

## All homogeneous pure radiation spacetimes satisfy the Einstein–Maxwell equations

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## NOTE

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## Abstract

It is shown that all homogeneous pure radiation solutions to the Einstein equations admit electromagnetic sources. This corrects an error in the literature.

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A spacetime  $(M, g)$  is said to be a homogeneous pure radiation solution of the Einstein equations if it admits a transitive group of isometries and if there exists a function  $\Phi$  and a 1-form  $k$  such that the Einstein tensor  $G$  is of the form

$$G_{ab} = \Phi^2 k_a k_b, \quad \text{where} \quad g^{ab} k_a k_b = 0. \quad (1)$$

A classification of homogeneous pure radiation solutions is given in [1] based upon results of [2] and [3]. Such solutions are all pp waves, and they are either plane waves or are diffeomorphic to

$$g = dx \otimes dx + dy \otimes dy + du \odot dv - 2 e^{2\rho x} du \otimes du, \quad \rho = \text{const}. \quad (2)$$

The plane waves are known to arise from electromagnetic sources. It is asserted in [1, 3], based upon results of [4], that the metric (2) does not arise from an electromagnetic source. However, the metric (2) *does* arise from an electromagnetic source. In fact, infinitely many electromagnetic fields can serve as the source for the spacetime defined by (2).

For any function  $f(u)$  the null 2-form

$$F = 2\rho e^{\rho x} [\cos(\rho y + f(u)) du \wedge dx - \sin(\rho y + f(u)) du \wedge dy], \quad (3)$$

its Hodge dual  $\tilde{F}$  and the metric (2) satisfy the source-free Maxwell equations  $dF = 0 = d\tilde{F}$  and the Einstein equations:

$$G_{ab} = \frac{1}{2} (F_{ac} F_b{}^c + \tilde{F}_{ac} \tilde{F}_b{}^c) = 4\rho^2 e^{2\rho x} \nabla_a u \nabla_b u. \quad (4)$$

The electromagnetic field given in (3) is ‘non-inheriting’, that is, it is not invariant under the isometry group of  $(M, g)$ . For example,  $F$  is not invariant under translations in  $y$  for any choice of  $f(u)$ .

Thus, granted theorem 12.6 of [1], all homogeneous pure radiation solutions of the Einstein equations arise from electromagnetic sources.

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## References

- [1] Stephani H, Kramer D, MacCallum M, Hoenselaers C and Herlt E 2003 *Exact Solutions of Einstein's Field Equations* 2nd edn (Cambridge: Cambridge University Press)
- [2] Wils P 1989 *Class. Quantum. Grav.* **6** 1243
- [3] Steele J 1990 *Class. Quantum. Grav.* **7** L81
- [4] Sippel R and Goenner H 1986 *Gen. Rel. Grav.* **18** 1229