Suitable Strategies for In-plane Orbit Acquisition Using Micro-thrusters

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Abstract:

The deviations in the injection orbital parameters, resulting from launcher dispersions, need to be corrected through a set of acquisition maneuvers to achieve the desired nominal parameters. When multiple satellites are injected into a single orbital plane, as a part of constellation establishment, they have to positioned in the plane with appropriate semi-major axis '*a*' and mean anomaly '*M*'. In this paper, three strategies are studied for achieving orbit acquisition. The first strategy is by deriving an analogy to the Linear Quadratic Regulator (LQR). The state dynamics and the control law are of the form $\dot{X} = \mathbf{A}X + \mathbf{B}U$ and $U = -\mathbf{K}X$. The feedback gain **K** is calculated by minimizing the cost function. Under this strategy the thrust (N) and velocity increment (ΔV) are functions of time and only the matrix **K** needs to be up-linked. Any revision in the current or the target states, will then lead to a simple re-calculation of **K** and up-linking them. The second strategy assumes that ΔV is same for each maneuver and calculates the number of maneuvers and the ΔV required for each maneuver. If the maneuvers are stopped for reasons like orbit assessment, and thruster performance evaluation, the strategy can be restarted easily without having any penalty on the overall ΔV. Besides these two strategies, a third strategy based on the application of Fuzzy Modified Potential Function is also studied for autonomous orbit acquisition with constraints in the path. By adding Fuzzy logic to the potential function it is shown that, maneuvers can be changed gradually ahead of the constraints. Onboard implementation related aspects are also briefly addressed for all the strategies.

Introduction

Due to the increasing cost of the ground operations, worldwide interest is towards autonomy in spacecraft operations. Thanks to the recent advances in VLSI and MEMS technologies, onboard autonomy is becoming a reality. Missions like PROBA (Project for Onboard Autonomy), as the name suggests, are aimed at demonstrating the autonomous operations in space. A fully autonomous system would make use measurements that are from passive sources and carryout estimation and control with no dependence on ground systems. The autonomy could be in terms of house keeping operations, attitude control and finally orbit control and maintenance. Autonomous orbit control or maintenance is a mission critical operation as it involves fuel expenditure and any unexpected anomaly can lead to mission catastrophes. Any autonomous orbit correction strategy should therefore have very small corrections at each step and be able to revise the strategy with minimal cost on the total fuel.

Besides the advances in technology, many launchers are now offering piggyback launches for micro and nano-satellites. This has enabled many academic institutions to take up design and development of small satellites[1]. Towards

establishing global coverage for earth observations and communication from Low Earth Orbit (LEO) altitudes, many constellations have been proposed till date and many more are expected. All these factors make satellite autonomy an essential feature. This paper addresses some suitable in-plane orbit correction strategies with features like – (1) small corrections, (2) ability to review and restart the strategy without fuel penalty and (3) ability to handle constraints. The orbit corrections are aimed at realizing the required nominal altitude and in-plane separation between the satellites in a given plane. The paper describes three strategies as given below.

1. Using the analogy of Linear Quadratic Regulator (LQR), the gain **K** for orbit correction is calculated a-priori and up-linked. An autonomous orbit estimation module monitors the orbit and if the need arises, updates the gain **K.**
2. For situations, where the orbit corrections need to be carried out only at apogee or perigee (half orbit interval), a strategy that provides fixed velocity impulse (ΔV) is described. It also has the restart capability.
3. The third strategy utilizes the concepts of Fuzzy Logic and Potential Functions to achieve orbit acquisition in the presence of constraints.

The case studies using the typical thrust level of ~1mN realizable with micro engines[2] indicate that orbit acquisition takes about 5 to 6 weeks to correct for the injection errors. The performance of the third strategy, which is fuzzy logic based, is well suited for situations wherein a satellite has to be maneuvered past another operational satellite in order to replace a failed satellite in the constellation. Considering the advantages of integer arithmetic, simple onboard implementation schemes are also analyzed towards the end.

In-plane Orbit Acquisition

The primary goal of orbit acquisition in a constellation is to slowly correct the errors in semi-major axis, 'a' (due to errors at injection) so that, the satellite is placed in its final orbit with proper in-plane separation ΔM with respect to other satellites in the plane.

As shown in Figure-1, the satellite A is initially at an angle of ΔM_0 with respect to B, which is its final slot in the nominal orbit. Through a series of small and autonomous corrections, the satellite is to be brought to the nominal orbit and positioned relative to F. Under maneuver free conditions, an offset in Δa results in a perturbation in the mean motion η, given by,

$$\eta^2 = \frac{\mu}{a^3}\ ,\ \Delta\eta = \frac{-3\eta\Delta a}{2a} \qquad (1)$$

Defining $\overline{X} = [\Delta M \quad \Delta\eta]^T$ as the state, the state dynamics due to a tangential thrust τ is given by,

$$\frac{d}{dt}\left[\overline{X}\right] = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\overline{X} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \left[-\frac{3\tau}{am} \right]$$

where m is the mass of the satellite. The above equation can be written in the classical form,

$$\dot{\overline{X}} = \mathbf{A}\overline{X} + \mathbf{B}U \qquad (2)$$

Any autonomous algorithms that compute U based on the current deviations with respect to the target, can then lead to successful orbit acquisition. The following sections describe some such techniques.

LQR Analogy

The orbit acquisition problem given by equation (2) can be solved by using the Linear Quadratic Regulator (LQR), which computes the control force U using the feedback law, $U = -K\overline{X}$. The gain K is calculated so as to minimize the cost function,

$$J = \int_0^\infty \left[X^T \mathbf{Q} X + U^T \mathbf{R} U \right] \cdot dt$$

where **Q** and **R** are the weight matrices for X and U. The choice of **Q** and **R** decides the feedback gain K and hence the closed loop response. The weights are chosen based on methods like[3],

- State weighting
- Control weighting
- Pole positioning
- Cross over frequency and close loop bandwidth
- Closed loop time response

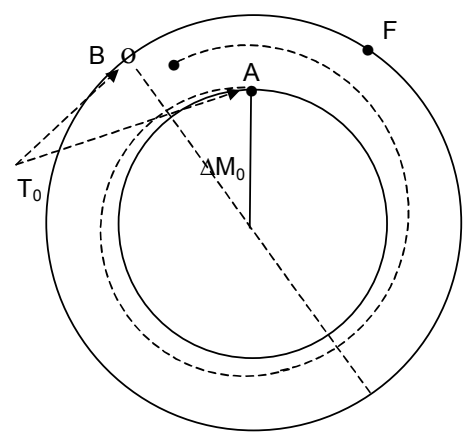

Fig-1 Orbit Acquisition Geometry

In the context of orbit acquisition, the gain K is chosen to ensure that the state $\overline{X}$ asymptotically reduces to zero. In other words, there are no over correction in terms of Δa. Substituting for U in (2), the closed loop dynamics becomes,

$$\frac{d}{dt}\left[\overline{X}\right]=\begin{bmatrix}0 & 1\\ -k_1 & -k_2\end{bmatrix}\cdot\overline{X} \qquad (3)$$

It can be shown that, $\overline{X}$ reduces asymptotically to zero, when the roots of the closed loop system are real and negative. This leads to $k_2^2 > 4k_1$. Since the control force U = 0 at t=0, we further get,

$$U = -K\overline{X} = -\left[k_1\Delta M_0 + k_2\Delta\eta_0\right] = 0$$

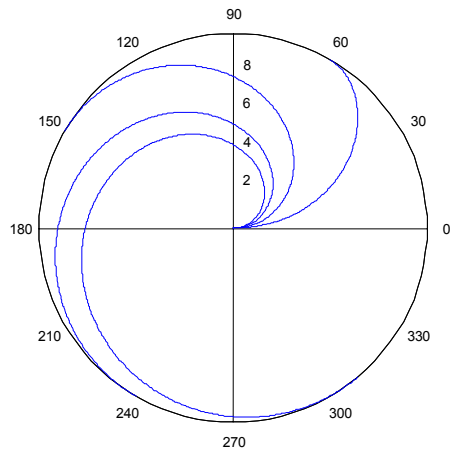

Fig-2 Orbit Acquisition by LQR Analogy

Combining all these, k_1 and k_2 are found to be,

$$k_1 = -k_2\left(\frac{\Delta\eta_0}{\Delta M_0}\right) \text{ and } k_2 > -4\left(\frac{\Delta\eta_0}{\Delta M_0}\right)$$

Case Study: Assume that Δa_0 (injection error in semi-major axis) is 10 km and that the phase angle error ΔM_0 is 60^0. Let the target orbital radius be 7200 kms. We then get ΔM_0 = 1.047 rad and $\Delta\eta_0$ = -2.156e-6 rad/sec. Figure-2 shows the variation of orbital altitude and phase angle. Different spirals are for different initial values of ΔM. It can be seen that the LQR strategy is reaching the target from all initial conditions. Time history of Δa, thrust, cumulative ΔV are shown in Figure-3. Since initial Δa is 10 kms, the ΔV required for orbit correction is about 5.167 m/sec. The thrust required is around 1mN, realizable by micro-thrusters. The ΔV required by the LQR strategy is very close to the theoretical value thus establishing that the strategy does not cause any fuel penalty for phase acquisition**.**

Onboard Implementation: For the case Δa=10 kms and ΔM=60 deg the feedback gain K is calculated as,

K = [1.713386e-011 8.3199046867e-006]

This results in the following discrete state transition matrix **Φ** for the system given by (3),

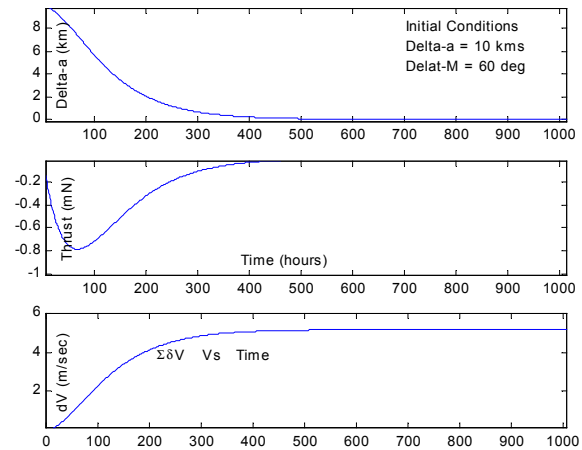

Fig-3 Time history of Δa, ΔV, and Thrust

$$\Phi = \begin{bmatrix} 9.9989e-001 & 3.54649e+003 \\ -6.07651e-008 & 9.703836e-001 \end{bmatrix}$$

An easy strategy for implementation is to use integer arithmetic and realize multiplication and division by bit shift operations. If we express ΔM and Δη as integers in micro-radians and nano-radians/sec respectively, then the above state transition matrix can be rewritten as

$$\Phi = \begin{bmatrix} 0.99989 & 3.54649 \\ -6.07651e-5 & 0.9703836 \end{bmatrix}$$

After converting the above into equivalent binary form, the state dynamics takes the following form.

$$\begin{bmatrix}\Delta M\\ \Delta\eta\end{bmatrix}_{k+1} = 2^{-20}\begin{bmatrix}FFF8C & 38BE6C\\ 0 & F86B0\end{bmatrix}\cdot\begin{bmatrix}\Delta M\\ \Delta\eta\end{bmatrix}_k$$

The above form can easily be realized by using bit shift left/right operations. Similarly the gain matrix K can also be implemented in integer arithmetic form after suitable scaling. The errors due to truncation will result as an error in ΔM and Δη, increasing with time. One remedy is to refresh the parameters at regular intervals based on the latest orbital information and uplink them. Even otherwise refreshing will be required to assess the performance of the thrusters and the effect of neglected perturbations. Thus, the LQR strategy requires only the state transition matrix **Φ** and the gain matrix K to be up-linked to carryout orbit corrections autonomously.

Equal Impulse Strategy

The LQR strategy described earlier assumes that the micro-thrusters are on continuously, which may not be feasible always. Under situations of power

and thermal constraints, the thrusters may have to be operated only for a short span in each orbit. A typical scenario could be when the thrusters are operated only around apogee and perigee alternatively. That is, the interval between maneuvers is an odd multiple of half-orbit. This scenario is depicted in Figure-4.

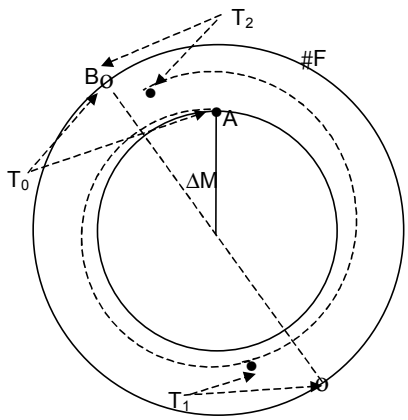

Fig-4 Equal Impulse Strategy

At time T_0 the satellite is in the initial orbit at an angle of ΔM with respect to its final position B. The maneuvers are carried out with reference to the orbital period of the nominal orbit. As shown in the figure T_0, T_1, T_2, …. T_n are the maneuver times spaced half-orbit apart. If ΔV is the velocity increment at each maneuver, the phase difference ΔM and altitude difference Δa after each maneuver is given by,

$$\begin{bmatrix}\Delta M\\ \Delta a\end{bmatrix}_{k+1} = \begin{bmatrix}1 & p\\ 0 & 1\end{bmatrix}\begin{bmatrix}\Delta M\\ \Delta a\end{bmatrix}_{k} + \begin{bmatrix}0\\ q\end{bmatrix}\Delta V \qquad (4)$$

which is again written in the classical form as,

$$X_{k+1} = \mathbf{A}X_k + \mathbf{B}U_k \qquad (5)$$

$$X_k \triangleq [\Delta M \quad \Delta a]^T,\ \mathbf{A} = \begin{bmatrix}1 & p\\ 0 & 1\end{bmatrix},\ \mathbf{B} = \begin{bmatrix}0\\ q\end{bmatrix}, U = \Delta V,$$

$$p = -\frac{3}{2}\left(\frac{\eta}{a}\right)\left(\frac{T}{2}\right)\ ,\ q = \frac{2a}{V}$$

The task is then to calculate ΔV and the number of corrections k. The initial condition is $[\Delta M_0\ \Delta a_0]^T$ and the final condition is $[0\ \ 0]^T$. Using the recursion, equation (5), can be written as,

$$\begin{aligned} X_{k+1} &= \mathbf{A}X_k + \mathbf{B}\Delta V_k \\ &= \mathbf{A}(\mathbf{A}X_{k-1} + \mathbf{B}\Delta V_{k-1}) + \mathbf{B}\Delta V_k \\ &= \mathbf{A}^2 X_{k-1} + (\mathbf{A}+\mathbf{I})\mathbf{B}\Delta V, \quad \text{since } \Delta V_k = \Delta V_{k-1} \\ &= \mathbf{A}^3 X_{k-2} + (\mathbf{A}^2 + \mathbf{A} + \mathbf{I})\mathbf{B}\Delta V \\ &= \mathbf{\Phi}X_0 + \mathbf{\Gamma}\Delta V, \quad \text{where } \mathbf{\Phi} = \mathbf{A}^{k+1} \text{ and} \end{aligned}$$

$$\mathbf{\Gamma} = (\mathbf{A}^k + \mathbf{A}^{k11} + \ldots + I)B$$

Further, $\mathbf{A}^{k+1} = \begin{bmatrix}1 & (k+1)p\\ 0 & 1\end{bmatrix}$

$$\mathbf{A}^k + \mathbf{A}^{k-1} + \ldots + I = \begin{bmatrix}k+1 & \left(\frac{k(k+1)}{2}\right)p\\ 0 & k+1\end{bmatrix}$$

Therefore, linking the initial and final conditions, we get,

$$\begin{bmatrix}0\\ 0\end{bmatrix} = \mathbf{\Phi}\begin{bmatrix}\Delta M\\ \Delta a\end{bmatrix}_0 + \mathbf{\Gamma}\Delta V$$

From the above, ΔV and k can be calculated.

$$k = 2\left[\frac{-\Delta M_0}{p\cdot\Delta a_0} - 1\right];\quad \Delta V = -\frac{\Delta a_0}{(k+1)\cdot q} \qquad (6)$$

If ΔV is high for the thruster to realize, k can be recalculated by modifying ΔM_0 with extra cycles i.e. adding or subtracting 360^0 or its multiples to ΔM_0.

Case studies:

For $\Delta a_0 < 0$ and $\Delta M_0 < 0$: Assume Δa_0=-30 kms and $\Delta M_0 = -60^0$ and the nominal orbital radius be 7200 kms. This results in p=-6.545e-4 and q = 1.935 kms per m/sec of ΔV. Then number of corrections, k=104 and ΔV= 0.1476 m/sec per correction. Figure-5 shows the typical trajectory for this case. Curve A1 is for $\Delta M_0 = -60^0$. If ΔM_0 is changed by

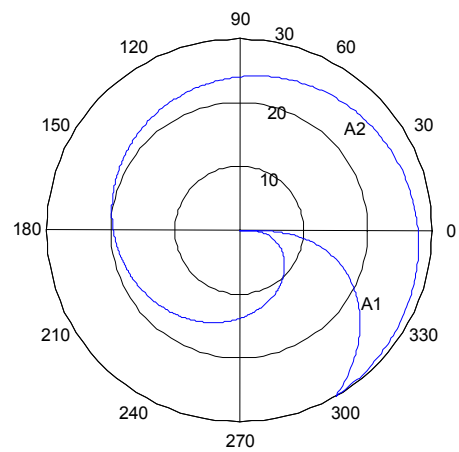

Fig-5 Equal Impulse Strategy-Polar Plot

one cycle, we get the trajectory A2 for which k=744 and ΔV=0.0208 m/sec per correction. Performance of the strategy has also been investigated for other initial conditions. Figure-6 depicts the performance under different initial conditions.

Table-1 Total ΔV for *Equal Impulse Strategy*

Δa (Kms)	ΔM (Deg)	K (half orbits)	$\lvert\Delta V\rvert$ (m/sec)	$\Sigma\lvert\Delta V\rvert$ (m/sec)
-30	-60	104	0.1476	15.35
-30	-60-360	744	0.0208	15.47
30	-60	531	0.0291	15.47
30	60	104	0.1476	15.35
-30	60	531	0.0291	15.47

Other details like number of corrections, ΔV per correction and total ΔV are shown in Table-1. The last column of the table indicates a nearly constant

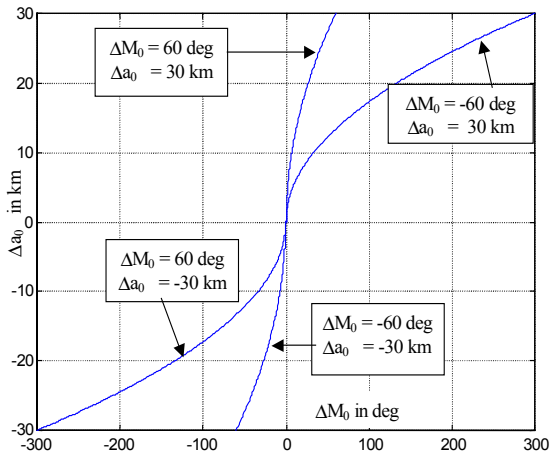

Fig-6 Equal Impulse Strategy-Performance with different initial conditions

value, which is equal to the total ΔV for correcting the altitude offset. It is thus established that the strategy does not impose fuel penalty for realizing the required phase separation. Since any orbit correction strategy using micro-thrusters is bound to take a long time it is important to review the orbital parameters at regular intervals and restart a strategy if needed. Under such circumstances, the corrections will be stopped and then started again after tracking and orbit determination. Figure-7 demonstrates the restart capabilities of the strategy. Even when the maneuvers are stopped for tracking, orbit determination and started again, there is no over-corrections in Δa, which in turn confirms that there is no fuel penalty.

Fuzzy Modified Potential Function Strategy

The LQR strategy and the Equal Impulse strategies are shown to be suitable for in-plane orbit acquisitions and they are also suitable for situations wherein the maneuvers need to be stopped and started again for reviewing the orbit correction through tracking and orbit determination. As the satellite is maneuvered to its final location, there may be constraints to be satisfied. For, instance, proximity to another operational satellite may have to be avoided to eliminate interference or possible collisions. This translates into a set of bounds on Δa and ΔM to be taken care of. A simple method is to formulate the constraints as repulsive potential functions while the normal trajectory to the desired destination is formulated as an attractive potential function. Such a technique, based on Fuzzy Logic is explained in the following sections.

Different types of potential functions have been studied by many authors[4-6]. The fuzzy strategy proposed here is similar to a human being deciding to slow down and take diversion as he approaches the obstacle.

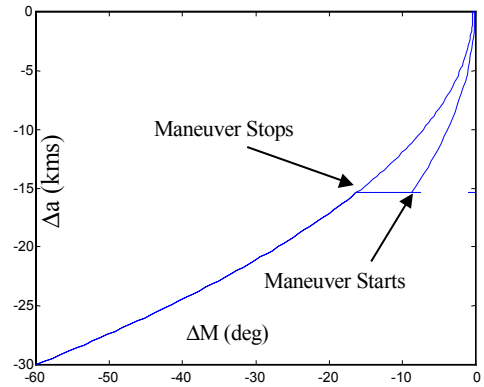

Fig-7 Equal Impulse Strategy-Restart Features

Potential Functions

One of the widely used potential is the quadratic potential well[7] described as,

$$\phi = \frac{1}{2}\overline{X}^T \mathbf{K}\overline{X} \tag{7}$$

where $\overline{X}$ represents the position vector. Assuming that the ith obstacle is located at $\overline{X}_i$, the potential function representing the free space as well as the obstacles, is given by,

$$\phi = \frac{1}{2}\overline{X}^T \mathbf{K}\overline{X} + \frac{1}{2}Pe^{-(\overline{X}-\overline{X}_i)^T \mathbf{F}(\overline{X}-\overline{X}_i)} \tag{8}$$

For a surface defined by the above potential, the control strategy is such that the object moves along the negative gradient. Before discussing the fuzzy logic strategy, the crisp strategy is briefly described.

Crisp Strategy: For a potential surface defined by (8), the rate of change of potential[7] is given by, $\dot{\phi} = \nabla\phi \cdot \overline{V}$, where $\overline{V}$ is the velocity of the vehicle. Under the crisp strategy, the control force is switched on at $\dot{\phi} = 0$. The switching action $s(t)$ is,

$$\begin{aligned} s(t) &= 0 \quad \text{when } \dot{\phi} < 0 \\ &= 1 \quad \text{when } \dot{\phi} \geq 0 \end{aligned} \tag{9}$$

When $s(t)$ is 1, the control law is such that, the velocity V^+ shortly after applying the control force is given by,

$$V^+ = V^- + \Delta V = -k(X,V)\cdot\frac{\nabla\phi}{|\nabla\phi|} \tag{10}$$

In the above equation, V^- is the velocity just prior to the control force. This leads to,

$$\dot{\phi} = \nabla\phi \cdot \overline{V}^+ = -k(X,V)|\nabla\phi| \tag{11}$$

Equation (11) ensures that $\dot{\phi}$ is negative definite, implying that the object moves towards the minimum. One natural question that arises is -

since $\dot{\phi}$ is continuously evaluated why not take a decision to initiate control action even before $\dot{\phi}$ becomes equal to zero? The solution to this is the application of fuzzy logic as discussed below.

Fuzzy Strategy: Let $\mu(\dot{\phi})$ be the fuzzy membership function that replaces equation (9). Thus we have,

$$s(t) = \mu(\dot{\phi}) \quad \text{when } \dot{\phi} < 0 \qquad (12)$$
$$= \quad 1 \quad \text{when } \dot{\phi} \geq 0$$

The membership function is so chosen such that, in the proximity of $\dot{\phi} = 0$, $\mu(\dot{\phi})$ changes from 0 to 1. Rewriting equation (10),

$$\Delta V = -V^- - k(X, V) \cdot \frac{\nabla\phi}{|\nabla\phi|} \qquad (13)$$

The actual velocity increment δV, to be given is decided by the fuzzy membership function. That is,

$$\delta V = \mu(\dot{\phi}) \cdot \Delta V \qquad (14)$$

The velocity following the control action is then,

$$V^+ = V^- + \delta V = \left(1 - \mu(\dot{\phi})\right)V^- - \mu(\dot{\phi})k\frac{\nabla\phi}{|\nabla\phi|} \qquad (15)$$

After the application of δV, $\dot{\phi}$ becomes,

$$\dot{\phi}^+ = \nabla\phi \cdot \overline{V}^+ = \left(1 - \mu(\dot{\phi})\right)\dot{\phi}^- - \mu(\dot{\phi})k|\nabla\phi| \qquad (16)$$

When $\dot{\phi}$ is small negative, the first term is negative as $\mu(\dot{\phi})$ <1 and the second term is also negative definite. When $\dot{\phi} \geq 0$, the first term vanishes as $\mu(\dot{\phi})$ =1. Then equation (16) is same as (10).

Case Study: As shown in Figure-1, let the satellite A be injected in to the 'initial orbit' which is away from its target orbit by Δa in orbital radius and let the initial angular position be away by ΔM. Under the orbit acquisition strategy, both Δa and ΔM are to be brought to zero. The dynamics of $\Delta\eta$ and ΔM are given by,

$$\Delta\eta_{k+1} = \Delta\eta_k$$
$$\Delta M_{k+1} = \Delta M_k + \Delta\eta_k \tau$$

Assume that there is a constraint centered at $(\Delta a_c, \Delta M_c)$ that is to be avoided as the satellite is brought into its final orbit. Let the state be defined as,

$$\overline{X} \underset{=}{\Delta} [\Delta M \quad \Delta\eta]^T$$

The total potential is given by,

$$\phi = \frac{1}{2}\overline{X}^T\left(F_1^T F_1\right)\overline{X} + F_2 e^{-\frac{1}{2}X_i^T\left(F_3^T F_3\right)X_i} \qquad (17)$$

For the total potential described above, the control strategy is to change Δa that leads to a change in $\Delta\eta$ by in-plane maneuvers. The membership function is assumed to be of the form,

$$\mu(\dot{\phi}) = e^{-F_4\dot{\phi}^2} \quad \text{if } \dot{\phi} < 0 \qquad (18)$$
$$= 1 \quad \text{if } \dot{\phi} \geq 0$$

Using state dynamics, potential function and the fuzzy functions defined above, the post maneuver velocity is given by (15). Figures 8 through 10 show the variation of $\Delta\eta$ and ΔM under crisp and fuzzy strategies. For the case used in the simulation the initial value of Δa is 15 km and ΔM is 180°. The target orbital radius is assumed to be 7178 km (800 km altitude). Let the constraint be located at (Δa_c = 18 kms, ΔM_c = 45°). This could be the location of another satellite that is likely to cause interference during orbit acquisition. The goal is to reduce Δa and ΔM to zero. Figure-8 indicates the

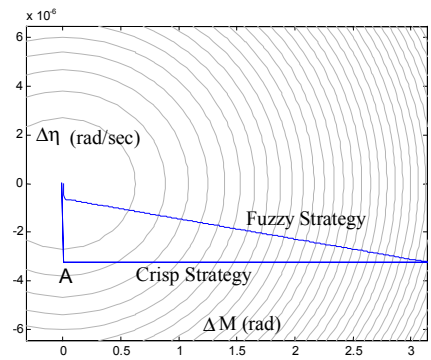

Fig-8 Potential Function with no constraint

performance when there are no constraints in the path. The potential surface is represented by iso-potential contours where the potential increases with the distance from the origin. The origin represents the targeted position of the satellite. Under crisp strategy, no maneuver is carried out as long as the object is moving along the negative gradient, i.e. $\dot{\phi} < 0$. The satellite motion under maneuver free conditions is described by a constant $\Delta\eta$ and hence a linear ΔM. This is depicted by the horizontal line. At 'A', $\dot{\phi}$ is zero and hence the

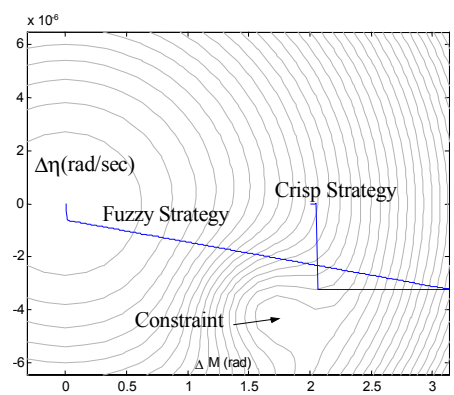

Fig-9 Potential Function with constraint

maneuver is carried out to reduce Δa to zero in one operation. On the other hand, the fuzzy

strategy attaches a small weight to maneuvers even when the potential gradient is negative and this results in small maneuvers to change Δa at regular intervals. Hence the trajectory is a continued motion towards then origin as shown in Figure-8. The performance is further analyzed in Figure-9 when thee are constraints in the path. In the present simulation the constraint is assumed to be at $\Delta a = 18$ km, $\Delta M = 90^0$. From Figures 9 and 10, it is seen that, under crisp strategy, the satellite goes up to the foot of the potential hill and then it is maneuvered abruptly in terms of Δa. Since Δa is fully corrected, the trajectory gets trapped at its current ΔM. On the other hand, under fuzzy strategy, the satellite always undergoes a small

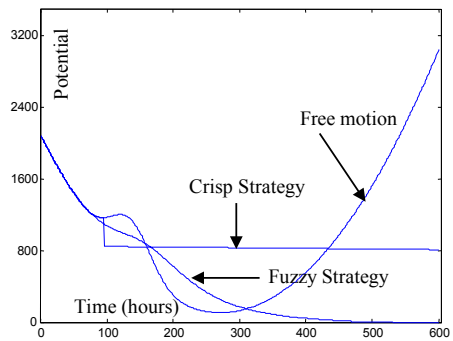

Fig-10 Variation of Potential Function along the trajectory

maneuver (although $\dot{\phi} < 0$) and hence smoothly passes by the hill. Figure-10 shows the time history of potential as the satellite moves. From the figure it can be seen that, both the crisp strategy and the fuzzy strategy ensure that the object moves along the negative gradient. The curve marked 'Free motion' indicates the path that would result had there been no maneuver at all.

Conclusion

The in plane orbit acquisition is one of the crucial mission operations for constellation establishment. To meet such demands, a few strategies suitable for autonomous low thrust maneuvers have been formulated and analyzed for their performance. Since the acquisition phase with low thrust engines (thrust of the order of 0.1 to 1 mN) may last for several weeks, issues related to stopping, reviewing and re-starting capabilities are also briefly addressed. Among the strategies proposed, in case of orbit acquisition with constraint, combining the potential function and fuzzy membership functions helps to initiate control ahead of reaching the obstacle. This results in smaller maneuvers, which is advantageous for space vehicles, as the operations can be carried out with smaller capacity thrusters.

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