DESIGNING GOOD PARTIAL COVERAGE SATELLITE CONSTELLATIONS*

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Abstract

Design of satellite constellations providing partial coverage of certain ground regions is becoming more important as small low-altitude satellites receive increased attention. The purpose of this study is to develop the procedures necessary for deriving the best constellations for partial coverage. The term best is defined here as the minimum number of satellites at the minimum possible inclination with the smallest possible maximum time gap. This paper analyzes circular orbit constellations and shows that repeating ground track orbits yield better results than non-repeating ground track orbits. This theory provides methods for designing the best constellations. Results, comparing the new, non-standard satellite constellations with the best standard designs, show the new designs generally yield better results and require significantly less computation. Results are given for ground points at different latitudes, different minimum elevation angles for viewing a ground point, different satellite altitudes, and various gap time requirements. The paper develops theory for extending the methods to minimum pass time duration requirements and to coverage of extended areas. Results are presented in these cases. The new methods will potentially save money in satellite production and launch costs and allow for increased coverage capabilities.

Introduction

A number of government agencies and private users have current interest in satellite constellations that provide partial coverage. This interest stems, in part, from studies of lightweight satellite designs that provide for quick updates and enhanced responsiveness. In particular, DARPA, the Air Force, the Navy, and the Army are studying "LIGHTSAT" concepts. These satellites are generally considered to reside in low-earth orbits and to provide partial coverage. Partial coverage typically means coverage of certain regions of the earth, with gap times in coverage no longer than some specified maximum.

Numerous researchers have studied satellite constellation design from the point of view of continuous coverage. Some of these studies considered global coverage (for example, References 1-9), while others were concerned with coverage of certain regions (for example, References 7-10). The literature on low-altitude satellite constellations that provide non-continuous coverage of a small part of the globe is sparse. Studies that have been done (for example, References 11-12) analyzed standard constellations for their partial coverage properties.

This study examines ways to configure partial coverage constellations in order to minimize the number of satellites (and therefore the cost) necessary for various coverage requirements. This paper is organized as follows.

First, the authors describe the coverage criteria used and the types of satellite constellations considered. Next, the analysis compares constellations involving repeating ground tracks with those whose orbits do not repeat over the same ground points. Then some comparisons are given for circular versus elliptical constellations. After this, the method for determining the best ascending node location for each satellite is described. The paper then develops methods for choosing the best constellation. Results show comparisons of standard constellations with the newly-developed constellations for various coverage criteria (coverage of single ground points) and satellite altitudes. Following these results is development of the modified theory necessary when one considers minimum time duration requirements for each satellite pass and coverage of extended areas. Results showing the sensitivities for these cases follow the theoretical discussion. The final section includes a summary and suggestions for further work.

Coverage Criteria

This analysis assumes that the coverage criteria take the following form. A particular region of the globe must be covered with no longer than a certain maximum time gap. Coverage is defined by the minimum elevation angle to the satellite measured from the local horizontal of the ground point. There might also be restrictions placed on the time duration of the pass and requirements for coverage of extended areas. These latter two criteria will be discussed after the general theory is developed for coverage of a single ground point with no minimum time duration for each pass.

Constellation Types

The derived constellations use circular orbits. Satellite inclinations will be allowed to vary, with the restriction that all satellites have the same inclination. Inclinations up to 90 degrees will be considered. Any constellation without equal inclinations would not provide

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coverage that is consistent with time, since the satellite orbit planes would rotate with respect to each other, due to the earth's oblateness. For the same reason, satellite altitudes will be the same for all satellites. A system using a combination of different altitudes and inclinations that yield similar oblateness effects will also give a constellation whose coverage is not consistent with time, because satellite phasing varies.

Elliptical orbits offer the advantage of longer duration passes, although the number of passes from any given satellite during one day will not be significantly affected by use of elliptical orbits. The disadvantage to elliptical orbits is that they are restricted to an inclination near 63.4 degrees so that the argument of perigee will not rotate significantly. As will be shown, coverage properties depend strongly on orbit inclination, so that the restriction to a specific inclination will usually be very detrimental to the coverage. Examples of elliptical constellations are provided which demonstrate the benefits of circular orbits at varying inclinations.

The issue here is to choose the inclination, ascending node location, and initial mean anomaly for each satellite that results in the minimum number of satellites needed to meet the coverage criteria.

Repeating versus Non-Repeating Ground Tracks

The first requirement in determining how to design partial coverage constellations is to determine whether ground tracks that repeat every day are necessary. Repeating tracks offer the advantage that the coverage will not change from day to day. Use of repeating tracks limits the altitude/inclination combinations that can be used. In particular, there are daily repeating ground tracks from satellites at roughly 500 km, 800 km, and 1200 km altitudes, as well as at other altitudes higher up. They repeat after 15, 14, and 13 orbits, respectively.

Table 1 gives a comparison of repeating and non-repeating constellations for coverage of two ground points, one at 30 degrees latitude and the other at 50 degrees latitude, for a minimum elevation angle of 5 degrees. The following sequence of steps were used to find the optimal constellation. First, minimize the number of satellites to meet the coverage criteria. Then find the minimum inclination of these satellites (at or above 28 degrees - the latitude of Kennedy Space Center) that still meets the time requirement. Minimum inclination will maximize launch vehicle payload capabilities. Finally, with fixed number of satellites and fixed inclination, minimize the longest time gap.

For purposes of comparison, an ANSER-developed program was modified to iterate over Walker-type[1] satellite constellations (Ref. 1) to find the best repeating and non-repeating constellations of this standard design. J_2 effects on orbit plane precession and on orbit period were included in this analysis. The table shows that repeating ground-track constellations do as well as non-repeating constellations in all cases, usually better, and sometimes better at a lower altitude. For this reason, and since the object is to maximize the coverage with the minimum number of satellites, repeating ground tracks will be analyzed for the remainder of this paper. Note that results for higher minimum elevation angles yield differences that are even more pronounced. With smaller coverage circles, there will be a higher percentage gain with one extra pass per spacecraft per repeat time interval over the ground site.

Elliptical Constellations

As mentioned before, elliptical constellations offer the advantage of longer dwell times over ground points, while maintaining the disadvantage of restriction to 63.4° inclination. A number of elliptical cases were run to compare with the data in Table 1. Perigee was restricted to be above 200 km altitude. The data shows that limiting the inclination to 63.4° is inefficient for large gap time requirements, as most of the gap requirements are met with large margins. For smaller gap requirements, more satellites were added instead of finding an optimal inclination. This 63.4° inclination becomes less limiting and more advantageous for coverage of ground points closer to 63.4 degrees. The theory developed below for constellation design could be easily extended for elliptical constellations, but was not done for this report.

Choosing Ascending Node Location

Since repeating ground tracks will be used, the object will be to find the initial ascending node location that maximizes the number of passes of the satellite over the desired ground point. One would also like to maximize the duration of the shortest of these passes. Figure 1, made using ANSER's Ascending Node versus Time (ANT) program, shows the variations in coverage that can occur. With equator crossing at time zero, one can read the time of coverage as a function of the longitude of ascending node from the graph. Ground tracks go vertically up the page in this display. Later orbits are vertical lines displaced to the left by the motion that the ground track has in each orbit.

Two things are evident from the figure. First, coverage varies widely as a function of orbit inclination. This fact will be important in the rest of this analysis. Second, choosing the right longitude of ascending node is important because it can add a pass to the coverage of an individual satellite. Figure 2 shows the gain from correct choice of ascending node.

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[1] A Walker design has all satellites at the same altitude and inclination. Satellites are spaced evenly, so that the total number of satellites is the number per plane times the number of planes. Orbit planes are spaced evenly around the equator, and the relative phasing for when satellites in adjacent planes cross the equator is the same for all orbit planes.

Table 1. Comparison of Repeating and Non-Repeating Ground Tracks.*

a. Ground Point at 30° Latitude

Gap Constraint (hours)	Non-Repeating Ground Tracks Altitude (km)		
	500	900	1200
1/2	7,39,0.5	6,37,0.5	6,28,0.5
1	5,28,1.0	4,33,0.9	4,28,0.9
2	3,28,1.5	2,32,1.9	2,28,1.8
4	2,34,3.4	2,28,2.7	2,28,1.8
8	2,28,4.2	2,28,2.7	2,28,1.8
12	1,73,12.0	1,70,11.3	1,28,11.9
24	1,28,16.8	1,28,14.7	1,28,11.9

Gap Constraint (hours)	Repeating Ground Tracks Approximate Altitude (km)		
	500	800	1200
1/2	7,38,0.5,490.7	5,50,0.5,834.5	5,55,0.5,1212.3
1	4,35,0.8,488.5	4,29,1.0,819.6	4,28,0.8,1195.1
2	2,35,1.8,488.5	2,28,1.7,819.1	2,28,1.7,1195.1
4	2,28,3.4,483.9	2,28,1.7,819.1	2,28,1.7,1195.1
8	2,28,3.4,483.9	2,28,1.7,819.1	2,28,1.7,1195.1
12	1,52,12.0,503.8	1,49,11.1,833.6	1,28,11.9,1195.1
24	1,28,15.1,483.9	1,28,12.8,819.1	1,28,11.9,1195.1

b. Ground Point at 50° Latitude

Gap Constraint (hours)	Non-Repeating Ground Tracks Altitude (km)		
	500	900	1200
1/2	7,57,0.5	6,59,0.5	5,74,0.5
1	4,64,1.0	4,55,1.0	3,78,1.0
2	2,64,1.9	2,60,1.6	2,56,1.7
4	2,48,3.6	2,45,3.8	2,41,3.1
8	2,40,6.8	2,33,7.4	2,28,7.8
12	1,85,10.6	1,71,11.5	1,74,10.4
24	1,33,23.6	1,28,22.1	1,28,19.7

Gap Constraint (hours)	Repeating Ground Tracks Approximate Altitude (km)		
	500	800	1200
1/2	6,64,0.5,518.2	5,73,0.5,859.9	5,72,0.5,1229.1
1	4,55,0.9,507.2	3,73,0.9,859.9	3,73,0.9,1230.3
2	2,55,2.0,507.2	2,52,1.6,836.3	2,50,1.7,1208.2
4	2,48,3.6,499.7	2,43,3.0,828.6	2,40,3.1,1201.3
8	2,38,7.6,490.7	2,33,6.4,821.8	1,84,6.9,1243.4
12	1,74,10.7,532.2	1,62,11.4,846.6	1,62,10.5,1218.7
24	1,28,23.5,483.9	1,28,21.7,819.1	1,28,17.8,1195.1

* Table values are number of satellites, minimum inclination, and actual minimum gap. In the repeating case, the actual altitude is also listed. Constellations are for 5° minimum elevation. Standard constellations only.

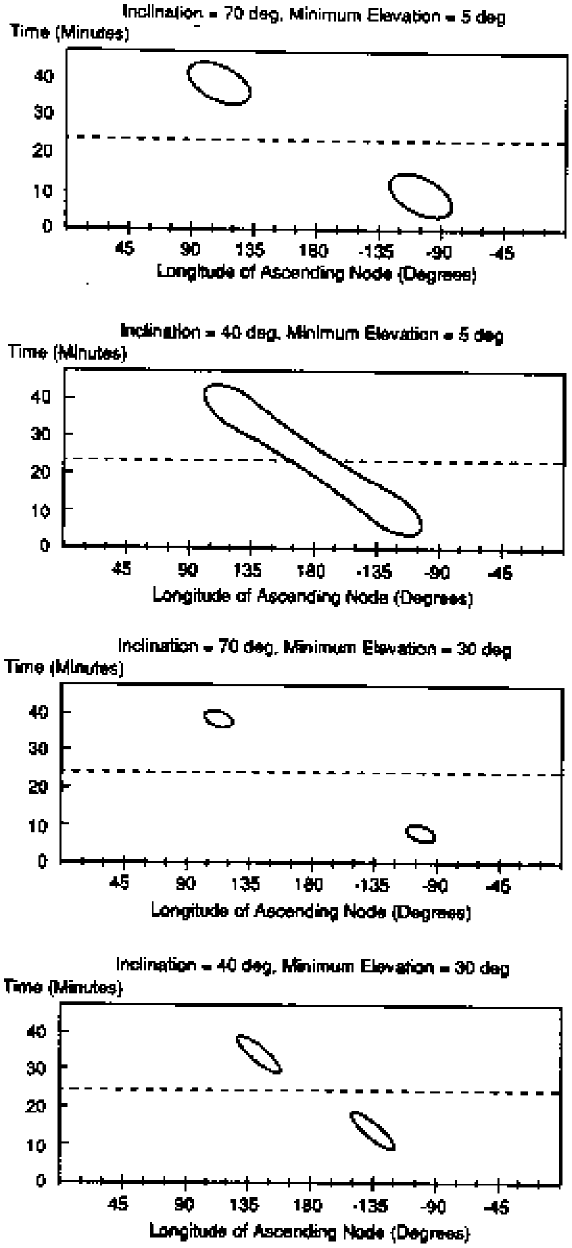


Fig. 1. Longitude of Ascending Node-Versus-Time Examples (Altitude = 500km, Ground Point at 30° Latitude, -90° Longitude).

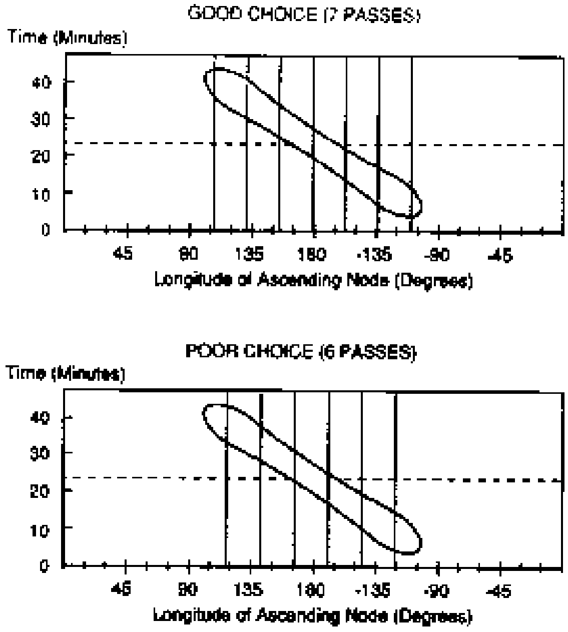


Fig. 2. Choosing Good Ascending Nodes.

The earth-central angle of coverage for a satellite covering a ground point is given by

$$\beta = \pi/2 - \sigma - \sin^{-1} [R_e \sin (\sigma + \pi/2)/r] \qquad (1)$$

where σ is the minimum elevation angle, r is the satellite orbit radius, and R_e is the earth's radius.

If the satellite enters a circle around the ground point with this earth-central angle as its radius, then the satellite will see the ground point. Since the highest latitude that the satellite ground track reaches is equal to its inclination, there will be no coverage for inclinations below $\phi - \beta$, where ϕ is the ground point latitude (assuming the coverage circle angle is less than the latitude).

One may calculate the longitudes of ascending node corresponding to the edges of the lobes in Figure 1. As an example see Figure 3, which shows the satellite ground track for a non-rotating earth. The difference in longitudes between the ground point and the ascending node for the case shown is

$$b = \sin^{-1} [\tan \phi/\tan (\xi + i)] \qquad (2)$$

where i is the orbit inclination and ξ is given by

$$\tan \xi = \sin i/(\sin\phi/\sin\beta - \cos i) \qquad (3)$$

The angle c from equator crossing to the point of tangency A is given by

$$\sin c = \tan \beta/\tan \xi \ . \qquad (4)$$

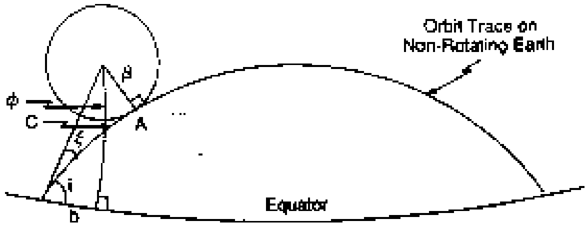


Fig. 3. Calculating the Ascending Node for the Right Edge of a Single Lobe.

Similar equations hold for the other edges of the lobe. A modified earth rotation rate (in degrees per day), taking orbit precession into account, is given by

$$\text{Rate} = \omega_e + 9.9639\ (R_e/r)^{3.5} \cos i \qquad (5)$$

where ω_e is the earth's rotation rate and R_e is the radius of the earth. The orbit period, τ, is given by

$$\tau = 2\pi \sqrt{r^3/\mu}\ \left[1 - 0.001624\ R_e^2 \{1 + (4 - 5\sin^2 i)/2\}/r^2\right] \qquad (6)$$

One may calculate the time for the satellite to travel the angle ξ using Equation 6 and the actual difference in longitude (b) that includes earth rotation and oblateness effects (Eq. 5).

The coverage will consist of either one or two lobes (Fig. 1) depending on the values of β, σ, and i. If $i > \beta + \sigma$, there will be two lobes of coverage, and only one lobe if $i \leq \beta + \sigma$. All the ascending nodes and mean anomalies for the edges of the coverage regions are found in a similar manner to the one just demonstrated.

The number of passes possible from a single satellite is determined by the width of the lobes. As an example, suppose the satellites are near 500 km altitude. Then there will be 24 degrees between adjacent ground tracks. If the lobe is greater than seven ground tracks wide (7x24 degrees), then 8 passes from one satellite in a day can be assured. For single lobes, the best ascending node choice is one which maximizes the number of passes and equalizes the parts of the lobe to the right of all passes and to the left of all passes. This will maximize the duration of the shortest passes, which generally are the ones on the outer edges of the lobe.

For the case of two lobes, the method is a little more complex. For a degree or so above $i = \beta + \phi$, the lobes are close enough together that the gap in the middle does not cause a loss of a pass. Above this inclination, one must choose the ascending node that causes the largest number of satellite passes. This is a function not only of the lobe width, but also of the separation between lobes. Sometimes both lobes will get the same number of passes, and other times one lobe will have one pass more than the other lobe.

The single lobes increase in width as the satellite inclination increases, so that the coverage generally improves with increasing inclination, up to the point that the lobe splits into two. When there are two lobes, some inclinations are good and some are not, depending on what happens to the separation of the lobes relative to the ground track motion per orbit. The two lobes increase their separation until it is maximized at an inclination of 90 degrees.

Choosing the Best Constellation

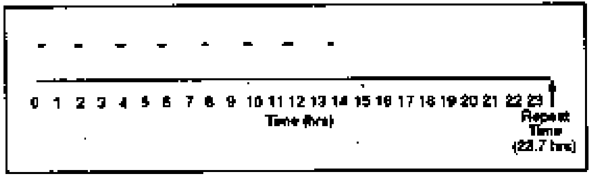
For large coverage gap constraints (near 24 hours), the choice of constellation is easy. The constellation consists of one satellite. Pick the lowest acceptable inclination above $\phi - \beta$.

Calculate the in-time of the first pass (using a simple coverage model) and the out-time of the last pass, using the ascending node locations already chosen. Knowing the total time of coverage from the number of passes that give coverage, subtract this value from the total repeat time to obtain the gap time. The repeat time is the number of orbits until repeat occurs multiplied by the orbit period of the satellite.

When the gap constraint becomes smaller (near 12 hours), two lobes are generally required, since they split the total repeat time nearly in half. One lobe usually does not extend far enough in longitude. Again, one satellite should be enough. Using the out-time from one lobe and the in-time to the next lobe (from a simple coverage model), and counting the orbits in between, one may find the gap time. Gaps must be calculated from the first lobe to the second and then from the second to the first, since the largest gap is not always between the same lobes.

As the gap constraint is further reduced, the constellation requires more than one satellite. Now the problem changes to the placement of satellites relative to each other. Since the longitude of ascending node is fixed to the best one for coverage of a given ground point, one may calculate time into and out of coverage for that satellite, based on zero time for entry into coverage

a. Satellite at 834.5km, 50 degree inclination, covering ground point at 30 degrees latitude, 5 degrees minimum elevation.



b. Optimal placement for 5 satellites, maximum gap time 0.496 hours.

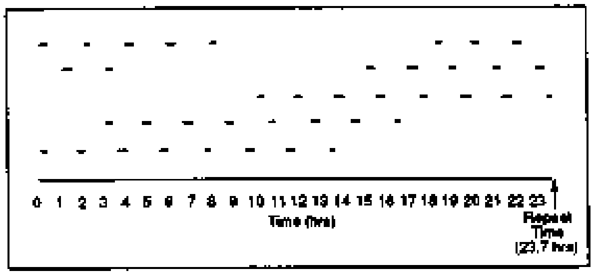


Fig. 4. Timelines of Coverage.

on the first pass. Figure 4a shows one satellite's coverage times on a timeline that extends from zero to the repeat time. Optimal constellations are obtained by placing individual satellites along this timeline in the best fashion, since all satellites will have the same (maximized) coverage properties (Fig. 4b). Once the relative positions of the satellite timelines are determined, the initial inertial ascensions of ascending node (Ω_o, with respect to the Greenwich Meridian) and initial mean anomalies (M_o) may be found according to

$$(\Omega_o)_n = \mathrm{MOD}(\Omega^* + t_n\, G/P, 2\pi) \tag{7}$$

and

$$(M_o)_n = \mathrm{MOD}\ (2\pi - 2\pi\ [t_n - \mathrm{INT}(t_n/P)P]/P, 2\pi) \tag{8}$$

In these equations, Ω^* is the ascending node of the first satellite calculated with an initial mean anomaly of 0 degrees, t_n is the in-time of the first pass through the lobe for the nth satellite, G is the ground-track motion in one orbit, and P is the orbit period. The MOD function assures that the result will be between 0 and 2π.

Initially, the authors attempted to place satellite timelines in a fashion that would minimize coverage gaps, using logical rules for placement. This method quickly becomes prohibitive as one proceeds to four satellites and beyond, especially for coverage with two lobes. The next obvious procedure is to iterate over time and satellite number, trying all possibilities for placement of each satellite's timeline. This was done with a time increment of one minute and yields optimal configurations for up to five satellites. The problem with extending to a larger number of satellites is computer run time. Run time increases by factors of thousands for each additional satellite, since for every possible relative position of four satellites to a fifth, all possible positions of the sixth satellite must be tried. Run times for 6 satellites on a SUN workstation extended into weeks.

One factor that assists in analyzing constellations with many satellites is the disadvantage of two lobes. As inclination increases with one lobe, the covered region becomes wider. This effect continues until the lobe splits into two. After that, the lobes, initially separate quickly as inclination increases. Beyond a degree or so above the point where the lobe splits into two, the maximum number of passes available from two lobes decreases below that available from the best one-lobe inclination. The passes are all fixed in relative position. Thus two lobes will always give a worse result than one lobe whenever there are enough satellites to cause at least one pass for each orbit during the repeat period. Consequently, if the inclination with the most passes for one lobe multiplied by the number of satellites is greater than or equal to the number of orbits in the repeat period, two lobes can be ignored above the inclination where the gap between lobes does not cause loss of a pass.

The key to analysis of many-satellite constellations is judicious choice of relative satellite timeline positions. This becomes possible to do when the number of satellites is large enough so that one can consider only one-lobe timelines. In the first case, the gap time requirement is larger than the largest gap between passes on adjacent revolutions of a single satellite (generally true for gap time requirements of two hours or greater). In this case, the first pass in-times should be placed evenly between 0 and the repeat time. One may then use Equations 7 and 8 to get the final constellations.

If the gap time requirement is smaller than the largest gap between passes on adjacent revolutions of a single satellite, timelines from two satellites must be meshed together. The first pass of the second satellite can be placed anywhere that causes some meshing of passes from the second satellite with those of the first satellite. As an example, let j be an integer that defines the relative position of second-satellite passes to first-satellite passes. Figure 5 shows a few examples of how the meshing occurs.

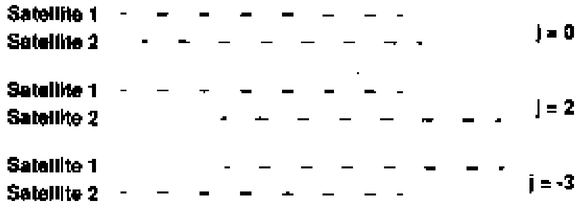


Fig. 5. Meshing Satellite Passes.

For cases of $j \geq 0$, the first pass of the second satellite should be spaced after the end of the associated first-satellite pass by exactly the time requirement. This spacing will maximize the coverage from individual satellites in order to minimize the number of satellites required. One then must check gaps between the end of the rest of the first-satellite passes and the beginning of the second-satellite passes. If any of these gaps are greater than the time requirement, the second-satellite timeline must shift to the left to cause all gaps to be less than or equal to the requirement. For cases of $j < 0$, one starts by placing the first available second-satellite pass to the right of the end of the first pass for the first satellite, by exactly the time requirement. After that, the procedure is the same as for $j \geq 0$. Meshes of two satellites are generally adequate to fill gaps between adjacent passes for time gap requirements of one hour.

Following placement of the second satellite, the designer must check all gaps after second-satellite passes and before the next first-satellite passes. If any of these are greater than the time requirement, a third satellite must be added to the mesh. It is placed in a similar fashion with respect to the second satellite as the second is with respect to the first. Thus another integer defines the relative position of the third satellite with respect to the second. If time gaps are still too large, one may add more satellites to the mesh. Three satellites are generally adequate to fill gaps between adjacent passes for time gap requirements of one-half hour.

Once a mesh is complete, one may add additional meshes to meet the gap requirement. This begins by placing the first additional satellite to the right of the last fully-meshed set of passes by exactly the time requirement. Other satellites are placed similarly to the first mesh until another mesh is complete. Gaps resulting must continue to be checked as certain values for the integers determining the mesh configuration cause gaps to open up later on. This occurs because of the nonuniformity of the length of the various passes.

This process continues until satellite passes start to overlap with the first mesh. When this overlapping occurs, one must look at gaps caused during the overlap. A satellite that overlaps with one placed in the first mesh cannot be moved further to the right, since that would cause a gap larger than the time requirement to open up. It can, however, be moved to the left. Sometimes moving a timeline to the left reduces gaps with the overlapped region to the point where gaps become less than the requirement and an additional satellite is avoided. The process continues until all gaps are filled.

At this point, the designer has a constellation that uses the minimum number of satellites that fulfill the time requirement for a given satellite inclination. One would then like to minimize the gap. Usually, there is some overlap at the end that can be reduced, thereby reducing the maximum time gap. In order to reduce all the maximum gaps by the same amount, each satellite after the first moves to the left by the desired gap reduction increment multiplied by the satellite number minus one. Thus satellite n+1 is closer to satellite n by the same reduction increment as satellite 2 is closer to satellite 1. In doing this squashing, the designer must check all of the gaps to the right of satellite passes and assure that they will not become larger than the maximum gap. A gap to the right becomes larger by the time movement multiplied by the difference in satellite numbers.

Satellites that have already moved to the left to close gaps from overlap with early satellites should not be moved further to the left as quickly. This is because the gap that needs reduction in this case is between the already-squashed timeline and a much earlier satellite timeline. Movement of this satellite by nsat - 1 multiplied by the reduction causes the gap to reduce more quickly than other gaps. By equating the gaps to the left after squashing between initial satellites and gaps to the right between a late satellite and one of the early satellites, one obtains for the maximum gap reduction increment due to a given gap to the right

$$[y(n) - y(n-m) - s + T]/(m + 1) \qquad (9)$$

Here, s is the gap of interest to the right, T is the gap time requirement, and m is the difference in satellite numbers between the satellite to be moved leftward (n) and the earlier satellite that it is sharing the gap with. The variable y is the amount that a satellite has already moved to the left to close a gap resulting from overlapping and is a function of satellite number. y includes the amount this single satellite has moved plus the amount the previous satellite that it was originally meshed with had moved. In squashing satellites together, one should examine not only the satellite passes immediately to the right of a given pass, but ones near it to the left that it may pass when the time increment occurs.

As a last test, to make sure the largest gap is as small as possible with this configuration, one can take all the timelines and look at each gap and the satellite numbers that correspond to the start and end of the gap. Next record the largest gap for each combination (e.g., satellite 4 on left and 2 on right have a maximum gap of...). Knowing the gaps that are nearly equal to the maximum gap, determine the satellites that could be separated without exceeding the maximum gap. Figure 6 shows this situation. Here, satellite two is on the right of a maximum gap, with satellite one on the left. Satellite five is not connected to any satellite to its right. Hence satellite two can be moved towards one, three towards two by twice as much, etc., thereby reducing all the gaps. Note that if the arrow between satellites two and four went in the opposite direction, so that a pass from satellite two was to the right of one from satellite four, moving satellite four towards two would increase the gap rather than decrease it.

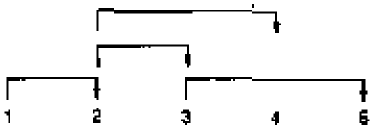


Fig. 6. Maximum Gap Connections (Arrow Indicates Satellite on Right Side of Gap).

Once it is determined which satellites must move, one must find the amount of movement that will reduce the largest gap by the maximum amount in all cases where it occurs without opening any larger gaps. One can do this by looking at all cases of adjacent passes where the satellite on the right is to the right of the gap to be widened, and the satellite to the left is on the left of the gap to be widened (e.g., 5-1, 4-2, and 5-3 in Fig. 6, but not 2-4). In Fig. 6 the satellite combination that restricts the gap reduction of the maximum gaps the most is the one to use.

After all this has been done, one final option remains that might yield better constellations in some cases. Typically the first and last passes from a satellite are the shortest and hence the most restrictive in setting up a meshing. In cases where there is full meshing (j=0 and similarly for other satellites needed for the mesh), one could ignore the first and last passes in setting up a mesh. The next set of satellites that meshes with this set starts earlier than where it would normally start, in order to fill the gaps resulting from ignoring the last passes. This procedure continues until the last set of meshing satellites fills the gaps on the left side of the first set of meshing satellites. This method only works in cases where there is enough overlap in the earlier constellation to allow for the initial squashing of sets of satellites without opening up large gaps.

Other alternative constellations may be found by choosing in-times for Equations 7 and 8 that cause mean anomalies to be equal to Walker values and right ascensions that are as close as possible to Walker values while still assuring the best ascending nodes as calculated before. For one satellite per plane, mean anomalies would be exact Walker values. For multiple satellites per "plane," timelines are chosen so that the gaps between satellites are equal to the gap time requirements. For gap times down to one-half hour, one would never want more than three satellites per "plane." Satellites will not be in exactly the same plane, but rather will have the same longitude of ascending node. For multiple satellites per "plane," satellite separation within a plane can be deduced, once a working constellation is found. These constellations may be compared with the ones found earlier.

Results

Table 2 lists altitudes for the repeating ground track constellations. The values listed are the three lowest repeating ground track altitudes high enough to remain in orbit for any significant period of time. Tables 3-8 give results for coverage of single ground points at 30 degrees latitude and at 50 degrees latitude. The 0 degree longitude value is arbitrary. One may apply the results to any longitude by shifting the ascending node values. Results are given for minimum elevation angles of 5 degrees, 30 degrees, and 60 degrees; and for satellite altitudes near 500 km, 800 km, and 1200 km (the same cases as in Table 2). The criteria for choosing the best constellation was to first minimize the number of satellites for meeting the gap requirement, then to find the minimum inclination for these satellites (at or above 28 degrees), and finally to minimize the largest gap.

Results obtained using the methods in this paper are compared with the best constellations obtainable using standard constellation designs (described earlier in the repeating versus non-repeating section). The Walker (Ref. 1) T/P/F designator is given for each standard constellation. T is the total number of satellites, P is the number of orbit planes, and F is the relative phasing between satellites in adjacent planes in units of 360/T. For the standard designs, a computer program iterated through all possible ascending nodes (through one ground-track motion, when the coverage repeats) to find the one providing the best coverage. Increments were one degree in ascending node and one degree in inclination.

Table 2. Repeating Ground Track Altitudes.

Inc. (deg)	Altitudes for Repeating Ground Track (km)		
28	483.911	819.102	1195.08
29	494.499	819.607	1195.51
30	485.108	820.131	1195.95
31	485.738	820.672	1196.41
32	486.390	821.232	1196.88
33	487.062	821.810	1197.37
34	487.755	822.407	1197.88
35	488.470	823.021	1198.40
36	489.205	823.655	1198.94
37	489.963	824.308	1199.50
38	490.740	824.979	1200.07
39	491.539	825.668	1200.66
40	492.359	826.376	1201.26
41	493.200	827.103	1201.88
42	494.062	827.848	1202.52
43	494.945	828.611	1203.17
44	495.849	829.395	1203.84
45	496.773	830.195	1204.52
46	497.718	831.015	1205.23
47	498.685	831.853	1205.94
48	499.672	832.710	1206.68
49	500.679	833.584	1207.43
50	501.708	834.478	1208.20
51	502.756	835.389	1208.98
52	503.825	836.319	1209.78
53	504.915	837.268	1210.60
54	506.024	838.234	1211.43
55	507.153	839.218	1212.28
56	508.302	840.221	1213.14
57	509.471	841.241	1214.02
58	510.658	842.278	1214.92
59	511.866	843.333	1215.83
60	513.092	844.406	1216.76
61	514.338	845.496	1217.71
62	515.603	846.604	1218.66
63	516.885	847.728	1219.64
64	518.187	848.869	1220.63
65	519.506	850.026	1221.64
66	520.843	851.200	1222.66
67	522.197	852.390	1223.69
68	523.569	853.597	1224.74
69	524.958	854.818	1225.81
70	526.364	856.056	1226.88
71	527.786	857.308	1227.98
72	529.225	858.576	1229.08
73	530.678	859.859	1230.20
74	532.147	861.155	1231.34
75	533.631	862.466	1232.48
76	535.130	863.791	1233.64
77	536.644	865.129	1234.82
78	538.171	866.481	1236.00
79	539.712	867.845	1237.20
80	541.266	869.222	1238.41
81	542.833	870.610	1239.63
82	544.411	872.011	1240.86
83	546.002	873.423	1242.10
84	547.604	874.846	1243.35
85	549.217	876.279	1244.62
86	550.841	877.723	1245.89
87	552.474	879.176	1247.17
88	554.117	880.638	1248.46
89	555.769	882.109	1249.76
90	557.429	883.589	1251.07

Table 3. Constellation Results
Ground Point 30°, 0°; Minimum Elevation 5°.

Alt. Range (km)	Max Gap Req't (hrs)	NEW METHODS					STANDARD DESIGN				
		No. of Sats	Inc. (deg)	Gap (hrs)	Ascending Nodes (deg)	Initial Mean Anomalies (deg)	No. of Sats	Inc. (deg)	Gap (hrs)	Walker T/P/F	First Asc. Node (deg - M_0=0)
Near 500	1/2	6	39	0.50	348.17,22.83,57.50 168.57,203.23, 237.90	0,201.21,42.42, 180.0,21.21, 222.42	7	38	0.50	7/7/4	10
	1	4	35	0.82	348.18,52.87, 168.60,232.28	0.0,111.95,180.0, 291.95	4	35	0.82	4/4/1	12
	2	2	35	1.80	348.18,168.51	0,181.27	2	35	1.79	2/2/1	12
	4	2	28	3.39	336.16,156.64	0,179.47	2	28	3.38	2/2/1	0
	6	2	28	3.39	336.16,156.64	0,179.47	2	28	3.38	2/2/1	0
	8	2	28	3.39	336.16,154.64	0,179.47	2	28	3.38	2/2/1	0
	12	1	52	12.00	12.15	0	1	52	12.0	N/A	0
	18	1	28	15.16	336.16	0	1	28	15.1	N/A	0
	24	1	28	15.16	336.16	0	1	28	15.1	N/A	0
Near 800	1/2	5	49	0.50	6.60,78.73,150.86, 222.99,295.12	0,72,144,216,288	5	50	0.50	5/5/1	6
	1	4	28	0.82	4.00,351.76, 178.57,191.61	216.90,39.18, 298.49,116.21	4	29	1.0	4/2/2	6
	2	2	28	1.68	353.76,174.16	0,0	2	28	1.67	2/2/2	19
	4	2	28	1.68	353.76,174.16	0,0	2	28	1.67	2/2/2	19
	6	2	28	1.67	353.76,174.16	0,0	2	28	1.67	2/2/2	19
	8	2	28	1.67	353.76,174.16	0,0	2	28	1.67	2/2/2	19
	12	1	48	11.18	6.60	0	1	49	11.14	N/A	6
	18	1	28	12.82	353.76	0	1	28	12.79	N/A	19
	24	1	28	12.82	353.76	0	1	28	12.79	N/A	19
Near 1200	1/2	5	50	0.50	169.71,61.94,14.01 247.60,296.19	136.18,97.90,0.0, 208.22,297.58	5	55	0.50	5/5/1	0
	1	4	28	0.74	0.18,15.50,191.04, 206.36	0.0,161.30,43.80, 205.10	4	28	0.80	4/2/2	0
	2	2	28	1.74	0.18,180.55	0,180	2	28	1.74	2/2/2	0
	4	2	28	1.74	0.18,180.55	0,180	2	28	1.74	2/2/2	0
	6	2	28	1.74	0.18,180.55	0,180	2	28	1.74	2/2/2	0
	8	2	28	1.74	0.18,180.55	0,180	2	28	1.74	2/2/2	0
	12	1	28	11.88	0.18	0	1	28	11.86	N/A	0
	18	1	28	11.88	0.18	0	1	28	11.86	N/A	0
	24	1	28	11.88	0.18	0	1	28	11.86	N/A	0

Table 4. Constellation Results
Ground Point 30°, 0°; Minimum Elevation 30°.

Alt. Range (km)	Max Gap Req't (hrs)	NEW METHODS					STANDARD DESIGN				
		No. of Sats	Inc. (deg)	Gap (hrs)	Ascending Nodes (deg)	Initial Mean Anomalies (deg)	No. of Sats	Inc. (deg)	Gap (hrs)	Walker T/P/F	First Asc. Node (deg – M_0=0)
Near 500	1/2	11	34	0.45	324.13,356.93,29.73, 62.54,95.34,128.14, 160.95, 193.75, 226.55,259.36, 292.16	0,229.09,98.18, 327.27, 196.36, 65.45,294.55, 163.64,32.73, 261.82,130.91	11	34	0.45	11/11/7	12
	1	6	34	0.81	324.13,88.41,212.70, 336.66,100.94, 225.23	0,300,240,172.48, 112.48,52.48	7	33	0.94	7/7/2	0
	2	3	34	1.63	324.13,84.41,204.68	0.0,0.0,0.0	3	34	1.62	3/3/3	12
	4	3	29	2.84	312.10,72.39,192.69	0,360.0,360.0	3	29	2.84	3/3/3	0
	6	2	34	5.12	324.13,144.55	0,179.90	2	34	5.91	2/2/2	12
	8	2	29	6.77	312.10,132.54	0,180.0	2	29	7.56	2/2/2	0
	12	2	28	8.40	300.07,120.51	0,180.15	2	28	9.20	2/2/2	20
	18	1	34	16.90	324.13	0	1	34	16.86	N/A	12
	24	1	28	20.17	300.07	0	1	28	20.13	N/A	8
Near 800	1/2	10	33	0.50	327.98,4.06,40.13, 76.21,112.29,148.37, 184.44,220.52,256.60, 292.67	0,216,72,288,144, 0,216,72,288,144	10	34	0.50	10/10/6	19
	1	6	31	0.82	327.98,342.02,90.21, 104.24,212.44, 226.47	0,163.97,92.55, 256.52,185.09, 349.06	6	32	0.83	6/3/2	19
	2	3	31	1.74	327.98,88.24,208.51	0,120,240	3	32	1.72	3/3/1	19
	4	3	28	2.42	315.10,75.37,195.64	0,120,240	3	28	3.01	3/3/3	2
	6	2	31	4.63	327.98,148.32	0,0.73	2	32	4.59	2/2/2	19
	8	2	28	6.37	315.10,135.63	0,358.30	2	28	6.35	2/2/2	6
	12	2	28	6.37	315.10,135.63	0,358.30	2	28	6.35	2/2/2	6
	18	1	31	16.44	327.98	0	1	32	16.38	N/A	19
	24	1	28	18.16	315.10	0	1	28	18.12	N/A	6
Near 1200	1/2	8	40	0.49	346.29,113.29, 151.30,158.96, 166.62,293.61, 331.62,339.28	0,151.98,18.76, 279.40,180, 331.98,198.76, 99.38	10	34	0.46	10/10/2	7
	1	4	40	1.00	346.29,31.96,166.62, 212.28	0,127.40,180, 307.40	5	32	0.99	5/5/2	0
	2	3	28	1.94	318.56,78.80,199.05	0,240,120	3	28	1.94	3/3/2	13
	4	2	31	3.99	332.43,152.86	0,179.06	2	30	4.00	2/2/1	0
	6	2	28	5.88	318.56,138.93	0,179.97	2	28	5.87	2/2/1	13
	8	2	28	5.88	318.56,138.93	0,179.97	2	28	5.87	2/2/2	13
	12	1	84	10.40	13.97	0	1	84	10.38	N/A	14
	18	1	28	17.72	318.56	0	1	28	17.68	N/A	13
	24	1	28	17.72	318.56	0	1	28	17.68	N/A	13

Table 5. Constellation Results
Ground Point 30°, 0°; Minimum Elevation 60°.

		NEW METHODS					STANDARD DESIGN				
Alt. Range (km)	Max Gap Req't (hrs)	No. of Sats	Inc. (deg)	Gap (hrs)	Ascending Nodes (deg)	Initial Mean Anomalies (deg)	No. of Sats	Inc. (deg)	Gap (hrs)	Walker T/P/F	First Asc. Node (deg - M_0=0)
Near 500	1/2	18	31	0.50	300.07,340.17,20.26, 60.36,100.46,140.55, 180.65,220.74,260.84 307.34,347.44,27.53, 67.63,107.73,147.82, 187.92,228.01,268.11	0,120,240,0,120, 240,0,120,240, 251.20,11.20, 131.20,251.20, 11.20,131.20, 251.20,11.20, 131.20	18	32	0.46	18/18/12	6
	1	9	31	0.93	300.07,340.17,20.26, 60.36,100.46,140.55 180.65,220.74,260.84	0,120,240,0,120, 240,0,120,240	9	31	0.94	9/9/3	11
	2	5	31	1.65	300.07,12.24,84.42, 156.59, 228.76	0,0,0,0,0	5	31	1.64	5/5/5	12
	4	4	31	2.55	300.07,30.20,120.48, 210.83	0,91.29,180.37 268.29	4	31	2.54	4/4/1	11
	6	3	31	4.51	300.07,60.47,180.72	0,358.27,358.88	3	31	4.49	3/3/3	12
	8	2	34	6.78	312.10,132.43	0,180.32	2	34	7.59	2/2/2	12
	12	2	28	11.77	276.02,96.41	0,180.69	2	28	11.74	2/2/1	9
	18	1	40	16.93	324.12	0	1	40	16.89	N/A	12
	24	1	28	23.54	276,02	0	1	28	23.48	N/A	9
Near 800	1/2	15	33	0.49	315.10,50.18,52.74 60.24,67.73,102.82, 165.38,172.87,180.37 215.45, 278.02, 285.51 293.00,327.32,29.88	0,229.83,75.85, 331.16,226.47, 96.30,302.31, 197.62,92.93, 322.77,168.78, 64.09,319.40, 189.23,35.25	16	34	0.50	16/16/2	6
	1	8	33	0.87	315.10,328.89,50.79, 64.59,146.48,160.28, 242.18,255.97	0,167.26,103.15, 270.41,206.30, 13.56,309.45, 116.71	8	33	0.95	8/4/2	0
	2	4	33	1.78	315.10,45.29,135.48, 225.67	180,0,180	4	33	1.84	4/4/3	0
	4	3	33	2.49	315.10,75.30,195.60	0,120.80,240.17	3	33	3.66	3/3/2	15
	6	3	30	4.26	302.21,62.59,182.80	0,118.45,239.18	3	30	4.25	3/3/1	19
	8	2	33	6.42	315.10,135.50	0,359.18	2	34	6.39	2/2/2	6
	12	2	28	9.98	289.33,109.72	0,0.21	2	28	9.96	2/2/2	4
	18	1	40	16.46	327.97	0	1	40	16.43	N/A	19
	24	1	28	21.79	289.33	0	1	28	21.74	N/A	3
Near 1200	1/2	12	33	0.49	318.56,356.11,63.40, 71.09,78.79,116.35, 183.63,191.33, 199.02,236.58, 303.87,311.56	0,232.69,79.62, 339.81,240, 112.69,319.62, 219.81,120, 352.69,199.62, 99.81	14	33	0.48	14/7/2	0
	1	6	33	0.99	318.56,3.81,78.79, 124.04,199.02, 244.27	0,132.87,240, 12.87,120,252.87	8	28	0.98	8/8/3	0
	2	3	36	1.99	304.69,34.87,125.05, 215.23	0,270,180,90	3	36	2.00	3/3/2	14
	4	3	28	3.99	304.69,65.02,185.24	0,238.77,119.12	3	28	3.98	3/3/2	0
	6	2	33	5.98	318.56,138.81	0,181.21	2	33	5.97	2/2/1	14
	8	2	28	7.93	304.69,124.99	0,180.75	2	28	7.92	2/2/1	0
	12	2	28	7.93	304.69,124.99	0,180.75	2	28	7.92	2/2/1	0
	18	1	33	17.82	318.56	0	1	33	17.79	N/A	14
	24	1	28	19.76	304.69	0	1	28	19.72	N/A	0

Table 6. Constellation Results
Ground Point 50° , 0° ; Minimum Elevation 5°.

		NEW METHODS					STANDARD DESIGN				
Alt Range (km)	Max Gap Req't (hrs)	No. of Sats	Inc. (deg)	Gap (hrs)	Ascending Nodes (deg)	Initial Mean Anomalies (deg)	No. of Sats	Inc. (deg)	Gap (hrs)	Walker T/P/F	First Asc. Node (deg – M_0=0)
Near 500	1/2	6	64	0.50	166.11,0.68,7.90 199.36,207.18,41.16	34.67,0.86,243.83, 256.62,139.59, 105.78	6	64	0.47	6/2/1	0
	1	4	55	0.92	348.14,53.20,168.47 233.53	0,105.86,180, 285.86	4	55	0.92	4/4/1	12
	2	2	55	1.97	348.14,168.47	0,180	2	55	1.99	2/2/1	12
	4	2	48	3.59	336.13,156.49	0,180	2	48	3.58	2/2/1	0
	6	2	42	5.21	324.12,144.50	0,180	2	42	5.98	2/2/2	0
	8	2	38	6.83	312.09,132.50	0,180	2	38	7.61	2/2/2	0
	12	1	74	10.73	12.15	0	1	74	10.71	N/A	12
	18	1	42	17.02	324.12	0	1	42	16.98	N/A	12
	24	1	33	23.54	276.01	0	1	33	23.49	N/A	0
NEAR 800	1/2	5	73	0.47	19.43,91.53, 163.63,235.73, 307.83	0,72,144,216,288	5	73	0.47	5/5/1	19
	1	3	73	0.89	19.43,139.60, 259.76	0,120,240	3	73	0.90	3/3/1	19
	2	2	52	1.62	353.72,174.03	0,0	2	52	1.62	2/2/2	19
	4	2	43	2.99	340.85,161.20	0,0	2	43	2.98	2/2/2	6
	6	2	37	4.70	327.98,148.35	0,0	2	37	4.70	2/2/2	19
	8	2	32	6.49	315.10,135.49	0,0	2	33	6.43	2/2/2	7
	12	1	62	11.44	6.58	0	1	62	11.42	N/A	6
	18	1	37	16.52	327.98	0	1	37	16.49	N/A	19
	24	1	28	21.79	289.33	0	1	28	21.74	N/A	5
Near 1200	1/2	5	71	0.50	307.93,146.70, 14.34,241.53,108.72	143.36,76.79,1.17, 285.55,209.93	5	72	0.50	5/5/3	25
	1	3	74	0.87	27.84,156.71,285.57	0,126.74,253.47	3	73	0.95	3/3/1	0
	2	2	50	1.73	0.14,180.43	0,180	2	50	1.73	2/2/2	0
	4	2	40	2.25	346.29,166.62	0,180	2	40	3.13	2/2/2	0
	6	2	29	5.98	318.56,138.92	0,180	2	28	6.00	2/2/1	13
	8	1	84	6.95	41.69	0	1	84	6.94	N/A	14
	12	1	62	10.47	13.99	0	1	62	10.46	N/A	14
	18	1	28	17.84	318.56	0	1	28	17.81	N/A	13
	24	1	28	17.84	318.56	0	1	28	17.81	N/A	13

Table 7. Constellation Results
Ground Point 50° , 0° ; Minimum Elevation 30°.

		NEW METHODS					STANDARD DESIGN				
Alt Range (km)	Max Gap Req't (hrs)	No. of Sats	Inc. (deg)	Gap (hrs)	Ascending Nodes (deg)	Initial Mean Anomalies (deg)	No. of Sats	Inc. (deg)	Gap (hrs)	Walker T/P/F	First Asc. Node (deg - M_o=0)
Near 500	1/2	11	55	0.47	324.10,356.89,29.67 62.46,95.25,128.03, 160.82,193.61, 226.39,259.18, 291.97	0,229.09,98.18, 327.27,196.36, 65.45,294.55, 163.64,32.73, 261.82,130.91	11	55	0.48	11/11/7	12
	1	6	55	0.79	324.10,88.32,212.55 336.51,100.74, 224.96	0,300,240,174.17 114.17,54.17	6	55	0.95	6/6/3	12
	2	3	55	1.60	324.10,84.32,204.54	0,0,0	3	55	1.60	3/3/3	12
	4	3	50	2.92	312.08,72.31,192.55	0,0,0	3	50	2.92	3/3/3	0
	6	2	55	5.25	324.10,144.52	0,178.57	2	55	5.23	2/2/1	12
	8	2	50	6.87	312.08,132.45	0,179.77	2	50	7.66	2/2/2	0
	12	2	44	11.79	276.01,96.46	0,178.90	2	44	11.77	2/2/1	8
	18	1	55	17.09	324.10	0	1	55	17.06	N/A	11
	24	1	44	23.60	276.01	0	1	44	23.55	N/A	7
Near 800	1/2	9	60	0.45	340.83,15.16,49.50, 109.60,143.94, 178.28,212.62, 272.71,307.05	0,240,120,0,240, 120,0,240,120	11	56	0.50	11/11/7	10
	1	5	60	0.90	340.83,47.79,114.75 207.46,274.43	0,144,288,72,216	6	52	0.84	6/3/2	19
	2	3	52	1.71	327.96,88.16,208.37	0,120,240	3	52	1.71	3/3/1	19
	4	2	60	3.02	340.83,161.12	0,359.87	2	60	3.01	2/2/2	6
	6	2	52	4.78	327.96,148.33	0,359.08	2	52	4.76	2/2/2	19
	8	2	47	6.51	315.08,135.47	0,359.18	2	47	6.49	2/2/2	6
	12	1	80	11.53	6.55	0	1	80	11.51	N/A	6
	18	1	52	16.63	327.96	0	1	52	16.60	N/A	19
	24	1	41	21.87	289.32	0	1	41	21.82	N/A	6
Near 1200	1/2	9	59	0.47	346.26,23.24,60.21, 97.19,134.17,198.87 235.85,272.83, 309.80	0,240,120,0,240, 120,0,240,120	9	63	0.48	9/9/2	0
	1	5	51	0.90	332.41,44.52, 116.63,188.75, 260.86	0,144,288,72,216	5	51	0.90	5/5/2	0
	2	3	51	1.81	332.41,92.60,212.79	0,240,120	3	51	1.81	3/3/3	0
	4	2	59	2.33	346.26,166.47	0,180.71	2	59	3.22	2/2/2	0
	6	2	51	4.18	332.41,152.66	0,180.35	2	51	5.09	2/2/2	0
	8	2	40	8.00	304.68,124.94	0,180.81	2	40	7.98	2/2/2	0
	12	1	81	10.61	13.97	0	1	81	10.59	N/A	14
	18	1	44	17.96	318.55	0	1	44	17.93	N/A	14
	24	1	37	23.67	276.94	0	1	37	23.63	N/A	0

Table 8. Constellation Results
Ground Point 50°, 0°; Minimum Elevation 60°.

		NEW METHODS					STANDARD DESIGN				
Alt Range (km)	Max Gap Req't (hrs)	No. of Sats	Inc. (deg)	Gap (hrs)	Ascending Nodes (deg)	Initial Mean Anomalies (deg)	No. of Sats	Inc. (deg)	Gap (hrs)	Walker T/P/F	First Asc. Node (deg – M_0=0)
Near 500	1/2	18	51	0.48	300.06,340.13,20.21, 60.29,100.36,140.44, 180.52,220.60,260.67, 307.06,347.06,27.69, 67.53,107.61,147.92 188.00,228.07,268.15	0,120,240,0,120, 240,0,120,240, 248.06,8.06, 128.06,248.06, 8.06,128.06, 248.06,8.06, 128.06	18	52	0.48	18/18/12	6
	1	9	51	0.97	300.06,340.13,20.21, 60.29,100.36,140.44, 180.52,220.60,260.67	0,120,240,0,120. 240,0,120,240	9	51	0.97	9/9/3	12
	2	5	51	1.63	300.06,12.20,84.33, 156.47,228.61	0,0,0,0,0	5	51	1.62	5/5/5	12
	4	4	51	2.62	300.06,30.27,120.59, 210.67	0.0,89.38, 177.20,268.59	4	51	2.61	4/4/1	12
	6	3	51	4.58	300.06,60.37,180.66	0,358.79,357.83	3	51	4.57	3/3/3	12
	8	2	54	6.90	312.08,132.32	0,181.38	2	54	6.88	2/2/1	0
	12	2	48	11.82	276.01,96.29	0,181.14	2	48	11.79	2/2/1	10
	18	1	59	17.13	324.09	0	1	59	17.10	N/A	12
	24	1	48	23.64	276.01	0	1	48	23.59	N/A	12
Near 800	1/2	14	54	0.46	315.08,355.55,361.79, 102.26,168.50,208.97, 275.21,322.36,2.84, 69.07,109.55,175.78, 216.26,282.49	0,154.29,308.57, 102.86,257.14, 51.43,205.71, 258.15,52.43, 206.72,1.01, 155.29,309.58 103.86	17	54	0.36	17/17/3	5
	1	7	54	0.94	315.08,355.55,61.79, 102.26,168.50,208.97, 275.21	0,154.29,308.57, 102.86,257.14, 51.43,205.71	9	50	0.89	9/9/4	19
	2	4	54	1.76	315.08,45.23,135.40, 225.53	0,180,0,180	4	54	1.75	4/4/2	6
	4	3	54	2.58	315.08,75.39,195.56	0,118.42,238.89	3	54	2.57	3/3/1	5
	6	3	50	4.34	302.20,62.43,182.64	0,119.80,239.78	3	50	4.33	3/3/1	19
	8	2	54	6.53	315.08,135.27	0,1.55	2	54	6.52	2/2/2	7
	12	2	47	10.07	289.32,109.70	0,359.39	2	47	11.80	2/2/2	16
	18	1	60	16.66	327.95	0	1	60	16.63	N/A	19
	24	1	47	21.91	289.32	0	1	47	23.62	N/A	16
Near 1200	1/2	13	53	0.49	318.54,355.36,61.37, 68.80,76.24,113.07, 179.07,186.51, 193.95,230.77,296.78, 304.22,311.65	0,242.00,105.28, 8.73,272.18, 154.18,17.47, 280.92,184.37, 66.37,289.65, 193.10,96.55	14	53	0.50	14/7/2	0
	1	7	53	0.94	331.98,318.19,72.69, 87.03,188.64,202.98 304.59	85.45,11.66, 318.32,132.11, 253.31,67.10, 188.30	8	52	1.00	8/8/3	0
	2	4	53	1.89	318.54,48.68,138.81, 228.95	0,270,180,90	4	53	1.88	4/4/1	0
	4	3	53	2.17	318.54,78.68,198.89	0,240.53,120.24	3	53	3.41	3/3/3	23
	6	3	46	5.98	290.80,50.94,171.28	0,240.85,119.05	3	46	5.96	3/3/2	14
	8	2	49	8.00	304.67,124.88	0,181.06	2	49	7.99	2/2/1	0
	12	2	45	11.85	276.94,97.23	0,180.20	2	45	11.83	2/2/1	0
	18	1	54	17.98	318.54	0	1	54	17.96	N/A	13
	24	1	45	23.71	276.94	0	1	45	23.67	N/A	1

Two advantages of the new methods are evident. First, and most important, they produced equal or better results for nearly all cases evaluated. In some cases, as many as two satellites are saved. In others, one does not save satellites, but can launch into a lower inclination or obtain a smaller maximum gap. In a number of cases, the standard constellations are the best and one cannot improve on them using the new methods. Cases where there is a difference of one degree in inclination or .01 hours in gap time may be due to computer roundoff.

The second advantage of the new methods is that they are much less computationally intensive than the standard approach. Table 9 shows SUN workstation run-times (on a time-shared system) for a selection of the cases in Tables 3-8. Note that computer run-times for the new method decrease for higher elevation limits, while the standard approach run-times significantly increase. Run-times for the new method decrease for higher elevation limits because there are fewer passes per satellite, so there are fewer possible orientations to analyze. The standard method requires much more run-time because more satellites are needed to meet the requirement.

Table 9. Computer Run-Times (Min) for 800km Altitude and 50° N Ground Point.

Gap (Hrs)	Minimum Elevation		
	New Method		
	5°	30°	60°
1/2	17.1	5.3	2.8
1	2.6	0.8	0.4
24	0.2	0.2	0.2
	Standard Design		
	5°	30°	60°
1/2	506	5764	16084
1	271	1803	6559
24	6	51	68

Table 10. Results with Time Duration Limits. Ground Point 50° N.

Max Gap Req't (hrs)	Alt Range (km)	5° Minimum Elevation								30° Minimum Elevation							
		No Requirement		2-minute Minimum		5-minute Minimum		10-minute Minimum		No Requirement		2-minute Minimum		5-minute Minimum		10-minute Minimum	
		No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)
1/2	NEAR 500	6	64	6	65	7	58	Coverage not long enough at any inclination		11	55	12	52	Coverage not long enough at any inclination		Coverage not long enough at any inclination	
1		4	55	4	56	4	58			6	55	7	51				
2		2	55	2	56	2	58			3	55	4	51				
4		2	48	2	48	2	51			3	50	3	51				
6		2	42	2	43	2	45			2	55	2	56				
8		2	38	2	38	2	41			2	50	2	51				
12		1	74	1	75	1	77			2	44	2	45				
18		1	42	1	43	1	45			1	55	1	56				
24		1	33	1	34	1	36			1	44	1	45				
1/2	NEAR 800	5	73	5	73	6	54	6	61	9	60	9	60	11	53	Coverage not long enough at any inclination	
1		3	73	3	73	4	54	4	61	5	60	5	60	7	52		
2		2	52	2	52	2	54	2	61	3	52	3	53	4	52		
4		2	43	2	44	2	45	2	52	2	60	2	60	3	52		
6		2	37	2	37	2	39	2	45	2	52	2	53	2	58		
8		2	32	2	33	2	34	2	40	2	47	2	48	2	52		
12		1	62	1	62	1	64	1	72	1	80	1	80	2	45		
18		1	37	1	37	1	39	1	45	1	52	1	53	1	58		
24		1	28	1	28	1	29	1	35	1	41	1	41	1	45		
1/2	NEAR 1200	5	71	5	71	5	71	5	71	9	59	9	60	10	54	Coverage not long enough at any inclination	
1		3	74	3	74	3	75	4	46	5	51	5	51	5	54		
2		2	50	2	50	2	51	2	56	3	51	3	51	3	54		
4		2	40	2	40	2	41	2	41	2	59	2	60	3	43		
6		2	29	2	29	2	29	2	33	2	51	2	51	2	51		
8		1	84	1	84	1	85	2	29	2	40	2	41	2	51		
12		1	62	1	62	1	63	1	68	1	81	1	82	1	85		
18		1	28	1	28	1	29	1	33	1	44	1	45	1	47		
24		1	29	1	28	1	28	1	28	1	37	1	38	1	40		

Time Duration Minimum

For the calculation of optimal constellations when there is a time duration minimum for each pass, the only change necessary is to find the ascending nodes where a pass gives the specified time duration minimum exactly. With the lobes suitably reduced in extent, the procedure is otherwise unchanged.

To find an ascending node location that gives exactly the specified duration, consider the location of the ground point as

$$\vec{r}_G = R_e \cos\theta \cos\phi \, \hat{i} + R_e \sin\theta \cos\phi \, \hat{j} + R_e \sin\phi \, \hat{k} \qquad (10)$$

where

$$\theta = \theta_o + Rate*t \, . \qquad (11)$$

Here R_e is the earth radius, ϕ is the latitude, and θ is the longitude, modified by the rotation since t=0. t is measured from nodal crossing, and Rate is the earth's angular velocity modified by oblateness effects (Equation 5). The satellite position is given by

$$\begin{aligned} \vec{r}_s = {} & r(\cos E \cos\Omega - \sin E \sin\Omega \cos i)\hat{i} \\ & + r(\cos E \sin\Omega + \sin E \cos\Omega \cos i)\hat{j} \qquad (12) \\ & + r(\sin E \sin i)\, \hat{k} \end{aligned}$$

where r is the satellite radius, Ω is the inertial ascending node location with respect to Greenwich, i is the inclination, and E is the eccentric anomaly, given by

$$E = 2\pi t/\tau \qquad (13)$$

for circular orbits, where τ (Equation 6) is the period. Taking the dot product of $\vec{r}_G$ and the vector locating the satellite from the ground, one obtains ζ, the complement of the minimum elevation angle:

$$\cos\zeta = \frac{\vec{r}_G \cdot (\vec{r}_s - \vec{r}_G)}{R_e \left| \vec{r}_s - \vec{r}_G \right|} \qquad (14)$$

ζ is a known, fixed quantity at entry and exit times. Thus one may write two equations that have solutions t_1 and t_2 for entry and exit times for a given ascending node location. At the ascending node value that gives exactly the required time duration, the times are related by

$$t_2 - t_1 = C \qquad (15)$$

where C is the specified time duration. Substituting for t_2 in the second equation yields two equations in the two unknowns t_1 and Ω. By taking the derivatives of both equations with respect to t_1 and Ω, and using Newton's method in the form

$$\vec{x}_{k+1} = \vec{x}_k - [F(\vec{x}_k)]^{-1} \; \vec{g}(\vec{x}_k), \qquad (16)$$

one may find the appropriate values for Ω. In Equation 16, $\vec{x}$ is the vector with elements t_1 and Ω. $\vec{g}$ is the vector of Equation 14, written for t_1 and t_2 in a form so that $\vec{g}=0$ yields a solution. F is the matrix of partial derivatives of each g equation with respect to t_1 and Ω. Table 10 compares results for some of the earlier cases as the time duration limit increases.

Coverage of Extended Areas or Several Ground Points

There are two obvious ways to pose the problem of coverage of extended areas. One is that the entire area is seen simultaneously by one satellite. This problem might be appropriate for a communications system that does not have crosslink capability. The alternate problem is one that says each point in the extended area must meet the gap requirement, but with no restrictions as to simultaneity or which satellite sees the ground point. This requirement might be more appropriate for some kind of observing system.

Simultaneous Coverage from One Satellite

In this case, one may set up a matrix of ground points covering the perimeter of the area to be covered, with some additional points in the center. If one desires to cover a set of points rather than an area, then these points would be input. One only needs to examine a longitude range (in the extended area) of one orbit's ground track motion since the pattern is repeating. Next calculate the ascending node limits for each ground point and choose the intersection of these. Then find in/out times for passes for each ground point and find the intersection of these. The resulting timeline of passes can be treated as before in deriving the best constellation.

One problem associated with simultaneous coverage of an extended region results from the coverage created by the intersection of the coverage for each point in the region. For extended regions, some points may have single lobe coverage, while others have two lobe coverage. The resulting intersection may have either. As previously stated for single point coverage, the maximum number of passes occurs where the lobe splits into two lobes. For large extended regions, however, there may not be any single lobe coverage. This creates a problem for short gap time requirements, as the theory does not yet support two lobe analyses for short gaps.

Table 11 gives some results for the case of simultaneous coverage of an extended region. Two extended regions were used as examples; they are Pennsylvania and India. Pennsylvania is a relatively small area, about 2.5 degrees by 5.5 degrees. India is larger, about 21.4 degrees by 27.5 degrees. The results are optimal in the sense that the new theory in this paper is used to derive the constellations, using the timelines found by the intersection of all points in the region.

Non-Simultaneous Coverage from Any Satellite

Applying the previous methods to this problem is much more difficult and will not be attempted in this paper. However, if the ground area is at least one orbit's ground track motion wide in longitude (about 24 degrees or so for the cases considered here), then there is nothing to be gained by picking specific longitudes of ascending node. Differences in mean anomaly could cause some gains in coverage over a standard constellation. The gains over standard constellations are probably not as large as previous gains.

Table 11. Results for Coverage of Extended Areas.*

Alt. Range (km)	Gap Time Req't (hrs)	Coverage of Pennsylvania								Coverage of India							
		Simultaneous Coverage by Same Satellite				No Coverage Restrictions				Simultaneous Coverage by Same Satellite				No Coverage Restrictions			
		5° Minimum elevation		30° Minimum elevation		5° Minimum elevation		30° Minimum elevation		5° Minimum elevation		30° Minimum elevation		5° Minimum elevation		30° Minimum elevation	
		No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)	No of Sats	Inc. (deg)
1/2	NEAR 500	7	52	14	43	7	50	+	+	++	++	No simultaneous coverage at any inclination		11	28	+	+
1		4	52	7	43	4	52	7	46	++	++			7	28	15	31
2		2	52	4	43	2	52	4	43	++	++			4	28	7	29
4		2	42	3	43	2	42	3	43	4	28			3	28	5	32
6		2	35	2	49	2	35	2	49	3	28			2	28	4	29
8		2	31	2	43	2	31	2	43	2	28			2	28	3	33
12		1	77	2	36	1	77	2	36	2	28			1	74	2	33
18		1	35	1	49	1	35	1	49	1	33			1	28	1	39
24		1	28	1	36	1	28	1	36	1	28			1	28	1	33
1/2	NEAR 800	6	49	11	47	6	55	12	47	10	28	No simultaneous coverage at any inclination		8	28	+	+
1		4	48	6	47	4	50	6	47	6	28			5	28	12	28
2		2	48	3	47	2	48	3	47	3	28			3	28	6	28
4		2	37	3	40	2	37	3	40	2	30			2	28	4	28
6		2	29	2	47	2	30	2	47	2	28			2	28	3	28
8		2	28	2	40	2	28	2	40	2	28			2	28	3	28
12		1	62	2	33	1	62	2	33	1	74			1	64	2	30
18		1	29	1	47	1	30	1	47	1	28			1	28	1	37
24		1	28	1	33	1	28	1	33	1	28			1	28	1	30
1/2	NEAR 1200	5	63	10	45	6	47	10	47	8	28	No Simultaneous Coverage at any inclination		7	28	+	+
1		4	33	5	45	4	39	5	45	4	28			4	28	8	28
2		2	46	3	45	2	46	3	45	2	28			2	28	4	28
4		2	28	3	33	2	28	3	33	2	28			2	28	3	28
6		2	28	2	38	2	28	2	38	2	28			2	28	2	28
8		2	28	2	33	2	28	2	33	2	28			2	28	2	28
12		1	63	1	86	1	62	1	85	1	55			1	33	2	28
19		1	28	1	38	1	28	1	38	1	28			1	28	1	28
24		1	28	1	30	1	28	1	29	1	28			1	28	1	28

* Simultaneous coverage uses new theory; coverage with no restrictions uses standard constellations
\+ Computer run times in excess of 2 weeks precluded inclusion of this data
++ New theory does not yet support solution for two lobe coverage with short gap requirements

Table 11 shows results for standard constellations covering Pennsylvania and India. Note that there is not much difference in the results between the simultaneous coverage constellations and the non-simultaneous standard constellations for the Pennsylvania region. Pennsylvania is small enough that requiring simultaneous coverage by a given satellite is not a limiting restriction. The larger region of India, however, is a much more restricting case. Simultaneous coverage of India always required at least an equal constellation, or one at a higher inclination or number of satellties.

Summary

The analysis in this paper shows how one can methodically design satellite constellations to optimize partial coverage by minimizing the number of satellites required, minimizing their orbit inclination, and minimizing the resulting maximum gap time. Repeating ground track constellations are shown to give better results over those with non-repeating ground tracks. Satellites with elliptical orbits are compared to those with circular orbits. The theory presented in the paper is for design of the best constellations using circular orbits. This theory shows how to pick the best ascending node location and initial mean anomaly for each satellite, and how to design the best constellation.

Results using the new methods and resulting in non-standard constellations are compared with the best standard constellations and are usually better. Results are given for ground points at different latitudes, different minimum elevation angles for viewing a ground point, different satellite altitudes, and various gap time requirements. The new design methods require significantly less computer run time as compared to standard methods.

This paper also develops theory for extending the methods to minimum pass time duration requirements and to coverage of extended areas. Tables in the paper give results for these cases.

Study of elliptical as well as circular orbits to find the cases where elliptical results are better is a topic that deserves future consideration. The theory for elliptical constellations would be very similar to that for circular constellations. Once timelines are found for one satellite, the rest of the theory would be identical. More extensive results for time duration limitations and coverage of extended areas would be useful. Additional analyses to solve the extended region case for two lobe coverage with short gap requirements would also be useful. The theory presented here was not extended to the case of extended area coverage that is non-simultaneous and is not from the same satellite. Such an extension would be beneficial.

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