Arc-Minute Attitude Stability on a Nanosatellite: Enabling Stellar Photometry on the Smallest Scale

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ABSTRACT

The precision of astronomy and stellar photometry missions is strongly influenced by the attitude stability of the instrument platform. Recent developments in the miniaturization of star trackers and reaction wheels have opened the possibility of performing precise stellar photometry with nanosatellites. The BRIght Target Explorer (BRITE) mission uses a constellation of six nanosatellites that will photometrically observe the brightest stars in the sky. Each BRITE satellite will use a CCD imager with a 3-cm aperture telescope. The small telescope is capable of making photometric measurements with precision of 0.1%. This photometric precision is in part made possible by reacquiring target stars using the same set of pixels for multiple observations. This reacquisition requirement implies arc-minute attitude stability. To accomplish this requirement the attitude is controlled by an orthogonal set of three reaction wheels, and estimated with a star tracker, developed jointly by Sinclair Interplanetary, Ryerson University's SAIL facility and the Space Flight Laboratory. This paper focuses on the challenges of and solutions to three-axis arc-minute pointing stability on the nanosatellite scale. Special attention is given to the effect of reaction wheel jitter, the practical limitations associated with miniaturized star trackers, and attitude estimation without the use of rate gyros. The solutions presented apply to small satellites in general, including BRITE constellation. The first satellites in BRITE Constellation are scheduled to launch in late 2011.

INTRODUCTION

There is a growing need for high precision attitude control for small spacecraft [1, 2, 3, 4]. As the miniaturization of scientific and commercial instruments continues, small satellites are being utilized to fly missions faster and cheaper. While the size and power needs of these advanced payloads are shrinking, many still have a fundamental requirement for accurate and stable pointing.Achieving a high level of pointing stability is vital to the continued advancement of operational small satellite platforms.

Over the last decade, the attitude determination and control capabilities of small satellites have improved through advances in technology and a shift in design approaches that are tailored towards the levels of risk tolerated at this end of the spacecraft scale spectrum. Despite recent advances, very fine pointing is still a challenging endeavor on the small scale. While it is possible to benefit from the wealth of experience in attitude determination and control from larger spacecraft, there exist challenges and techniques unique to small satellites. This paper covers some of the challenges associated with very fine attitude control and provides some techniques to make these challenges tractable with a real-world example. The BRITE constellation stellar photometery mission, designed at the Space Flight Laboratory, will implement fine pointing at the nanosatellite scale through the application of the practices described in this paper.

BRITE Constellation Overview

The BRIght-star Target Explorer, or BRITE mission consists of a constellation of six nanosatellites which will make photometric observations of some of the apparently brightest stars in the sky. The constellation has been developed by the University of Toronto Institute for Aerospace Studies Space Flight Laboratory (UTIAS-SFL), with contributions from partners in Austria and Poland. BRITE is a complementary mission to the MOST mission**,** launched in 2003[5],but will focus on the most massive of stars, those with a visual magnitude of +3.5 or brighter (286 stars). These stars are also the most luminous and massive stars, with typical life cycles a thousand times shorter than solar-type starsand are of interest for their role in producing the heavier elements in the universe. The byproducts of these stars contribute the interstellar medium and enrich it with heavier ions. Studying these stars will test and expand our knowledge of how their heavy ion enrichment would have been crucial to the evolution of the early universe.

The science goal of the BRITE mission is to measure variations in the brightness of stars and to use those variations to deduce the internal behaviour of those stars, a science known as asteroseismology. The BRITE missions aim to measure the brightness variations to an accuracy of 0.1%.

Each BRITE satellite is equipped with a wide angle telescope with an approximately 24°x19° field of view. The Charge-Coupled Device (CCD) imager used has an active region of 4008x2672 pixels, which leads to approximately 30 arc-seconds per pixel. With a field of view this large, there will be several regions of interest (ROI) in each image. Up to 15 of these ROIs, will be chosen in advance by the science team for on board processing, with the rest of the image discarded to reduce the data load. The optical design of the instrument is slightly defocused to spread the star's light over a small number of pixels. The resulting point spread function will help avoid undersampling and improve the photometric accuracy.

To meet the ambitious scientific objectives pixel-to-pixel variation on the CCD must be minimized. This is accomplished by holding the attitude stabile to within a few pixels, the point spread function of each target star will be captured by the same set of pixels throughout the observation campaign. To meet the scientific objectives, the centre of the point spread shall be stabilized within 2 pixels RMS, or 1.0'. It is also desired that the image be smeared slightly, to effectively smooth the image on each exposure. This requirement translates to ensuring some attitude motion during imaging, and that that motion be radially symmetric over the course of the exposures.

The BRITE satellites will be some of the smallest astronomy satellites flown to date. The advanced attitude control employed by the satellite, just a few years ago would have required more power and volume than a nanosatellite of this size could have provided. Currently there are six BRITE satellites planned, with UniBRITE and BRITE Austria prepared for launch later this year. Flight assembly has been completed on both these satellites and they are currently undergoing system level testing. Figure 1 shows UniBRITE preparing for thermal vacuum testing at the David Florida Laboratory.

The BRITE satellites are based on the successful Generic Nanosatellite Bus (GNB) developed at the Space Flight Laboratory. The inaugural flight of the GNB was the AISSat-1 mission, which had launched in July 2010, providing one-year of flight heritage to date. The GNB has a 20cm cubic profile, a 6.5-kg mass and a modular payload bay. Shown in Figure 2 is an

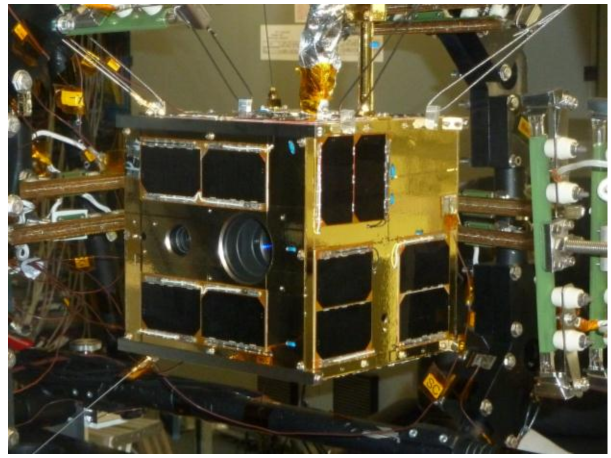

Figure 1: UniBRITEat system level TVAC at DFL

exploded view of the GNB platform with the BRITE telescope and star tracker berthed in the payload bay. The attitude subsystem of the GNB is tailored to be modular in nature, and is able to accept a range of actuators and sensors. This modular design can meet the needs of a very wide variety of missions, ranging from those only requiring coarse knowledge and control through permanent magnets and hysteresis rods to high-performance systems requiring arc-minute level control using low-power star trackers and miniature reaction wheels. Much of this technology has been flight proven by over combined seven-years of successful operation aboard SFL's CanX-2 [6], NTS[7]and AISSat-1[8].

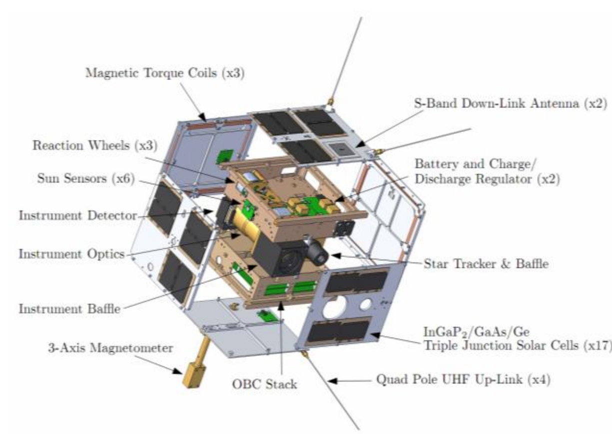

Figure 2: BRITE Satellite Exploded View

In order to minimize complexity and cost, redundancy on the bus is implemented only in areas where prudent to improve reliability of the platform. For instance, the bus includes two on-board low power computers which are configured in such a way that either computer can perform the function of the other in the event of a failure. Further, the spacecraft features redundancy in energy storage and regulation with two lithium-ion batteries and peak-power tracking battery charge & discharge regulators. These batteries store power generated by advanced triple junction solar cells, which have a peak power generation of 11-watts.

SMALL SCALE ATTITUDE DETERMINATION AND CONTROL

The attitude determination and control subsystem (ADCS) of a satellite is responsible for estimating and controlling the orientation of the spacecraft. Recent advancements in miniaturization and on orbit processing power have opened the door to highly capable attitude systems on board nanosatellites. Several trailblazers have proven the possibility of three-axis pointing on such a small platform [6, 8, 9]. Improved attitude capabilities will enable the next generation of nanosatellite missions. However, attitude control for small satellites has several additional challenges when compared to larger satellites. While newly available hardware has opened the door to high performance attitude control for nanosatellites, the attitude estimation and control techniques need to similarly evolve. The small moments of inertia typical of small satellites make them very susceptible to disturbance torques that could quickly move the satellite's attitude off target. Focused attitude state estimation and aggressive control techniques are often needed to counteract the inherent mobility of such a small craft.

The overall pointing performance of a satellite is a function of the quality of the attitude determination and the precision of attitude control and actuation. The next sections will breakdown attitude performance into determination, disturbance environment, actuator performance and fine control schemes. Each section will describe some of the challenges unique to small satellites and offer solutions shown to be effective for BRITE constellation.

ATTITUDE DETERMINATION

Attitude estimation performance is proportional to the accuracy of the sensor suite. While sun sensors, magnetometers and horizon sensors suitable for nanosatellites exist, their overall accuracy is limited. Sun sensors for satellite missions are typically no better than 0.1° due to the need for wide-angle fields of view [10] and provide no orientation knowledge about the sun-vector. Magnetometers as attitude sensors are only as accurate as the knowledge of the geomagnetic field, which changes constantly in response to space weather. These limitations often lead to the requirement for a star tracker. Star trackers are capable of very fine precision attitude determination, independent of orbital determination and changing ephemeris. Until recently, there were no star trackers available that were practical for nanosatellites. Existing units for larger spacecraft had volume and power requirements, as well as cost, to beyond what most nanosatellite programs could afford. In recent years, several star trackers designed for small satellites have come on the market. These include the ComTech Aero Astro Miniature Star Tracker (MST) [11], and the Sinclair-SAIL-SFL Star Tracker (S3S) [12]. These and other similar sensors are enabling very fine pointing precision on nanosatellite scales.

Placement of a star tracker can be critical to mission success. Most star trackers require a view of at least three stars unpolluted by stray light from the Sun, Moon, Earth or even some planets. In addition, the unsymmetrical accuracy of the star tracker must be considered when deciding placement. Star trackers can have relatively poor resolution in resolving roll about their boresight. This error can be up to an order of magnitude greater compared to the two transverse axes. For astronomy missions, such as the BRITE satellites, the best star tracker placement is typically co-aligned with the telescope boresight. The telescope has similar stray light limitations as a star tracker and the same poor resolution about its boresight. Co-alignment reduces the number of satellite faces sensitive to stray light and matches the lowest accuracy axis, the roll axis, with the axis with the lowest requirement.

Star Tracker Performance

The first two BRITE satellites to launch, UniBRITE and BRITE Austria, will use the MST as their primary attitude sensor while subsequent BRITE satellites will make use of the newer S3S tracker for attitude determination. The update rate ofthe MST limits the cadence of the attitude determination and control cycle to 0.5Hz. This slow cadence will have a significant impact on the overall attitude performance, but it will be shown that this can be mitigated through the use of high bandwidth attitude filter and controller.

In order to model the impact of the MST on the BRITE mission, field tests were conducted to characterize performance metrics such as measurement noise, drop-out frequency, and stray-light sensitivity. Specifically, these tests involved recording the star tracker's output as it tracked a night sky and comparing that to the expected results determined from ephemeris data. As shown in Figure 3, the star tracker's performance was determined to be 23" RMS in each of the transverse axes and 114" RMS in roll about the boresight[13]. Due to co-alignment between the instrument and the star tracker, pitch and yaw error from the star tracker maps directly to pointing error of the imager's boresight. The error in roll about the boresight is less sensitive since it only maps to slight skewing of the image, where the effect is most pronounced at the outer corners of the CCD. However, since BRITE will be imaging stars in all parts of its field of view, the pointing budget must account for the worst case region of interest. The outer corner of the CCD field of view is 12° from the centre. From the spherical cosine law,the apparent transverse motion of the target star at the outer corner due to a roll about the centre of 114" is 24.9".

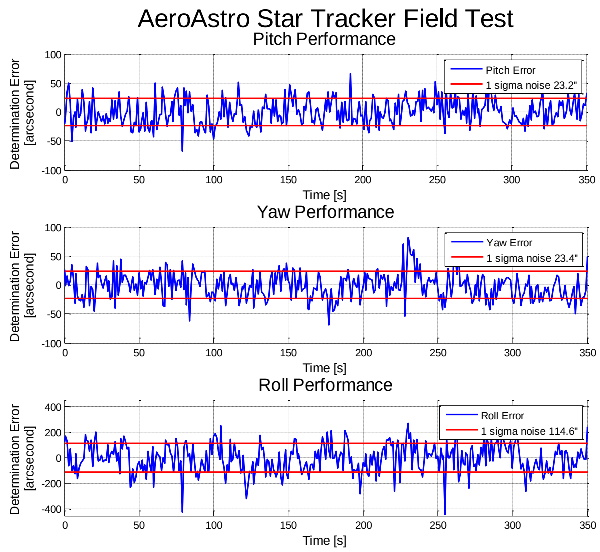

Figure 3: MST Field Test Data

Attitude Rate Determination

In addition to attitude position determination, the attitude rates must be estimated for use in the attitude controller. It will be shown below that poor attitude rate estimates can have a significant impact on pointing performance. Attitude rate sensors are often used to measure the attitude rates of a satellite, even in the Nanosatellite class. For missions with fine pointing requirements, and especially for missions with targets fixed in the inertial frame like astronomy missions, the attitude rates involved are very small. For an astronomy mission with arc-minute stability, such as BRITE, the angular rates are typically only 10's of arc-seconds per second. An accurate measurement of rates of this magnitude requires rate sensors and signal processing with mechanical, power and cost requirements that can exceed the envelope of many nanosatellite missions. Estimating the satellite's body rates from a single star tracker is possible when using an appropriate state estimator, such as an Extended Kalman Filter (EKF). The details of an EKF are out of the scope of this paper, however, it is sufficient to say that the EKF will propagate the attitude rate estimation based on the estimated control and disturbance torques, and then correct this estimate by comparing it to the finite difference between sequential attitude position measurements from the star tracker. From this we can see that the quality of the body rate estimation can be derived from the quality of the plant dynamics model implemented and the noise from the star tracker. Based on the MST field tests, the noise in the star tracker is much greater than the uncertainty in the plant model, and therefore the star tracker updates are only able to correct for low frequency bias drifts in the rate estimation. Note, the S3S star tracker estimates rates in addition to the inertial-to-tracker frame quaternion, therefore rate estimation performance on BRITE spacecraft with the S3S star tracker is expected to be better than those with the MST star tracker.

The change in the attitude body rates due to applied torques active over a single control frame can be approximated as

$$\boldsymbol{\omega}_{k+1} = \int_{t_o} \mathbf{I}^{-1}\boldsymbol{\tau}\cdot dt + \boldsymbol{\omega}_k \qquad (1)$$

where $\boldsymbol{\omega}_k$ is the angular body rate vector at time k, $\mathbf{I}$ is the moment of inertia tensor, $\boldsymbol{\tau}$ is the sum of the control and disturbance torques applied to the spacecraft and t_o is the time the torque is applied, typically the time between control frames. From this relation we can estimate the uncertainty in the attitude rate estimate, $\Delta\boldsymbol{\omega}$, from the uncertainty in the moment of inertia, $\Delta\mathbf{I}$, and applied torque, $\Delta\boldsymbol{\tau}$. The uncertainty of the derived quantity is related to its sensitivity of the measured quantities. This sensitivity is computed by taking the partial derivative of (1) with respect to the uncertainty sources, τ and I. Since we are only interested in approximating the degree of uncertainty, we can further simplify by examining a single axis case, and find that the uncertainty can be expressed as

$$\Delta\omega = \frac{t_o}{I}\Delta\tau + \frac{\tau \cdot t_o}{I^2}\Delta I \qquad (2)$$

Where the derivatives of the base quantities are the uncertainties:

$$\frac{d}{d\tau}\tau \equiv \Delta\tau \qquad (3)$$

$$\frac{d}{dI} I \equiv \Delta I \quad (4)$$

The moment of inertia of the satellite is typically estimated from the mechanical design. The accuracy of this is limited by the knowledge of each component. Assumptions and allocations must be made for components that are difficult to model, such as the wiring harness. It is possible to measure the satellite's moment of inertia through ground testing. This is done by placing the satellite on a horizontal pendulum and measuring the frequency of oscillation after perturbing the setup. This frequency is proportional to the moment of inertia about the axis of rotation. When using a calibrated horizontal pendulum with the axis of spin along the principle axes, the moment of inertia can be measured to within 0.2%, however even a simple device can measure the inertia to within a few percent. Techniques also exist to determine the moment of inertia on orbit based on simple test maneuvers [14].

It will be shown that for the BRITE missions, the sum of the torque uncertainty is 1.4μNm. The typical commanded torque is on the order of 8μNm. Based on the estimated moment of inertia and the controller update rate of 2.0s the error in attitude rate estimate is expected to be 15.6"/s from the torque uncertainty and 1.8"/s from the moment of inertia uncertainty, for a total error of 17.4"/s.

Attitude State Estimation

An attitude state estimator is used to filter some of the noise from the star tracker, improving the attitude determination from the raw star tracker measurements. A stiffer filter would provide smoother attitude estimates, however, nanosatellites tend to be very susceptible to small disturbances, which will cause the satellite to accelerate. For the BRITE missions specifically, some angular velocity during observations is desired to soften the stellar point spread function. The bandwidth of the estimator must be high enough to capture this induced motion. Otherwise, the additional phase lag in the estimation could have a detrimental impact on the overall pointing performance. This bandwidth requirement severely limits how aggressively the state estimation can be filtered. A state estimator that is more responsive to drifts caused by disturbance torques must necessarily trust the star tracker measurements more heavily, causing the noise from the star tracker to have a large impact the overall attitude performance.

It is possible to more aggressively filter the noisier roll measurements about the star tracker's boresight. The star tracker outputs a quaternion measurement which describes the rotation between the inertial frame to star tracker frame, and is fed into the attitude state estimator. To account for the additional roll noise would require continuously adjusting the EKF filter parameters as the boresight axis moves in the inertial frame, however, this would be computationally intensive. Instead the unsymmetrical filter parameters are applied to the angular rate estimates, which can be aligned with the star tracker's reference frame. The estimation of the angular rate about the star tracker boresight is set to trust the internal propagation more heavily so that the magnitude of the body rate estimation error is approximately equal in all axes.

BRITE Attitude Determination Performance

The performance of the BRITE attitude state estimator is characterized using SFL's high-fidelity simulator environment. The position and rate estimation performance in terms of estimated attitude, actual attitude, and the raw star tracker measurements, converted into the roll, pitch and yaw about the target, when using the MST, are shown in Figure 4 and Figure 5. The attitude filter has improved the attitude estimates about the boresight to an RMS value of 75" from the raw MST outputs of 114" RMS. The estimation of the transverse axes are limited by the rate estimation performance, and remain only as good as the MST raw output of 23" RMS. The angular body rates are accurate to 16.5"/s RMS about the star tracker boresight and 13.5"/s RMS about each of the transverse axes. This is slightly better than the predicted accuracy based on the state propagation uncertainty, indicating that the attitude filter is performing well.

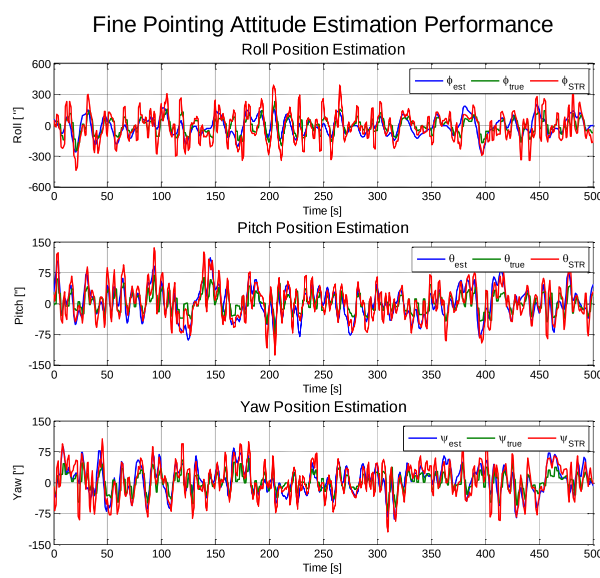

Figure 4: Attitude Estimation Performance for BRITE Satellites using MST

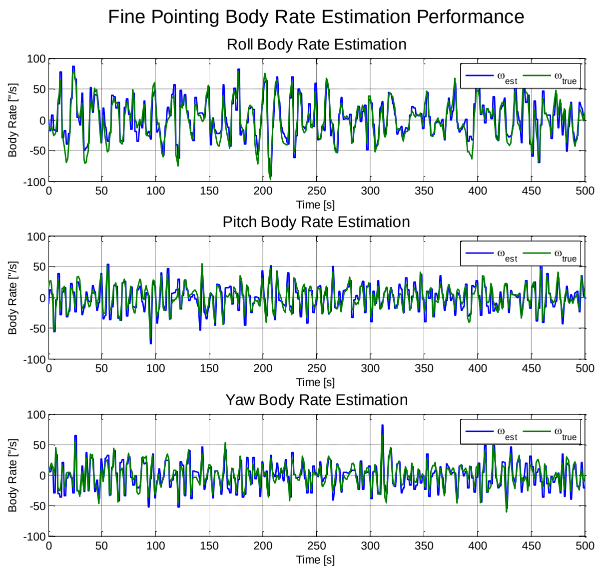

Figure 5: Attitude Rate Estimation Performance for BRITE Satellites using MST

DISTURBANCE REJECTION

When the stabilization requirements are so close to the limit of the estimator's performance, as they often are when pushing the limits of technology, it is insufficient to correct for disturbances after they have affected the attitude of the satellite. Once the disturbances have accelerated the satellite sufficiently to be detectible by the state estimator, any action the attitude control could take may be too late to prevent the attitude from drifting out of the required bounds. This is especially true if the controller update rate is slow, as it is for the BRITE missions. Instead what is suggested is estimation of the disturbance environment and having the controller act on these disturbances a priori. The difficulty then lies in accurately estimating the disturbances.

Disturbance torques applied to the satellite will act to accelerate the satellite's attitude away from the target until the attitude controller has a chance to correct them on the subsequent control cycle. The angular distance traveled in that time is:

$$\phi_i = \iint_{t_O} \frac{\Delta\tau}{I} d^2 t = \frac{\Delta\tau}{2I} t_O^2 \tag{5}$$

Environmental Disturbance Torque Estimation

For a cubic nanosatellite, such as the BRITE satellites, the moment of inertia is very symmetric, and the centre of mass is very near the centre of volume. This makes most of the typical environmental disturbances, specifically gravity gradient, aerodynamic and solar radiation pressure, very small. These are still estimated, and applied to the state estimation filter, but they are typically on the order of 10^{-9}Nm and can be consider negligible. The torque caused by the interaction between the satellite and the geomagnetic field, though, can be significant as the residual magnetic dipole of the spacecraft can be considerable. The primary sources of this dipole come from the permanent magnets in the reaction wheels and the current flowing through the satellite's electronics. Considerable effort is spent on unit and system level testing to determine the satellite's dipole. However this dipole is dynamic in nature as it varies with operation mode as well as reaction wheel speed and acceleration. This variability makes it very difficult to accurately determine the dipole over all cases of interest. Since the dipole is expected to vary, as will the geomagnetic field, it is necessary to perform this estimation in near real time, on orbit. It is not expected that the dipole will change rapidly; rather a slow change over the course of minutes is expected. A solution is to implement a controller that includes an integrator term to zero the steady state error. Careful selection of the integral gain will allow this integrator to converge on and continue to track the net disturbance torque. While this gain selection may not be the best for zeroing the steady state error, having it estimate the external disturbance is worth the small diversion from optimal gain selection. This converged integrator term is then used by the state estimator as the model for the external disturbances. The results of numerical simulations, shown in Figure 6, show that the disturbance estimation converges to the true disturbance quickly, and continues to track the changing disturbance to within 0.1μNm RMS. The state estimator for the BRITE missions is still bandwidth limited, and thus it is crucial that any secular disturbances that are not modeled be minimized. If there are constant disturbances corrected by the integral control but not applied to the state estimator, the rate estimate will be biased. This bias in the rate estimate will translate to a bias in the overall pointing performance.

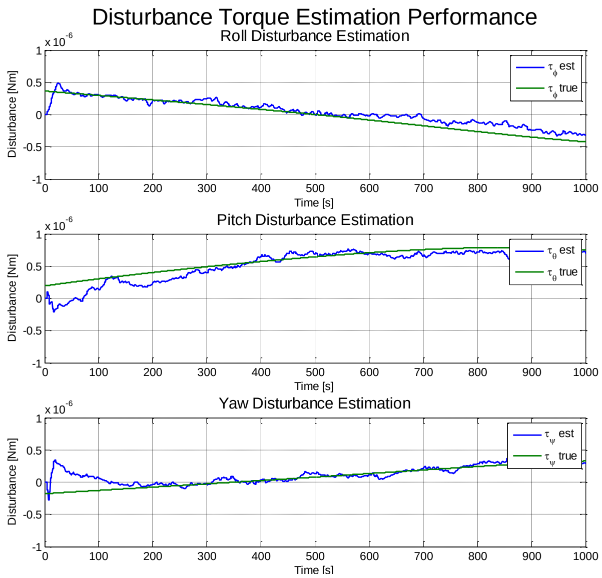

Figure 6: Disturbance Torque Estimation

Internal Disturbance Torque Estimation

Another major source of disturbance on the satellite comes from the cross-coupling torque caused by the angular momentum stored in the three orthogonal reaction wheels. Euler's equation written in the satellite's body frame states

$$\mathbf{I}\dot{\boldsymbol{\omega}} + \dot{\mathbf{h}}_W + \boldsymbol{\omega}^{\times}\left(\mathbf{I}\boldsymbol{\omega} + \mathbf{h}_W\right) = \mathbf{g} \tag{6}$$

where $\mathbf{I}\boldsymbol{\omega}$ is the angular momentum of the satellite's structure, $\mathbf{h}_W$ is the angular momentum stored in the reaction wheels and $\mathbf{g}$ is the total of the external torques. The reaction wheel stiffening term, $\boldsymbol{\omega}^{\times}\mathbf{h}_W$, can be very significant, often dominating the overall dynamics. Reaction wheels tend to perform better with some bias momentum. The Sinclair-SFL reaction wheels are kept above 4mNms to avoid wheel jitter near zero rotor speed. When the attitude has fully converged to the fine pointing operations, the body rates of the BRITE satellite are on the order of 50"/s or less. With this rate, and the default bias momentum, the satellite experiences a disturbance torque on the order of 1.75μNm. This is an order of magnitude larger than the environmental disturbances acting on the BRITE spacecraft and, if left uncorrected, would drive the attitude performance out of the required stability bound. The correction involves adding the estimate of this torque to the plant dynamics model in the state estimator and adding this estimate to the control effort as a feed-forward term. Since the body rate estimation is not perfect, the feed-forward correction has some error associated with it. To estimate the magnitude of this uncertainty we take the derivative of the cross term from (6) with respect to the significant sources of uncertainty. The reaction wheel angular momentum as well as the spacecraft's moment of inertial is well known compared to uncertainty in the angular body rates estimate. Neglecting the terms with low uncertainty the magnitude of this error can be estimated by:

$$\Delta\boldsymbol{\tau}_{FF} \approx \Delta\boldsymbol{\omega}^{\times}(\mathbf{h}_W + \mathbf{I}\boldsymbol{\omega}) + \boldsymbol{\omega}^{\times}\mathbf{I}\Delta\boldsymbol{\omega} \tag{7}$$

From the previous estimation and numerical simulation, it was shown that the body rate estimate is accurate to within 16.5"/s RMS about the worst-case axis. This error results in an inaccuracy estimation of the feed-forward correction of 0.3μNm RMS, or 15% of the correction, which is a large improvement over no correction. This disturbance acts on three orthogonal axes, which can be accounted for by taking the Euclidean norm of this estimate applied to all three axes.

ATTITUDE ACTUATORS

Fine attitude control requires that the attitude actuators are capable of delivering the required control authority with sufficient resolution and sufficiently free of additional disturbance. Reaction wheels are the most common actuator choice for precision pointing on a nanosatellite. Magnetic control is unable to control any roll about the geomagnetic field lines and reaction jets tend to be larger than is practical for small satellite attitude actuation.

The BRITE satellites will make use of the Sinclair-SFL 30mNms reaction wheels. These highly capable reaction wheels have over three years of flight heritage onboard the CanX-2 satellite, one year of heritage aboard the AISSat-1 satellite and continue to operate without incident. The reaction wheels are capable of storing more than 30mNms of angular momentum and delivering torques up to 2mNm.

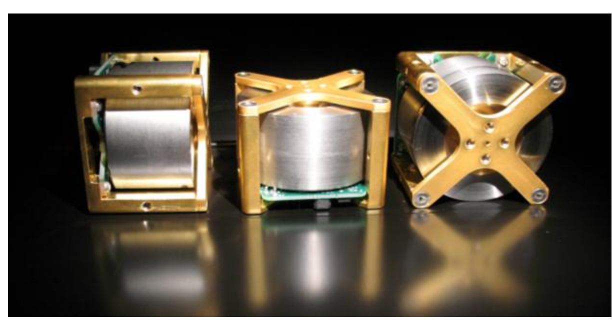

Figure 7: Sinclair-SFL Reaction Wheels [15]

Reaction wheel jitter is a common attitude disturbance that must be characterized and minimized when developing precision attitude control subsystems. Reaction wheel jitter has two chief sources, radial forces caused by rotor imbalance and torque jitter caused by non-idealities in the reaction wheel's driver.

We know from the equations of motion that a torque applied to the spacecraft will result in acceleration. To see what frequency of jitter-induced noise will most affect the satellite, we are interested in transforming this into the frequency domain. If we assume all initial conditions are zero, the Fourier transform of the equation of motion for a single axis will be

$$\mathcal{F}\left\{I\ddot{\theta}(t)\right\} = \mathcal{F}\left\{g(t)\right\} \tag{8}$$

$$-I\omega^2\theta(\omega) = g(\omega) \tag{9}$$

$$\theta(\omega) = \frac{g(\omega)}{-I\omega^2} \tag{10}$$

where θ is the angular wander due to the disturbance g. Note that here ω is a measure of frequency, specifically $2\pi f$, not the angular velocity. From (8) we can see that the magnitude of the pointing error drops off with the square of the frequency at which the disturbance is applied. This indicates that very high frequency disturbances have negligible impact on the attitude of the satellite.

Any imbalance in the spinning reaction wheels will impart a disturbance onto the satellite. This disturbance will by oscillatory, however it will move the satellite's attitude at high frequencies. The mass imbalance in the rotor of the reaction wheels will cause a centrifugal force when the wheel is spinning. The magnitude of the force due to imbalance is given by [16]:

$$\mathbf{F}_{Wb} = \omega_W^2 \cdot \mathbf{d}_W \tag{11}$$

where $\mathbf{d}_W$ is the direction and magnitude of the mass imbalance in the reaction wheel's frame and ω_W is the reaction wheel rotor angular velocity. If the rotor is offset from the centre of mass of the satellite, this force will result in a torque. The torque resulting from this force is the cross product of the force vector and the vector drawn from the centre of mass to the reaction wheel's rotor:

$$\boldsymbol{\tau}_{Wb} = \sum_w \mathbf{r}_w^{\times}\mathbf{C}_{bW}\mathbf{F}_{Wb} \tag{12}$$

where $\mathbf{C}_{bw}$ is the rotation between the reaction wheel's frame and the satellite's body frame, $\mathbf{r}_w$ is the position vector between the satellites centre of mass and each of the reaction wheel's centre of rotation, expressed in the body frame. The direction of the mass imbalance rotates about the reaction wheel's rotor with each rotation of the wheel according to

$$\mathbf{d}_W = d_W \cdot \begin{bmatrix} 0 \\ \sin(\omega_W t + \theta_d) \\ \cos(\omega_W t + \theta_d) \end{bmatrix} \tag{13}$$

where d_W is the magnitude of the imbalance, θ_d is the angular position of the imbalance, and t is the time the wheel has been spinning and $\mathbf{d}_W$ is expressed in the reaction wheel frame, described by the axis of rotation followed by the two transverse axes. What (13) indicates is that the disturbance in the two transverse axes oscillates with the frequency of the reaction wheel's rotation. Because of this we can combine (10)through (13) to estimate the impact on the pointing cause by the imbalance of a single wheel:

$$\theta_{Wb} \approx \frac{r_w \cdot d_W}{I} \tag{14}$$

It is interesting to note that the dependency on the reaction wheel speed has cancelled out. The impact of the disturbance torque will approach zero asthe wheel speed approaches zero, but is well approximated by (14) for non-zero speeds. To account for multiple wheels mounted orthogonally, we take the Euclidean of the three contributions. Note, the Sinclair-SFL reaction wheel used on BRITE are individually balanced, thus minimizing the impact of this disturbance. All the reaction wheels to be used on BRITE satellites have an imbalance of less than $0.51\text{x}10^{-6}$kg m. This results in an additional pointing error of 0.8" or less from the reaction wheel imbalance.

The other source of reaction wheel jitter comes from the reaction wheel imperfectly tracking the commanded torque command. Shown in Figure 8 is the speed and torque response of the Sinclair-SFL reaction wheel tracking a torque command, captured at 100Hz. While there is some high frequency noise overlaid onto the response, there is very little low frequency noise, just under 0.5μNm RMS. This represents the minimum torque resolution of the reaction wheel and the disturbance felt by the satellite. There are three orthogonal reaction wheels on each BRTIE satellite, so we take the Euclidian norm of three wheels at 0.47μNm each to get the total reaction wheel jitter of 0.81μNm. By applying (5), we can estimate the contribution to the pointing error from the reaction wheel jitter for BRITE spacecraft at 9.3".

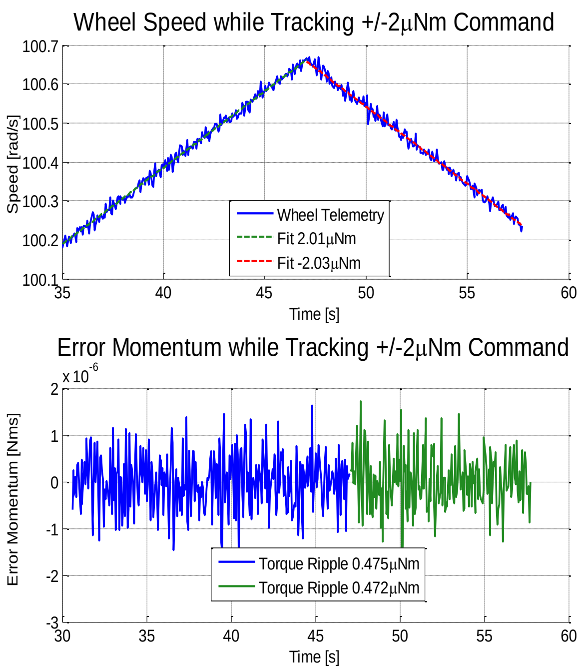

Figure 8: Torque Tracking Performance of Sinclair-SFL Reaction Wheel

PRECISION ATTITUDE CONTROLLER

The objective of the attitude controller is to counteract disturbances while maneuvering and stabilizing the attitude at the target. There are many controllers available to be used to maneuver and stabilize the attitude of the satellite [17]. For simplicity of implementation and analysis, a conventional PID controller was selected for the BRITE attitude controller. This simplistic controller is able to meet the demanding attitude requirements largely as a result of the feed-forward control terms discussed in the previous section. The addition of these feed-forward terms effectively linearizes the system. The reaction wheels can provide substantial control effort, so there is no need to implement a torque optimal controller. The major requirement of the controller is to minimize the pointing error in the face of noisy determination information, while keeping the attitude rates between 20-40"/s.

The control effort calculated by the PID controller with quaternion feedback[18], in the satellites body frame, can be written as:

$$\mathbf{u} = \mathbf{K}_P \boldsymbol{\varepsilon}_e \operatorname{sgn}(\eta_e) + \mathbf{K}_D \boldsymbol{\omega} \tag{15}$$

where $\mathbf{K}_P$ and $\mathbf{K}_D$ are the gain matrixes, $\boldsymbol{\varepsilon}_e$ is the vector portion of the quaternion error and η_e is the scalar portion. The sign of the scalar portion of the error is included in the proportional control term to avoid any sign ambiguities that could cause the controller to go the long way around to the target. There is an additional integral control term not shown in (15) since it is considered part of the feed forward disturbance rejection.

The controller gains, $\mathbf{K}_P$ and $\mathbf{K}_D$, are selected to shape the dynamic response of the satellite, and can be selected by [17]:

$$\mathbf{K}_P = 2 \cdot \mathbf{I} \cdot \omega_n^2 \tag{16}$$

and

$$\mathbf{K}_D = 2 \cdot \mathbf{I} \cdot \xi \cdot \omega_n \tag{17}$$

where ξ is the damping ratio and ω_n is the dynamic natural frequency. Choice of the controller's natural frequency and damping involves careful balance of several key factors. The control must be fast enough to correct for movement caused by unmodeled disturbances before they cause the attitude to drift significantly. The controller must also respond to updated state estimates. It is possible to use a slower controller that will effectively filter out more of the estimation noise, however, the phase lag associated with this often results in far worse pointing performance. For the BRITE missions specifically, it is important to have similar pointing and smearing from exposure to exposure. This requires that the pointing error distribution be symmetrical over the course of an exposure, rather than slowly drifting about the intended target. Since the exposures for the BRITE instrument can last from 0.1s to 100s, it is advantageous to have the pointing error appear symmetric in as short of time period as possible.

For a discrete time controller, the angular distance the satellite's attitude moves based on the controller's output in a single control cycle is proportional to the control gain and the initial estimated attitude error. By using the small angle approximation, and assuming zero initial body rates, equation (15) can be combined with (5) to get the angular distance traveled as a result of the control effort:

$$\phi_{traveled} = \frac{\omega_n^2 t_O^2}{2}\phi_e \tag{18}$$

where ϕ_e is the initial estimated angular distance to the target and $\phi_{traveled}$ is the distance traveled in one control frame, t_O. It is inadvisable for the controller to command an angular motion greater than the initial distance:

$$\phi_{traveled} \le \phi_e \tag{19}$$

If (19) is not obeyed, the attitude will consistently overshoot the target, leading to large oscillations and pointing errors. This puts an upper bound on the natural frequency of the controller based on the controller cadence:

$$\omega_n \le \frac{\sqrt{2}}{t_O} \tag{20}$$

Selecting the controller's natural frequency right at this boundary provides the fastest response with minimal overshoot. Correction of any attitude error caused by the unmodeled disturbances can be accomplished with no additional lag, other than from the attitude estimator, if the controller's natural frequency is as large as permitted by (20). The angular error caused by the disturbance torques over a control cycle is:

$$\phi_{dist} = \frac{\tau_{dist} t_O^2}{2 \cdot I} \tag{21}$$

To correct for this pointing error in a time optimal sense, the angular distance traveled as the result of the controller's commands during the next control cycle should be the same.

$$\phi_{traveled} \approx \phi_{dist} \tag{22}$$

Applying (18) to (22) we see that the ideal controller behavior attempts to move the entire distance in a single control frame. From this, the natural frequency is determined from the controller cadence:

$$\omega_n \approx \frac{\sqrt{2}}{t_O} \tag{23}$$

The previous discussion for gain selection is valid for the telescope transverse axes, where absolute pointing precision is most critical. For the roll about the telescope's boresight, the attitude precision is less sensitive. Rather, reducing the body rates about the roll axis has more of an impact on the overall pointing performance. The nonlinear dynamics, specifically the cross torques described in (6), increase in magnitude with increased angular body rates about any axis. The selected speed of the controller dictates the typical body rates of the satellite:

$$\omega \propto \omega_n^2 t_O \cdot \phi_{est} \tag{24}$$

Where ω is the spacecraft's body rates. For the telescope transverse axes, the larger body rates associated with a large ω_n is a tolerable cost for quick response. However, for the roll axis, the star tracker's measurement noise is much larger, meaning that for the same ω_n, the body rates will be much higher. To correct for this, the controller's natural frequency about this axis should be reduced. This has the advantage of adding additional smoothing of the noisier state estimate and reducing the body rates at the cost of larger pointing errors cause by disturbance induced drift.

The controller damping is nominally selected to be fractionally less than critically damped. This reduces the overshoot and angular velocity without slowing the dynamic response significantly. An additional source of disturbance torque comes from the controller acting on the body rate estimates rather than the true body rates. This erroneous torque, referred to as damping error, can be estimated from:

$$\Delta\boldsymbol{\tau}_{\text{control}} = \mathbf{K}_D \Delta\boldsymbol{\omega} \tag{25}$$

where $\Delta\boldsymbol{\omega}$ is the uncertainty in the body rate estimation. The EKF for the BRITE missions typically overestimates the actually body rates, as can be seen in Figure 5; a result of the filter smoothing the estimates. To account for this, the damping ratio used in the controller is reduced proportionally.

BRITE STELLARPHOTOMETRY POINTING PERFORMANCE

The pointing performance of the BRITE satellites is a culmination of the determination and control performances discussed throughout. The various determination and disturbance errors all contribute to the overall pointing error of the satellite. For the BRITE mission the various error sources and their estimated magnitudes are listed in Table 1.

Table 1: Pointing Performance Estimation

Source	Error Type	Magnitude
Star Tracker Pitch Noise	Determination	23.0"
Star Tracker Yaw Noise	Determination	23.0"
Star Tracker Roll Noise	Determination	39.4"*
Reaction Wheel Controller Jitter	Actuator Disturbance	9.3"
Reaction Wheel Imbalance	Actuator Disturbance	0.8"
Environmental Disturbances	Plant Model	1.2"
Damping Error	Determination	12.3"
Feed Forward Error	Determination	6.0"
Combined Error (RSS)	-	**53.8''**

*Since the telescope pointing is less sensitive to roll, the roll error is scaled by what would be seen by the worst-case corner pixel.

The estimate of the total pointing performance from these independent sources is computed from the root sum squared (RSS) of the various components. These error sources are independent, and for the most part act orthogonally to each other, which is modeled by RSS.

Table 1 shows that the majority of the attitude error sources for the BRITE satellites are the result of determination uncertainty. The other major source comes from the performance of the attitude actuators used. Correcting for either of these beyond the feed-forward and rapid controller used requires either a better star tracker or faster controller cycles. Faster estimator cycles could filter the attitude state estimates to a larger extent, rejecting more of the noise from the star tracker. In addition the majority of the error sources listed in Table 1 come from propagating an erroneous or disturbance torque between control frames, as per (5). The error contribution from these sources will reduce with the square of the reduction in time between control cycles. For the first set of BRITE satellites, the MST limits the attitude estimation and control cadence. However subsequent BRITE satellites will make use of the S3S, and will be capable of faster control cadence.

To confirm this pointing performance estimation, in-depth numerical simulations were performed. The simulation included the orbital and attitude dynamics, as well as detailed models of the sensors and actuators. The sensor models take the true state from the dynamics simulation and convert them to outputs similar to what the real sensors would output, including the addition of noise and error. These signals are given to the attitude determination and control flight software, which runs in an emulator, and returns commands to the reaction wheel models to close the loop. The results of these simulations are shown in Figure 9 and Figure 10. Both plots show the same simulation, Figure 9 plots the complete 15 minute observation to show the long term behavior. Figure 10 focuses on a small segment of that to better show the short-term behavior. The overall pointing performance is 51.3" RMS, which is in very good agreement with the performance estimation presented above. What the simulations show, is the speed of the controller to recover from a poor estimate. This is especially evident in the brief sequence plot, where the attitude starts to drift away from the target. Within 1-2 control cycles however, the attitude recovers to within the 30", where it holds until the next outlying star tracker measurement. This responsiveness also creates a reasonably symmetric spread of attitudes about the target, as per the BRITE requirements. With the exception of a few outliers, this symmetry is maintained over time frames as short as 10 seconds.

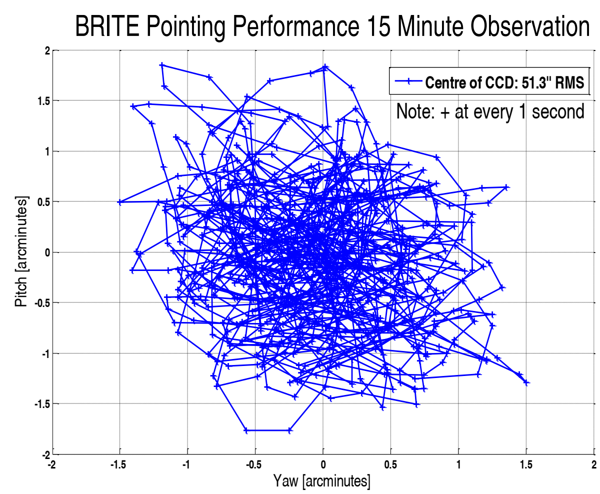

Figure 9: BRITE Telescope Pointing Performance with MST - Full Observation

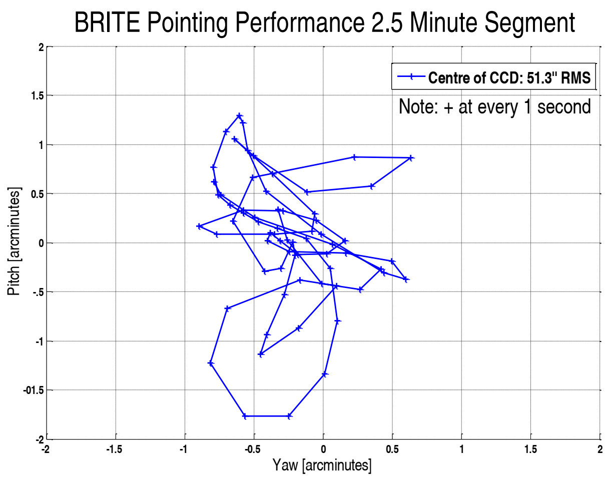

Figure 10: BRITE Telescope Pointing Performance with MST – Observation Segment

CONCLUSIONS

Arc-minute level pointing with small satellites is a challenging but achievable objective. This level of precision is made possible thanks to recent advancements in star tracker technology. The AeroAstro MST and the S3S as well as other high performance sensors tailored to nanosatellites are enabling fine determination for small satellite missions.

Another important technology enabling fine pointing of nanosatellites are reaction wheels. Jitter caused by torque output oscillation and to a lesser extent, rotor imbalance can cause significant pointing degradation. The Sinclair-SFL reaction wheels have been shown to produce very low amounts of jitter, and would be capable of meeting the performance requirements of future sub arc-minute missions.

Small satellites are very susceptible to disturbance torques. With small moments of inertia, a given torque can cause significant angular disturbance. This susceptibility can be mitigated by the use of a high bandwidth state estimator and controller. It is also advisable to estimate the disturbances and apply the appropriate correction to the state estimator and controller a priori. This is especially important for the cross-coupling terms, which are very large and can be modeled with reasonable accuracy. Such a priori correction can significantly reduce the required estimator and controller bandwidth for a given performance requirement.

By harnessing the improvements in star tracker and reaction wheel technologies and adopting the attitude estimation and control techniques discussed above, the BRITE stellar photometry mission is expected to yield arc-second level pointing and stability, and the first is scheduled launch within Q4 2011 to Q1 2012.

ACKNOWLEDGMENTS

The authors would like to acknowledge the entire BRITE team at UTIAS-SFL and around the world, working tirelessly to assemble and integrate the satellites. In addition, we acknowledge the gracious support of The University of Vienna, Graz University of Technology, The Institute of Communications and Radio Frequency Engineering at Vienna University of Technology, Austrian Research Promotion Agency, Ontario Centre for Excellence, Canadian Space Agency, and Sinclair Interplanetary.

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