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INTERACTIVE MODIFICATION OF QUADRATIC MULTIOBJECTIVE WATER RESOURCES PLANNING STRATEGIES

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WATER RESOURCES MANAGEMENT LABORATORY
DEPARTMENT OF AGRICULTURAL ENGINEERING

PAUL J. KILLIAN

RICHARD C. PERALTA

ARKANSAS WATER RESOURCES RESEARCH CENTER
UNIVERSITY OF ARKANSAS
223 OZARK HALL
FAYETTEVILLE, ARKANSAS 72701

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INTERACTIVE MODIFICATION OF QUADRATIC MULTIOBJECTIVE WATER RESOURCES PLANNING STRATEGIES

Paul J. Killian and Richard C. Peralta

Authors are Research Assistant and Assistant Professor, Agricultural Engineering Dept., University of Arkansas, Fayetteville, Arkansas.

ABSTRACT

An interactive method is presented for modifying a multiobjective water resources planning strategy by changing constraining conditions on regional objectives and local variables. The method is illustrated by modifying a conjunctive use, sustained groundwater withdrawal strategy for minimizing the cost of meeting regional water demand on the Arkansas Grand Prairie. The strategy was developed using a model in which the finite difference form of the two-dimensional groundwater flow equation is embedded in an optimization process. The quadratic optimization is accomplished by utilizing the General Differential Algorithm to obtain values of drawdown, pumping, and recharge in each finite difference cell. Results from the formal optimization process are submitted to a separate program for interactive evaluation and modification. The interactive algorithm applies the constraint method and constrained derivatives of the objective function to develop the noninferior solution and tradeoff functions. The modification procedure is extended to determining the influence on the regional objectives for repeated changes in several local decision variables.

INTRODUCTION

The development of a regional water resources management strategy often includes the application of optimization theory to determine the allocation plan that most effectively satisfies a desired objective. The two major components of any optimization problem are the objective function and the variables. In this paper, an objective function is a statement of the desired goal of a regional water management strategy. The variables in the optimization problem represent local conditions which affect attainment of the regional objectives. When a finite difference technique is used in a water management model, the conditions at each node or finite difference cell are considered "local" variables.

Within the complex arrangement of legislative, sociologic, and economic goals influencing water resources management, it is difficult, if not impossible, to optimize a single objective function without adversely affecting other regional objectives or the values of local variables. Because opposing interests and ideas cannot be ignored in a realistic optimization procedure, there is a need for a technique of rapidly modifying the constraining conditions and determining the resulting effect on multiple regional objectives.

Because several decision makers are usually involved in the strategy selection process, the modification method should be interactive. Interactive techniques of multiobjective analysis have been used in the past to improve the coordination of subjective decision makers with an objective numerical process

(Monarchi and others, 1973; Haimes and Hall, 1974, Datta and Peralta, 1985). With an interactive procedure, the decision makers can actively participate in: (1) moving through the decision space defined by a multiobjective analysis to decide on a compromise between regional objectives, and; (2) changing the bounds on decision variables to reflect local considerations.

When conflicting objectives exist in the same problem, no single solution is available in which all aspects are optimally attained. However, through the application of generating techniques (Cohon and Marks, 1975) a noninferior set of solutions can be created. This solution set is also referred to as a "nondominated" set, the "Pareto Optimum", the "transformation curve" or the "efficiency" curve. A feasible solution is noninferior if no other feasible solution exists that will cause one objective to improve without forcing at least one other objective to degrade (Cohon 1978). At each noninferior solution, the relationship between competing goals is expressed in terms of a tradeoff function. The tradeoff function describes the amount of one objective that must be sacrificed in order to improve attainment of another objective.

Every decision variable also exhibits a relationship with the objective functions. Dual values, or constrained derivatives, describe the relative worth of each local decision variable on the regional objective. In the development of water management strategies, the objective functions applied to a region are frequently a maximization or a minimization of the aggregate effects on subareas within the region. This utilitarian approach provides for regional optimization at the expense of local development. By knowing how local changes affect regional optimality, changes in local variables can be considered in regional management decisions. Peralta and Killian, (1984) illustrate a method of refining an optimal regional solution in which only a single change to one decision variable is made. Their method however, is not interactive and is inadequate for analyzing continued changes in several decision variables.

One purpose of this paper is to present a method and example that utilizes quadratic parametric programing techniques in an interactive manner to develop the noninferior solution set and tradeoff functions. The second purpose is to demonstrate how this method may be extended to rapidly determine the effect on the objective functions due to repeated changes in any number of decision variables.

As a developmental step in the Grand Prairie Water Supply Project, (Peralta and others 1984a), the interactive method is demonstrated through application to the bicriterion problem of developing a conjunctive use, sustained yield pumping strategy for the Grand Prarie region of Southeast Arkansas. Opposing objectives considered in this example include a linear function to maximize regional groundwater withdrawal and a quadratic expression to minimize the total cost of supplying regional water demand. These objective functions are simultaneously evaluated within the same framework of physical and institutional constraints.

Simulation is performed by applying the finite difference form of the two-dimensional steady-state groundwater flow equation, (Pinder and Bredehoeft, 1968) as part of the constraining conditions in the optimization model. This technique of linking the simulation to the optimization model is referred to as the embedding method (Gorlick, 1983).

In the illustrative example, local variables subject to management constraint include the drawdown, pumping, and recharge in each finite difference cell. (Several considerations for determining limitations on these variables are listed by Bear (1979).) Drawdown is defined as the difference in elevation between a horizontal datum, located above the potentiometric surface, and the potentiometric surface. Groundwater pumping refers to the volume of groundwater removed from the system by a well penetrating the aquifer, and recharge represents the volume of water entering the groundwater system from outside the region. The net sum of pumping and recharge in each cell is referred to as excitation.

The development of the interactive modification method is explained by first describing the objective functions and the constraining conditions used in the example application. The necessary theory is then presented through discussion of: (1) the generation technique used to construct the noninferior solution set; (2) the General Differential Algorithm, and; (3) constrained derivatives. This is followed by a presentation of the interactive procedure used to construct the noninferior solution set, make repetitive local changes, and determine the influence of local changes on regional objectives. The conditions under which the method may be applied are also detailed.

OBJECTIVES FOR THE GRAND PRAIRIE

The quadratic objective function applied in the example, is unique in that it estimates the cost of maintaining a sustained yield by minimizing the cost of both groundwater and surface water required to satisfy regional demand. A complete derivation of this objective function and the factors involved is presented by Peralta and Killian (1984). For the purposes of this paper the following general representation is satisfactory.

minimize (1)

$$z_1 = \sum_{i=1}^{N} c_e(i)\, p(i)\, f(s(i)) + c_m(i)\, p(i) + c_a(i)\, p_a(i)$$

where:

- z_1 = the total annual cost of water supply, (\$/year);
- N = the total number of finite difference cells in which drawdown and pumping are variable;
- $c_e(i)$ = the cost associated with raising a unit volume of groundwater one unit distance, (\$/L4);
- $p(i)$ = the annual volume of groundwater pumped from cell i, (L3/year);
- $f(s(i))$ = a linear function of drawdown which describes the total dynamic head at cell i, (L);
- $c_m(i)$ = the cost associated with a unit volume of groundwater pumped, (\$/L3);
- $c_a(i)$ = the cost per unit volume of alternative water supplied in cell i, (\$/L3);
- $p_a(i)$ = the annual volume of alternative water use at cell i, (L3/year).

Because water requirements of each cell are satisfied by the conjunctive use of groundwater and an alternative water source, the following relationship is used to replace $p_a(i)$ in equation (1).

$$p_a(i) = w(i) - \bar{p}(i) \quad \text{for } i=1,N \tag{2}$$

where:

$w(i)$ = the annual water requirements in cell i, (L3/year).

The linear objective function used to maximize regional groundwater pumping is similiar to the formulation used by Aguado and others (1974), Alley and others (1976), and Elango and Rouve (1980). This is described as follows.

$$\text{maximize} \quad z_2 = \sum_{i=1}^{N} p(i) \tag{3}$$

where:

z_2 = the total volume of groundwater annually withdrawn from the region, (L3/year).

The bicriterion problem consisting of both objective functions is a two dimensional vector within a solution space of dimension 2N + M, where M is the total number of constant head cells. The following notation is used to describe this situation.

$$\text{optimize} \quad z = \{z_1, z_2\}. \tag{4}$$

Because it is not possible to maximize or minimize this problem without either prior knowledge or numerical representation of management preference, the term "optimize", as it appears in equation (4), refers to accurately defining the set of noninferior solutions.

The regional goals expressed by the objective functions are dependent on the drawdown, pumping, and recharge in each finite difference cell. Each of these local variables is limited by an upper and lower bound. The bounds on these variables delineate the feasible region, or solution space. The feasible region for the bicriterion example problem is defined by the following constraints.

$$p(i) = \sum_{j=1}^{K} - t(i,j)\ s(j) \qquad \text{for } i=1,N \qquad (5)$$

$$r(m) = \sum_{j=1}^{K} - t(m,j)\ s(j) \qquad \text{for } m=1,M \qquad (6)$$

$$s_{min}(i) \le s(i) \le s_{max}(i) \qquad \text{for } i=1,N \qquad (7)$$

$$p_{min}(i) \le p(i) \le p_{max}(i) \qquad \text{for } i=1,N \qquad (8)$$

$$r_{min}(m) \le r(m) \le r_{max}(m) \qquad \text{for } m=1,M \qquad (9)$$

where:

$$t(i,i) = \sum_{\substack{j=1 \\ j \neq i}}^{K} - t(i,j)\ ;$$

$t(i,j)$ = the transmissivity between finite difference cell i and cell j, for i = j, (L2/year);

K = the total number of cells in the study area, also the total number of inequality constraints, K = N + M ;

M = the total number of constant head cells in the region;

$s_{min}(i)$ = the lower limit on drawdown in cell i, (L);

$s_{max}(i)$ = the upper limit on drawdown in cell i, (L);

$p_{min}(i)$ = the lower limit on annual groundwater pumping in cell i, (L3/year);

$p_{max}(i)$ = the upper limit on annual groundwater pumping in cell i, (L3/year);

r(m) = the annual recharge at constant head cell m, (L3/year);

$r_{min}(m)$ = the lower limit on annual recharge in constant head cell k, (L3/year);

$r_{max}(k)$ = the upper limit on annual recharge in constant head cell k, (L3/year).

Equality constraints (5) and (6) are substituted into the objective functions and constraints (8) and (9) such that the only explicitly defined variable is drawdown. Pumping and recharge are defined in terms of the slack variables associated with constraints (8) and (9), respectively.

THEORY

Generation Technique

The method used in this paper to generate the noninferior solution set is referred to by Cohon and Marks (1975) as the constraint method. Under the constraint method, all but one objective become additional constraints. The single, or principal objective is optimized by conventional methods while the constrained objectives are limited by a chosen value. The selection of a principal objective does not indicate management preference.

To construct the noninferior solution set, the limiting value for a particular constrained objective is varied and the principal objective optimized at each new point. This is generally defined by the following formulation.

$$\text{min/max} \quad z_p = f(x) \qquad (10)$$

subject to:

$$z_h \geq L_h \qquad \text{For } h=1,H \qquad (11)$$

where:

z_p = value of the principal objective function;

z_h = value of objective constraint h;

L_h = the limiting value of objective constraint h;

H = total number of objective constraints.

For the bicriterion example, the linear objective function, equation (3), becomes an objective constraint and the problem description is represented in the operational form:

$$\text{minimize} \quad z_1 = g(s) \qquad (12)$$

Subject to the conditions of the feasible region as previously defined by (5), (6), (7), (8), (9), and the following additional condition.

$$z_2 \geq L_2 \qquad (13)$$

where:

$g(s)$ = equation (1) expressed in terms of drawdown alone;

L_2 = the minimum allowable total groundwater annually withdrawn from the aquifer underlying the region.

At each value of L_2, a new value of z_1 is computed. Within the feasible region of the solution space, the objective constraint will be binding. Therefore, a noninferior solution exists as a set of N drawdown values, at which z_2 is equal to L_2.

The values of L_2 represent the minimum allowable regional pumping imposed by a management decision. The range of L_2 for which the objectives will be conflicting and the corresponding range of regional cost values are defined by the following limits.

$$z_2 \text{ at } \min z_1 \leq L_2 \leq \max z_2 \tag{14}$$

for:

$$\min z_1 \leq z_1 \leq z_1 \text{ at } \max z_2$$

For values of L_2 less than z_2 at min z_1, the constrained objective and the principal objective are not in opposition, the objective constraint is not binding and the value of z_1 resulting from the optimization is equal to min z_1.

A systematic approach to developing the noninferior solution set varies the value of L_2 from one extreme to the other, covering the entire range in a predetermined number of steps. By using a controlled interactive method, only areas of the solution set which are of particular interest to the decision makers need be examined. Thus, by ignoring areas of the region which are of little concern, such as the extreme ends of the feasible range, each decision maker can accurately pinpoint his or her best-compromise solution with minimal computational effort. By using a differential algorithm in this interactive procedure, tradeoff functions for each regional objective and each local decision variable are readily available.

General Differential Algorithm

The General Differential Algorithm, developed by Wilde and Beightler (1967) and discussed in detail by Morel-Seytoux (1972), is a direct climbing method of locating the optimal solution through a systematic gradient search routine. The interactive technique presented in this paper uses an extension of the General Differential Algorithm to evaluate the change in the value of the principal objective function and the system response resulting from a change in the optimal solution set.

To aid in the explanation of the General Differential Algorithm consider the minimization of a quadratic objective function with N variables subject to K inequality constraints. During any iteration in the search process, the problem will consist of K equations and N+K variables, (K of these variables are slack variables introduced to transform the inequality constraints into equality conditions). The constraining equations are separable and as such, K variables are expressed as a function of N independent variables. N independent variables are initially referred to as decision variables while K dependent variables are referred to as solution or state variables. The specific separation of variables into state variables and decision variables is known as the partition of the system.

The functional equivalents of the state variables are directly substituted into the objective function such that the objective function is an unconstrained expression of N decision variables and no state variables. During each iteration in the optimization process, one decision variable is changed to improve the value of the objective function. A change in any decision variable will cause every state variable related by the K equality conditions to change.

In the example problem, a decision variable is either a drawdown variable, or a slack variable corresponding to one of the inequality conditions described by constraints (8), (9), and (13). At the optimum, all decision variables that are limited by a binding constraint are associated with a non-zero constrained derivative. Assuming a minimization process, if a decision variable is against an upper limit, the related constrained derivative must be negative. A decision variable has a positive constrained derivative associated with it if the lower limit is binding. If the value of a decision variable is not equal to a limiting condition, the corresponding constrained derivative is zero and any change in the decision variable does not improve the value of the objective function. This is simply a non-dogmatic explanation of the Kuhn-Tucker conditions.

<u>Constrained Derivatives</u>

The change in the value of the unconstrained form of the principal objective function, for a given change in a particular decision variable, is expressed in terms of the gradient of the unconstrained objective function. The gradient of the objective function is the vector of first partial derivatives with respect to the decision variables. Each first partial derivative is referred to as a constrained derivative. ("Constrained" derivative implies that the constraining conditions have been substituted into the objective function.) The constrained derivative describes the direction and magnitude of a change in the value of the objective function for an instantaneous change in the value of the decision variable.

Because the objective function described in this application is a quadratic expression, each constrained derivative of the objective function is a linear function of decision variables. Thus, for a change in the value of a single decision variable, the values of all related constrained derivatives also change. The change in the value of each constrained derivative is determined by evaluating the vector of second partial derivatives of the objective function with respect to the decision variables. For a quadratic objective function, this will be a vector of constant terms. The change in the constrained derivatives of the principal objective function for a change in decision variable i is described in terms of the second partial derivatives as follows.

$$\Delta v(j) = b(j,i) \Delta x_d(i) \qquad \text{for } j=1,N \text{ and } i=1,N \qquad (15)$$

where:

$\Delta v(j)$ = the change in the value of the constrained derivative.

$b(j,i)$ = the second partial derivative of z_p taken first with respect to decision variable j and again with respect to decision variable i.

Utilizing equation (15), the change in the value of the objective function for a change in one decision variable is expressed in terms of both the first order and second order partial derivatives as

$$\partial z_p / \partial x_d(i) = v(i) + b(i,i)\,(x'_d(i) - x_d(i)) \qquad (16)$$

for i=1,N

where:

$v(i)$ = the constrained derivative of z_p with respect to decision variable $x_d(i)$;

$b(i,i)$ = the second partial derivative of z_p with respect to decision variable $x_d(i)$.

$x'_d(i)$ = the new value of decision variable i;

$x_d(i)$ = the value of decision variable i, prior to increasing or decreasing the value.

For a specific change in a decision variable the above equation is integrated over $\Delta x_d(i)$ to yield

$$\Delta z_p = \{ v(i) + 0.5\, b_d(i,i)\, (\Delta x_d(i)) \}\, (\Delta x_d(i)) \qquad (17a)$$

for i=1,N

where:

Δz_p = the change in the value of the principal objective function;

$\Delta x_d(i)$ = the specific change in the decision variable i, or the difference between $x'_d(i)$ and $x_d(i)$.

For a specific change in the decision variable associated with an objective constraint, equation (17b) describes the tradeoff function.

$$\Delta z_p = \{ v(h) + 0.5\ b(h,h)\ (\Delta x_d(h)) \}\ (\Delta x_d(h)) \qquad (17b)$$

for h=1,H

Equations (15), (16), (17a) and (17b) are valid when the change in the decision variable does not cause a repartitioning of system variables. This limitation is discussed in detail in a subsequent section.

The change in all system variables in response to a change in the value of a single decision variable is referred to as the system response. Because all decision variables are independent, a change to one decision variable will not effect the value of the remaining decision variables. Every state variable however, is expressed as a function of decision variables and is therefore affected. By evaluating the gradients of the state variables, the change to the state variables in response to a change in the value of a single decision variable is determined.

In the bicriterion example, the constraints are linear and the resultant state gradients are vectors of constants. Therefore, the first partial of a state variable with respect to each decision variable is valid for any arbitrary change in a single decision variable, not merely an incremental change. The system response to a change in the value of a single decision variable is represented by the following formulation.

$$\triangle x_s(k) = d(k,i) \triangle x_d(i) \qquad \text{for } k=1,K \qquad (18)$$

where:

$\triangle x_s(k)$ = the change in state variable k;

$d(k,i)$ = the first partial derivative of state variable k with respect to decision variable i;

$\triangle x_d(i)$ = the change in decision variable i.

The partial derivatives of the state variables, d(k,i), are revised each time the system variables are repartitioned.

The concepts described indicate how the value of the principal objective function and the system variables change for a given change in a single decision variable. These methods are applied in the development of the interactive procedure.

THE INTERACTIVE PROCEDURE

The bicriterion example problem is formulated as it appears in equations (12) and (13) with L_2 set equal to any feasible value of total regional pumping. This problem is initially solved by a quadratic programing procedure written by Leifsson and others (1981) which uses the General Differential Algorithm to determine the optimal solution. The optimal set of N drawdown values, N pumping values, and M recharge values that result from the initial optimization represent one noninferior solution. These values, along with the values of the first and second order partial derivatives are transferred to a separate program for interactive evaluation.

In a constrained optimization, the decision variables are generally tight variables with nonzero constrained derivatives. To modify the original noninferior solution, any decision variable may be changed by modifying its upper or lower bound to expand or reduce the original size of the solution space. This effectively forces the decision variable to assume a desired value when the problem is optimized under the revised conditions.

Moving Through the Noninferior Solution Set

To generate the set of noninferior solutions, several changes to the binding limit, L_2, of the objective constraint are input, one at a time, to the interactive program. This modifies the value of the slack variable associated with constraint (13). The system response to each change is determined by equation (18) and the new value of the principal objective function is determined by equation (17b). The values of the constrained derivatives are revised by equation (15) and the system is checked for optimality. If the solution is not optimal, the interactive program performs the iterations necessary to make the solution noninferior.

At any point in the noninferior solution set, the relationship between regional objectives is described by the constrained derivative of the principal objective function with respect to the decision variable associated with each objective constraint. Once a favorable relationship is achieved and a compromise solution agreed upon, the resulting values of all local variables may be examined.

In examining the local variables, a group of decision makers may identify areas at which the variable values of drawdown, pumping, or recharge are unsatisfactory. To refine the compromise strategy and address local concerns, the interactive program is utilized as explained in the following section.

Local Influence on Regional Objectives

At a noninferior solution, each local variable is either a state variable, or a decision variable. The constrained derivative of the principal objective function with respect to a state variable is zero, indicating the independence between the principal objective function and the state variables. A change to a local condition represented by a state variable may be made by changing a decision variable, (or several decision variables), such that the desired effect on the particular state variable, (described by equation (18)), is achieved. Several examples of this are discussed by Peralta and Killian, (1984). To change the value of a decision variable representing drawdown, pumping or recharge, the binding limit is appropriately changed.

A change in the bound on a local decision variable changes the feasible region of the solution space common to both the principal objective and the objective constraints. Depending on the extent of the change, the noninferior solution that exists prior to changing a local bound is not necessarily optimal after the bound has been re-established. In other words, the solution may become inferior. At an inferior solution, one objective can be changed without adversely affecting the other objectives. Using the interactive procedure, the decision makers may choose the regional dimension in which to move such that the solution becomes noninferior.

Equation (16a) is used to determine the change in the principal objective function resulting from a specific change in the value of a decision variable. In making this change the objective constraints remain fixed and a new solution set results. At the new solution, the change in the value of an objective constraint, needed to insure that the principal objective retains its original value, may be calculated by solving equation (16b) for $\triangle x_d(h)$. This value is then input to the interactive program such that the original value of the objective function is obtained.

<u>Conditions Under Which the Procedure may be Utilized</u>

To change the value of a decision variable, the limiting bound is replaced with a value that either expands or reduces the size of the solution space. This effectively creates a new problem. Depending on the extent of the change to the bound, the new problem may require subsequent iterations to achieve optimality.

The solution that exists prior to changing the bound (the old optimal solution) is the starting point for the new problem and must be feasible within the new solution space. If a change in a bound increases the size of the solution space (if the upper limit is increased or the lower limit is decreased) the old solution is always a feasible starting point. If however, the solution space is reduced (a lower bound is increased or an upper bound is decreased) the extent of the change to the bound on a decision variable is limited by feasibility criteria. A reduction in the size if the solution space that causes the old optimal solution to be infeasible within the new solution space is not permitted with the interactive procedure.

The magnitude of the feasible change is determined by the constraints imposed on the involved variables. A decision variable is allowed to increase or decrease until it, or another variable, encounters a limiting condition. Since the bound on the decision variable itself is dictated by the user, the feasible positive and negative deviation is controlled by the first state variable to reach its upper or lower limit. The value of the feasible deviation is found by solving equation (18) for $\triangle x_d$ with $\triangle x_s(i)$ defined as the difference between the state variable and its approaching bound.

If the change in the bound on a decision variable is within, or equal to the feasible deviation, the corresponding change in the value of the decision variable is equal to the change in the bound. The constraint remains tight, and the system response is feasible, though not necessarily optimal.

Optimality is affected if a single decision variable is changed such that application of equation (16) causes one of the constrained derivatives to change signs. The maximum absolute change in the value of a decision variable such that none of the nonzero constrained derivatives change sign is referred to as the optimal deviation. To change sign, a constrained derivative must first change from a positive or negative value, to zero.The optimal deviation is determined by applying equation (15) with _

$\Delta v(j)$ defined as the difference between the value of the constrained derivative and zero. If the change in the bound on a decision variable is within both the optimal deviation and the feasible deviation, the change in the value of the decision variable is equal to the change in the bound and the resulting strategy is optimal.

The bound on a decision variable can be changed in excess of the feasible and optimal deviation if the change increases the size of the feasible region. In such a case, a state variable reaches its bound and the initial change in the decision variable is less than the input change in the bound. A re-partitioning of the variables is performed such that the tight state variable becomes a decision variable and the loose decision variable becomes a state variable. Additional iterations may be necessary to make the feasible solution optimal as well.

In summary : (1) the interactive process may be used to modify an existing strategy when a change in the limiting bound on any decision variable decreases the size of the solution space if the change to the bound is within the feasible deviation determined through the use of the constrained derivatives; (2) the interactive modification method may not be used to change a bound in excess of the feasible deviation if the change decreases the size of the solution space; (3) the method can analyze any arbitrary change in the limiting bound on a decision variable if the change increases the size of the solution space. When the change in the solution space exceeds the optimal deviation, additional iterations are necessary if the optimal result is desired. These iterations are performed by the interactive program by utilizing the same subroutines developed for the interactive process.

APPLICATION AND DISCUSSION

Site Description

The quadratic and linear objective functions for minimizing total cost and maximizing total regional groundwater withdrawal are applied in the multiobjective format to the Grand Prairie of southeastern Arkansas. Figure 1 shows the Grand Prarie subdivided into 204 finite difference cells. Of the 204 total cells, 52 are constant head cells used to simulate conditions along the periphery of the study area.

The Grand Prairie is an extensively cultivated and irrigated agricultural area and one of the prime rice producing regions of the country (Griffis 1972). A heavy layer of clay underlies the topsoil and prevents infiltration from recharging the aquifer. The only apparent sources of recharge are the rivers which border the area and extensions of the aquifer outside the study area. Extensive pumping and limited recharge has resulted in a declining water table and water shortages in this Quaternary aquifer.

Aquifer characteristics used for simulation are those reported by Peralta and others (1984b). These data include the elevation of the top and bottom of the aquifer, (used in determining the saturated thickness), and a hydraulic conductivity of 82 meters per day, (270 feet per day).

The drawdown and pumping in the non-constant head cells are bounded by an upper and a lower limit. The lower limit on drawdown represents the average ground surface elevation in each cell. The upper limit on drawdown is such that 6 meters (20 feet) of saturated thickness is guaranteed in each cell. The lower limit on pumping is zero (to prevent physically unrealistic internal recharge from being computed) and the upper limit on pumping is equal to the current average annual groundwater withdrawals. The variable recharge in constant head cells is limited such that maximum annual observed recharge from outside the system is never exceeded.

Cost coefficients used in the quadratic objective function are estimated from information received from the U.S. Army Corps of Engineers, (personal communication with Joe Clements, Dwight Smith, and Stony Burke). In areas where no surface water is available for use as an alternative source, the opportunity cost associated with reduced production is used as the alternative water cost.

The matrix of second partial derivatives in the least-cost objective function, equation (1), consists of groundwater cost coefficients and transmissivity values. Before optimization, this Hessian matrix was examined and found to be positive-definite, thus insuring that the resulting solution is the global optimum. Details of this are discussed by Peralta and Killian (1984).

Noninferior Solution Set

Figure 2 displays the resulting set of noninferior solutions interactively generated as outlined previously. Shown with every exact noninferior solution is the corresponding tradeoff function expressed by the the first order partial derivatives in units of dollars per cubic decameter. Although the total range defined by (14) is presented in Figure 2, in actual practice it is not necessary to produce the entire set of solutions.

From the noninferior solution set, the best-compromise solution may be determined by implementing the surrogate worth tradeoff method introduced by Haimes and Hall (1974) and adopted for interactive development of a conjunctive use, sustained yield strategy by a group of decision makers (Datta and Peralta, 1985). For illustrative purposes, solution set A is chosen as a compromise solution, though not necessarily the best compromise solution. For solution A, the total annual regional groundwater pumping is maintained at 138,000 cubic decameters, (112,000 acre feet). The total regional cost of the conjunctive use strategy is 9.3 million dollars and the average combined cost of groundwater and alternative water (including opportunity cost) is 26 dollars per cubic decameter, (32 dollars per acre foot).

Local Change

At the compromise solution, the local groundwater pumping in cell (3,4) is equal to its lower limit, which is 0.0 . In other words, for the benefit of the region as a whole, no groundwater withdrawal is permitted at this cell and in fact, no water needs are satisfied. Assuming that a group of decision makers wish to improve the equity of the compromise solution to groundwater users in cell (3,4), the lower limit on groundwater pumping in cell (3,4) is increased, and the regional effect analyzed.

The constrained derivative for the pumping in cell (3,4) is

32 dollars per cubic decameter, (40 dollars per acre foot). For every cubic decameter increase in groundwater pumping in cell (3,4), the regional cost increases by 32 dollars. Because the second partial derivative of the objective function with respect to the pumping is a positive 0.008 dollars per cubic decameter per cubic decameter, (0.012 dollars per acre foot per acre foot), the constrained derivative, (32 dollars per cubic decameter), will increase as the local pumping increases.

The most that pumping can be increased in cell (3,4) and still maintain feasiblility is 237 cubic decameters, (192 acre feet), at which point the pumping in cell (5,5) reaches its lower limit. Because the change will reduce the size of the solution space, the limit of 237 cubic decameters must be recognized. If the desired increase in the pumping at cell (3,4) is greater than 237 cubic decameters, the original problem must be reformulated and submitted for execution using standard optimization code.

Assume that the decision makers agree to increase pumping in cell (3,4) by 224 cubic decameters, (183 acre feet). In accordance with equation (17a), the modification causes the total regional cost to increase by 7,430 dollars. The change of 224 cubic decameters also causes the values of some of the constrained derivatives to change sign, thus making the solution inferior. The interactive program requires 5 subsequent iterations and about two minutes of processing time to calculate the optimal solution. At the revised optimum, the increase in total regional cost is 7,390 dollars and the pumping in cell (3,4) is 224 cubic decameters.

This new noninferior solution is point B on Figure 3, an enlarged section of Figure 2 in the vicinity of the compromise solution. At point B, the total regional pumping is still 138,000 cubic decameters but the cost is 7,390 dollars greater than the cost of solution point A.

The decision makers may also want to know how the total regional pumping of strategy A is affected by a local increase of 224 cubic decameters in cell (3,4), if the total cost remains constant. At point B, the constrained derivative of the principal objective with respect to the constrained objective, (the instantaneous tradeoff function), is 30 dollars per cubic decameter (37 dollars per acre foot), and the corresponding second partial derivative is 0.002 dollars per cubic decameter per cubic decameter, (0.003 dollars per acre foot per acre foot). Solving equation (17b) for Δx_d with Δz_p equal to -7,390 dollars results in a reduction in total regional pumping of 250 cubic decameters, (202 acre feet). Because this increase in the size of the feasible region is less than the maximum feasible deviation, the first and second partial derivatives remain valid. This means that in order to increase groundwater availability at cell (3,4) from 0 to 224 cubic decameters, while maintaining total regional cost at 9.3 million dollars, a total of 474 cubic decameters of groundwater must be forsaken in all remaining cells. Implementing this change results in the noninferior solution indicated by point C in Figure 3.

At point C, the total cost is the original 9.3 million dollars, but the total regional pumping has decreased by 250 cubic decameters. The curve connecting points B and C indicates a portion of the set of noninferior solutions for the new solution space. At any point on the revised curve, the minimum amount of groundwater pumping at cell (3,4) is 224 cubic decameters.

The extension of the noninferior solution set in a local dimension is possible at any compromise solution with any decision variable. Therefore, for the 152 decision variables in this example, the total number of possible decision directions, including the two regional dimensions, is 154.

SUMMARY

An interactive parametric programing method is introduced in the form of a computer program to effectively and efficiently evaluate several conflicting objectives. With this technique, the user is able to interactively investigate any area of the feasible solution space and utilize both regional and local tradeoff functions in selecting and designing a regional water management strategy.

By applying this method, decision makers may interactively modify a management strategy in both the regional and local decision dimensions. Regional changes are made by moving through the set of noninferior solutions to locate a compromise solution and regional tradeoff functions. Local changes, or modifications in the finite difference variables, are accomplished by changing the constraining conditions on local decision variables. The constrained derivatives are available for evaluating the response of regional objectives to repeated changes in local decision

variables.

In the field example the procedure is used to modify an optimal regional conjunctive use, sustained groundwater withdrawal strategy. The strategy is initially obtained from a management model that minimizes the cost of meeting water needs from the conjunctive use of groundwater and surface water while maintaining a sustained yield. The optimization process uses the finite difference form of a two dimensional groundwater flow equation as part of the constraining conditions. For multiobjective analysis, a second objective function that maximizes the total regional groundwater withdrawal under sustained yield conditions is included in the original problem as an additional constraint. The results of the formal optimization include local variables representing the drawdown, pumping, and recharge in each finite difference cell. The initial results also include a decision variable that represents the total regional groundwater withdrawal under the optimum strategy.

The results of the formal optimization are input to an interactive computer program and the set of noninferior solutions is generated. At any feasible solution, the tradeoff function between competing objectives is given to aid in locating a compromise solution. The procedure also provides information on the response of the regional objectives to a change in any local decision variable. This information is used for modifying the compromise solution with respect to local concerns.

The interactive modification method may be applied for any change in a bound on a decision variable, when the change increases the size of the feasible region. For the given example of 152 decision variables and 204 inequality constraints, if a change in the bound on a decision variable is less than the maximum feasible deviation, the optimal solution is calculated with a few additional iterations and about two minutes of processing time. If the change in the bound causes a re-partitioning of the system variables, it may take more than a hundred iterations and considerably more processing time to arrive at an optimum.

When a change in a bound decreases the size of the feasible region, the change is limited by the feasible deviation determined by utilizing constrained derivatives. The interactive procedure is not appropriate if a desired change decreases the size of the feasible region in excess of the feasible deviation. In such a case the problem must be re-submitted and solved by a standard optimization code.

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LIST OF FIGURES

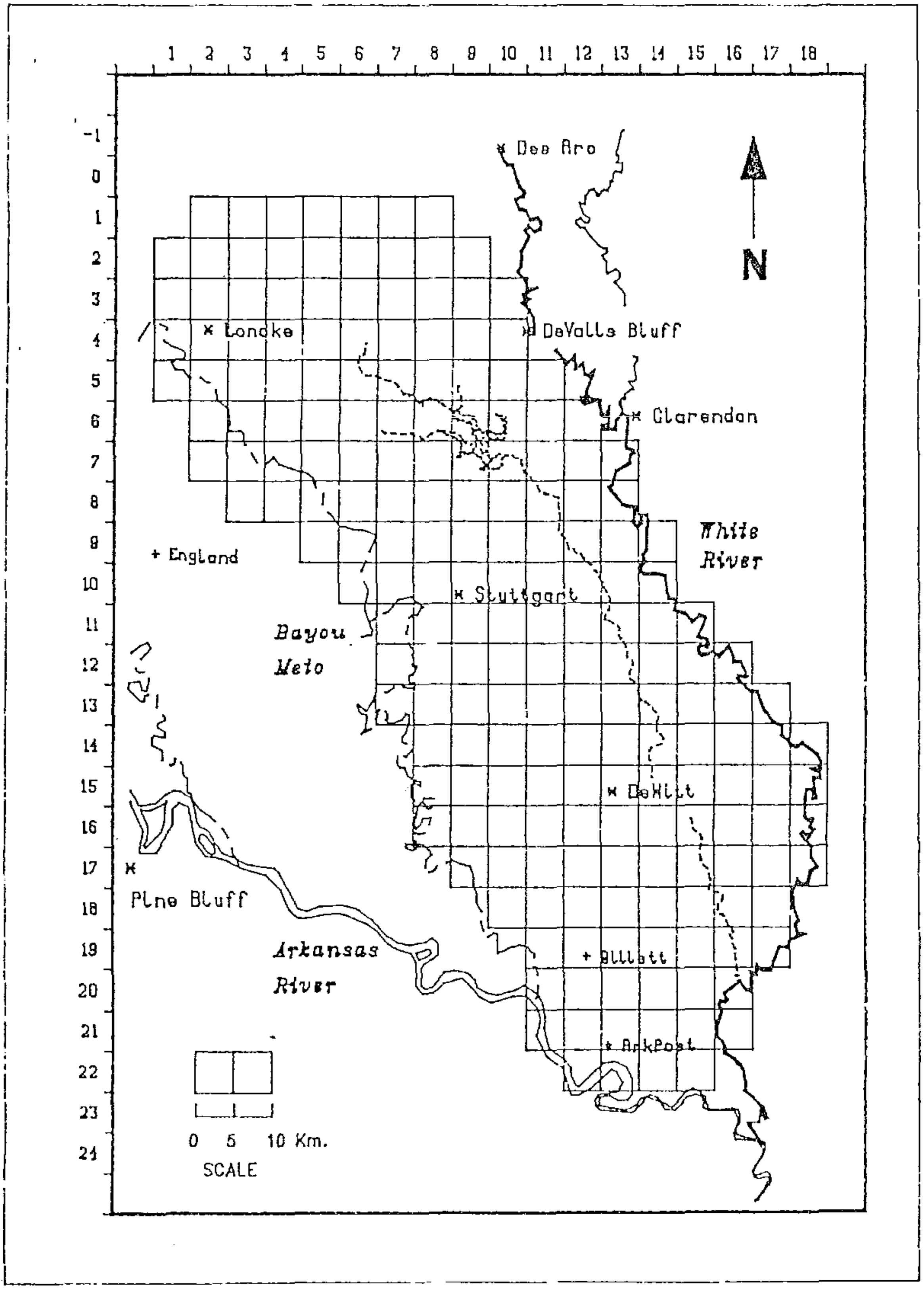
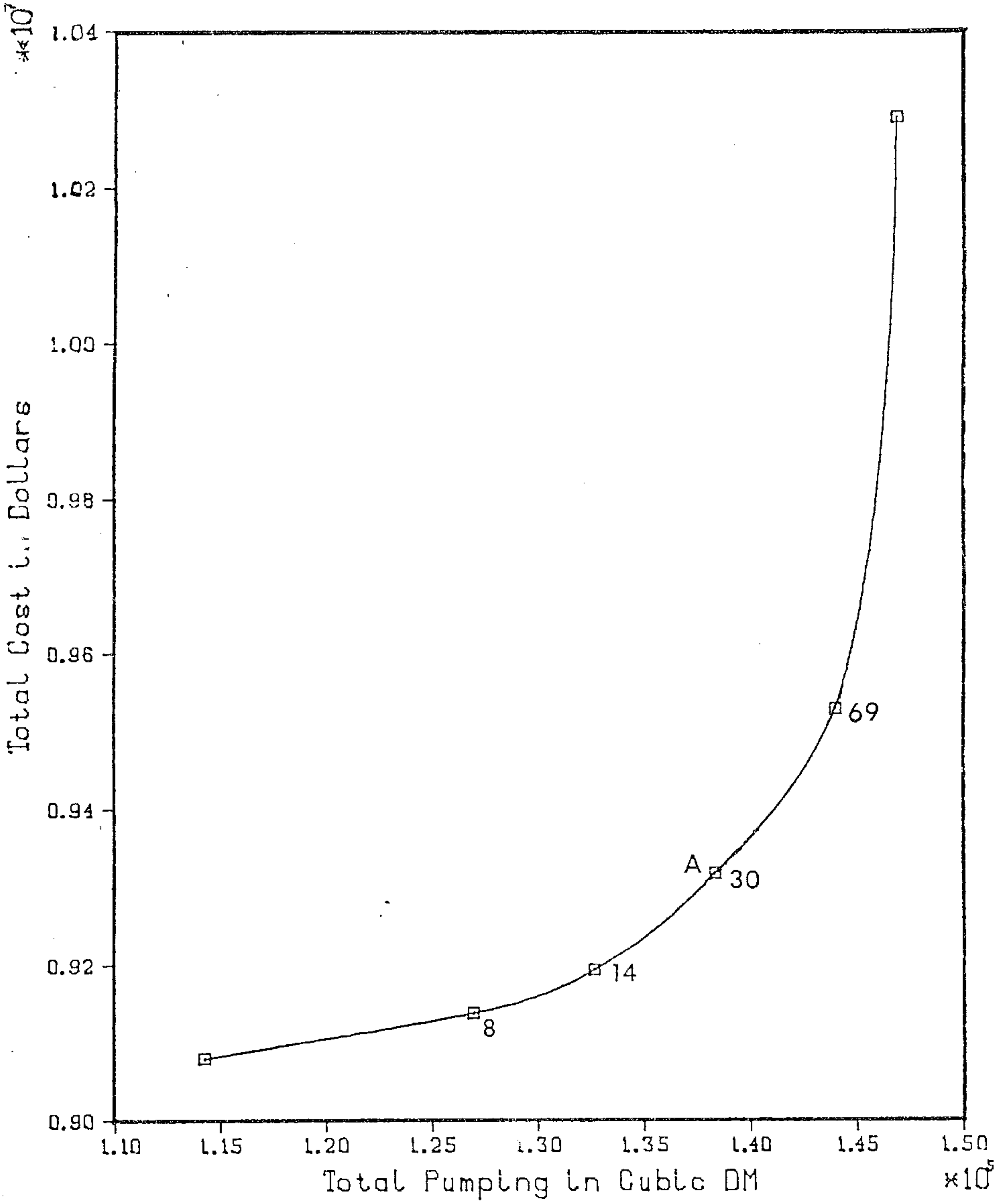

Des Arc
N
Lonoke
DeValls Bluff
Clarendon
White River
England
Stuttgart
Bayou Meto
DeWitt
Pine Bluff
Arkansas River
Gillett
ArkPost
0 5 10 Km.
SCALE

FIGURE 1



$\times 10^7$
1.04
1.02
1.00
0.98
0.96
0.94
0.92
0.90
Total Cost in Dollars
1.10
1.15
1.20
1.25
1.30
1.35
1.40
1.45
1.50
Total Pumping in Cubic DM
$\times 10^5$
8
14
A
30
69

FIGURE 2

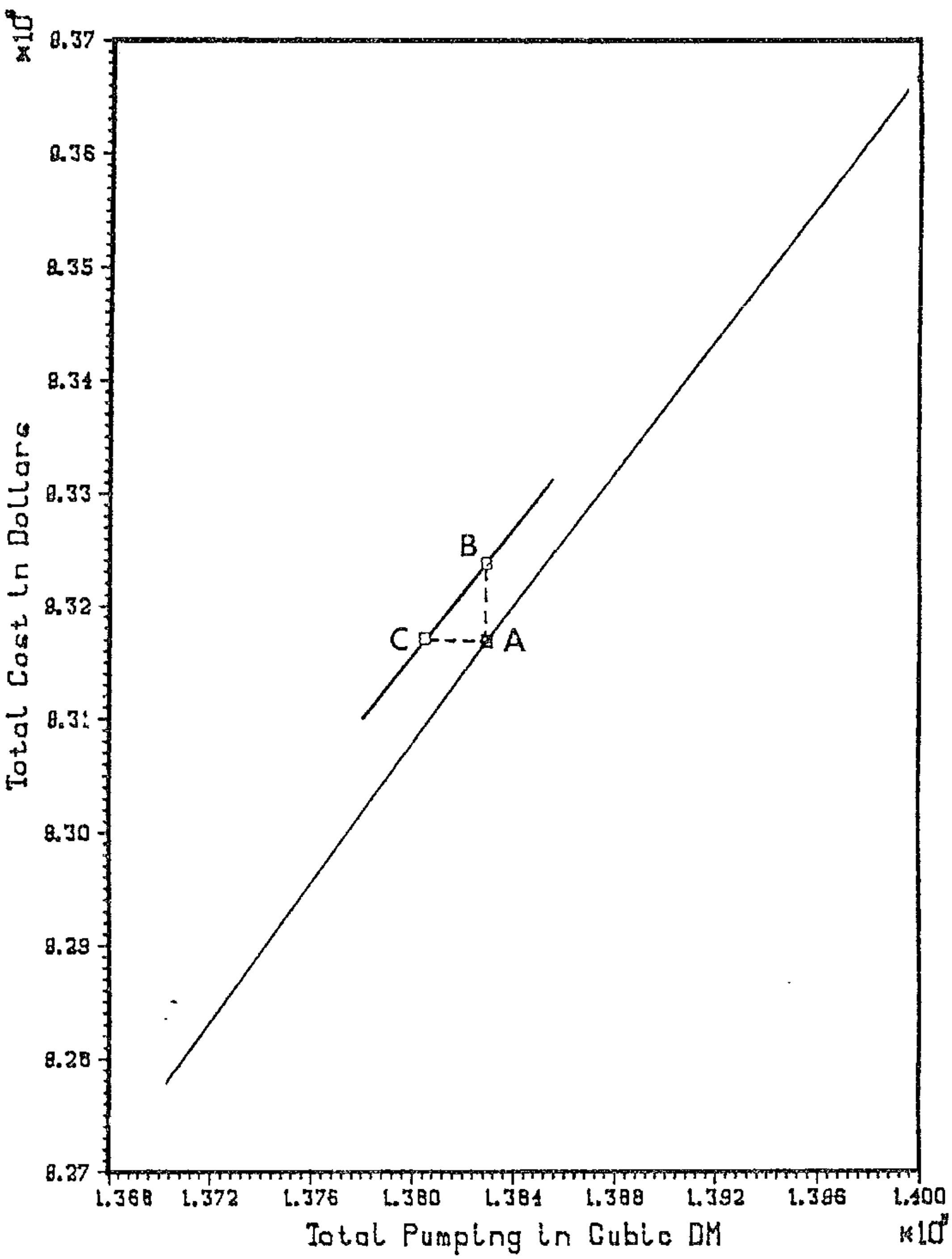

x10^8
8.37
8.36
8.35
8.34
8.33
8.32
8.31
8.30
8.29
8.28
8.27
Total Cost in Dollars
B
C
A
1.368
1.372
1.376
1.380
1.384
1.388
1.392
1.396
1.400
Total Pumping in Cubic DM
x10^7

FIGURE 3