SSC99-VI-2

Phase Acquisition and Formationkeeping of A New Power Consumption Monitoring Satellite Constellation

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Abstract

A new satellite constellation proposed for global monitoring of electrical power consumption is described in the paper. The optimal small satellite constellation structure as well as its control accuracy required for serving the mission objective throughout the designed life span is examined. The orbital dynamics is analysed for the purposes of optimal phase acquisition and formationkeeping strategy design. A low-cost strategy for spreading all satellites onto their prescribed positions under both time and fuel consumption constraints is explained. The separation errors due to control system uncertainties are analysed, and the system requirements for the constellation phase acquisition are specified. A control strategy is investigated for keeping of the relative pattern of the constellation in spite of the perturbation effects from atmospheric drag and the potential harmonics of the non-spherical Earth, and fuel expenditure is minimised. The system feasibility is demonstrated via simulation results. The control system relies upon low-cost, practical flight-proven sensing and actuating systems for small satellite missions.

Introduction

The E-SAT Inc., which is jointly owned by DBS Industries and Echostar in the US, is proposing a 6-satellite LEO communications system targeted initially at the gas and electric utility industry for its subsidiary Global Energy Metering Service, Inc. (GEMS). It will employ a combined TDMA and CDMA protocol to allow it to share the spectrum with other little-LEO users. In April 1999 DBSI contracted Eurockot with two launches with a value of US$30m (1999), and SSTL (UK) with the supply of the six spacecraft with a value of US$17m. Supply of the communications payload has not yet been contracted.

The spacecraft supplier and operator, Surrey Satellite Technology Limited (SSTL), is a University owned company, set up in 1985 to transfer academic research and development into industrial space technology, and provides rapid and cost-effective satellite missions based on the latest research and development performed under the same roof of the Surrey Space Centre. Under the UoSAT 'trade-mark', it has pioneered the design and operation of the modern microsatellite for a wide range of applications. By mid-1999, 14 microsatellites weighting 60kg or less, and a 300kg mini-satellite have been launched. A further ten microsatellites are currently planned for launch in 1999 through to 2001, including 6 to form the ESAT constellation. The table below lists the missions SSTL and the Surrey Space Centre have been directly responsible for. The shaded area indicates spacecraft under constructor or ready for launch.

The orbit manoeuvre operation after the launch stage is divided into two stages - the phase acquisition and formationkeeping. The phase acquisition stage spread the spacecraft in a bunch onto their prescribed positions relative to other satellites within the same plane, and relative to those in the different plane. Any residual error after the phase acquisition will be fine tuned, and the formation of the constellation will be kept throughout its life span in spite of astrodynamic disturbances by the formationkeeping stage.

In this paper, we analyse the equation of motion in convenient form for each stage separately. The relative motion under continuous thrust is analysed for the phase acquisition stage. The continuous tangential thrust programme is applied in the orbit transfer to separate the relative phase between satellites. The relative motion under Earth oblateness and

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atmospheric drag is analysed, and an analytic control algorithm is then designed with minimised fuel expenditure.

#	Launch	Satellite	Orbit	Mission	Primary customer
1	'81	UoSAT-1	540km, 97.5° (SS)	Education	Univ. of Surrey
2	'84	UoSAT-2	690km, 98° (SS)	Comms	Univ. of Surrey
3	'90	UoSAT-3	780km, 98° (SS)	Comms	VITA, SSTL
4	'90	UoSAT-4	780km, 98° (SS)	Imaging	SSTL
5	'91	UoSAT-5	780km, 98° (SS)	Comms	SSTL
6	'92	KitSat-1	1325km, 66°	Tech.Demo	KAIST (S.Korea)
7	'92	S80/T	1325km, 66°	Comms	MMS (France)
8	'93	KitSat-2*	810km, 98.6° (SS)	Tech.Demo	KAIST
9	'93	HealthSat-2	810km, 98.6° (SS)	Comms	Satelife, VITA, DataTrax (US)
10	'93	PoSat-1	810km, 98.6° (SS)	Tech.Demo	PoSAT consortium (portugal)
11	'95	CERISE	670km, 98.1° (SS)	ELINT	Alcatel Espace (France)
12	'95	FASat-Alfa	675km, 82.5°	Tech.Demo	Chilean Air Force
13	'98	FASat-Bravo	821km, 98.6° (SS)	Tech.Demo	Chilean Air Force
14	'98	TMSat	821km, 98.6° (SS)	Tech.Demo	TMC (Thailand)
15	'99	UoSAT-12	650km, 64.5°	Imaging	SSTL, ESA, NTU (Singapore)
16	'99	TiungSat	670km, 98.1° (SS)	Tech.Demo	Malaysian Government
17	'00	Clementine	670km, 98.1° (SS)	ELINT	Alcatel Espace (France)
18	'99	PICOSat	To be manifested	Tech.Demo	SMC/TEL, FCT (USAF)
19	'00	SNAP	To be manifested	Tech.Demo	SSTL
(6)	'00-'01	ESAT	800km, polar	S&F comms	DBSI, US

Table 1.University of Surrey Microsatellite Missions
* built in Korea using SSTL platform & KAIST payload

The control problem of a satellite constellation in circular orbits has been considered by a number of authors. Lamy and Pascal[1] who have pointed out the importance of orbit choice and margins to be considered for the formationkeeping. The state variables are phases of each satellite with respect to the mean satellite constellation. Glickman[2] developed a time-destination approach (TIDE) to constellation formationkeeping in which individual satellite flight path errors are indirectly controlled by closing control loops on timing and position errors in reaching a series of precomputed equatorial destinations. Yuri Ulybyshev[3] applied a Linear-Quadratic Controller to control a prescribed mean constellation configuration. The configuration is represented by a tree digraph. The state variables to be controlled are mean along-track displacement of satellites relative to prescribed intersatellite spacing and displacements of satellite orbital periods with respect to the reference orbit period. A method of autonomous ring formation for planar constellations of satellites based on the concept of potential functions was presented by McInnes, et al.[4] Unlimited forms of constellation configuration may employ this strategy to keep a particular formation as a selected artificial potential function will automatically form a satellite path constraint and the control law tries to keep a satellite motion away from high potential positions and settle its position at a desired

low potential position. However, none of these methods consider the J2 and higher order effects on relative motion and control analytically.

In this paper, we present strategies for phase acquisition and formationkeeping which takes account the Earth oblateness and atmospheric drag, and the controller design is based upon the analytic solutions of the motion. The algorithm are, therefore, rather simple and require very small computation burden. The motion trajectory can be determined so that both magnitudes and frequency for delta-V firings are minimised. Indeed, the simplicity of the strategy allows the control system to be easily implemented by using standard low-cost technology on small satellites.

In the next section we shall describe the structure of the E-SAT constellation. The analysis and control of relative motion between spacecraft during the phase acquisition and formationkeeping will be described in the consecutive sections, as well as the simulation result will be shown.

Description of The E-SAT Constellation

The ESAT constellation is a 6/2/1 constellation, with 6 satellites in 2 planes, with a 60-degree phase difference between node crossings in adjacent planes.

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The 130kg satellites are box shaped measuring 800x600x600mm, with avionics based on the smaller SSTL microsatellites. They will be deployed in bunches of three, and have to be on-station within three weeks. The last two weeks are preserved for the phase acquisition manoeuvre. Satellites are required to be nadir stabilised to within ±2.5 degrees, and have a lifetime in excess of 5 years. One of the major challenges in the system design was the requirement for a cost-effective solution to the in-orbit deployment and station keeping. Gravity gradient stabilisation with active magnetic control, and passive thermal control by yaw spin would be the most effective solution for the attitude control on-station, and delivers the highest reliability and lifetime. However the propulsion requirements complicate this solution.
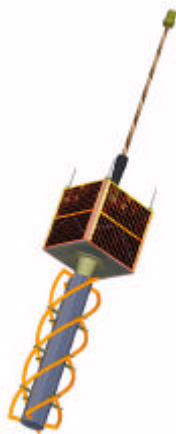
Figure 1. ESAT Satellite

For a standard Hohmann transfer, high thrust in the order of 5-10N would be required, leading to a challenging attitude control problem in order to control the disturbance torques induced by mis-alignments and COG variations. These were found to drive requirements for actuators and sensors that were only required during the station-acquisition phase. Similarly the thrust level did not lend itself to simple cost-effective solutions, favouring bi-propellant engines. Indeed the system might have to be complemented with a lower thrust orbit maintenance system. For that reason, low thrust transfers were considered, reducing the attitude control requirements, so that a cost-effective cold-gas system could be employed. It was determined that following a week of platform commissioning, a sequence of two single orbit low thrust spiral transfers could separate the satellites within two weeks, leaving a drift phase between which could be employed for payload commissioning. A 0.1N thrust cold gas system with a

rectangular cluster of thrusters is to be employed for both attitude and orbit control, simplifying the system significantly, hence reducing cost.

The Stationkeeping is required to avoid a multi-visibility of the satellites on the service area and gateways. Consequently, the actual control accuracy requirements are not too tight, with a ±2.5° window, however there are advantages to be gained in the proposed communications system by having finer control, to within a ±30km or ±0.25° along track. As such, an SSTL GPS receiver unit is carried offering precise timing and position knowledge.

Constellation Phase Acquisition

Three satellites in each orbital plane are planned to be launched by the same vehicle. Their initial conditions are assumed to be the same, although slight differences will be intentionally introduced during injection to avoid collision. One of the satellites will be assigned as the head of the group. The other two are then aimed to be separated leading and lagging 120° from the head satellite, respectively. Principally, the relative phase drift between two satellites occurs when their nodal periods are different. The magnitude of the drift is simply equal to the difference in mean motion, Δn, and the phase separation between satellites, Δq, can be expressed as :

$$\Delta q = \Delta q_0 + \Delta nt$$

(1)

Therefore, to separate two satellites which are initially at the same condition, we need to create a differential mean motion between them accordingly to the desired drift rate and direction.

The relative in-plane motion between satellites may be explained by using the coordinate system in Fig.2.
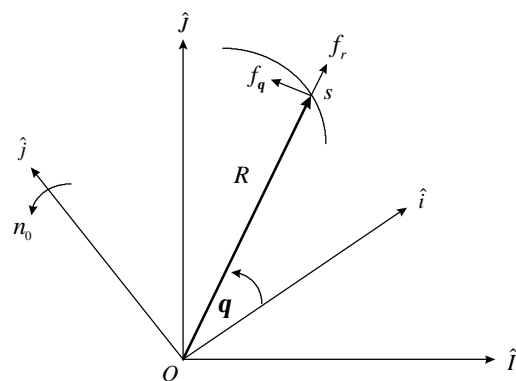
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The motion of a satellite under perturbing acceleration viewed from a coordinate (i, j) whose origin is at the centre of the Earth, and is rotating with a constant rate can be expressed in polar form as :

$$\ddot{R} - R(n_0 + \dot{q})^2 + \frac{m}{R^2} = f_r$$

(2)

$$R\ddot{q} + 2\dot{R}(n_0 + \dot{q}) = f_q$$

(3)

where R and q are radius and phase of the satellite, respectively, measured in the rotating frame, n_0 is a constant rotation rate of the frame, m is the gravitational constant and f_r and f_q are perturbing accelerations along the radial and azimuthal directions, respectively.

Assuming that the reference satellite is orbiting with mean motion of n_0 and a constant radius R_0, where $R_0^3 n_0^2 = m$, and assuming a small radius deviation, r, of the observed satellite from R_0, we can linearise the equations to first order as :

$$\ddot{r} - 2R_0 n_0 \dot{q} - 3n_0^2 r = f_r \qquad (4)$$

$$R_0\ddot{q} + 2n_0\dot{r} = f_q \qquad (5)$$

If the acceleration components are constant thrusts, the equations can be immediately solved and the solutions can be formed as :

$$r = \left(\frac{f_r}{n_0^2} + \frac{2f_q t}{n_0}\right) + \Delta a_0 - R_0 e_0 \cos(n_0 t - w) \qquad (6)$$

$$q = -\left(\frac{f_r t}{R_0 n_0} + \frac{3}{2}\frac{f_q t^2}{R_0}\right) - \frac{3}{2}\frac{n_0}{R_0}\Delta a_0 t + 2e_0 \sin(n_0 t - w) + (q_0 - 2e_0 \sin w)$$

(7)

where

$$\Delta a_0 = \left(\frac{2R_0 \dot{q}}{n_0} + 4r_0\right) \qquad (8)$$

$$e_0 = \sqrt{\left(\frac{f_r}{R_0 n_0^2} + \frac{2\dot{q}_0}{n_0} + \frac{3r_0}{R_0}\right)^2 + \left(\frac{\dot{r}_0}{R_0 n_0} - \frac{2f_q}{R_0 n_0^2}\right)^2} \qquad (9)$$

$$\tan w = \frac{2f_q - \dot{r}_0 n_0}{3r_0 n_0^2 + 2\dot{q}_0 R_0 n_0 + f_r} \qquad (10)$$

The change in mean radius due to continuous thrust can be found from the terms in the bracket of the radius solution. It has been shown that the tangential subclass continuous Keplerian thrust is practically optimal for transfer between two coplanar orbits[5]. Also the orbital shape (eccentricity, e, and argument of perigee, w) is preserved after the transfer if the thrust level is low enough. In the general case, however, the osculating e and w at any time after switching off the thruster can be found from Eq.9 and Eq.10 by setting $f_r = f_q = 0$. The orbital shape, especially eccentricity, can be preserved regardless of the thrust level by applying continuous burns along the velocity direction for a duration of an integer number of orbital period.

With a thrust level and transfer time (P) constraints given, we can combine Eq.1 and Eq.6 and solve for the optimal (minimise fuel) firing duration as :

$$t_{fire} = \frac{1}{3}\frac{a_0 \Delta q}{P f_q}$$

(11)

Fig.3 demonstrates the phase acquisition sequence of a satellite by using optimal thrust. The satellite altitude is raised up first to make a differential mean motion of the satellite relative to the head satellite. The relative phase of the satellite then shifts backward as it has a longer orbital period. When the desired separation, -120°, is achieved the satellite altitude is then brought back to its reference altitude by applying continuous thrust for the same duration, but in the opposite direction. In the simulation, we assume the circular orbit before the acquisition. The actual mission thrust level available for continuous firing is 0.1 N. Therefore, we require the total delta-V requirement of 8.13 m/s for 2 weeks acquisition time. Note that, to preserve the eccentricity, it is necessary to start the second firing at the same orbital phase as that at the end of the first leg.

An alternative strategy, which does not minimise fuel, is to have a continuous thrust for exactly one orbital

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period. The advantage of this approach is that eccentricity does not grow during phase acquisition (see Fig.4). This strategy requires a delta-V of 12.10 m/s, but the acquisition period reduces to 9.38 days.
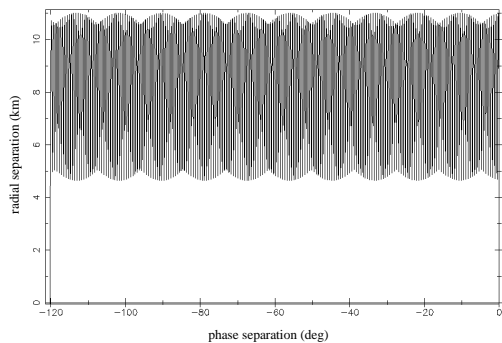

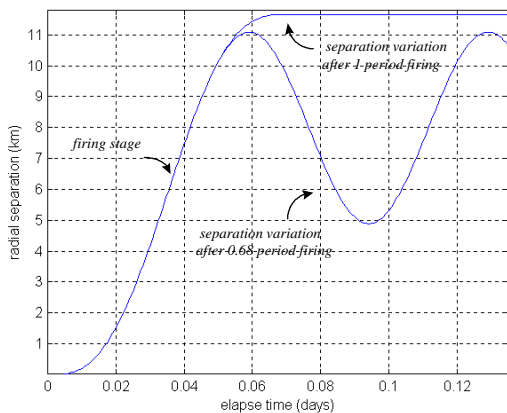**Figure 3.** Relative phase acquisition by optimal continuous thrust

Figure 4. Comparison of radial separation variation after orbit raising stage.

A similar strategy can be applied for the separation of the another satellite in the same plane. To make a positive phase separation, the satellite altitude must be lowered first to force the relative phase shifting forward. The satellite is then brought back to the nominal altitude when leading 120° in relative separation has been achieved .

The argument of latitude (AOL) will be used as a key variable in the control of relative phase separation between satellites, especially those in the different planes, since it has a common meaning, the phase of a satellite measured from the equator crossing, regardless of orbital plane. Once the phase of the head satellite is determined, the phases to be referred to for

all other satellites in the system can be immediately assigned, as shown in Fig.5. Another merit of using AOL as the control variable is that it is easier to be determined than the true anomaly, because the eccentricity and argument of perigee are ill-defined for satellites in near circular orbits.
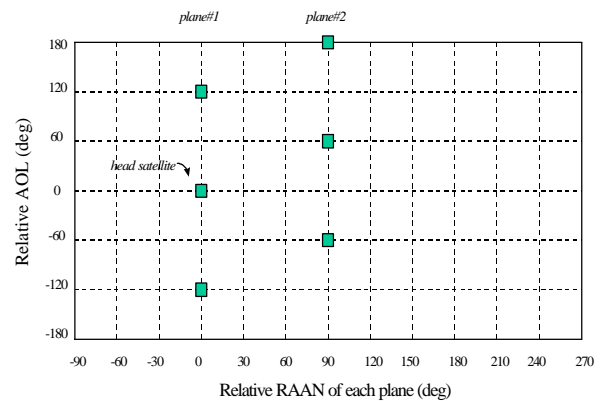

Figure 5. Constellation relative phase separation

The strategy described above allows the other three satellites, which will be launched on a separate vehicle, to be placed on their assigned slots by using a similar strategy to that used for the in-plane separation. All satellites are controlled by referring to the same head satellite. The relative initial conditions of these satellites depend upon the epoch of the second launch. Note that the constraint in altitude of the transfer orbits should be introduced now to avoid the theoretical collisions between satellites in the different planes during the transfer.

The astrodynamic perturbations will not cause any significant effects during the phase acquisition stage, since their time constants are generally much longer than the transfer duration. Also possible errors due to system uncertainties are not so critical, because the aim of this stage is actually to achieve coarse separations between satellites. Trim manoeuvres will used once the formationkeeping stage is started. Eq.11 suggests that 16.5^o pitch error , 2.8 minutes firing duration error and 4.1% thrust level error individually during the firing still gives a final phase separation within the desired control tolerance of $\pm 5^o$. These specifications are redundant for the navigation and control system used in the mission. The effect from the lost mass due to the propellant consumption is also negligible.

Formationkeeping Strategy

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The formationkeeping routine will be conducted after the phase acquisition is finished. Any residual phasing error from the early stages will be fine tuned and the constellation formation pattern will be kept in spite of the astrodynamic disturbances.

Like the phase acquisition phase, all satellites in the system are controlled relative to the head satellite. At the mean time, the whole constellation global structure, especially mean altitude, is maintained periodically.

Most of the fuel expenditure will be used for in-plane motion control against atmospheric drag. Cross-track displacements, i.e. inclination and right ascension of ascending node (RAAN), are not significant in our case because the operational orbits are exactly polar orbits, which the variations are very small (see Fig.6 which simulates the variation of inclination and RAAN of a satellite throughout a year by using a high fidelity orbit propagator[6] with the Earth oblateness and the Sun and the Moon gravitational effects are taken account).
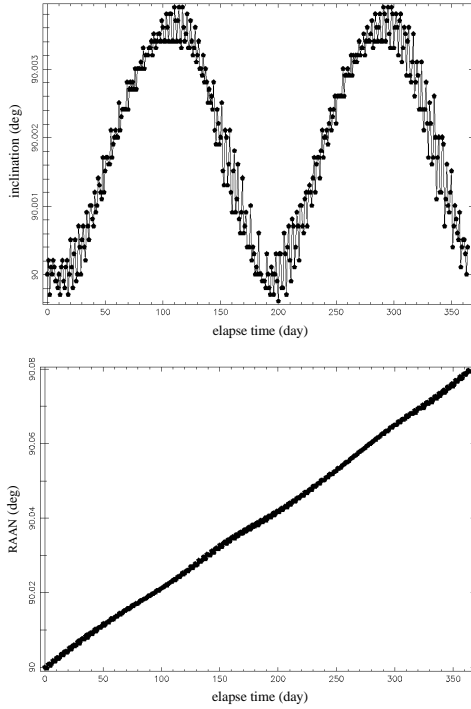

Figure 6. Inclination and RAAN variations

In small impulsive thrust control, where the delta-V magnitude may be of comparable order to those induced by the geopotential harmonics, the Earth oblateness effects have to be taken into account in the control strategy design.

The in-plane motion of a satellite about an axisymetric oblate Earth with secular, long-period and short-period perturbation terms included may be represented by using an epicycle coordinate as[7] :

$$r = a(1 + \boldsymbol{r}) - A\cos(\boldsymbol{a} - \boldsymbol{w}_0) + a\boldsymbol{c}\sin \boldsymbol{b} + \boldsymbol{d}\cos 2\boldsymbol{b} \quad (12)$$

$$\boldsymbol{l} = \boldsymbol{b} + 2e\left[\sin(\boldsymbol{a} - \boldsymbol{w}_0) + \sin \boldsymbol{w}_0\right] + 2\boldsymbol{c}(1 - \cos \boldsymbol{b}) + \boldsymbol{s}\sin 2\boldsymbol{b}$$

$$(13)$$

$$\boldsymbol{b} = (1 + \boldsymbol{k})\boldsymbol{a}$$

(14)

where r is instantaneous orbital radius, $\boldsymbol{l}$ argument of latitude, $A \cong ae$ is epicycle amplitude, $\boldsymbol{a}$ is mean anomaly measured from the equator crossing, $\boldsymbol{r}$ and $\boldsymbol{k}$ are secular perturbation terms caused by J_2 :

$$\boldsymbol{r} = -\frac{1}{2}J_2\left(\frac{R_e}{a}\right)^2\left(1 - \frac{3}{2}\sin^2 I_0\right)$$

(15)

$$\boldsymbol{k} = 3J_2\left(\frac{R_e}{a}\right)^2\left(1 - \frac{5}{4}\sin^2 I_0\right)$$

(16)

$\boldsymbol{d}$ and $\boldsymbol{s}$ are short-period perturbation terms due to J_2:

$$\boldsymbol{d} = \frac{1}{4}J_2\left(\frac{R_e}{a}\right)^2\sin^2 I_0$$

(17)

$$\boldsymbol{s} = -\frac{3}{4}J_2\left(\frac{R_e}{a}\right)^2\left(1 - \frac{7}{6}\sin^2 I_0\right) \quad (18)$$

$\boldsymbol{c}$ is the long period perturbation term which is the summation of effects of all odd harmonics. Note that only J_2 is retained for even harmonics since it is of order 10^{-3} while all higher even J_n are of order 10^{-6} or smaller.

The analysis must take account of the secular and long period variations. The short period variations can, consequently, be averaged over half an orbital period to obtain :

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$$r_{avg} = a(1+\boldsymbol{r}) - A^* \cos(\boldsymbol{g}-\boldsymbol{w}_0) + a\boldsymbol{c}^* \sin\boldsymbol{g}$$

(19)

$$\boldsymbol{l}_{avg} = (1+\boldsymbol{k})\boldsymbol{g} + 2e\left[\sin(\boldsymbol{g}-\boldsymbol{w}_0)+\sin\boldsymbol{w}_0\right]+2\boldsymbol{c}^*(1-\cos\boldsymbol{g})$$

(20)

where $A^* = \dfrac{2A}{\boldsymbol{p}}$, $\boldsymbol{c}^* = \dfrac{2\boldsymbol{c}}{\boldsymbol{p}}$ and $\boldsymbol{g}$ is an arbitrary phase.

For control purposes, long-period disturbance due to the coupling term between J_2 and odd zonal harmonics may be also ignored since its time constant is considered much longer than our control cycle period (it normally takes several months for a period, whereas our formationkeeping routine is planned to be monitored at least once a week). Furthermore, the periodic phase variation due to the epicyclic term can be ignored at this stage as we are interested in control of mean phase of the satellite. The mean relative phase separation between two spacecraft, $\Delta\boldsymbol{l}$, then becomes proportional to the differential nodal frequency between them :

$$\Delta\boldsymbol{l} = \Delta\boldsymbol{l}_0 + \Delta n^* . t$$

(21)

where $n^* \cong (1+\boldsymbol{k})n$ is nodal frequency.

When the mean phase separation between satellites exceeds the control tolerance, a delta-V is required to change the nodal frequency and bring the spacecraft back to its nominal separation. The delta-V magnitude can be determined from the specified transfer time interval, Δt, as follows.

The required impulsive change in nodal frequency, Δn_c^*, to bring the satellite back to the target phase in a time interval Δt is :

$$\Delta n^* + \Delta n_c^* = \frac{\Delta\boldsymbol{l}}{\Delta t}$$

(22)

where $\Delta\boldsymbol{l}$ is the phase separation at the instant of firing from the target phase, and Δn is the difference in nodal frequency between the satellite and the reference.

The delta-V can be evaluated from the change in orbital energy:

$$\Delta E = V_r . \Delta V_r + V_q . \Delta V_q$$

(23)

and hence, to first order,

$$\Delta a = \frac{2}{an^2}\left(V_r . \Delta V_r + V_q . \Delta V_q\right)$$

(24)

where V_r and V_q are velocity components in radial and azimuthal direction, respectively, obtained from the differentiation of Eq.12 and Eq.13 :

$$V_r = n\left[A\sin(\boldsymbol{a}-\boldsymbol{w}_0)+a\boldsymbol{c}\cos\boldsymbol{b}-2\boldsymbol{d}\sin 2\boldsymbol{b}\right]$$

(25)

$$V_q = (1+\boldsymbol{k})n\left[a(1+\boldsymbol{r})+A\cos(\boldsymbol{a}-\boldsymbol{w}_0)+2a\boldsymbol{c}\sin\boldsymbol{b}+(2\boldsymbol{s}+\boldsymbol{d})\cos 2\boldsymbol{b}\right]$$

(26)

ΔV_r and ΔV_q are impulsive delta-V components. The change in mean semi-major axis according to a small impulsive delta-V then becomes :

$$\Delta a = \frac{2\Delta V_q}{n}(1+\boldsymbol{r}+\boldsymbol{k})$$

(27)

The change in nodal frequency, Δn^*, relates to a small change in a through :

$$\Delta n^* = -\frac{3}{2}\frac{n}{a}(1+\frac{7}{3}\boldsymbol{k})\Delta a$$

(28)

Substituting Eq.(27) into Eq.(28), we obtain

$$\Delta n^* = -\frac{3\Delta V_q}{a}(1+\boldsymbol{r}+\frac{10}{3}\boldsymbol{k})$$

(29)

This allows the delta-V required for setting the initial conditions to be evaluated as :

$$\Delta V_q = \frac{a}{3}\left(1-\boldsymbol{r}-\frac{10}{3}\boldsymbol{k}\right)\left(\Delta n^* - \frac{\Delta\boldsymbol{l}}{\Delta t}\right)$$

(30)

Another delta-V is required to eliminate the drift once the nominal separation is achieved. The magnitude of Δn_c^* must be equal to the current Δn^*, but opposite in direction, and the required change in along-track velocity to kill the drift is:

$$\Delta V_q = \frac{a\Delta n^*}{3}\left(1-\boldsymbol{r}-\frac{10}{3}\boldsymbol{k}\right)$$

(31)

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The new set of epicycle parameters after delta-V firing can be calculated by solving the following equations.

$$\bar{r} = r \tag{32}$$

$$\bar{V}_r = V_r + \Delta V_r$$
(33)

$$\bar{V}_q = V_q + \Delta V_q \tag{34}$$

where the bars denote new variables after firing. For an impulsive thrust, the new orbital parameters are :

$$\bar{A}\sin(\boldsymbol{a} - \bar{w}_0) = A\sin(\boldsymbol{a} - w_0) + \Delta C_r + \frac{\Delta V_r}{n}$$
(35)

$$\bar{A}\cos(\boldsymbol{a} - \bar{w}_0) = A\cos(\boldsymbol{a} - w_0) + \Delta C_q + \frac{2\Delta V_q}{n} \tag{36}$$

where

$$\Delta C_r = (\bar{a}\bar{c} - ac)\cos\boldsymbol{a} - 2(\bar{d} - d)\sin 2\boldsymbol{a} \tag{37}$$

$$\Delta C_q = 3(\bar{c} - c)\sin\boldsymbol{a} - 2(\bar{a}\bar{s} + \bar{d} - as - d)\cos 2\boldsymbol{a} \tag{38}$$

,and finally we can solve for new e and w after firing through :

$$\bar{A} = \sqrt{x^2 + h^2} \tag{39}$$

$$\bar{w}_0 = \boldsymbol{a} - \tan^{-1}\frac{h}{x}$$

(40)
where

$$h = A\sin(\boldsymbol{a} - w_0) + \Delta C_r + \frac{\Delta V_r}{n} \tag{41}$$

$$x = A\cos(\boldsymbol{a} - w_0) + \Delta C_q + \frac{2\Delta V_q}{n}$$

(42)

The simulation result in Fig.7 demonstrates the control performance. In the simulation, a scenario is set up at the starting point so that the controlled satellite is drifting away from its nominal phase relative to the head satellite, . The first delta-V firing is invoked when the relative phasing exceeds 5^0 tolerance. The relative phase between satellite is then drifting backward with a controlled drift rate and brought back to the nominal separation. The second firing is applied when the satellite reaches its target point. We can see

from Fig.7 (b) that the satellite orbiting with a small circle about the target point after the control.

Mean Altitude Maintenance

The constellation needs to maintain its mean altitude in spite of atmospheric drag. In this section we estimate the delta-V required for the mean altitude maintenance, as well as the delta-V strategy to control the global structure of the constellation. A simplified drag model is employed here by assuming that the atmosphere of the Earth is spherically symmetric and there is no time variation in the density.
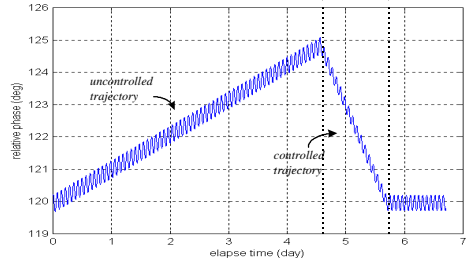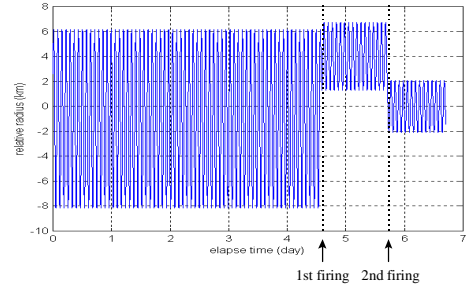
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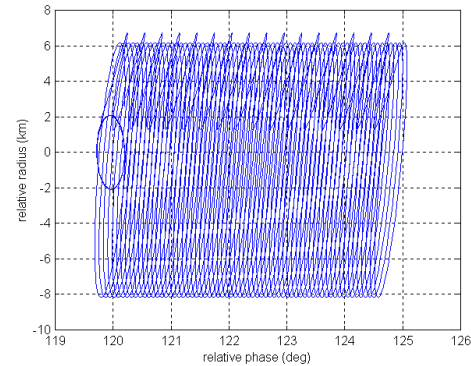
b.

Figure 7. In-plane relative motion during a control cycle

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The decay rate of mean semi-major axis due to atmospheric drag may be formulated as[8]:

$$\frac{da}{dt} = -\frac{2p}{T_r}\left(\frac{A}{m}\right)C_D a_r^{3/2} \boldsymbol{r}_r a^{1/2} \exp\{\boldsymbol{b}(a_r - a)\} \qquad (43)$$

where $\boldsymbol{r}$ is the density of the ambient atmosphere at the orbital radius, A is a reference area, frequently chosen as the cross-sectional area of the object perpendicular to the direction of motion, m is mass of the spacecraft, C_D is the drag coefficient, T is the orbital period and $\boldsymbol{b} = 1/H$ where H is the density scale height. The subscripts r denotes the value at the reference altitude.

In the region around the reference altitude, the air density may be approximated to be uniform throughout the control region. The decay rate then becomes

$$\frac{da}{dt} = -D\sqrt{a_r a}$$

$\qquad (44)$

where $\quad D = \frac{2p}{T_r}\left(\frac{A}{m}\right)C_D a_r \boldsymbol{r}_r$.

The drift period from the reference altitude to the minimum tolerance altitude can be found by directly integrating Eq.(44)

$$\int_{a_r}^{a_r(1-\boldsymbol{e})} \frac{da}{\sqrt{a}} = -D\sqrt{a_r}\int_0^P dt \qquad (45)$$

where $\boldsymbol{e}$ is a factor of mean altitude deviation and P is the control period. If $\boldsymbol{e}$ is considered small compared to 1, the integration yields

$$P = \frac{\boldsymbol{e}}{D}$$

(46)

An along-track delta-V is required for recovering the altitude of each satellite at each control period, and the delta-V magnitude is equal to that is taken out by atmospheric drag :

$$\Delta V_q = \frac{1}{2}\boldsymbol{e} n a_r \qquad (47)$$

Note that a drag-free orbit model may be used to propagate the target points of each spacecraft. The spacecraft are then controlled with respect to their reference points in space instead of with respect to the head satellite described above. By using an absolute control strategy, the initial altitude of the satellite has to be initialised appropriately at the starting point of the control cycle, so that its motion relative to the target point can be kept within the control tolerance and the firing frequency can be minimised. The absolute control strategy is depicted by a phase diagram in Fig.8.

Assuming a spherical Earth and using the drag model above, we can formulate the mean phase separation equation as a function of a which is decaying due to drag as :
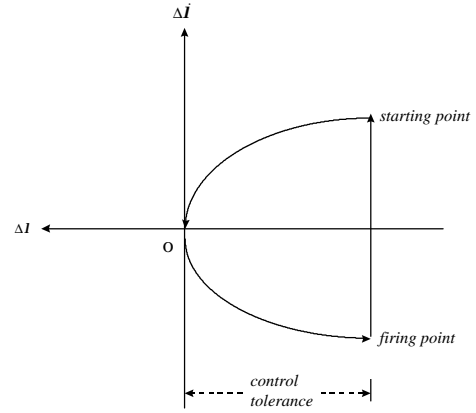

Figure 8. Absolute control cycle

$$d(\Delta \boldsymbol{l}) = \left(-\frac{\sqrt{\boldsymbol{m}}}{D\sqrt{a_r}a^2} + \frac{n_r}{D\sqrt{a_r}\sqrt{a}}\right)da \qquad (48)$$

Integrating through the equation yields

$$\Delta \boldsymbol{l} = \Delta \boldsymbol{l}_0 + \frac{\sqrt{\boldsymbol{m}}}{D\sqrt{a_r}\boldsymbol{x}^2} + \frac{2n_r \boldsymbol{x}}{D\sqrt{a_r}} \qquad (49)$$

where $\boldsymbol{x} = \sqrt{a}$.

At the target point we know that $\Delta \boldsymbol{l} = 0$ and $\boldsymbol{x} = \boldsymbol{x}_r$, therefore the in-track equation becomes

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$$\Delta l = \frac{\sqrt{m}}{D\sqrt{a_r}\,x_r^2}\left[\left(\frac{x_r}{x}\right)^2 + 2\frac{x}{x_r} - 3\right] \qquad (50)$$

Introducing a state $x = x/x_r$, the states at the control tolerance, s , can be found by solving the cubic equation :

$$2x^3 - (3+a)x^2 + 1 = 0 \qquad (51)$$

where $a = sD/n_r$.

Only two positive real roots (which represent the initial and final orbit altitudes of one control cycle) are required. Note that a is generally small compared to one. Therefore the desired roots must be close to one. Solving Eq.(51) in the region of $x = 1$, we obtain the desired initial and final states, x_i and x_f :

$$x_i = \frac{\sqrt{3}}{\sqrt{3} - \sqrt{a}} \qquad \text{and} \qquad x_f = \frac{\sqrt{3}}{\sqrt{3} + \sqrt{a}} \,. \qquad (52)$$

The delta-V needed for transfer from the final state, x_f , of the present cycle to the initial state, x_i , of the next cycle, can be obtained from equation :

$$\Delta V_q = \frac{n_f(a_i - a_f)}{2} = \frac{1}{2}\sqrt{\frac{m}{a_r}}\frac{(x_i^2 - x_f^2)}{x_f^3} \qquad (53)$$

or, as a function of a ,

$$\Delta V_q = 2V\sqrt{a}\left[\frac{\left(\sqrt{3} - \sqrt{a}\right)}{\left(\sqrt{3} - \sqrt{a}\right)^2}\right] \qquad (54)$$

where V is the circular velocity of the reference orbit. Control cycle period, P , can be obtained by integrating Eq.(44) :

$$P = \frac{2}{D\sqrt{a_r}}(\sqrt{a_i} - \sqrt{a_f}) = \frac{2}{D}(x_i - x_f) \qquad (55)$$

or

$$P = \frac{4}{D}\frac{\sqrt{3a}}{3-a} \qquad (56)$$

Let's introducing a cost function, J , as the ratio of delta-V per cycle to the cycle period:

$$J = \frac{\Delta V_q}{P} \qquad (57)$$

we obtain

$$J = \frac{1}{2}\frac{DV}{\sqrt{3}}\left[\frac{\left(\sqrt{3} + \sqrt{a}\right)^2}{\left(\sqrt{3} - \sqrt{a}\right)}\right] \qquad (58)$$

The initial and final altitude become more asymmetric around the reference altitude when the decay rate is bigger and, hence, the cost per cycle is growing as s becomes wider. For a small value of a , on the other hand, the motion profile is approximated to be symmetric around the reference altitude, and the equations above can be reduced as :

$$\Delta V_q = 2V\sqrt{sD/3n_r} \qquad (59)$$

$$P = \frac{4}{D}\sqrt{sD/3n_r} \qquad (60)$$

$$J = \frac{1}{2}DV \qquad (61)$$

The estimated decay rate is 3.83 m/day at the altitude around 800 km during the operational period which is of high solar activity. Graphs in Fig.9 show the in-plane motion for one control cycle. The cycle period is 98.6 days and the delta-V requirement for each cycle is 19.6 cm/s (0.726 m/s/year).

Conclusion

The orbital configuration of a new small-satellite constellation for global monitoring of electrical power consumption has been described, as well as its orbit control requirement. Convenient forms of the equations of relative motion have been derived for both the purposes of phase acquisition and formationkeeping of the constellation. Minimised fuel

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consumption strategies have been proposed for spreading all satellite from the launch vehicles onto their prescribed positions, and for keeping the relative formation between satellites throughout the mission life span in spite of astrodynamic perturbations. The simplicity of the strategy allows the control system to be easily implemented by using standard low-cost technology on small satellites.
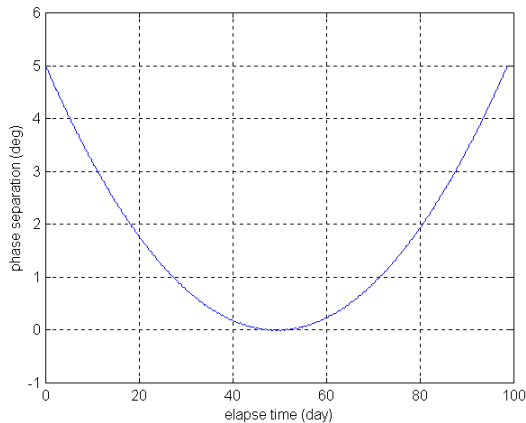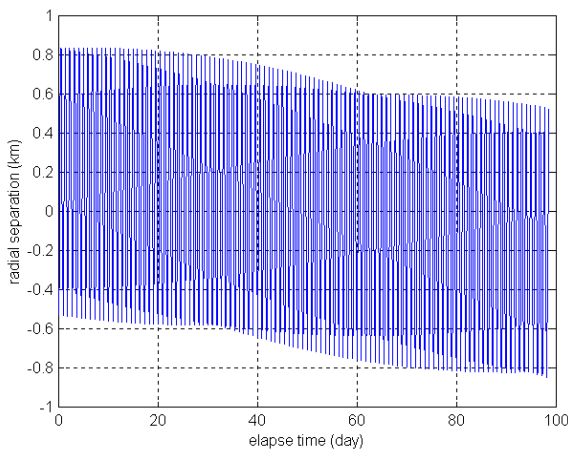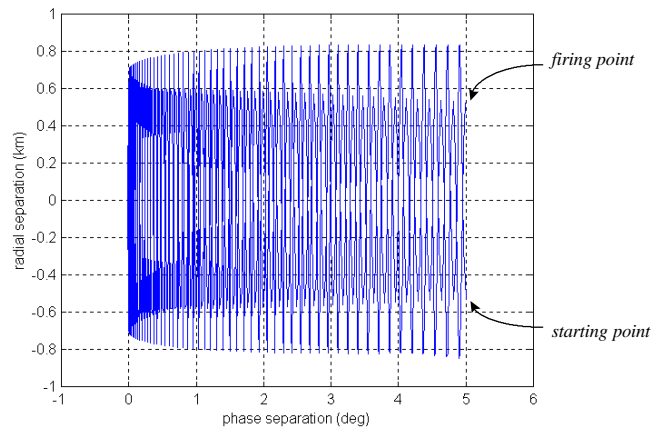
c.

Figure 9. In-plane relative motion during a cycle of absolute control strategy

a.

b.

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