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Precise Relative Orbit Determination of Low Earth Orbit Formation Flights Using GPS Pseudorange and Carrier-Phase Measurements

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Abstract: Formation flying is emerging as an important technology on achieving the tight mission requirements of imaging and remote sensing systems, especially radio interferometry and synthetic aperture radar (SAR) applications. A higher absolute and relative position and orbit knowledge is always sought in these kinds of applications. Such requirements can be met to a large extent by manipulation of GPS data. Carrier-phase Differential GPS (CDGPS) measurements can also be used to further increase the accuracy in relative position and orbit determination dramatically.

Using a geometric model has a clear advantage of generality and wide applicability, independent of complex dynamic models for different types of platforms. Hence, the proposed approach uses input from GPS receiver on the master satellite and pseudorange based absolute position estimates from the slave satellites. In addition, single-difference (SD) phase measurements between the master and the slave satellites are also required, which provide very accurate relative distance information. SD information is input into a Kalman filter to determine the relative orbits within the formation to a higher precision.

In this paper, we present a geometrical approach to relative orbit determination and present an algorithm for the refinement of position estimates through combining carrier-phase and pseudorange data.

1. Introduction

Formation flying is acknowledged as a crucial technology for many planned space missions. In most, if not all, of these missions relative orbit determination is of very high importance. These missions include proposed stellar interferometry formation under NASA's New Millennium Program and LEO missions for atmosphere and gravity modelling and for coordinated Earth observing[1].

Global Positioning System (GPS) is proven as an accurate positioning and orbit determination tool; furthermore, it leads to not only a reduction in mass, cost and the power requirements but also an increased satellite autonomy, thus reduced ground operation costs[4]. Carrier-phase differential GPS (CDGPS) is demonstrated to be useful in relative positioning and orbit determination in indoor and outdoor experimental settings[4,5], although it is yet to be proven in a space mission.

The conventional GPS based precise orbit determination strategies rely on data from a network of terrestrial GPS receivers as well as the spaceborne receiver. The estimation procedure is complex and lengthy, consisting of integration of the GPS data with accurate dynamic models of the low Earth orbit (LEO) satellite[2, 3]. These strategies rely greatly on the GPS measurement strength, especially for the low altitude spacecraft. However, a geometric approach with no complex dynamic modelling has the advantage of simplicity, wide applicability and increased autonomous operation.

In view of the above discussion, the theme for the project has been chosen as "relative orbit determination in a LEO formation using GPS measurements". The work includes a basic GPS simulator for real-time pseudorange and carrier-phase data processing (position determination) and processing the data using an Extended Kalman Filter for relative orbit determination. The entire procedure is geometry based, autonomous and essentially independent of the platform.

The main objective is to properly evaluate the method to determine the relative orbit with a reasonable accuracy, autonomously and in real-time. The method is to be properly evaluated for precision and possibility for an onboard application. The only required input is the GPS signal; however, intense intersatellite communications is also vital. Note that, an important design driver is to minimise computational load, given the fact that processing power is a valuable commodity on the orbit.

The real-time nature of the problem dictates that the integer ambiguity resolution for accurate CDGPS measurements should also be carried out in real-time. Although the details of the ambiguity resolution techniques are beyond the scope of this paper, a brief overview of the ongoing research is presented.

2. System Description

This paper studies the orbit determination problem for two satellites in formation; however, the basic principles do not rely on the number of satellites in the formation. Hence, the solution to the orbit determination problem can be extended to any number of satellites.

The formation is assumed to be in a near-circular low Earth orbit (LEO) and within close proximity of each other (<20km). This region is particularly convenient since the number of GPS satellites in view is very high (11 to 17), providing a favourable geometry (or a low geometric dilution of precision (GDOP)). The LEO region is to host a number of GPS based formation flying missions in the near future.

The formation is assumed to comprise two identical satellites, one of which is the master and the other is the slave. This designation is arbitrary and, since the hardware is assumed to be identical they can switch roles any time in the mission. At this point, the formation architecture is not important; however, as the number of satellites increase, "decentralised" control, where phase differences are between assigned pairs in the formation, or "centralised" control, where all phase differences are between the master and each slave satellite, become an important parameter. In the former approach the computational load is distributed between the satellites, in the latter the master satellite handles the entire computational load. Corazzini et al. addressed the issue of centralised vs. decentralised approaches in a basic three satellite setting, open to extrapolation using the results as building blocks[8].

It is possible to have fully autonomous absolute orbit determination by each member of the formation, since this relies on pseudorange measurements by individual satellites. However, relative position determination requires the transfer of phase information data between the satellites. Therefore, continuous communication between the two satellites is crucial.

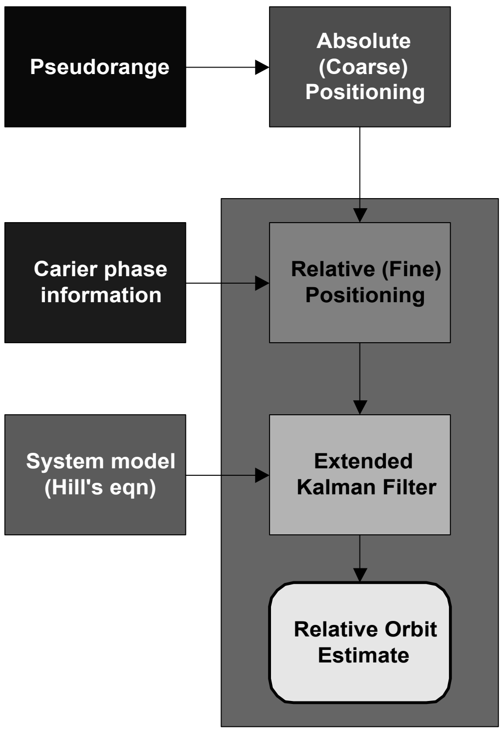

Figure 1 System Block Diagram

A layered approach is used to determine the relative orbit (Figure 1). Absolute state is determined from the pseudorange measurements for each satellite (Coarse Positioning). Carrier-phase information is then evaluated using a simple least squares estimator, to find the relative position estimates. The resulting high accuracy relative position estimates are input to an Extended Kalman Filter (EKF) to determine the relative orbit. The EKF comprises small matrices in simple forms. Furthermore, convergence is quicker. This step-by-step approach is different from the common procedure in the literature, where the measurements themselves are input to a more complex Extended Kalman Filter and are solved simultaneously[5,8].

2.1 Absolute (Coarse) Positioning

Coarse Positioning is the absolute position determination module of the algorithm. Position determination is carried out in discrete time using simulated pseudorange measurements, solved by the least squares method. The details of the procedure are documented in many sources[9, 10, 11] and will not be presented here.

The GPS receivers are assumed to be tracking four GPS satellites simultaneously, while continuously monitoring the geometric dilution of precision (GDOP) parameter. The tracked satellite set is changed frequently, keeping minimum GDOP at all times.

Absolute position ranging noise is estimated to be 5m (based on error budgets presented in[11], no Selective Availability).

2.2 Relative (Fine) Positioning

Fine Positioning is the relative position determination module of the algorithm and is very similar to Coarse Positioning in structure and solution method. Relative positioning is carried out in discrete time using simulated carrier-phase difference measurements solved by the least squares method. (See Figure 2 for the geometry of the problem.)

Formulation begins with the well-known single difference (SD) equation in its general form[10]:

$$SD = \phi_{km} + N_{km} + S_{km} + f\tau_{km} \qquad (1)$$

where
k and m refer to the receiving satellites
ϕ is the transmitted satellite signal phase as a function of time
N is the unknown integer number of carrier cycles from the source to the receiver
S is phase noise due to all sources (e.g. receiver, multipath)
f is the carrier frequency
τ is the associated satellite or receiver clock bias.

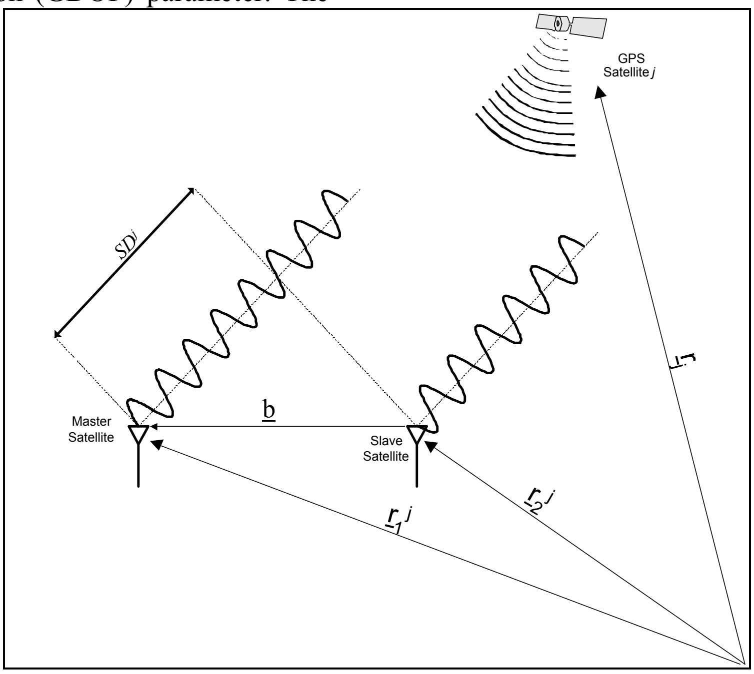

Figure 2 CDGPS Geometry (Not to Scale)

The ϕ term is supplied by the phase counter of the GPS receiver and is known. *N* term is the integer ambiguity and is assumed known to a very good accuracy (see section 2.4 for integer ambiguity resolution). *S* is the phase noise term and simulated as white gaussian noise with σ=20mm. Finally, τ is assumed to be negligible, since Absolute Positioning algorithm corrects the system clocks of the formation with respect to the GPS system clocks to a very good accuracy. Thus, combining all the noise affects in the phase noise term, the SD term is assumed known to 20mm accuracy.

An equation similar to Eq.(1) can be written for each received GPS satellite.

Baseline vector definition in the vector notation is:

$$\underline{SD}^j = \left(\underline{b} \cdot \frac{\underline{r}_1^j}{\left|\underline{r}_1^j\right|} \right) \cdot \frac{\underline{r}_1^j}{\left|\underline{r}_1^j\right|} \qquad (2)$$

where $\underline{r}_1^j$ is the range vector from the master LEO satellite from the origin in Earth Central Inertial (ECI) coordinate system and $\underline{b}$ is the baseline vector, defined from slave to master LEO satellite. The definitions in vector form are,

$$\underline{r}_1^j = \underline{r}_j - \underline{r}_1 \qquad (3)$$

$$\underline{b} = \underline{r}_2 - \underline{r}_1 \qquad (4)$$

Inserting the two vector definitions into (2) yields the magnitude,

$$\left|\underline{SD}^j\right| = \frac{(\underline{r}_2 - \underline{r}_1)\cdot(\underline{r}_j - \underline{r}_1)}{\left|\underline{r}_j - \underline{r}_1\right|} \qquad (5)$$

Writing the above equation in scalars (where $\underline{r}$ = x.$\underline{i}$+y.$\underline{j}$+z.$\underline{k}$) and carrying out the dot product operation in scalar form,

$$\left|\underline{SD}^j\right| = \frac{(x_2 - x_1)\cdot(x_j - x_1) + (y_2 - y_1)\cdot(y_j - y_1) + (z_2 - z_1)\cdot(z_j - z_1)}{\sqrt{(x_j - x_1)^2 + (y_j - y_1)^2 + (z_j - z_1)^2}} \qquad (6)$$

Defining,

$$\begin{aligned} x_{rel} &= x_2 - x_1 \\ y_{rel} &= y_2 - y_1 \\ z_{rel} &= z_2 - z_1 \end{aligned} \qquad (7)$$

and inserting into (6),

$$\left|\underline{SD}^j\right| = \frac{x_{rel}\cdot(x_j - x_1) + y_{rel}\cdot(y_j - y_1) + z_{rel}\cdot(z_j - z_1)}{\sqrt{(x_j - x_1)^2 + (y_j - y_1)^2 + (z_j - z_1)^2}} \qquad (8)$$

The SD equation is a function of the baseline (or relative position), the position of master LEO satellite and the GPS satellite. The position estimate of the master LEO is available from the Coarse Positioning; GPS position is known with a reasonable accuracy and baseline is the unknown quantity, for which an initial estimate available from the Coarse Positioning solution for the slave LEO satellite. There are as many such equations as the number of received GPS satellites.

Rather than solving this set of non-linear equations, linearisation works better and is computationally less expensive. The linearised forms of the vectors (shown in scalar notation) is simply the combination of the estimated value and the error part,

$$\begin{aligned} x_{rel} &= \hat{x}_{rel} + \Delta x_{rel} \\ y_{rel} &= \hat{y}_{rel} + \Delta y_{rel} \\ z_{rel} &= \hat{z}_{rel} + \Delta z_{rel} \end{aligned} \qquad (9)$$

Note that, $\underline{b}$ = $x_{rel}.\underline{i}+y_{rel}.\underline{j}+z_{rel}.\underline{k}$. Thus, the following is the linearisation of SD vector with respect to the baseline vector $\underline{b}$,

$$\begin{aligned} \underline{SD}^j &= \underline{\hat{SD}}^j + \Delta\underline{SD}^j \\ \Delta\underline{SD}^j &= \frac{\partial \underline{SD}^j}{\partial \hat{b}} \cdot \Delta\underline{b} \end{aligned} \qquad (10)$$

$$\Delta \underline{SD}^j = [\frac{\partial \underline{SD}^j}{\partial \hat{x}_{rel}} \quad \frac{\partial \underline{SD}^j}{\partial \hat{y}_{rel}} \quad \frac{\partial \underline{SD}^j}{\partial \hat{z}_{rel}}] \cdot \begin{bmatrix} \Delta x_{rel} \\ \Delta y_{rel} \\ \Delta z_{rel} \end{bmatrix} \quad (11)$$

The last equation (Eq.(11)) requires the determination of the partial derivatives of the SD vector in the x_{rel} , y_{rel} and z_{rel} directions.

$$\frac{\partial \underline{SD}^j}{\partial \hat{x}_{rel}} = \frac{x_j - x_1}{\sqrt{(x_j - x_1)^2 + (y_j - y_1)^2 + (z_j - z_1)^2}}$$

$$\frac{\partial \underline{SD}^j}{\partial \hat{y}_{rel}} = \frac{y_j - y_1}{\sqrt{(x_j - x_1)^2 + (y_j - y_1)^2 + (z_j - z_1)^2}} \quad (12)$$

$$\frac{\partial \underline{SD}^j}{\partial \hat{z}_{rel}} = \frac{z_j - z_1}{\sqrt{(x_j - x_1)^2 + (y_j - y_1)^2 + (z_j - z_1)^2}}$$

Note that, the GPS satellite position in the equation is known and master LEO position is estimated from the Coarse Positioning solution; hence, the partial differential terms themselves are estimated quantities.

Rearranging (11) and introducing the δ of the parameter for simplicity, which is the difference between the estimated and the measured value (i.e., the correction applied in the iteration);

$$\left| \delta SD^j \right| = \left[\frac{\partial \underline{SD}^j}{\partial \hat{x}_{rel}} \quad \frac{\partial \underline{SD}^j}{\partial \hat{y}_{rel}} \quad \frac{\partial \underline{SD}^j}{\partial \hat{z}_{rel}} \right] \cdot \begin{bmatrix} \delta x_{rel} \\ \delta y_{rel} \\ \delta z_{rel} \end{bmatrix} \quad (13)$$

This equation can be written for all received GPS satellites. Combining all SD equations in matrix form,

$$\delta SD = H \cdot \delta X_{rel} \quad (14)$$

The equation is then solved for δX_{rel} iteratively until convergence.

The solution procedure detailed above is very similar to pseudorange based position determination. However, since the noise term is on the order of a few tens of millimetres, the resulting error is also very small.

2.3 Integer Ambiguity Resolution

There exists a vast literature on integer ambiguity resolution in carrier-phase differencing. Following is strictly not a comprehensive evaluation of all possible methods; it is intended to merely give an idea of the current real-time ambiguity resolution schemes.

Kim & Langley[6] have reviewed the various ambiguity estimation and validation schemes. The ambiguity resolution techniques are classified in three groups:
a) In the measurement domain: This is the simplest technique where L1 and L2 signal frequencies are used in combination to smooth the C/A or P-code pseudoranges, which, in turn are used to estimate the integer ambiguities.
b) In the coordinate domain: This is the oldest resolution technique and involves using only the fractional value of the instantaneous carrier-phase measurement. It provides relatively poor computational efficiency.
c) In the ambiguity domain: This technique attempts to evaluate the resolution in three steps; the float solution, integer ambiguity estimation and the fixed solution.

Although the first two groups are relatively straightforward, there are many different techniques in the third group. Some representative techniques are as follows[6]:

- Least Squares Ambiguity Search Technique (LSAST)
- Fast Ambiguity Resolution Approach (FARA)
- Modified Cholesky decomposition method
- Least-squares Ambiguity Decorrelation Adjustment (LAMBDA)
- Null space method
- Fast Ambiguity Search Filter (FASF)
- Optimal Method for Estimating GPS Ambiguities (OMEGA)

The aforementioned techniques differ from each other in terms of the computational efficiency of the search process[6].

The success of the ambiguity resolution technique decreases with the increasing baseline distance, the presence of multipath as well as increased ionospheric activity[7].

The success rate of the ambiguity resolution will depend on the technique applied, visible GPS satellite geometry, baseline distance, multipath and other environmental effects, particularly the ionosphere. However, validated solutions for 98-100% of the time are reported[7].

For the purposes of this paper, integer ambiguity is assumed resolved perfectly for all the measurements.

2.4 Extended Kalman Filter (EKF) Design

The solver algorithm chosen for the relative orbit determination task is the Extended Kalman Filter. Recursive and real-time, it does not require storage of large amounts of data. It is relatively simple and computationally inexpensive to implement. Furthermore, it takes into account the dynamic model of the system.

The estimated states are the relative state of the slave satellite with respect to the master satellite.

$$\begin{array}{ll} x'_{rel} = x'_2 - x'_1 & \dot{x}'_{rel} = \dot{x}'_2 - \dot{x}'_1 \\ y'_{rel} = y'_2 - y'_1 & \dot{y}'_{rel} = \dot{y}'_2 - \dot{y}'_1 \\ z'_{rel} = z'_2 - z'_1 & \dot{z}'_{rel} = \dot{z}'_2 - \dot{z}'_1 \end{array} \quad (15)$$

Note that x', y', z' are the radial, along-track and cross-track curvilinear coordinates (prime is introduced to prevent confusion with the previous inertial coordinate notation).

Linearised system dynamics model is based on Hill's equations with a simple, circular, two-body reference orbit:

$$\begin{bmatrix} \dot{x}' \\ \dot{y}' \\ \ddot{x}' \\ \ddot{y}' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3\omega^2 & 0 & 0 & 2\omega \\ 0 & 0 & -2\omega & 0 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ \dot{x}' \\ \dot{y}' \end{bmatrix} + \begin{bmatrix} f_{x'} \\ f_{y'} \\ f_{\dot{x}'} \\ f_{\dot{y}'} \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} \dot{z}' \\ \ddot{z}' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} z' \\ \dot{z}' \end{bmatrix} + \begin{bmatrix} f_{z'} \\ f_{\dot{z}'} \end{bmatrix} \quad (17)$$

In Hill's equations, x' and y' are coupled to each other; however, z' is not coupled to the other axes. It is possible to take advantage of this property in a computational economy sense. Setting up a single EKF for the six states will require frequent usage of 6x6 matrices. Instead, it is more advantageous to set up two EKF, one for the coupled x' and y' states and one for the uncoupled z' states. This will require 4x4 and 2x2 matrix operations in each step.

Finally, the outputs of the Relative Positioning module are input to the EKF as measurements, which simplify the measurement equation to a great extent:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{H(x',y')} \begin{bmatrix} x' \\ y' \\ \dot{x}' \\ \dot{y}' \end{bmatrix} + \begin{bmatrix} \upsilon_{x'} \\ \upsilon_{y'} \end{bmatrix} \quad (18)$$

$$z' = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{H(z')} \begin{bmatrix} z' \\ \dot{z}' \end{bmatrix} + \upsilon_{z'} \quad (19)$$

3. Preliminary Results and Analysis

To evaluate the performance of the system delineated above, several tests are conducted using a numerical simulator. It simulates the GPS constellation as well as the formation satellites, however the simulator does not include an orbit propagator and uses the ephemerides data from the commercial Satellite Tool Kit © software. The simulator is still under development and the results should be treated as preliminary.

The simulation comprises the full GPS constellation and two satellites at 600km altitude sun-synchronous orbits. Sampling time is 10 seconds and the total simulation duration is 12 hours. Noise magnitudes for Absolute (Coarse) and Relative (Fine) Positioning modules are assumed to be 5m and 20mm, respectively.

Although assessing the accuracy of the Coarse Positioning algorithm is not one of the principle aims, it is still required to check whether it yields acceptable performance, since the output of this module will be input to the Fine Positioning module. Figure 3 illustrates the performance of Coarse Positioning module under different noise magnitudes. The error corresponding to 5m noise is approximately 5m per axis, which is consistent with what the literature suggests (no Selective Availability)[11]. The figure also shows the clock bias error term (shown as C.DB) in terms of distance.

From the relative orbit point of view, more important is the error in the Relative Positioning module. **Figure 4** presents the relative positioning error with respect to the noise magnitude. Although noise modelling is not highly articulated (single difference measurements corrupted with noise), it demonstrates that cm level accuracy is achievable with CDGPS solution. Relative

Positioning algorithm requires three or four iterations to achieve this result.

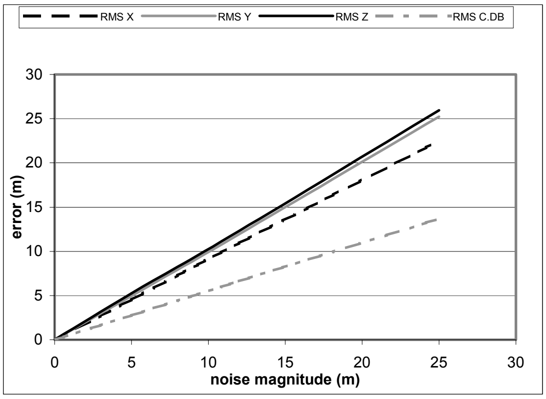

Figure 3 Error vs. Pseudorange Noise Magnitude for the Absolute Positioning Module.

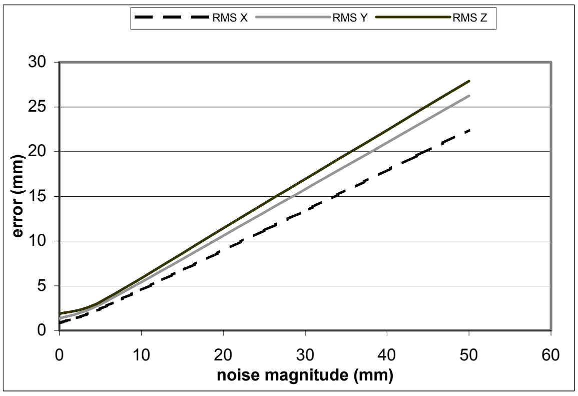

Figure 4 Error vs. SD Noise Magnitude for the Relative Positioning Module

The error in relative position in inertial coordinates after 20mm noise magnitude is shown in Table 1.

Table 1 Position Error for Relative Positioning Module (in ECI Coordinates)

Error	St. Dev.
X Axis Relative Position (mm)	8.88
Y Axis Relative Position (mm)	10.61
Z Axis Relative Position (mm)	11.30
Total 3D Positioning Error (mm)	17.86

It should be noted, however, Relative Positioning algorithm is not very sensitive to the absolute positioning error. To provide a challenge for the code, pseudorange noise has been increased from 5m to 50m per axis. Following the linearity observed in Figure 3, absolute error has also increased to ten times the original absolute positioning error. On the other hand, relative positioning error increased by approximately 22.8%, 24.7% and 65.0% on inertial X, Y and Z axes, respectively. Nevertheless, cm level accuracy is maintained.

Once the Relative Positioning module solves for the relative positions in the inertial frame, the relative position estimates are transformed into the curvilinear along-track, cross-track and radial coordinates for use with Hill's equations in the Extended Kalman Filter (EKF).

Although cm level relative positioning accuracy generated by the least squares solution is sufficient for many applications, relative orbit determination requires not only the positions but also the velocities. The velocities are to be evaluated in the EKF, completing the orbit definition.

The EKF module is tested with external input from STK. The external relative position data is corrupted with noise and input as the measurements. The total 3D magnitude of the error from the Relative Positioning module has been determined as 17.86mm (Table 1). Thus, an equivalent noise of 10.31mm per axis has been applied. The results are presented in Table 2.

Table 2 Position and Velocity Error for EKF Module (in Relative Coordinates)

Error	St. Dev.
Relative Radial Position (mm)	10.35
Relative Along-track Position (mm)	10.10
Relative Cross-track Position (mm)	10.18
Total 3D Relative Error (mm)	17.68
Relative Radial Velocity (mm/sec)	2.15
Relative Along-track Velocity (mm/sec)	2.11
Relative Cross-track Velocity (mm/sec)	2.06

As previously noted, the main aim of the EKF module is to determine the relative velocities accurately. Due to the high confidence in the relative position measurements, EKF does not attempt to improve relative positions further with the system equation. Hence, the relative position estimates are improved only marginally. However, the velocities are determined with very high accuracy, on the order of 2mm/sec. The algorithm does not employ Doppler or any other velocity measurements; nevertheless it provides reasonably accurate velocity information.

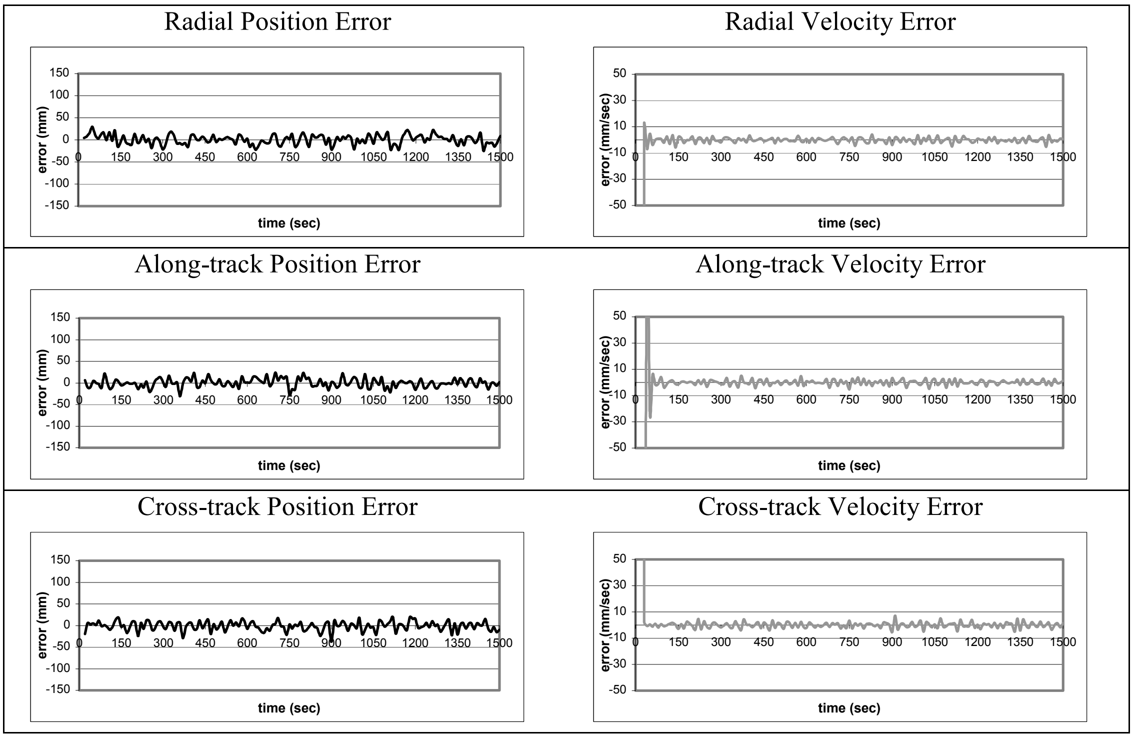

Figure 5 Relative Position and Velocity Error History for the First 1500 Second

There is one other aspect of the proposed algorithm which is not revealed by Table 2. Figure 5 illustrates the behaviour of the error in relative positions and velocity. The relative positions converge instantly and the relative velocities converge at the third timestep at the latest. This is a significant advantage over slower converging systems, since it will be possible to obtain high precision orbit data without delay. Even though GPS signal is lost for a while, once the signal is reacquired, EKF initialisation takes little time and relative orbit information becomes available very quickly. Notice, however, this analysis does not take into account the effects of signal loss on the integer ambiguity resolution scheme. Nonetheless, using the state-of-the-art ambiguity resolution schemes, initialisation delay time for ambiguity-free and accurate carrier-phase information is minimised.

4. Conclusions

This paper is part of the ongoing research to establish and evaluate a relative orbit determination algorithm. It is designed for real time operation onboard the satellite, with the emphasis on convergence and speed of computation. Within this context, a geometrical, CDGPS based approach for precise relative positioning has been outlined. This algorithm is incorporated in the layered system structure, to provide the measurements for the Extended Kalman Filter. The resulting algorithm is essentially independent of the platform and is not limited by the LEO setting, although good GPS coverage is essential.

Preliminary numerical simulation results have been presented and are shown to provide cm level accuracy in relative position and mm/sec level accuracy in relative velocity determination. Favourable convergence characteristics and the simple yet powerful algorithm and precise orbit determination are the promising highlights of the method.

5. Acknowledgements

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