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Experimental Analysis of the Effects of Manipulations in Weighted Voting Games

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EXPERIMENTAL ANALYSIS OF THE EFFECTS OF MANIPULATIONS IN WEIGHTED VOTING GAMES

by

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ABSTRACT

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Weighted voting games are classic cooperative games which provide compact representation for coalition formation models in human societies and multiagent systems. As useful as weighted voting games are in modeling cooperation among players, they are, however, not immune from the vulnerability of *manipulations* (i.e., dishonest behaviors) by strategic players that may be present in the games. With the possibility of manipulations, it becomes difficult to establish or maintain trust, and, more importantly, it becomes difficult to assure fairness in such games. For these reasons, we conduct careful experimental investigations and analyses of the effects of manipulations in weighted voting games, including those of manipulation by *splitting*, *merging*, and *annexation*. These manipulations involve an agent or some agents misrepresenting their identities in anticipation of gaining more power or obtaining a higher portion of a coalition's profits at the expense of other agents in a game.

We consider investigation of some criteria for the evaluation of game's robustness to manipulation. These criteria have been defined on the basis of theoretical and experimental analysis. For manipulation by splitting, we provide empirical evidence to show that the three prominent indices for measuring agents' power, *Shapley-Shubik*, *Banzhaf*, and *Deegan-Packel*, are all susceptible to manipulation when an agent splits into several false identities. We extend a previous result on manipulation by splitting in *exact unanimity* weighted voting games to the Deegan-Packel index, and present new results for *excess unanimity* weighted voting games. We partially resolve an important open problem concerning the bounds on the extent of power that a manipulator may gain when it splits into several false identities in *non-unanimity* weighted voting games. Specifically, we provide the first three non-trivial bounds for this problem using the Shapley-Shubik and Banzhaf indices. One of the bounds is also shown to be asymptotically tight.

Furthermore, experiments on *non-unanimity* weighted voting games show that the three indices are highly susceptible to manipulation via annexation while they are less susceptible to manipulation via merging. Given that the problems of calculating the Shapley-Shubik and Banzhaf indices for weighted voting games are NP-complete, we show that, when the manipulators' coalitions sizes are restricted to a small constant, manipulators need to do only a polynomial amount of work to find a much improved power gain for both merging and annexation, and then present two enumeration-based pseudo-polynomial algorithms that manipulators can use. Finally, we argue and provide empirical evidence to show that despite finding the optimal beneficial merge is an NP-hard problem for both the Shapley-Shubik and Banzhaf indices, finding beneficial merge is relatively easy in practice. Also, while it appears that we may be powerless to stop manipulation by merging for a given game, we suggest a measure, termed *quota ratio*, that the game designer may be able to control. Thus, we deduce that a high quota ratio decreases the number of beneficial merges.

(128 pages)

PUBLIC ABSTRACT

Ramoni Olaoluwa Lasisi

This dissertation investigates weighted voting games and three methods of manipulating those games, called *splitting*, *merging*, and *annexation*. The manipulations involve an agent or some agents misrepresenting their identities in anticipation of gaining more power over the outcomes of games. Indeed, in open anonymous environments, manipulation can be easy and cheap to achieve. We provide clear and sufficient discussion on related work and backgrounds to motivate this research topic, which certainly deserves attention. Weighted voting games are among key cooperative games, and the manipulations considered in this dissertation are natural, and have practical applications, that are likely to raise interests in the game theory and artificial intelligence communities.

We provide interesting theoretical and simulation results for the two broad classes of weighted voting games - *unanimity* and *non-unanimity* weighted voting games. In the anaylses and experiments, the following prominent power indices are considered to measure the influence of players: *Shapley-Shubik*, *Banzhaf*, and *Deegan-Packel*. The results we propose in this work fit under the models of deception and fraud, as well as models and mechanisms for establishing identities. If an agent can increase its power, as evidenced by a power index, it is more likely to employ any of these forms of manipulation, and it becomes difficult to maintain trust. Thus, awareness of various levels of power indices susceptibility to manipulation allows users to informatively select a power index. This provides some assurance of identity, which is crucial for establishing and maintaining trustworthy interactions. This study also increases our indepth understanding of these manipulations in weighted voting games and their effects, which may provide insights that are needed in the development of methods to reduce the effects of the menace in the future.

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Ramoni.

To my teacher - who, among other things, is my spiritual father and mentor.

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CHAPTER 1
INTRODUCTION

1.1 Overview

Weighted voting games (WVGs) are important in multiagent systems because of their usage in automated decisions making. One way of modeling cooperation among players for making joint decisions that is frequently found in the real world is via the use of WVGs [1]. WVGs are widely studied [2–4]. Prominent real-life situations where WVGs have found applications include the United Nations Security Council, the Electoral College of the United States, the International Monetary Fund [5, 6], the Council of Ministers, and the European Community [2]. Other areas of application of WVGs include economics, political science, neuroscience, threshold logic, reliability theory, distributed systems [7], and multi-robot team formation for distributed coverage [8]. Also, Nordmann and Pham [9] have considered reliability and cost evaluation of weighted dynamic-threshold voting-systems. These systems are used in target detection, pattern recognition, safety monitoring, and human organization systems. The issue of WVGs design has also recently received the attention of many reseachers in the field. See the work of [10–12] for some examples of WVGs design issues.

WVGs represent mathematical abstractions of voting systems. In a WVG, a *quota* is given and each voter has an associated *weight*. Voters express their opinions through their votes by electing candidates to represent them or influence the passage of bills. A subset of agents, called a *coalition*, whose total weight meets or exceeds the quota, is said to be *winning*. However, in the more traditional homogeneous voting system [13], which is a special case of voting in which all voters have unit weight, the winning coalition is determined by the majority of the agents. A voter's weight in a game is the number of votes controlled by the voter, and this is the maximum number of votes she is permitted to cast. The weights of agents may correspond to resources or skills available to the agents, while the quota is the amount of resources or skills required for a task to be accomplished. For example, in *search and rescue*, robotic agents put their resources (i.e., weights) together in large natural disaster environments to reach the necessary levels (i.e., quota) to save life and property.

It is natural to naively think that the numerical weight of an agent directly determines the corresponding strength of the agent in a WVG. The measure of the strength of an agent in a game is termed its *power*. This is the ability of an agent to influence the decision-making process. Although a larger weight by an agent makes it more likely that an agent affects the outcome of a WVG, the weight of an agent in a game is not typically proportional to its power [14]. Consider, for example, a WVG of three voters, a_1, a_2, and a_3 with respective weights $6, 3$, and 1. When the quota for the game is 10, then a coalition consisting of all the three voters is needed to win the game. Thus, each of the voters is of equal importance in achieving the winning coalition. Hence, they each have equal power irrespective of their weight distribution, in that every voter is necessary for a win. The three prominent power indices for computing agents' power found in the literature are the *Shapley-Shubik* [15], *Banzhaf* [16, 17], and *Deegan-Packel* [18] indices. These power indices are used in this dissertation to analyze the effects of manipulations in WVGs. Other lesser known power indices are the *Holler-Packel* [19] and *Johnston* [20] power indices.

The Shapley-Shubik, Banzhaf, and Deegan-Packel indices satisfy the axioms that characterize a power index, have gained wide usage in political arena, and are the main power indices found in the literature [3]. The indices have been defined on the framework of subsets of winning coalitions in the game they seek to evaluate. A wide variation in the results they provide can be observed. This is due to the different methods of computation of the associated subsets of the winning coalitions.These indices measure the influence of voters differently.

Computing the Shapley-Shubik, Banzhaf, and Deegan-Packel indices of players in WVGs is NP-complete [4, 21], and admits pseudo-polynomial algorithms using dynamic programming. Efficient exact algorithms using generating functions [22, 23] also exist for both the Shapley-Shubik and Banzhaf indices, where the weights of all agents in the WVGs are restricted to integers. Deng and Papadimitriou [24] have shown that computing the Shapley values of players in WVGs is #P-complete. We also note that efficient approximation algorithms exist for computing the Shapley-Shubik and Banzhaf indices. The work of Bachrach et al. [25] and Fatima et al. [26] provide examples on approximating power indices in WVGs.

The problem of insincere and manipulative behaviors in votings or elections among agents is a fundamental problem that has received attention of many reseachers. Different forms of insincere and manipulative behaviors have been considered. See the recent work of [27–29]. According to Zuckerman et al. [30], these insincere behaviors include *manipulation* (dishonest behavior by voters), *control* (dishonest behavior by the election authority), and *bribery* (dishonest behavior by an outside party). We are concerned in this dissertation with only the first form of insincere behavior from above, i.e., manipulations involving dishonest behaviors by players in a game.

Manipulations have been considered in non-cooperative games such as auctions [31,32], cooperative games [33], and in open anonymous environments [34], such as the internet. The maiden study of this behavior in the context of weighted voting is the work of [35]. WVGs are useful in modeling cooperation among players for making joint decisions, but they are not immune from the vulnerability of manipulations by strategic players in the games. With the possibility of manipulation, it becomes difficult to establish or maintain trust, and it becomes difficult to assure fairness in such games. We provide a detailed study and analysis of the effects of three forms of manipulation in WVGs, called *splitting*, *merging*, and *annexation* [2, 35–37].

In manipulation by splitting, an agent, termed a *manipulating agent*, may alter a WVG by splitting its identity into multiple names and distributing its weight across all the associated names. This misrepresentation may allow the manipulating agent to gain more power over the outcome of a game. The *altered* game consists of the non-manipulating agents in the *original* game and the several identities, termed *false agents*, into which a manipulating agent splits. The power of the manipulating agent is thus the sum of the power of all its associated false agents in the altered game. The manipulating agent hopes that the value of its accumulated power (from the false agents) will be greater than its value in the original game. Manipulation occurs when manipulating agents achieve an increase in power with respect to the original WVGs they manipulate.

WVGs are also subject to another method of manipulation, called merging, which is also known as *alliance* or *collusion*. This manipulation involves coordinated action among some agents who come together to form a *bloc* by merging their weights into a single weight in order to have more power over the outcomes of games [2, 37]. In a beneficial merge, merged agents are compensated commensurate with their share of the power gained by the bloc. The agents whose weights are merged into a bloc are referred to as *assimilated* agents since they can no more vote as individual voters in the new game. Agents in the merged bloc are assumed to be working cooperatively and have transferable utility. Thus, proceeds from merging can easily be distributed among the manipulators without bickering.

Finally, in manipulation by annexation, an agent, termed an *annexer*, usurps the voting weights of some other agents in a game [2,37]. We also refer to agents whose voting weights were acquired as the assimilated agents. The new game consists of the previous agents in the original game whose weights were not annexed and the *bloc* of agents made up of the annexer and the assimilated agents. The annexer also incurs some compensation that is made to the assimilated agents to forfeit their weights.

The major difference between manipulation by merging and annexation is that, in merging, the assimilated agents must be compensated commensurate with their share of the power gained by the bloc if the merging is beneficial, while in annexation, only the annexer is compensated for the participation of the group. Annexed agents are assumed to either voluntarily forfeit their weight or be compensated on a one-time basis that is not related to power.

1.2 Motivation

Manipulation in open anonymous environments, such as the internet, is hard to detect. According to Yokoo et al. [34], the manipulation can come in several forms which include collusion or *false-name*, where a single agent is acting as many agents i.e., splitting. Thus, the increased use of online systems (such as trading systems or peer-to-peer networks where WVGs are also applicable) mean that manipulation remains an important challenge that calls for attention. Bachrach and Elkind [35] have also interpreted identity splitting in WVGs as agents trying to obtain a higher share of the grand coalition's gain or obtaining more political power by splitting a political party into several other parties with similar political platforms. The authors then conjectured that false-name manipulation by splitting is widespread in the real world and stands to pose serious issues in multiagent environments. Furthermore, Felsenthal and Machover [2, 37] consider a real life example of annexation where a shareholder buys the voting shares of some other shareholders in a firm in order to use them for her own interest. Clearly, this action allows the annexer to possess more shares and makes it easier for her to affect the outcomes of decisions in the firm.

WVGs can be viewed as a form of competition among agents to share the available *fixed* power whose total value is always 1. Agents may thus resort to a form of manipulation (splitting, merging, or annexation) to improve their influence in anticipation of gaining more power. From the foregoing, it is not difficult to see that manipulation by splitting, merging, and annexation in WVGs is undesirable. The power increase achieved by manipulating agents is at the expense of other agents in a game who are being denied the utility that is due to them.

This research is primarily motivated by the reasons stated above and the need to provide insights into understanding the details of the problem of insincere and manipulative behaviors. Furthermore, previous work [14,36] has shown that the problem of finding beneficial split, merge, or annexation is NP-hard for both the Shapley-Shubik and Banzhaf power indices, and leave the impression that this is indeed so in practice. Although this worst case complexity for manipulation is daunting enough to discourage would-be manipulators, nonetheless, it is possible that real life instances of WVGs are easy to manipulate. We note that real WVGs are small enough that exponential amount of work may not deter manipulators from participating in manipulations. According to [12], the number of players in most real life examples of WVGs is between 10 and 50. Hence, there may be little deterrent to manipulation in practice considering the NP-hardness results of the previous work.

1.3 Organization of the Dissertation

The remainder of this dissertation is organized as follows. Chapter 2 presents preliminaries to provide necessary backgrounds in WVGs, power indices, and formal problem definition of manipulation by splitting, merging, and annexation in WVGs. Chapter 3 considers susceptibility of power indices to manipulation by splitting. We present new bounds on manipulation by splitting into several false identities in WVGs in Chapter 4. Chapters 5 and 6 respectively consider experimental investigation and analyses of the effects of manipulation by merging and annexation in WVGs. We conclude in Chapter 7 and give directions for future work.

CHAPTER 2
PRELIMINARIES

2.1 Definitions and Notation

We give the following definitions and notation that are used throughout the dissertation. Let $I = \{1, \ldots, n\}$ be a set of $n \in \mathbb{N}$ agents. Let $\{w_1, \ldots, w_n\}$ be the corresponding weights of these agents. The non-empty subsets, $C \subseteq I$, are called *coalitions*. The set I of all agents is also referred to as the *grand coalition*.

Definition 1. *Weighted Voting Game.*

A WVG with *quota* $q \in \mathbb{R}$ involving agents I is represented as $[w_1, \ldots, w_n; q]$. Denote by $w(C)$, the weight of a coalition, C, derived as the summation of the weights of agents in C, i.e., $w(C) = \sum_{j \in C} w_j$. A coalition, C, wins in a game if $w(C) \geq q$, otherwise it loses. WVGs belong to the class of *simple voting games*. In simple voting games, each coalition, C, has an associated function $v : C \rightarrow \{0, 1\}$. The value 1 implies a win for C and 0 implies a loss. So, $v(C) = 1$ if $w(C) \geq q$ and 0 otherwise.

The weights of the agents are given in non-increasing order, i.e., $w_1 \geq w_2 \geq \ldots \geq w_n$. Note that this assumption does not affect the definition of the game or the generality of our results. See also the work of [4, 30] where this assumption has also been used.

Definition 2. *Dummy and Critical Agents.*

An agent i in a coalition C is *dummy* if its weight is not needed for C to be a winning coalition, i.e., $w(C \backslash \{i\}) \geq q$. Otherwise, it is *critical* to C, i.e., $w(C) \geq q$ and $w(C \backslash \{i\}) < q$.

Definition 3. *Unanimity and Non-Unanimity Weighted Voting Games.*

A game is a unanimity WVG if there is a single winning coalition in the game, and every agent in the game is critical to the coalition, otherwise, it is termed a non-unanimity

WVG. We use the terms *exact unanimity*, for unanimity WVGs having total weight of agents, $w(I)$, in the game equal to the quota i.e., $w(I) = q$, and *excess unanimity*, for unanimity WVGs with total weight of agents greater than the quota i.e., $w(I) > q$.

Definition 4. *Minimal Winning Coalition.*

A winning coalition C is a *minimal winning coalition* if every proper subset of C is a losing coalition, i.e. $w(C) \geq q$ and $\forall T \subset C$, $w(T) < q$.

Definition 5. *Power Vectors.*

The power vector of a WVG of n agents is simply an n-dimensional vector $v \in \mathbb{R}^n$ of the power of each of the agents in the game listed in order. The power of the agents is computed using a power index.

Definition 6. *Shapley-Shubik Power Index.*

The Shapley-Shubik power index quantifies the marginal contribution of an agent to the grand coalition. Each permutation of the agents is considered. We term an agent *pivotal* in a permutation if the agents preceding it do not form a winning coalition, but by adding this agent, a winning coalition is formed. The Shapley-Shubik index assigns power to each agent based on the proportion of times (in all permutations) it is pivotal . We formally specify the computation of the power index using notation of [35]. Denote by π a permutation of the agents, so $\pi : \{1, \ldots, n\} \to \{1, \ldots, n\}$ and by Π the set of all possible permutations. Denote by $S_\pi(i)$ the predecessors of agent i in π, i.e., $S_\pi(i) = \{j : \pi(j) < \pi(i)\}$. The Shapley-Shubik index, $\varphi_i(G)$, for each agent i in a WVG G is given by

$$\varphi_i(G) = \frac{1}{n!} \sum_{\pi \in \Pi} [v(S_\pi(i) \cup \{i\}) - v(S_\pi(i))]. \tag{2.1}$$

For example, consider a WVG $G = [10, 7, 6, 4; 18]$ of four agents. We compute the Shapley-Shubik power of the first agent (with weight 10) as follows. The agent is pivotal in 12 of the $4! = 24$ permutations in the game. See Figure 2.1 for the 24 permutations in this game. So, the Shapley-Shubik index of the agent, $\varphi_1(G) = \frac{12}{24} = 0.50$. Similarly,

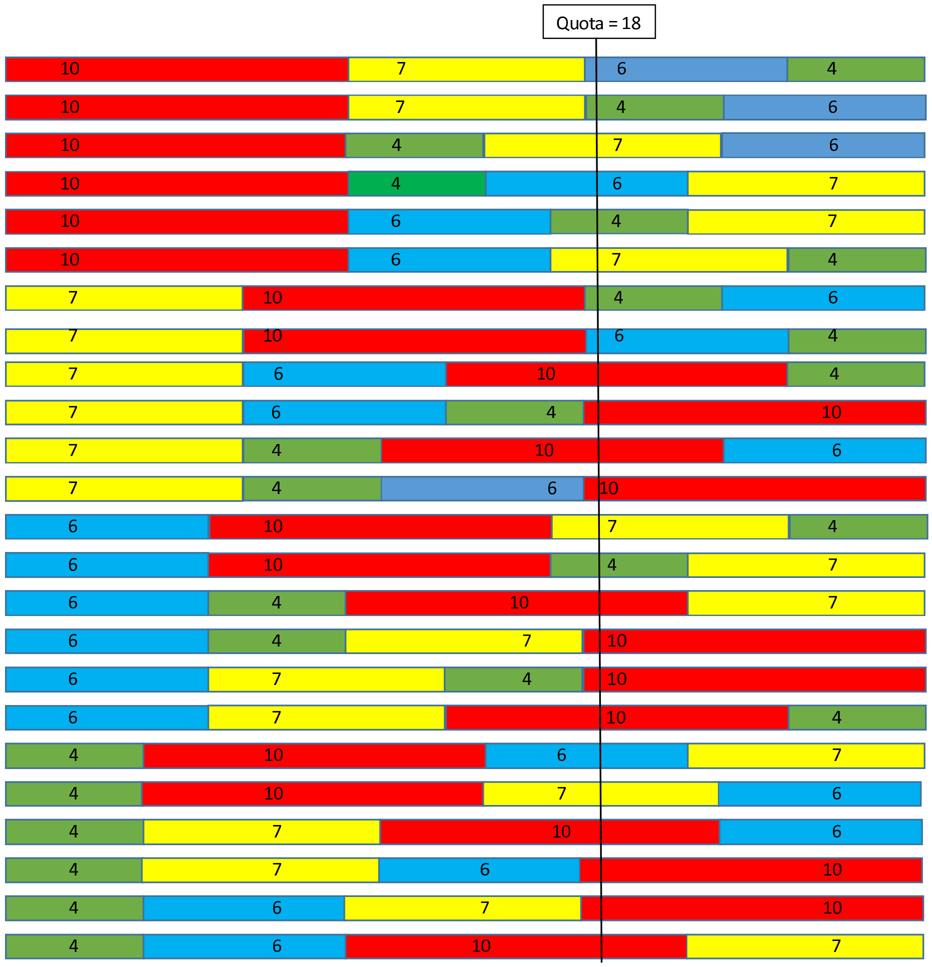

Figure 2.1. The 24 permutations in the game $G = [10, 7, 6, 4; 18]$ with the quota indicated as a line through all the permutations indicating the pivotal positions in the permutations.

the second (weight 7), third (weight 6), and fourth (weight 4) agents are pivotal in only four permutations each. Thus, the power of each of these agents is $\frac{4}{24} = 0.167$. The agent crossing the quota line is pivotal at that position. The power vector for this game using the Shapley-Shubik power index is $[0.500, 0.167, 0.167, 0.167]$.

Definition 7. *Banzhaf Power Index.*

The Banzhaf power index computation for an agent i is the proportion of times i is critical compared to the total number of times any agent in the game is critical. The Banzhaf index, $\beta_i(G)$, for each agent i in a WVG G is given by

$$\beta_i(G) = \frac{\eta_i(G)}{\sum_{j \in I} \eta_j(G)} \tag{2.2}$$

where $\eta_i(G)$ is the number of winning coalitions in which agent i is critical in game G.

Consider using the Banzhaf index to compute the power of the first agent in the WVG $G = [10, 7, 6, 4; 18]$. There are four winning coalitions in the game: $\{\{\underline{10}, \underline{7}, \underline{6}\}$, $\{\underline{10}, \underline{6}, \underline{4}\}$, $\{\underline{10}, \underline{7}, \underline{4}\}$,$\{\underline{10}$,7,6,4$\}\}$. The critical agents in each of the winning coalitions are underlined. The first agent (weight 10) is critical four times. The second (weight 7), third (weight 6), and fourth (weight 4) agents are critical in only two winning coalitions each. So, the Banzhaf index of the first agent, $\beta_1(G) = \frac{4}{4+2+2+2} = 0.40$. Similarly, the power of each of the second, third, and fourth agent is $\frac{2}{4+2+2+2} = 0.20$. The power vector for this game using the Banzhaf power index is $[0.40, 0.20, 0.20, 0.20]$.

Definition 8. *Deegan-Packel Power Index.*

The Deegan-Packel power index limits consideration for the computation of agents' power to the minimal winning coalitions in a game. Let MWC be the set of all minimal winning coalitions in a WVG G and MWC$_i$ be the set of all the minimal winning coalitions in game G that include agent i. The Deegan-Packel power index, $\gamma_i(G)$, for each agent i in game G is given by

$$\gamma_i(G) = \frac{1}{|MWC|} \sum_{C \in MWC_i} \frac{1}{|C|}. \tag{2.3}$$

We compute the Deegan-Packel power of the first agent for WVG $G = [10, 7, 6, 4; 18]$. There are three minimal winning coalitions in this game: $\{\{10, 7, 6\}$, $\{10, 6, 4\}$, $\{10, 7, 4\}\}$, and the first agent (with weight 10) belongs to all of them. The Deegan-Packel index of the agent, $\gamma_1(G) = \frac{1}{3}\left(\frac{1}{3} + \frac{1}{3} + \frac{1}{3}\right) = 0.33$. Similarly, the second (weight 7), third (weight 6), and fourth (weight 4) agents belong to two of the minimal winning coalitions each. Thus, the power of each of these agents is $\frac{1}{3}\left(\frac{1}{3} + \frac{1}{3}\right) = 0.22$. The power vector for this game using the Deegan-Packel power index is $[0.33, 0.22, 0.22, 0.22]$.

Observe that the power vectors computed by the three indices are different. The importance of power indices as solution concepts for measuring voting power and in the distribution of payoffs make them natural targets for manipulators [30].

2.2 Formal Problem Definition

We provide formal definitions of manipulation by splitting, merging, and annexation in WVGs. Let $k \in \mathbb{N}$. Let ϕ be any of Shapley-Shubik, Banzhaf, or Deegan-Packel index. Denote by $(\phi_1(G), \ldots, \phi_n(G)) \in [0,1]^n$ the power of agents in a WVG G of n agents.

2.2.1 Manipulation by Splitting

When an agent engages in manipulation by splitting, the number of agents in the new game increases by the number of false agents formed by splitting. The quota and weights of other agents in the new game remain the same. The sum of the power of the false agents becomes the power of the manipulator in the new game. The extent of power gained or lost for a manipulator can be measured as a fraction of power of the manipulator in the new game and its power in the original game.

Let agent $i \in I$ with weight w_i in game G be a manipulating agent. Suppose agent i splits its weight among $k \geq 2$ false agents, $i_1, \ldots, i_k$, having corresponding weights, $w_{i_1}, \ldots, w_{i_k}$, such that $w_i = \sum_{j=1}^{k} w_{i_j}$ and $w_{i_j} > 0$ for all j. We have a new set of agents after splitting: $I' = \{1, \ldots, i-1, i_1, \ldots, i_k, i+1, \ldots, n\}$. The initial game G of n agents has been altered by agent i to give a new WVG G' of $n+k-1$ agents. Thus, for a manipulating agent i with power $\phi_i(G)$ in G, the sum of the power of the $k \geq 2$ false agents in G' that the manipulating agent splits is $\sum_{j=1}^{k} \phi_{i_j}(G')$. The ratio $\tau = \frac{\sum_{j=1}^{k} \phi_{i_j}(G')}{\phi_i(G)}$ compares the sum of the power of the false agents in the altered game G' to the power of the manipulator (before it splits) in the original game G. τ gives a factor of the power gained or lost by agent i when it alters game G to give G'. We say that ϕ is susceptible to manipulation by splitting if there exists a game G' such that $\tau > 1$; the split is termed *advantageous* or *beneficial*. If $\tau < 1$, then the split is *disadvantageous* or *non-beneficial*, while the split is *neutral* when $\tau = 1$.

2.2.2 Manipulation by Merging

Manipulation by merging is the voluntary coordinated action of would-be manipulators who come together to form a bloc. Agents in the bloc are assimilated voters since they can no more vote as individual voters in the new game, rather as a bloc. The new game consists of the agents in the original game that are not assimilated as well as the bloc formed by the assimilated voters. The power of the bloc in the new game is compared to the sum of the individual powers of all the members of the bloc in the original game.

Let $2 \leq k \leq n$. Consider a manipulators' coalition C of k agents which is a k-subset of the n-set I. We assume that C contains k distinct elements chosen from I. Suppose the manipulators in C voluntarily merge into a single bloc denoted by $\&C$, i.e., agents $i \in C$ have been assimilated into the bloc $\&C$, then, we have a new set of agents in the game after merging. Thus, the initial game G of n agents has been altered by the manipulators to give a new WVG G' consisting of the bloc and other agents not in the bloc i.e., $I \backslash C$. Thus, for the manipulating agents $i \in C$ with power $\phi_i(G)$ in game G, the sum of the power of the k manipulators is $\sum_{i \in C} \phi_i(G)$, while that of the bloc formed by the manipulators in game G' is $\phi_{\&C}(G')$. The ratio $\tau = \frac{\phi_{\&C}(G')}{\sum_{i \in C} \phi_i(G)}$ compares the power index of the assimilated bloc formed by merging in the altered game G' to the sum of the original power indices of the agents in the merged bloc. τ gives a factor of the power gained or lost by the manipulators when they alter game G to give G'. The power index, ϕ, is said to be susceptible to manipulation by merging in game G if there exists a game G' such that $\tau > 1$; the merging is termed *advantageous* or *beneficial*. If $\tau < 1$, then the merging is *disadvantageous* or *non-beneficial*, while the merging is *neutral* when $\tau = 1$.

2.2.3 Manipulation by Annexation

Let $1 \leq k \leq n$. Consider a manipulators' coalition C of k agents which is a k-subset of the n-set I. We assume that C contains k distinct elements chosen from I. Suppose agent $i \notin C$ alters game G by annexing coalition C i.e., i assimilates the agents in C to form a bloc denoted by $\&(C \cup \{i\})$, then, we have a new set of agents in the game after annexation. Thus, the initial game G of n agents has been altered by the annexer to give a new WVG G' consisting of the bloc and other agents not in the bloc i.e., $I \backslash (C \cup \{i\})$. The ratio $\tau = \frac{\phi_{\&(C \cup \{i\})}(G')}{\phi_i(G)}$ compares the power of the assimilated bloc formed by annexation in the altered game G' to the power of the annexer in the original game G. τ gives a factor of the power gained or lost by annexer i when it alters game G to give G'. We say that ϕ is susceptible to manipulation by annexation if there exists a game G' such that $\tau > 1$; the annexation is termed *advantageous* or *beneficial*. If $\tau < 1$, then the annexation is *disadvantageous* or *non-beneficial*, while the annexation is *neutral* when $\tau = 1$. For the sake of simplicity in our discussion and analysis, we also refer to the factor of increment as power gain or benefit.

2.3 Simulating Weighted Voting Games

In this section, we provide a description of the methods used in generating the WVGs that are used for all our experiments, and the reasons why the methods have been employed. In view of the computational complexity of computing the power indices exactly from their definitions, we consider only WVGs with weights of the players in such games restricted to integers. We have thus implemented the efficient exact pseudo-polynomial methods of computing the Shapley-Shubik and Banzhaf indices for integer weights using generating functions [22, 23]. The Deegan-Packel power index is computed directly from its definition by using an efficient algorithm to find the subsets of agents in a game. The algorithm to compute the subsets defines a search known as a *Hamiltonian walk* that visits each corner of an n-cube exactly once, where n is the number of elements in the set [38].

When creating a new game, the quota, q, of the game is randomly generated such that $\frac{1}{2}w(I) < q \leq w(I)$, where $w(I)$ is the sum of the weights of agents I in the game. Quota has thus been defined as needing at least half of the total weight by a coalition to win in a game. The number of agents, n, in each of the WVGs is chosen uniformly at random from the set $\{10, 11, \ldots, 20\}$. We have limited the number of agents to 20 for reasons of complexity in the computation of the Deegan-Packel index for large games. Although, the efficient exact pseudo-polynomial methods for computing the Shapley-Shubik and Banzhaf indices described above are appropriate for computing the indices for large voting bodies [39], where the numbers of players in the games are greater than 20. However, this is not the case for the method used in computing the Deegan-Packel index. This choice of the number of agents in our experiments is consistent with the empirical evaluations of previous research, such as [25] and [14], where the number of agents in their experiments are respectively 13 and 24.

2.4 Statistical Distributions to Generate Weights

The integer weights of agents in each of the WVGs are generated using uniform, normal, and Poisson distributions. The following subsections give descriptions of how agents' weights are generated using the three distributions.

Weights Generation Using Uniform Distribution

The weights of agents in each game are chosen such that all weights are integers and drawn from a uniform distribution, $U(2, W)$, where $W \in \{10, 20, \ldots, 90\}$. $U(a, b)$ defines a uniform distribution over the interval (a, b), where both a and b are finite.

Weights Generation Using Normal Distribution

The weights of agents in each game are chosen such that all weights are integers and drawn from a normal distribution, $N(\mu, \sigma^2)$, where μ and σ^2 are the mean and variance. We use a mean of $\mu = 50$ and values of standard deviation σ from the set $\{5, 10, \ldots, 40\}$.

Weights Generation Using Poisson Distribution

The weights of agents in each game are chosen such that all weights are integers and drawn from a Poisson distribution, $Poisson(\lambda)$, where parameter λ is the mean of the distribution. We use a mean of $\lambda = 50$.

We note that the parameters in these distributions are sufficient to see patterns of behaviors in our experiments and as well provide some generalization on the evaluation of the effects of manipulations in the WVGs. Furthermore, these weights are reflective of realistic voting procedures as the weights of agents in real votings are not too large [35]. For example, the highest weight of the members of the Electoral College of the United States is 55 (held by the state of California) in the 2012 electoral votes.

2.5 Main Contributions

Until now, the experimental investigation and analysis of the effects of manipulation in WVGs that we consider in this dissertation is yet to be researched. We provide a summary of our major contributions for the three forms of manipulation in WVGs.

2.5.1 Results for Manipulation by Splitting

Previous work has considered manipulation by splitting in WVGs. Upper and lower bounds on the extent of power that a manipulator may gain exist for the case when a manipulator splits into *exactly two* false identities for both the Shapley-Shubik and Banzhaf indices. The bounds on the case when an agent splits into more than two false identities has remained open for the three power indices. Our main contributions for the manipulation when an agent split into several false identities are highlighted below. The publications associated with these results are [40, 41].

- We extend a previous result of Aziz and Paterson [36] on exact unanimity WVGs to the Deegan-Packel index. Specifically, we show that in an exact unanimity WVG, if the Deegan-Packel index is used to compute agents' power, it is advantageous for an agent to split up into several false agents.

- We present new results for excess unanimity WVGs using the Shapley-Shubik and Banzhaf indices. Specifically, we propose new bounds for situations when the original games are excess unanimity WVGs and the resulting games after splitting are non-unanimity WVGs.

- We provide empirical evidence to show that the three indices (i.e., Shapley-Shubik, Banzhaf, and Deegan-Packel) are all susceptible to manipulation in non-unanimity

WVGs when an agent splits into several false identities. However, that the Deegan-Packel index is more susceptible than the Shapley-Shubik and Banzhaf indices.

- Finally, we provide the first *three* non-trivial bounds when a manipulator splits into $k > 2$ false identities using the Shapley-Shubik and Banzhaf indices. One of the bounds is also shown to be asymptotically tight (i.e., there exists at least a game in which an agent achieves the proposed bounds by splitting into several false identites).

2.5.2 Results for Manipulations by Annexation and Merging

Little work exists on manipulation via annexation and merging in WVGs. Our main contributions on the experimental analysis of the effects of annexation and merging in WVGs are highlighted below. The publications associated with these results are [42–46].

- For unanimity WVGs of n agents:

  a. Contrary to Aziz and Paterson [36] that for both the Shapley-Shubik and Banzhaf indices it is advantageous for a player to annex, we show that this is not true in its entirety. Apart from the fact that manipulation by annexation always increases the power of other agents that are not annexed by the same factor of increment as an annexer achieves, the annexer also incurs some compensation that is made to the assimilated agents to forfeit their weights, thus, reducing the benefit the annexer thought it gained.

  b. The upper bound on the extent to which an annexer may gain while annexing other agents in an altered game is at most n times the power of the agent in the original game. This result holds for the Shapley-Shubik, Banzhaf, and Deegan-Packel power indices.

- The Shapley-Shubik, Banzhaf, and Deegan-Packel indices are all *highly* susceptible to manipulation via annexation in non-unanimity WVGs. On the other hand, the three indices are all *less* susceptible to manipulation via merging in non-unanimity

WVGs. The Shapley-Shubik index is the most susceptible of the three indices for both manipulation via annexation and merging.

- We show that manipulators need to do only a *polynomial* amount of work to find a much improved power gain over a simple random approach to manipulation and present two enumeration-based pseudo-polynomial algorithms that manipulators can use. Our enumeration-based methods are shown to achieve significant improvement in benefits for manipulators in several numerical experiments. Thus, unlike the simple random approach (where merging has little or no benefits for the manipulators using the Shapley-Shubik and Banzhaf indices), results from our experiments suggest that merging can be *highly* effective for both the Shapley-Shubik and Banzhaf indices.

- We propose simple heuristics for manipulation by annexation in WVGs, and as well present careful analysis of the performance of the heuristics.

- Finally, despite finding the optimal beneficial merge is an NP-hard problem for both the Shapley-Shubik and Banzhaf indices, results from our experiments show that finding a beneficial merge is relatively easy in practice. While it appears impossible to stop manipulation by merging for a given game, controlling a suggested measure, termed *quota ratio*, is desirable. We deduce that a high quota ratio reduces the percentage of beneficial merges, and then conclude that the Banzhaf index may be more desirable to avoid merging, especially for high quota ratios.

CHAPTER 3
SUSCEPTIBILITY OF POWER INDICES TO MANIPULATION BY SPLITTING

3.1 Overview

Research in manipulation in WVGs has been active [14,35,36]. However, in this earlier research, false-name manipulation by splitting has been restricted to the case of when a manipulating agent splits into exactly *two* false identities, and using only the Shapley-Shubik and Banzhaf indices to evaluate agents' power. Many of the NP-hardness results of Aziz et al. [14] for splitting into exactly two false identities (using the Shapley-Shubik and Banzhaf indices) also hold for splitting into several false identities, however, results quantifying the extent of power gained by a manipulating agent when it splits into several false identities for the three power indices were unspecified, until our work.

In this chapter, we provide examples to illustrate manipulation by splitting into several false identities. Second, we extend a previous result on exact unanimity WVGs to the Deegan-Packel index, and present new results for excess unanimity WVGs. Specifically, we propose new bounds for the Shapley-Shubik and Banzhaf power indices on the extent of gains a manipulator may achieve when the original game is an excess unanimity WVG and the resulting game after splitting is a non-unanimity WVG. Third, we demonstrate a simple approach to weight splitting into several false identities for manipulating agents : *simple sampling procedure*, which simplifies manipulation by splitting in non-unanimity WVGs. Fourth, we consider experimental investigation and analysis of manipulation by splitting in non-unanimity WVGs using the three power indices to compute agents' power. Finally, we conduct normality tests on some real-world weighted voting systems to provide some justifications on the use of the normal distribution to generate agents' weights for experiments in the remaining chapters of the dissertation.

3.2 Examples of Manipulation by Splitting

For the sake of simplicity, we assume that only one of the agents is engaging in manipulation by splitting.

Example 1. *Advantageous split.*

Let $G = [6, 5, 4, 4, \mathbf{3}; 12]$ be a WVG of five agents. Consider the fifth agent (with weight 3) shown in bold. The power of the agent in this game using the three indices are:

Shapley-Shubik index: $\varphi_5(G) = 0.167$

Banzhaf index: $\beta_5(G) = 0.172$

Deegan-Packel index: $\gamma_5(G) = 0.185$

Suppose the agent is a manipulator that splits into three false identities with weights 1 each. We have a new game $G' = [6, 5, 4, 4, \mathbf{1}, \mathbf{1}, \mathbf{1}; 12]$ with the false agents shown in bold. The power of each of the false agents and the sum of the power of the false agents for each of the power indices in G' are:

Shapley-Shubik index: $\varphi_{5_1}(G') = \varphi_{5_2}(G') = \varphi_{5_3}(G') = 0.057$. The sum of the power of the false agents, $\sum_{j=1}^{3} \varphi_{5_j}(G') = 3 \times 0.057 = 0.171$

Banzhaf index: $\beta_{5_1}(G') = \beta_{5_2}(G') = \beta_{5_3}(G') = 0.059$. The sum of the power of the false agents, $\sum_{j=1}^{3} \beta_{5_j}(G') = 3 \times 0.059 = 0.177$

Deegan-Packel index: $\gamma_{5_1}(G') = \gamma_{5_2}(G') = \gamma_{5_3}(G') = 0.116$. The sum of the power of the false agents, $\sum_{j=1}^{3} \gamma_{5_j}(G') = 3 \times 0.116 = 0.348$

The factor by which the manipulator gains using the three indices are as follows.

Shapley-Shubik index: $\frac{0.171}{0.167} = 1.02$

Banzhaf index: $\frac{0.177}{0.172} = 1.03$

Deegan-Packel index: $\frac{0.348}{0.185} = 1.88$

Example 2. *Disadvantageous split.*

Let $G = [9, 9, \mathbf{8}, 2, 1; 20]$ be a WVG of five agents. Consider the third agent (with weight 8) shown in bold. The power of the agent in this game using the three power indices are as follows.

Shapley-Shubik index: $\varphi_3(G) = 0.183$

Banzhaf index: $\beta_3(G) = 0.182$

Deegan-Packel index: $\gamma_3(G) = 0.208$.

Suppose the agent splits into five false identities with weights $2, 2, 2, 1, 1$. We have a new game $G' = [9, 9, \mathbf{2}, \mathbf{2}, \mathbf{2}, 2, \mathbf{1}, \mathbf{1}, 1; 20]$ with the false agents shown in bold. The power of each of the false agents and the sum of the power of the false agents for each of the power indices in game G' are as follows.

Shapley-Shubik index: $\varphi_{3_1}(G') = \varphi_{3_2}(G') = \varphi_{3_3}(G') = 0.038$ and $\varphi_{3_4}(G') = \varphi_{3_5}(G') = 0.032$. The sum of the power of the false agents, $\sum_{j=1}^{5} \varphi_{3_j}(G') = 0.177$

Banzhaf index: $\beta_{3_1}(G') = \beta_{3_2}(G') = \beta_{3_3}(G') = 0.021$ and $\beta_{3_4}(G') = \beta_{3_5}(G') = 0.014$. The sum of the power of the false agents, $\sum_{j=1}^{5} \beta_{3_j}(G') = 0.092$

Deegan-Packel index: $\gamma_{3_1}(G') = \gamma_{3_2}(G') = \gamma_{3_3}(G') = 0.065$ and $\gamma_{3_4}(G') = \gamma_{3_5}(G') = 0.083$. The sum of the power of the false agents, $\sum_{j=1}^{5} \gamma_{3_j}(G') = 0.361$

The factor by which the manipulator loses (for the Shapley-Shubik and Banzhaf indices) and gains (for the Deegan-Packel index) are as follows.

Shapley-Shubik index: $\frac{0.177}{0.183} = 0.97$

Banzhaf index: $\frac{0.092}{0.182} = 0.50$

Deegan-Packel index[1]: $\frac{0.361}{0.208} = 1.74$

[1] Note that this game is advantageous using the Deegan-Packel power index.

We have shown that a manipulator may gain or lose power using the three power indices when it engages in manipulation by splitting into several false identities. Similar examples where a manipulator neither gains nor loses power exist for the three indices.

3.3 Unanimity Weighted Voting Games

We recall that a WVG in which there is a single winning coalition and such that every agent is critical to the coalition is a unanimity WVG (see Section 2.1).

3.3.1 Exact Unanimity Weighted Voting Games

For exact unanimity WVGs, Aziz and Paterson [36] proved the following results:

Proposition 1. *In a unanimity WVG with $q = w(I)$, if Banzhaf indices are used as payoffs of agents in a WVG, then it is beneficial for an agent to split up into several agents. The same holds for Shapley-Shubik power index.*

We extend these results for the Deegan-Packel power index as follows.

Theorem 1. *In an exact unanimity WVG with $q = w(I)$, if the Deegan-Packel index is used to compute the power of agents, then, it is advantageous for an agent to split up into several false identities.*

Proof. Let G be an exact unanimity WVG of n agents with quota $q = w(I)$. It is easy to see that the Deegan-Packel power index of every agent i in game G, $\gamma_i(G) = \frac{1}{n}$. Suppose agent 1 splits into $m + 1$ false agents, then we have a new exact unanimity game, G', of $n + m$ agents. The Deegan-Packel power index of every agent i in G', $\gamma_i(G') = \frac{1}{n+m}$. Hence, the new Deegan-Packel power index of agent 1 in game G' is $\gamma_1(G') = \frac{m+1}{n+m} > \frac{1}{n}$ for $n > 1$. □

The following theorem is immediate from Proposition 1 and Theorem 1:

Theorem 2. *Let G be an exact unanimity WVG of n agents with quota $q = w(I)$. Suppose an agent i splits into several false agents in a new game G'. Using the Shapley-Shubik, Banzhaf, and Deegan-Packel power indices, the power of the manipulating agent assigned via each of the power indices increases as the number of splits increases.*

Corollary 1. *Let G be an excess unanimity WVG with $w(I) > q$. Let an agent i split into several false agents in a new game G'. Suppose the new game G' is also a unanimity WVG, then the splitting is advantageous for i if any of Shapley-Shubik, Banzhaf, and Deegan-Packel power index is used to compute the agent's power.*

3.3.2 Excess Unanimity Weighted Voting Games

From the assumption of Corollary 1, the new game that results when an original excess unanimity WVG is altered may not necessarily remains unanimity. Consider the following examples.

Example 3. *Advantageous Split for Excess Unanimity*

Let $G = [5, 3, 3; 10]$ be an excess unanimity WVG of three agents. The Shapley-Shubik power of each agent in this game is 0.3333. Suppose the first agent alters G by splitting its weight among four false agents with weights, $2, 1, 1, 1$, such that the new game $G' = [3, 3, \mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1}; 10]$. Note that G' is non-unanimity. The sum of the Shapley-Shubik power of the false agents, $\varphi_{1_1}(G') + \sum_{j=2}^{4} \varphi_{1_j}(G') = 0.2667 + 3 \times 0.0667 = 0.4668 > \varphi_1(G)$. The agent benefits from the split action and increases its power index by a factor, $\tau = \frac{0.4668}{0.3333} = 1.40$, of the original index.

Example 4. *Disadvantageous Split for Excess Unanimity*

Let $G = [5, 5, 5; 11]$ be an excess unanimity WVG of three agents. The Shapley-Shubik power of each agent in this game is 0.3333. Suppose the third agent alters G by splitting its weight among three false agents with weights, $2, 2, 1$, such that the new game $G' = [5, 5, \mathbf{2}, \mathbf{2}, \mathbf{1}; 11]$. Note that G' is non-unanimity. The sum of the Shapley-Shubik power of the false agents, $\sum_{j=1}^{3} \varphi_{3_j}(G') = 3 \times 0.0333 = 0.0999 < \varphi_3(G)$. The agent incurs a decrease in power, and its power decreases by a factor of $\tau = \frac{0.0999}{0.3333} = 0.30$.

It is interesting to observe that, unlike in exact unanimity WVGs where splitting into several false identities is typically advantageous for a manipulator, the situation is not the same for the case of excess unanimity WVGs. It is clear from the examples above that it may or may not be advantageous for a manipulator to split into several false identities if the original game is an excess unanimity WVG. This, as seen, is a positive result for game designers in that splitting into several identities in this type of game is not guaranteed to always be advantageous for manipulators. We now provide bounds for the Shapley-Shubik and Banzhaf indices on the extent of gains a manipulator may achieve when the original game is an excess unanimity WVG and the resulting game after splitting is non-unanimity WVG.

Theorem 3. *Let $G = [w_1, \ldots, w_n; q]$ be an excess unanimity WVG of n agents. If an agent i alters G by splitting into $k \geq 2$ false agents in a new game G', and such that G' is not a unanimity WVG, then, the power index of the agent in the new game using the Shapley-Shubik index is at most $\frac{k(k-1)}{n+k-1}\varphi_i(G)$.*

Proof. It is clear that the only winning coalition in game G consists of all the agents in the game since G is unanimity. So, $\varphi_x(G) = \frac{1}{n}$, for all agents $x \in I$ in game G. Suppose agent i splits into $k \geq 2$ false identities, $i_1, \ldots, i_k$, in game G', and such that G' is not unanimity. There are now $n + k - 1$ agents in game G'. Observe that since G is unanimity, all the remaining agents in G (i.e., excluding agent i) cannot meet the quota of the game in G'. Thus, the only possibility for a particular false agent i_j to be pivotal in any permutation of the agents in G' is this:

> All the non-manipulating agents and zero or more false agents precede agent i_j, and the remaining false agent(s) appear after i_j. Note that at least one false agent *must* appear after i_j since G' is non-unanimity.

Let $\Pi_{G'}$ be the set of all permutations of agents in game G'. Consider any permutation $\pi \in \Pi_{G'}$. Let the false agent i_j be pivotal at location r in π. When there is no false agent before location r, there are at most $(n-1)! \cdot C(k-1, 0) \cdot 1 \cdot (k-1)!$ possible permutations[2] for agent i_j to be pivotal, since there are $n-1$ non-manipulating agents before r and $k-1$ false agents after r. Similarly, when there is only one false agent before location r, there are

[2] $C(n, r) = \frac{n!}{r!(n-r)!}$.

at most $n! \cdot C(k-1,1) \cdot 1 \cdot (k-2)!$ possible permutations for agent i_j to be pivotal, since there are now n agents ($n-1$ non-manipulating agents and 1 false agent) before r, $C(k-1,1)$ ways of chosing the false agent that appear before r, and the remaining $k-2$ false agents appear after r. We continue this process until there are at most $k-2$ false agents before r (analogously, until there is only 1 false agent after r). So, in general, it easy to see that the false agent i_j will be pivotal in at most the following number of permutations:

$$\sum_{p=0}^{k-2}(n+p-1)! \cdot C(k-1,p) \cdot (k-p-1)!$$

By the same argument, each of the k false agents is pivotal for the same number of permutations. So, we have[3]

$$\begin{aligned}
& k\sum_{p=0}^{k-2}(n+p-1)! \cdot C(k-1,p) \cdot (k-p-1)! \\
= \quad & k\sum_{p=0}^{k-2}(n+p-1)! \cdot \frac{(k-1)!}{p!} \\
= \quad & k!\sum_{p=0}^{k-2}\frac{(n+p-1)!}{p!} \cdot \frac{(n-1)!}{(n-1)!} \\
= \quad & k! \cdot (n-1)!\sum_{p=0}^{k-2}C(n+p-1,p) \\
= \quad & k! \cdot (n-1)! \cdot C(n+k-2,k-2) \\
= \quad & k! \cdot (n-1)! \cdot \frac{(n+k-2)!}{(k-2)! \cdot n!} \\
= \quad & \frac{k \cdot (k-1) \cdot (n+k-2)!}{n}.
\end{aligned}$$

Hence, the sum of the Shapley-Shubik power of the false agents in G',

[3] We use this combinatorial identity to simplify the summation: $\sum_{p=0}^{k} C(n+p,p) = C(k+n+1,k)$.

$$\begin{aligned}
\sum_{j=1}^{k} \varphi_{i_j}(G') &\leq \frac{k \cdot (k-1) \cdot (n+k-2)!}{n \cdot (n+k-1)!} \\
&= \frac{k \cdot (k-1)}{n \cdot (n+k-1)} \\
&= \frac{k \cdot (k-1)}{n \cdot (n+k-1)} \cdot n \cdot \frac{1}{n} \\
&= \frac{k(k-1)}{n+k-1} \varphi_i(G).
\end{aligned}$$

□

Theorem 4. *Let $G = [w_1, \ldots, w_n; q]$ be an excess unanimity WVG of n agents. If an agent i alters G by splitting into $k \geq 2$ false agents in a new game G', and such that G' is not a unanimity WVG, then, the power index of the agent in the new game using the Banzhaf index is at most $\frac{nk}{n+k-1}\beta_i(G)$.*

Proof. Recall that $\eta_x(G)$ is the number of winning coalitions for which an agent x is critical in a WVG G. It is clear that the only winning coalition in game G consists of all the agents in the game since G is unanimity. So, $\eta_x(G) = 1$ and $\beta_x(G) = \frac{1}{n}$, for all agents $x \in I$ in game G. Suppose agent i splits into $k \geq 2$ false identities, $i_1, \ldots, i_k$, in an altered game G', and such that G' is not unanimity. The sum of the Banzhaf power of the false agents in the altered game G', is given as

$$\sum_{j=1}^{k} \beta_{ij}(G') = \frac{\sum_{j=1}^{k} \eta_{ij}(G')}{\sum_{j=1}^{k} \eta_{ij}(G') + \sum_{x \in I \setminus \{i\}} \eta_x(G')} \tag{3.1}$$

We first bound the number of coalitions in the non-unanimity WVG G' for which each of the non-manipulating agents is critical. Since G is unanimity, any winning coalition in G' must include all of the $n-1$ non-manipulators and one or more of the false agents. There are $2^k - 1$ ways of selecting one or more of the false agents and adding them to the coalition consisting of only the $n-1$ non-manipulating agents. Thus, each of the non-manipulating agents can be in at most $2^k - 1$ coalitions consisting of the false agents. However, since not all of these possible coalitions will be winning, we let the number of winning coalitions in G' be $m \leq 2^k - 1$. Hence, $\sum_{x \in I \setminus \{i\}} \eta_x(G') \leq m(n-1) \leq (2^k - 1)(n-1)$.

We now bound the number of coalitions for which the false agents are critical. We partition the false agents into two groups: *critical splinters* and *excess splinters*. Critical splinters are a minimal set of false agents that when added to a set of non-manipulators create a winning coalition. Excess splinters are false agents that are not critical (or make some critical splinters not to be critical) when added to a winning coalition of non-manipulators and critical splinters. Since we are not interested in non-winning coalitions, we only need to consider the cases where there are enough critical splinters to make the coalitions winning. Let c be the average number of critical splinters in each of the winning coalitions. Note that c is bounded by k, so, all the false agents can be critical in at most km coalitions, i.e., $\sum_{j=1}^{k} \eta_{ij}(G') = cm \leq km$.

Thus, Equation 3.1, becomes

$$
\begin{aligned}
\sum_{j=1}^{k} \beta_{ij}(G') &= \frac{cm}{cm + m(n-1)} \\
&\leq \frac{km}{km + m(n-1)} \\
&= \frac{k}{n+k-1} \\
&= \frac{k}{n+k-1} \cdot n \cdot \frac{1}{n} \\
&= \frac{nk}{n+k-1} \cdot \beta_i(G).
\end{aligned}
$$

□

3.4 Non-Unanimity Weighted Voting Games

Manipulation by splitting in non-unanimity WVGs is more interesting as it provides complex and realistic scenarios that are not well-understood. As the structure of the WVGs changes due to splitting by a manipulating agent, the number of winning coalitions as well as the minimal winning coalitions in the games change. Consider a WVG with quota q, involving agents I. If any agent $i \in I$ has weight $w_i \geq q$, then the agent will always win without forming coalitions with other agents. The more interesting games we consider are those for which $w_i < q$ for all i, and such that q satisfies the inequality $q \leq w(I) - m$, where m is the weight of the smallest agent in the game. When the grand coalition emerges, it will always contain at least one agent that is not critical in the coalition. It is easy to see that the resultant games are non-unanimity WVGs.

3.4.1 Weight Splitting into Several False Identities

It is important to observe that for a certain manipulating agent, i, with weight, w, there are different possible ways the agent can split w among $k > 2$ false agents. Since it is not clear which of the possible weights combinations of the false agents gives an improved power, a straightforward approach will be to simply enumerate all such possible splits of agent i into k false agents and check the factors of power gained for a split that gives the highest improved power. All such possible splits correspond to the population of splits of the manipulating agent's weight (i.e., all possible altered WVGs) that the agent needs to consider. Unfortunately, enumerating all such possible splits of agent i into k false agents is non-trivial even if the weights are restricted to integers. This is actually the well-known problem of *partitioning* of integer w into k parts or less [38]. Thus, it is impractical to examine all the splits of the false agents in the population even for small values of w and k.

We propose a *simple sampling procedure*, to address this problem. The procedure provides a huge simplification in computational effort and significant benefits for manipulating agents to engage in manipulation by splitting in non-unanimity WVGs.

Simple Sampling Procedure

A simple approach we employ here is to consider *samples* of WVGs from the population of altered WVGs created by the splits of the manipulating agent's weight. The information contained within the samples is then used to investigate properties of the population from which the samples are drawn [47].

We consider a WVG G of n agents having a distinct manipulating agent i with integer weight w. For agent i to split its weight w among k false agents, we generate the population of the splits for weight w as the set $P(w, k)$, containing *all* j-way splits, $s = (w_1, \dots, w_j)$, of false agents such that $3 \leq j \leq k$. A random sample, $S \subset P(w, k)$, of splits of a particular *size* (determined as described below) is then taken from the population. For each split $s \in S$, the procedure first removes w from WVG G, adds the split $s = (w_1, \dots, w_j)$ to create an altered WVG G', and then computes the factor of the power gained or lost by the agent. We then conduct an exhaustive search of S for the split having the highest factor of increment and return the split as the way the agent should split its weight in game G. We provide a simple justification for using this approach. Using the definition of $P(w, k)$ above, we note that $|P(10, 10)| = 36$, whereas, $|P(50, 10)| = 55,461$. So, for the trivial cases when $w \leq 10$, examining every element of $P(w, k)$ in the partition to determine the split with the highest factor of increment seems reasonable, and can be done exactly by simple enumeration. However, this is not the case as w increases. A crucial factor that determines the performance of this method is the sample size $|S|$ taken from the population. For our experiments, we have used sample sizes determined as 10% of the population size.

3.4.2 Simulation Settings

In order to evaluate the behavior of the power indices for the non-unanimity WVGs, we conduct experiments to analyze the effects of manipulation by splitting using each of the three indices when a strategic agent splits its weight in the games. The values of the power indices of the several false agents into which the agent splits are added and compared with the power index of the agent in the original game. We have used a total of 100 original WVGs for each experiment, allow a manipulating agent to split its weight in each of the games using the Simple Sampling procedure, and finally compute the factor of increment over the entire set of games for the three indices. For each game, the number, k, of false agents into which a manipulating agent, i, splits is randomly drawn from a uniform distribution over the range $[3, 7]$, while ensuring that $k \leq w$, where w is the weight of the agent.

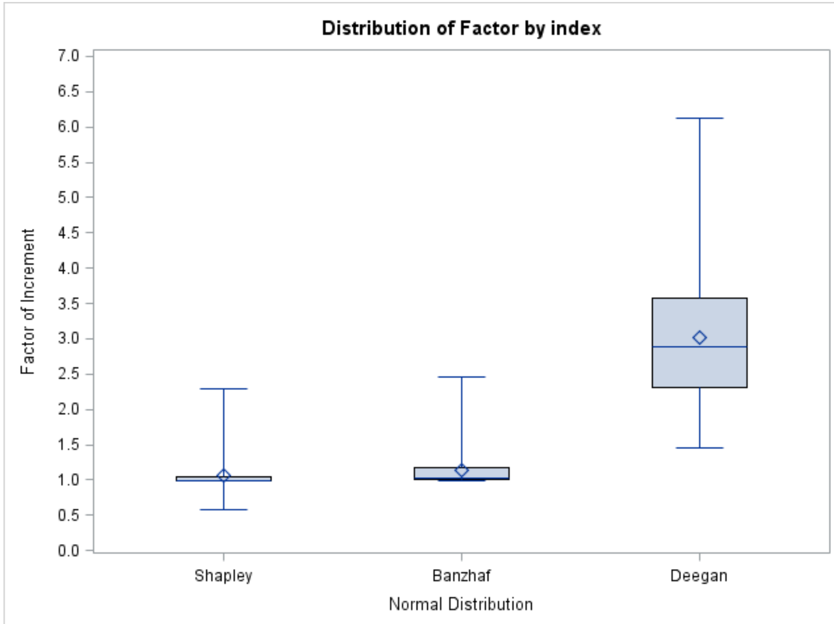

Figure 3.1. Estimation of the extent of susceptibility of the power indices to manipulation by splitting into several false identities using the Simple Sampling procedure. WVGs are generated by normal distribution.

3.4.3 Empirical Results and Discussion

Results from the experiments show the existence of beneficial splits when agents engage in splitting into several false identities for the three power indices. However, the extent to which agents gain varies with the indices. We use *box plots* in the presentation of our results. Box plots are graphical representations used to illustrate the center and spread of group of data, such as the lower and upper limits, quartiles (lower, median, and upper quartiles), and mean. We show in Figures 3.1, 3.2, and 3.3, box plots summarizing the results of the extent of susceptibility of the three power indices to manipulation by splitting into several false identities using the Simple Sampling Procedure for 100 WVGs. This is achieved by comparing the population of the factor of increment attained by agents in different games for each indices. The results are respectively for WVGs generated by normal (Figure 3.1), uniform (Figure 3.2), and Poisson (Figure 3.3) distributions. The x-axes show the three power indices while the y-axes are the factors of increment over the samples of WVGs achieved by manipulating agents.

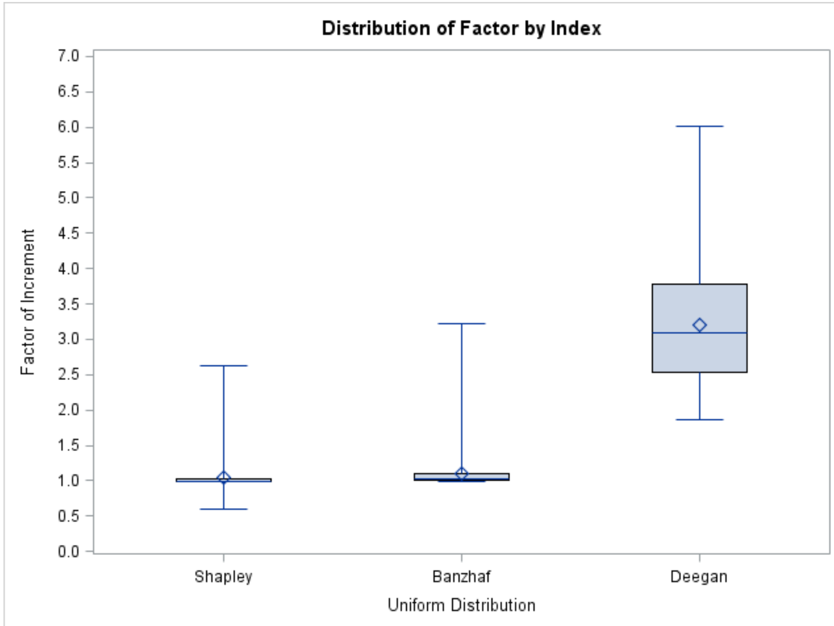

Figure 3.2. Estimation of the extent of susceptibility of the power indices to manipulation by splitting into several false identities using the Simple Sampling procedure. WVGs are generated by uniform distribution.

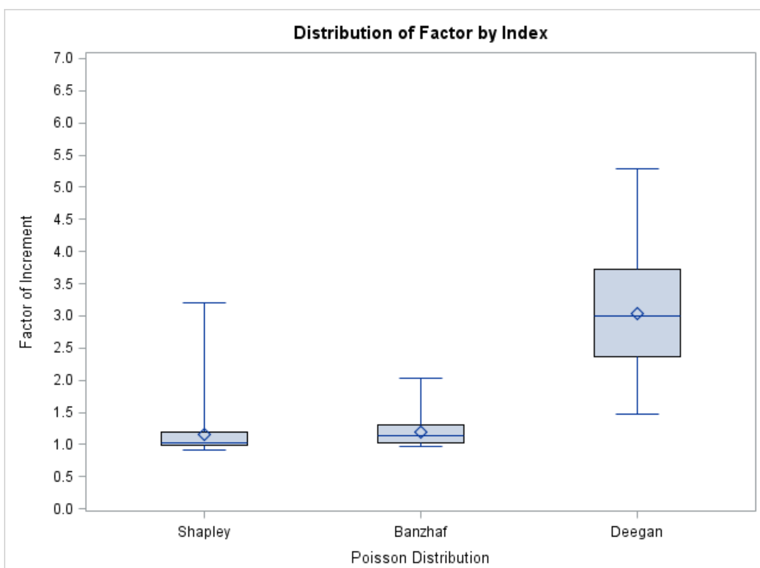

Figure 3.3. Estimation of the extent of susceptibility of the power indices to manipulation by splitting into several false identities using the Simple Sampling procedure. WVGs are generated by Poisson distribution.

The high susceptibility of the Deegan-Packel index to manipulation by splitting can be observed from the figures. The maximum gain attained by manipulating agents while using the Deegan-Packel index is about 6.1 (normal), 6.0 (uniform), and 5.3 (Poisson) times of their original values. On the other hand, the maximum gain attained while using any of the Shapley-Shubik or Banzhaf index is less than a factor of 3.5 of their original values for the three distributions. Furthermore, while the average[4] factor of increment for manipulation is high for the Deegan-Packel index (~ 3.0) for the three distributions, those of the Shapley-Shubik (~ 1.1 (normal), ~ 1.0 (uniform), ~ 1.2 (Poisson)) and Banzhaf (~ 1.1 (normal & uniform), ~ 1.2 (Poisson)) are a little bit low. On average, splitting does not appear to improve power using the Shapley-Shubik and Banzhaf indices, especially for both the normal and uniform distributions. We also show (in Table 3.1), the standard deviations of the factor of power gained by manipulators from the data of Figures 3.1, 3.2, and 3.3 for the normal, uniform, and Poisson distributions. These results suggest that the Deegan-Packel index is more susceptible to manipulation by splitting into several false identities than the Shapley-Shubik and Banzhaf indices in non-unanimity WVGs. They are also indicative of the ease by which each of the power indices is manipulable.

Table 3.1. The standard deviations of the factor of increment from the data of Figures 3.1, 3.2, and 3.3 for the three distributions.

	Shapley-Shubik index	Banzhaf index	Deegan-Packel index
Normal Distribution	0.23	0.29	0.94
Uniform Distribution	0.21	0.26	0.79
Poisson Distribution	0.33	0.23	0.87

Finally, the number of games that are advantageous and disadvantageous for the three power indices are analyzed. Figures 3.4, 3.5, and 3.6, show the number of advantageous and disadvatageous games from the samples considered for our experiments for the normal, uniform, and Poisson distributions respectively. All the games are advantageous for the Deegan-Packel index across the three distributions. There are 64 and 90 games that are advantageous respectively for the Shapley-Shubik and Banzhaf indices for the Poisson

[4] The diamond symbols in the box plots indicate the mean values of the datasets.

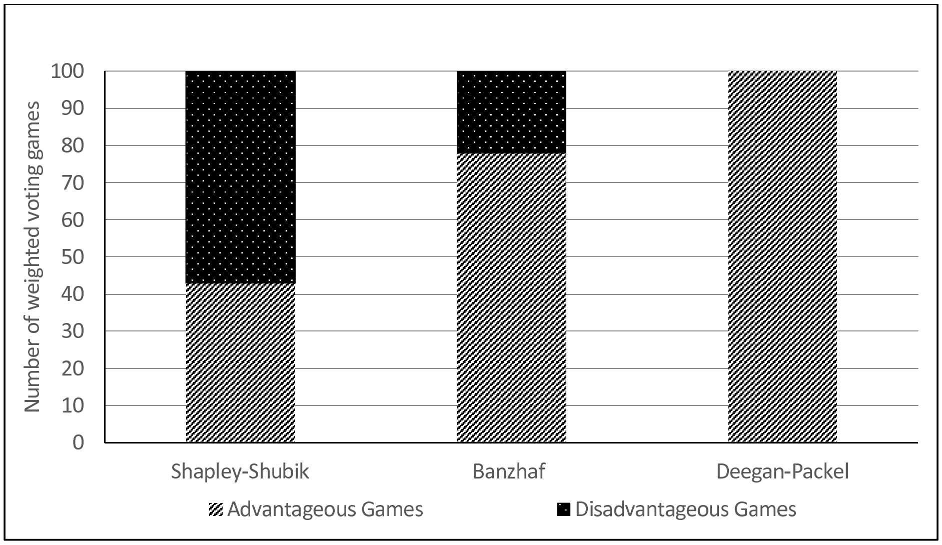

Figure 3.4. The number of advantageous and disadvantageous games among the three indices for manipulation by splitting into several false identities using the Simple Sampling procedure in 100 non-unanimity WVGs. WVGs are generated by normal distribution.

distribution. See Figure 3.6. However, for both the normal and uniform distributions only 43 and 36 are advatageous for the Shapley-Shubik index. Lastly, there are 78 and 73 games that are advantageous, respectively, for the normal and uniform distributions using the Banzhaf index. See Figures 3.4 and 3.5. Clearly, the Deegan-Packel index is more susceptible to manipulation by splitting into several false identities than the Shapley-Shubik and Banzhaf indices.

These results are important for the following reasons. First, since beneficial weight splitting into many identities can easily be achieved with the Simple Sampling procedure, this may provide some motivation for agents to engage in manipulation especially using the Deegan-Packel index. Second, no previous work has considered the evaluation of the extent of the effects of splitting into several identities in non-unanimity WVGs. Lastly, we see that there are no significant differences in all the results for the WVGs (generated by the normal, uniform, and Poisson distributions) that were used in the experiments.

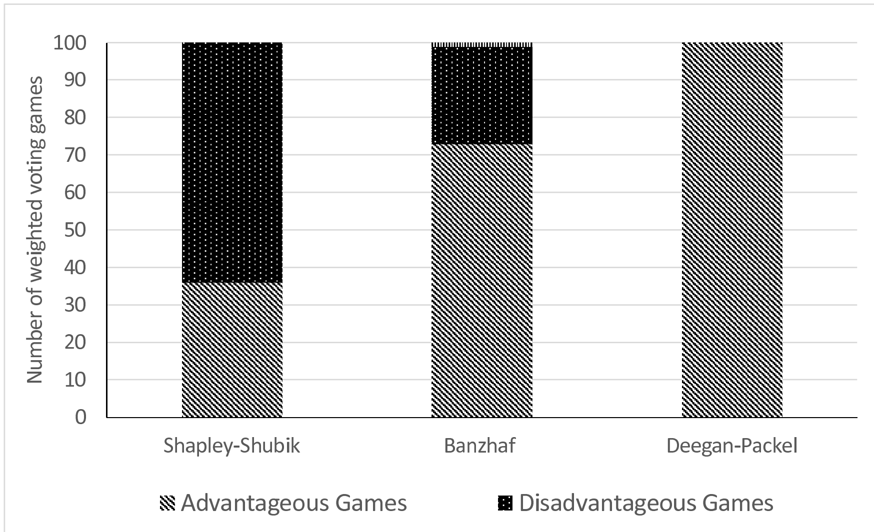

Figure 3.5. The number of advantageous and disadvantageous games among the three indices for manipulation by splitting into several false identities using the Simple Sampling procedure in 100 non-unanimity WVGs. WVGs are generated by uniform distribution.

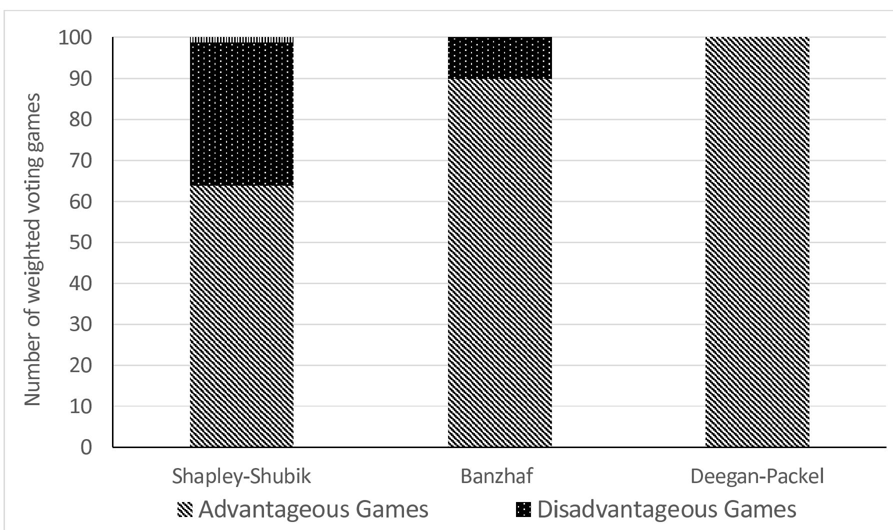

Figure 3.6. The number of advantageous and disadvantageous games among the three indices for manipulation by splitting into several false identities using the Simple Sampling procedure in 100 non-unanimity WVGs. WVGs are generated by Poisson distribution.

3.5 Normal Distribution and Normality Tests on Real-world Voting Systems

Since we found no significant difference in the results we obtained for the three distributions (i.e., normal, uniform, and Poisson) as demonstrated in the previous sections, we use the normal distribution in the remaining part of the dissertation to generate agents' weights in our experiments. Our use of the normal distribution is further motivated by some normality tests we carried out on three real-world weighted voting systems. We conducted normality checks (using *graphical checks* and *numerical tests* from statistics [48]) on these voting systems and found that the weights of the games are normally distributed. We illustrate some of the results of our normality checks on recent datasets (i.e., as at 2012) from the European Union (EU), the Electoral College (EC) of the United States, and the International Monetary Funds (IMF). We provide evidence of appropriateness of relevant assumptions by using the graphical checks for normality and our conclusions are as follows:

Histograms with Normal Curves : The histograms with normal curves for the three datasets (i.e, the EU, EC, and IMF) are normal or almost normal (in some cases), suggesting that the weights are normally distributed. See Figure 3.7 for the histogram with normal curve for the 2012 Electoral College's Weights of the United States. The weights are included in Appendix A for completeness.

Normal Probability Plots : Normal probability plots compare observed values to expected (normal) values. The plots are approximately linear for the three datasets, so we conclude that the weights of the players in the voting systems are normally distributed for each of the datasets. See Figure 3.8 for the normal probability plot for the 2012 Electoral College's weights of the United States. Also see the appendices for similar curves and plots for the European Union (Appendix B), and International Monetary Funds (Appendix C) datasets.

Numerical Test for Normality : We went further to confirm the results of the graphical

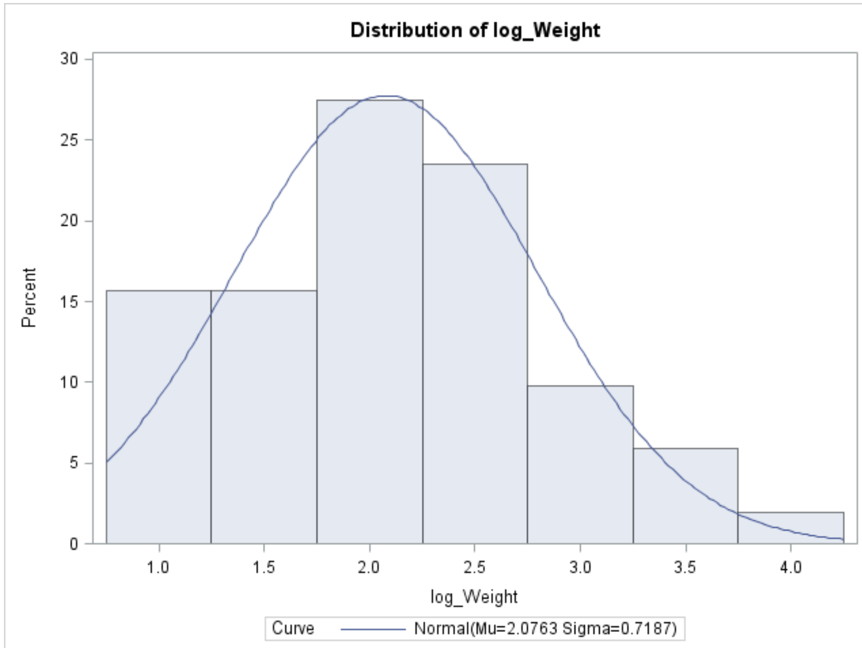

Figure 3.7. The histogram with normal curve for the 2012 Electoral College's weights of the United States.

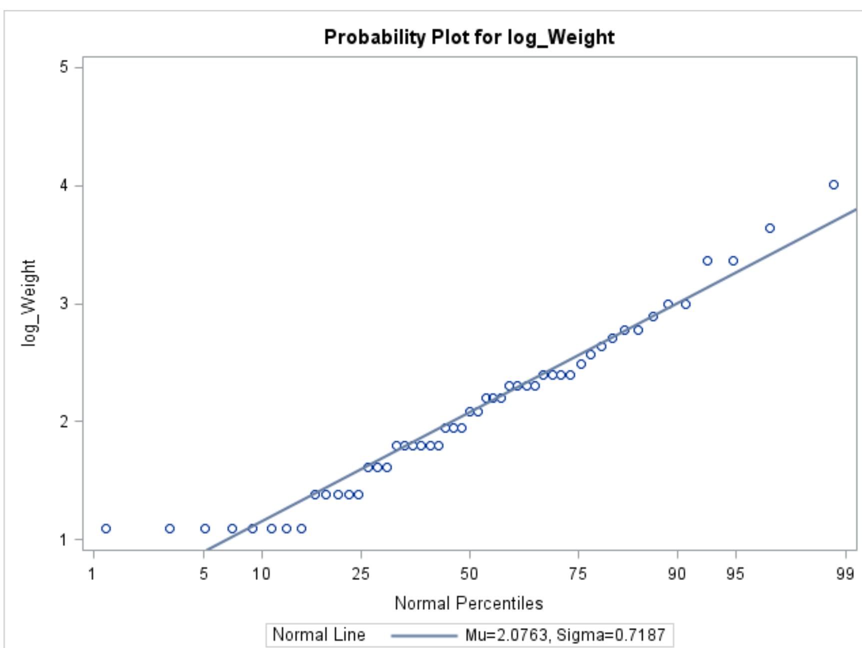

Figure 3.8. The probability plot for the 2012 Electoral College's weights of the United States.

checks above by performing numerical tests for normality for the three datasets. The notable Kolmogorov-Smirnov test provides p-values of > 0.150 (for EC), > 0.150 (for EU), and 0.136 (for IMF) which are all greater than the significance level, $\alpha = 0.05$. Thus, we fail to reject the null hypothesis that the data values, i.e., the weights of the players in the games are normally distributed.

CHAPTER 4
NEW BOUNDS ON SPLITTING INTO SEVERAL FALSE IDENTITIES

4.1 Overview

We continue the investigation of manipulation by splitting into several false identities in this chapter. Since our experimental results of the previous chapter have suggested ideas on the extent to which the indices are susceptible to manipulation by splitting in the general case of non-unanimity WVGs, we complement those results with new theoretical bounds, thereby generalizing earlier results on this problem.

Bachrach and Elkind [35] consider the computational aspects of manipulation by splitting in WVGs. They use the Shapley-Shubik power index to evaluate agents' power and consider the case when an agent splits into exactly $k = 2$ false identities. The extent to which agents may increase or decrease their Shapley-Shubik power is also bounded. The authors show that for any WVG of n players, a manipulator cannot gain more than a factor of $\frac{2n}{n+1} < 2$, while the agent cannot lose more than a factor of $\frac{n+1}{2}$ of its original power by splitting into two false agents. Similar results were obtained by [36] using the Banzhaf power index. They show that the maximum gain a manipulating agent can achieve is at most twice its value in the original game. Recently, Aziz et al. [14] provide a lower bound on manipulation by splitting using the Banzhaf index. Their bound shows that for any split of a manipulator into two false agents, the agent can hope to get at least $\frac{1}{n}$ of its original power in a new game.

The two papers [35,36], left as an open problem, the bounds on the case when an agent splits into $k > 2$ false identities for both the Shapley-Shubik and Banzhaf indices. This problem, until now, also remains open in the recent work of [14]. In this chapter, we partially resolve this open problem by providing the first *three* non-trivial bounds on the effect of manipulation by splitting into several false identities for both the Shapley-Shubik and Banzhaf indices. One of the bounds is also shown to be asymptotically tight (i.e., there exists at least a game in which an agent can achieve the proposed factors of increment by splitting into several false identites).

4.2 Shapley-Shubik Power Index Bounds

We propose new bounds for manipulation by splitting into several identities using the Shapley-Shubik power index.

Theorem 5. *(Upper Bound). Let $G = [w_1, \ldots, w_n; q]$ be a WVG of n agents. If an agent i alters G by splitting into $k \geq 2$ false agents in a new game G', then, the power index of the agent in the new game using the Shapley-Shubik index is at most $\frac{nk}{n+k-1}\varphi_i(G)$. Moreover, this bound is asymptotically tight.*

Proof. Let an agent i be a distinguished manipulator that splits into k false agents, $i_1, \ldots, i_k$. Let Π_{G-i} be the set of all permutations of the remaining $n-1$ agents in game G (i.e., not including agent i). We define the term, *i-pivotal-basis*, to be a permutation $\pi \in \Pi_{G-i}$ such that it is possible to insert agent i into π to make i pivotal in game G. We refer to the resulting permutation as being *i-pivotal*. See Figures 4.1 (a) and (c) for visual illustrations of these definitions. Define also π' to be a *morphed* permutation in game G' as one formed from inserting the false agents, $i_1, \ldots, i_k$, into permutation π in any order. Suppose there are P_i i-pivotal permutations that can be formed from the set Π_{G-i}, then, the Shapley-Shubik index of agent i in game G, $\varphi_i(G) = \frac{P_i}{n!}$.

Our hope is that for every case where the insertion of agent i into a permutation π makes i pivotal in game G we can create $X \in \mathbb{N}$ permutations in which a false agent is pivotal in the corresponding morphed permutation π' of game G'. There are two ways we can get permutations in game G' in which one of the false agents is pivotal:

A. Insert the false agents, $i_1, \ldots, i_k$, into a permutation in game G for which agent i is not pivotal, but a false agent is now pivotal in the altered game G'.

B. Insert the false agents, $i_1, \ldots, i_k$, into an i-pivotal-basis permutation π. Those seem like good candidates, because the resulting permutation after inserting i into π is i-pivotal, so a false agent may be pivotal in the morphed permutation π' of game G'.

Case A: We show that there are no permutations in this case. Suppose there exists a morphed permutation of game G' in which a false agent i_j is pivotal, but the permutation $\pi^* \in \Pi_{G-i}$ in game G from which it is formed is not i-pivotal-basis. This permutation in G' has the following form:

$$C \quad i_j \quad D$$

where, C and D, are respectively the left and right sides of the morphed permutation from the pivotal false agent i_j. Taking this permutation, create a new permutation by sliding all the false agents from C right towards i_j. Also, slide all the false agents from D left towards i_j. Now, all the false agents occur together with i_j still pivotal in this new permutation. This shows that the insertion of agent i into the permutation π^* makes i pivotal in game G. Thus, π^* is i-pivotal-basis. Since this is a contradiction to our assumption, there are no permutations in this case.

Case B: Consider a certain i-pivotal-basis permutation $\pi \in \Pi_{G-i}$ for which the insertion of agent i makes i pivotal in game G. We need to insert the false agents, $i_1, \ldots, i_k$, into this permutation. This is done by the following steps:

1. Decide on which of the k false agents should be pivotal in the newly created permutation. There are $C(k, 1) = k$ ways[1] of doing this.

2. Order the remaining false agents, and call this ordering, ρ. There are $(k-1)!$ ways of doing this.

3. Now, merge ρ with π without changing the order of elements in ρ or π to create a new permutation π'. To understand how to count the ways of doing this, realize that

[1] $C(n, r) = \frac{n!}{r!(n-r)!}$.

there are $n-1$ items in π and $k-1$ items in ρ. To get a complete ordering, we form a new permutation by taking the next element of π or ρ. This is simply permutations with repetition[2], which gives, $\frac{(n-1+k-1)!}{(n-1)!(k-1)!}$.

4. To complete the new permutation, we place the agent selected in step 1 at the appropriate spot in the permutation. There is at most one possibility. Either we find a place to insert the agent to make it pivotal, or we cannot.

Now, from steps 1 to 4 above, we see that there are at most $\frac{k\cdot(k-1)!\cdot 1\cdot(n-1+k-1)!}{(n-1)!(k-1)!} = \frac{k\cdot(n+k-2)!}{(n-1)!}$ possible ways of finding a new permutation in which a false agent is pivotal in π'. We repeat the process for each of the P_i i-pivotal-basis permutations $\pi \in \Pi_{G-i}$. Hence, the sum of the Shapley-Shubik power of the false agents in game G',

$$\begin{aligned}
\sum_{j=1}^{k} \varphi_{i_j}(G') &\leq \frac{P_i \cdot k \cdot (n+k-2)!}{(n-1)!(n+k-1)!} \\
&= \frac{P_i \cdot k}{(n-1)!(n+k-1)} \\
&= \frac{k}{(n-1)!(n+k-1)} \cdot n! \cdot \frac{P_i}{n!} \\
&= \frac{nk}{(n+k-1)} \varphi_i(G).
\end{aligned}$$

We now prove that this bound is asymptotically tight. Let $G = [k, k, \ldots, k; nk]$ be a WVG of n agents. It is clear that the only winning coalition consists of all the agents. So, $\varphi_i(G) = \frac{1}{n}$ for all agents $i \in I$ in the game. Suppose the last agent splits into k false identities each with weight 1, we have a new game $G' = [k, k, \ldots, k, \underbrace{1, 1, \ldots, 1}_{k \text{ times}}; nk]$ of $n+k-1$ agents. Then, $\varphi_i(G') = \frac{1}{n+k-1}$ for each agent i in the altered game G'. Hence, $\sum_{j=1}^{k} \varphi_{n_j}(G') = \frac{k}{n+k-1} = \frac{nk}{n+k-1}\varphi_n(G)$. □

[2] Consider a *director* who decides whether to take from π or ρ. He says "original" to take from π or "splinter" to take from ρ. He must say "original" $n-1$ times and "splinter" $k-1$ times.

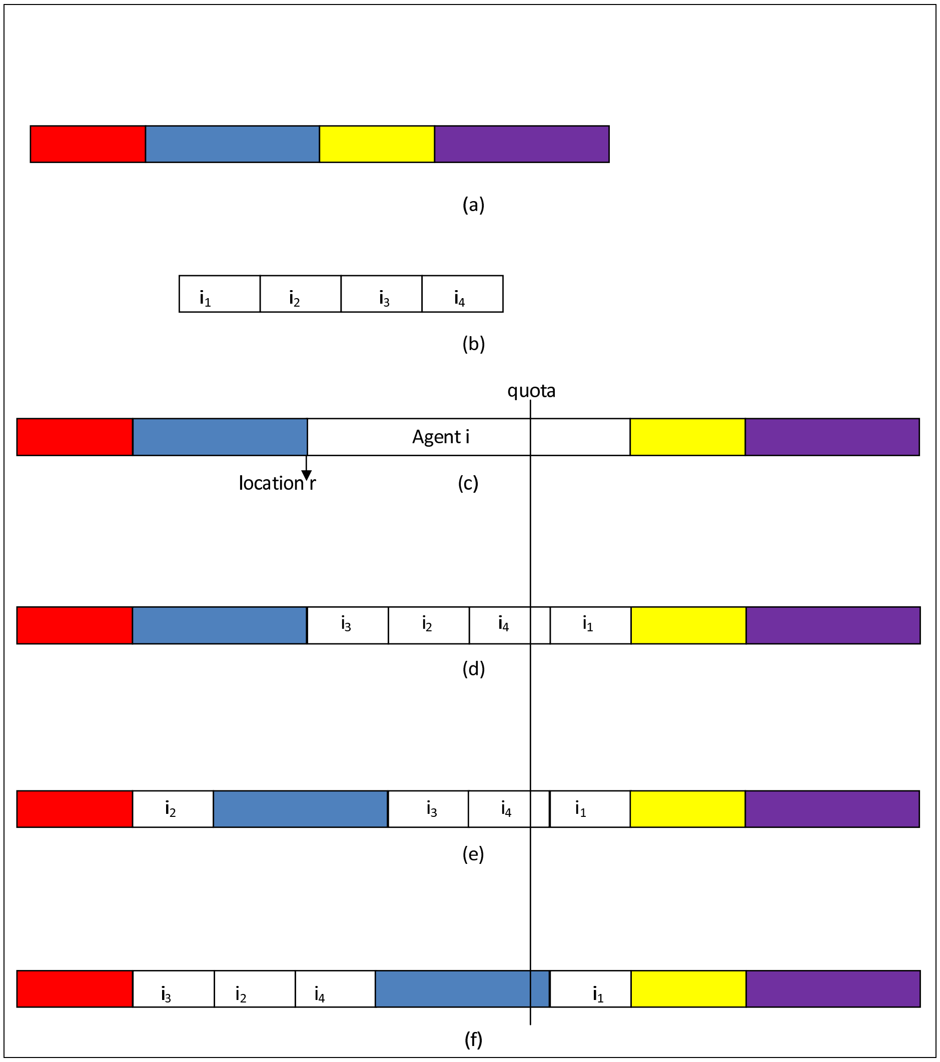

Figure 4.1. Illustration of definitions of terms for proofs of Theorems 2 and 3: (a) An i-pivotal-basis permutation with agent i removed. (b) Four false agents i_1, i_2, i_3, and i_4 that manipulating agent i splits. (c) An i-pivotal permutation with manipulating agent i pivotal starting from location r. (d) All the false agents are adjacent at location r of the i-pivotal-basis permutation with one of them pivotal. (e) The false agents are not all adjacent from location r and one of them is pivotal. (f) The false agents are not all adjacent from location r and none of them is pivotal.

Theorem 6. *(Lower Bound). Let $G = [w_1, \ldots, w_n; q]$ be a WVG of n agents. If an agent i alters G by splitting into $k \geq 2$ false agents in a new game G', then, the power index of the agent in the new game using the Shapley-Shubik index is at least $\frac{k}{C(n+k-1,k-1)}\varphi_i(G)$.*

Proof. Let agent i be a manipulator that splits into k false agents, $i_1, \ldots, i_k$, in an altered game G'. Consider any i-pivotal-basis permutation $\pi \in \Pi_{G-i}$ of agents in game G. Define r such that when agent i is inserted into π at location r, i is pivotal (see Figures 4.1 (a) and (c)). Now, consider a morphed permutation π' in game G' formed from inserting the false agents into π, in which a false agent is pivotal. While there are many permutations π' that can be morphed from π in which a false agent is pivotal, at the very least, we know that if all false agents *are adjacent* (in any order) and are inserted at location r in π, one of the false agents must be pivotal (see Figure 4.1 (d)). Thus, for each π, there are $k!$ such permutations that can be morphed from π. Notice, that we have ignored all other cases where the false agents are not adjacent in the permutation and one of the false agents is also pivotal (see Figure 4.1 (e)).

Suppose there are P_i i-pivotal permutations that can be formed from the set Π_{G-i} in game G, then, the sum of the Shapley-Shubik power of the false agents in game G',

$$\begin{aligned}
\sum_{j=1}^{k} \varphi_{i_j}(G') &\geq \frac{k! \cdot P_i}{(n+k-1)!} \\
&= \frac{k!}{(n+k-1)!} \cdot n! \cdot \frac{P_i}{n!} \\
&= \frac{k!n!}{(n+k-1)!}\varphi_i(G) \\
&= k \cdot \frac{(k-1)!n!}{(n+k-1)!}\varphi_i(G) \\
&= \frac{k}{C(n+k-1,k-1)}\varphi_i(G).
\end{aligned}$$

□

It is important to note that when the number of false agents is $k = 2$, our bounds (for the Shapley-Shubik power index) that we propose above agree with those found in [14].

4.3 Banzhaf Power Index Bound

We propose new bound for manipulation by splitting into several identities using the Banzhaf power index.

Theorem 7. *(Upper Bound). Let $G = [w_1, \ldots, w_n; q]$ be a WVG. If an agent i alters G by splitting into $k \geq 2$ false agents in a new game G', then, the Banzhaf power index of the agent in the new game can be as much as $k \cdot \beta_i(G)$.*

Proof. Let i be a manipulator that splits into k false agents, $i_1, \ldots, i_k$, with corresponding weights, $w_{i_1}, \ldots, w_{i_k}$, in a new game G'. We assume without loss of generality that $w_{i_1} \leq \cdots \leq w_{i_k}$. Define a *base coalition*, S_{G-i}, to be a set of agents from a winning coalition in a WVG G for which agent i is removed. Note that there are three possibilities for any agent to be critical in a winning coalition in game G:

1. Winning coalitions which do not contain agent i.
2. Winning coalitions which contain agent i, but in which i is not critical.
3. Winning coalitions in which agent i is critical.

Now, we need to transform each winning coalition in G to coalitions in the game G', and then count the number of critical agents in each transformed coalition.

Case 1: Let X_1 be the total number of winning coalitions in game G which do not contain agent i. Let X_2 be the average number of the critical agents in each of these winning coalitions. Since agent i is not present in any of these winning coalitions, the winning coalitions are not changed by transformation. Thus, the total number of critical agents from this case in the new game, G', is $X_1 \cdot X_2$.

Case 2: Let Y_1 be the total number of winning coalitions which contain agent i, but in which i is not critical. Let Y_2 be the average number of critical agents in each of these coalitions. To create coalitions in G', we add 1 or more of the false agents to the base coalition S_{G-i} to create a new winning coalition $S'_{G'-i}$. There are $2^k - 1$ ways of selecting 1 or more of the false agents. No false agent will be critical, since agent i was not critical in S_{G-i}, but every critical agent in S_{G-i} will still be critical in $S'_{G'-i}$. Thus, we have a total of $Y_1 \cdot Y_2 \cdot (2^k - 1)$ critical agents.

However, as we remove some of the false agents from the coalition, we could create new critical agents. For example, in the game [5,5,5;10], coalition, $\{5,5,5\}$, has no critical agents, but coalition $\{5,5,4\}$ has two critical agents where none were critical before. Let Y_3 be the average number of new critical agents created in each transformed coalition. Thus, the number of critical agents in this case can be counted as $Y_1 \cdot Y_2 \cdot (2^k - 1) + Y_1 \cdot Y_3 \cdot (2^k - 2)$.

Case 3: Let Z_1 be the total number of winning coalitions in which agent i is critical. Let Z_2 be the number of critical agents in each of these winning coalitions (not counting agent i). Note that we do not expect Z_2 to be the same for each winning coalition, but for simplicity, we assume Z_2 is the average number. To create a coalition in G', we add 1 or more of the false agents to the base coalition, S_{G-i}, to create a new winning coalition $S'_{G'-i}$. There are $2^k - 1$ ways of selecting 1 or more of the false agents. Since i was critical in the original coalition, we must add enough false agents to S_{G-i} to make $S'_{G'-i}$ winning. For example, if the sum of the weights of agents in S_{G-i} is w, and the quota of game G is q, the false agents which are added must be of a cumulative weight of at least $w - q$. We call this needed weight from the false agents, the *i-need*:

a. If the sum of the weights of false agents added is less than i-need, the coalition is losing and no critical agents will be contributed from this case.

b. If the sum of the weights of the false agents is as close to i-need without having excess false agents, the transformed coalition will be winning and every false agent will be critical. The critical agents of S_{G-i} will also be critical in $S'_{G'-i}$. For simplicity of analysis, we assume that the false agents are all of the same weight. Let p be the minimal number of false agents that are required to meet the i-need. There are $C(k,p)$ ways of selecting which false agents are present in the transformed coalition. The number of winning coalitions in which the false agents are critical is $Z_1 \cdot p \cdot C(k,p)$, since there are Z_1 base coalitions, $C(k,p)$ ways of deciding which of the p false agents to include, and all the p false agents will be critical. However, in each of these transformed coalitions, the agents which were critical in the base coalition are still critical. Thus, the total number of critical agents is: $Z_1 \cdot p \cdot C(k,p) + Z_1 \cdot Z_2 \cdot C(k,p)$.

c. If the added false agents exceed i-need, the transformed coalition will be winning but it is possible that none of the false agents is critical. The critical agents of S_{G-i} will also be critical in $S'_{G'-i}$. We must count how many false agents are critical and how many total agents are critical from this case. For simplicity, we assume that all false agents are of the same weight, so that by adding an extra false agent, none of the false agents is critical. There are $C(k,p+1)+\ldots+C(k,k) = \sum_{j=p+1}^{k} C(k,j)$ ways of selecting $p+1$ or more of the false agents. Thus, the total number of critical agents are $Z_1 \cdot Z_2 \cdot \sum_{j=p+1}^{k} C(k,j)$.

Putting it altogether, the total number of critical agents (including the k false agents):

$$\begin{aligned} X_1 \cdot X_2 &+ Y_1 \cdot Y_2 \cdot (2^k - 1) + Y_1 \cdot Y_3 \cdot (2^k - 2) + Z_1 \cdot p \cdot C(k,p) + Z_1 \cdot Z_2 \cdot C(k,p) \\ &+ Z_1 \cdot Z_2 \cdot \sum_{j=p+1}^{k} C(k,j) \\ &= X_1 \cdot X_2 + Y_1 \cdot Y_2 \cdot (2^k - 1) + Y_1 \cdot Y_3 \cdot (2^k - 2) + Z_1 \cdot p \cdot C(k,p) + Z_1 \cdot Z_2 \cdot \sum_{j=p}^{k} C(k,j). \end{aligned}$$

Now, the original Banzhaf power of agent i in game G is

$$\beta_i(G) = \frac{Z_1}{Z_1 + X_1 \cdot X_2 + Y_1 \cdot Y_2 + Z_1 \cdot Z_2}$$

.

Similarly, the new power of agent i in game G' (which is the sum of the power of the false agents) is

$$\sum_{j=1}^{k} \beta_{i_j}(G') = \frac{Z_1 \cdot p \cdot C(k,p)}{X_1 \cdot X_2 + Y_1 \cdot Y_2 \cdot (2^k - 1) + Y_1 \cdot Y_3 \cdot (2^k - 2) + Z_1 \cdot p \cdot C(k,p) + Z_1 \cdot Z_2 \cdot \sum_{j=p}^{k} C(k,j)}.$$

The ratio, $\tau = \frac{\sum_{j=1}^{k} \beta_{i_j}(G')}{\beta_i(G)}$, gives,

$$\frac{p \cdot C(k,p)(Z_1 + X_1 \cdot X_2 + Y_1 \cdot Y_2 + Z_1 \cdot Z_2)}{X_1 \cdot X_2 + Y_1 \cdot Y_2 \cdot (2^k - 1) + Y_1 \cdot Y_3 \cdot (2^k - 2) + Z_1 \cdot p \cdot C(k,p) + Z_1 \cdot Z_2 \cdot \sum_{j=p}^{k} C(k,j)}.$$

Since we want to find the highest possible ratio, we need to determine cases which maximize the ratio. Note that the ratio of increase of $power_i$ of agent i is bounded by, $\frac{1}{power_i}$, since the new power of the agent can be at most 1. We note that if p is small, $\sum_{j=p}^{k} C(k,j)$ approaches 2^k. Since a large denominator makes for a small ratio, terms in the denominator which are multiplied by 2^k without terms in the numerator being multiplied by a large number drive down the ratio. Consider the case in which $Y_1 \cdot Y_2 = 0$ and $Y_1 \cdot Y_3 = 0$. Let $p = k$. Now our ratio becomes: $\frac{k(X_1 \cdot X_2 + Z_1 + Z_1 \cdot Z_2)}{X_1 \cdot X_2 + k \cdot Z_1 + Z_1 \cdot Z_2}$ which is bounded by k. Thus, there are cases where splitting into several false identities improves the power of a manipulator by a factor of as much as k.

We next show the existence of such a case. Let $G = [w_1, w_2, \ldots, w_n; q]$ be a unanimity WVG of n agents such that $q = \sum_{i=1}^{n} w_i$. It is clear that the only winning coalition consists of all the agents. So, $\beta_i(G) = \frac{1}{n}$ for all agents $i \in I$ in the game. Suppose the last agent splits into k false identities, we have a new game $G' = [w_1, w_2, \ldots, w_{n-1}, w_{n_1}, w_{n_2}, \ldots, w_{n_k}; q]$ of $n+k-1$ agents. Then, $\beta_i(G') = \frac{1}{n+k-1}$ for each agent i in the altered game G'. The ratio of the new power to the original power of the manipulator is $\frac{nk}{n+k-1}$. So, as n goes to infinity, the denominator approaches n. Thus, the ratio goes to k.

Similarly, as $k \to \infty$, the denominator approaches k, and the ratio goes to n. Thus, we see that if the original power is $\frac{1}{n}$, then, we cannot improve by more than a factor of n. $\square$

We conclude this chapter with a summary of the updated results on the bounds for manipulation by splitting into $k \geq 2$ false identities in WVGs in Table 4.1.

Table 4.1. Summary of the bounds for the Shapley-Shubik and Banzhaf indices when an agent i in a WVG G of n agents splits into $k \geq 2$ false agents, $i_1, \ldots, i_k$, in an altered WVG G' of $n+k-1$ agents.

Bounds	Shapley-Shubik index	Banzhaf index
Upper (k = 2)	$\varphi_{i_1}(G') + \varphi_{i_2}(G') \leq \frac{2n}{n+1}\varphi_i(G)$ [35]	$\beta_{i_1}(G') + \beta_{i_2}(G') \leq 2\beta_i(G)$ [36]
Lower (k = 2)	$\varphi_{i_1}(G') + \varphi_{i_2}(G') \geq \frac{2}{n+1}\varphi_i(G)$ [35]	$\beta_{i_1}(G') + \beta_{i_2}(G') \geq \frac{1}{n}\beta_i(G)$ [14]
Upper ($k > 2$)	$\sum_{j=1}^{k} \varphi_{i_j}(G') \leq \frac{nk}{n+k-1}\varphi_i(G)$	$\sum_{j=1}^{k} \beta_{i_j}(G') \leq k\beta_i(G)$
Lower ($k > 2$)	$\sum_{j=1}^{k} \varphi_{i_j}(G') \geq \frac{k}{C(n+k-1,k-1)}\varphi_i(G)$	?

CHAPTER 5
MANIPULATION BY MERGING IN WEIGHTED VOTING GAMES

5.1 Overview

Felsenthal and Machover [37] characterize situations when it is advantageous or disadvantageous for agents to merge into a bloc, and show that using the Shapley-Shubik index, merging can be advantageous or disadvantageous. Aziz and Paterson [36] focus on the complexity of finding advantageous merging. They show that for unanimity WVGs, it is disadvantageous for a coalition to merge using the Shapley-Shubik index to compute payoff. Also, finding an optimal beneficial merge is NP-hard for both the Shapley-Shubik and Banzhaf indices [14]. It is important to note that none of these papers deal with the experimental evaluation and analysis of the type of beneficial merging that we consider here.

In this chapter, we provide examples to illustrate manipulation by merging in WVGs. Second, we consider the difficulty of predicting beneficial merges in WVGs. Third, we provide experimental investigation and analysis of manipulation by merging in WVGs using the three indices. Fourth, given that the problem of computing the Shapley-Shubik and Banzhaf power indices of agents is NP-complete [4, 21], we present an enumeration-based pseudo-polynomial time algorithm for merging that manipulators may use to find a much improved power gain. Finally, we consider evaluation of the opportunities for beneficial merging in WVGs. Here, we argue and provide empirical evidence to show that despite finding the optimal beneficial merge is an NP-hard problem for both the Shapley-Shubik and Banzhaf power indices, finding beneficial merge is relatively easy in practice.

5.2 Examples of Manipulation by Merging

We have used the Deegan-Packel index as a reference for these examples. The effects of this manipulation are summarized in a table for each example using the three power indices. The manipulating agents are all shown in bold.

Example 5. *Advantageous Merging for Deegan-Packel index.*

Let $G = [8, \mathbf{7}, 4, \mathbf{4}, \mathbf{2}, 1, 1; 28]$ be a WVG of seven agents. Let the manipulators' coalition C consists of the three agents in bold i.e., with weights 7, 4, and 2. The cumulative power of these manipulators, $\sum_{i \in C} \gamma_i(G) = 0.452$. Suppose these manipulators merge their weights to form a single bloc $\&C$ and alter G to give a new game $G' = [\mathbf{13}, 8, 4, 1, 1; 28]$. The power of the bloc is $\gamma_{\&C}(G') = \gamma_1(G') = 0.500 > 0.452$. The factor of power gained by the manipulators is $\tau = \frac{0.500}{0.452} = 1.11$.

Note that we have implicitly assumed that agents in the blocs formed are working cooperatively and have transferable utility. Thus, proceeds from merging can easily be distributed among the manipulators. For instance, in this example, each manipulator may first be assign a payoff equal to what it would get in the original game G, then, the gain (i.e., $0.500 - 0.452 = 0.048$) derived from the altered game G' can then be distributed among the members of C using different solution concepts for revenue distribution from coalitional game theory.

Table 5.1. Advantageous merging showing the cumulative power of the assimilated agents in the original game $G = [8, \mathbf{7}, 4, \mathbf{4}, \mathbf{2}, 1, 1; 28]$, the power of the bloc in the altered game $G' = [\mathbf{13}, 8, 4, 1, 1; 28]$, and the factor of increment for the three power indices.

Power Index	Original game G	Altered game G'	Factor (Increment)
Shapley-Shubik	0.488	0.667	1.37
Banzhaf	0.485	0.600	1.24
Deegan-Packel	0.452	0.500	1.11

Not all manipulation by merging is beneficial. Example 6 illustrates a non-beneficial merge for manipulators.

Example 6. *Disadvantageous Merging for Deegan-Packel index.*

Let $G = [\mathbf{9}, \mathbf{8}, 5, \mathbf{5}, 4, 3, 1; 30]$ be a WVG of seven agents. Let the manipulators' coalition $C = \{1, 2, 4\}$. The cumulative power of these manipulators, $\sum_{i \in C} \gamma_i(G) = 0.508$. Suppose these manipulators merge their weights to form a single bloc $\&C$ and alter G to give a new game $G' = [\mathbf{22}, 5, 4, 3, 1; 30]$. The power of the bloc is $\gamma_{\&C}(G') = \gamma_1(G') = 0.306 < 0.508$. The factor of power lost by the manipulators is $\tau = \frac{0.306}{0.508} = 0.60$.

Table 5.2. Disadvantageous merging showing the cumulative power of the assimilated agents in the original game $G = [\mathbf{9}, \mathbf{8}, 5, \mathbf{5}, 4, 3, 1; 30]$, the power of the bloc in the altered game $G' = [\mathbf{22}, 5, 4, 3, 1; 30]$, and the factor of decrement for the three power indices.

Power Index	Original game G	Altered game G'	Factor (Decrement)
Shapley-Shubik	0.676	0.467	0.69
Banzhaf	0.579	0.368	0.64
Deegan-Packel	0.508	0.306	0.60

We have shown that manipulators may gain or lose power using the three power indices when they engage in manipulation by merging. Similar examples where manipulators neither gain nor lose power exist for the three indices.

5.3 Difficulty of Predicting Beneficial Merge and Power Vectors

We provide visual description of manipulation by merging in WVGs to explain the intricacies of what goes on during manipulation, and examines the effects of small changes in the weights of agents on their corresponding power in WVGs using power vectors [49].

5.3.1 Difficulty of Beneficial Merge Prediction in Weighted Voting Games

A visual description clarifies manipulation by merging in WVGs (see Figure 5.1). We use the Shapley-Shubik power index for illustration. Consider a WVG of three agents denoted by the following patterns: Agent 1 (■), Agent 2 (□), and Agent 3 (▤). The weight of each agent in the game is indicated by the associated length of the pattern. A box in the pattern corresponds to a unit weight. Each row represents a permutation. Suppose all permutations of the three agents are given as shown in Figure 5.1. We can use the same figure to consider a range of quotas from 1 to 6 for the game. The Shapley-Shubik indices of the three agents

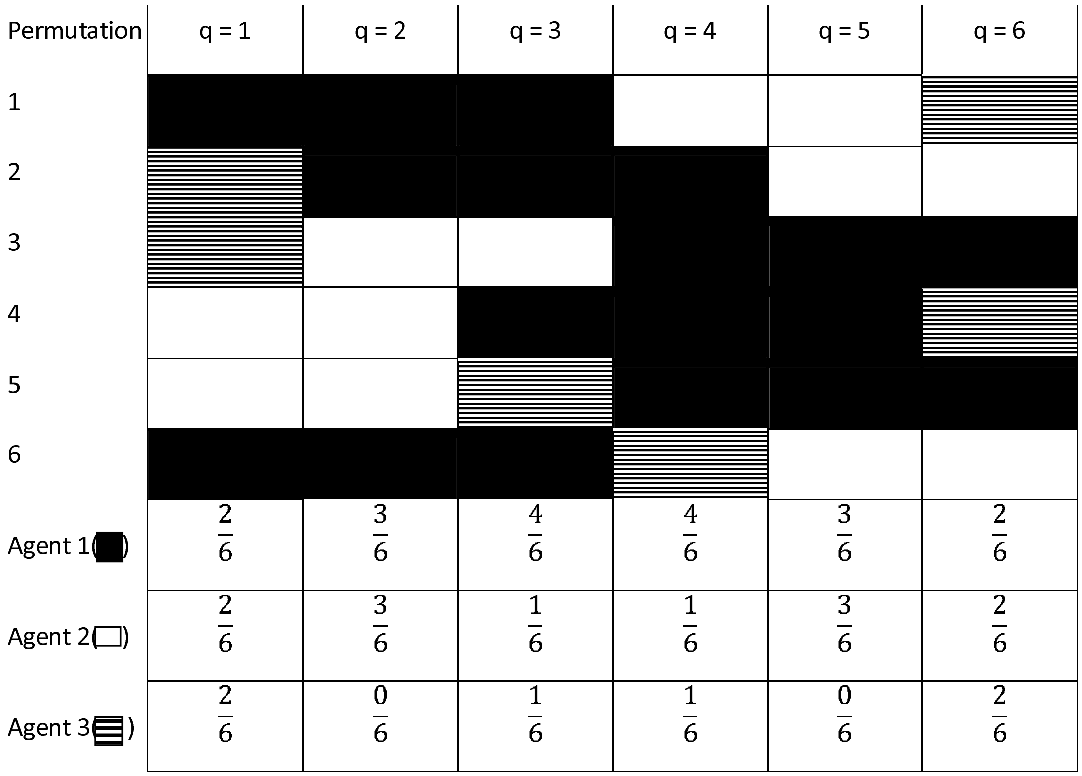

Figure 5.1. Six permutations of 3 agents and the power indices of the agents for values of quota from $q = 1$ to $q = 6$.

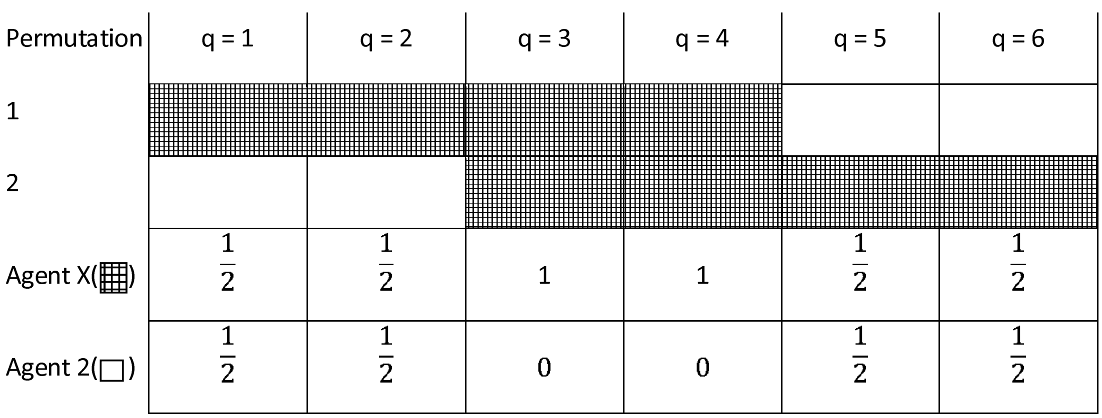

Figure 5.2. Manipulation by merging between Agent 1 and Agent 3 (from Figure 5.1) to form a new Agent X. The indices of Agent X and Agent 2 computed by Shapley-Shubik index after merging for various values of quota are also shown.

are computed from the figure and shown in the associated table of the figure. These power indices for the agents in the game correspond to using various values of the quota for the same weights of the agents.

Consider a manipulation where Agent 1 and Agent 3 merge their weights to form a new agent, Agent X. Here, Agent 1 and Agent 3 cease to exist since they have been assimilated by Agent X. Thus, we have only two agents (Agent X and Agent 2) in the altered WVG. Figure 5.2 shows the results of the merging between Agent 1 and Agent 3. Notice that the number of rows has been reduced to two, as there are now only two possible orderings. Consider the cases when the quota of the game is 1 or 6, the power of the assimilated agents for Agent X from Figure 5.1 shows that Agent 1 and Agent 3 each has a power of $\frac{1}{3}$ for a total power of $\frac{2}{3}$. The power of Agent X which assimilates these two agents in the two cases is each $\frac{1}{2} < \frac{2}{3}$. Also, the power of the manipulators stays the same for the cases where the quota is either 2 or 5. Specifically, the sum of the powers of Agent 1 and Agent 3 is $\frac{1}{2}$ for these cases. This is also true of Agent X for these cases. Finally, for the cases where the quota of the game is 3 or 4, the power of Agent X is 1 which is greater than $\frac{5}{6}$, the sum of the powers of Agent 1 and Agent 3 in the original game.

Note the difficulty of predicting what will happen when manipulators engage in merging. This illustration also shows that the choice of the quota of a game is crucial in determining the distribution of power of agents in a WVG. An apparent question that concerns the manipulators from the illustration above is the following : *Can effective merging heuristics be found even though predicting beneficial merging is difficult?*

5.3.2 Using Power Vectors

Recall the definition of power vector (from Section 2) of a WVG of n agents as an n-dimensional vector $v \in \mathbb{R}^n$ of the power of each of the agents in the game listed in order. Using power vectors, we investigate the effects of small changes in the weights of agents on their corresponding powers in WVGs. The Shapley-Shubik index is used in this illustration. The small changes in the weights of agents are related to weights changes when two or more agents merge their weights to form a bloc, thus providing some insights

(a) $q = 12$

Possible weights of the second agent

Possible weights of the first agent

	0	1	2	3	4	5	6	7	8	9	10	11	12
8									1				
9									1	1			
10								1	1	1	1		
11								1	1	1	1	1	
12							3	3	3	3	3	3	2
13							4	4	4	4	4	4	
14						4	4	4	4	4	4		
15						4	4	4	4	4	4		
16					4	4	4	4	4				
17					4	4	4	4					
18				4	4	4	4						
19				4	4	4							
20			4	4	4								
21			4	4									
22		4	4										
23		4											
24	4												

(b) $q = 16$

Possible weights of second agent

Possible weights of the first agent

	0	1	2	3	4	5	6	7	8	9	10	11	12
8									1				
9									3	2			
10								3	3	2	2		
11								3	3	2	2	2	
12							3	3	3	2	2	2	2
13							3	3	3	2	2	2	
14						3	3	3	3	2	2		
15						3	3	3	3	2	2		
16					4	4	4	4	4				
17					4	4	4	4					
18				4	4	4	4						
19				4	4	4							
20			4	4	4								
21			4	4									
22		4	4										
23		4											
24	4												

(c) q = 18

Possible weights of the second agent

Possible weights of the first agent

	0	1	2	3	4	5	6	7	8	9	10	11	12
8									1				
9									1	2			
10								1	2	2	2		
11								2	2	2	2	2	
12							3	2	2	2	2	2	2
13							3	2	2	2	2	2	
14						3	3	2	2	2	2		
15						3	3	2	2	2	2		
16					3	3	3	2	2				
17					3	3	3	2					
18				4	4	4	4						
19				4	4	4							
20			4	4	4								
21			4	4									
22		4	4										
23		4											
24	4												

Figure 5.3. Using power vectors to illustrate the effects of small changes in the weights of agents on their corresponding Shapley-Shubik powers in WVGs.

into merging. A crucial observation is that we can have many games having the same power vector. For example, the following WVGs: $[11, 9, 4; 12]$, $[11, 8, 5; 12]$, $[11, 7, 6; 12]$, $[10, 9, 5; 12]$, $[10, 8, 6; 12]$, and $[10, 7, 7; 12]$ all have the same power vector $[0.33, 0.33, 0.33]$, even though the weights' distribution of agents in the games differ.

In Figure 5.3, we consider all WVGs of 3 agents such that the total weights of the agents in each game is 24. Figures $5.3(a), 5.3(b)$, and $5.3(c)$ are for the cases when the quotas q of the games are 12, 16, and 18, respectively. The y-axes indicate the possible weights of the first agent while the x-axes indicate the possible weights of the second agents in the games. Note that since agents weights are given in non-increasing order, the possible weights for the second agent are dependent on the weights of the first agent. The possible weights of the third agents are not shown since they can implicitly be derived from the weights of the first two agents. Only 4 power vectors are attainable in the WVGs of 3 players using the Banzhaf index [49], there are also 4 different power vectors for these games when the number of agents is 3 and using the Shapley-Shubik index. These power vectors are coded as $1, 2, 3$, and 4 below:

$$
\begin{aligned}
1 &: [0.33,\ 0.33,\ 0.33] \\
2 &: [0.50,\ 0.50,\ 0.00] \\
3 &: [0.67,\ 0.17,\ 0.17] \\
4 &: [1.00,\ 0.00,\ 0.00]
\end{aligned}
$$

with the games of each power vector representing appropriate regions shaded in Figure 5.3. The following facts from the figures have impact on weight changes as it relates to merging in the games:

- The number of different power vectors is a function of the number of agents, n, in the games.
- The size of the region (associated with a particular power vector) changes with the quota.
- Some power vectors are volatile to changes with respect to small changes in weight (such as $[11, 7, 6; 18]$) while others are not (such as $[12, 11, 1; 18]$).

5.4 Susceptibility of Power Indices to Manipulation by Merging

Previous work [36], has shown that for unanimity WVGs, and for both the Shapley-Shubik and Banzhaf indices: *it is disadvantageous for a coalition to merge*. In view of this positive result (as game designers are interested in discouraging agents' participation in manipulation), and the fact that strategic agents are interested in only mergings that improve their power, we consider manipulation by merging for only the non-unanimity WVGs. In order to evaluate the behavior of the three power indices for merging in non-unanimity WVGs, we conduct experiments to analyze the effects of merging when two or more agents merge into a bloc. We first provide a description of the simulation settings and then present the empirical evidence gathered from the experiment.

5.4.1 Simulation Settings

The WVGs were generated as described in Section 2.3. For each game, we determine the three power indices of the agents in the game. Since manipulation by merging requires coordinated action of the manipulators, strategic agents are selected among the agents in the WVGs to form the blocs of manipulators. The assimilated blocs of the manipulators are formed by $k \in \mathbb{N}$ agents in each game; we refer to k as the *bloc size* or the number of assimilated agents. The blocs sizes and the members of the blocs are chosen uniformly at random[1] from the set $\{2, 3, \ldots, 10\}$, while also ensuring that $k \leq n$, where n is the number of agents in a game. We compute the new power index of the bloc in the altered games next. Then, we determine the factor of increment by which the bloc gains or loses in the manipulation for the corresponding bloc sizes. The power of the bloc is compared to the sum of the original powers of the assimilated agents in the bloc.

For our study, we generated $2,000$ original WVGs of various bloc sizes, allow manipulation by the bloc of manipulators, and compute the factors of increment over all the games. The factors of increment provides the extent of susceptibility to manipulation by

[1] We note that randomly generating members of the blocs fails to consider the benefits of a more strategic approach to manipulation. We address this issue in the next section.

each of the three indices. We estimate this relationship among the three indices by comparing their factors of increment simultaneously in similar games.

5.4.2 Empirical Results and Discussion

Experiments confirm the existence of advantageous merging for the non-unanimity WVGs. However, the extent to which agents gain varies among the indices. We provide a comparison of susceptibility to manipulation by merging among the three indices by comparing the population of factors of increment attained by strategic agents in different games for each of the indices. A summary of susceptibility to manipulation by merging among the three indices is shown in the box plot of Figure 5.4. The x-axis indicates the power indices while the y-axis is the factor of increment achieved by agents in the $2,000$ WVGs.

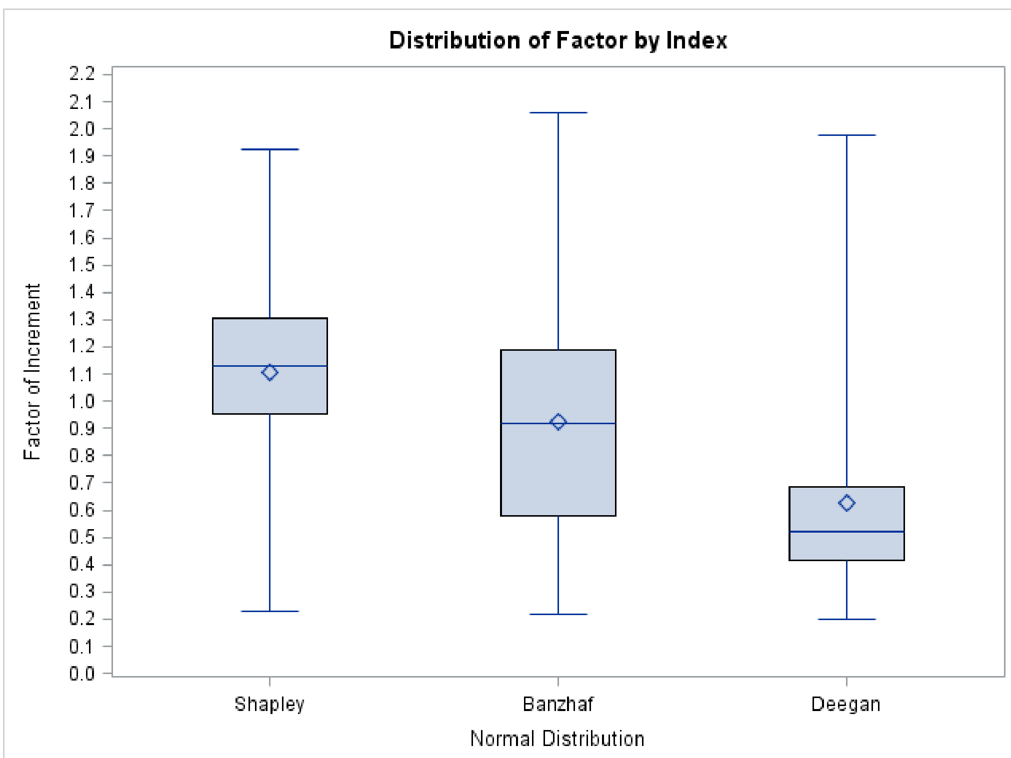

Figure 5.4. Susceptibility to manipulation by merging among the Shapley-Shubik, Banzhaf, and Deegan-Packel indices for non-unanimity weighted voting games.

From the figure, the highest gain attained by manipulation blocs for the three indices are $\sim$ 1.9 for Shapley-Shubik, $\sim$ 2.1 for Banzhaf, and $\sim$ 2.0 for the Deegan-Packel index. Since the highest factors of increment are extreme values, they are not indicative of the true behavior to compare merging in the three power indices. Let us consider the mean value of the data (which serves as a central measure) from this experiment. We note that, on average, only the Shapley-Shubik index appears to be susceptible to manipulation by merging for this type of game. The average factor of increment for the three power indices are $\sim$ 1.12 for Shapley-Shubik, $\sim$ 0.90 for Banzhaf, and $\sim$ 0.60 for the Deegan-Packel index. An important positive result that is observable from Figure 5.4 is that the highest average factor of increment for the three indices is less than a factor of 1.2, and the corresponding absolute highest value is less than a factor of 2.2. We shall see (in the next chapter) the reason for the importance of these values when we compare with the factor of increments for annexation, where similar measures are typically very high. To further see the relationship among the three indices on their susceptibility to manipulation by merging, we compare the percentages of advantageous, disadvantageous, and neutral games in the 2,000 WVGs.

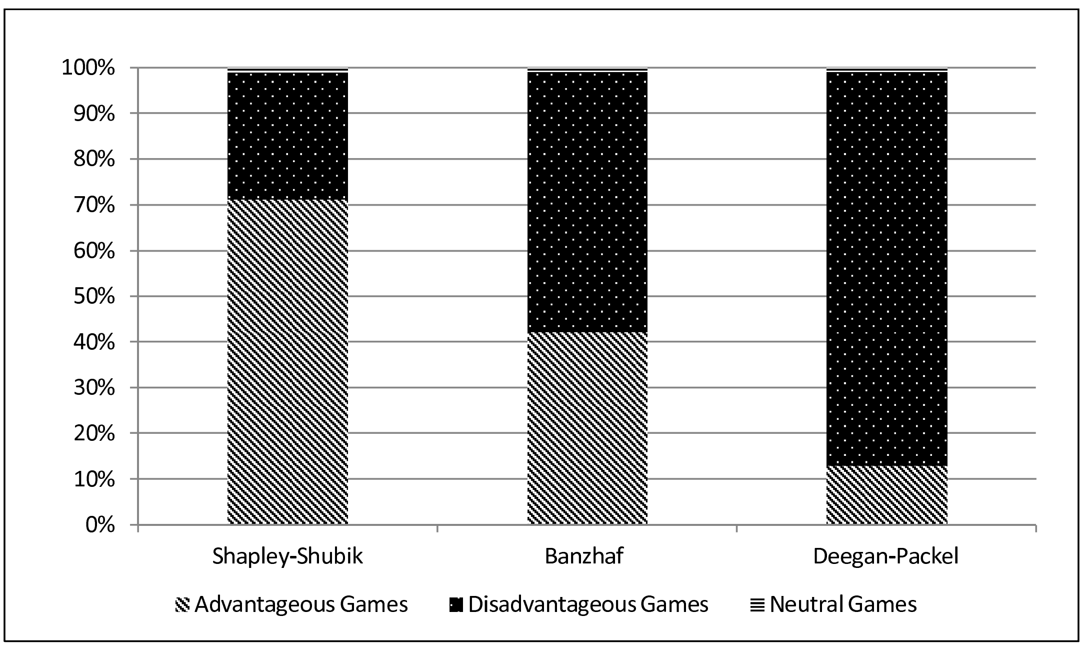

Figure 5.5. Percentage of advantageous, disadvantageous, and neutral games among the power indices for manipulation by merging in 2,000 non-unanimity weighted voting games.

Figure 5.5 shows the percentage of advantageous, disadvantageous, and neutral games for manipulation by merging among the three indices. Examination of the $2,000$ WVGs reveals that more than 71% of the games are advantageous for Shapley-Shubik index, about 42% for the Banzhaf index, and less than 14% of the games for the Deegan-Packel index. Even for the cases where the games are advantageous for the three indices, the factor of increment achieved by the blocs of manipulators are not very high, and in most cases are less than a factor of 2. This analysis suggests that the Shapley-Shubik index is more susceptible to manipulation by merging than the Banzhaf and Deegan-Packel power indices for non-unanimity WVGs, even though the factor of increment is not high. Now, since only the Shapley-Shubik index is more susceptible to manipulation by merging, and also, since the factor by which the bloc of manipulators gains is very low, this may provide less motivation for strategic agents to generally engage in manipulation by merging for non-unanimity WVGs when they are being evaluated using any of the three power indices, and in particular, when the Deegan-Packel index is employed.

We have some comments regarding these results. First, we found that our results of manipulation by merging for the non-unanimity WVGs are consistent with those of manipulation by splitting into several false identities of the previous chapter. A scenario where splitting by a strategic agent is disadvantageous corresponds to a scenario where it is advantageous for several strategic agents to merge. Second, we have assumed that the assimilated agents in the blocs can easily distribute the gains from their collusion among themselves in a fair and stable way. Thus, making them agree to engage in the manipulation if it is profitable. This assumption is strong. Even at that, we see that the three power indices are less vulnerable to manipulation by merging.

5.5 Algorithm for Merging in Weighted Voting Games

We note that randomly selecting the agents to be assimilated for merging as proposed in the previous experiment fails to consider the benefits of a more strategic approach to manipulation. We propose an enumeration-based approach for merging. In considering our enumeration-based approach to merging, we have implemented a pseudo-polynomial manipulation algorithm. Given that computing the Shapley-Shubik and Banzhaf indices is

NP-complete, and only pseudo-polynomial or approximation algorithms are available to compute agents' power, it is reasonable that the manipulation algorithm we propose is also pseudo-polynomial, since we necessarily need to use these power indices in computing agents' benefits during manipulation. Aziz et al. [14] have also shown that determining if there exists a beneficial merge is NP-hard using either the Shapley-Shubik or Banzhaf indices.

5.5.1 Manipulation Algorithm for Merging

A straightforward approach to determine a coalition that yields the most improved benefit in merging in a WVG is to simply enumerate all the possible coalitions of agents in the game and then compute the benefit for each of these coalitions. We can then output the coalition with the highest value. Unfortunately, enumerating all the possible coalitions is exponential in the number of agents. Also, computing the power indices naively from their definitions means that we have two exponential time problems to solve. We provide an alternative approach.

Let procedure *PowerIndex*(G, i) be a pseudo-polynomial algorithm for computing the power index of an agent i in a WVG G of n agents for Shapley-Shubik or Banzhaf index according to any of [4,22,23]. We first use *PowerIndex*(G, i) as a subroutine in the construction of a procedure, *GetMergeBenefit*(G, S). Procedure *GetMergeBenefit*(G, S) accepts a WVG G and a would-be manipulators' coalition, S. It first computes the sum of the individual power index of the assimilated agents in S using *PowerIndex*(G, i). Then, it alters G by replacing the sum of the weights of the assimilated agents in G with a single weight in a new game G' before computing the power of the bloc $\&S$ in G'. Finally, *GetMergeBenefit*(G, S) returns the factor of increment of the merged bloc $\&S$. Let $A(G)$ be the pseudopolynomial running time of *PowerIndex*(G, i). Now, since $|S| \leq |I| = n$, procedure *GetMergeBenefit*(G, S) takes at most $O(n \cdot A(G))$ time which is pseudopolynomial.

We now use *GetMergeBenefit*(G, S) to construct an algorithm that manipulators can use to determine a coalition that yields a good benefit in merging. We first argue that manipulators tend to prefer coalitions which are small in size because they are easier to form and less likely to be detected. Also, intra-coalition coordination, communication, and other overheads increase with coalition size. Thus, we suggest a limit on the size of the manipulators' coalitions since it is unrealistic and impractical that *all* agents in a WVG will belong to the manipulators' coalition. This reasoning is consistent with the assumptions of the previous work on merging [14] as well as coalition formation [50]. We note, however, that limiting the manipulators coalitions' size does not change the complexity class of the problem as finding the coalition that yields the most improved benefit remains NP-hard.

Consider a WVG of n agents. Suppose the manipulators' coalitions, S, have a limit, $k < n$, on the size of the coalitions, i.e., S, are bounded as $2 \leq |S| \leq k$. In this case, the number of coalitions that the manipulators need to examine is at most $O(n^k)$ which is polynomial in n. Specifically, the total number of these coalitions is:

$$\binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{k} = \sum_{j=2}^{k} \binom{n}{j}. \tag{5.1}$$

So, we have

$$\begin{aligned} \sum_{j=2}^{k} \binom{n}{j} &= \sum_{j=2}^{k} \frac{n(n-1)\cdots(n-j+1)}{j!} \\ &\leq \sum_{j=2}^{k} \frac{n^j}{j!} \\ &\leq \sum_{j=2}^{k} \frac{n^j}{2^{j-1}} \\ &= \frac{n^2}{2^1} + \frac{n^3}{2^2} + \cdots + \frac{n^k}{2^{k-1}} \\ &= O(n^k). \end{aligned}$$

Running *GetMergeBenefit*(G, S) while updating the most[2] improved benefit found so far from each of these coalitions requires a total running time of $O(n^k \cdot A(G))$ which is

[2] We refer to the most improved benefit among the $O(n^k)$ polynomial coalitions and not from the original 2^n coalitions since we have restricted each manipulators' coalition size to a constant $k < n$.

pseudopolynomial, and thus becomes reasonable to compute for small k.

5.5.2 Evaluation of the Manipulation Algorithm for Merging

It is impractical that strategic agents would employ the random simulation approach of the previous section (i.e., Subsection 5.4.1) to engage in manipulation by merging. This is because simply guessing a particular coalition from among all the exponential possible coalitions does not provide a good chance for strategic agents to benefit significantly in the manipulation. We note also that this chance of success by the manipulators decreases as the number of agents in a game becomes large. Now, since it is impractical to exhaustively consider all the exponential possible coalitions, we have presented a pseudo-polynomial manipulation algorithm where we have restricted the sizes of coalitions to be considered by the manipulators to a constant k which is less than the number n of the agents in the game. Our idea is for the manipulators to sacrifice optimality for good merging. By doing so, the manipulation algorithm potentially bypass a lot of search. Although, this manipulation algorithm is incomplete, nonetheless, it considers more search space than the random approach, and hence, is guaranteed to find a much improved factor of increment than the random simulation method.

We perform experiments to confirm the above hypothesis. First, we make a simple modification to the random simulation approach which provides manipulators with higher average factor of increment. The modification involves the selection of the best factor of increment from three random choices (which we refer to as the *best-of-three* method). We compare results of our enumeration-based method with those of the best-of-three method. Unlike the random simulation approach where merging has little or no benefits for manipulators using both the Shapley-Shubik and Banzhaf indices, results from our experiments suggest that manipulation via merging can be *highly* effective.

The WVGs for this experiment were generated as described in Section 2.3 while the simulation setting is similar to those of Section 5.4.1. We have used a total of $2,000$ original WVGs for this experiment and compute the average factor of increments over the entire set of games for both the Shapley-Shubik and Banzhaf power indices. The evaluation is carried out for the proportion of agents in manipulators' blocs in the WVGs. We compare the results of the enumeration-based method with those of the best-of-three method.

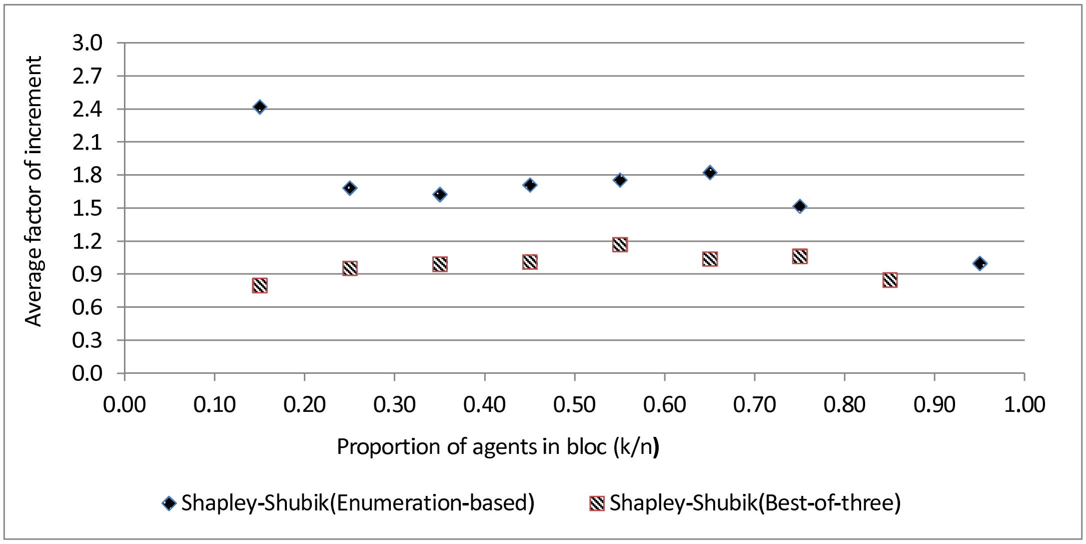

Figure 5.6. The average factor of increment for merging for the enumeration-based and best-of-three methods using various values of size of bloc divided by the number of agents in the games (Shapley-Shubik).

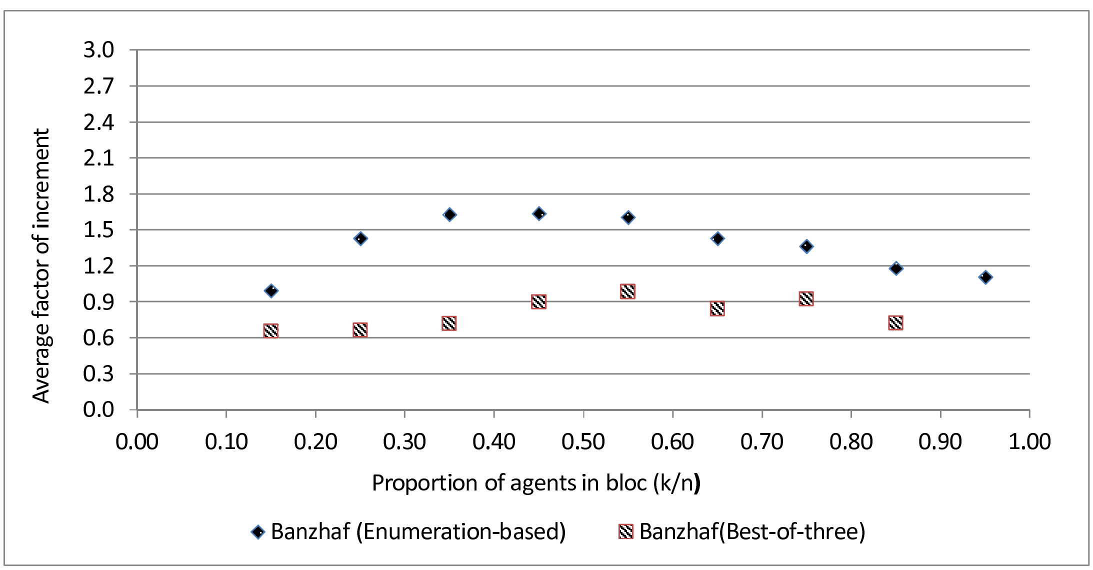

Figure 5.7. The average factor of increment for merging for the enumeration-based and best-of-three methods using various values of size of bloc divided by the number of agents in the games (Banzhaf).

Figures 5.6 and 5.7 show the benefits from merging for both the best-of-three and enumeration-based methods using the Shapley-Shubik and Banzhaf indices. The x-axes indicates the proportion of agents in the manipulators' bloc (i.e. $\frac{|S|}{|I|} = \frac{k}{n}$) whose factor of increment were reported while the y-axes are the average factor of increment achieved by manipulating agents in those coalitions. For the Shapley-Shubik power index, Figure 5.6 shows that manipulating agents achieved improved power using the enumeration-based approach than using the best-of-three method. There are cases where the manipulators achieved more than 2.4 times as much as the original power for the enumeration-based method, and in general the average factor of increment is between 1.5 and 2.7 times the original power of the manipulators. Whereas, the best-of-three method has only minor effects for the manipulators as the average factors of increment in these tests are below 1.3. Similar trends between the enumeration-based and best-of-three methods are observed for the case of Banzhaf index too (see Figure 5.7). However, the average factor of increment is lower for the two methods using the Banzhaf index. On average, merging does not appear to significantly improve power using the best-of-three method for either the Shapley-Shubik or Banzhaf index, and in most cases is harmful for the agents. Thus, finding a good manipulation is non-trivial. We conclude that since improvement in power over the best-of-three method can be achieved with only a polynomial amount of work by considering small coalition size, then, manipulators are more likely to seek a much improved power gain in merging using the enumeration-based approach.

5.6 Evaluation of Opportunities for Beneficial Merging

We conduct further experimental investigations to provide understanding and analysis of the opportunities for beneficial merging available for strategic agents in WVGs. Previous work [14] has shown that finding a beneficial merge is NP-hard for both the Shapley-Shubik and Banzhaf power indices, and leaves the impression that this is indeed so in practice. Although this worst case complexity for manipulation by merging is daunting, it is possible that real instances of WVGs are easy to manipulate. We note that real WVGs are small enough that exponential amount of work may not deter manipulators from participating in manipulation by merging. Hence, manipulations may, in some cases, be achieved in practice.

Also, even though real life WVGs may consist of small number of agents, to date, a careful investigation of effective heuristics for manipulating such games by merging are yet to be researched [36]. This, we argue, may be primarily due to the inherent difficulty of the problem. This is because the ability to find beneficial merging depends on the characteristics of the game. Some games have little opportunity for merging while others could have many beneficial merges. So, in contrast to the work of [14], we study experimental evaluation of the effects of merging using parameters of the games to analyze opportunities for beneficial merging for the manipulators. This, we believe, will provide insight into understanding of the problem and also guide the decisions of game designers.

5.6.1 Simulation Settings

The WVGs for the experiments here were generated as described in Section 2.3. However, for clarity of presentation, the number of agents, n, in each of the original WVGs is 10. We have also restricted the number of assimilated agents, k, in each game to 2. This is consistent with the assumptions of previous work on merging [14, 45] and coalition formation [50], as manipulators' blocs of small sizes are easier to form, and more importantly to the manipulators, they are less likely to be detected by other agents in the games. Apart from this, we also believe that an indepth understanding of this case (i.e., $k = 2$), will provide necessary background in understanding of the general case of when $k > 2$.

We have used a total of $2,000$ distinct WVGs for our experiments. For each game, we vary the quota of the game from $\frac{1}{2}w(I) + 1$ to $w(I)$ in steps of 10, where $w(I)$ is the sum of the weights of all agents in the game. We then compute the factor of increment for each assimilated bloc of size 2 in a game using the Shapley-Shubik and Banzhaf indices. The evaluation is carried out for the proportion of beneficial merges in a game and the quota ratio, $\frac{q}{w(I)} = \frac{\text{quota}}{\text{total weight}}$. The quota ratios for the experiments range from 0.5 to 1.0, and indicate the fraction of the total weight needed for the quotas. A quota ratio of 1.0 suggests the existence of a unanimity WVGs, where all agents in a game are needed to form a winning coalition. Thus, a winning coalition always exists in the games. In other words, the quota ratio is a measure of the percentage of weight needed to form a winning coalition.

We consider all possible manipulators' blocs of size 2. The percentage of beneficial merge for a quota ratio is the fraction of cases whose factors of increment is greater than a specified value of τ, i.e., a factor of power gain in a game. We tweak τ using different values to see how the percentage of beneficial merges varies and provide discussions of the effects noticed below. The pseudocode to compute the percentage of beneficial merges for each quota ratio is given in Figure 5.8.

```
percentBeneficialMerge(Agents I, WVG G, τ) {
for quota q of G from ½w(I)+1 to w(I) step 10

    successCount = 0;
    totalCount = 0;
    for each manipulators' bloc b of size 2
      compute factor of increment f for b

          if f > τ  then
              successCount++;

      totalCount++;
    end for
    quotaRatio = q / w(I);
   percentBenefit = successCount / totalCount;
 end for
}
```

Figure 5.8. Pseudocode to compute the percentage of beneficial merges for each quota ratio.

5.6.2 Empirical Results and Discussion

We present the results of our experiments. Figures 5.9 and 5.10 are indications of the opportunities for beneficial merging available for the manipulators. The x-axes are the quota ratios and the y-axes are the percentage of beneficial merging available to the manipulators when a beneficial merge is defined as $\tau = 1$, i.e., a factor of power gain greater than 1.

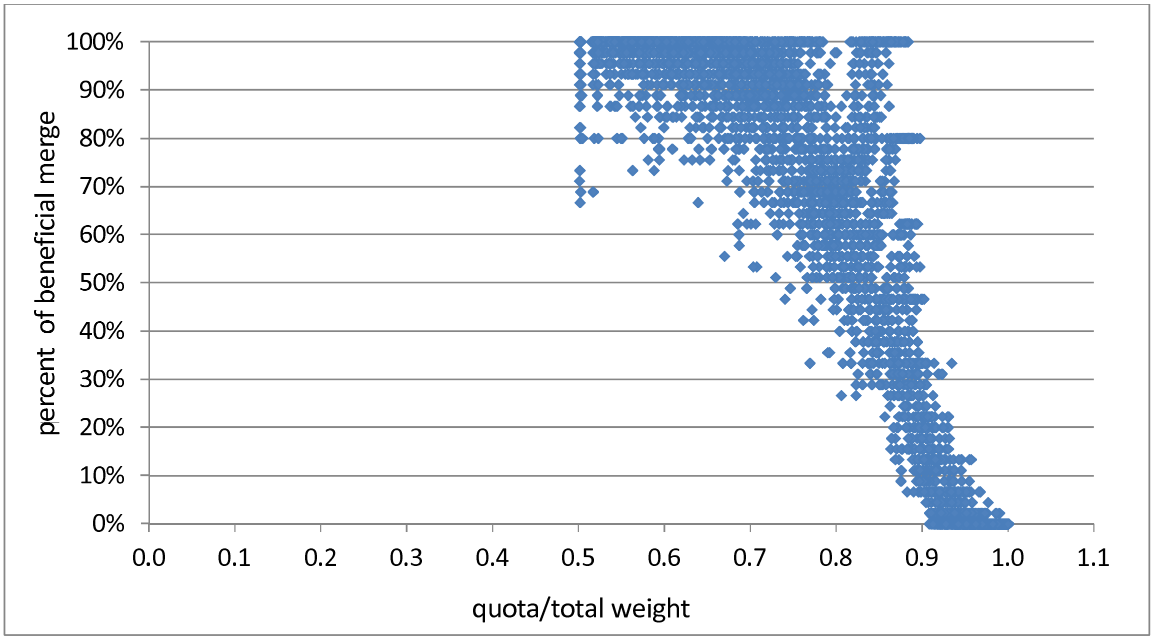

Figure 5.9. Percentage of beneficial merging for various values of quota ratio when a beneficial merge is defined to have a factor of power gain greater than 1.0 (Shapley-Shubik).

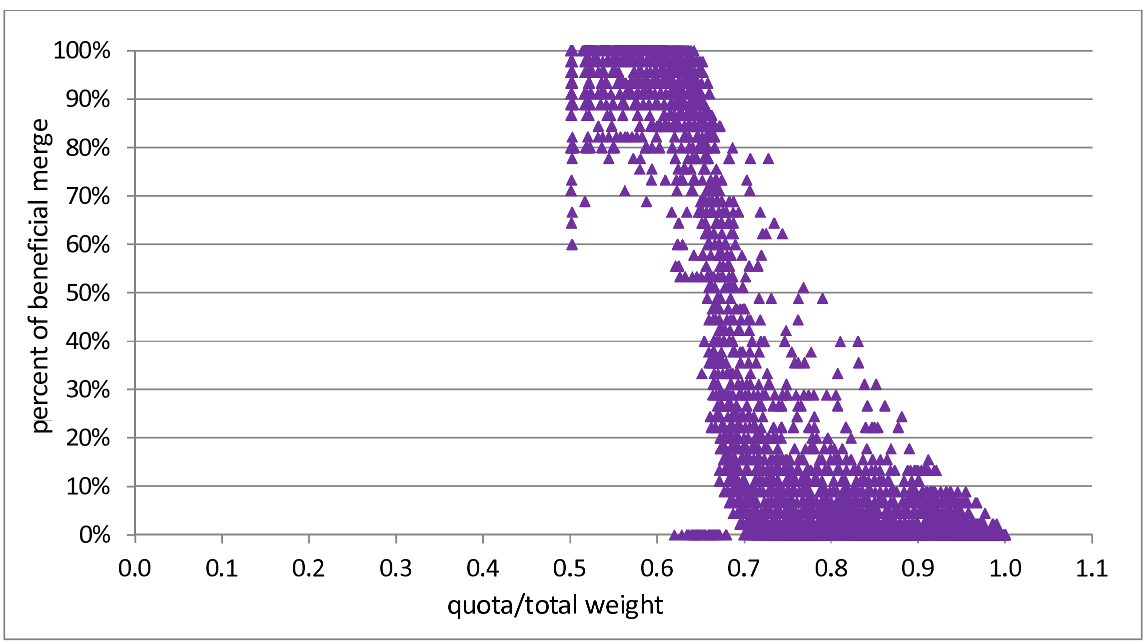

Figure 5.10. Percentage of beneficial merging for various values of quota ratio when a beneficial merge is defined to have a factor of power gain greater than 1.0 (Banzhaf index).

Figures 5.9 and 5.10 show that finding a beneficial merge is relatively easy in practice, at least for the WVGs we considered, and restricting each manipulators' blocs to size 2. In reality, finding the best merging may not even be desirable, as it assumes every agent will be willing to merge. Manipulators cannot petition every agent to see if they are willing to merge, as the manipulators would have announced their intent to cheat. However, a dishonest agent may first discover opportunities for beneficial merging before suggesting such merge to other would-be manipulators.

The figures show that beneficial mergings are easy to find and almost impossible to stop. While it appears from the figures that we may be powerless to stop merging for a given game, the game designer may be able to control the quota. Thus, a high quota ratio reduces the opportunities for dishonesty as the percentage of beneficial merges goes down. Using both the Shapley-Shubik and Banzhaf indices to compute agents' power, we can deduce from the figures that the Banzhaf index is more desirable to avoid cheating especially for high quota ratios. This is because the percentages of beneficial mergings for high values of the quota ratio using the Banzhaf index are smaller compare to those of the Shapley- Shubik index. Table 5.3 shows the means and standard deviations of the factor of power gained by manipulators from Figures 5.9 and 5.10. This shows that, on average, manipulation by merging is easier using the Shapley-Shubik index than using the Banzhaf index. This also indicates that the Banzhaf index may be more desirable to avoid manipulation in this situation.

Table 5.3. The means and standard deviations of the factor of power gained for Figures 5.9 and 5.10 using the Shapley-Shubik and Banzhaf indices.

Power Indices	Shapley-Shubik Index	Banzhaf Index
Mean	1.142	1.062
Standard deviation	0.182	0.057

For the second set of experiments, we consider a more realistic scenario for the manipulators. Even though we have defined a beneficial merge as a merge in which manipulators have a power gain with $\tau > 1$, manipulators may only be interested in beneficial merge with appreciable gains as the risks of being detected by the mechanism may exceed the

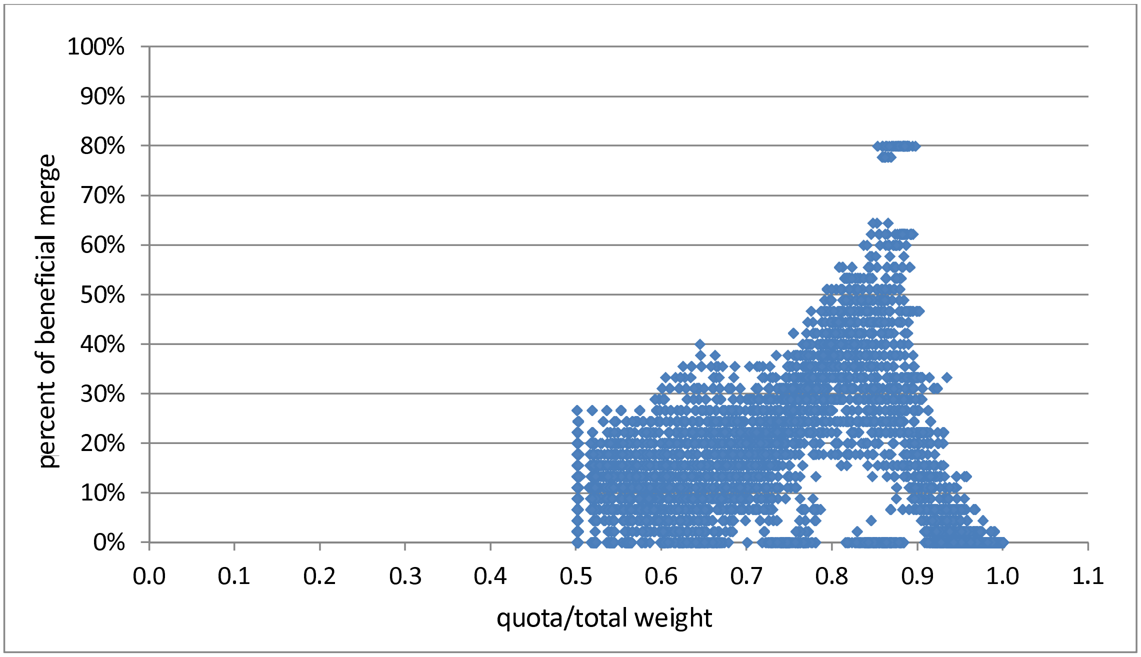

Figure 5.11. Percentage of beneficial merging for various values of quota ratio when a beneficial merge is defined to have a factor of power gain greater than 1.15 (Shapley-Shubik).

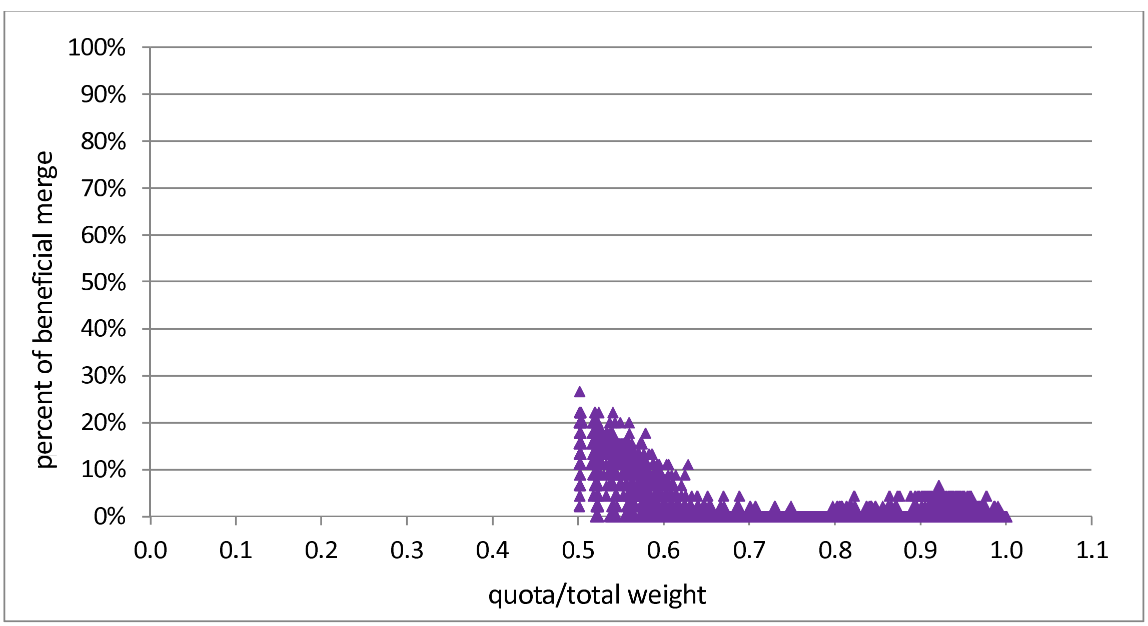

Figure 5.12. Percentage of beneficial merging for various values of quota ratio when a beneficial merge is defined to have a factor of power gain greater than 1.15 (Banzhaf index).

anticipated benefits. Now, we restrict the minimal beneficial rate to $\tau = 1.05$ or 1.10, and do not notice appreciable change in the percentage of beneficial merging compared with those of Figures 5.9 and 5.10. Thus, we do not report them here.

However, for value of $\tau = 1.15$, which represents at least a 15% anticipated increment from the original power of the manipulators, we noticed a sharp contrast from earlier results. This is an interesting and positive result for the designer of a game as it shows that the percentage of beneficial merges drops for both the Shapley-Shubik and Banzhaf power indices. See Figures 5.11 and 5.12. The opportunities for beneficial merge for the manipulators using the Shapley-Shubik index may still be high, even when the factor of power gained has been increased to $\tau = 1.15$. However, for the case of the Banzhaf index (see Figure 5.12), the maximum percentage of beneficial merge available for the manipulators is considerably less. We argue that it is not unlikely that low percentage of beneficial merge may discourage manipulators in engaging in manipulation by merging if these conditions that we describe prevail and also using the Banzhaf power index to compute agents' power.

CHAPTER 6

MANIPULATION BY ANNEXATION IN WEIGHTED VOTING GAMES

6.1 Overview

We recall the definition of annexation in Section 2, the power of the assimilated bloc in an altered WVG is compared to the power of the annexer in the original game. By this definition, intuition suggests that annexation should always be advantageous. This intuition is indeed true using the Shapley-Shubik index. However, there exists situations where annexation is disadvantageous using the Banzhaf index. This is true of the Deegan-Packel index too. See [14, 36, 37] for different examples of WVGs where annexation is disadvantageous using the Banzhaf index. Example 7 below provides an illustration of where annexation is disadvantageous for the Deegan-Packel index. This situation where annexation results in power decrease for the annexer is referred to as the *bloc paradox* [37].

Felsenthal and Machover [2, 37] originally studied annexation in WVGs. They consider when the blocs formed by annexation are advantageous or disadvantageous. Also, Aziz et al. [14] consider the computational aspects of the problem of annexation in WVGs. They show that finding optimal beneficial annexation in a WVG is NP-hard using the Banzhaf index. Our work differs from those of these authors as we consider the extent to which the agents involved in annexation may gain using the three power indices.

In this chapter, we provide examples to illustrate manipulation by annexation. Second, we investigate the susceptibility of the three indices to annexation. Third, we present an enumeration-based pseudo-polynomial algorithm for annexation that manipulators may use to find a much improved power gain. Finally, we propose and evaluate annexation heuristics. These heuristics require little computational efforts and provide good information for annexers to make decisions on how to annex in WVGs.

6.2 Examples of Manipulation by Annexation

We have used the Deegan-Packel index as a reference for these examples. The effects of this manipulation are summarized in a table for each example using the three power indices. The annexer as well as other manipulating agents are shown in bold.

Example 7. *Advantageous Annexation for Deegan-Packel index.*

Let $G = [8, \mathbf{5}, \mathbf{4}, 4, 3, 3, 2; 18]$ be a WVG of seven agents. Let agent 2 (with weight 5) be an annexer. The power of this agent, $\gamma_2(G) = 0.172$. Suppose the agent annexes agent 3 with weight 4. An assimilated bloc of weight 9 is formed in the new game $G' = [\mathbf{9}, 8, 4, 3, 3, 2; 18]$. The power index of the bloc formed by the annexer, $\gamma_1(G') = 0.260 > \gamma_2(G)$. The agent gains from the annexation and increases its power index by a factor, $\tau = \frac{0.260}{0.172} = 1.51$, of the original index.

Table 6.1. Advantageous annexation showing the annexer's power in the original game $G = [8, \mathbf{5}, \mathbf{4}, 4, 3, 3, 2; 18]$, the altered game $G' = [\mathbf{9}, 8, 4, 3, 3, 2; 18]$, and the factor of increment for the three power indices.

Power Index	Original game G	Altered game G'	Factor (Increment)
Shapley-Shubik	0.171	0.350	2.04
Banzhaf	0.171	0.340	1.99
Deegan-Packel	0.172	0.260	1.51

Example 8. *Disadvantageous Annexation*[1] *for Deegan-Packel index.*

Let $G = [9, 9, 9, \mathbf{8}, 7, 5, \mathbf{3}; 29]$ be a WVG of seven agents. Let agent 4 (with weight 8) be an annexer. The power of this agent, $\gamma_4(G) = 0.171$. Suppose the agent annexes agent 7 with weight 3. An assimilated bloc of weight 11 is formed in the new game $G' = [\mathbf{11}, 9, 9, 9, 7, 5; 29]$. The power index of the bloc formed by the annexer, $\gamma_1(G') = 0.159 < \gamma_4(G)$. Even though the weight of the annexer increases (from 8 to 11), the annexer loses from the annexation and decreases its power index by a factor, $\tau = \frac{0.159}{0.171} = 0.93$, of the original index.

[1] This is an example of the bloc paradox for the Deegan-Packel power index.

Table 6.2. Disdvantageous annexation showing the annexer's power in the original game $G = [9, 9, 9, \mathbf{8}, 7, 5, \mathbf{3}; 29]$, the altered game $G' = [\mathbf{11}, 9, 9, 9, 7, 5; 29]$, and the factor of increment/decrement for the three power indices.

Power Index	Original game G	Altered game G'	Factor (Increment/Decrement)
Shapley-Shubik	0.179	0.217	1.21
Banzhaf	0.177	0.217	1.22
Deegan-Packel	0.171	0.159	0.93

6.3 Susceptibility of Power Indices to Manipulation by Annexation

We assume that only one of the agents is engaging in the annexation.

6.3.1 Unanimity Weighted Voting Games

Aziz and Paterson [36] have shown that for unanimity WVGs in both the Shapley-Shubik and Banzhaf indices: *it is advantageous for a player to annex other players*. We observe that this result naturally extends to the Deegan-Packel index too. This is because for unanimity WVGs, the definitions of the Shapley-Shubik, Banzhaf, and the Deegan-Packel indices using Formulas 2.1, 2.2, and 2.3, respectively, are equivalent. In fact, the power of all agents in any unanimity WVGs is the same for the three indices.

Note that Aziz and Paterson have not considered the bounds on the extent to which strategic agents may gain with respect to games they manipulate. This is important as it provides motivation for strategic agents to engage in manipulation when derivable gains are appreciable. The gains (i.e., the factor of increments) show the extent of susceptibility to manipulation among the power indices. The magnitude of this gain for unanimity WVGs, as we shall see shortly, depends on the number of agents in the original game, the number of agents the annexer is able to annex, as well as some compensation that is made to the assimilated agents to forfeit their weights. Example 9 illustrates a unanimity WVG where an annexer achieves three times its original power annexing some other agents in a game.

Example 9. *Advantageous Annexation for Unanimity Weighted Voting Games*

Consider $G = [9, \mathbf{9}, 8, \mathbf{8}, \mathbf{7}, 6, \mathbf{6}, \mathbf{5}, 4, \mathbf{3}, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}; 73]$, a unanimity WVG of 15 agents $I = \{1, 2, \ldots, 15\}$ in order. The Deegan-Packel index of any agent in the game is $\frac{1}{15} = 0.0667$.

Suppose the second agent with weight 9, annexes the 10-subset, $C = \{4, 5, 7, 8, 10, 11, 12, 13, 14, 15\}$, of agents in the game (i.e., the agents in bold). The new game $G' = [\mathbf{46}, 9, 8, 6, 4, ; 73]$. The annexer has improve its weight to 46. The Deegan-Packel index of the annexer in G' is $\gamma_1(G') = \frac{1}{5} = 0.2000$. The agent benefits from the annexation and increases its power by a factor of 3.

Table 6.3. Annexation in unanimity WVGs showing the annexer's power in the original game $G = [9, \mathbf{9}, 8, \mathbf{8}, \mathbf{7}, 6, \mathbf{6}, \mathbf{5}, 4, 4, \mathbf{3}, \mathbf{2}, \mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}; 73]$, the altered game $G' = [\mathbf{46}, 9, 8, 6, 4, ; 73]$, and the factor of increment for the three power indices.

Power Index	Original game G	Altered game G'	Factor (Increment)
Shapley-Shubik	0.067	0.200	3.00
Banzhaf	0.067	0.200	3.00
Deegan-Packel	0.067	0.200	3.00

It appears that the annexer has achieved a gain of three times its original power while annexing some other agents, but this is not true in its entirety. We provide the following arguments. Since the original and the altered games are unanimity, the power of all agents in each game is the same. While the annexer has improved its weight, and consequently its power by three times its original power, other agents that were not assimilated have also had their power increased by the same factor, even though their weights in the original and altered games remain the same. Clearly, these agents do not incur any compensation that is made to the assimilated agents to forfeit their weights like the annexer, whose improved weight and power must have been achieved at some compensation. The compensation made by the annexer reduces the benefits the agent thought it gained, making the annexer's benefit worse than the benefits of other agents not engaging in annexation. This weakens Aziz and Paterson's [36] result that for unanimity WVG and for both the Shapley-Shubik and Banzhaf indices it is advantageous for a player to annex.

Now, suppose we assume that the annexer still accrues some gains even after the application of some compensation, then these gains are the same for the three indices. We see that the extent of susceptibility to manipulation among the three indices are the same. Hence, for any unanimity WVGs, the manipulability of any one index does not dominate the manipulability of other indices.

Finally, the generalization of the upper bound on the extent to which a strategic agent may gain with respect to games it manipulate in any unanimity WVGs follows from [36]. For any unanimity WVG of n agents, the power of each agent is $\frac{1}{n}$. If a strategic agent annexes $k-1$ other agents, the power of the strategic agent as well as that of the other agents in the new game is $\frac{1}{n-k+1}$. Hence, the factor of increment for each agent is $\frac{n}{n-k+1}$. This factor of increment is the same for the three indices. When $k = 1$, (i.e., the strategic agent is not annexing any other agent), then the factor of increment is 1. Whereas, when $k = n$, the strategic agent is able to annex the remaining $n-1$ agents in the original game, then the factor of increment is n times the power of the agent in the original game. This is the upper bound on the extent to which a strategic agent may achieve while annexing other agents in any unanimity WVG. This bound holds for the three power indices.

6.3.2 Empirical Results for Non-Unanimity Weighted Voting Games

We conduct experiments to evaluate the effects of manipulation when an agent annexes some other agents in non-unanimity WVGs. The simulation settings for the experiments in this section are similar to the previous settings described in Section 5.4. We consider annexation of at least one agent in the game by the annexer, while the weights of other agents not annexed remain the same in the altered games. For a particular game, the annexer may annex $1 \leq k \leq 10$ other agents, k is the *bloc size* or the number of assimilated agents.

We present the results of our simulations. Experiments confirm the existence of advantageous annexation for non-unanimity WVGs when agents engage in manipulation using the three indices. We provide a comparison of susceptibility to manipulation by annexation among the three indices by comparing the population of factors of increment attained by strategic agents in different games for each of the indices. A summary of susceptibility to manipulation by annexation among the three power indices for $2,000$ WVGs is shown in Figure 6.1. The x-axis indicates the bloc sizes while the y-axis is the average factor of increment achieved by agents in the $2,000$ WVGs for corresponding bloc sizes.

The effect of manipulation by annexation is pronounced for the three power indices, as all the indices are highly susceptible to manipulation. However, the higher susceptibility

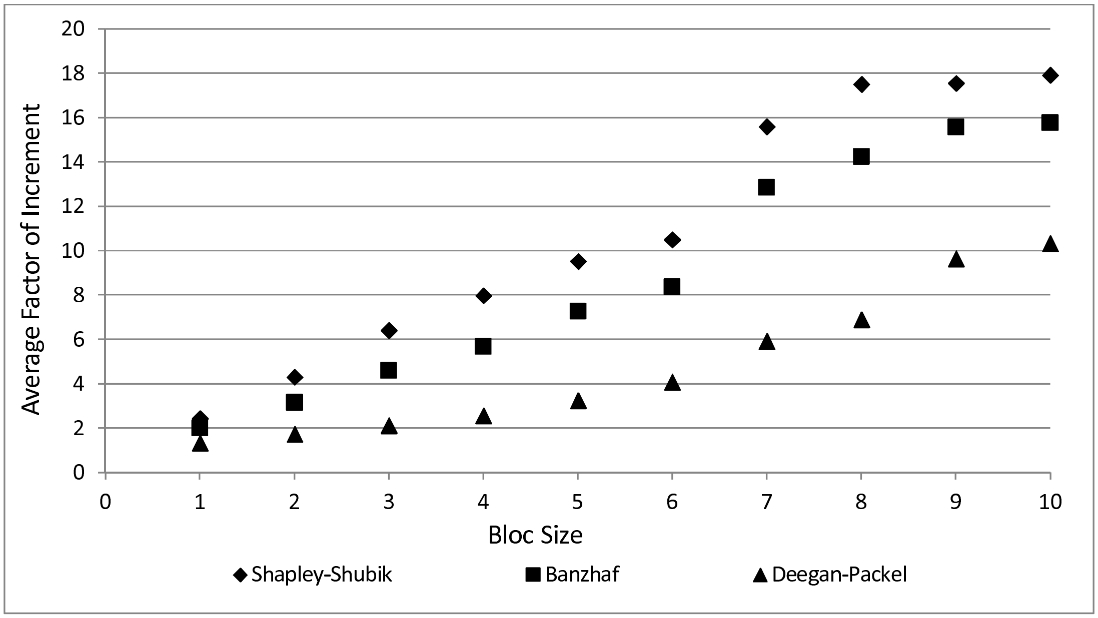

Figure 6.1. Susceptibility to manipulation by annexation among the Shapley-Shubik, Banzhaf, and Deegan-Packel indices for non-unanimity weighted voting games.

of the Shapley-Shubik and Banzhaf indices than the Deegan-Packel index can be observed from Figure 6.1. While the average factor of increment for manipulation rapidly grows with the bloc sizes for the Shapley-Shubik and Banzhaf indices, that of the Deegan-Packel index grows more slowly. Furthermore, there appears to be a positive correlation between the average factor of increment and the bloc sizes for the three indices. The figure shows that the average factor of increment increases with the bloc sizes.

This analysis suggests that the Shapley-Shubik and Banzhaf power indices are more susceptible to manipulation via annexation than the Deegan-Packel power index. Since all the three power indices are susceptible to manipulation via annexation, this may provide some motivation for strategic agents to generally engage in such manipulation for non-unanimity WVGs when they are being evaluated using any of the three power indices, and in particular, when the Shapley-Shubik index is employed.

6.4 Algorithm and Heuristics for Annexation in Weighted Voting Games

We present manipulation algorithm and heuristics that annexers may use to engage in annexation in WVGs.

6.4.1 Manipulation Algorithm for Annexation

Our pseudo-polynomial manipulation algorithm for annexation provides a modification of the manipulation algorithm for merging of Section 5.5. Recall that G is a WVG of n agents and S is a would-be manipulators' coalition. As before, let $A(G)$ be the pseudo-polynomial running time of $PowerIndex(G, i)$, which computes the power index of an agent i in a WVG G of n agents for Shapley-Shubik or Banzhaf index according to any of [4,22,23]. We first replace $GetMergeBenefit(G, S)$ with another procedure called, $GetAnnexationBenefit(G, i, S)$. $GetAnnexationBenefit(G, i, S)$ accepts a WVG G, an annexer, i, and a coalition S to be assimilated by i. The procedure then returns the factor of increment of the assimilated bloc $\&(S \cup \{i\})$.

We use $GetAnnexationBenefit(G, i, S)$ to construct an algorithm that the annexer can use to determine the coalition that yields the most improved benefit in annexation. The method of construction of the algorithm is the same as that of the manipulation algorithm for merging with the exception that we add the weight of an annexer i to the weight of each coalition S and compare the power index $\Phi_{\&(S \cup \{i\})}(G')$ of the assimilated bloc in a new game G' to the power index $\Phi_i(G)$ of the annexer in the original game G. The annexer examines a polynomial number of coalitions of the agents assuming a limit $k < n$ on the size of each coalition. Since the annexer belongs to a coalition it annexes, the total number of coalitions examined by the annexer is:

$$\binom{n-1}{1} + \cdots + \binom{n-1}{k-1} = \sum_{j=1}^{k-1} \binom{n-1}{j}. \tag{6.1}$$

Bounding this equation using similar approach as in Equation 5.1 shows that Equation 6.1 is $O(n^k)$. Thus, as before, the manipulation algorithm for annexation also runs in pseudo-polynomial time, with a total running time of $O(n^k \cdot A(G))$.

6.4.2 Manipulation Heuristics for Annexation

Unlike the manipulation algorithm for annexation above where we have n and k as the number of agents and the number of assimilated agents in a WVG, manipulating agents may not be interested in achieving the most improved power gain among the $O(n^k)$ polynomial coalitions described before. This is because the number of these coalitions may be large even for small values of n and k. Thus, we propose heuristics that agents may use for annexation in WVGs. Considering the basic requirements for a good heuristic as its ease of computation and to be as informative as possible [51], the heuristics we propose are designed to first avoid the enumeration-based approach of the manipulation algorithm above. Second, we avoid the simple random approach of Section 6.3.2 to ensure that our heuristics provide good information for manipulating agents to make decisions on how to annex in WVGs.

Recall from Equation 6.1 that an annexer needs to examine a polynomial number of assimilated coalitions of size at most $k-1$ to find the most improved power gain among these coalitions whose sizes we have restricted to k. We note also that the assimilated coalitions with maximal weights among these coalitions are those of sizes $k-1$. Thus, it is enough for a particular annexer to check only the assimilated coalitions of size exactly $k-1$ in order for the annexer to find the coalition with the most improved benefit using the two power indices. This indeed is the case for the Shapley-Shubik index. For the case of Banzhaf index, we conduct a test to check the highest factor of increment among all the coalitions of $2,000$ different WVGs. We found that the coalitions with maximal weights (i.e., those having sizes of $k-1$) yield the highest possible factor of increment in over 82% of the games. The remaining highest factor of increment are archieved by manipulators' blocs with lower number of agents in them. In none of these two situations do we experience the bloc paradox.

Now, there are only $\begin{pmatrix} n-1 \\ k-1 \end{pmatrix}$ such assimilated coalitions to be considered by this annexer. As seen, the amount of work carried out by the annexer is still polynomial, however, this heuristic requires smaller computational effort compared to that of the enumeration-based method and thus makes it more useful in practice. We can even do better using the idea in the following heuristic:

MaximalWeights heuristic:

Given: A WVG of n agents with a distinguished annexer i.

Procedure: Since agents' weights are given in non-increasing order, let agent i annex the first $k-1$ agents from the remaining $n-1$ agents in the game.

Again, let $A(G)$ be the pseudopolynomial running time of Shapley-Shubik or Banzhaf power index of an agent according to [22, 23]. The running time of the *MaximalWeights* heuristic is $O(n + A(G))$. Here is a brief analysis. We just need to sum the weights of the next $k-1$ agents from the remaining $n-1$ agents to that of the annexer at a cost of $O(k)$. In the worst case, it takes $O(n)$ to sum the weights when $k = n$, i.e., the annexer is able to annex all the remaining $n-1$ agents. Computing the power of the annexer in each of the original and altered games takes $O(A(G))$. Therefore, the total running time of the heuristic is $O(n + A(G))$. Now, apart from the fact that this heuristic appears to find the most improved gain, its running time is by far less than the $O(n^k \cdot A(G))$ running time of the manipulation algorithm for annexation of Subsection 6.4.1 and the above heuristic especially when k is large. Thus, the *MaximalWeights* heuristic is more useful in practice for the annexer. The benefits achievable by manipulating agents using these heuristics compare with those of the enumeration-based method which serves as upper bound.

CHAPTER 7
CONCLUSIONS

7.1 Summary

We investigate the effects of the following forms of manipulation - *splitting*, *merging*, and *annexation*, in weighted voting games. We consider three prominent power indices, *Shapley-Shubik, Banzhaf,* and *Deegan-Packel* indices, that are used in evaluating agents' power in such games. Our focus is on the characterization of the extent to which strategic agents may gain engaging in such manipulations, and showing how susceptibility to manipulation among the three indices compares for unanimity and non-unanimity weighted voting games.

We started with manipulation when an agent splits into several false identities. We extend a previous result on exact unanimity weighted voting games to the Deegan-Packel index and present new results for excess unanimity WVGs. Specifically, we propose new bounds for the Shapley-Shubik and Banzhaf power indices on the extent of gains a manipulator may achieve when the original game is an excess unanimity weighted voting game and the resulting game after splitting is a non-unanimity weighted voting game. We illustrate the susceptibility of the three power indices to manipulation by splitting through simulations of a large number of weighted voting games with a manipulating agent in each of the games. Results from our experiments show that the three indices are susceptible to manipulation when an agent splits into several false identities. However, the Deegan-Packel index is more susceptible than the Shapley-Shubik and Banzhaf indices, with the Shapley-Shubik being the least susceptible. Hence, using Shapley-Shubik index to evaluate weighted voting games reduces agents' motivation towards manipulation by splitting. Second, since our experimental results have suggested ideas on the extent to which each of the indices are susceptible to manipulation by splitting, we went further to partially resolve an open problem concerning the bounds on the extent of power that a manipulator may gain when it splits into $k > 2$ false identities. We provide the first three non-trivial bounds for this problem using the Shapley-Shubik and Banzhaf indices. One of the bounds is also shown to be asymptotically tight. The analyses of these novel results not only increase our understanding on the extent of power that manipulators may gain while they engage in manipulation by splitting in WVGs, they also provide further insights into the problem which we believe may reveal methods on how to reduce the effects of the menace in the future.

Furthermore, we consider the effects of manipulation by merging and annexation. We provide visual illustrations of merging in weighted voting games to give some insights into why it is difficult to predict how to merge. Given that the problems of calculating the Shapley-Shubik and Banzhaf indices for weighted voting games are NP-complete, we show that, when the manipulators' coalitions sizes are restricted to a small constant, manipulators need to do only a polynomial amount of work to find a much improved power gain for both merging and annexation, and then present two enumeration-based pseudo-polynomial algorithms that manipulators can use. Finally, we examine the effects of small changes in the weights of agents on their corresponding powers in weighted voting games. This is illustrated by showing that power vectors are often unchanged. Then, we argue and provide empirical evidence to show that despite finding the optimal beneficial merge is an NP-hard problem for both the Shapley-Shubik and Banzhaf power indices, finding beneficial merge is relatively easy in practice. Hence, there may be little deterrent to manipulation by merging in practice using the NP-hardness results. Also, while it appears that we may be powerless to stop manipulation by merging for a given game, we suggest a measure, termed *quota ratio*, that the game designer may be able to control. Thus, we deduce that a high quota ratio decreases the number of beneficial merges. Using the Shapley-Shubik and Banzhaf indices to compute agents' power, we conclude that the Banzhaf index may be more desirable to avoid manipulation by merging, especially for high values of quota ratios.

7.2 Future Work

There are several areas of ongoing research on these problems. Here are some directions for future work. It will be interesting to provide a generalized upper bound for Theorem 5 on manipulation by splitting into several false identities. Also, the lower bound when a manipulator splits into $k > 2$ false agents using the Banzhaf power index is still open. Furthermore, since our experimental results have suggested ideas on the extent of susceptibility for manipulation by annexation and merging, other future work we plan to consider are the bounds on manipulation by annexation and merging in weighted voting games. We also seek to expand our experimental evaluations on the opportunities for beneficial merging to understand the general case of manipulators' blocs of size greater than 2. Again, we seek to provide careful investigations of effective heuristics for manipulation by merging in weighted voting games. Finally, there are still several other interesting open problems on false-name manipulation in weighted voting games from [14].

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APPENDICES

Appendix A

ELECTORAL COLLEGE'S WEIGHTS OF THE UNITED STATES

Table A.1. 2012 Electoral College's Weights of the United States.

s/n	States	Weights
1	California	55
2	Texas	38
3	Florida	29
4	New York	29
5	Illinois	20
6	Pennsylvania	20
7	Ohio	18
8	Georgia	16
9	Michigan	16
10	North Carolina	15
11	New Jersey	14
12	Virginia	13
13	Washington	12
14	Arizona	11
15	Indiana	11
16	Massachusetts	11
17	Tennessee	11
18	Maryland	10
19	Minnesota	10
20	Missouri	10
21	Wisconsin	10
22	Alabama	9
23	Colorado	9
24	South Carolina	9
25	Kentucky	8
26	Louisiana	8
27	Connecticut	7
28	Oklahoma	7
29	Oregon	7
30	Arkansas	6
31	Iowa	6
32	Kansas	6
33	Mississippi	6
34	Nevada	6
35	Utah	6
36	Nebraska	5
37	New Mexico	5
38	West Virginia	5
39	Hawaii	4

Table A.2. 2012 Electoral College's Weights of the United States.

s/n	States	Weights
40	Idaho	4
41	Maine	4
42	New Hampshire	4
43	Rhode Island	4
44	Alaska	3
45	Delaware	3
46	D.C.	3
47	Montana	3
48	North Dakota	3
49	South Dakota	3
50	Vermont	3
51	Wyoming	3

Appendix B

RESULTS OF NORMALITY CHECK ON THE 2012 INTERNATIONAL MONETARY FUNDS DATA

The Executive Board of the IMF at present has 24 executive directors, and is chaired by a Managing Director in a non-voting capacity. The member countries and thier voting power are listed in pages $93-99$. The $*$ in front of a country name indicates the chair for a multi-country constituencies. The actual weight of each country or group is computed using the country voting power as a percentage of the total voting power of all the countries or groups. For example, the voting weight of Saudi Arabia is $\frac{70{,}592}{2{,}515{,}703} \times 100 = 2.8061$ and that of the United States is $\frac{421{,}961}{2{,}515{,}703} \times 100 = 16.7731$. Similarly, the voting weight of the group consisting of Denmark*, Estonia, Finland, Iceland, Latvia, Lithuania, Norway, and Sweden is $\frac{85{,}615}{2{,}515{,}703} \times 100 = 3.4032$. A complete list of the countries and their computed corresponding voting weights used in this dissertation is shown in Table B.1.

INTERNATIONAL MONETARY FUND

COMPOSITION OF THE EXECUTIVE BOARD

NOVEMBER 1, 2012

	Constituency Member(s) (* indicates chair for multi-country constituencies)	Voting Power
Ahmed Abdulkarim ALKHOLIFEY	Saudi Arabia	70,592
		70,592
Benny ANDERSEN	Denmark *	19,651
	Estonia	1,676
	Finland	13,375
	Iceland	1,913
	Latvia	2,158
	Lithuania	2,576
	Norway	19,574
	Sweden	24,692
		85,615
Kossi ASSIMAIDOU	Benin	1,356
	Burkina Faso	1,339
	Cameroon	2,594
	Central African Republic	1,294
	Chad	1,403
	Comoros	826
	Congo, Democratic Republic of the	6,067
	Congo, Republic of	1,583
	Côte d'Ivoire	3,989
	Djibouti	896
	Equatorial Guinea	1,260
	Gabon	2,280
	Guinea	1,808
	Mali	1,670
	Mauritania	1,381

	Mauritius	1,753
	Niger	1,395
	Rwanda	1,538
	São Tomé and Príncipe	811
	Senegal	2,355
	Togo *	1,471
		39,069
Der Jiun CHIA	Brunei Darussalam	2,889
	Cambodia	1,612
	Fiji, Republic of	1,440
	Indonesia	21,530
	Lao People's Democratic Republic	1,266
	Malaysia	18,476
	Myanmar	3,321
	Nepal	1,450
	Philippines	10,930
	Singapore *	14,817
	Thailand	15,142
	Tonga	806
	Vietnam	5,344
		99,023
Ambroise FAYOLLE	France	108,122
		108,122
Pablo GARCÍA-SILVA	Argentina	21,908
	Bolivia	2,452
	Chile *	9,298
	Paraguay	1,736
	Peru	7,121
	Uruguay	3,802
		46,317
Alexander GIBBS	United Kingdom	108,122
		108,122

Thomas A. HOCKIN	Antigua and Barbuda	872
	Bahamas, The	2,040
	Barbados	1,412
	Belize	925
	Canada *	64,429
	Dominica	819
	Grenada	854
	Ireland	13,313
	Jamaica	3,472
	St. Kitts and Nevis	826
	St. Lucia	890
	St. Vincent and the Grenadines	820
		90,672
Meg LUNDSAGER	United States	421,961
		421,961
Rakesh MOHAN	Bangladesh	6,070
	Bhutan	800
	India *	58,952
	Sri Lanka	4,871
		70,693
Mohammad Jafar MOJARRAD	Afghanistan	2,356
	Algeria	13,284
	Ghana	4,427
	Iran, Islamic Republic of *	15,709
	Morocco	6,619
	Pakistan	11,074
	Tunisia	3,602
		57,071
Daikichi MOMMA	Japan	157,022
		157,022

Andrea MONTANINO	Albania	1,337
	Greece	11,755
	Italy *	79,560
	Malta	1,757
	Portugal	11,034
	San Marino	961
		106,404
Aleksei V. MOZHIN	Russia	60,191
		60,191
Paulo NOGUEIRA BATISTA, JR.	Brazil *	43,242
	Cape Verde	833
	Dominican Republic	2,926
	Ecuador	4,215
	Guyana	1,646
	Haiti	1,556
	Nicaragua	2,037
	Panama	2,803
	Suriname	1,658
	Timor-Leste	819
	Trinidad and Tobago	4,093
		65,828
Johann PRADER	Austria *	21,876
	Belarus	4,601
	Czech Republic	10,759
	Hungary	11,121
	Kosovo	1,327
	Slovak Republic	5,012
	Slovenia	3,487
	Turkey	15,295
		73,478
José ROJAS RAMIREZ	Colombia	8,477
	Costa Rica	2,378
	El Salvador	2,450

	Guatemala	2,839
	Honduras	2,032
	Mexico	36,994
	Spain	40,971
	Venezuela, Républica Bolíviriana de *	27,328
		123,469
Momodou Bamba SAHO	Angola	3,600
	Botswana	1,615
	Burundi	1,507
	Eritrea	896
	Ethiopia	2,074
	Gambia, The *	1,048
	Kenya	3,451
	Lesotho	1,086
	Liberia	2,029
	Malawi	1,431
	Mozambique	1,873
	Namibia	2,102
	Nigeria	18,269
	Sierra Leone	1,774
	South Africa	19,422
	South Sudan	1,967
	Sudan	2,434
	Swaziland	1,244
	Tanzania	2,726
	Uganda	2,542
	Zambia	5,628
	Zimbabwe	4,271
		82,989
A. Shakour SHAALAN	Bahrain	2,087
	Egypt *	10,174
	Iraq	12,621
	Jordan	2,442
	Kuwait	14,548
	Lebanon	3,401
	Libya	11,974
	Maldives	837
	Oman	3,107

	Qatar	3,763
	Syrian Arab Republic	3,673
	United Arab Emirates	8,262
	Yemen	3,172
		80,061
Menno SNEL	Armenia	1,657
	Belgium	46,789
	Bosnia & Herzegovina	2,428
	Bulgaria	7,139
	Croatia	4,388
	Cyprus	2,319
	Georgia	2,240
	Israel	11,348
	Luxembourg	4,924
	Macedonia, FYR	1,426
	Moldova	1,969
	Montenegro	1,012
	Netherlands *	52,361
	Romania	11,039
	Ukraine	14,457
		165,496
Hubert TEMMEYER	Germany	146,392
		146,392
René WEBER	Azerbaijan	2,346
	Kazakhstan	4,394
	Kyrgyz Republic	1,625
	Poland	17,621
	Serbia	5,414
	Switzerland *	35,322
	Tajikistan	1,607
	Turkmenistan	1,489
		69,818
Jong-Won YOON	Australia	33,101
	Kiribati	793

	Korea, Republic of *	34,401
	Marshall Islands	772
	Micronesia, Federated States of	788
	Mongolia	1,248
	New Zealand	9,683
	Palau	768
	Papua New Guinea	2,053
	Samoa	853
	Seychelles	846
	Solomon Islands	841
	Tuvalu	755
	Uzbekistan	3,493
	Vanuatu	907
		91,302
Tao ZHANG	China	95,996
		95,996

Table B.1. Weights of the International Monetary Fund members

	Countries	Weights
1	Saudi Arabia	2.8061
2	Denmark	3.4032
3	Togo	1.5530
4	Singapore	3.9362
5	France	4.2979
6	Chile	1.8411
7	United Kingdom	4.2979
8	Canada	3.6042
9	United States	16.7731
10	India	2.8101
11	Iran, Islamic Republic of	2.2686
12	Japan	6.2417
13	Italy	4.2296
14	Russia	2.3926
15	Brazil	2.6167
16	Austria	2.9208
17	Venezuela, Republica Boliviriana de	4.9079
18	Gambia, The	3.2988
19	Egypt	3.1825
20	Netherlands	6.5785
21	Germany	5.8191
22	Switzerland	2.7753
23	Korea, Republic of	3.6293
24	China	3.8159

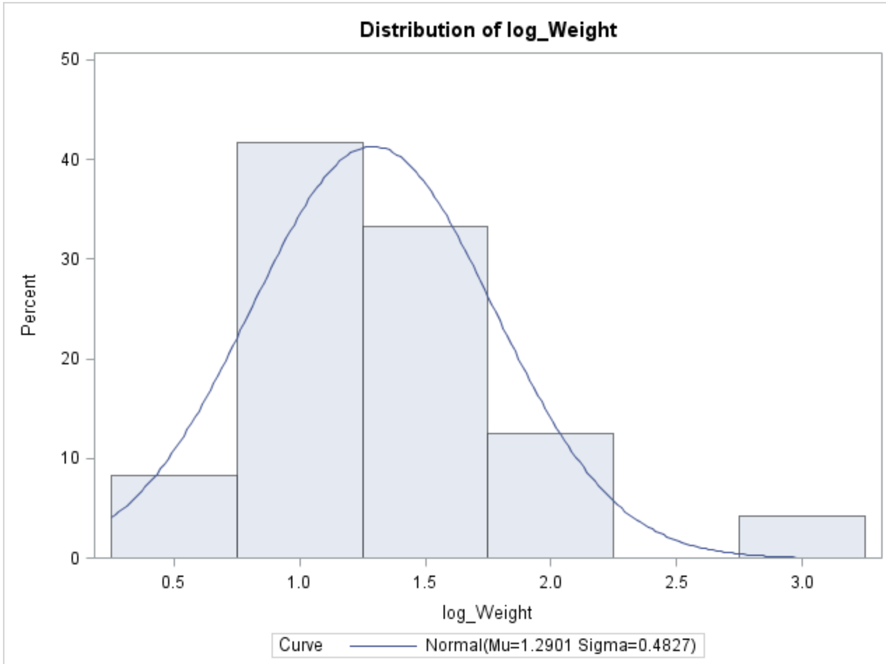

Figure B.1. The histogram with normal curve for the International Monetary Funds weights.

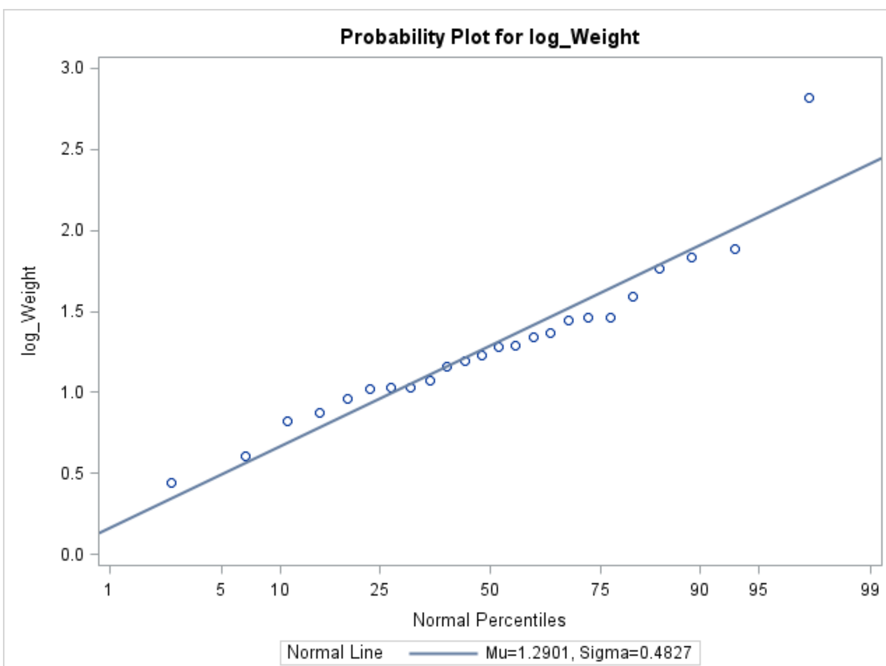

Figure B.2. The probability plot for the International Monetary Funds weights.

Appendix C

RESULTS OF NORMALITY CHECK ON THE EUROPEAN UNION DATA

Table C.1. Voting weights of the European Union members

	Countries	Weights
1	Germany	29
2	Italy	29
3	United Kingdom	29
4	France	29
5	Spain	27
6	Poland	27
7	Romania	14
8	Netherlands	13
9	Portugal	12
10	Hungary	12
11	Czech Rep	12
12	Belgium	12
13	Greece	12
14	Bulgaria	10
15	Sweden	10
16	Austria	10
17	Slovakia	7
18	Denmark	7
19	Finland	7
20	Ireland	7
21	Lithuania	7
22	Cyprus	4
23	Latvia	4
24	Slovenia	4
25	Estonia	4
26	Luxembourg	4
27	Malta	3
	Total	345

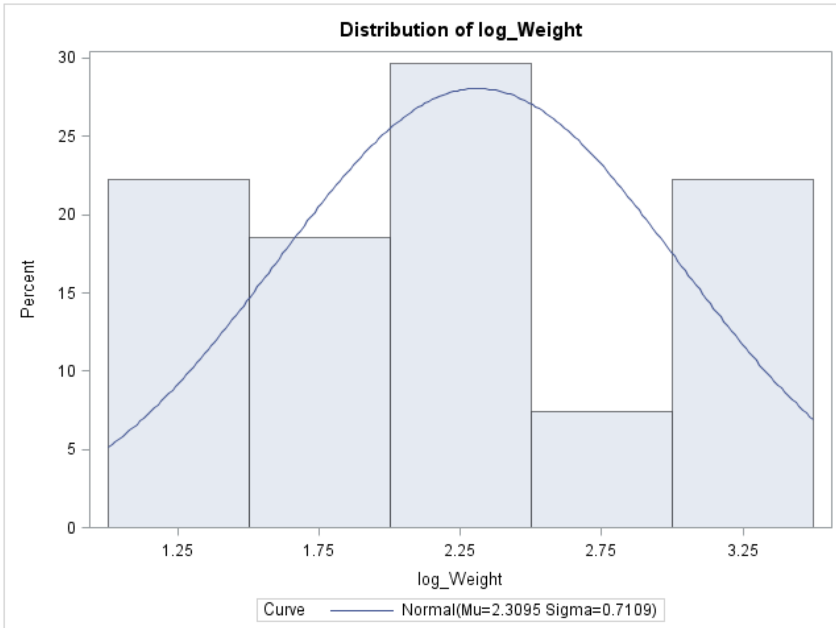

Figure C.1. The histogram with normal curve for the European Union Members' weights.

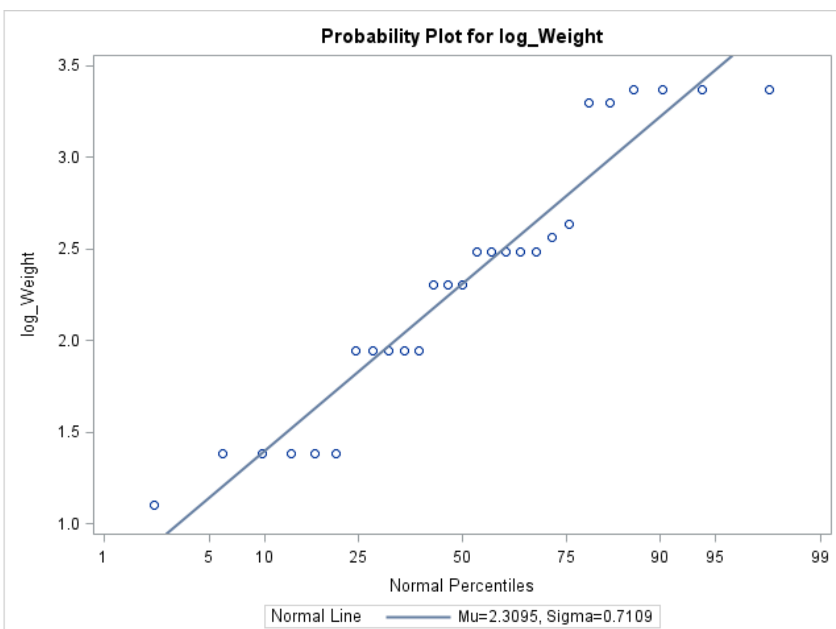

Figure C.2. The probability plot for the European Union Members' weights.

Appendix D

PERMISSIONS

All permissions relating to the inclusion of published articles in this dissertation are presented in subsequent pages.

April 5, 2013

Department of Computer Science

Utah State University, USA

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The Secretariat,

International Conference on Agents and Artificial Intelligence (ICAART)

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2910-595 Setúbal - Portugal

To the Editors, Proceedings of ICAART 2011, 2012, and 2013:

I am preparing my dissertation in the department of Computer Science at Utah State University. I hope to complete my degree in the summer of 2013. The following articles, of which I am the first author, and which appeared respectively in the proceedings of ICAART 2011, 2012, and 2013, report essential part of my dissertation research:

1. **Ramoni O. Lasisi** and Vicki H. Allan. Annexations and Merging in Weighted Voting Games: The Extent of Susceptibility of Power Indices. In proceedings of the 3rd International Conference on Agents and Artificial Intelligence (ICAART 2011), Italy, Jan. 28--30, 2011, pp. 124--133.

2. **Ramoni O. Lasisi** and Vicki H. Allan. A Search-based Approach to Annexation and Merging in Weighted Voting Games. In proceedings of the 4th International Conference on Agents and Artificial Intelligence (ICAART 2012), Portugal, Feb. 6--8, 2012, pp. 44--53.

3. **Ramoni O. Lasisi** and Vicki H. Allan. Experimental Evaluation of the Effects of Manipulation by Merging in Weighted Voting Games. In proceedings of the 5th International Conference on Agents and Artificial Intelligence (ICAART 2013), Spain, Feb. 15--18, 2013, pp. 196--203.

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I am preparing my dissertation in the department of Computer Science at Utah State University. I hope to complete my degree in the summer of 2013. The following articles, of which I am the first author, and which appeared respectively in the Book Chapters of Agents and Artificial Intelligence, report essential part of my dissertation research:

1. **Ramoni O. Lasisi** and Vicki H. Allan. Manipulation of Weighted Voting Games and the Effect of Quota. Agents and Artificial Intelligence. Communications in Computer and Information Science Volume 271, 2013, pp 413-428.

2. **Ramoni O. Lasisi** and Vicki H. Allan. Manipulation of Weighted Voting Games via Annexation and Merging. Agents and Artificial Intelligence. Communications in Computer and Information Science Volume 358, 2013, 364--378.

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2. **Ramoni O. Lasisi** and Vicki H. Allan. Manipulation of Weighted Voting Games via Annexation and Merging. Agents and Artificial Intelligence. Communications in Computer and Information Science Volume 358, 2013, 364--378.

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Ramoni Lasisi,

Department of Computer Science

Utah State University, USA

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Ramoni O. Lasisi

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---

RESEARCH INTERESTS

My primary research interests lie on issues at the intersection of computer science (artificial intelligence, multiagent systems, and algorithms), and economics (game theory, mechanism design, and social & economic networks). My secondary research interests are in bionformatics and computational biology, including RNA secondary structures predictions.

EDUCATION

July 2013 **Ph.D., Computer Science**, Utah State University, USA
Dissertation: Experimental Analysis of the Effects of Manipulations in Weighted Voting Games
Dissertation Advisor: Dr. Vicki H. Allan

Jan. 2008 **M.Sc., Computer Science**, University of Lagos, Nigeria

May 2002 **B.Sc., Mathematical Sciences**, University of Agric., Abeokuta, Nigeria

PROFESSIONAL APPOINTMENTS

2012 - Present **Programmer**
Utah Water Research Laboratory, Utah State University.
Re-writing the Ground Control Station software of the Unmanned Area Vehicles (UAVs) for the AggieAir project of the Utah Water Research Laboratory. Supporting the research efforts of the AggieAir Project.

2009 - 2013 **Graduate Research Assistant**

Multiagent Systems Research Group, Dept. of Computer Science, Utah State University.

Conducted research on the investigation of the computational and combinatorial aspects of cooperative game theory, including *coalition formation, computational social choice*, and *multiagent resource allocation*. Wrote several programs, designed, and ran experiments to support research efforts in the group. Mentored undergraduate students in the research group.

2008 - 2009 **Graduate Research Assistant**

Computational Geometry & Bioinformatics Algorithms Lab., Dept. of Computer Science, Utah State University.

Conducted research in theoretical computer science. Specifically involved in the design and analysis of efficient exact and approximation algorithms for bioinformatics and computational biology problems involving RNA secondary structures prediction with pseudoknots. Wrote several programs, designed, and ran experiments to support research efforts in the lab.

2006 - 2008 **Head of Technology**

IQID Identification Solutions, Lagos, Nigeria.

Led a team involved in the development and maintenance of a cheque issuer confirmation system, *SecureCheque*. The system eliminates the inherent problems in the manual cheque processing by applying standard biometric and bar code technology to the issuer identity and cheque detail verification process. I also supported the *Secure Time & Attendance* application of the firm as requested by users/customers.

2004 - 2006 **Senior Software Engineer**

IQID Identification Solutions, Lagos, Nigeria.

I was a member of a team involved in the design, implementation, and support of the Lagos State Government's Lands Bureau e-registry using Livelink ECM from Open Text Corporation. I was also a member of a team that developed a biometric time and attendance application used to track attendance details of employees using fingerprints.

Achievement: I wrote more than half of the stored procedures for the projects in MS SQL and later converted to PL/SQL scripts; jointly wrote and continously update the functional, design, and program specifications/documentations for stored procedures with another team member; and successfully led the deployment of the applications at ten clients' sites and resolve post implementation issues, including bug fixes, trainings, and specialized customization requests.

2003 - 2004 **Software Engineer**

Computer Systems Associates (CSA), Lagos, Nigeria.

I was a member of a team involved in the maintenance of the retail core module of a banking application (Equinox Banking System).

Achievement: Traced and fixed bugs reported by clients and successfully implement many of the customization requests for the retail core module of the application to meet banks specifications by writing stored procedures, designing and coding of front ends to achieve the customizations.

TEACHING

Teaching Assistant

Spring 2012 CS 6100 - Multiagent Systems, taught by Dr. Vicki Allan, USU.

Laboratory Tutor

Spring 2012 CS 1400–Introduction to Computer Science 1– CS1 (using C++), USU.

CS 1410–Introduction to Computer Science 2– CS2 (using C++), USU.

CS 2420 –Algorithms and Data Structures–CS3 (using C++), USU.

Guest lectures

Spring 2011 CS 6100 - Multiagent Systems, taught by Dr. Vicki Allan, USU.

USU 1320–Lang. & Identity, taught by Dr. Karin Jonge-Kannan, USU.

Teaching Workshop

Summer 2011 Completed the International Teaching Assistants' Workshop Pre-Fall 2011 with a recommendation for a **Teaching Assistantship without limitation**. Evaluation of the workshop is based on participants comprehensibility in a teaching role using the following factors: pronunciation, fluency, organization, and classroom interaction.

Other Teaching

2002 -2003: **Center Head**, NIIT Education & Training Centre, Abeokuta, Nigeria. I was the head of the computer training center. I engaged in IT training/teaching, including the Futurz, eTechnology, and Oracle 9i curriculum of NIIT. Specifically, I trained students in Programming Approaches and Techniques, Object Oriented Concepts, C/C++, Java, SQL Server 2000, Oracle 9i SQL and PL/SQL.

REFEREED PUBLICATIONS

1. Ramoni O. Lasisi and Vicki H. Allan. False-Name Manipulation in Weighted Voting Games - Empirical and Theoretical Analysis (Submitted).

2. Ramoni O. Lasisi and Vicki H. Allan. Experimental Evaluation of the Effects of Manipulation by Merging in Weighted Voting Games. In 5th Intl. Conf. on Agents and Artificial Intelligence (ICAART 2013), Feb. 15–18, 2013, pp. 196–203. (22% acceptance rate for short papers).

3. Ramoni O. Lasisi and Vicki H. Allan. A Search-based Approach to Annexation and Merging in Weighted Voting Games. In 4th Intl. Conference on Agents and Artificial Intelligence (ICAART 2012), Portugal, Feb. 6–8, 2012, pp. 44–53. (14% acceptance rate for full papers). **Best student paper award nominee.**

4. Ramoni O. Lasisi and Vicki H. Allan. Annexations and Merging in Weighted Voting Games: The Extent of Susceptibility of Power Indices. In proceedings of the 3rd International Conference on Agents and Artificial Intelligence (ICAART 2011), Italy, Jan. 28–30, 2011, pp. 124–133. (9% acceptance rate for full papers).

5. Ramoni O. Lasisi and Vicki H. Allan. False Name Manipulations in Weighted Voting Games: Susceptibility of Power indices. In 13th Workshop on Trust in Agents Societies of AAMAS Conference, Canada, pp. 130–150, May 9–14, 2010.

6. Minghui Jiang, Pedro J. Tejada, Ramoni O. Lasisi, Shanhong Cheng, and D. Scott Fechser. K-partite RNA Secondary Structures. Journal of Computational Biology, 17: 915–925, 2010.

7. Minghui Jiang, Pedro J. Tejada, Ramoni O. Lasisi, Shanhong Cheng, and D. Scott Fechser. K-partite RNA Secondary Structures. In proc. of the 9th Workshop on Algorithms in Bioinformatics, volume 5724 of Lecture Notes in Bioinformatics, pp. 157–168, Springer, Sept. 12–13, 2009.

INVITED BOOK CHAPTERS

8. Ramoni O. Lasisi and Vicki H. Allan. Manipulation of Weighted Voting Games via Annexation and Merging. Agents and Artificial Intelligence. Communications in Computer and Information Science, Vol. 358, 2013, pp. 364–378.

9. Ramoni O. Lasisi and Vicki H. Allan. Manipulation of Weighted Voting Games and the Effect of Quota. Agents and Artificial Intelligence. Communications in Computer and Information Sc., Vol. 271, 2013, pp. 413-428.

OTHER PUBLICATION

10. Ramoni O. Lasisi. A Research Note on the Shapley Values of *Single Large Party* Voting Games.

AWARDS AND HONORS

- 2013 –Utah State University research and projects grant proposal award.
- 2002 – Best graduating student in the department of Mathematical Sciences (all options), Federal University of Agriculture, Abeokuta, Nigeria.
- 2001 – Federal Government of Nigeria's Scholarship Award.