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Garrett G. Smith

American Option Pricing: A Simulated Approach

Utah State University

# **American Option Pricing: A Simulated Approach**

Garrett G. Smith

## Abstract

This study examines methods of pricing American style options, moving from the binomial model to the Black Scholes method and finishing with simulated method of option pricing. A simulated approach is based off the work established by Longstaff and Schwartz (2001) and extended by Rambharat and Brockwell (2010). Downfalls of these methods are discussed, as are ways to improve upon them. Using Monte Carlo methods and particle filtering will lead to a platform where options are priced with greater detail. Also, these simulated methods lead to faster computing time allowing for a more efficient use of resources and a theoretical framework of pricing.

## Introduction

In Episode V of the Star Wars series Yoda teaches an uncertain Luke Skywalker “Always in motion is the future.” Just as Luke has to make a decision to act based on partial information so must investors. The future is unknown; what the future stock price will be is equally unknown. Using past data and current market conditions can lead to an educated guess; however; it is still a guess.

The closer a model can get to uncertainty the better that model will be at reflecting real life. By making fewer assumptions, such as, what will the volatility of a stock be in the future this uncertainty can be brought within reach. Unknown volatility can be estimated by simulating the stock path based off of stochastic processes. Then the question will rise, how much is the option to buy or sell in the future worth today?

## Options

There are two major types of options, European and American options. The names are not based on where they are traded but based on the exercise options that come with them.

(Other option styles, such as Bermudan, exist but only these two styles will be discussed here.)

To demonstrate the difference a given option with expiration date  $t$ :

1. A European option can only be exercised at time  $t$ .
2. An American option can be exercised any time between now and time  $t$ . (McDonald, 2006)

Inherent in both of these options is the right to exercise but not the necessity. If an option is purchased there is no requirement to use the option. All of these factors are important to keep in mind when looking to price an option. The freedom to exercise at anytime, as in an American

option, should lead to a higher price than that of a European option. If an investor purchases a put option and the underlying stock skyrockets then the investor over paid for the put because the stock price never drop below the strike price of the option. Given all the uncertainty how can the true price be found?

Price can be found through arbitrage pricing theory, as was done by Black and Scholes (1973). Another approach is simulation as done by Longstaff and Schwartz (2001). Their work was great though it was with its shortcomings.

### **Black-Scholes Model**

Fischer Black and Myron Scholes originally argued “If options are correctly priced in the market, it should not be possible to make sure profits by creating portfolios of long and short positions in options and their underlying stocks.” (Black, 1973) This is the idea of put call parity. Buying various options and creating different long and short positions, arbitrage opportunities should not be present in the market.

Black and Scholes go on to present a way to price an option using this parity. The general understanding is taking the discreteness of the binomial model extended it to infinity. In addition, the inputs for the Black-Scholes model are constant. The current price, the strike price, the volatility, the short term or risk free interest rate, time to expiration, and dividend payments are decided before the calculation is done.

Using these basic premises Black and Scholes derived valuable equations to price a European put and call. Their model was based on a series of the following assumptions:

1. Continuously compounded returns on the stock are normally distributed and independent over time. (No jumps)
2. The volatility of continuously compounded returns is known and constant.
3. Future dividends are known, either as a dollar amount or as a fixed dividend yield.
4. The risk-free rate is known and constant.
5. There are no transaction costs or taxes.
6. It is possible to short-sell costlessly and to borrow at the risk-free rate. (McDonald, 2006)

It is clear, with the assumptions used by Black and Scholes, there is some money left on the table. For one, not allowing for jumps in the stock market is very different from reality. This is where Longstaff and Schwartz step in to help assist with some of these assumptions.

### **Longstaff-Schwartz Method**

Francis A. Longstaff and Eduardo S. Schwartz introduced a method in 2001 that is quickly becoming as accepted as the Black-Scholes model. Their goal is to approximate the value of an American option through simulation. The simulation technique they used is the Monte Carlo method. This led to an accurate measure of option value. It also opened the door for the ability to add more complex models and overcome the shortfalls of previous methods. (Longstaff, 2001)

Monte Carlo sampling uses a base of random numbers to generate outcomes. In the case of the Longstaff-Schwartz method paths are generated. These paths are potential value paths that a stock could take. In addition, Monte Carlo simulations can take place very quickly and generate tens of thousands of paths in a short amount of time. After these paths are generated a

simple regression can be estimated to value the probability and magnitude of a path.

(Rambharat, 2010)

The power of the Longstaff-Schwartz method is in the customization. There are many discussions on Monte Carlo simulations and regression based pricing as to not be included here. The model can be built to include more and more finite things. Rambharat and Brockwell took a similar method of Monte Carlo simulations for more accurate pricing. (Rambharat, 2010)

### **Rambharat and Brockwell**

Rambharat and Brockwell (2010) begin their paper with the sentence “We introduce a new method to price American-style options on underlying investments governed by stochastic volatility models.” (Rambharat, 2010) Longstaff and Schwartz use a simple Brownian motion simulate the stock price matrix. Rambharat and Brockwell (2001) evolve price paths according to the Ito stochastic differential equations that were extended to the discrete time approximations. In addition to stochastic volatility Rambharat and Brockwell (2010) implement the use of a particle filter.

Two major differences of Rambharat and Brockwell’s (2010) work is the use of stochastic volatility in pricing American options and the use of a particle filter

### **Stochastic Volatility**

“Arguably, stochastic volatility models are the most realistic models to date for underlying equities” states Rambharat and Brockwell (2010). Allowing for stochastic volatility overcomes the shortcomings of past models because it allows for jumps in the stock price and

changing volatility. In addition, fewer assumptions have to be made about how past volatility effects future volatility.

Starting with Ito stochastic differential equations and working under the risk-neutral measure, asset prices can be evolved. These can be driven by various factors such as the risk free rate, the mean reversion rate, and the volatility of volatility.

### **Particle Filter**

The aspect of a particle filter is a great step forward from Rambharat and Brockwell (2010). Particle filtering, or sequential Monte Carlo estimation, is a way to estimate a process through simulation. A particle filter is to stochastic processes and Bayesian statistics as the Kalman filter is to Gaussian and linear distributions.

The process for a particle filter can be broken into three parts: 1) Simulate forward 2) Weight the appropriate draws 3) Resample based on the new particles. A close process to the particle filter is importance sampling. Using this process volatility can be inferred though unobserved. The power of this method cannot be understated. By using a particle filter all types of processes can be developed such as the pricing of other exotic options.

### **Data**

To show the power of the complete model four comparisons will be made. The four models are a recreation of the results performed by Rambharat and Brockwell (2010). The effort is made to restructure their results to authenticate the results. The main difference in the four models is how volatility is measured. The process moves from constant volatility to realized volatility, to unobserved volatility, and finishing with the actual volatility through observing the



stock price path. The experiments are based off three stocks, Dell Inc., The Walt Disney Company, and Zerox Corporation. The four methods are:

Method A: Basic LSM. Simply put, the Longstaff Schwartz method of pricing an option.

Method B: Realized Volatility. The magnitude of changes to volatility is factored in.

Method C: Monte Carlo and Grid Method. Rambharat developed a method of gridding and using vectors to estimate values. This is incorporated with the particle filter and stochastic volatility.

Method D: Observable Volatility. The option price is determined off the actual stock price path. This is the benchmark.

To determine the posterior distributions of the underlying parameters Markoff Chain Monte Carlo is used. Due to the sensitive nature of these estimates I will begin with their already found parameters.

Table 1

*Explanation of the inputs to option pricing models:  $\alpha$  Volatility Mean Reversion Rate,  $\beta$  Volatility Mean Reversion Level,  $\gamma$  Volatility of Volatility,  $\lambda$  Volatility Risk Premium,  $\rho$  Co-dependence between the share price and volatility process,  $\sigma_0$  Initial Volatility,  $S_0$  Initial Share Price,  $T$  Days to Expiration,  $K$  Strike Price,  $r$  Risk Free Rate Number of particles 1,000, Number of paths 15,000*

<b>Experiment No.</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>Parameters</b>									
$\rho$	-0.055	-0.035	-0.09	-0.01	-0.03	-0.017	-0.075	-0.025	-0.05
$\alpha$	3.3	0.25	0.95	0.02	0.015	0.0195	0.015	0.035	0.025
$\beta$	log(0.55)	log(0.20)	log(0.25)	log(0.25)	log(0.35)	log(0.70)	log(0.75)	log(0.15)	log(0.25)
$\gamma$	0.5	2.1	3.95	2.95	3	2.5	6.25	5.075	4.5
$\lambda$	-0.1	-1	-0.025	-0.0215	-0.02	-0.0155	0	-0.015	-0.015
<b>Option Inputs</b>									
$K$	23	17	16	27	100	95	16	18	19
$T$	10	20	14	50	50	55	17	15	25
$r$	0.055	0.0255	0.0325	0.03	0.0225	0.0325	0.0325	0.055	0.025
$S_0$	20	15	15	25	90	85	15	20	17
$\sigma_0$	0.5	0.35	0.3	0.5	0.35	0.75	0.35	0.2	0.35

After running the data, the following results are obtained:

Table 2

*Option price results of the various methods.*

<b>Experiment</b>	<b>A (Basic LSM)</b>	<b>B (Realized Volatility)</b>	<b>C (MC/Grid)</b>	<b>D (Observable Volatility)</b>
<b>1</b>	3.071 (0.011)	3.066 (0.012)	3.079 (0.012)	3.064 (0.011)
<b>2</b>	2.158 (0.01)	2.168 (0.01)	2.164 (0.01)	2.179 (0.01)
<b>3</b>	1.235 (0.008)	1.258 (0.009)	1.279 (0.009)	1.279 (0.009)
<b>4</b>	3.658 (0.026)	4.256 (0.038)	4.699 (0.044)	4.756 (0.045)
<b>5</b>	13.176 (0.078)	14.694 (0.113)	14.694 (0.113)	16.174 (0.137)
<b>6</b>	18.623 (0.121)	21.476 (0.168)	22.758 (0.184)	22.996 (0.185)
<b>7</b>	1.613 (0.0123)	1.856 (0.0189)	1.997 (0.0214)	2.0297 (0.0222)
<b>8</b>	0.11 (0.004)	0.1473 (0.006)	0.1703 (0.007)	0.1845 (0.0073)
<b>9</b>	2.507 (0.014)	2.77 (0.021)	2.881 (0.023)	2.921 (0.024)

Comparing these results to the R&B data they are close but not exact. This can be do to the none exact nature of statistical estimation.

## Extensions & Conclusion

The next step to be completed is developing an advanced particle filter. The method Rambharat and Brockwell use is a basic filter and can be overloaded to only return one variable. Therefore, with the help of Dr. Todd Moon an advanced particle filter will be put in place.

Another large step will be to streamline the code and parallelize it. Allowing for computations to happen across many GPU's will open the door to do tens of thousands of simulations in a second. The particle filter and parallelized code will allow for the ability to stream real time stock quotes, run through a simulation and regression process, and get a option quote in an instant.

Modeling, not arbitrage, is the direction of option pricing. Being built on the back of giants like Black and Scholes, Longstaff and Schwartz, and Rambharat and Brockwell allows for a process unlike any other. Using these tools greater and more accurate models can be produced,

computational times will decrease, and more option varieties can be price. As a result accurate option prices can be obtained in the blink of an eye.

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