

# New Approach to Achieving Stand Alone GPS Attitude Determination using Dual Short Baselines for Small-Satellite

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## Abstract

This paper proposes a new approach to GPS (Global Positioning System) attitude determination for small satellite application in LEO (low Earth orbit). Prior knowledge of attitude and integer resolution is not required. The methodology of the new approach includes integer ambiguity search, initial estimation of attitude and line bias, attitude initialisation, path difference estimation and fine attitude determination. The observable is the carrier phase difference measurement between two GPS antennas. A dual short baseline (typical baseline length up to 30 cm) is assumed in this research.

The key point to initialising attitude is to estimate the attitude of individual baseline vectors with respect to the reference frame. Elimination of integer ambiguity is a simple task. Two sets of vectors are required to determine an initial attitude. Once attitude is initialised, an estimation algorithm based on the extended Kalman filter starts to determine the attitude. The integer ambiguities and cycle slips can be resolved properly. The filter now is converged and, fine attitude is estimated. The robustness of the filtering estimator is tested with simulated anomalous conditions.

## Nomenclature

$\mathbf{A}$	: transformation attitude matrix
$\mathbf{b}$	: baseline vector
$\mathbf{s}$	: line-of-sight unit vector to GPS satellite
$l$	: L1 carrier wavelength
$h$	: integer ambiguity
$k$	: integer ambiguity including effects of error
$b$	: line bias
$w$	: measurement noise
$j$	: real phase difference (- to + )
$\underline{r}$	: actual modulo path difference (- /2 to + /2)
$\underline{r}_{(m)}$	: measured modulo path difference
$r$	: actual path difference
$r_{(m)}$	: path difference including error
$f$	: roll angle
$q$	: pitch angle
$y$	: yaw angle
sup $T$	: transpose of matrix
sub $B$	: body-fixed frame
sub $R$	: reference frame
hat $\hat{\phantom{x}}$	: estimate

## Introduction

Spacecraft attitude determination is generally based on the use of traditional attitude sensors such as Sun sensors, Earth sensors, star sensors, inertial sensors, and magnetometers. However, since GPS has been successfully used for spacecraft navigation, a new approach using GPS attitude determination has been rapidly developing for space applications.

In 1993, GPS attitude determination was demonstrated for the first time on the RADCAL satellite<sup>1</sup>. Using an onboard GPS receiver and multiple GPS antennas, GPS attitude sensing can be achieved through carrier phase difference measurements between two antennas. The measured scalar phase difference is used as an observable for the attitude algorithm for determining spacecraft orientation. However, there are several problems involved in the system implementation.

Firstly, as a GPS receiver can measure only a fraction of carrier phase cycle (- to + in radian or in equivalent - /2 to + /2 in range), the number of full

cycles are unknown. Moreover, if the fraction of phase measurement is very close to the edge, offset errors (e.g. line bias and multipath effects) and noise may cause the unknown integer number to slip to another number. This will magnify the difficulty of resolving the integer ambiguity.

Secondly, once the integer ambiguities are resolved, the spacecraft attitude can be estimated. There are several optimal attitude algorithms that can be applied for spacecraft applications. However, the particular choice of optimal algorithm depends on the mission requirements and practical spacecraft detail such as computation time available, baseline configuration and receiver hardware design. For example, it has been reported that the three orthogonal baseline configuration is the best configuration. However, most satellites can only accommodate a coplanar or near coplanar configuration.

Thirdly, once the attitude is determined, then the performance, reliability and accuracy of GPS attitude determination need to be improved. This requirement depends on the efficiency of the method used to mitigate the measurement error.

This paper investigates the feasibility of the use of a new approach to estimate spacecraft attitude without prior attitude knowledge. The GPS attitude information is measured from dual baselines in the presence of measurement noise and line bias. The integration of three algorithms: a novel `block ambiguity search`; followed by a standard algebraic algorithm and the extended Kalman filter.

The block ambiguity search consists of a computation of the attitude pointing of each baseline individually, without regard to their relative attitude within the spacecraft, and initially not even to each other.

With this approach and by exploiting the over-determination of the attitude solution using four (or more) satellites it is a relatively simple matter to solve the so-called ‘integer ambiguity problem’ with respect to each baseline. As is well known the problem is to identify the initially unknown integer wavelengths that must be inserted into each path difference measurement: typically in simulation using our new method only one or a few possible solutions emerge as likely candidates.

Then by looking at a pair wise combination of possible solutions the best and correct solution is obtained given knowledge of the actual angle between baselines. The correct number of integer

insertions into each path difference measurement has now been established.

Having obtained a correct solution to the pointing of the baselines (always in reference space) is then a simple matter to compute the actual attitude of the satellite, in the second phase, using the standard TRIAD algorithm.

Having found what we now call an `initial attitude` this information now feeds a standard extended Kalman filter to enable continued attitude determination now exploiting the dynamics of the spacecraft.

This third step is attitude refinement using an extended Kalman filter. However, the measured path differences must continue to be checked and modified by an appropriate integer multiples of wavelength during filtering. This paper describes a technique to perform this process; and other refinements to prevent divergence; so that the overall process may be called extended Kalman filtering with integrity check.

## 2 Background

In this section, the fundamentals of phase difference measurement and the problems in GPS attitude determination are briefly introduced.

### 2.1 Fundamentals of GPS Attitude Measurement

The fundamental measurement in GPS attitude determination is the path difference measurement between two antennas separated by the baseline length. For baseline  $i$  and GPS satellite  $j$ , the actual path difference,  $r_i^{(j)}$ , can be expressed as

$$r_i^{(j)} = \mathbf{b}_{iR} \cdot \mathbf{s}_R^{(j)} = \underline{r}_i^{(j)} + \mathbf{h}_i^{(j)} \mathbf{I} \quad (1)$$

where  $\underline{r}_i^{(j)} \equiv \mathbf{j}_i^{(j)} \mathbf{I} / 2p$ .

As the GPS measurement is always perturbed by measurement noise and bias, the realistic equation can be expressed as

$$r_{i(m)}^{(j)} = \left( \underline{r}_i^{(j)} + \mathbf{b}_i + w_i^{(j)} \right) + \mathbf{h}_i^{(j)} \mathbf{I} \equiv \underline{r}_{i(m)}^{(j)} + k_i^{(j)} \mathbf{I} \quad (2)$$

where bias in equivalence with path difference is considered to be within  $\pm \lambda/2$ .

For Equation (2), only the  $\underline{r}_{i(m)}^{(j)}$  term is given by a GPS receiver. The  $k$  ambiguity cycles need to be resolved.

Note that the original integer resolutions may slip one cycle due to the error in measurement.

## 2.2 Problems in GPS Attitude Determination

To achieve an attitude solution from GPS measurements, several problems must be addressed. These include integer ambiguity resolution, error sources, and the choice of the operational attitude algorithm.

The problem of integer ambiguity has been introduced earlier. Several methods based on either ambiguity search or motion techniques have been proposed to resolve the integer ambiguity problem.

The general concept of the ambiguity search technique for attitude determination is to search for the true integer ambiguity set that minimises the error between the estimate baseline length and the known length<sup>2</sup>. The use of the double phase difference approach in ambiguity search not only removes the bias, but also reduces ambiguity search space as well<sup>3,4</sup>.

The motion-based methods operate on a batch of measurements collected over a given period of time, which it is assumed that the integer ambiguities still remain constant over the given period<sup>5,6,7</sup>.

Error sources in GPS attitude sensing consist of multipath, line bias, receiver noise, phase centre variation, and geometry of selected GPS satellites.

Multipath is usually considered the dominant error source<sup>8</sup> in GPS attitude determination. A GTD (Geometrical Theory Diffraction) method can be used to model and verify the differential carrier phase error caused by multipath effects.<sup>9</sup> However, multipath error can be mitigated by using signal to noise ratio information to correct multipath errors in carrier phase difference measurements<sup>10</sup> or by using a calibration method<sup>11</sup>.

Line bias is a phase offset between two antenna chains, caused simply by different length cables or by different RF front ends. Possible methods to remove line bias include the use of double phase differences or a calibration signal source.

Antenna phase centre is one factor that affects the carrier phase measurement. Generally, the physical centre of the antenna does not coincide with the

received point of GPS signals on the antenna. Furthermore, these received signal points will vary with the elevation angle, and also from one antenna to another antenna.<sup>12</sup>

The geometry of the GPS satellites is another factor in achieving high quality results. The geometry changes with time due to the relative motion of the GPS satellites. A measure for the geometry is generally described as the Dilution of Precision (DOP) factor. In GPS attitude sensing, the uncertainty of the given attitude solutions will depend upon the geometry of selected GPS satellites and the antenna baseline configuration. To predict or evaluate the given attitude solutions, the numerical mean called ADOP (Attitude Dilution of Precision) factor can be used to evaluate the attitude solution.

This paper concentrates on the integer ambiguity resolution, line bias and path difference estimation and operational attitude algorithms for dual baseline configurations, accounting for measurement noise and line bias. The known angle between baseline vectors will be taken into account in rejecting the wrong solution from ambiguity search techniques.

## 2.3 Approximate Attitude Error

A rough approximation to the rms attitude error,  $s_{\theta}$ , for effective baseline length,  $L$ , can be estimated by<sup>13</sup>

$$s_{\theta} \cong \frac{s_r}{L} \frac{2p}{1} \quad \text{radian} \quad (3)$$

where  $s_r$  is the rms of path difference error.

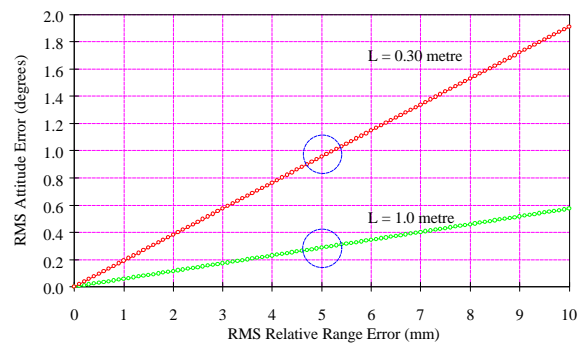


Figure 1: Approximate attitude error of GPS-based attitude determination

Typically, the expected path difference error is in the order of 5 mm approximately<sup>14</sup>. The expected pointing error of 1 metre baseline is then about 0.3 degrees as shown in Figure 1. The attitude error will increase to 1.0 degree when the baseline length is reduced to 30 cm.

## 3 GPS Attitude Estimation from Dual Baseline

### 3.1 Initial Step Estimation - the block ambiguity search

This section describes the baseline vector estimation using block ambiguity search.

The aim is to determine the attitude of each baseline individually and directly in reference space.

The measurements from each baseline  $i$  and four selected GPS satellites can be expressed as a matrix equivalent to Equation (2).

$$\mathbf{S}\mathbf{y} = \mathbf{r}_{(m)} + \mathbf{I}\mathbf{k} \quad (4)$$

where

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_R^{(1)} & 1 \\ \mathbf{s}_R^{(2)} & 1 \\ \mathbf{s}_R^{(3)} & 1 \\ \mathbf{s}_R^{(4)} & 1 \end{bmatrix}_{(4 \times 4)} \quad \mathbf{r}_m = \begin{bmatrix} r_{-i(m)}^{(1)} \\ r_{-i(m)}^{(2)} \\ r_{-i(m)}^{(3)} \\ r_{-i(m)}^{(4)} \end{bmatrix}_{(4 \times 1)} \quad (5)$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{b}_{iR} \\ \mathbf{b}_i \end{bmatrix} \equiv \begin{bmatrix} x_{iR} \\ y_{iR} \\ z_{iR} \\ \mathbf{b}_i \end{bmatrix}_{(4 \times 1)} \quad \mathbf{k} = \begin{bmatrix} k_i^{(1)} \\ k_i^{(2)} \\ k_i^{(3)} \\ k_i^{(4)} \end{bmatrix}_{(4 \times 1)}$$

The inverse matrix of  $\mathbf{S}$  matrix can be expressed as

$$\mathbf{S}_{(4 \times 4)}^{-1} \equiv \begin{bmatrix} \mathbf{U} \\ \mathbf{v} \end{bmatrix} \quad (6)$$

where  $\mathbf{U}$  is a  $(3 \times 4)$  matrix and  $\mathbf{v}$  is a  $(1 \times 4)$  vector.

Given a trial set of integers in  $\mathbf{k}$ , an estimated baseline vector  $\hat{\mathbf{b}}_{iR}$  can be obtained by

$$\hat{\mathbf{b}}_{iR} = \mathbf{U}(\mathbf{r}_{(m)} + \mathbf{I}\mathbf{k}) \quad (7)$$

An initial line bias can be computed in parallel with Equation (9) from

$$\hat{\mathbf{b}}_i = \mathbf{v}(\mathbf{r}_{(m)} + \mathbf{I}\mathbf{k}) \quad (8)$$

For each trial choice of values in  $\mathbf{k}$  a trial baseline vector is computed and also a trial line bias. The most likely correct solution(s) may be determined by monitoring two effects

1. a computed length of a trial baseline should be close to the actual baseline length.
2. a computed trial bias value should not be contradicted by whatever information is known about that bias.

From detailed simulation with plausible values to typical measurement errors we show that in most cases a unique and correct solution is found to each baseline vector.

For medium baseline lengths the range of search in  $k$  values typically lies between -2 and +2 and does not cause an excessive load for computation in a systematic search, requiring only 625 trials.

In some worst case solutions more than one contender emerges as a solution to each baseline vector.

To get round this problem we exploit the fact that the process so far has estimated each baseline vector independently.

So we exploit the known information of the angle between the two baselines by a pair wise comparison. The correct pair is identified with a computed angle between them which is closest to the true angle of the actual baselines.

Even if information between computed trial bias and known bias is not used there is a high degree of success in the operation and processing subsequent epochs will ensure 100% acquisition eventually.

The computed bias is used in any case to update and monitor the progress of the Kalman filter, once the correct solution has been identified.

Simulations show that this combination of procedures within the block ambiguity search is always successful, leading to a unique and correct solution to the pointing of the baselines .

### 3.2 Estimation of attitude

The block ambiguity search has now identified the correct pointing (with some error) of the two baselines, expressed in reference space.

The next step is to determine the actual attitude  $\hat{\mathbf{A}}_0$  of the satellite. A novel feature here is now to use the baseline vectors computed in reference space as observation vectors.

An attitude, to the actual satellite , can therefore be determined using a standard TRIAD algorithm<sup>15</sup>., using the further knowledge of the pointing the direction of these same vectors relative to the body frame of the satellite.

The top level diagram of the initialisation step is shown in Figure 2.

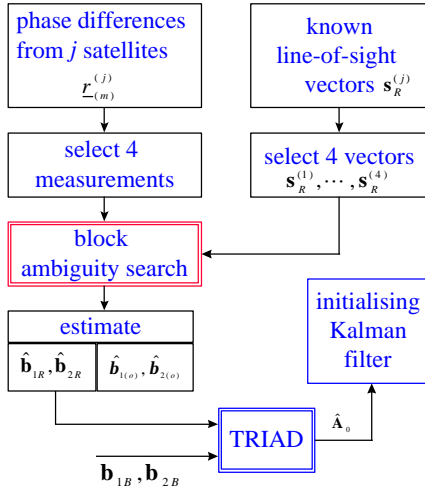


Figure 2: Initial attitude estimation

It is important to be note that once the initial attitude and initial line bias are estimated, the block ambiguity search will not be required again until re-initialisation is required.

### 3.3 Attitude Estimation using Kalman Filter

This section describes the attitude estimation using the extended Kalman filter. The filter is initialised by the attitude knowledge derived from the initial step.

For the first epoch of filtering, the knowledge of attitude and line bias from the initial step will be used to estimate path differences and feed into the filter.

An estimated attitude from the Kalman filter will be used to estimate and update line bias. The updated line bias and estimated attitude also will be used to estimate path differences in the next epoch. The process diagram of the filtering estimation is shown in Figure 3.

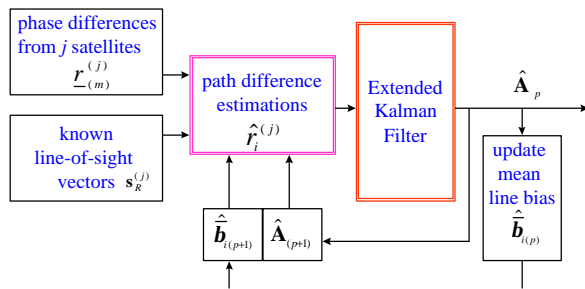


Figure 3: Filtering estimation process

In this section, the equation models of the extended Kalman filter are briefly introduced in the first sub section. The technique of path difference estimation is described in the second sub section. The re-estimation of line bias is explained in the second sub section.

#### 3.3.1 Equation Models for Kalman Filter

The Kalman filter presented here is implemented to estimate spacecraft attitude and its dynamics for small rotation angles. The equation models are based on Euler angle representation.

##### State Vector

The filter state vector consists of three estimated Euler angles and their rates.

##### System Equation

Since the development of the filter follows the standard UoSat microsatellite platform<sup>16</sup> which is a gravity gradient stabilised/nadir pointing spacecraft, the system model of the filter can be derived from the dynamics model of Earth-pointing spacecraft for small rotation angles<sup>17</sup>.

$$\ddot{\mathbf{f}} + 4\mathbf{k}^2 \mathbf{w}_o^2 \mathbf{f} = (1 - \mathbf{k}^2) \mathbf{w}_o \dot{\mathbf{y}} \quad (9a)$$

$$\ddot{\mathbf{q}} + 3\mathbf{k}^2 \mathbf{w}_o^2 \mathbf{q} = 0 \quad (9b)$$

$$\ddot{\mathbf{y}} + \mathbf{w}_o \dot{\mathbf{f}} = 0 \quad (9c)$$

where  $\mathbf{w}_o$  is the constant orbital angular velocity of the spacecraft (assuming in circular orbit),  $\mathbf{k}^2 = (1 - I_z/I_t)$ ,  $I_z$  is a principal moment of inertia of z axis, and  $I_t$  is a transverse moment of inertia.

##### GPS Measurement Equation

The path difference measurement from baseline  $i$  and GPS satellite  $j$  can be expressed as

$$r_i^{(j)} = \mathbf{b}_{iB}^T \mathbf{A} \mathbf{s}_R^{(j)} \quad (10)$$

where  $\mathbf{b}_{iB} = [x_{iB} \ y_{iB} \ z_{iB}]^T$ ,  $\mathbf{s}_R^{(j)} = [s_{xR}^{(j)} \ s_{yR}^{(j)} \ s_{zR}^{(j)}]^T$

It is assumed that spacecraft is slowly rotating (e.g.10 minutes per revolution) about the yaw axis for thermal control and solar power management purposes. Therefore, a small roll and pitch can be assumed. The GPS measurement equation then can be given by

$$r_i^{(j)} = V_1 \cos \mathbf{y} + V_2 \sin \mathbf{y} + V_3 \quad (11)$$

where

$$V_1 = x_{iB} (s_{xR}^{(j)} - s_{zR}^{(j)} \mathbf{q}) + y_{iB} (s_{yR}^{(j)} + s_{zR}^{(j)} \mathbf{f}) \quad (12a)$$

$$V_2 = x_{iB} (s_{yR}^{(j)} + s_{zR}^{(j)} \mathbf{f}) - y_{iB} (s_{xR}^{(j)} + s_{zR}^{(j)} \mathbf{q}) \quad (12b)$$

$$V_3 = z_{iB} (s_{xR}^{(j)} \mathbf{q} - s_{yR}^{(j)} \mathbf{f} + s_{zR}^{(j)}) \quad (12c)$$

The state transition matrix and process noise matrix of the developed filter are implemented following the standard Kalman filter formulation.

### 3.3.2 Path Difference Estimations

Phase difference is provided by the GPS receiver, while the knowledge of attitude and line bias is provided by the filtering estimation.

During operation of the Kalman filter there is a continual need to estimate and update the necessary integer corrections on each phase difference measurement.

Fig 3 shows in block diagram form that this is accomplished in two ways.

Using the filter a predicted attitude is used to compute predicted path difference measurements to be compared against the current measured differences.

Unlike the standard Kalman filter this innovation need to be adjusted when necessary by integer shifts in each component of the observation vector when a cycle shift has been detected.

By backwards computation also the proposed integer corrections can be checked to see if a consistent bias value is generated, and if not a further correction made.

### 3.4 Integrity Check

Several circumstances may cause the filtering estimation to diverge. The developed Kalman filter presented in this paper includes a protection function to prevent the divergence or high error in estimated solution. This function is designed to be activated after the filter has been running for a while. A simple diagram of the loop process of this Kalman filter and integrity check is shown in Figure 4.

In normal operation, the updated covariance matrix and updated state vector are used in the propagation state for the next epoch estimation, and this information will be written into the buffer memory if the difference of estimated attitude between two epochs is satisfied within the set boundary.

If this difference is outside the set boundary, the updated information will not be saved into the buffer memory. Simultaneously, the previous updated information will be used in the propagation state. In this protection operation, the propagated covariance matrix and propagated state vector will be written

back to the buffer memory again. The reason is that the anomalous operation may last longer than one epoch. However, after a long period of this anomaly, the propagated information may diverge from current spacecraft dynamics. Therefore, a re-initialisation operation will be required under this condition.

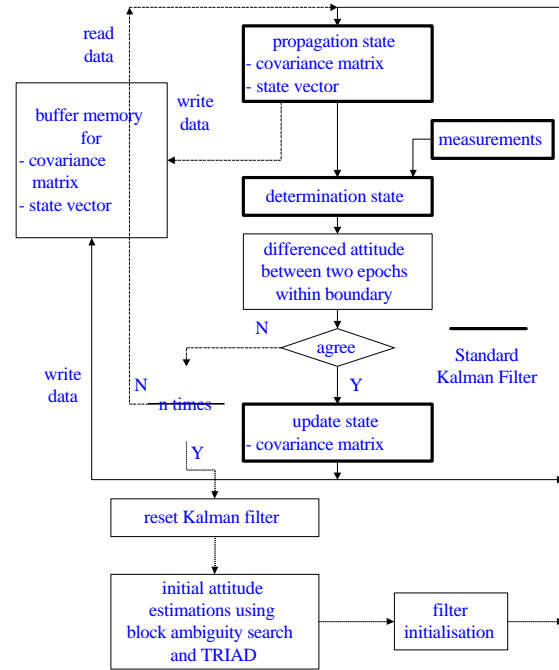


Figure 4: Loop process of Kalman filter with divergence protection

In such a re-initialisation operation, the block ambiguity search and TRIAD will estimate again an attitude for initialising Kalman filter. It may not necessary to re-estimate the line bias again. We can use the back up figure which was saved before resetting the filter, and re-estimate after the estimated attitude is given by the filtering estimator. With experience gained of the actual line bias this may also speed up the re-acquisition during the first step of the block ambiguity search.

## 4 Simulation Results

All simulation results presented in this paper are based on a gravity gradient satellite in a Sun-synchronous circular orbit, 98 degrees inclination angle, and 800 km altitude. The nominal simulation parameters are given in Table 1.

The six hours of simulated GPS measurements are used as the input file. The estimated attitude error (the estimated attitude from the extended Kalman filter compares to the reference attitude from simulation) in roll, pitch and yaw are plotted in XY format in which the X axis is the time.



It is important to note that in this paper, the measurement error is assumed as white Gaussian with 5 mm rms, while the line bias is assumed as constant.

Table 1: Nominal Simulation Parameters

initial attitude degrees	roll 10	pitch 10	yaw 0
moment of inertia kg m <sup>2</sup>	x axis 100.0	y axis 100.0	z axis 1.0
initial angular velocity deg/sec	$w_x$ 0.0	$w_y$ -0.059	$w_z$ 0.6
baseline coordinates [x y z] metre	$b_{1B}$ [03 0 0]	$b_{2B}$ [0 03 0]	
line bias metre	$b_1$ 0.09	$b_2$ -0.08	

Two anomalous operations are simulated for testing the robustness of the filtering estimator. The first anomaly (Case I) assumes that an error in the data memory may possibly caused by a single event upset and it introduces the single errors to the filter. The second anomaly (Case II) assumes that the simulated GPS measurement is perturbed by excessive noise for a short period. The later circumstance may be similar to some cases under real operation. For example a high gain antenna of an onboard RF transmitter is transmitting high power signals to communicate with the ground station.

#### 4.1 Attitude Acquisition from block ambiguity search

Simulation follows a simpler approach in the block ambiguity search that the general procedure outlined in section 3.1 : the approach consists simply in selecting the one solution for each baseline which is closest in length to the known length. Bias information is only used as far as prediction of its sign.

This procedure includes the test of known angle between baselines against angle between trial baselines.

This simplified approach sometimes rejects correct solutions on the first epoch. The remedy is simply to repeat the acquisition on the next epoch block of data until successful acquisition.

This simplified approach sometimes accepts incorrect solutions on the first epoch. The remedy is simply to

repeat the acquisition on the next epoch of data and test for consistency of attitude is delivered.

In a typical heuristic test the angle between baseline is 90 degree and tolerance between baseline angles was set at  $\pm 15$  degrees. Then it was seen that 75% of solutions from the block ambiguity search were correct from the very first epoch of data used The method was of course always successful in eventually acquiring the attitude in the few following epochs for those 25 % of trials that were initially unsuccessful.

Figures and tabulation show the success in direct attitude determination on a successful acquire epoch.

The estimated error in roll is shown in Figure 5. For the estimated error in pitch and yaw, the rms figure shown in Table 2 is used to represent the characteristics instead of XY plot. The estimated figure of initial line bias is shown in Figure 6.

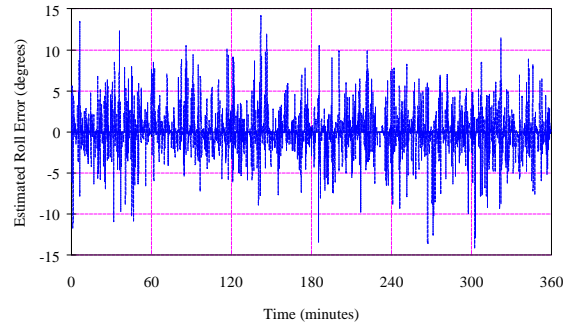


Figure 5: Estimated roll error of initial attitude

Table 2: RMS Error of Initial Attitude

angle	roll	pitch	yaw
error (1 )	3.23	3.41	0.88
degree			

From the rms error as shown in the table 2, it can be seen that the accuracy of initial attitude knowledge is adequate to initialise Kalman filter.

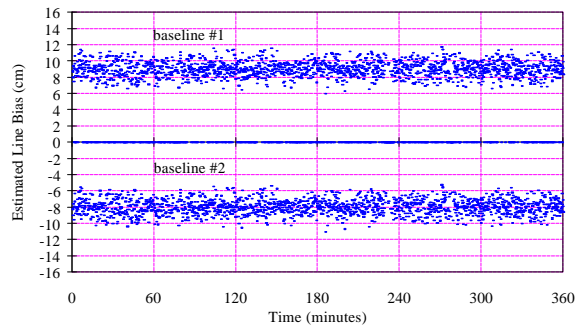


Figure 6: Initial line bias from block ambiguity search

From these graphs, the error of initial line bias is within  $\pm 2$  cm. This confirms that the knowledge of the initial line bias can be used in path difference estimation for the first epoch of filtering estimation.

#### 4.2 Estimation under Kalman Filtering

This section shows the simulated results of filtering estimation under normal condition. The estimated attitude error is shown in Figure 7, and the estimated line bias is shown in Figure 8.

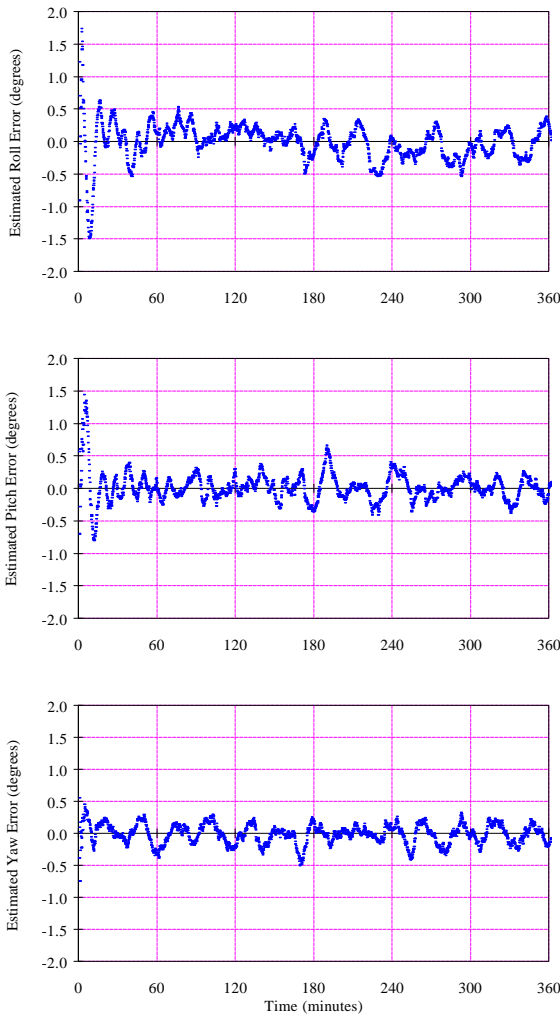


Figure 7: Estimated attitude error (normal condition)

Table 3: RMS Error of Filtering Estimation

angle	roll	pitch	yaw
error (1 ) degree	0.36	0.31	0.20

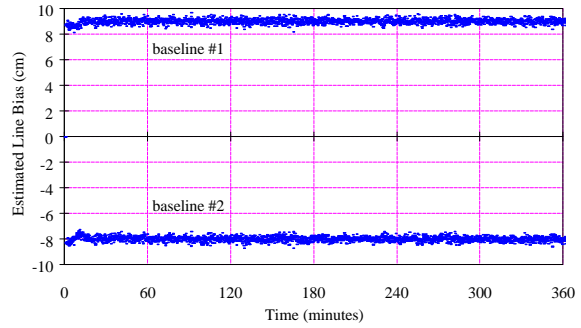


Figure 8: Estimated line bias (normal condition)

From these graphs, it can be seen that the filtering estimation performs very well. The error of estimated line bias is within  $\pm 1$ cm.

#### 4.3 Anomalous Conditions without Integrity Check

This section shows the simulated results of the filtering without integrity check. Two examples of anomaly as described previously are simulated.

##### 4.3.1 Case I : Single Error

In this example, it assumes that a single errors occur at 120<sup>th</sup> minute and 240<sup>th</sup> minute of the operation. For the first event, it is assumed that each estimated Euler angle from the filtering estimation is perturbed by an error of 20 degrees ( $\sim 34.5$  degrees of pointing error). For the second event, an error of 30 degrees for each angle is used for the same purpose. The RMS of attitude error in roll, pitch and yaw will be shown in Table 4. The estimated attitude errors in roll is shown in Figure 9. The estimated line bias is shown in Figure 10.

Table 4: RMS Error of Case I without Integrity Check

angle	roll	pitch	yaw
error (1 ) degree	22	16	50

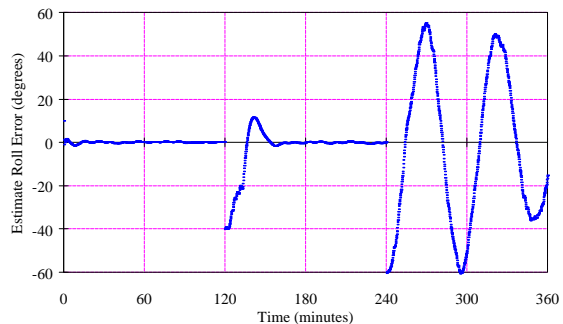




Figure 9: Estimated roll error of Case I without integrity check

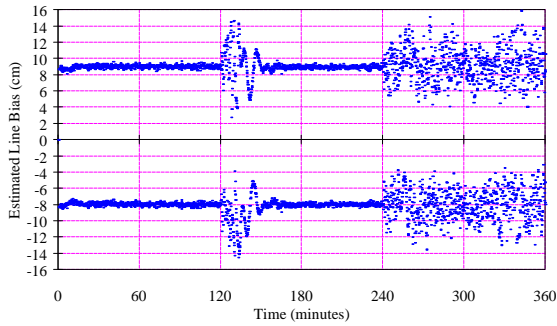


Figure 10: Estimated line bias of Case I without integrity check

From these graphs, it can be seen that the filter can survive from the first event, however it takes time to re-converge again. But in the second event upset, the estimation completely diverges.

#### 4.3.2 Case II : Continuous Error

In this example, two periods of 5 minutes perturbation are simulated. The first period is since 120<sup>th</sup> minute to 125<sup>th</sup> minute, and the second period is since 240<sup>th</sup> minute to 245<sup>th</sup> minute.

Table 5: RMS Error of Case II without Integrity Check

angle	roll	pitch	yaw
error ( )	8	9	83
degree			

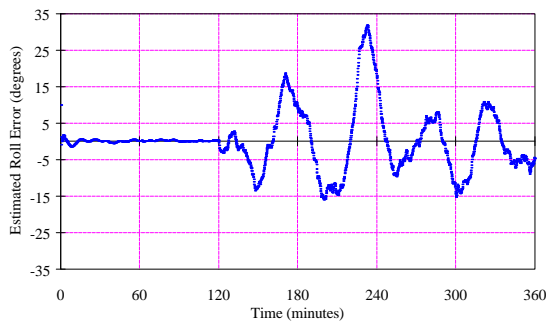


Figure 11: Estimated roll error of Case II without integrity check

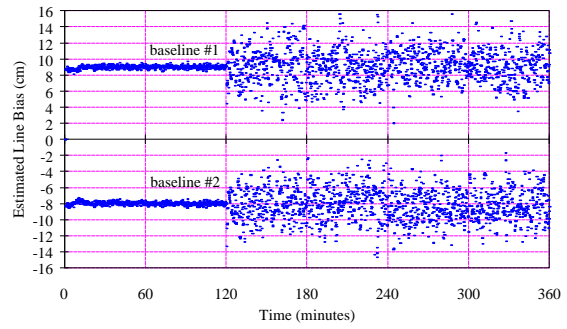


Figure 12: Estimated line bias of Case II without integrity check

From these graphs, it can be seen that in the presence of excessive noise for short periods, the filter diverges after the first period of perturbation and continues to diverge.

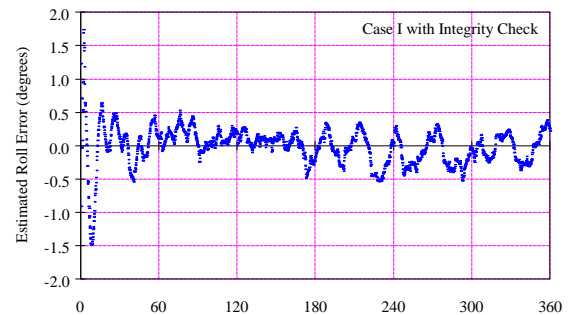
#### 4.4 Anomalous Conditions with Integrity Check

This section shows the improved performance of the filtering estimation under divergence protection. The example in section 4.3.1 and section 4.3.2 are re-demonstrated.

##### 4.4.1 Case I : Single Errors

With the integrity check in place it can be seen that the filter recovers not only from the first but also the second single upset event.

As the large error caused by a single event errors can be detected by the integrity check, the filtering estimation can be expected to continue normally. The estimated attitude error is shown in Figure 13. The rms of estimated attitude error is shown in Table 6.



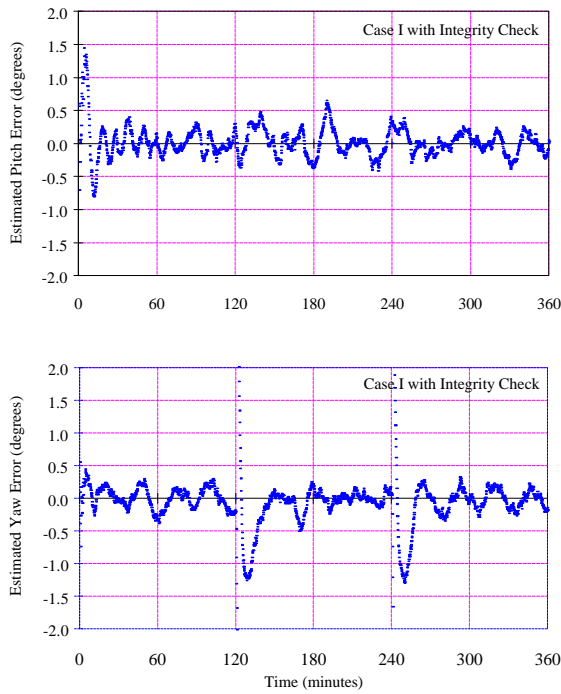


Figure 13: Estimated attitude error of Case I with integrity check

Table 6: RMS Error of Case I with Integrity Check

angle	roll	pitch	yaw
error (1 )	0.36	0.31	0.53
degree			

From these graphs, it can be seen that the attitude error in roll and pitch is the same as the results under normal conditions. The reason is that the rate of change in roll and pitch between two epochs is not so high, and the updated covariance matrix of previous epoch can cope with this rate. However, there is a short period of fluctuation in yaw as shown in Figure 13. The reason is that the yaw rate is quite high, and the use of previous information may not cope with the current yaw rate perfectly.

#### 4.4.2 Case II : Continuous Errors

In this case, the anomalous operation will continue to occur with the set count in the integrity check regularly being exceeded. So regular initialisation is called and for and supplied by the overall algorithm

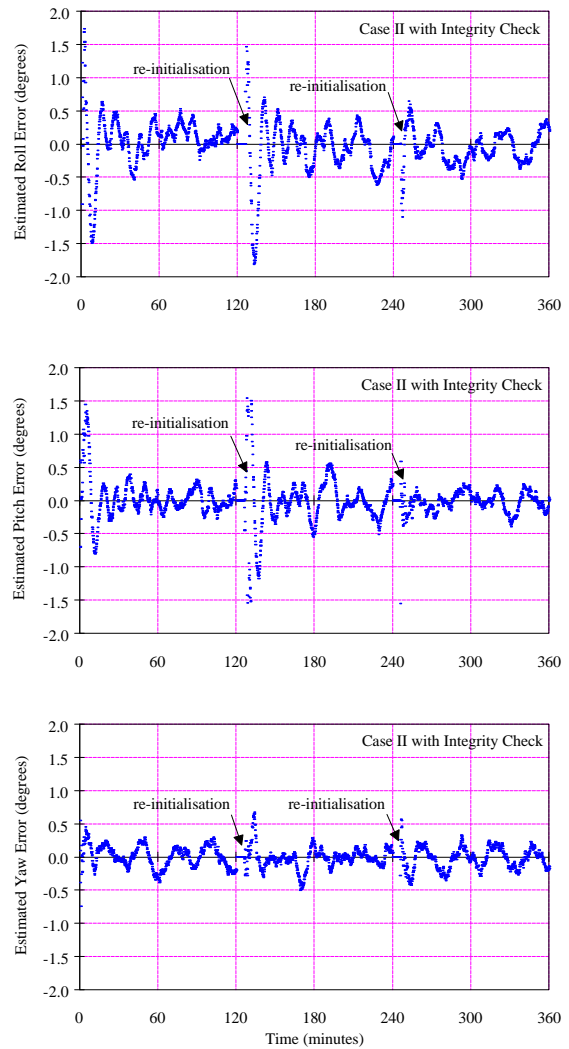


Figure 14: Estimated attitude error of Case II with integrity check

As we expected, several periods of anomalous operations are detected by integrity check. Once the number of the detection exceeds the limit of set count (e.g. 10 epochs), the protection function resets the Kalman filter, and requiring the initial step to estimate the attitude to re-initialising Kalman filter.

The simulated results in this sub section and the previous sub section indicate that the developed Kalman filter with integrity check provides a robust solution, and the filtering itself including the initial step estimation performs very robustly under anomalous conditions.

## 5. Conclusions

In this paper, the principle and problems of attitude determination using GPS signals were described. A new approach based on the integration of three

algorithms: block ambiguity search; TRIAD; and extended Kalman filter was presented. A technique to

resolve integer ambiguities and cycle slips caused by noise and line bias was developed for path difference estimations. Divergence protection was implemented as an integrity check on the filtered solution.

The new approach requires only short dual baselines which can be accommodated on small satellites. The block ambiguity search requires only four selected GPS satellites for initial line bias and baseline vector estimation. The technique of resolving integer ambiguities included cycle slips allows total attitude error up to  $\pm 18$  degrees for 30 cm baseline.

The filtering estimation was tested in the presence of measurement noise and bias, and under simulated anomalous conditions. Simulated results indicate that the filtering estimator provides a robust solution in presence of single errors, and re-initialises itself after continuous errors.

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