5-1964

Simplified Approach to the Problem of Stability of Soil Slopes Under Horizontal Earthquake and Pore Pressure

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SIMPLIFIED APPROACH TO THE PROBLEM OF STABILITY OF SOIL SLOPES UNDER HORIZONTAL EARTHQUAKE AND PORE PRESSURE

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Civil Engineering

Approved:

Thesis Director

Major Professor

Head of Department

Dean of Graduate Studies

UTAH STATE UNIVERSITY
Logan, Utah

1964

ACKNOWLEDGMENTS

The present study is carried out in the field of soil mechanics under the direction of Dr. D. F. Peterson, Dean of the College of Engineering, Dr. Reynold K. Watkins, Head of the Mechanical Engineering Department, Professor Fred W. Kiefer and Dr. Irving S. Dunn of the Civil Engineering Department at the Utah State University.

The author wishes to express his gratitude to them for their assistance and many valuable suggestions. Appreciation is also expressed to other members of the committee and to the other staff members of the Engineering College for their assistance and cooperation.

Dalim K. Majumdar

TABLE OF CONTENTS

LIST OF SYMBOLS

1. Angle of internal friction	ϕ
2. Acceleration due to gravity	g
3. Arc length	$\widehat{L}$
4. Buoyant unit weight of soil	γ_b
5. Construction pore pressure factor	ϵ
6. Chord length	$\bar{L}$
7. Earthquake acceleration coefficient	α
8. Equivalent unit weight	γ_E
9. Friction angle modified due to earthquake (horizontal)	ϕ_{mE}
10. Friction angle modified due to pore pressure	ϕ_{mp}
11. Force due to cohesion	C
12. Factor of safety	F
13. Horizontal earthquake force	F_E
14. Initial angle of internal friction without earthquake or pore pressure	ϕ_i
15. Moment arm ratio	ℓ/d
16. Saturated unit weight of soil	γ_s
17. Specific gravity of soil	G
18. Slope of embankment	i
19. Sudden draw-down factor	β
20. Seepage factor	δ
21. Stability number	$c/F\gamma H$
22. Unit cohesion	c
23. Unit weight of soil at any moisture content	γ_t

24. Unit weight of soil . . . . γ

25. Unit weight of water . . . . γ_w

26. Void ratio . . . . e

27. Weight of sliding mass . . . . W

LIST OF FIGURES

LIST OF TABLES

CHAPTER I

INTRODUCTION

A problem that has confronted engineers for a great number of years is that of determining the stability of soil slopes. Many factors introduce complications into stability analysis. Most embankments contain heterogeneous soils, often of several types. This and other complications usually necessitate the use of a simplified cross section. At the same time, it is often necessary to adopt simplified average soil characteristics which represent actual characteristics as best as possible. Those steps which bear on the choice of the simplified section and the simplified soil characteristics are always important in stability analysis work. These steps are independent, however, of the actual analysis. From this point on, simplified conditions only are considered, since the main object herein is to explain methods of analysis once the simplifications are made. In this analytical approach the following simplified conditions and assumptions are carried out:

1. An average or typical cross section is used. It is assumed that no shearing stresses act on the plane of the section and, therefore, that a two-dimensional case exists. The entire mass is assumed to be composed of one type of uniform soil.
2. It is assumed that the shearing strength of each individual soil occurring in the cross section may be represented by an expression in the form of Coulomb's empirical law:

   $$S = c_e + \bar{\sigma} \tan \phi_e$$

   where c_e and ϕ_e are the effective cohesion and the effective friction angle, respectively, that apply for each soil under the existing conditions. The normal pressure $\bar{\sigma}$ is the intergranular pressure on the failure plane.

3. The intergranular pressure used in the shearing strength expression is obtained by deducting the neutral pressure from the total pressure existing at that point, provided that the earthquake is not considered. To get the effective intergranular pressure with the horizontal earthquake being considered it is necessary to subtract the normal component of the earthquake pressure from the above intergranular pressure. The neutral pressure or the pore pressure at any point is given by the flow-net in case of seepage, by the full hydrostatic pressure in case of sudden draw-down and by $\frac{U_a \quad \Delta V}{V_{ao} + hV_w - \quad \Delta V}$ in case of consolidation during construction (10). In the above expression

    U_a = Initial pore pressure (usually considered as atmospheric pressure--absolute pressure)

    V_{ao} = Initial volume of free air in the soil mass (per cent of total initial volume)

    ΔV = $V_{ao} - V_{ac}$ = Volume change from sonsolidation (per cent of total initial volume)

    V_w = Volume of water in the soil mass (per cent of total initial volume)

    h = Capacity of water to dissolve air from Henry's Law (approximately 0.02)

All stability analyses are based on the concept that an embankment fails unless the resultant resistance to shear on every surface traversing the embankment is greater than the resultant shearing force exerted on that surface by the mass above, along with the earthquake force when this is to be considered. The surface which is most liable to fail is called the critical surface.

Proposed research

The ϕ- circle method developed by the late D. W. Taylor provides a relation between the stability number ($c/F\gamma H$) and slope angle i for different values of ϕ (angle of internal friction). But, Taylor in his mathematical analysis did not consider the possibility of failure of slopes from other considerations such as

1. Earthquake (horizontal)
2. Pore pressure due to construction.

So the major investigation of this dissertation is to find some relationships such that one can utilize Taylor's stability charts even when one is to consider horizontal earthquake or pore pressure or both. The writer here analyzes the following cases:

1. Earthquake (horizontal)
2. Pore pressure
    a. Due to sudden draw-down
    b. Due to seepage
    c. Due to construction.

This study is limited to a simple slope with ideal conditions.

CHAPTER II

REVIEW OF LITERATURE

Various methods of stability computations have been developed. All assume homogeneous soil, constant angle of slope, level top surface and shearing strength as expressed by Coulomb's equation. The most common methods may be summarized as follows:

1. The Culmann method assumes rupture will occur on a plane. It is of interest only as a classical solution, since actual failure surfaces are invariably curved.
2. The Resal-Frontard method assumes the soil mass to act as a slope of infinite extent, and by use of conjugate stress relationships results in an equation for the rupture surface. This method has been criticized because the results indicate that the mass above the rupture surface is not in static equilibrium at the point of failure with all shearing strength being utilized.
3. Brahtz method of stability analysis: In the development of the science of soil mechanics, as in the development of most of the physical sciences, the gap between theory and application has at some points been wide and difficult to bridge. One school of thought has developed methods of analysis which, though easily applied, fail to take into account some of the important physical properties of the soil mass. A second school of thought has deduced methods, which although they take into account many of the physical properties of the materials, involve intricate mathematical computation, and

are, therefore, somewhat abstruse and difficult of application. Brahtz' method of stability analysis belongs to the second group.

4. The spiral method assumes the rupture to be a logarithmic spiral.

5. The circular arc method was first proposed by K. E. Petterson, based on his study of the failure of a quay wall in Goeteborg in 1915 or 1916. The justification that circular arcs are close approximations of actual rupture surfaces comes from field investigations of a large number of actual slides, especially in railroad cuts by the Swedish Geotechnical Commission. This method has been widely accepted as satisfactory, and several methods of procedure based on the circular arc have been proposed, among which are the following:

   a. Method of slices (7): This method was advanced by the Swedish Geotechnical Commission and developed in quite some detail by Professor W. Fellenius. The main objection to this or any other graphical method such as that given by A. W. Bishop (11) and later developed by M. Arnold (15) rests in the fact that the most dangerous circle of an infinite number of possible circles must be found, and, thus, the graphical procedure must be repeated for a large number of circles.

   b. ϕ-circle method (1): This method was proposed some years ago by Professors Glennon Gilboy and Arthur Casagrande, its initial use being in the development of a completely graphical solution of the slope problem. In the mathematical solutions, the late D. W. Taylor set up a relation between the dimensionless number $(c/F\gamma H)$ and slope angle i for different values of ϕ (angle of internal friction). In that above dimensionless number, F is the factor of safety with respect to cohesion, c is the actual unit cohesion for the soil in question, and γ is the effective unit weight of soil. This basic dimensionless expression is called the "stability number." This method is a simple convenient bridge between theory and application. But, Taylor in his mathematical analysis did not consider all possibilities such as earthquake, pore pressure, etc. The purpose of this analysis is to develop a simple, clear and objective solution for the horizontal earthquake case and various types of pore pressure cases.

CHAPTER III

DIMENSIONAL AND MATHEMATICAL ANALYSIS

<u>Dimensional analysis</u>

Stability of a soil slope is dependent on

$H \equiv$ height $\doteq L$

$i \equiv$ slope angle $\doteq$ (dimensionless)

$c \equiv$ soil cohesion $\doteq FL^{-2}$

$\phi \equiv$ friction angle of soil $\doteq$ (dimensionless)

$\gamma \equiv$ total unit weight of soil $\doteq FL^{-3}$

π-term relationship to determine the critical height H at which a landslide will occur is given by:

$$H = f_1 (c, \gamma, i, \phi)$$

ie., $H = C_\alpha (c^{c_1} \gamma^{c_2} i^{c_3} \phi^{c_4})$

where

c_1, c_2, c_3, c_4 are constant indices.

So, $L = (FL^{-2})^{c_1} (FL^{-3})^{c_2}$

Then,

$L : 1 = -2C_1 - 3C_2$ So, $C_1 = 1$

$F : 0 = C_1 + C_2$ $C_2 = -1$

So, $H = C_\alpha \left(c^1 \gamma^{-1} i^{C_3} \phi^{C_4} \right)$

Or, $\frac{c}{\gamma H} = f(i, \phi)$ . . . . (1)

If one introduces F, the factor of safety with respect to cohesion, then above equation (1) becomes:

$$\frac{c_e}{F\gamma H} = f(i, \phi) = \text{function of } i \text{ and } \phi.$$

This dimensionless quantity, $\frac{c_e}{F\gamma H}$ is called the stability number. It depends on slope angle i and friction angle ϕ, and one will write it as $\frac{c}{F\gamma H}$. Two different factors of safety had been proposed for use in stability problems. Fellenius and most other investigators used the ratio of actual shearing strength to critical shearing strength, which was in agreement with the usual concept of factor of safety and was called the true factor of safety. The other factor had been used by Jaky and Rendulic, and might be described as the ratio between actual cohesion and critical cohesion.

Of the two items which comprise the shearing strength, namely cohesion and friction, the ratio between actual and critical cohesion has bearing on the cohesion only and will be called herein the factor of safety with respect to cohesion. In setting up mathematical solutions, the use of this factor proves to be simpler, thus it is used in the derivations given herein. However, the true factor of safety may easily be adapted in the results of these solutions. The friction angle, ϕ, as used here, is the developed obliquity. If a true factor of safety, F_T is desired to apply to both cohesion and friction, then the developed obliquity must be ϕ/F_T. Expressed mathematically the above may be summarized as follows:

$$\frac{c}{F\gamma H} = f\ (i\ ,\phi) \text{ and } \frac{c}{F_T\gamma H} = f\ (i\ ,\frac{\phi}{F_T})$$

Mathematical analysis

The mathematical solution was given by D. W. Taylor (1), first for the case of circles passing through the toe of the slope, and followed by the case of circles passing below the toe of the slope. In this analysis the forces considered were:

1. The weight of the mass, W, acting vertically downward through the center of gravity of the mass.

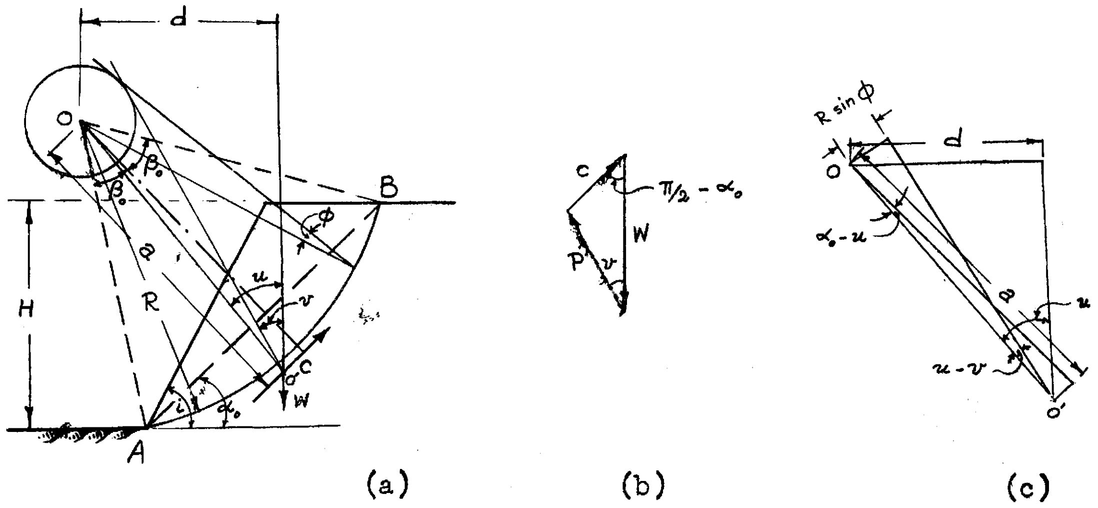


Figure 1. Sketches showing elements of the ϕ-circle method for circle passing through the toe of slope.

2. The resultant cohesion, C:- Its magnitude is $c_1\bar{L}$, where c_1 is the unit cohesion required for equilibrium and $\bar{L}$ is the length of the chord AB. Its line of action is parallel to the chord AB and its moment arm, a, is described by

$$c_1\bar{L}a = c_1\widehat{L}R \qquad \text{or} \qquad a = \frac{R\widehat{L}}{\bar{L}}$$

where $\widehat{L}$ is the length of arc AB. Thus, the line of action of C may be found, and its position is independent of the magnitude of the cohesion.

3. The resultant force, P, is transmitted from grain to grain of the soil across the arc AB. The force polygon is as shown in the sketch.

From Figure 1 (a)

$$C = c_1\bar{L} = \frac{c\bar{L}}{F} = \frac{2c}{F}R \operatorname{Sin}\beta_o \quad . \; . \; . \; . \; . \; . \; . \quad (1)$$

where, F = factor of safety.

$$W = \gamma R^2\beta_o - \gamma R^2 \operatorname{Sin}\beta_o \operatorname{Cos}\beta_o + \frac{\gamma H^2}{2}(\operatorname{Cot}\alpha_o - \operatorname{Cot} i) \quad (2)$$

$$R = \frac{H}{2} \operatorname{Cosec}\alpha_o \operatorname{Cosec}\beta_o \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

$$Wd = \left[\gamma R^2\beta_o\right]\left[\frac{2R}{3}\frac{\sin\beta_o}{\beta_o}\sin\alpha_o\right] - \left[\gamma R^2 \sin\beta_o \cos\beta_o\right]$$
$$\times\left[\frac{2R}{3}\cos\beta_o \sin\alpha_o\right] - \left[\frac{\gamma H^2}{2}\cot\alpha_o\right]\left[\frac{H}{3}\cot\alpha_o + R\sin\left(\alpha_o-\beta_o\right)\right]$$
$$-\left[\frac{\gamma H^2}{2}\cot i\right]\left[\frac{H}{3}\cot i + R\sin\left(\alpha_o-\beta_o\right)\right]$$

which reduced and combined with (3) gives

$$Wd = \frac{\gamma H^3}{12}\left[1 - 2\cot^2 i + 3\cot i \cot\alpha_o - 3\cot i \cot\beta_o + 3\cot\alpha_o \cot\beta_o\right] \quad . \quad . \quad . \quad . \quad (4)$$

From Figure 1 (c)

$$00' = d \operatorname{Cosec} u \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$00' = a \operatorname{Sec}(\alpha_o - u) = R\beta_o \operatorname{Cosec}\beta_o \operatorname{Sec}(\alpha_o - u) \quad . \quad (6)$$

$$00' = R \sin\phi \operatorname{Cosec}(u - v) \quad . \quad . \quad . \quad . \quad (7)$$

From Figure 1 (b)

$$\frac{W}{C} = \frac{\cos(\alpha_o - v)}{\sin v} = \cos\alpha_o \cot v + \sin\alpha_o \quad . \quad . \quad . \quad (8)$$

Substituting (3) in (2) and the result in (4)

$$\frac{H}{2d} = \frac{1/2 \operatorname{Cosec}^2\alpha_o(\beta_o \operatorname{Cosec}^2\beta_o - \cot\beta_o) + \cot\alpha_o - \cot i}{1/3\,(1 - \cot^2 i) + \cot i\,(\cot\alpha_o - \cot\beta_o) + \cot\alpha_o \cot\beta_o} \quad (9)$$

From (5), (6) and (3)

$$\cot u = \frac{H\beta_o}{2d}\operatorname{Sec}\alpha_o \operatorname{Cosec}\alpha_o \operatorname{Cosec}^2\beta_o - \tan\alpha_o \quad . \quad . \quad (10)$$

From (5), (7) and (3)

$$\sin(u - v) = \frac{H}{2d}\sin u \operatorname{Cosec}\alpha_o \operatorname{Cosec}\beta_o \sin\phi \quad . \quad (11)$$

Placing (3) in (1) and (2), then setting (8) equal to the ratio of (2) and (1)

$$\frac{c}{F\gamma H} = \frac{1/2 \operatorname{Cosec}^2\alpha_o(\beta_o \operatorname{Cosec}^2\beta_o - \cot\beta_o) + \cot\alpha_o - \cot i}{2\cot\alpha_o \cot v + 2} \quad (12)$$

The solution for the case where the rupture surface passes below the toe of the slope is almost the same as for the above case. The only important difference is that another variable enters which is designated by n, and is shown in Figure 2.

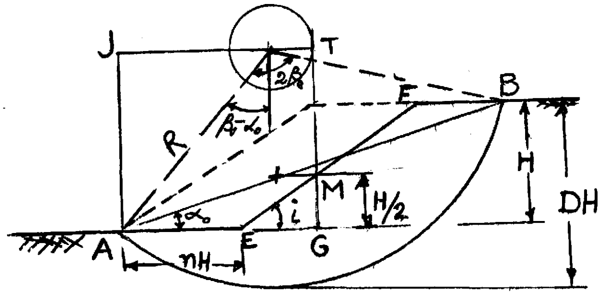


Figure 2. Elements of ϕ-circle method for circle passing below the toe of the slope.

Here, $n = \frac{1}{2} (\text{Cot}\,\alpha_o - \text{Cot}\,\beta_o - \text{Cot}\,i + \text{Sin}\,\phi\,\text{Cosec}\,\alpha_o\,\text{Cosec}\,\beta_o)$ (13)

$$\frac{c}{F\gamma H} = \frac{1/2\ \text{Cosec}^2\alpha_o(\beta_o\text{Cosec}^2\beta_o - \text{Cot}\,\beta_o) + \text{Cot}\,\alpha_o - \text{Cot}\,i - 2n}{2\ \text{Cot}\,\alpha_o\,\text{Cot}\,i + 2} \quad (14)$$

$D = \frac{1}{2} (\text{Cosec}\,\alpha_o\,\text{Cosec}\,\beta_o - \text{Cot}\,\alpha_o\,\text{Cot}\,\beta_o + 1)$ . . . . . (15)

(Depth factor)

Figure 7 gives the chart for stability number vs slope angle for different angles of internal friction. Figure 8 gives the chart showing effect of depth limitation, DH on stability number $\frac{c}{F\gamma H}$ for $\phi = 0$. This DH is defined as the maximum depth of a point through which the slip circle passes. Now the question is: can these charts be used when horizontal earthquake or pore pressure is introduced in the problem? The writer has analyzed the above cases in the following way:

Case I: Earthquake (horizontal)

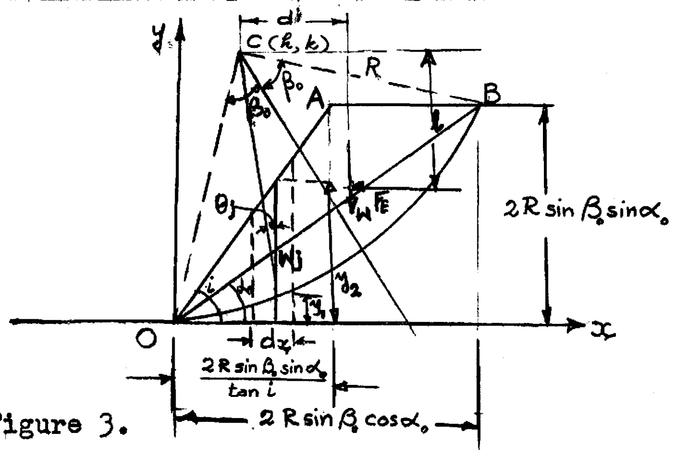


Figure 3.

If one analyzes the effect of horizontal earthquake force on the stability of soil slopes, it can be shown that it increases the overturning force and decreases the resisting force. If now, one considers the effective unit weight of the sliding mass as γ_E , then the effective weight W_E ($= \gamma V$) will give the same overturning moment as by the horizontal earthquake fore, F_E ($= \alpha W = \alpha \gamma V$) and the weight, W.
In other words, from Figure 16, by taking moment about the center of the circle

$$W_E d = Wd + F_E \ell$$

Or $$\gamma_E Vd = \gamma Vd + \alpha \gamma V \ell$$

Or $$\gamma_E = \gamma + \alpha \gamma \ell / d = \gamma (1 + \alpha \ell / d) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Where V is the volume of the sliding mass, γ is the given unit weight of the soil, ℓ is the moment arm of the force F_E and d is the moment arm of W, the weight of sliding mass.

It is this γ_E which will be used in the stability number. Now, if one analyzes a small elementary strip on the sliding mass with weight, W_j and horizontal earthquake force of αW_j, then it can be seen that the resisting force other than cohesion is ($W_j \text{ Cos } \theta_j - \alpha W_j \text{ Sin } \theta_j$) $\tan \phi_i$, where $\tan \theta_j$ is the slope of the sliding circle at the jth element, and ϕ_i is the given angle of internal friction.

So, the total resisting force is

$$\sum (W_j \text{ Cos } \theta_j - \alpha W_j \text{ Sin } \theta_j) \tan \phi_i$$

i.e., $$\gamma \sum [(y_2 - y_1)_j \text{ Cos } \theta_j - \alpha (y_2 - y_1)_j \text{ Sin } \theta_j] \Delta x \tan \phi_i \quad . \quad . \quad (2)$$

The total effect due to earthquake force being taken care of by equivalent unit weight, γ_E and the modified friction angle, ϕ_{mE}, the resisting force is given by $\gamma_E \sum [(y_2 - y_1)_j \text{ Cos } \theta_j] \Delta x \tan \phi_{mE} \quad . \quad . \quad . \quad (3)$

Now equating (2) with (3),

$$\gamma \sum [(y_2-y_1)_j \cos \theta_j - \alpha (y_2-y_1)_j \sin \theta_j] \tan \phi_i$$
$$= \gamma_E \sum [|y_2-y_1|_j \cos \theta_j] \tan \phi_{mE}$$

Or, $$m = \frac{\tan \phi_{mE}}{\tan \phi_i} = \frac{\gamma}{\gamma_E} [1 - \frac{\alpha \sum |y_2-y_1|_j \sin \theta_j}{\sum |y_2-y_1|_j \cos \theta_j}] \quad . \quad . \quad . \quad . \quad (4)$$

Where m is the ratio of the tangent of the modified friction angle to the tangent of the initial or given friction angle. So from (1) and (4),

$$m = \frac{1}{(1+\alpha(l/d))} [1 - \frac{\alpha \sum (y_2-y_1)_j \sin \theta_j}{\sum (y_2-y_1)_j \cos \theta_j}] \quad . \quad . \quad . \quad . \quad . \quad (5)$$

So, for different values of α, different values of m will be obtained for a given ϕ_i and a given slope. Now, from this m, ϕ_{mE} can be obtained. And, it is with this value of ϕ_{mE} one will look into Taylor's chart when horizontal earthquake is to be considered. The slope with the new slip circle (obtained with modified ϕ) with equivalent unit weight γ_E will give the same resisting moment and overturning moment as given with the original slip circle (obtained with initial ϕ) with the weight W and the earthquake force, F_E.

For a wide range of α, the value of ϕ_{mE} lies between ϕ_i and 0 ie., $\phi_i \geqslant \phi_{mE} \geqslant 0$. It can be seen that the failure circle given by i and ϕ_i and that by i and 0 are very close to each other, or in other words, the discrepancy involved is very small. So, $L_i \approx L_m$, then C_i (cohesive force) $\approx C_m$ (cohesive force) for small values of c (unit cohesion). The total resisting moment for the initial case with earthquake is given by equation (2) and that given for the modified case by equation (3). From equations (1), (2) and (3) one can get the equation (5), and from there one gets the values of ϕ_{mE} (modified friction angle due to earthquake). By successive approximation one will also get the same result. For example, let $i = 30°$ and $\phi_i = 20°$ and for which $l/d = 1.77$ from Figure 9. From equation (5), the solution of which is shown in Appendix under heading, $i = 30°$, $\phi_i = 20°$

$$m = \frac{1-0.48\alpha}{1+1.77\alpha} = \frac{\tan\phi_{mE}}{\tan\phi_i}$$

when $\alpha = 0.10$, then $m = 0.81$, and then $\phi_{mE} = 16.5°$.

Now, using $\phi = 16.5°$, $l/d = 1.82$ (for Figure 9.)

For $\alpha = 0.10$, $m = 0.80$.

Then $\tan\phi_{mE} = 0.80 \times \tan 20° = 16.5°$ (approximate).

The equation (5) can be expressed in the form of integration as follows:

$$m = \frac{1}{(1+\alpha l/d)}\left[1-\alpha\frac{\int(y_2-y_1)\,\mathrm{Sin}\,\theta\,dx}{\int(y_2-y_1)\,\mathrm{Cos}\,\theta\,dx}\right]$$

Or, $$m = \frac{1}{(1+\alpha l/d)}\left[1-\alpha\frac{\text{Numerator}}{\text{Denominator}}\right] \quad . . . . (5')$$

Equation of the circle is

$$(x-h)^2 + (y-K)^2 = R^2 \quad . . . . (6)$$

Equation of the line OA is

$$y = x \tan i \quad . . . . (7)$$

Equation of the line AB is

$$y = (2R \;\mathrm{Sin}\,\beta_o \;\; \mathrm{Sin}\,\alpha_o) \quad . . . . (8)$$

Where, $h = R\,\mathrm{Sin}\,(\beta_o - \alpha_o)$

$K = R\,\mathrm{Cos}\,(\beta_o - \alpha_o)$

From equation (6)

$$\frac{dy}{dx} = -\frac{x-h}{y-K} = \tan\theta$$

So, $\mathrm{Sin}\,\theta = \frac{x-h}{R}$; $\mathrm{Cos}\,\theta = -\frac{y-K}{R}$

$$= \frac{\sqrt{R^2-(x-h)^2}}{R}$$

So, in equation (5')

$$\text{Numerator} = \int_0^{2R\,\mathrm{Sin}\,\beta_o\,\mathrm{Cos}\,\alpha_o}(y_2-y_1)\,\mathrm{Sin}\,\theta\,dx$$

$$\text{Numerator} = \int_0^{2R \sin\beta_o \sin\alpha_o/\tan i} [x \tan i - (K - \sqrt{R^2-(x-h)^2})] \frac{x-h}{R}\, dx$$

$$+ \int_{2R \sin\beta_o \sin\alpha_o/\tan i}^{2R \sin\beta_o \cos\alpha_o} [2R \sin\beta_o \sin\alpha_o - (K - \sqrt{R^2-(x-h)^2})] \frac{x-h}{R}\, dx$$

$$\text{Numerator} = \frac{\tan i}{R} \left[\frac{x^3}{3} - h\frac{x^2}{2}\right] \Bigg|_0^{\frac{2R \sin\beta_o \sin\alpha_o}{\tan i}}$$

$$- \frac{K}{R} \frac{(x-h)^2}{2} \Bigg|_0^{\frac{2R \sin\beta_o \sin\alpha_o}{\tan i}}$$

$$+ \frac{1}{R} \left[-\frac{1}{3} \left(R^2-(x-h)^2\right)^{3/2}\right] \Bigg|_0^{2R \sin\beta_o \sin\alpha_o/\tan i}$$

$$+ \left(\frac{2R \sin\beta_o \sin\alpha_o - K}{R}\right) \frac{(x-h)^2}{2} \Bigg|_{\frac{2R \sin\beta_o \sin\alpha_o}{\tan i}}^{2R \sin\beta_o \cos\alpha_o}$$

$$+ \frac{1}{R} \left[-\frac{1}{3} \left(R^2-(x-h)^2\right)^{3/2}\right] \Bigg|_{\frac{2R \sin\beta_o \sin\alpha_o}{\tan i}}^{2R \sin\beta_o \cos\alpha_o}$$

$$\text{Numerator} = (\tan i)\, R^2 \left[\frac{8}{3} \frac{\sin^3\beta_o \sin^3\alpha_o}{\tan^3 i} - \sin(\beta_o - \alpha_o) \right.$$

$$\left. 2\frac{\sin^2\beta_o \sin^2\alpha_o}{\tan^2 i}\right]$$

$$- \frac{\cos(\beta_o - \alpha_o)}{2} R^2 \left[\left(\frac{2\sin\beta_o \sin\alpha_o}{\tan i} - \sin(\beta_o - \alpha_o)\right)^2 - \sin^2(\beta_o - \alpha_o)\right]$$

$$- \frac{1}{3} R^2 \left[\cos^2(\beta_o - \alpha_o) + \frac{4 \sin\beta_o \sin\alpha_o}{\tan i} \sin(\beta_o - \alpha_o) \right.$$

$$-\frac{4\,\sin^2\beta_o\,\sin^2\alpha_o}{\tan^2 i}\Big)^{3/2} - \cos^3(\beta_o-\alpha_o)] - \frac{\cos(\beta_o+\alpha_o)}{2} R^2[\sin^2(\beta_o+\alpha_o)$$

$$-\left(\frac{2\,\sin\beta_o\,\sin\alpha_o}{\tan i} - \sin(\beta_o-\alpha_o)\right)^2]$$

$$-\frac{R^2}{3}\Big[\Big(\cos^2(\beta_o-\alpha_o) - 4\sin^2\beta_o\cos^2\alpha_o + 4\times \sin\beta_o\cos\alpha_o\sin(\beta_o-\alpha_o)\Big)^{3/2} - \Big(\cos^2(\beta_o-\alpha_o) - \frac{4\sin^2\beta_o\sin^2\alpha_o}{\tan^2 i} + 4\,\frac{\sin\beta_o\sin\alpha_o}{\tan i}\times \sin(\beta_o-\alpha_o)\Big)^{3/2}\Big]$$

Denominator $= \int_0^{2R\sin\beta_o\cos\alpha_o} (y_2-y_1)\cos\theta\, dx$

$$= \int_0^{2R\sin\beta_o\sin\alpha_o/\tan i} [x\tan i - (K-\sqrt{R^2-(x-h)^2})]\,\frac{\sqrt{R^2-(x-h)^2}}{R}\,dx$$

$$+\int_{2R\sin\beta_o\sin\alpha_o/\tan i}^{2R\sin\beta_o\cos\alpha_o} [2R\sin\beta_o\sin\alpha_o - (K-\sqrt{R^2-(x-h)^2})]\times\frac{\sqrt{R^2-(x-h)^2}}{R}\,dx$$

Denominator $= \int_0^{2R\sin\beta_o\sin\alpha_o/\tan i} \tan i\,(x-h)\frac{\sqrt{R^2-(x-h)^2}}{R}\,dx + \int_0^{2R\sin\beta_o\sin\alpha_o/\tan i}\frac{\tan i\,(h)-K}{R}\times\sqrt{R^2-(x-h)^2}\,dx$

$$+\frac{1}{R}\int_0^{2R\sin\beta_o\sin\alpha_o/\tan i}\left(R^2-(x-h)^2\right)dx$$

$$+\frac{2R\sin\beta_o\sin\alpha_o-K}{R}\int_{2R\sin\beta_o\sin\alpha_o/\tan i}^{2R\sin\beta_o\cos\alpha_o}\sqrt{R^2-(x-h)^2}\,dx$$

$$+\frac{1}{R}\int_{2R\sin\beta_o\sin\alpha_o/\tan i}^{2R\sin\beta_o\cos\alpha_o}[R^2-(x-h)^2]\,dx$$

Denominator $= \frac{\tan i}{R}\left[-\frac{1}{3}\left(R^2-(x-h)^2\right)^{3/2}\right]\Big|_0^{2R\sin\beta_o\sin\alpha_o/\tan i} + \frac{\tan i\,(h)-K}{R}\times$

$$\frac{1}{2}\left[(x-h)\sqrt{R^2-(x-h)^2} + R^2\sin^{-1}\left(\frac{x-h}{R}\right)\right]\Big|_0^{2R\sin\beta_o\sin\alpha_o/\tan i}$$

$$+ \frac{1}{R}\left[R^2 x - \left(\frac{x-h}{3}\right)^3 \right] \Bigg|_{0}^{\frac{2R \sin\beta_0 \sin\alpha_0}{\tan i}}$$

$$+ \frac{2R \sin\beta_0 \sin\alpha_0 - R \cos(\beta_0 - \alpha_0)}{R} \; \frac{1}{2}\Big[(x-h) \times$$

$$\sqrt{R^2-(x-h)^2} + R^2 \sin^{-1}\left(\frac{x-h}{R}\right)\Big] \Bigg|_{\frac{2R \sin\beta_0 \sin\alpha_0}{\tan i}}^{2R \sin\beta_0 \cos\alpha_0}$$

$$+ \frac{1}{R}\left[R^2 x - \frac{(x-h)^3}{3}\right] \Bigg|_{2R \sin\beta_0 \sin\alpha_0/\tan i}^{2R \sin\beta_0 \cos\alpha_0}$$

Denominator $= \frac{\tan i}{3} R^2 \Big[\Big(\cos^2(\beta_0 - \alpha_0) + 4 \frac{\sin\beta_0 \sin\alpha_0}{\tan i} \times$

$$\sin(\beta_0 - \alpha_0) - 4 \frac{\sin^2\beta_0 \sin^2\alpha_0}{\tan^2 i} \Big)^{3/2}$$

$$-\cos^3(\beta_0 - \alpha_0)\Big]$$

$$+ \frac{\sin(\beta_0 - \alpha_0)\tan i - \cos(\beta_0 - \alpha_0)}{2} \times$$

$$R^2 \Big[\Big(2\frac{\sin\beta_0 \sin\alpha_0}{\tan i} - \sin(\beta_0 - \alpha_0)\Big) \times$$

$$\Big(\cos^2(\beta_0 - \alpha_0) - 4 \frac{\sin^2\beta_0 \sin^2\alpha_0}{\tan^2 i}$$

$$+ 4 \frac{\sin\beta_0 \sin\alpha_0}{\tan i} \sin(\beta_0 - \alpha_0)\Big)^{1/2}$$

$$+ \sin^{-1}\Big(2\frac{\sin\beta_0 \sin\alpha_0}{\tan i} - \sin(\beta_0 - \alpha_0)\Big)$$

$$+ \sin(\beta_0 - \alpha_0)\cos(\beta_0 - \alpha_0) + (\beta_0 - \alpha_0)\Big]$$

$$+ R^2 \Big[\frac{2 \sin\beta_0 \sin\alpha_0}{\tan i} - \frac{1}{3}\Big(\frac{2 \sin\beta_0 \sin\alpha_0}{\tan i} - \sin$$

$$(\beta_0 - \alpha_0)\Big)^3 - \frac{1}{3}\sin^3(\beta_0 - \alpha_0)\Big]$$

$$- \frac{\cos(\beta_0 + \alpha_0)}{2} R^2 \Big[\sin(\beta_0 + \alpha_0)$$

$$\cos^2(\beta_o - \alpha_o) - 4\sin^2\beta_o \cos^2\alpha_o + 4\sin\beta_o$$
$$\cos\alpha_o \sin(\beta_o - \alpha_o)\Big)^{1/2} + \sin^{-1}$$
$$\Big(\sin(\beta_o + \alpha_o)\Big) - \Big(\frac{2\sin\beta_o \sin\alpha_o}{\tan i}$$
$$- \sin(\beta_o - \alpha_o)\Big) \Big(\cos^2(\beta_o - \alpha_o)$$
$$- \frac{4\sin^2\beta_o \sin^2\alpha_o}{\tan^2 i} + 4\,\frac{\sin\beta_o \sin\alpha_o}{\tan i} \times$$
$$\sin(\beta_o - \alpha_o)\Big)^{1/2} - \sin^{-1}\Big(\frac{2\sin\beta_o \sin\alpha_o}{\tan i}$$
$$- \sin(\beta_o - \alpha_o)\Big)\Big]$$
$$+ R^2\Big[2\sin\beta_o \cos\alpha_o - \frac{1}{3}\sin^3(\beta_o + \alpha_o)$$
$$-2\,\frac{\sin\beta_o \sin\alpha_o}{\tan i} + \frac{1}{3}\Big(\frac{2\sin\beta_o \sin\alpha_o}{\tan i}$$
$$- \sin(\beta_o - \alpha_o)\Big)^3\Big]$$

Let $A = \sin\beta_o \dfrac{\sin\alpha_o}{\tan i}$

$B = \sin(\beta_o - \alpha_o)$; $C = \cos(\beta_o - \alpha_o)$

$D = \sin(\beta_o + \alpha_o)$; $E = \cos(\beta_o + \alpha_o)$

$F = \sin\beta_o \cos\alpha_o$; $G = \sin\beta_o \sin\alpha_o$

$G/A = \tan i$

Then,

$$\frac{\text{Numerator}}{R^2} = \Big[\frac{8}{3}\frac{G}{A}A^3 - 2.\frac{G}{A}\,.\,B\,.\,A^2\Big] - \frac{C}{2}\big[(2A-B)^2 - B^2\big]$$
$$- \frac{1}{3}\Big[\Big(C^2 + 4AB - 4A^2\Big)^{3/2} - C^3\Big] - \frac{E}{2}\Big[D^2 -$$
$$\Big(2A-B\Big)^2\Big] - \frac{1}{3}\Big[\Big(C^2 - 4F^2 + 4FB\Big)^{3/2} - \Big(C^2 - 4A^2$$
$$+ 4AB\Big)^{3/2}\Big]$$

$$\frac{\text{Numerator}}{R^2} = [8/3\,GA^2 - 2ABG] - [2CA^2 - 2ABC] - \frac{1}{3}\Big[\Big(C^2 + 4AB - 4A^2\Big)^{3/2}$$
$$- C^3\Big] - \frac{E}{2}\big[D^2 - (2A-B)^2\big] - \frac{1}{3}\Big[(C^2 - 4F^2 + 4FB)^{3/2}$$
$$- \Big(C^2 - 4A^2 + 4AB\Big)^{3/2}\Big]$$

$$\frac{\text{Denominator}}{R^2} = -\frac{G}{3A}\left[\left(C^2 + 4AB - 4A^2\right)^{3/2} - C^3\right] + \frac{BG-AC}{2A}\left[(2A-B)\times\left(C^2-4A^2+4AB\right)^{1/2} + \sin^{-1}(2A-B) + BC + \sin^{-1}(B)\right] + \left[2A - \frac{1}{3}(2A-B)^3 - \frac{1}{3}B^3\right] - \frac{E}{2}\left[D\left(C^2-4F^2+4FB\right)^{1/2} + \sin^{-1}D - (2A-B)\left(C^2-4A^2+4AB\right)^{1/2} - \sin^{-1}(2A-B)\right] + \left[2F - \frac{D^3}{3} - 2A + \frac{1}{3}(2A-B)^3\right]$$

$$\frac{\text{Denominator}}{R^2} = -\frac{G}{3A}\left[\left(C^2 + 4AB-4A^2\right)^{3/2} - C^3\right] + \frac{BG-AC}{2A}\left[(2A-B)\times\left(C^2-4A^2+4AB\right)^{1/2} + \sin^{-1}(2A-B) + BC + \sin^{-1}(B)\right] - \left[\frac{1}{3}B^3\right] - \frac{E}{2}\left[D\left(C^2-4F^2+4FB\right)^{1/2} + \sin^{-1}D - (2A-B)\times\left(C^2-4A^2+4AB\right)^{1/2} - \sin^{-1}(2A-B)\right] + \left[2F-\frac{D^3}{3}\right].$$

And, thus, the solution of equation (5) is obtained.

Case II: Pore pressure

(a) Pore pressure due to sudden draw-down:

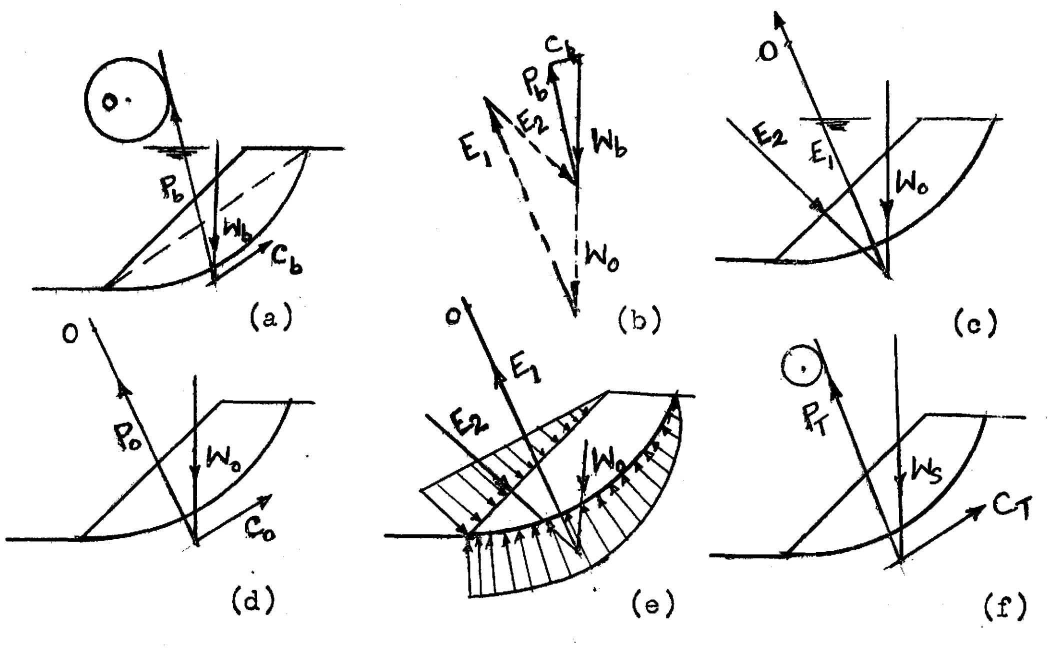


Figure 4. Draw-down case.

The force W of Figure 1 (b) represents the effective weight of the sliding mass. For the submerged or buoyant slope, Figure 4 (a) shows the forces acting according to the ϕ-circle solution, wherein W_b, the effective weight for this case, equals the product of the area of the sliding mass and the buoyant unit weight. The force polygon is shown in full lines in Figure 4 (b). The weight, W_o, which is equal to the weight of a mass of water of the same total volume as the sliding mass, must be present in the submerged case, but it has no effect since it is just balanced by forces E_1 and E_2, the resultant water pressures across the rupture arc and the slope, respectively. The lines of action of these three forces W_o, E_1 and E_2 are shown in Figure 4 (e). It may be noted that the moments of W_o and E_2 about 0 must just balance each other. It is correct to speak of W_o as an overturning force, but in this instance its overturning effect is just counterbalanced by the resisting effect of E_2.

The submerged case may be transformed into the sudden draw-down case by the sudden removal of the force E_2. Since the moment of E_2 just balances that of W_o, removal of E_2 introduces an additional overturning tendency equal to the moment of W_o. The weight W_o at the instant of sudden draw-down is carried by a temporary excess of pressure in the water, and intergranular stresses can replace this hydrostatic excess only as fast as the necessary strains in the mass can develop. So, with the assumption that W_o is not carried by the soil skeleton, and, moreover, since E_1 being normal to the slip circle passes through the center of it, no friction can be developed to help in resisting the shearing stresses it induces. Thus, the overturning forces acting are W_o which is resisted by cohesion and friction together. The force diagram for W_b alone is shown in Figure 4 (a) while that for W_o is shown in Figure 4 (d). Thus, the cohesion required to overcome the combined overturning effects of W_b and W_o is the sum of C_b and C_o of Figures 4 (a) and 4 (d). (1)

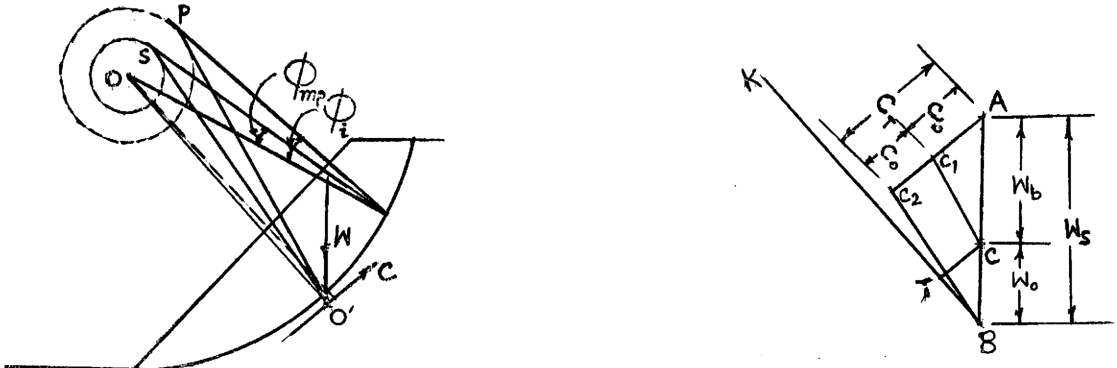


Figure 5. Force diagram for sudden draw-down case.

The writer adopts the following graphical method:

In Figure 5 (b) $\overline{AC} = W_b$ (buoyant weight). This W_b is balanced by $\overline{CC_1}$ and $\overline{C_1A}$ (cohesion required for W_b). $\overline{CB} = W_o$ (weight of a mass of water of the same volume as the sliding mass).

$$\overline{AC} + \overline{CB} = W_b + W_o = W_s$$

From B, BK is drawn parallel to 0'0. From C, CT is drawn parallel to C_1A to meet BK at T. Then CT is the cohesion required to give resisting moment to overcome the overturning moment due to W_o. Then $\overline{TC}$ ($\overline{C_2C_1}$) is added to $\overline{C_1A}$. So, $C_T = \overline{C_2C_1} + \overline{C_1A} = C_o + C_b$. This C_T is the total cohesion to overcome the effect of total weight, $W_s = W_b + W_o$. Then BC_2 gives rise to ϕ_{mP}, the new friction angle obtained from pore pressure due to sudden draw-down. From point 0', 0'S is drawn parallel to BC_2. Then a small circle is drawn such that 0'S is a tangent to this circle. This small circle gives rise to ϕ_{mP} (modified friction angle under pore pressure due to sudden draw-down). In Figure 5 (b),

$$\frac{\overline{CB}}{\overline{AB}} = \frac{W_o}{W_s} = \frac{\gamma_w}{\gamma_s} = \beta$$ (sudden draw-down factor), as defined by the writer.

So, for different values of β, one will get different values of ϕ_{mP}/ϕ_i, and it is this value of ϕ_{mP} one uses in Taylor's chart. Modified slip circle corresponding to ϕ_{mP} can be used in the analysis. And, the unit weight to be used in stability number is the saturated unit weight, γ_s.

(b) Pore pressure due to seepage: Its effect is similar to that of sudden draw-down. Here, in place of β, one can use seepage factor as defined by the writer.

$$\delta = \frac{\gamma_W a_1}{\gamma_t (A-a) + \gamma_s a} = \text{seepage factor}$$

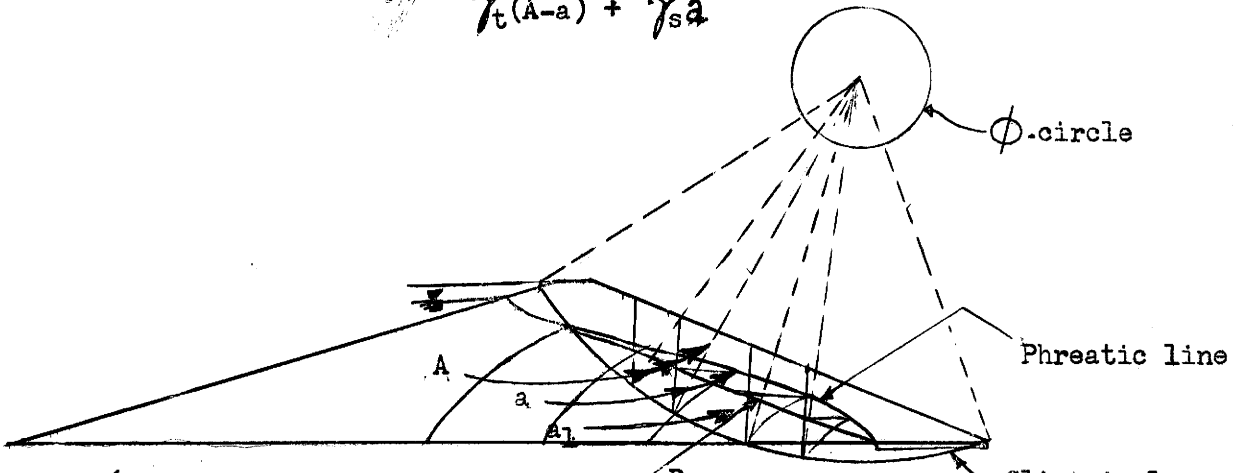


Figure 6. Effect of seepage.

Where

γ_w = Unit weight of water

γ_t = Unit weight of soil at any moisture content

γ_s = Saturated unit weight of soil

a_1 = The area between the pore pressure curve and slip circle obtained by ϕ-circle method

a = The area between the phreatic line and the slip circle

A = Total area of sliding mass.

In a similar manner to that of sudden draw-down, a chart of δ vs ϕ_{mP}/ϕ_i can be obtained for the case of pore-pressure due to seepage. It is this value of ϕ_{mP} that one will use in Taylor's chart. And, the unit weight to be used in stability number is $\gamma_E = \frac{\gamma_t (A-a) + \gamma_s a}{A}$ as suggested by the writer.

(c) Pore pressure due to construction: Its procedure is also similar to that of sudden-draw-down. Only instead of β, one uses ϵ which is expressed as a ratio of the pore pressure force to the total weight on the rupture plane. If the height of the soil above any point on the rupture plane is h, and the height due to pore water pressure is h_1 above the same point, then as defined by the writer

$$\epsilon = \frac{\gamma_w h_1}{\gamma_t h}$$

Where

γ_w = Unit weight of water

γ_t = Unit weight of soil at any moisture content

ϵ = Construction pore pressure factor.

If the effect of pore pressure due to construction is 10 per cent of the full hydrostatic pressure then

$$\epsilon = 0.10 \frac{\gamma_w}{\gamma_t} .$$

So then, like those of Cases II (a) and II (b) a chart of ϵ vs ϕ_{mP}/ϕ_i can be obtained. This chart will give the friction angle modified due to construction pore pressure. And the unit weight of soil in this case is γ_t.

CHAPTER IV

GENERAL DESCRIPTION AND PROCEDURE

Taylor's stability chart gives the relation between stability number, $c/F\gamma H$ and slope angle, i for different values of ϕ. From this relationship one can find factor of safety, F, for a given soil and a given slope with a given height or one can find the height of the embankment where the factor of safety is specified. The writer has tried to develop certain relationships with the help of which one can still use the Taylor's chart even when one is to consider either horizontal earthquake or pore pressure effect in the problem of slope stability.

Case I: Effect of horizontal earthquake

The writer has analyzed the effect of horizontal earthquake in 40 different cases and as a typical example only one is shown in Figure 16, for $i = 30°$, $\phi_i = 10°$, and $H = 50'$.

In all these examples two different heights of embankment, namely 50' and 75', were chosen. The results obtained were the same for the two heights as shown in the charts of l/d vs i, with a given ϕ_i (initial angle of friction without the effect of earthquake). This shows the validity of the relationship of the dimensionless quantities.

The procedure that the writer has adopted in this analysis is as follows and the graphical construction is as shown in Figure 16.

First, the earth embankment is drawn to a scale. Then, corresponding to the given slope, i, and the given initial friction angle, ϕ_i, the critical slip circle is drawn with the help of Table I (given by

Taylor). This circle has its center at 0 which is determined by the values of α_o and β_o (as given in Table I), and it passes through the toe, A, and meets the embankment at B. And so, AB is the rupture arc. At any point L on the slip circle, angle OLK is drawn such that angle OLK = ϕ_i (given initial friction angle when there is no earthquake).

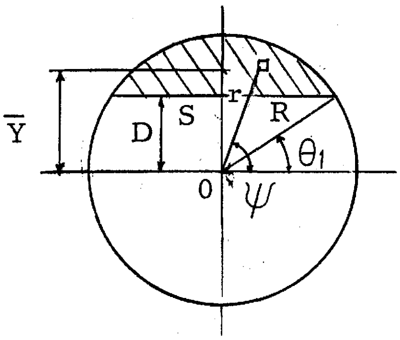
Then the ϕ-circle is drawn with its center at 0, such that it touches LK at K. After the critical slip circle and the friction circle are drawn, the next step is to find the center of gravity of the combined sliding mass; and this is done in the following way:

Referring to Figure 16, from 0, OS is drawn perpendicular to AB, the chord of length $\bar{L}$.

$$OB = \text{Radius},\ R$$

$$OS = D$$

Now, one is to find the center of gravity of the circular segment.



$$x^2 + y^2 = R^2$$

$$x = r'\cos\psi$$

$$Y = r'\sin\psi$$

$$R \sin\theta_1 = D$$

$$\text{or,}\quad \theta_1 = \sin^{-1}\frac{D}{R}$$

$\bar{y}$ (the distance of the c-g from the center 0 along OS) = $\dfrac{\iint y\,dy\,dx}{\iint dy\,dx}$

$$= \frac{\int_{\theta_1}^{\pi/2}\left[\int_{D\operatorname{Cosec}\psi}^{R} r^2\,dr\right]\sin\psi\,d\psi}{\int_{\theta_1}^{\pi/2}\left[\int_{D\operatorname{Cosec}\psi}^{R} r\,dr\right]d\psi}$$

$$\text{ie.,}\quad \bar{y} = \frac{2}{3}D\,\frac{(R/D)^3\cos\theta_1 - \cot\theta_1}{(R/D)^2(\pi/2 - \theta_1) - \cot\theta_1}$$

This $\bar{y}$ is then marked at a_2 along OS. Then the next step is to find the C. G. of the triangular portion by graphical method and then marked at

a_1. Let the distance $a_1a_2 = d$. The combined C. G. lies at a_o on a_1a_2 at a distance of $\overline{x} = \left(\frac{A_1 d}{A_1 + A_2}\right)$ from a_2, where

A_1 = Area of the triangular portion

A_2 = Area of the circular sector

Thus, the weight, W, and the horizontal earthquake force, F_E (α W) act at a_o. The moment arm of W is d and that of F_E is l. After knowing l and d chart for l/d vs i is prepared. Then $\gamma_E = \gamma \times (1 + \alpha l/d)$ is determined. Now, according to the analysis of Case I in Chapter III,

$$m = \frac{\tan \phi_{mE}}{\tan \phi_i} = \frac{1 - \alpha \frac{\text{Numerator}}{\text{Denominator}}}{1 + \alpha l/d}$$

is calculated.

For example, given i = 30° and $\phi_i = 15°$, Figure 9 gives $l/d = 1.808$. Then to calculate $\left[1 - \alpha \frac{\text{Numerator}}{\text{Denominator}}\right]$ one has to take the values of α_o and β_o corresponding to the given i and ϕ_i. In this case, $\alpha_o = 27°$, $\beta_o = 39°$ from Table I. So,

$$A = \frac{\sin \beta_o \sin \alpha_o}{\tan i} = \frac{\sin 39° \sin 27°}{\tan 30°} = 0.494$$

$B = \sin(\beta_o - \alpha_o) = \sin 12° = 0.2079$

$C = \cos(\beta_o - \alpha_o) = \cos 12° = 0.9781$

$D = \sin(\beta_o + \alpha_o) = \sin 66° = 0.9135$; $E = \cos(\beta_o + \alpha_o) = 0.4067$

$F = \sin \beta_o \cos \alpha_o = \sin 39° \cos 27° = 0.561$

$G = \sin \beta_o \sin \alpha_o = \sin 39° \sin 27° = 0.285$

$\frac{8}{3} GA^2 - 2ABG = 0.126$

$2CA^2 - 2ABC = 0.275$

$C^2 + 4AB - 4A^2 = 0.389$

$2A - B = 0.780$

$C^2 - 4F^2 + 4FB = 0.165$

$BG - AC = -0.424$

$\sin^{-1}(2A-B) = 51.3^\circ = \frac{\pi}{180} \times 51.3 = 0.895$ radian

$BC = 0.2079 \times 0.9781 = 0.203$

$\sin^{-1}(B) = 12^\circ = \frac{\pi}{180} \times 12 = 0.210$ radian

$\sin^{-1}(D) = 66^\circ = \frac{\pi}{180} \times 66 = 1.150$ radian

$\frac{\text{Numerator}}{R^2} = 0.0934$; So, Numerator $= R^2\,(0.0934)$

$\frac{\text{Denominator}}{R} = 0.200$; So, Denominator $= R^2\,(0.200)$

Then,

$$1 - \frac{\text{Numerator}}{\text{Denominator}}\alpha = 1 - \frac{R^2(0.0934)}{R^2(0.200)}\alpha = 1-0.467\,\alpha$$

So, $$m = \frac{1-0.467\,\alpha}{1+1.808\,\alpha}$$

Hence, for different values of α, different values of m will be obtained. These are shown in Figures 10 to 14.

Case II: Effect of pore pressure

For the effect of pore pressure in general, the writer has analyzed four different cases, namely

1. With $i = 30^\circ$
   $\phi_i = 15^\circ$
   $H = 50'$
2. With $i = 45^\circ$
   $\phi_i = 20^\circ$
   $H = 50'$
3. With $i = 30^\circ$
   $\phi_i = 20^\circ$
   $H = 75'$
4. With $i = 30^\circ$
   $\phi_i = 20^\circ$
   $H = 50'$

Figure 17 is shown as a typical one. In all these cases, the main idea is to find a relationship between the pore pressure factor and ϕ_{mP}/ϕ_i (ratio of modified friction angle to initial friction angle). All these four cases have given same result which shows it can be extended dimensionally to other values.

(a) <u>Pore pressure due to sudden draw-down</u>. The writer has adopted the following procedure in establishing a relationship between sudden draw-down factor, β (γ_w/γ_s) and ϕ_{mP}/ϕ_i (ratio of friction angle modified under pore pressure due to sudden draw-down to the initial friction angle). As in Figure 16, the C. G. of the sliding mass is determined and O' is located in this case. Procedure for the graphical construction is followed as given by the writer in Case II (a) of Chapter III. For different values of β, different values of ϕ_{mP}/ϕ_i are obtained. When these points are plotted (as shown in Figure 15), they are found to fit a straight line such that $\frac{\phi_{mP}}{\phi_i} + \beta = 1$

or
$$\frac{\phi_{mP}}{\phi_i} = 1 - \beta = 1 - \frac{\gamma_w}{\gamma_s} = \frac{\gamma_b}{\gamma_s}$$

or
$$\frac{\phi_{mP}}{\phi_i} = \frac{G-1}{G+e}$$

Where

G = The sp. gr. of soil,

e = The void ratio of soil.

(b) <u>Pore pressure due to seepage</u>. Similar procedure to that of sudden draw-down is adopted in this case to establish the relationship between the seepage factor, δ (as defined in Case II (b) of Chapter III) and ϕ_{mP}/ϕ_i (ratio of the friction angle modified under pore pressure due to seepage to the initial friction angle without seepage). Equivalent unit weight, γ_E is determined from the formula given by the writer in Case II (b) of Chapter III.

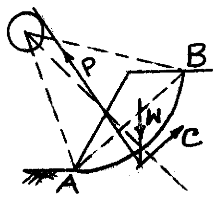
(c) Pore pressure due to construction. This procedure is also similar to that of sudden draw-down. A relationship is established between construction pore pressure coefficient, ϵ (as defined by the writer in Case II (c) of Chapter III) and ϕ_{mP}/ϕ_i (ratio of friction angle modified under construction pore pressure to the initial friction angle without pore pressure).

CHAPTER V

RESULTS OF THE ANALYSIS

The results of the stability analysis by ϕ-circle method considering only the external force W, are tabulated in Table I. These data given by Taylor (1) are verified with a digital 1620 computer. Results of the writer's analysis considering earthquake are tabulated in Table 2, and the results are plotted in Figure 10 to Figure 14 as m vs α for different ϕ_i's and different constant slope angle i. Also, the results of (ℓ/d) vs i are plotted for different ϕ_i's in Figure 9. Results of the analysis done by the writer for the case of pore pressure are tabulated in Table 3, and the results are plotted in Figure 15 as ϕ_{mP}/ϕ_i vs β (or δ or ϵ).

Table 1. Data on critical circles by the ϕ -circle method (1).

(1) i	(2) ϕ	(3) α_0	(4) β_0	(5) n	(6) D	(7) $c/F\gamma H$	(8) Corrected $c/F\gamma H$	(9) Remarks
90°	0°	47.6°	15.1°			0.261	0.261	
	5°	50°	14°			0.239	0.239	
	10°	53°	13.5°			0.218	0.218	
	15°	56°	13°			0.199	0.199	
	20°	58°	12°			0.182	0.182	
	25°	60°	11°			0.166	0.166	
75°	0°	41.8°	25.9°			0.219	0.219	The final
	5°	45°	25°			0.195	0.195	column is
	10°	47.5°	23.5°			0.173	0.173	after ϕ-
	15°	50°	23°			0.153	0.152	circle
	20°	53°	22°			0.135	0.134	correction
	25°	56°	22°			0.118	0.117	on the assumption
60°	0°	35.3°	35.4°			0.191	0.191	that
	5°	38.5°	34.5°			0.163	0.162	intensity
	10°	41°	33°			0.139	0.138	of p-force
	15°	44°	31.5°			0.118	0.116	equal to
	20°	46.5°	30.2°			0.098	0.097	zero at A
	25°	50°	30°			0.081	0.079	and B and varying
45°	0°	(28.2°)	(44.7°)		1.062	(0.170)	(0.170)	sinusoidally
	5°	31.2°	42.1°		1.026	0.138	0.136	between —
	10°	34°	39.7°		1.006	0.110	0.108	$K=\frac{1-(2\beta/\pi)^2}{\cos\beta_0}-1$
	15°	36.1°	37.2°		1.001	0.086	0.083	
	20°	38°	34.5°			0.065	0.062	The radius
	25°	40°	31°			0.046	0.044	of the ϕ-circle
30°	0°	(20°)	(53.4°)		1.301	0.156	0.156	is R sin ϕ.
	5°	(23°)	(48°)		1.161	0.112	0.110	If the
		20°	53°	0.29	1.332	0.113	0.110	corrected
	10°	25°	44°		1.092	0.078	0.075	circle of
	15°	27°	39°		1.038	0.049	0.046	slightly
	20°	28°	31°		1.003	0.027	0.025	larger radius
	25°	29°	25°			0.010	0.009	R Sin ϕ', is intro-
15°	0°	(10.6°)	(60.7°)		(2.117)	(0.145)	(0.145)	duced, the
	5°	(12.5°)	(47°)		(1.549)	(0.072)	(0.068)	following
		11°	(47.5°)	0.55	1.697	0.074	0.070	relation
	10°	(14°)	(34°)		1.222	0.024	0.023	R Sin ϕ'=
		14°	34°	0.04	1.222	0.024	0.023	R Sin ϕ(1+K) must hold
All Values	0	0	66.8			0.181	0.181	good.

Figures in parentheses are values for most dangerous circle through the toe when a more dangerous circle exists which passes below the toe.

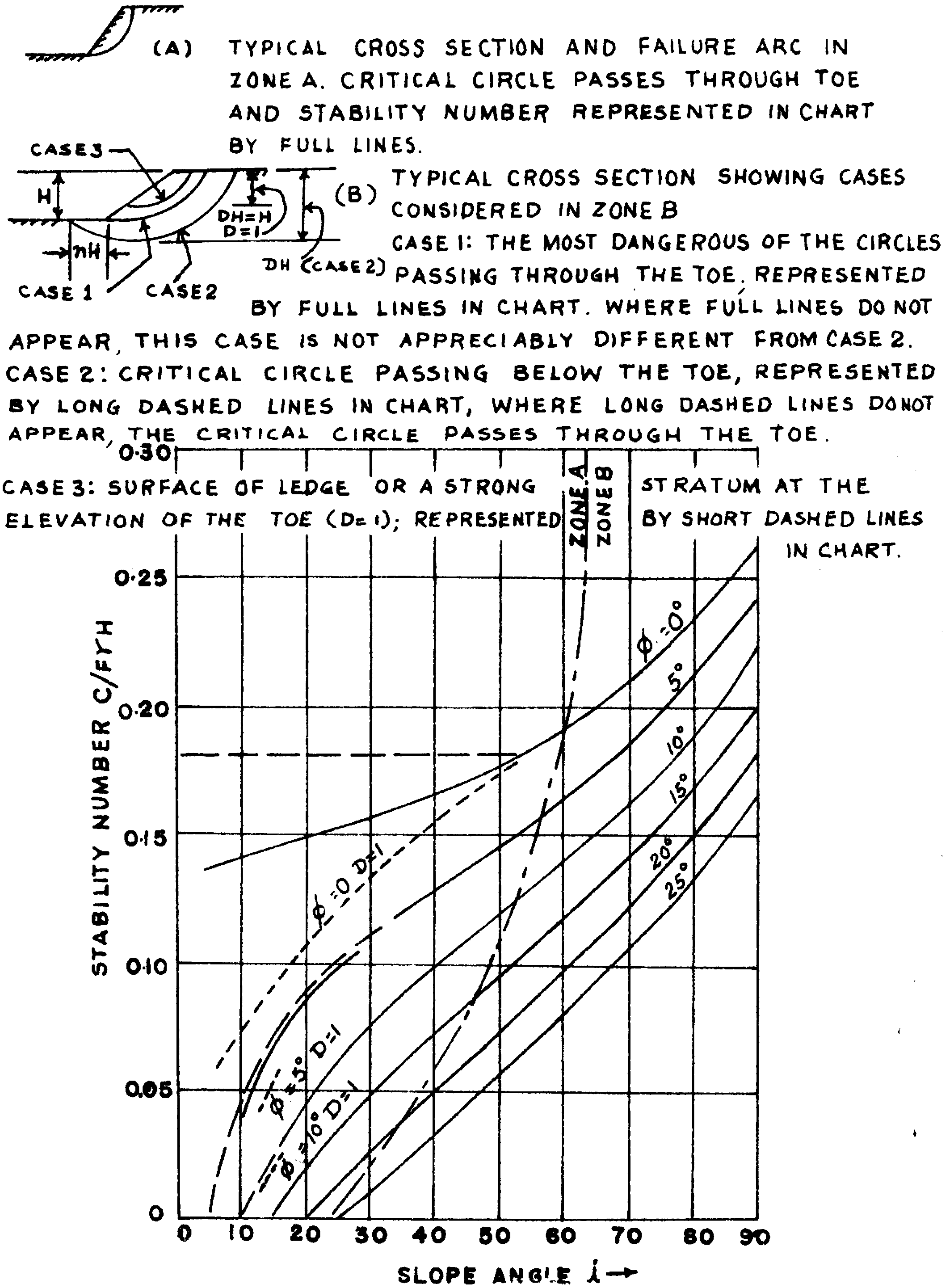


Figure 7. Chart of stability numbers.

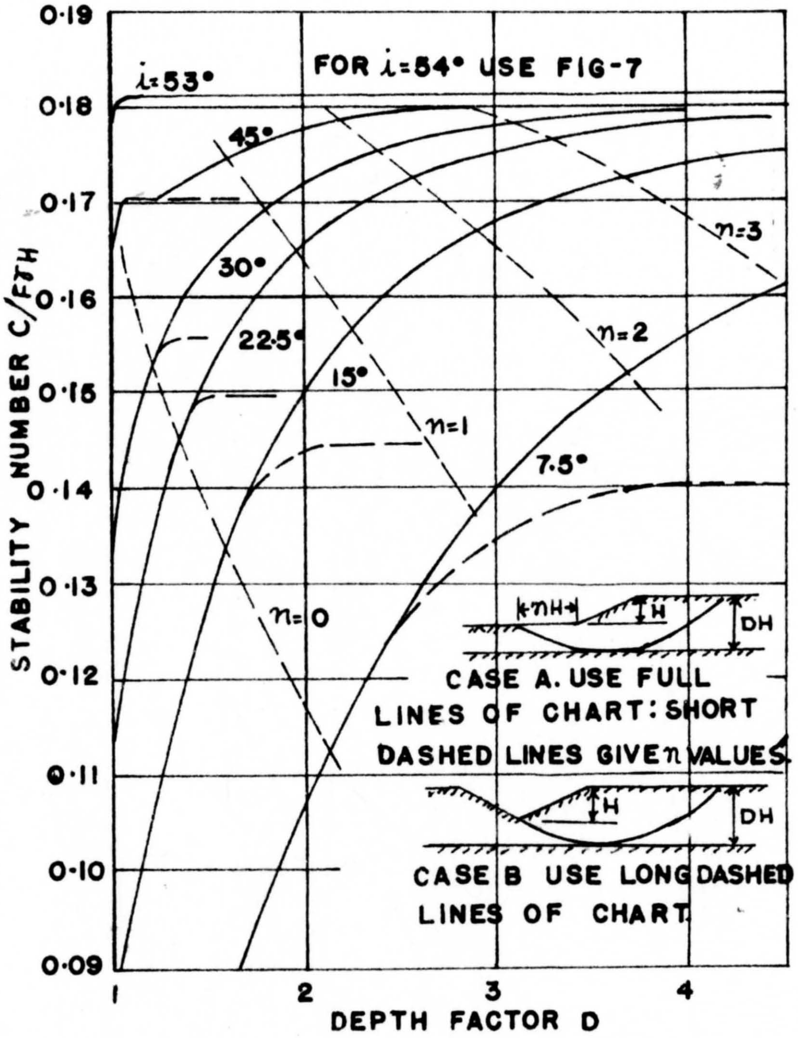


Figure 8. Chart of stability numbers for the case of zero friction angle and limited depth.

Table 2. It is furnished by the writer from graphical and mathematical analysis for the horizontal earthquake force with the help of data from Table 1. (Reference to Figure 16 and Appendix.)

(1) i (slope angle)	(2) ϕ_i (Initial friction angle)	(3) l/d (Moment-arm ratio)	(4) α (Coefficient of earthquake acceleration)	(5) m (Ratio of the Tangent of the friction angle modified due to earthquake to the initial friction angle)		(6) Remarks
			0.0	1.000		The values of l/d
75°	5°	0.946	0.1	0.840	$m=\frac{1-0.815\alpha}{1+0.946\alpha}$	are computed by
			0.2	0.700		taking average of
			0.3	0.586		the values on two
			0.4	0.490		dams of height 50'
			0.5	0.403		and 75'. Both the
			0.6	0.326		dams give almost
			0.7	0.258		the same results.
			0.8	0.200		This shows that the results can be
			0.0	1.000		extended for any
	10°	0.838	0.1	0.830		dimension.
			0.2	0.690		
			0.3	0.575		$m = \frac{1-0.94\alpha}{1+0.838\alpha}$
			0.4	0.467		
			0.5	0.375		
			0.6	0.290		
			0.7	0.215		
			0.8	0.148		
			0.0	1.000		
	15°	0.755	0.1	0.840		$m = \frac{1-0.97\alpha}{1+0.755\alpha}$
			0.2	0.700		
			0.3	0.578		
			0.4	0.470		
			0.5	0.375		
			0.6	0.288		
			0.7	0.210		
			0.8	0.140		
			0.0	1.000		
	20°	0.670	0.1	0.835		$m = \frac{1-1.08\alpha}{1+0.67\alpha}$
			0.2	0.690		
			0.3	0.560		

Table 2. Continued

(1) i	(2) ϕ_i	(3) l/d	(4) α	(5) m	(6) Remarks
			0.4	0.448	
			0.5	0.345	
			0.6	0.250	
			0.7	0.166	
			0.8	0.089	
			0.0	1.000	
	25°	0.590	0.1	0.825	$m = \frac{1-1.27\alpha}{1+0.59\alpha}$
			0.2	0.665	
			0.3	0.525	
			0.4	0.400	
			0.5	0.282	
			0.6	0.176	
			0.7	0.079	
			0.8	-	
			0.0	1.000	
60°	5°	1.130	0.1	0.850	$m = \frac{1-0.63\alpha}{1+1.13\alpha}$
			0.2	0.710	
			0.3	0.610	
			0.4	0.515	
			0.5	0.437	
			0.6	0.370	
			0.7	0.312	
			0.8	0.260	
			0.0	1.000	
	10°	1.028	0.1	0.845	$m = \frac{1-0.72\alpha}{1+1.028\alpha}$
			0.2	0.710	
			0.3	0.600	
			0.4	0.505	
			0.5	0.420	
			0.6	0.350	
			0.7	0.290	
			0.8	0.232	
			0.0	1.000	
	15°	0.934	0.1	0.840	$m = \frac{1-0.83\alpha}{1+0.934\alpha}$
			0.2	0.700	
			0.3	0.586	
			0.4	0.486	
			0.5	0.398	
			0.6	0.320	
			0.7	0.254	
			0.8	0.192	
			0.0	1.000	
	20°	0.838	0.1	0.830	$m = \frac{1-0.91\alpha}{1+0.838\alpha}$
			0.2	0.700	
			0.3	0.580	

Table 2. Continued

(1) i	(2) ϕ_i	(3) l/d	(4) α	(5) m	(6) Remarks
			0.4	0.475	
			0.5	0.385	
			0.6	0.301	
			0.7	0.228	
			0.8	0.163	
	25°	0.728	0.0	1.000	$m = \frac{1-1.56\alpha}{1+0.728\alpha}$
			0.1	0.790	
			0.2	0.600	
			0.3	0.437	
			0.4	0.291	
			0.5	0.161	
			0.6	0.045	
			0.7	-	
			0.8	-	
45°	5°	1.505	0.0	1.000	$m = \frac{1-0.493\alpha}{1+1.505\alpha}$
			0.1	0.830	
			0.2	0.690	
			0.3	0.590	
			0.4	0.500	
			0.5	0.430	
			0.6	0.370	
			0.7	0.320	
			0.8	0.275	
	10°	1.330	0.0	1.000	$m = \frac{1-0.59\alpha}{1+1.33\alpha}$
			0.1	0.830	
			0.2	0.695	
			0.3	0.590	
			0.4	0.500	
			0.5	0.425	
			0.6	0.360	
			0.7	0.305	
			0.8	0.255	
	15°	1.210	0.0	1.000	$m = \frac{1-0.66\alpha}{1+1.21\alpha}$
			0.1	0.830	
			0.2	0.700	
			0.3	0.590	
			0.4	0.495	
			0.5	0.416	
			0.6	0.350	
			0.7	0.290	
			0.8	0.240	
	20°	1.138	0.0	1.000	$m = \frac{1-0.71\alpha}{1+1.138\alpha}$
			0.1	0.830	
			0.2	0.695	
			0.3	0.595	

Table 2. Continued

(1) i	(2) ϕ_i	(3) l/d	(4) α	(5) m	(6) Remarks
			0.4	0.492	
			0.5	0.410	
			0.6	0.340	
			0.7	0.280	
			0.8	0.225	
			0.0	1.000	
	25°	1.080	0.1	0.830	$m = \frac{1-0.79\alpha}{1+1.08\alpha}$
			0.2	0.692	
			0.3	0.575	
			0.4	0.480	
			0.5	0.392	
			0.6	0.320	
			0.7	0.260	
			0.8	0.197	
			0.0	1.000	
30°	5°	2.100	0.1	0.795	$m = \frac{1-0.37\alpha}{1+2.10\alpha}$
			0.2	0.650	
			0.3	0.545	
			0.4	0.463	
			0.5	0.398	
			0.6	0.344	
			0.7	0.300	
			0.8	0.260	
			0.0	1.000	
	10°	1.960	0.1	0.800	$m = \frac{1-0.42\alpha}{1+1.96\alpha}$
			0.2	0.660	
			0.3	0.550	
			0.4	0.465	
			0.5	0.398	
			0.6	0.344	
			0.7	0.296	
			0.8	0.250	
			0.0	1.000	
	15°	1.808	0.1	0.805	$m = \frac{1-0.467\alpha}{1+1.808\alpha}$
			0.2	0.665	
			0.3	0.550	
			0.4	0.470	
			0.5	0.400	
			0.6	0.345	
			0.7	0.300	
			0.8	0.255	
			0.0	1.000	
	20°	1.770	0.1	0.810	$m = \frac{1-0.48\alpha}{1+1.77\alpha}$
			0.2	0.670	
			0.3	0.560	

Table 2. Continued

(1) i	(2) ϕ_i	(3) l/d	(4) α	(5) m	(6) Remarks
			0.4	0.470	
			0.5	0.400	
			0.6	0.340	
			0.7	0.300	
			0.8	0.255	
			0.0	1.000	
	25°	1.720	0.1	0.810	$m = \frac{1-0.492\alpha}{1+1.72\alpha}$
			0.2	0.670	
			0.3	0.560	
			0.4	0.470	
			0.5	0.400	
			0.6	0.340	
			0.7	0.300	
			0.8	0.255	
			0.0	1.000	
15°	5°	4.708	0.1	0.670	$m = \frac{1-0.20\alpha}{1+4.708\alpha}$
			0.2	0.495	
			0.3	0.390	
			0.4	0.320	
			0.5	0.270	
			0.6	0.230	
			0.7	0.200	
			0.8	0.176	
			0.0	1.000	
	10°	3.850	0.1	0.670	$m = \frac{1-0.64\alpha}{1+3.85\alpha}$
			0.2	0.492	
			0.3	0.374	
			0.4	0.294	
			0.5	0.230	
			0.6	0.186	
			0.7	0.150	
			0.8	0.120	

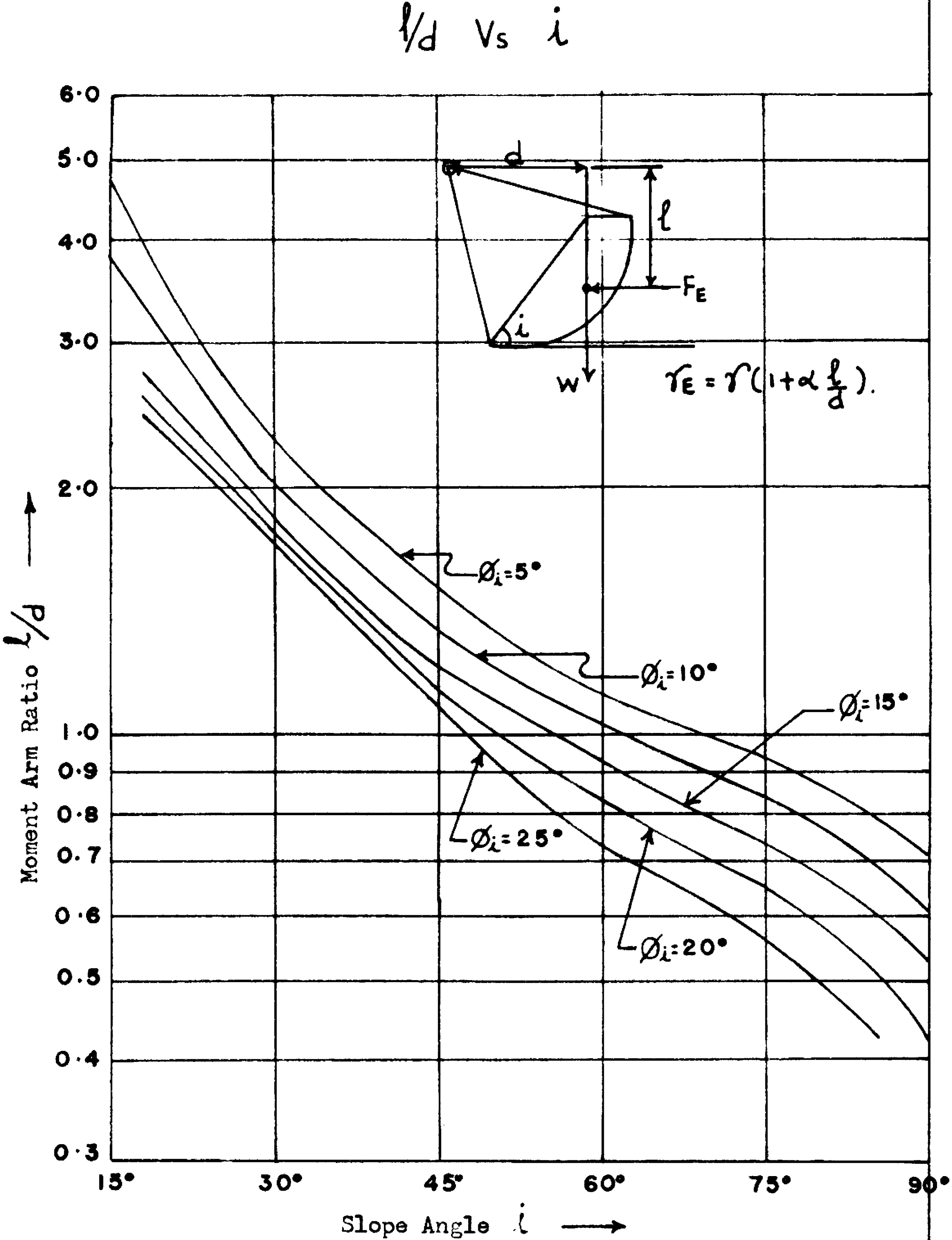


Figure 9. Moment arm ration l/d vs. slope angle i and initial friction angle ϕ_i.

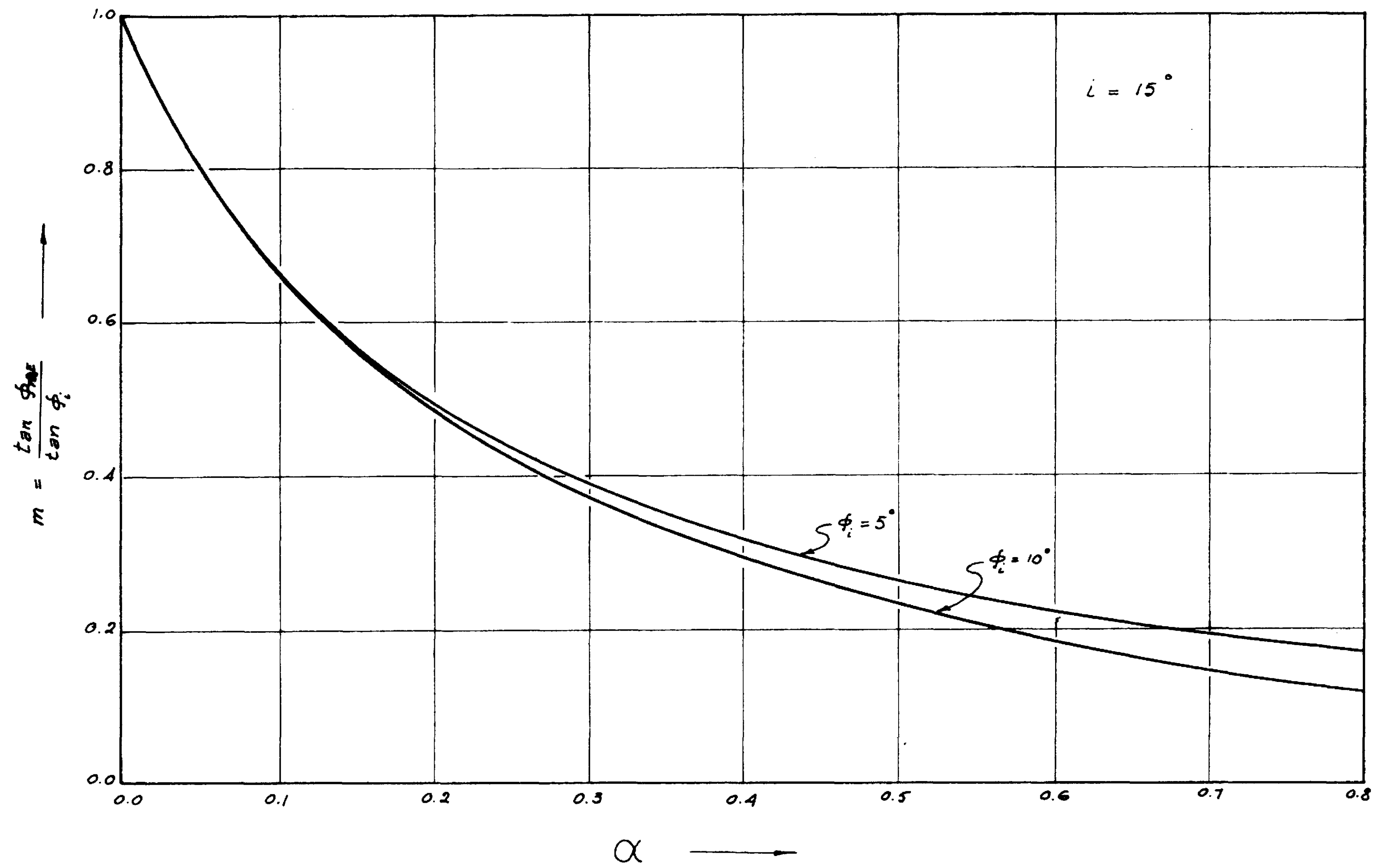


Figure 10. Friction angle ratio m vs. horizontal earthquake acceleration coefficient α for a slope of 15°.

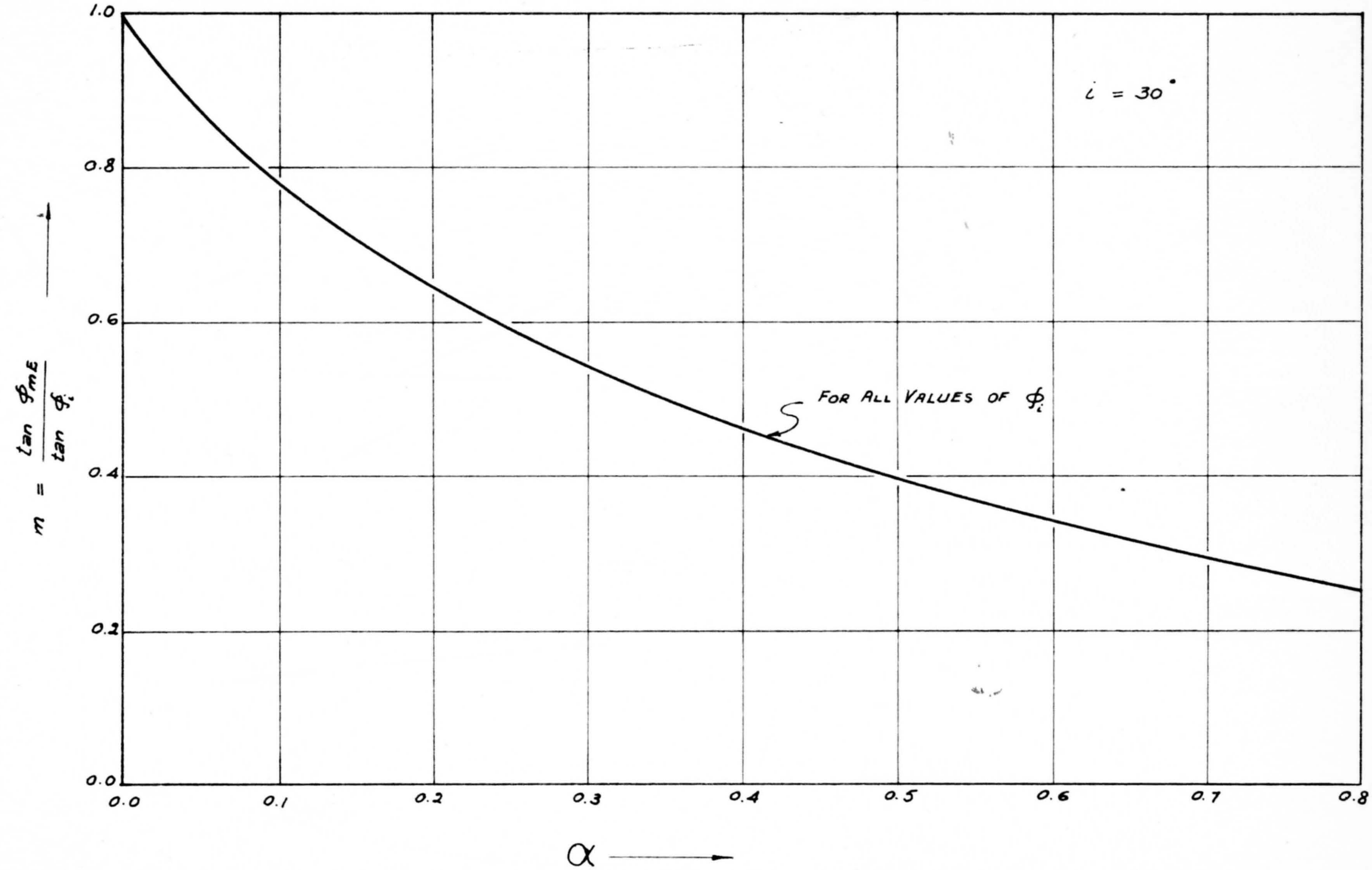


Figure 11. Friction angle ratio m vs. horizontal earthquake acceleration coefficient α for a slope of 30°.

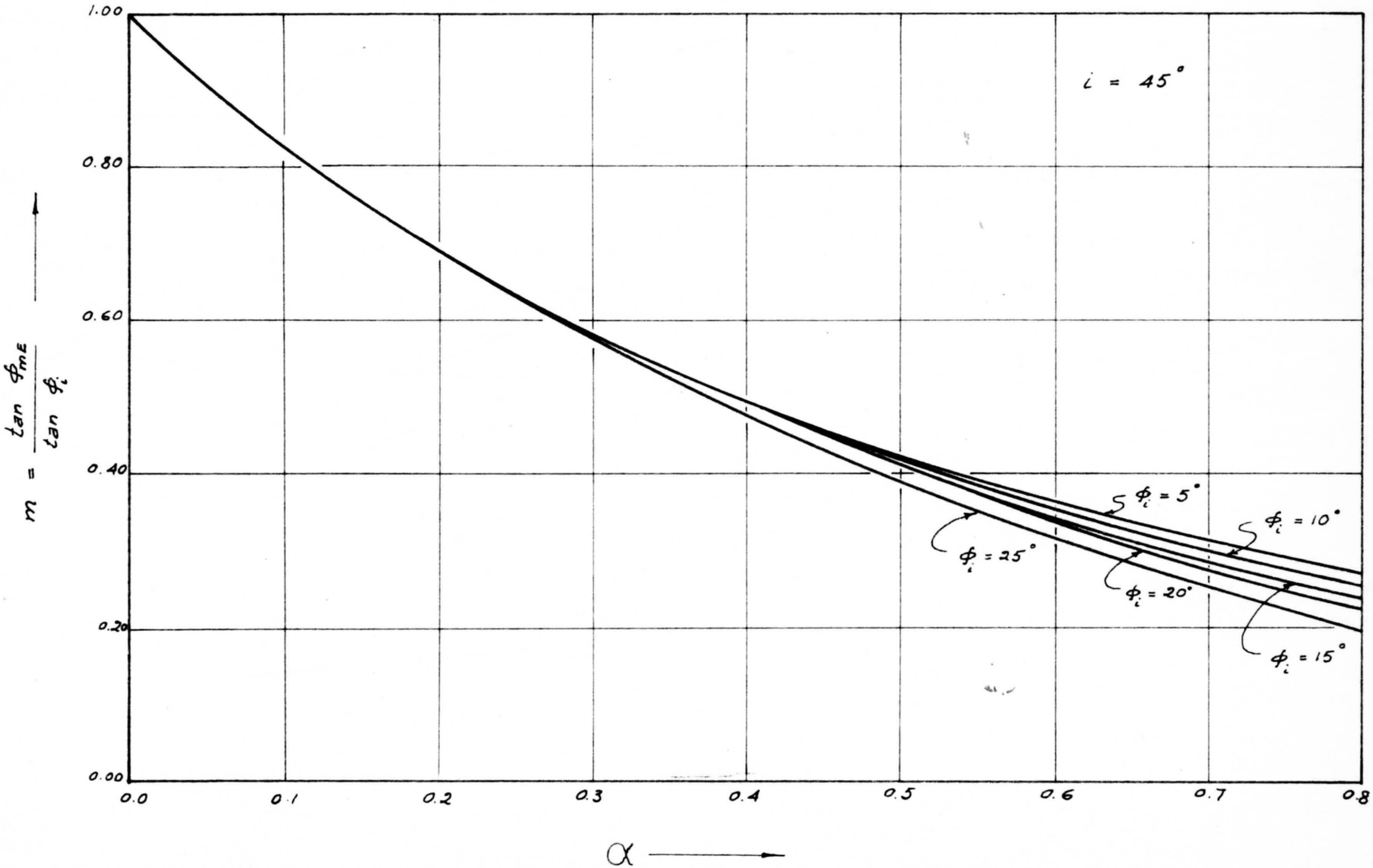


Figure 12. Friction angle ratio m vs. horizonatal earthquake acceleration coefficient α for a slope of 45°.

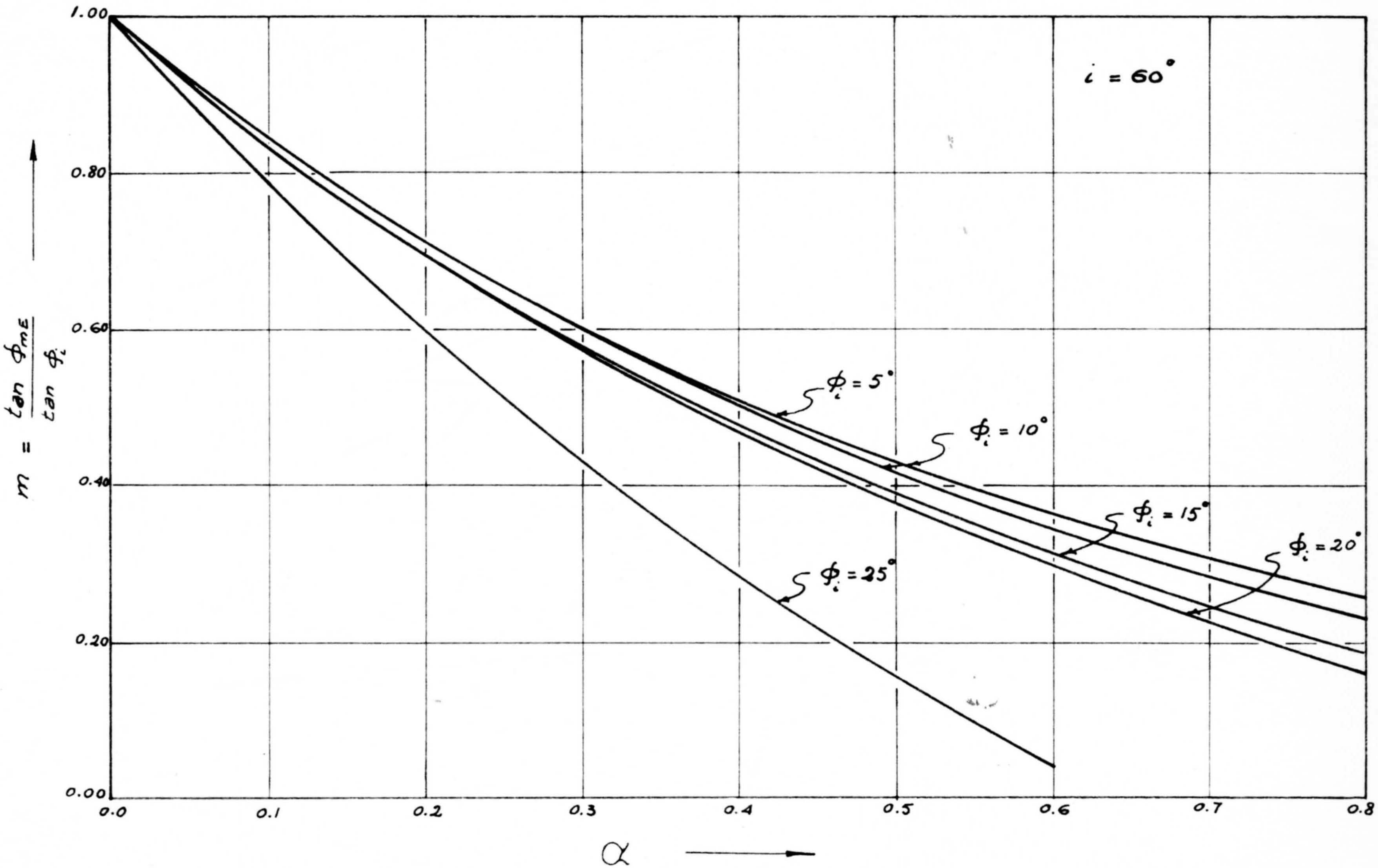


Figure 13. Friction angle ratio m vs. horizontal earthquake acceleration coefficient α for a slope of 60°.

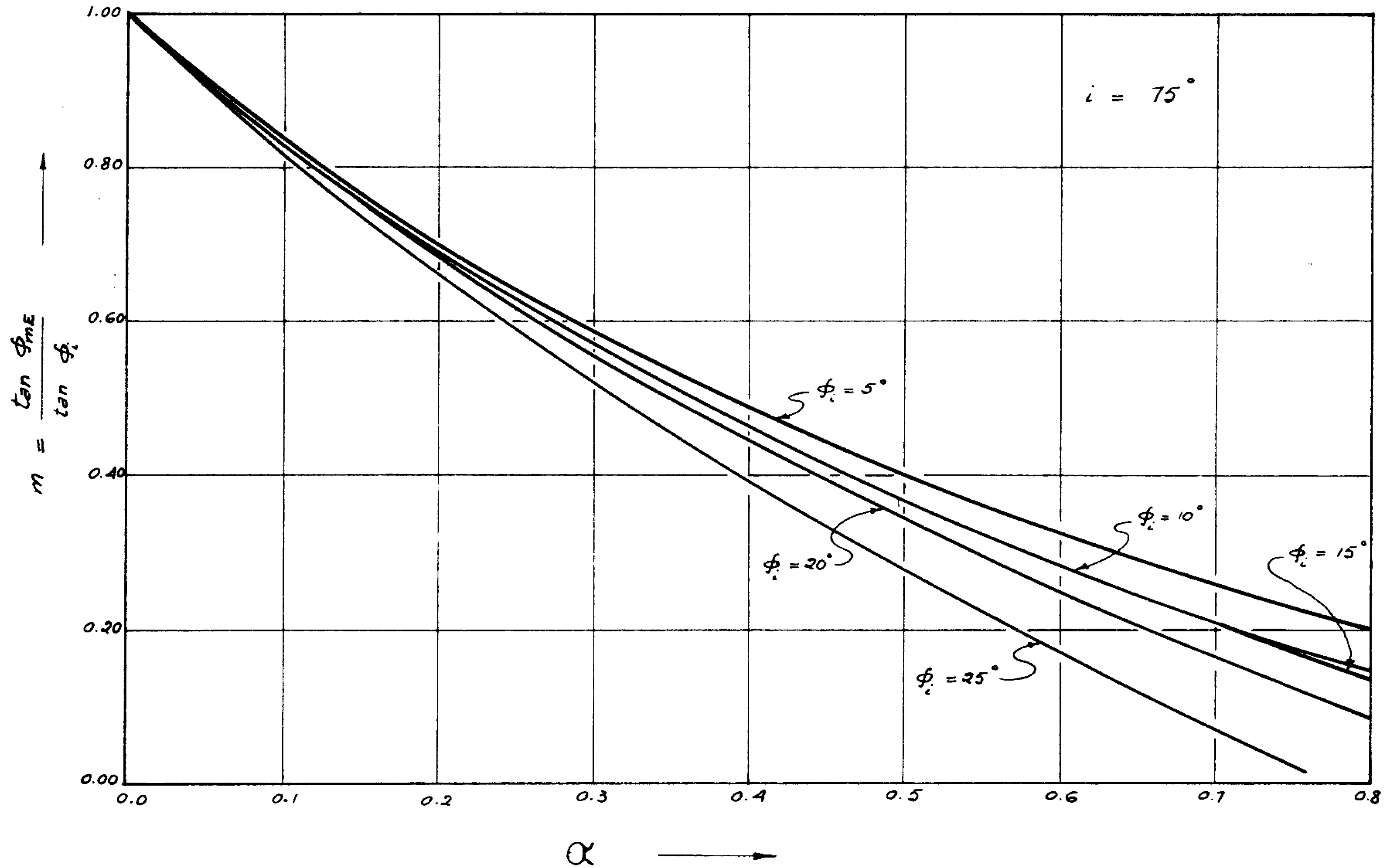


Figure 14. Friction angle ratio m vs. horizontal earthquake acceleration coefficient α for a slope of 75°.

Table 3. Results of the analysis considering pore pressure are tabulated below.

β or δ or ϵ	ϕ_{mP}/ϕ_i With $i=30^\circ$ $\phi_i=15^\circ$ H=50'	With $i=45^\circ$ $\phi_i=20^\circ$ H-20'	With $i=45^\circ$ $\phi_i=20^\circ$ H-75'	With $i=30^\circ$ $\phi_i=20^\circ$ H=50'	Remarks
0.0	1.00	1.00	1.00	1,00	
0.1	0.94	0.91	0.90	0.90	
0.2	0.82	0.83	0.80	0.80	
0.3	0.70	0.72	0.70	0.60	
0.4	0.60	0.62	0.62	0.62	
0.5	0.53	0.54	0.50	0.52	
0.6	0.42	0.40	0.40	0.41	
0.7	0.33	0.30	0.30	0.32	
0.8	0.13	0.20	0,20	0.22	
0.9	0.07	0.08	0.09	0.09	
1.0	-	-	-	-	

The above data are plotted on graph paper and they seem to fit a straight line passing through (1,0) and (0,1)

So, the equation of the line is

$$\beta + \frac{\phi_{mp}}{\phi_i} = 1 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1)$$

or $$\delta + \frac{\phi_{mp}}{\phi_i} = 1 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (2)$$

or $$\epsilon + \frac{\phi_{mp}}{\phi_i} = 1 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

From (1) $$\frac{\phi_{mp}}{\phi_i} = 1 - \beta = 1 - \frac{\gamma_w}{\gamma_s} = \frac{\gamma_b}{\gamma_s} = \frac{G-1}{G+e}$$

so, in case of sudden draw-down

$$\frac{\phi_{mp}}{\phi_i} = \frac{G-1}{G+e}$$

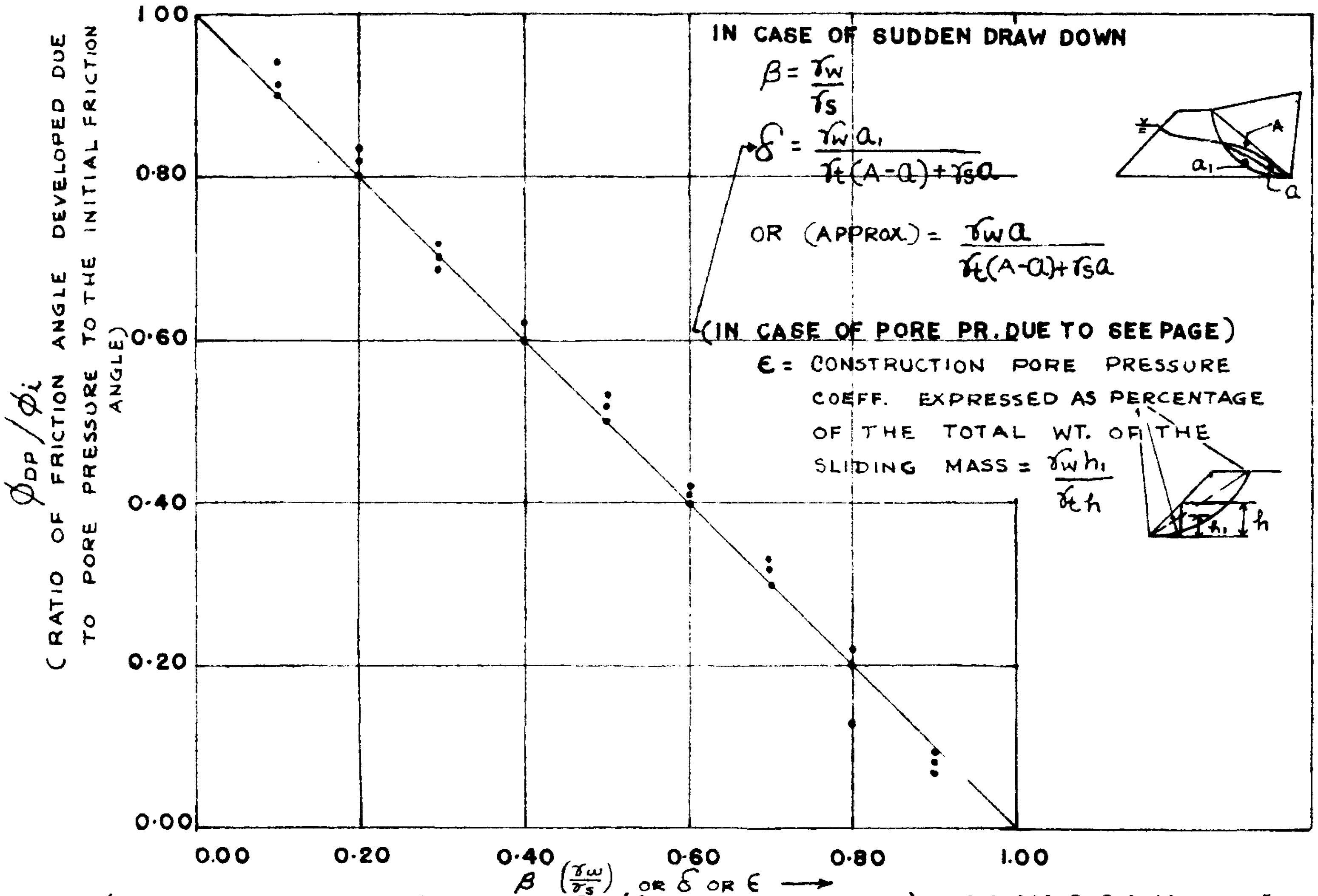


Figure 15. Ratio of developed friction angle (due to pore pressure) and initial friction angle vs. sudden draw-down or construction pore pressure coefficient.

116

CHAPTER VI

APPLICATION TO DESIGN

In the preceding chapters possible design procedures are outlined. As a means of further explaining and clarifying and at the same time verifying this method with the existing but time-consuming slip-circle slice method, the following design problem is taken.

The problem in Figure 18 is a typical section of a dam with the following soil properties

$\phi_i = 10^\circ$; $\gamma_t = 120$ p.c.f.; $\gamma_s = 134.8$ p.c.f.;

$c = 1000$ p.s.f.

Design number 1

The stability analysis of the upstream slope is to be done for the following cases, first by slip-circle slice method and then by ϕ-circle method.

Case I with earthquake ($\alpha = 0.1$)

Case II with pore pressure due to sudden draw-down

Case III with pore pressure due to construction ($\xi = 0.20$)

Design number 2

The stability analysis of the downstream slope is to be done for the case, namely, pore pressure due to seepage. For this one needs to draw the flow net and at least to draw the phreatic line and the equipotential lines. These in turn will give the areas A, a, a_1 in case of ϕ-circle method. Then, one can compute

$$\delta = \frac{\gamma_w a_1}{\gamma_t (A-a) + \gamma_s a}$$

Solution, Design Number 1

Case I. By slip-circle slice method: (Figure 18)

$$F \text{ (factor of safety)} = \frac{c\widehat{L} + \tan\phi_i\,[\gamma_t(\sum N - \sum E_n)]}{\gamma_t(\sum T + \sum E_T)}$$

$$= \frac{1{,}000 \times 317 + 0.1763\,[120\,(10{,}160-236)]}{120\,(2360 + 1{,}016)}$$

$$= 1.30$$

Case I. By ϕ-circle method:

From Figures 10 and 11 (by interpolation) corresponding to i (slope angle) = 16° and $\phi_i = 10°$, for $\alpha = 0.10$ $\tan\phi_{mE}/\tan\phi_i = 0.68$. So, $\phi_{mE} = 6.90$. Let the true factor of safety be F = 1.2, then ϕ_m-actual $= \frac{6.9}{1.02} = 5.8°$. Corresponding to this ϕ_m-actual and i = 16°, $c/F\gamma H = 0.05$ (from Figure 7) . . . . (1)
Now, γ_E (for $\alpha = 0.10$) $= \gamma_t(1 + \alpha\,\ell/d)$ and corresponding to $\phi_i = 10°$ and i = 16°, $\ell/d = 3.85$. So, $\gamma_E = 120\,(1 + 0.10 \times 3.85) =$ 166 p.c.f. So, from (1) $F = \frac{1000}{166 \times 100 \times 0.05} = 1.2$. So the true factor of safety is 1.2.

Case II. By slice method: (Figure 18)

$$F \text{ (factor of safety)} = \frac{c\widehat{L} + \tan\phi_i\,[\gamma_s \sum N - \gamma_w \sum pp\,]}{\gamma_s \sum T}$$

$$= \frac{1{,}000 \times 317 + 0.1763\,[134.8 \times 10{,}160 - 62.5 \times 11{,}880]}{134.8 \times 2{,}360}$$

$$= 1.34$$

Case II. By ϕ-circle method:

From the Figure 15, for $\beta = \frac{\gamma_w}{\gamma_s} = \frac{62.5}{134.8} = 0.465$, $\phi_{mp}/\phi_i = 0.535$. So, $\phi_{mp} = 0.535 \times 10 = 5.35°$. Let F (true factor of safety) by 1.25. Then ϕ_m-actual $= \frac{5.35}{1.25} = 4.27°$. Corresponding to this ϕ_m-actual and i = 16°, $\frac{c}{F\gamma H} = 0.06$ (from Figure 7), or $F = \frac{10000}{0.05 \times 134.8 \times 100} = 1.25$. So, true factor of safety = 1.25.

Case III. By slice method:

If $\epsilon = 0.2$, then $\frac{\gamma_w h_1}{\gamma_t h} = 0.2$, or $\frac{h_1}{h} = \frac{0.2 \times 120}{62.5} = 0.385$.

So, pore pressure due to construction is 38.5 per cent of full hydrostatic pressure. Now, F (factor of safety)

$$= \frac{317,000 + 0.1763\,[120 \times 10,160 - 62.5 \times 0.385 \times 11,880]}{120 \times 2,360}$$

$= 1.70$

Case III. By ϕ-circle method:

From Figure 15, corresponding to $\epsilon = 0.2$, $\frac{\phi_{mp}}{\phi_i} = 0.8$.

So, $\phi_{mp} = 8^\circ$. Let F = 1.55, then ϕ_m-actual $= \frac{8}{1.55} = 5.2^\circ$. Now, for $i = 16^\circ$ and ϕ_m-actual $= 5.2^\circ$, $c/F\gamma H = 0.054$. Or, $F = \frac{1000}{0.054 \times 120 \times 100}$

$= 1.55$. So, true factor of safety $= 1.55$.

Solution, design number 2

Pore pressure due to seepage by slip-circle slice method: (Figure 19)

$$F.S. = \frac{1,000 \times 264 + 0.1763\,[134.8 \times 2950 + 120 \times 3850 - 62.5 \times 2360]}{134.8 \times 1,004 + 120 \times 1,181}$$

$= 1.40$

Pore pressure due to seepage by ϕ-circle method: (Figure 20)

$$\delta = \frac{\gamma_w a_1}{\gamma_t (A-a) + \gamma_s a} = \frac{62.5 \times 3,250}{120\ (12,400 - 5,300) + 134.8 \times 5,300}$$

$= 0.13$, corresponding to $\delta = 0.13$, $\frac{\phi_{mp}}{\phi_i} = 0.87$. So, $\phi_{mp} = 0.87 \times 10 = 8.7^\circ$. Assuming $F = 1.25$, ϕ_m-actual $= \frac{8.7}{1.25} = 7^\circ$. So, corresponding to $i = 22^\circ$, ϕ_m-actual $= 70^\circ$, $c/F\gamma H = 0.063$ . . . (1)

Now, $\gamma_E = \frac{\gamma_t (A-a) + \gamma_s a}{A} = \frac{120\ (12,400 = 5,300) + 134.8 \times 5,300}{12,400}$

$= 126$ tbs/cft. From (1), $F = \frac{1000}{0.063 \times 126 \times 100} = 1.25$. Hence, true factor of safety is 1.25.

CHAPTER VII

SUMMARY AND CONCLUSION

The ϕ-circle method employed to get the relation between the stability number, $c/F\gamma H$ and slope angle, i, for the different ϕ's, makes the problem of stability analysis of earth slopes simpler compared to many other methods so far used in the design.

When the slope angle and the friction angle are known, $c/F\gamma H$ may be obtained directly from Figure 7, while for zero friction angle, if there is a limitation in the depth to which a rupture surface may extend, Figure 8 will furnish the value.

Now, these two figures, namely Figure 7 and Figure 8, are based only on the external load W, the weight of the sliding mass. If now the problem is complicated by introducing the following cases, namely (1) earthquake, (2) pore pressure, (a) due to sudden draw-down, (b) due to seepage, (c) due to construction, then the problem is to get the modified friction angles in those cases.

In case of earthquake Figures 10 to 14 as supplied by the writer from his mathematical and graphical analyses of 40 different cases give the relation between m (the ratio of the tangent of the modified friction angle due to earthquake to the tangent of the friction angle) and α (the coefficient of earthquake acceleration) for a given i (slope angle) and a given ϕ_i (initial friction angle). It is this modified friction angle which will be used to obtain the stability number in Figure 7. And, also in this case, the unit weight of soil in the stability number is the equivalent unit weight, γ_E, and, $\gamma_E = \gamma(1 + \alpha l/d)$ where l/d is the moment arm ratio. Figure 9 supplied by the writer gives the relation between (l/d) and slope angle (i) for different ϕ_i's.

In the case of pore pressure due to sudden drawdown, $\beta\,(\frac{\gamma_w}{\gamma_s}$ the sudden draw-down factor) is calculated and the corresponding ϕ_{mp}/ϕ_i (ratio of friction angle modified due to pore pressure to the initial friction angle) is obtained from Figure 15. The modified friction angle (ϕ_{mp}) is used to obtain the stability number in Figure 7. The unit weight in this case is the saturated unit weight of the soil. In this connection one thing can be observed. Figure 15 represents a straight line whose equation may be written as

$$\frac{\phi_{mp}}{\phi_i} + \beta = 1, \text{ or } \frac{\phi_{mp}}{\phi_i} = 1 - \beta = 1 - \frac{\gamma_w}{\gamma_s}$$

$$\text{or } \frac{\phi_{mp}}{\phi_i} = \frac{\gamma_b}{\gamma_s}$$

$$\text{or } \frac{\phi_{mp}}{\phi_i} = \frac{G-1}{G+e} \text{ (in case of sudden draw-down)}$$

where G is the specific gravity of the soil.

In case of pore pressure due to seepage, seepage factor δ $\left(\frac{\gamma_w a_1}{\gamma_t (A-A) + \gamma_s a} \text{ i.e. } \frac{w^a}{\gamma_t (A-a) + \gamma_s a} \text{ approximately}\right)$ is calculated after drawing the flow net, mainly the phreatic line and the equipotential lines, and the critical circle by ϕ-circle method with the help of Table I. Then, corresponding to this, δ, ϕ_{mp}/ϕ_i is obtained from Figure 15. And, this ϕ_{mp} is used in Figure 7 for the determination of the stability number. The equivalent unit weight (γ_E) to be used in the stability number, is given by $\gamma_E = \frac{\gamma_t (a-a) + \gamma_s a}{A}$.

In case of pore pressure due to construction, $\epsilon = \frac{\gamma_w h_1}{\gamma_t h}$ = construction pore pressure coefficient, where

γ_ω = unit weight of water

γ_t = unit weight of soil at any moisture content

h = full hydrostatic pressure head

h_1 = pore pressure head expressed as a per cent of full hydrostatic pressure head.

Knowing ϵ , ϕ_{mp}/ϕ_i can be obtained from Figure 15 and then with this ϕ_{mp}, stability number may be obtained from Figure 7. The unit weight used in this stability number is γ_t, the unit weight of soil mass at field moisture content. The dependability of the results of such solutions will depend entirely on the conditions. In a few ideal cases the results may be very dependable. More often there will be questionable items, such as the possibility of surface cracking at the top of the slope. In this case the results can be accepted only as rough indications. If good judgment is used in estimating and attempting to evaluate the unknown factors whatever the conditions may be, the results should be of value in arriving at logical conclusions.

For slight variations of soil properties within the sliding mass, average values for cohesion, unit weight and friction angle may be used.

Although stability problems involve more questionable items than most engineering problems, it is seldom feasible to make use of large factors of safety. Conditions are often such that about the largest factor of safety that may be chosen is 1.5. This method being a simple and handy one is advantageous to the designer for designing an earth embankment whether it is an earth dam, railway cut, highway embankment, canal embankment or any such earth slope.

APPENDIX

Information about Table 2 and all other pertinent data

Graphical analysis for the earthquake case done as in Figure 16, gives the following informations.

1. a. For $i = 15^\circ$

$\phi_i = 5^\circ$

H = 50'

α_o	β_o
11°	47.5°

Measured: R = 180 ft.

D = 123 ft.; $\theta_1 = 90^\circ - \beta_o = 42.5^\circ$

$$\bar{y} = \frac{2}{3} \times Dx \frac{(\frac{R}{D})^3 \cos\theta_1 - \cot\theta_1}{(\frac{R}{D})^2 (\frac{\pi}{2} - \theta_1) - \cot\theta_1}$$

= 146 ft. (This is fixed as a_1)

And, a_2 is fixed by the intersection of the medians of the triangle ABC.

$\overline{a_1 a_2} = 34'$;

A_1 = 1,915 sq. ft.
A_2 = 11,200 sq. ft.

Total 13,115 sq. ft.

$$\bar{X} = \frac{1{,}915 \times 34}{13{,}115} = 5 \text{ ft.}$$

$\theta = 2\,\beta_o = 95^\circ = 1.65$ radius

$\hat{L} = R\,\theta = 180 \times 1.65 = 297.5$ ft.

$$a = \frac{R\hat{L}}{L} = \frac{180 \times 297.5}{264} = 203.5 \text{ ft}$$

l	d	l/d
138.5 '	30'	4.616

b. For $i = 15^\circ$

$\phi_i = 5^\circ$

H = 75'

α_o	β_o
11°	47.5°

R = 272 ft.

D = 184 ft.

$\bar{y}$ = 220 ft.

$\overline{a_1a_2}$ = 52.5 ft.

A_1 = 4510 sq. ft., A_2 = 25,750 sq. ft.

$\bar{X} = \frac{4510 \times 52.5}{4510 + 25,750} = 7.8$ ft.

$\theta = 2\beta_o = 95° = 1.65$ radians

$\hat{L} = R\theta = 272 \times 1.65 = 450$ ft.

$a = \frac{R\hat{L}}{L} = \frac{272 \times 450}{401.5} = 306$ ft.

l	d	l/d
209'	43.5'	4.80

2\. a. For i = 15°

ϕ_i = 10°

H = 50'

α_o	β_o
14°	34°

R = 187 ft.

D = 155.5 ft.

$\bar{y}$ = 169 ft.

$\overline{a_1a_2}$ = 33.25 ft.

A_1 = 312 sq. ft; A_2 = 4,820 sq. ft.

$\bar{X} = \frac{312 \times 33.25}{312 + 4,820} = 2.02$ ft.

$\theta = 2\beta_o = 2 \times 34 = 65° = 1.18$ radians

$\hat{L} = R\theta = 187 \times 1.18 = 221$ ft.

$a = \frac{R\hat{L}}{L} = \frac{187 \times 221}{209.5} = 197$ ft.

l	d	l/d
16.25'	42'	3,870

b. For i = 15°

ϕ_i = 10°

α_o	β_o
14°	34°

H = 75'

R = 276 ft.

$D = 229.5$ ft.

$\bar{y} = 245$ ft.

$\overline{a_1a_2} = 41.6$ ft.

$A_1 = 700$ sq. ft.; $A_2 = 10{,}300$ sq. ft.

$$\bar{X} = \frac{700 \times 416}{700 + 10{,}300} = 2.65 \text{ ft.}$$

$\theta = 2\beta_o = 68^\circ = 1.18$ radians; $\widehat{L} = R\theta = 326$ ft.

$$a = \frac{R\widehat{L}}{\bar{L}} = \frac{276 \times 326}{308} = 294 \text{ ft.}$$

l	d	l/d
235'	61.5'	3.83

3. a. For $i = 30^\circ$

$\phi_i = 5^\circ$

α_o	β_o
23°	48°

H = 50'

$R = 86.2$ ft.

$D = 57.8$ ft.

$\bar{y} = 68.7$ ft.

$\overline{a_1a_2} = 19$ ft.

$A_1 = 768$ sq. ft.; $A_2 = 2{,}450$ sq. ft.

$$\bar{X} = \frac{768 \times 19}{768 + 2{,}450} = 4.55 \text{ ft.}$$

$\theta = 2\beta_o = 96^\circ = 1.67$ radians

$\widehat{L} = R\theta = 86.2 \times 1.67 = 144$ ft.

$$a = \frac{R\widehat{L}}{\bar{L}} = \frac{86.2 \times 144}{128.2} = 96.7 \text{ ft.}$$

l	d	l/d
59'	28'	210

4. a. For $i = 30$

$\phi_i = 10^\circ$

α_o	β_o
25°	44°

H = 50'

$R = 87'$

$D = 62.5''$

$\bar{y} = 67$ ft.

$\overline{a_1a_2} = 15.25$ ft.

$A_1 = 578$ sq. ft.; $A_2 = 2,028$ sq. ft.

$\bar{X} = \frac{578 \times 15.25}{578 + 2,028} = 3.38$ ft.

$\theta = 2\beta_o = 88^\circ = 1.53$ radians

$\hat{L} = R\theta = 87 \times 1.53 = 133.5$ radians

$a = \frac{R\hat{L}}{\bar{L}} = \frac{87 \times 133.5}{120.5} = 96'$

l	d	l/d
58.8'	30'	1.96

b. For $i = 30^\circ$

$\phi_i = 10^\circ$

α_o	β_o
25°	44°

$H = 75'$

$R = 127.5'$

$D = 92.5'$

$\bar{y} = 107$ ft.

$\overline{a_1a_2} = 27.5$ ft.

$A_1 = 1,140$ sq. ft.; $A_2 = 4,320$ sq. ft.

$\bar{X} = 5.8$ ft.

$\theta = 2\beta_o = 88^\circ = 1.53$ radians

$\hat{L} = R\theta = 127.5 \times 1.53 = 195$ ft.

$a = \frac{R\hat{L}}{\bar{L}} = \frac{127.5 \times 195}{176.5} = 141$ ft.

l	d	l/d
92'	48'	1.92

5. a. For $i = 30^\circ$

$\phi_i = 15^\circ$

α_o	β_o
27°	39°

$$H = 50'$$

$R = 88.5$ ft.

$D = 69.5$ ft.

$\bar{y} = 77.5$ ft.

$\overline{a_1a_2} = 18.36$ ft.

$A_1 = 327$ sq. ft.; $A_2 = 1{,}560$ sq. ft.

$\bar{X} = 3.16$ ft.

$\theta = 2\beta_o = 78^\circ = 1.36$ radians

$\widehat{L} = R\theta = 88.5 \times 1.36 = 120$ sq. ft.

$$a = \frac{R\widehat{L}}{\bar{L}} = \frac{88.5 \times 120}{110.5} = 96 \text{ ft.}$$

l	d	l/d
66'	36.8'	1.795

b. For $i = 30^\circ$

$\phi_i = 15^\circ$

α_o	β_o
27°	39°

$$H = 75'$$

$R = 133$ ft.

$D = 104$ ft.

$\bar{y} = 113$ ft.

$\overline{a_1a_2} = 23$ ft.

$A_1 = 780$ sq. ft.; $A_2 = 3{,}550$ sq. ft.

$\bar{X} = 4.15$ ft.

$\theta = 78^\circ = 1.36$ radians

$\widehat{L} = R\theta = 133 \times 1.36 = 181$ ft.

$$a = \frac{R\widehat{L}}{\bar{L}} = \frac{133 \times 181}{167} = 144 \text{ ft.}$$

l	d	l/d
96.5'	53'	1.820

6. a. For $i = 30°$

$\phi_i = 20°$

α_o	β_o
28°	31°

$H = 50'$

$R = 94.5$ ft.

$D = 89.4$ ft.

$\bar{y} = 94.5$ ft.

$\overline{a_1 a_2} = 16.5$ ft.

$A_1 = 238$ sq. ft.; $A_2 = 1,108$ sq. ft.

$\theta = 2\,\beta_o = 62° = 108$ radians

$\widehat{L} = R\,\theta = 104 \times 1.08 = 112$ ft.

$a = \frac{R\widehat{L}}{L} = \frac{104 \times 112}{107} = 109$ ft.

l	d	l/d
81'	46'	1.760

b. For $i = 30°$

$\phi_i = 20°$

α_o	β_o
28°	31°

$H = 75'$

$R = 159$ ft.

$D = 136.8$ ft.

$\bar{y} = 139$ ft.

$\overline{a_1 a_2} = 23.35$ ft.

$A_1 = 593$ sq. ft.; $A_2 = 2,600$ sq. ft.

$\bar{X} = 4.33$ ft.

$\theta = 2\,\beta_o = 62° = 108$ radians

$\widehat{L} = R\theta = 159 \times 1.08 = 171$ ft.

$a = \frac{R\widehat{L}}{L} = \frac{159 \times 171}{163} = 167$ ft.

l	d	l/d
120'	67.5'	1.780

7. a. For $i = 30^\circ$

$\phi_i = 25^\circ$

α_o	β_o
29°	25°

H = 50'

R = 122.5 ft.

D = 110.8 ft.

$\bar{y}$ = 111 ft.

$\overline{a_1 a_2}$ = 15 ft.

A_1 = 106.2 sq. ft.; A_2 = 785 sq. ft.

$\bar{X}$ = 1.79 ft.

$\theta = 2\,\beta_o = 50^\circ = 0.87$ radian

$\widehat{L} = R\,\theta = 106.2$ ft.

$$a = \frac{R\widehat{L}}{\bar{L}} = \frac{122.5 \times 106.2}{103.8} = 125 \text{ ft.}$$

l	d	l/d
95.5'	55.3'	1.72

8. a. For $i = 45^\circ$

$\phi_i = 5^\circ$

α_o	β_o
31.2°	42.1°

H = 50'

R = 72.8 ft.

D = 54.2 ft.

$\bar{y}$ = 61 ft.

$\overline{a_1 a_2}$ = 14.35 ft.

A_1 = 820 sq. ft.; A_2 = 1,208 sq. ft.

$\bar{X}$ = 5.32 ft.

$\theta = 2\,\beta_o = 84.2^\circ = 1.46$ radians

$\widehat{L} = R\,\theta = 72.8 \times 1.46 = 106$ ft.

$$a = \frac{R\widehat{L}}{\bar{L}} = \frac{72.8 \times 106}{97.5} = 79.2 \text{ ft.}$$

l	d	l/d
47'	3.13'	1.505

9. a. For $i = 45^\circ$

$\phi_i = 10^\circ$

α_o	β_o
34°	39.7°

H = 50'

R = 70.5 ft.

D = 54.5 ft.

$\bar{y} = 61.4$ ft.

$\overline{a_1a_2} = 14.2$ ft.

$A_1 = 562$ sq. ft. $A_2 = 1,060$ sq. ft.

$\bar{X} = 4.9$ ft.

$\theta = 2\beta_o = 79.4^\circ = 1.38$ radians

$\hat{L} = R\theta = 70.5 \times 1.38 = 97.5$ ft.

$a = \frac{R\hat{L}}{\bar{L}} = \frac{70.5 \times 97.5}{90} = 76.4$ ft.

l	d	l/d
46'	34.5'	1.330

b. For $i = 45^\circ$

$\phi_i = 10^\circ$

α_o	β_o
34°	39.7°

H = 75'

R = 106'

D = 81.7'

$\bar{y} = 90$ ft.

$\overline{a_1a_2} = 19.25$ ft.

$A_1 = 1,475$ sq. ft. $A_2 = 2,350$ sq. ft.

$\bar{X} = 7.43$ ft.

$\theta = 2\beta_o = 79.4^\circ = 1.38$ radians

$\hat{L} = R\theta = 143.5$ ft.

$a = \frac{R\hat{L}}{\bar{L}} = \frac{106 \times 143.5}{134.75} = 113$ ft.

l	d	l/d
67.3'	50.5'	1.330

10. a. For $i = 45°$

$\phi_i = 15°$

α_o	β_o
36.1°	37.2°

H = 50'

R = 70 ft.

D = 56.2 ft.

$\bar{y} = 59.3$ ft.

$\overline{a_1 a_2} = 11.5$ ft.

$A_1 = 495$ sq. ft.; $A_2 = 850$ sq. ft.

$\bar{X} = 4.24$ ft.

$\theta = 2\,\beta_o = 74.4° = 1.295$ radians

$\hat{L} = R\,\theta = 70 \times 1.295 = 90.6$ ft.

$a = \frac{R\hat{L}}{\bar{L}} = \frac{70 \times 90.6}{84.7} = 75$ ft.

l	d	l/d
44'	36.5'	1.200

b. For $i = 45°$

$\phi_i = 15°$

α_o	β_o
36.1°	37.2°

H = 75'

R = 104.8 ft.

D = 83.8 ft.

$\bar{y} = 94$ ft.

$\overline{a_1 a_2} = 21$ ft.

$A_1 = 1100$ sq. ft. $A_2 = 1,965$ sq. ft.

$\bar{X} = 7.55$ ft.

$\theta = 2\,\beta_o = 74.4° = 1.29$ radians

$\widehat{L} = R\theta = 104.8 \times 1.29 = 135$ ft.

$a = \frac{R\widehat{L}}{\overline{L}} = \frac{104.8 \times 135}{126.5} = 112$ ft.

l	d	l/d
68.2'	56'	1.22

11. a. For $i = 45^\circ$

$\phi_i = 20^\circ$

α_o	β_o
38°	34.5°

H = 50'

R = 71 ft.

D = 59 ft.

$\bar{y} = 63.5$ ft.

$\overline{a_1a_2} = 12.38$ ft.

$A_1 = 350$ sq. ft.

$A_2 = 718$ sq. ft.

$\bar{X} = \frac{350 \times 12.38}{1,068} = 4.05$ ft.

$\theta = 2\beta_o = 69^\circ = 1.20$ radians

$\widehat{L} = R\theta = 85.2$ ft.

$a = \frac{R\widehat{L}}{\overline{L}} = \frac{71 \times 85.2}{80.5} = 75.1$ ft.

l	d	l/d
46'	40.5'	1.135

b. For $i = 45^\circ$

$\phi_i = 20^\circ$

α_o	β_o
38°	34.5°

H = 75'

R = 108 ft.

D = 89.5 ft.

$\bar{y} = 93$ ft.

$\overline{a_1a_2} = 16.8$ ft.

$A_1 = 832$ sq. ft.; $A_2 = 1,620$ sq. ft.

$\bar{X} = 5.7$ ft.

$\theta = 2\beta_o = 69° = 120$ radians

$\widehat{L} = R\theta = 129.6$ ft.

$a = \frac{R\widehat{L}}{\bar{L}} = \frac{108 \times 129.6}{121.5} = 115$ ft.

l	d	l/d
68'	59.6'	1.140

12. a. For $i = 45°$

$\phi_i = 25°$

α_o	β_o
40°	31°

H - 50'

R = 75.6 ft.

D = 65 ft.

$\bar{y} = 66.5$ ft.

$\overline{a_1a_2} = 11.1$ ft.

$A_1 = 225$ sq. ft.; $A_2 = 531$ sq. ft.

$\bar{X} = 3.3$ ft.

$\theta = 2\beta_o = 62° = 1.08$ radians

$\widehat{L} = R\theta = 81.5$ ft.

$a = \frac{R\widehat{L}}{L} = \frac{75.6 \times 81.5}{78} = 79$ ft.

l	d	l/d
48'	44.4'	1.080

13. a. For $i = 60°$

$\phi_i = 5°$

α_o	β_o
38.5'	34.5°

H = 50'

R = 70 ft.

D = 58 ft.

$\bar{y} = 60.3$ ft.

$\overline{a_1a_2} = 10.5$ ft.

$A_1 = 826$ sq. ft.; $A_2 = 630$ sq. ft.

$\overline{X} = 5.98$ ft.

$\theta = 2\beta_o = 69° = 1.20$ radians

$\widehat{L} = R\theta = 70 \times 1.20 = 84$ ft.

$a = \frac{R\widehat{L}}{\overline{L}} = \frac{70 \times 84}{79.5} = 74$ ft.

l	d	l/d
41.2'	36.8'	1.12

b. For i = 60°

$\phi_i = 5°$

α_o	β_o
38.5°	34.5°

H = 75'

R = 106 ft.

D = 87.5 ft.

$\overline{y} = 93.7$ ft.

$\overline{a_1a_2} = 18$ ft.

$A_1 = 1,890$ sq. ft.; $A_2 = 1,436$ sq. ft.

$\overline{X} = 10.2$ ft.

$\theta = 2\beta_o = 69° = 1.2$ radians

$\widehat{L} = R\theta = 106 \times 1.2 = 127.2$ ft.

$a = \frac{R\widehat{L}}{\overline{L}} = \frac{106 \times 127.2}{120} = 112.2$ ft.

l	d	l/d
63.5'	55.8'	1.14

14. a. For i = 60°

$\phi_i = 10°$

α_o	β_o
41°	33°

H = 50'

R = 70.2 ft.

D = 59.2 ft.

$\overline{y} = 62.5$ ft.

$\overline{a_1a_2} = 10.8$ ft.

$A_1 = 760$ sq. ft.; $A_2 = 624$ sq. ft.

$\bar{X} = \frac{760 \times 10.8}{1,384} = 5.92$ ft.

$\theta = 2\beta_o = 66° = 1.145$ radians

$\widehat{L} = R\theta = 80.5$ ft.

$a = \frac{R\widehat{L}}{\bar{L}} = \frac{70.2 \times 80.5}{76} = 74.5$ ft.

l	d	l/d
41.2'	39.8'	1.035

b. For $i = 60°$

$\phi_i = 10°$

α_o	β_o
41°	33°

H = 75'

R = 105 ft.

D = 88 ft.

$\bar{y} = 92$ ft.

$\overline{a_1a_2} = 16$ ft.

$A_1 = 1,710$ sq. ft.; $A_2 = 1,385$ sq. ft.

$\bar{X} = 8.85$ ft.

$\theta = 2\beta_o = 66° = 1.145$ radians

$\widehat{L} = R\theta = 105 \times 1.145 = 120$ ft.

$a = \frac{R\widehat{L}}{\bar{L}} = \frac{105 \times 120}{114} = 110.5$ ft.

l	d	l/d
60.8'	59.5'	1.020

15. a. For $i = 60°$

$\phi_i = 15°$

α_o	β_o
44°	31.5°

H = 50'

R = 70 ft.

D = 60 ft.

$\bar{y}$ = 64.8 ft.

$\overline{a_1a_2}$ = 11.8 ft.

A_1 = 630 sq. ft.; A_2 = 523 sq. ft.

$\bar{X}$ = 6.5 ft.

$\theta = 2\beta_o = 63° = 1.1$ radians

$\hat{L} = R\theta = 70 \times 1.1 = 77$ ft.

$a = \frac{R\hat{L}}{\bar{L}} = \frac{70 \times 77}{72.4} = 74.5$ ft.

l	d	l/d
40.5'	43.6'	0.930

b. For i = 60°

ϕ_i = 15°

α_o	β_o
44°	31.5°

H = 75'

R = 104.5 ft.

D = 89.5 ft.

$\bar{y}$ = 94.5 ft.

$\overline{a_1a_2}$ = 16.25 ft.

A_1 = 1,410 sq. ft.; A_2 = 1,208 sq. ft.

$\bar{X}$ = 8.7 ft.

$\theta = 2\beta_o = 63° = 1.095$ radians

$\hat{L} = R\theta = 114.5$

$a = \frac{R\hat{L}}{\bar{L}} = \frac{104.5 \times 114.5}{109} = 110$ ft.

l	d	l/d
61'	64'	0.938

16. a. For i = 60°

ϕ_i = 20°

α_o	β_o
46.5°	30.2°

H = 50'

R = 68.2 ft.

D = 59 ft.

$\bar{y}$ = 61.7 ft.

$\overline{a_1a_2}$ = 10.25 ft.

A_1 = 487 sq. ft.; A_2 = 465 sq. ft.

$\bar{X}$ = 5.23 ft.

$\theta = 2\beta_o$ = 60.4 = 1.05 radians

$\widehat{L} = R\theta$ = 68.2 x 1.05 = 71.5 ft.

$a = \dfrac{R\widehat{L}}{\bar{L}} = \dfrac{68.2 \times 71.5}{69}$ = 70.5 ft.

l	d	l/d
37.8'	45'	0.840

b. For i = 60°

ϕ_i = 20°

α_o	β_o
46.5°	30.2°

H = 75'

R = 102 ft.

D = 88 ft.

$\bar{y}$ = 90.5 ft.

$\overline{a_1a_2}$ = 14.5 ft.

A_1 = 1,110 sq. ft.; A_2 = 1,050 sq. ft.

$\bar{X}$ = 7.45 ft.

$\theta = 2\beta_o$ = 60.4° = 1.05 radians

$\widehat{L} = R\theta$ = 107 ft.

$a = \dfrac{R\widehat{L}}{\bar{L}} = \dfrac{102 \times 107}{104}$ = 105 ft.

l	d	l/d
55'	65.8'	0.834

17. a. For i = 60°

ϕ_i = 25°

α_o	β_o
50°	30°

H = 50'

R = 65.5 ft.

D = 57 ft.

$\bar{y}$ = 58.5 ft.

$\overline{a_1a_2}$ = 9.62 ft.

A_1 = 330 sq. ft.; A_2 = 370 sq. ft.

$\bar{X}$ = 4.54 ft.

$\theta = 2\beta_o = 60° = 1,046$ radians

$\widehat{L} = R\theta = 65.5 \times 1.046 = 68.2$ ft.

$a = \frac{R\widehat{L}}{\bar{L}} = \frac{65.5 \times 68.2}{65} = 68.5$ ft.

l	d	l/d
32.8'	45.5'	0.720

b. For i = 60°

ϕ_i = 25°

α_o	β_o
50°	30°

H = 75'

R = 98.2 ft.

D = 85 ft.

$\bar{y}$ = 90.3 ft.

$\overline{a_1a_2}$ = 16.25 ft.

A_1 = 758 sq. ft.; A_2 = 872 sq. ft.

$\theta = 2\beta_o = 60° = 1.046$ radians

$\widehat{L} = R\theta = 102.5$ ft.

$a = \frac{R\widehat{L}}{\bar{L}} = \frac{98.2 \times 102.5}{98.2} = 102.5$ ft.

l	d	l/d
50.8'	69'	0.735

18. a. For i = 75°

ϕ_i = 5°

α_o	β_o
45°	25°

H = 50'

R = 83.5 ft.

D = 75.8 ft.

$\overline{y} = 115$ ft.

$\overline{a_1 a_2} = 14.25$ ft.

$A_1 = 2,215$ sq. ft.; $A_2 = 860$ sq. ft.

$\overline{X} = 10.25$ ft.

$\theta = 2\beta_o = 50° = 0.87$ radian

$\widehat{L} = R\theta = 110$ ft.

$a = \frac{R\widehat{L}}{\overline{L}} = \frac{126.5 \times 110}{106.5} = 131$ ft.

l	d	l/d
72.5'	77'	0.946

19. a. For $i = 75°$

$\phi_i = 10°$

α_o	β_o
47.5°	23.5°

H = 50'

R = 85 ft.

D = 78 ft.

$\overline{a_1 a_2} = 11$ ft.

$A_1 = 865.8$ sq. ft.; $A_2 = 344.2$ sq. ft.

$\overline{X} = 7.88$ ft.

$\theta = 2\beta_o = 47° = 0.82$ radian

$\widehat{L} = R\theta = 69.5$ ft.

$a = \frac{R\widehat{L}}{\overline{L}} = \frac{85 \times 69.5}{67.5} = 87.5$ ft.

l	d	l/d
46.6'	56'	0.832

b. For $i = 75°$

$\phi_i = 10°$

α_o	β_o
47.5°	23.5°

H = 75'

R = 128.8 ft.

D = 118.4 ft.

$\overline{a_1a_2}$ = 14.7 ft.

A_1 = 1,965 sq. ft. A_2 = 740 sq. ft.

$\bar{X}$ = 10.7 ft.

$\theta = 2\ \beta_o = 47^\circ = 0.82$ radian

$\widehat{L} = R\ \theta$ = 105.5 ft.

$a = \frac{R\widehat{L}}{\bar{L}} = \frac{128.8 \times 105.5}{101.8}$ = 133.5 ft.

l	d	l/d
71'	84'	0.845

20. a. For i = 75°

ϕ_i = 15°

α_o	β_o
50°	23°

H = 50'

R = 84 ft.

D = 77.4 ft.

$\bar{y}$ = 78.5 ft.

$\overline{a_1a_2}$ = 9.6 ft.

A_1 = 765 sq. ft. A_2 = 307 sq. ft.

$\theta = 46^\circ = 0.8$ radian

$\widehat{L} = R\ \theta$ = 67.2 ft.

$a = \frac{R\widehat{L}}{\bar{L}} = \frac{84 \times 67.2}{65.6}$ = 86.2 ft.

$\bar{X}$ = 6.85 ft.

l	d	l/d
44'	58'	0.756

b. For i = 75°

ϕ_i = 15°

α_o	β_o
50°	23°

H = 75'

R = 123 ft.

D = 113.4 ft.

$\bar{y} = 114$ ft.

$\overline{a_1a_2} = 13.75$ ft.

$A_1 = 1{,}700$ sq. ft.; $A_2 = 675$ sq. ft.

$\bar{X} = 9.85$ ft.

$\theta = 2\beta_o = 46° = 0.8$ radian

$\widehat{L} = R\theta = 98.4$ ft.

$a = \frac{R\widehat{L}}{\bar{L}} = \frac{123 \times 98.4}{97} = 125$ ft.

l	d	l/d
63.8'	84.6'	0.755

21. a. For $i = 75°$

$\phi_i = 20°$

α_o	β_o
53°	22°

H = 50'

R = 82.7 ft.

D = 76.7 ft.

$\bar{y} = 77.5$ ft.

$\overline{a_1a_2} = 9.30$ ft.

$A_1 = 660$ sq. ft.; $A_2 = 268$ sq. ft.

$\bar{X} = 6.62$ ft.

$\theta = 2\beta_o = 44° = 0.765$ radian

$\widehat{L} = R\theta = 63.4$ ft.

$a = \frac{R\widehat{L}}{\bar{L}} = \frac{82.7 \times 63.4}{62.4}$ 84 ft.

l	d	l/d
40.3'	61'	0.665

b. For $i = 75°$

$\phi_i = 20°$

α_o	β_o
53°	22°

$\underline{H = 75'}$

$R = 126.5$ ft.

$D = 117.5$ ft.

$\bar{y} = 118$ ft.

$\overline{a_1 a_2} = 13.1$ ft.

$A_1 = 1,460$ sq. ft.; $A_2 = 598$ sq. ft.

$\bar{X} = 9.25$ ft.

$\theta = 2\beta_o = 44° = 0.765$ radian

$\widehat{L} = R\theta = 96.5$ ft.

$a = \frac{R\widehat{L}}{\bar{L}} = \frac{126.5 \times 96.5}{93.8} = 130$ ft.

l	d	l/d
62'	92'	0.675

22. a. For $i = 75°$

$\phi_i = 25°$

α_o	β_o
56°	22°

$\underline{H = 50'}$

$R = 81.3$ ft.

$D = 75.5$ ft.

$\bar{y} = 77.5$ ft.

$\overline{a_1 a_2} = 9.75$ ft.

$A_1 = 554$ sq. ft. $A_2 = 242$ sq. ft.

$\bar{X} = 6.8$ ft.

$\theta = 2\beta_o = 44° = 0.765$ radian

$\widehat{L} = R\theta = 62$ ft.

$a = \frac{R\widehat{L}}{\bar{L}} = \frac{81.3 \times 62}{61} = 82.5$ ft.

l	d	l/d
37.5'	61.5'	0.61

22. b. For $i = 75^\circ$

$\phi_i = 25^\circ$

α_o	β_o
56°	22°

$\underline{H = 75'}$

$R = 121$ ft.

$D = 112.2$ ft.

$\bar{y} = 115$ ft.

$\overline{a_1 a_2} = 14.5$ ft.

$A_1 = 1{,}195$ sq. ft. $\qquad A_2 = 540$ sq. ft.

$\bar{X} = 10$ ft.

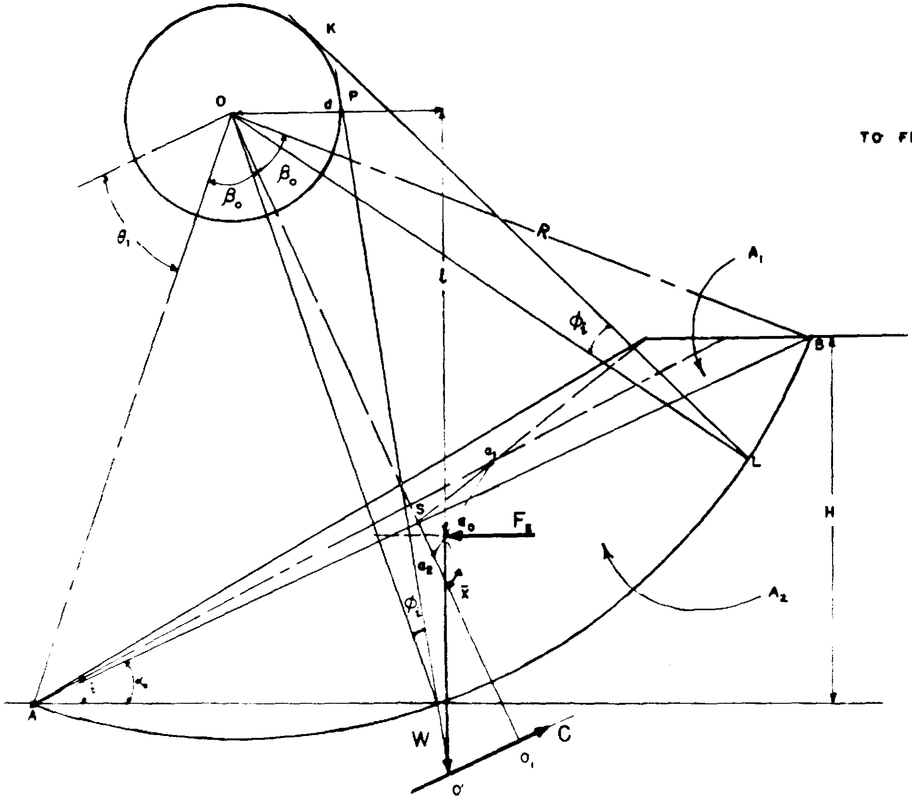
$\theta = 2\,\beta_o = 44^\circ = 0.765$ radian

$\widehat{L} = R\,\theta = 92.5$ ft.

$$a = \frac{R\widehat{L}}{\bar{L}} = \frac{121 \times 92.5}{91} = 123 \text{ ft.}$$

l	d	l/d
54.3'	92.5'	0.59

STABILITY ANALYSIS DUE TO EARTHQUAKE (HORIZONTAL)



INFORMATION :—

$i = 30°$

$\phi_i = 10°$

$H = 50'$

α_o	β_o
25°	44°

TO FIND THE C.G. OF THE CIRCULAR PORTION:

FROM THE DRAWING — $R = 87'$

$D = 62.5'$ (OS)

$\sin \theta_1 = \frac{D}{R} = 0.72$

SO, $\theta_1 = 46° = 0.805$ RADIAN $\left(\frac{\pi}{2} - \beta_o\right)$

$$\bar{Y} = \frac{2}{3} D \frac{\left(\frac{R}{D}\right)^3 \cos\theta_1 - \cot\theta_1}{\left(\frac{R}{D}\right)^2 \left(\frac{\pi}{2} - \theta_1\right) - \cot\theta_1}$$

$\bar{Y} = 67'$ ($= Oa_2$)

THIS GIVES POINT - a_2

THE C.G. (a_1) OF THE TRIANGULAR PORTION IS GIVEN BY THE INTERSECTION OF THE MEDIANS.

SO THEN a_1a_2 IS MEASURED AS 15.25 FT.

THE AREA OF THE TRIANGLE (A_1) = 578 FT²

THE AREA OF THE CIRCULAR PORTION (A_2) = 2,028 FT²

THE COMBINED C.G. IS GIVEN BY

$$\bar{x} = \frac{A_1(\overline{a_1a_2})}{(A_1 + A_2)} = 3.38'$$

TO CALCULATE OO_1 (THE LENGTH a)

$\theta = 2\beta_o = 88° = 1.53$ RADIAN

$\widehat{L} = R\theta = 133.5'$

$$a = \frac{R\widehat{L}}{L} = \frac{87 \times 133.5}{120.5} = 96'$$

l	d	l/d
58.8'	30'	1.960

FIGURE 16

SCALE

STABILITY ANALYSIS DUE TO PORE PRESSURE

INFORMATION –

$i = 45°$ $\quad$ α_o | β_o = 38° | 34.5°

$\phi = 20°$

$H = 50'$

$R = 71'$ $\quad$ $D = 59'$

$\bar{Y} = 63.5'$ $\quad$ $\bar{X} = 4.05'$

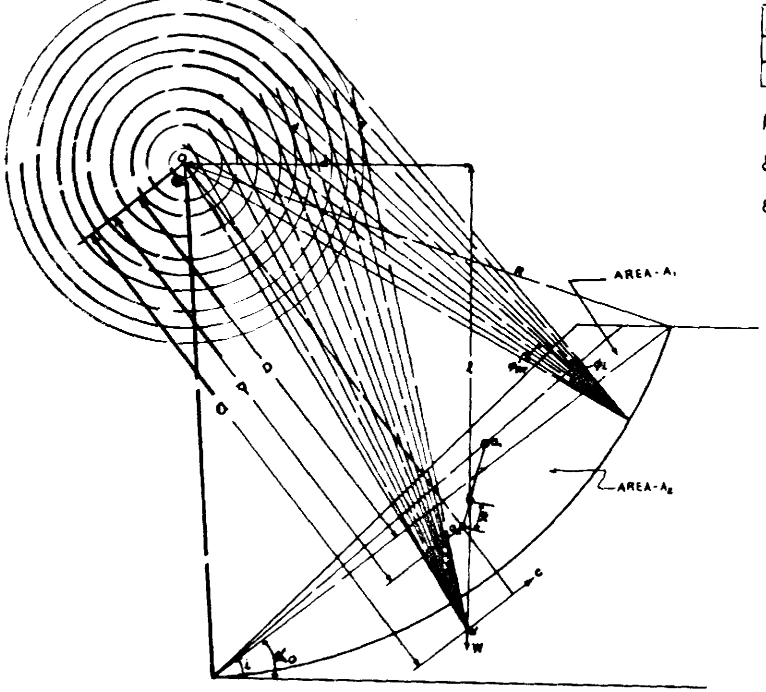
$a = 75.1'$

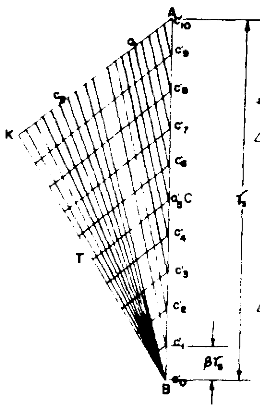
$\beta - \delta - \epsilon$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
ϕ_{D}	20°	18.1°	16.5°	14.3°	12.4°	10.8°	8.0°	6.0°	4.0°	1.5°	–
$\frac{\tan\phi_D}{\tan\phi}$	1.0	0.91	0.83	0.72	0.62	0.54	0.40	0.30	0.20	0.08	–

$\beta = \frac{\gamma_w}{\gamma_s}$ = SUDDEN DRAWDOWN FACTOR

$\delta = \frac{\gamma_w a_1}{\gamma_t (A - a) + \gamma_s a}$ = SEEPAGE FACTOR

$\epsilon = \frac{\gamma_w h_1}{\gamma_t h}$ = CONSTRUCTION PORE PRESSURE COEFFICIENT





IN CASE OF DRAWDOWN

$c_{10} c_0 = \gamma_s$

$c_1 c_0, c_2 c_0$ etc $= \gamma_w$

IN CASE OF PORE PR. DUE TO SEEPAGE

$c_{10} c'_0 = \gamma_t (A - a) + \gamma_s a$

$c_0 c'_1$ or $c'_2 c'_0$ etc $= \gamma_w a_1$

WHERE γ_t = MOIST DENSITY OF SOIL ABOVE PHREATIC LINE.

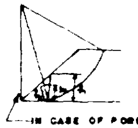
γ_s = SATURATED UNIT WT.

γ_w = UNIT WT OF WATER.

A = TOTAL AREA WITHIN SLIP CIRCLE.

a = TOTAL AREA WITHIN AND BELOW THE PHREATIC LINE.

a_1 = TOTAL AREA BETWEEN THE SLIP CIRCLE AND PORE PR. DIAGRAM



IN CASE OF PORE PRESSURE DUE TO CONSTRUCTION

$c_{10} c'_0$ = TOTAL WT. OF SOIL.

$c'_1 c'_0$ OR $c'_2 c'_0$ etc = PORE PR. EFFECT EQUIVALENT TO WT. CAUSING OVER TURNING MOMENT EXPRESSED AS A PERCENT OF TOTAL WT. OF SOIL.

$= \gamma_w h_1 = \epsilon \gamma_t h$

FIGURE 17

0 10

SCALE

STABILITY ANALYSIS OF U/S SLOPE WITH (a) EARTHQUAKE (b) PORE PRESSURE DUE TO SUDDEN DRAWDOWN (c) PORE PRESSURE DUE TO CONSTRUCTION

BY SLIP CIRCLE METHOD.

(TAKING E = 0·2)
(PORE PRESSURE = 0·2 TIMES THE WEIGHT ON THE SLICE ACTING IN THE DIRECTION OF THE NORMAL.)

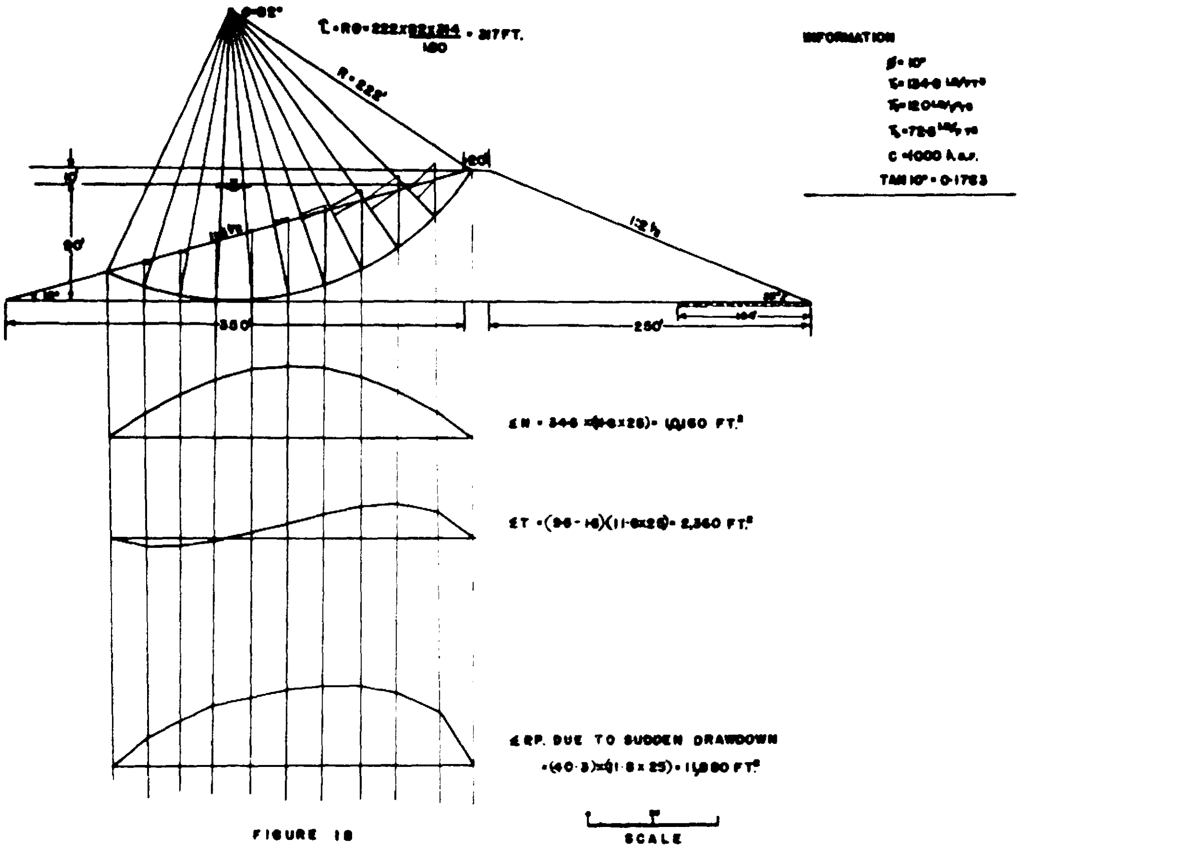


FIGURE 18

STABILITY ANALYSIS OF D/S SLOPE WITH PORE PRESSURE DUE TO SEEPAGE BY SLIP CIRCLE METHOD

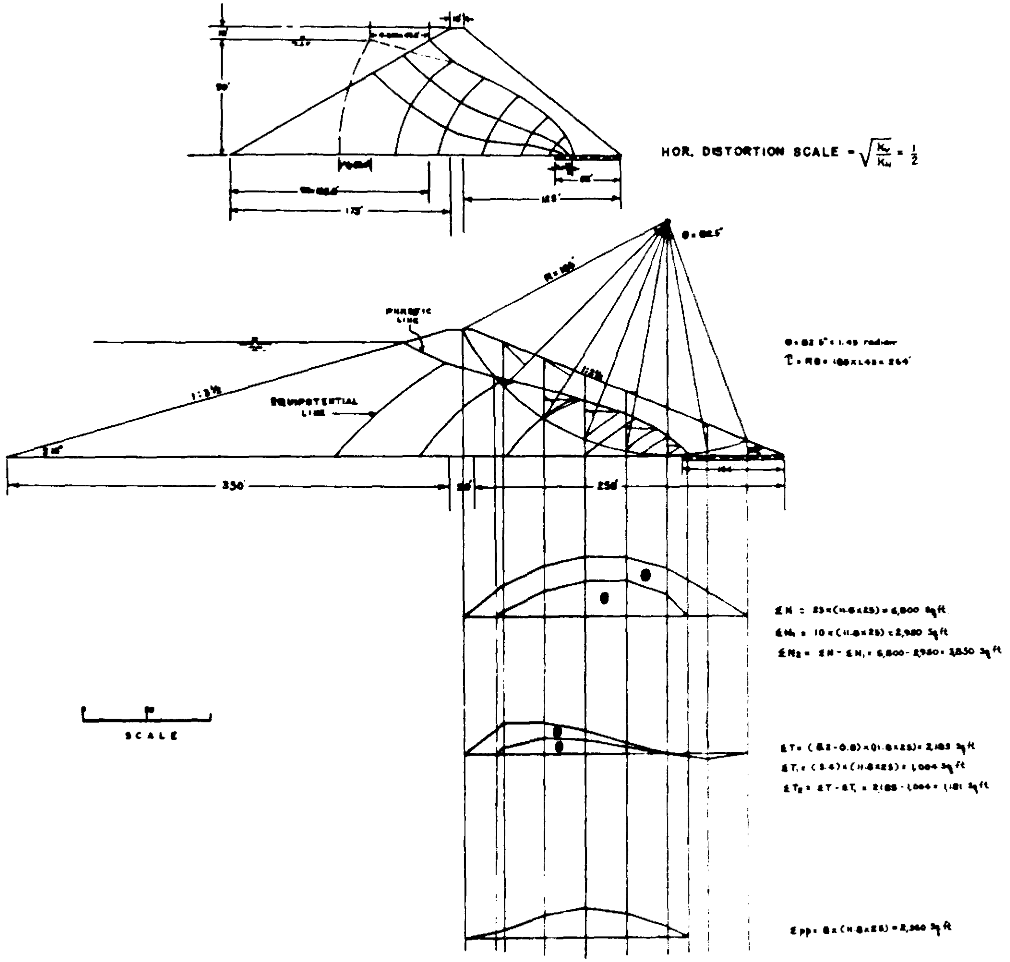


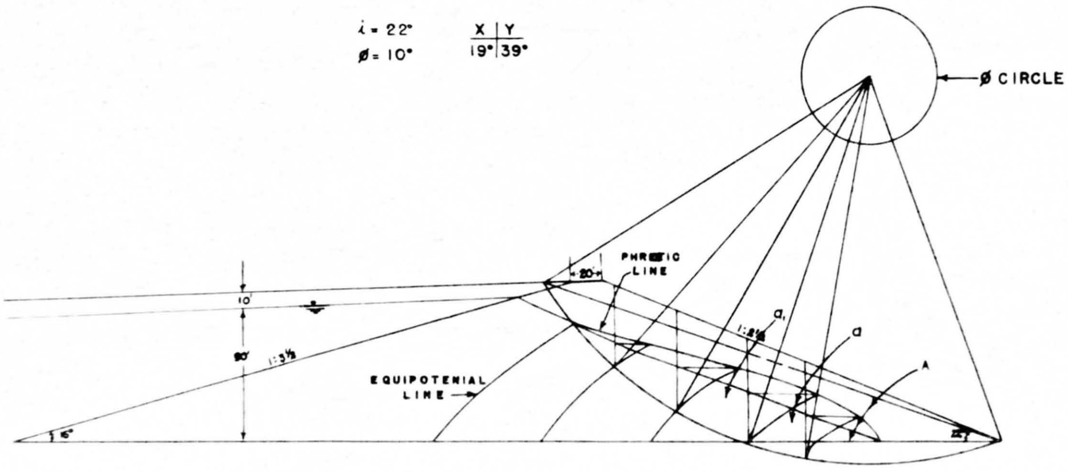
FIGURE 18

STABILITY ANALYSIS OF D/S SLOPE WITH PORE PRESSURE DUE TO SEEPAGE BY Ø-CIRCLE METHOD.

$i = 22°$

$\phi = 10°$

X	Y
19°	39°



CHECKING FOR SEEPAGE :—

$$\delta = \frac{\gamma_w a_1}{\gamma_t (A-a) + \gamma_s a} = 0.13$$

$$\gamma_c = \frac{\gamma_t (A-a) + \gamma_s a}{A} = 126 \text{ lbs/cft}$$

$\gamma_t = 120$ lbs/cft

$\gamma_s = 134.8$ lbs/cft

$\gamma_w = 62.5$ lbs/cft

AREA-A = 42 × (11·8 × 25) = 12,400 Sq ft

AREA-a = 18 × (11·8 × 25) = 5,300 Sq ft.

AREA-a_1 = 11 × (11·8 × 25) = 3,250 Sq ft

FIGURE 20

0 50
SCALE

VERIFICATION OF TABLE 1 (After D. W. Taylor)

BY 1620 COMPUTER PROGRAM

```
C C  DATA ON CRITICAL CIRCLES BY PHI-CIRCLE METHOD
   1 READ, AI,FI,X,Y
     RAD=3.1415927/180.
     AI=RAD*AI
     FI=RAD*FI
     X=RAD*X
     Y=RAD*Y
     COTX= (X)/SIN(X)
     COTY=COS(Y)/SIN(Y)
     COTI=COS(AI)/SIN(AI)
     SINFI=SIN(FI)
     SECX=1./COS(X)
     CSCX=1./SIN(X)
     CSCY=1./SIN(Y)
     AN=.5*(COTX-COTY-COTI+SINFI*CSCX*CSCY)
     IF(AN)3,4,4
   3 BN=AN
     AN=0.
     GO TO 5
   4 BN=AN
   5 UNUM=.5*CSCX*CSCX*(Y*CSCX*CSCY-COTY)+COTX-COTI-x.*AN
     UDEN=(1.-2.*COTI*COTI)/3.+COTI*(COTX-COTY)+COTX*COTY
     UDEN=UDEN+2.*AN*(AN-SINFI*CSCX*CXCY)
     COTU=UNUM*Y*SECY*CSCX*CSCY*CSCY/UDEN-1./COTX
     U=ATAN(1./COTU)
     SINU=SIN(U)
     SINB=UNUM*SINU*CSCX*CSCY*SINFI/UDEN
     COSB=SQRT(1.-SINB*SINB)
     B=ATAN(SINB/COSB)
     V=U-B
     COTV=COX(V)/SIN(V)
     CFWH=UNUM/(2.*COTX*COTV+2)
     D=.5*(CSCX*CSCY-COTX*COTY+1.)
     PUNCH, BN, D, CFWH
     GO TO 1
     END
```

DATA CARDS

i	ϕ	α_0	β_0
90.0	0.0	47.6	15.1
90.0	5.0	50.0	14.0
90.0	10.0	53.0	13.5
90.0	15.0	56.0	13.0
90.0	20.0	58.0	12.0
90.0	25.0	60.0	11.0
75.0	0.0	41.8	25.9
75.0	5.0	45.0	25.0
75.0	10.0	47.5	23.5
75.0	15.0	50.0	23.0
75.0	20.0	53.0	22.0
75.0	25.0	56.0	22.0
60.0	0.0	35.3	35.4
60.0	5.0	38.5	34.5
60.0	10.0	41.0	33.0
60.0	15.0	44.0	31.5
60.0	20.0	46.5	30.2
60.0	25.0	50.0	30.0
45.0	0.0	28.2	44.7
45.0	5.0	31.2	42.1
45.0	10.0	34.0	39.7
45.0	15.0	36.1	37.2
45.0	20.0	38.0	34.5
45.0	25.0	40.0	31.0
30.0	0.0	20.0	53.4
30.0	5.0	20.0	53.0
30.0	10.0	25.0	44.0
30.0	15.0	27.0	39.0
30.0	20.0	28.0	31.0
30.0	25.0	29.0	25.0
15.0	0.0	10.6	60.7
15.0	5.0	11.0	47.5
15.0	10.0	14.0	34.0

OUTPUT

n	D	$c/F\gamma H$
-1.3965	1.4070	2.6101E-01
-1.3507	1.5153	2.3873E-01
-1.2402	1.6125	2.1831E-01
-1.1346	1.7203	1.9938E-01
-1.0700	1.8659	1.8176E-01
-1.0048	2.0407	1.6517E-01
-6.0446E-01	1.0657	2.1908E-01
-5.6040E-01	1.1009	1.9490E-01
-5.3040E-01	1.1470	1.7303E-01
-4.6000E-01	1.1821	1.5302E-01
-4.2313E-01	1.2387	1.3461E-01
-3.5386E-01	1.2752	1.1746E-01
-2.8607E-01	1.0000	1.9056E-01
-2.6400E-01	1.0035	1.6333E-01
-2.4043E-01	1.0136	1.3924E-01
-2.3029E-01	1.0327	1.1772E-01
-2.0460E-01	1.0551	9.8256E-01
-1.8346E-01	1.9787	8.0241E-02
-7.2768E-02	1.0619	1.7035E-01
-1.0229E-01	1.0260	1.3779E-01
-1.1790E-01	1.0069	1.1003E-01
-1.0978E-01	1.0003	8.5923E-02
-9.7132E-02	1.0027	6.4806E-02
-9.7983E-02	1.0186	4.6304E-02
1.3638E-01	1.3007	1.5707E-01
2.9047E-01	1.2953	1.1312E-01
-1.5790E-02	1.0928	7.7681E-02
-4.9222E-02	1.0382	4.9269E-02
-5.0552E-02	1.0028	2.6530E-02
-4.9224E-03	1.0059	9.1584E-03
5.2511E-01	2.1175	1.5116E-01
5.5785E-01	1.6971	7.3763E-02
3.9890E-02	1.229	2.4334E-02

BIBLIOGRAPHY

1. Taylor, D. W., "Stability of Earth Slopes," Boston Society of Civil Engineers, JV24n3, July 1937.

2. Casagrande, Arthur, "Seepage Through Dams," Journal of New England Water Works Association V51n2, June 1937. (Reprinted in "Contributions to Soil Mechanics 1925-1940," Boston Society of Civil Engineers, 1940.)

3. Barber, E. S. and C. E. Mershone, "Graphical Analysis of Stability of Soil," Public Roads V21n8, October 1940.

4. Brahtz, J. H. A., "New Theories and Concepts in Analytical Soil Mechanics," Mines Mag. V30n12, December 1940.

5. Terzaghi, Karl, "Theoretical Soil Mechanics," John Wiley and Sons, Inc., New York, 1943.

6. Ek-Khoo Tan, "Stability of Soil Slopes," Am. Soc. of Civil Engrs., Proc. V73n1, January 1947.

7. Taylor, D. W., "Fundamentals of Soil Mechanics," John Wiley and Sons, Inc., New York, February 1948.

8. Terzaghi, Karl and Ralph B. Peck, "Soil Mechanics in Engineering Practice," John Wiley and Sons, Inc., New York, 1948.

9. "Treatise on Dams, Design Standard No. 2,1950," U. S. Bureau of Reclamation, Denver, Colorado.

10. "Stability Analysis, Construction Stage, Falcon Dam," Technical Memorandum 642, Section V, August 1951, U. S. Bureau of Reclamation Denver Federal Center, Denver, Colorado.

11. Bishop, A. W., "The Use of the Slip Circle in the Stability Analysis of Slopes," Geotechnique, V5n1, 1955.

12. Whitney Clark Huntington, "Earth Pressures and Retaining Walls," John Wiley and Sons, Inc., New York, 1957.

13. Meyer, Otto H., "Computation of the Stability of Slopes," Am. Soc. of Civil Engrs., Proc. V84nSM4, October 1958.

14. "Earth Manual," U. S. Bureau of Reclamation, Denver, Colorado, 1960.

15. Arnold, M., "Slope Stability Analysis by a New Graphical Method," Am. Soc. of Civil Engrs., Proc. V87nSM5, October 1961.