SSC01-VII-1

Low Thrust Trajectory Design for CRAFTI

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Abstract

The Canadian Robotic Asteroid Flyby and (Tentatively) Impact project (CRAFTI) proposes to accomplish an interplanetary mission to explore an asteroid while maintaining a budget of ~US$10 million. This cost target implies a launch as a secondary payload and hence a microsat-sized spacecraft. The mission objective is to send two spacecraft to encounter the near-Earth asteroid Toutatis in 2008; one of these will fly by Toutatis at close range, while the other impacts the asteroid.

An electric propulsion system is being considered to provide the thrust for the orbital manoeuvres. However, the thrust level achievable by this kind of system is very low, requiring long burn times that complicate the orbital analysis. A low-thrust orbit propagator that has been developed to support the CRAFTI mission analysis, and the preliminary trajectory design work that has been carried out, is described in this paper.

1.0 Introduction

The Canadian Space Agency's (CSA) current paradigm for space exploration favours the funding of several small projects over the funding of one large space mission. While this means that a greater number of organizations have access to government funds, the fund size is relatively small when contrasted with comparable programs in other countries. The result of such limited funding is the requirement to use innovative methods that produce interesting science projects.

One such project that is being studied under this low cost structure is the Canadian Robotic Asteroid Flyby and (Tentatively) Impact (CRAFTI) mission. CRAFTI proposes to use microsatellites to perform a flyby of a near Earth asteroid for a cost of around US $10 million including launch costs. Two identical satellites are launched as secondary payloads into a parking orbit then, at the appropriate time, directed on a trajectory to the target. Should both satellites arrive at the asteroid in a working condition, one of the satellites will be flown into the asteroid while the other satellite observes the impact. The scientific investigations of the structure of the asteroid will be performed using radar sounders and CCD cameras.

The nominal target for the mission is the Apollo class asteroid 4179 Toutatis. Toutatis is an excellent target for this study mainly owing to the closeness of its passage by Earth and the timing of the passage with regards to the time required to design and build the satellites.

One of the most significant systems that needs to be added to a microsatellite to accommodate an interplanetary trajectory is a propulsion system. However, the addition of the volatile chemicals normally associated with high thrust chemical thrusters will likely be at odds with the wishes of the primary payload on the launcher. Consequently, CRAFTI will employ a low thrust electric propulsion system. This choice of thrusters will also yield a decrease in the mass of propellant required, owing to the high I_{sp} of the engine, further reducing the cost of launch. The price of using the electric thrusters is the increased difficulty in the design of the orbital manoeuvres and trajectories. The difficulty arises from the long burn times associated with low thrust propulsion resulting in the inability to use impulsive thrust approximations when optimizing or even modelling the orbital dynamics.

2.0 Mission Phases

For planning purposes the mission will be divided into three segments: initial orbit injection, parking orbit, and asteroid interception. The position of the injected orbit is determined by the orbital requirements of the primary payload. It is advantageous, and more common, for the launcher to burn any remaining fuel in the tanks after the primary payload has been dropped off to place the secondary payloads in a higher energy super-synchronous transfer orbit. As a typical example of a super-synchronous orbit, Ariane V delivered Amsat's Phase 3D satellite to a (604 x 39334) km altitude with an inclination of 6.4 degrees [1]. This orbit was taken as the model for the initial injected orbit in the present study.

Since the primary payload dictates the launch time, it may be necessary for the satellites to be in a parking orbit about the Earth for some time while waiting for the appropriate time to commence the intercept trajectory. Furthermore, since the two satellites comprising the mission will be launched on separate vehicles, it will be necessary for the first satellite to wait in the parking orbit for the second satellite. The reason for launching on separate rockets is to provide extra insurance against launch failures as well as to allow any undetected bugs on the first satellite to be dealt with prior to the launch of the second. As it is conceivable that the satellites will need to remain in the parking orbit for a reasonably long period of time, possibly up to 6 months, it is necessary to consider ways of minimizing the effects of the Earth's radiation belts on the satellites' components. Since the highest energy particle fluxes exist near the equatorial plane, one method of dosage reduction is to increase the inclination of the orbit so that the time spent in the high-energy segment of the belts is reduced.

The asteroid interception phase begins with the satellites thrusting to increase the semi-major axis of the orbit until a hyperbolic escape trajectory to the flyby point has been achieved. The efficiency of the thrusting program in achieving the desired trajectory is the major factor in reducing the propellant required for the mission. The direction and magnitude of the thrust program will be optimized using methods of optimal control theory.

3.0 Optimization

3.1 System Dynamics

The framework for the optimization was set up such that the motion of the Earth and Toutatis are determined from heliocentric ephemeris [2,3]. The motion of the spacecraft was determined by integrating the gravitational accelerations acting on the satellite from the sun and the earth as well as the acceleration due to the propulsion system.

Figure 1 below illustrates the coordinate system used when defining the forces acting on the spacecraft. The inertial frame is J2000 with the x-coordinate pointing towards the first point of Aries.

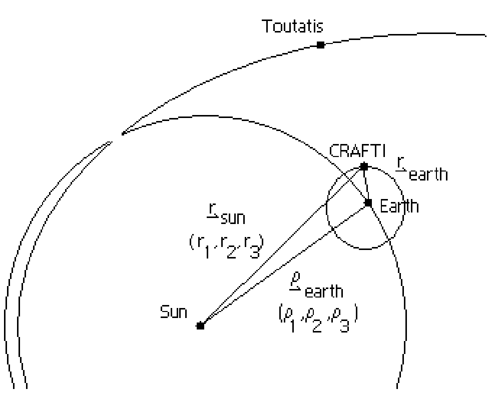

Figure 1: System Coordinates

The motion equation is

$$\ddot{\mathbf{r}}_{sun} = -\frac{\mu_{sun}}{r_{sun}^3}\mathbf{r}_{sun} - \frac{\mu_{earth}}{r_{earth}^3}\mathbf{r}_{earth} + \mathbf{u}(t) \qquad (1)$$

where μ_{sun} and μ_{earth} are the gravitational constants and $\mathbf{r}_{sun}$ and $\mathbf{r}_{earth}$ are the spacecraft position vectors relative to the earth and sun respectively and $\mathbf{u}(t)$ is the acceleration due to the thrusters.

The encounter date was taken to be November 9, 2008. This is the date that the Earth - Toutatis separation is a minimum at 0.0502 AU [4].

Allowing a transit time of 2 months to get from super-synchronous GTO to the asteroid defines the time interval of the asteroid interception phase. As a worse case scenario for the orbit initial conditions, the starting position in the super-synchronous GTO was taken to be perigee.

3.2 State Representation

The system dynamics were reformulated to conform to the standard state space model,

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) = \mathbf{a}(\mathbf{x}) + \mathbf{B}\mathbf{u}, \quad \mathbf{x}(t_0) = \mathbf{x_0} \qquad (2)$$

where $\mathbf{x}$ is the state vector $\mathbf{x} = [\mathbf{r}^T\ \mathbf{v}^T]^T$, $\mathbf{r}$ and $\mathbf{v}$ are the heliocentric position and velocities of the spacecraft and $\mathbf{u}$ is the control vector. The initial conditions for the state vector are defined by the initial orbit and time of departure, $t_0 = 0$.

The system dynamics matrix has the form of:

$$\mathbf{a}(\mathbf{x}) = \begin{bmatrix} \mathbf{v} \\ \mathbf{m}(\mathbf{r}) \end{bmatrix} \qquad (3)$$

where

$$\mathbf{m}(\mathbf{r}) = -\frac{\mu_{sun}}{r_{sun}^{3}}\mathbf{r}_{sun} - \frac{\mu_{earth}}{r_{earth}^{3}}\mathbf{r}_{earth} \tag{4}$$

The control influence matrix has the form of:

$$\mathbf{B} = \left[\mathbf{0}_{3x3} \quad \mathbf{1}_{3x3}\right]^{T} \tag{5}$$

The control vector, **u**, is composed of the accelerations that are to be adjusted to obtain the optimal solution. In fact, it is only the thrust that is being adjusted; the mass is a variable dependent on the thrust magnitude and the time over which the thrusting occurs. The propellant mass exhaust rate used is integrated as a state variable and is defined by

$$\dot{m} = \frac{F}{I_{sp}g} \tag{6}$$

where F is the thrust magnitude, I_{sp} is the specific impulse of the propellant being used and g is the local gravitational constant of the earth, 9.81 m/s^2.

Low-thrust electrothermal thrusters currently use propellants with an I_{sp} in the range of 150s to 1500s. As a design figure, a propellant with an I_{sp} of 815s was used [5].

Estimates of 200W of power, P, allocated to the propulsion system from the power budget, in conjunction with an estimate of 50% efficiency, η, for the power system allow an upper bound to be set on the thrust magnitude.

$$F_{max} = \frac{2\eta P}{I_{sp}g} \tag{7}$$

Use of Equation (7), determines that a maximum thrust of 25 mN is generated from the low thrust propulsion system [6]. This maximum value will be used to constrain the optimal control as it is being determined.

3.2 Two-Point Boundary Value Problem

The optimization follows from the minimization of the following quadratic cost function:

$$\mathbf{J} = \frac{1}{2}(\mathbf{x}-\mathbf{x}_d)^T\mathbf{S}(\mathbf{x}-\mathbf{x}_d)\Big|_{t=tf} + \frac{1}{2}\int_0^{tf}\left(\mathbf{u}^T\mathbf{u}\right)\,dt \tag{8}$$

where $t_0 = 0$.

The first term in the cost function applies a penalty to the error between the terminal state, **x**, and the desired final state, $\mathbf{x}_d$. The weighting matrix, **S**, is a positive semidefinite matrix used to enforce minimization of the terminal state error. Since the desired outcome of the intercept is a flyby and not a rendezvous, the weighting matrix will not weight the velocity components of the final state conditions thereby not forcing the velocity of the spacecraft to match the velocity of the target.

The quadratic control vector term in the integrand, $\mathbf{u}^T\mathbf{u}$, serves to minimize the control effort, and hence the fuel expended during the manoeuvres.

The Hamiltonian of the system is generated by adjoining the integrand of the cost function with the system dynamics:

$$H = \frac{1}{2}\mathbf{u}^T\mathbf{u} + \boldsymbol{\lambda}^T\left[\mathbf{a}(\mathbf{x}) + \mathbf{B}\mathbf{u}\right] \tag{9}$$

where $\boldsymbol{\lambda}$ represents the system costate vector.

The Euler-Lagrange equations are then used to determine the dynamics and final conditions of the costate equations as well as the condition for optimal control [7]:

$$\dot{\mathbf{x}} = \frac{\partial H}{\partial \boldsymbol{\lambda}}, \quad \mathbf{x}(0) = \mathbf{x}_0 \tag{10}$$

$$\dot{\boldsymbol{\lambda}}(t) = -\frac{\partial H}{\partial \mathbf{x}} \tag{11}$$

$$\boldsymbol{\lambda}(t_f) = \frac{\partial}{\partial \mathbf{x}}\left[\frac{1}{2}(\mathbf{x}-\mathbf{x}_d)^T \mathbf{S}(\mathbf{x}-\mathbf{x}_d)\right]\Bigg|_{t=t_f} \tag{12}$$

$$= \mathbf{S}[\mathbf{x}(t_f) - \mathbf{x}_d]$$

$$\frac{\partial H}{\partial \mathbf{u}} = \mathbf{0} \tag{13}$$

Evaluating Eq. (13) using the Hamiltonian in Eq. (10) reveals that the optimal control vector is only a function of the costate variables:

$$\mathbf{u}(t) = -\mathbf{B}^T \boldsymbol{\lambda}(t) \tag{14}$$

Since the state equations, Eq. (2) are defined by initial conditions at t_0 but the costate equations, Eqs. (11) & (12), are defined at the terminal time of the interval, t_f, the problem is a two-point boundary value problem (TPBVP).

3.3 TPBVP Solution Algorithm

Since the optimal control formulation is analytically very simple, the solution to the TPBVP will follow from the iteration of the control vector histories, **u**(t).

An initial guess of $\mathbf{u}^{(0)}$ is made. In general, at iteration n, we will have $\mathbf{u}^{(n)}$. Since the state equations are only functions of the state and control vectors, and the initial conditions are known, the state equations, Eq. (2), can be integrated forward in time to yield $\mathbf{x}^{(n)}$.

Having determined the evolution of **x**, the costate vector can be determined by integrating Eq. (11) backwards through time using the known terminal conditions, Eq. (12) to produce $\boldsymbol{\lambda}^{(n)}$.

A new **u**(t) is then calculated using $\mathbf{u}^{(n+1)}(t) = -\mathbf{B}^T\boldsymbol{\lambda}^{(n)}(t)$, and the process is iterated until the norm of the new **u**(t) differs by less than 0.01% from the norm of the previous **u**(t):

$$\frac{\sqrt{\int_0^{t_f} \left(\mathbf{u}^{n+1} - \mathbf{u}^n\right)^T \left(\mathbf{u}^{n+1} - \mathbf{u}^n\right)}}{\sqrt{\int_0^{t_f} \left(\mathbf{u}^n\right)^T \left(\mathbf{u}^n\right) dt}} < 1\text{x}10^{-4} \tag{15}$$

The numerical integration of the state and costate equations is performed using a standard Runge-Kutta algorithm of order 7(8).

4.0 *Propagate*

After the optimal thrust program has been determined, a closer examination of the orbital manoeuvres is desired to verify the space flight following the application of **u** as well as to determine the ΔV and fuel requirements for the mission. An orbit propagator, *Propagate*, was constructed to facilitate this need. Currently Propagate is only designed for the mission segments within the Earth's sphere of influence owing mainly to the lack of a solar gravitational model. Future versions of the program will permit the examination of the complete flight.

4.1 Orbit Propagation

The orbit propagator takes the orbital elements describing an initial orbit and a thrust profile as its inputs. It then uses Encke's method to calculate the final orbit at the conclusion of the thrust program. The outputs are the orbital elements of the final orbit, either elliptical or hyperbolic, the position and velocity vectors of the spacecraft at the Earth's sphere of influence and the total ΔV and fuel required by the spacecraft to complete the thrusting program.

4.2 Encke's Method

While it is perfectly valid to perform a straight integration of the equations of motion of the satellite to propagate the orbits, it is computationally inefficient and a solution using the method can be costly in time. A more computationally efficient method is to compute a solution to the Keplerian two body-problem then determine the difference between the two-body solution and the true solution and finally sum the Keplerian solution and the difference to find the true solution. This is known as Encke's method.

The Keplerian solution is also termed an osculating orbit, where to osculate means to kiss, an apt reference because the Keplerian orbit "kisses" the true orbit at the initial reference time. The Keplerian orbit is described by

$$\ddot{\boldsymbol{\rho}} + \frac{\mu}{\rho^3}\boldsymbol{\rho} = 0 \qquad (16)$$

where $\boldsymbol{\rho}$ is the position vector from the Earth to the osculating orbit.

The difference between the Keplerian orbit position vector and the true orbit position vector, $\mathbf{r}$, is defined by

$$\delta\mathbf{r} = \mathbf{r} - \boldsymbol{\rho} \qquad (17)$$

The true orbit is derived from the perturbed two-body problem where the perturbing acceleration can be due to other forces like those from planetary bodies or thrusters.

Combining Eqs. (16), (17) and the perturbed two-body problem yields Eq. (18), which describes the second time derivative of the differential as a function of the true and Keplerian solutions:

$$\delta\ddot{\mathbf{r}} = \mathbf{a}_p + \frac{\mu}{\rho^3}\left[\left(1 - \frac{\rho^3}{r^3}\right)\mathbf{r} - \delta\mathbf{r}\right] \qquad (18)$$

Since the $(\rho/r)^3$ term in Eq. (18) is very close to unity, another function is introduced to make the relation easier to solve [8]. The new function is f(q), where:

$$f(q) = q\left(\frac{3 + 3q + q^2}{1 + (1+q)^{3/2}}\right) \qquad (19)$$

and

$$q = \frac{(\delta\mathbf{r} - 2\mathbf{r})\cdot\delta\mathbf{r}}{r^2} \qquad (20)$$

Therefore, Eq. (18) becomes:

$$\delta\ddot{\mathbf{r}} = \mathbf{a}_p - \frac{\mu}{\rho^3}\left[f(q)\mathbf{r} + \delta\mathbf{r}\right] \qquad (21)$$

Integration of Eqs. (16) and (18) combined with Eq. (17) allows the true orbit to be calculated.

Figure 2 below, shows diagrammatically Encke's method.

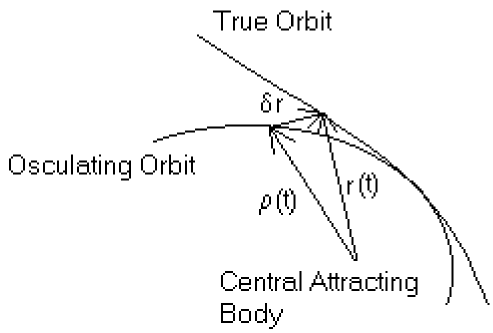

Figure 2: Encke's Method

The numerical integrations of Equations (16) and (18) are performed using a standard Runge-Kutta 7(8) algorithm.

4.3 Definition of Thrust Program

Propagate uses the true anomaly of the orbit to define the start and stop positions of the thrusting. This method of setting the start and stop thrust times was used to allow for ease of use when manually inputting a thrust profile. The input thrust directions are defined in terms of a body-centred coordinate system with the direction vectors aligned with the velocity vector, the perpendicular to the velocity vector in the orbital plane directed toward the orbit centre and the completion of the orthogonal set.

4.4 ΔV and Fuel Requirements

At the conclusion of each integration step, the ΔV is determined by integrating the accelerations due to the thrust as a state variable. The ΔV is then used in conjunction with the rocket equation to determine the propellant usage for the integration step:

$$m_p = m_0\left(1 - \exp\left(\frac{-\Delta V}{I_{sp} g}\right)\right) \tag{22}$$

where m_p is the mass of the propellant required during the integration step, m_0 is the mass of the spacecraft at the start of the step, I_{sp} is the specific impulse of the propellant and g is the gravitational constant of the Earth (~9.81 m/s^2).

The mass of the propellant used is then subtracted from the total mass of the satellite and propellant for the start of the next integration step. The acceleration due to thrusting is then calculated by Newton's second law as the force of the thrusters divided by the current total mass of the spacecraft and propellant.

The total m_p and ΔV used for the manoeuvre are determined by adding all of the constituent m_p and ΔV of the integration steps.

4.5 Radiation Models

While in an orbit that is close to the Earth, the satellite is exposed to large flux levels of high-energy electrons and protons from the Van Allen belts. These radiation effects can be detrimental to the spacecraft and its electronics and as such either the satellite or the orbits need to be designed to mitigate the dosage level. However, any shielding that is added to the satellite increases the mass of the satellite resulting in higher fuel and launch costs. Since a large portion of the mission is performed at a safe distance from the belts, it would be preferable to use only the minimum amount of shielding possible. The solution may come from the shape of the high energy belts themselves. The highest fluxes are concentrated close to the equator. Should the satellite's parking orbit be adjusted to a high inclination the flux levels will reduce and the region containing the highest energy particles will have been avoided.

The model implemented in the orbit propagator comes from the AE8 and AP8 missions performed by NASA from 1966 to 1980 [9].

5.0 Present Status

The activities described in this paper are all works in progress at varying stages of development.

5.1 Optimal Solution Algorithm

As is seen in Section 3, the mathematics describing the optimal solution has been developed. The computer implementation has begun and approximately 50% of the code has been written.

To facilitate the sequential verification of the code, the program is being organized into stand-alone functions. As each independent function is written, its output is compared with the anticipated result as derived from theoretical calculations or literature.

Matlab is being used as the programming language due to its built-in plotting and visualization sub-routines as well as its matrix manipulation capabilities.

5.2 *Propagate*

An initial version of the orbit propagator has been written and is presently being debugged.

Several verification checks have been built into the program to ensure the accuracy of the results. One of these checks is the simultaneous integration of the rate of change of the mass as described in Eq. (6) with the equations of motion. This allows a continual comparison with the propellant mass required as determined by the rocket equation to ensure that the ΔV is calculated correctly.

Verification is also being performed by comparing the analytical results of simple manoeuvres such as departure from a circular orbit under continuous thrust and inclination changes by thrusting normal to the orbital plane, with the numerical results determined by the program.

6.0 Future Work

Subsequent versions of both the optimal solution algorithm and the orbit propagator will require several modifications and additions to enhance the accuracy and utility of each.

6.1 Optimal Solution Algorithm

Currently, the optimal solution is generated using only the gravitational models of the sun and the earth. In order to provide a more realistic solution to the optimal control problem other perturbing forces must be included in the model. The forces that need to be included immediately are the Earth oblateness effects as well as gravitational models of the moon, Mars and Venus as they provide significant perturbing forces on the motion of the satellite. Ideally, every force that acts on the spacecraft should be modelled to provide the most accurate the solution. However, the zeal to add every available model must be tempered with the realization that every model that is included increases the computation time of the solution. Consequently, a threshold needs to be determined that will be used to determine the relevance of each of the gravitational models that are to be incorporated in the program.

The optimal thrust program for the plane inclination increase should be investigated to determine if the cost of moving out of the worst parts of the Van Allen belts is worth the gain in radiation dosage reduction.

6.2 *Propagate*

In order to model the entirety of the mission, gravity models of the sun, moon, Mars and Venus should be included. Also, to accurately model the orbits close to the earth, it is important to include J_2 perturbations and possibly even higher order J-effects.

Currently, the thrust programs are manually input into the orbit propagator. As one of the major purposes of the propagator is to closely examine the optimal thrust program generated by the optimal solution algorithm, a method of inputting the **u**(t) history in the form that it is generated by the optimal control algorithm would be useful. This involves placing the control history in the same reference frame as the equations of motion governing the satellite and converting the position-based thrust start and stop marks to time-based marks.

7.0 Conclusions

The CRAFTI project represents an ambitious yet achievable proposal to use a microsatellite to perform an interplanetary mission. With a budgetary target of $10 million, the mission will launch two 120 kg satellites for a flyby of the asteroid Toutatis in the autumn of 2008.

To minimize the fuel requirements and accommodate the additional safety requirements imposed by launching as a secondary payload, the propulsion will be provided by a low thrust system. However, the use of such a system requires a careful examination of mission planning and thrust programming because of the long burn times associated with low thrust engines.

The machinery of modern control theory will be used to generate the optimal thrust program that minimizes the fuel requirements for the mission. Additionally, an orbit propagator has been constructed to carefully examine the effects and requirements of the thrust program on the flight as well as model the potentially hazardous radiation fluxes interacting with the spacecraft.

In the past, microsatellites have been confined to terrestrial orbits and near Earth observations. CRAFTI will prove that these small satellites are capable of much more.

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