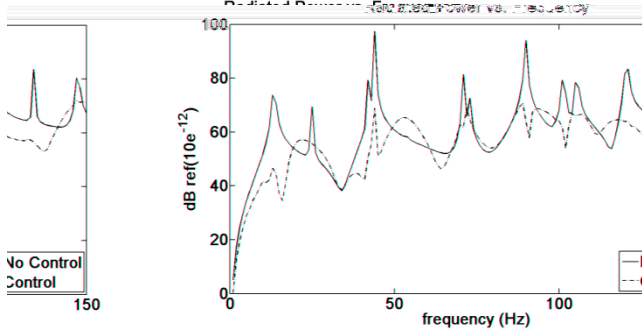

Figure 5: Comparison of radiated power before and after control of V_{comp}

As can be viewed in the results, controlling the quantity Vcomp did not always decrease the radiated power but does produce an overall desired result. The largest benefit of V_{comp} is that the control performance does not depend on sensor location. To validate this, the sensor was moved to multiple locations on the plate and the experiment repeated with comparable results at most all locations. It should be noted that when the sensor was placed in the corners, V_{comp} performed poorly.

Acoustic Radiation Modes

A reason for the overall success of Vcomp can be viewed when looking at acoustic radiation modes. In the past, controlling radiation modes has been an effective way to control the power radiated from a panel. However, structural vibrations must be known prior to this because sensors need to be placed in locations conducive to sensing all radiation modes present. In most cases, structural vibrations cannot be known without equipment such as an SLDV and if many modes are present, this technique requires the use of a large amount of sensors. Referring to the quantities used in V_{comp} and the first four acoustic radiation modes, there is a relationship between the two. The first radiation mode can be looked at like a transverse velocity, the second a bending in x, third a bending in y, and the fourth, a twisting term. Given the [R] matrix as solved for above, the acoustic radiation modes are given by

$$[R] = [Q]^T[\Lambda][Q] \quad (8)$$

Where [Q] is a matrix of orthogonal eigenvectors, [Λ] is a diagonal matrix of eigenvalues. The relative magnitudes of the radiation modes are given by the elements of [Λ], and the shape is given by the corresponding row of [Q]. The shapes of the first six acoustic radiation modes are shown in Figure 4.

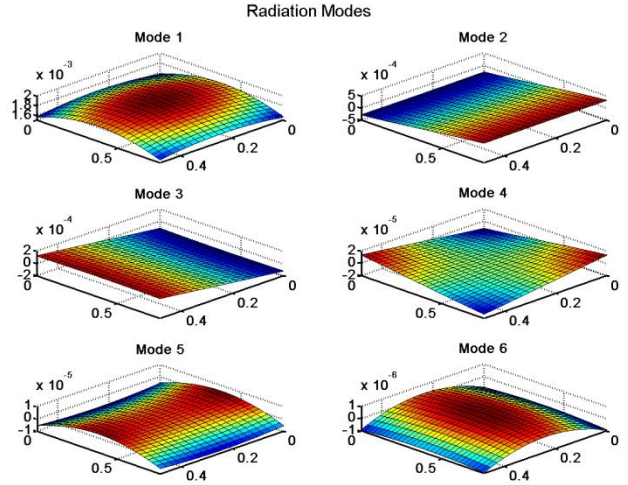Figure 6: Shape of the first six radiation modes

Using radiation modes, the overall power radiated is give by

$$\{\tilde{y}\} = [Q]\{\tilde{v}_e\} \quad (9)$$

$$\bar{P}(\omega) = \{\tilde{y}\}^H[\Lambda]\{\tilde{y}\} = \sum_{r=1}^{R} \lambda_r |\tilde{y}_r|^2 \quad (10)$$

Where R is the total number of elements and λ_r, $\tilde{y}_r$ are the components corresponding to the element of interest. It should be noted that the shape of the radiation mode is dependent on frequency. The higher the frequency, the higher the more curvature there is in the individual radiation modes. The power radiated by the individual acoustic radiation modes is given by

$$\bar{P}_m(\omega) = \lambda_m |\tilde{y}_m|^2 \quad (11)$$

with m being the individual mode. The control of the first six radiation modes was analyzed to see the effects. The individual power radiated from the modes was calculated using equation (11), referencing $10e^{-12}$ W/m^2. A comparison of the individual modes before and after control of V_{comp} is given in Figure 5.
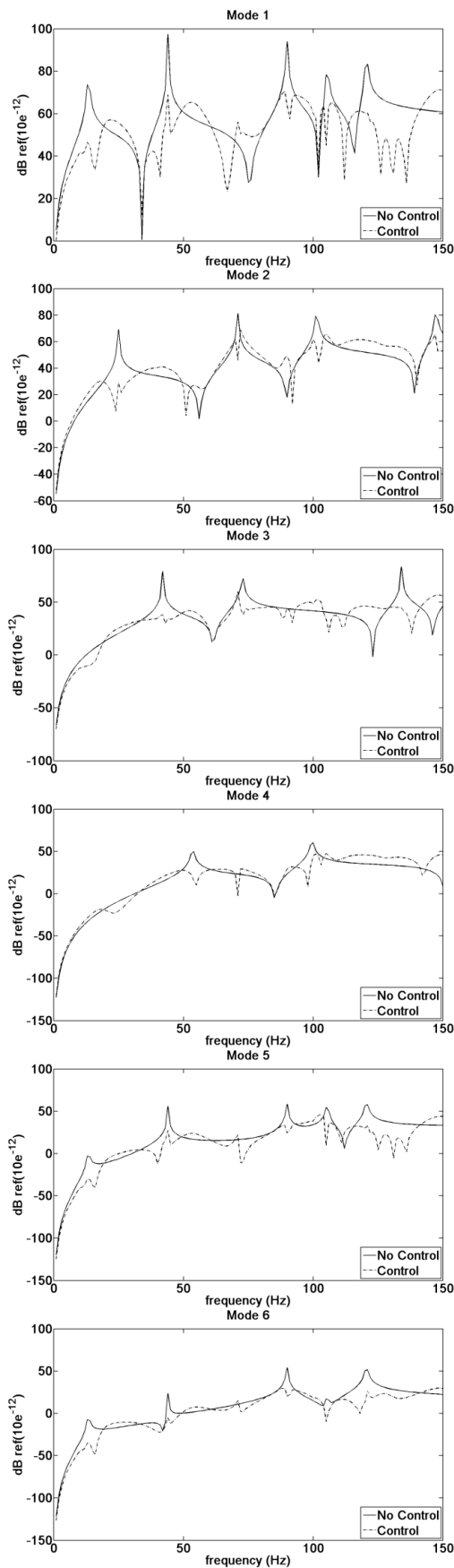
Figure 7: Control of the first six radiation modes

Using V_{comp}, most all of the peaks of the individual radiation modes were significantly reduced. While at some instances an individual radiation mode was increased, the overall effect was desirable.

The sum of the first six radiation modes should provide a good estimate of the overall power radiated by the plate. The following figure looks at the sum of the magnitudes of the first six radiation modes before and after control.
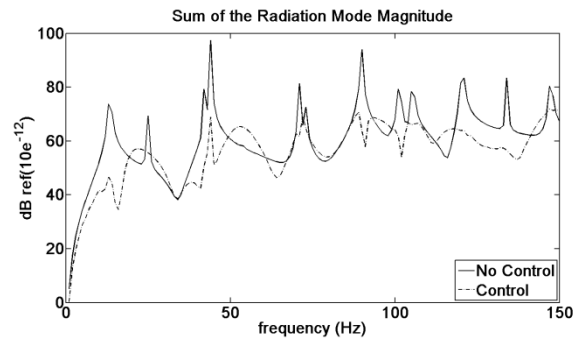

Figure 8: Sum of the first six radiation modes

When comparing figures 3 and 6, it can be seen that the first six radiation modes are a good estimate of the overall radiated power.

Summary and Conclusions

The new structural quantity, V_{comp}, produces desirable results in the active control of structures. The benefits include: control at higher structural modes, control independent of sensor location, and V_{comp} uses only a single point measurement with a compact sensor. The control at higher frequencies can be explained by the control of multiple acoustic radiation modes, not just a single one. While all of the desirable characteristics are benefits, the most highly sought after is that of the control being marginally independent of sensor location. Strictly speaking, this results in a lack of need to know the structural vibrations before placing the sensor. Although this new method does not attenuate the noise across a large spectrum and in some cases actually boosts the power, the overall result is promising in the area of active control of structural acoustics.

References

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