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The Effect of Negative Energy Shells on Schwarzschild Spacetime and their Penrose Diagrams

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The effect of negative energy shells on Schwarzschild spacetime

and their Penrose diagrams

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Oregon State University

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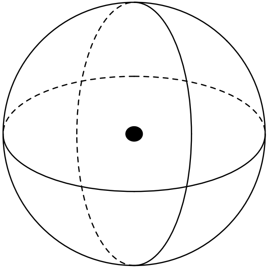
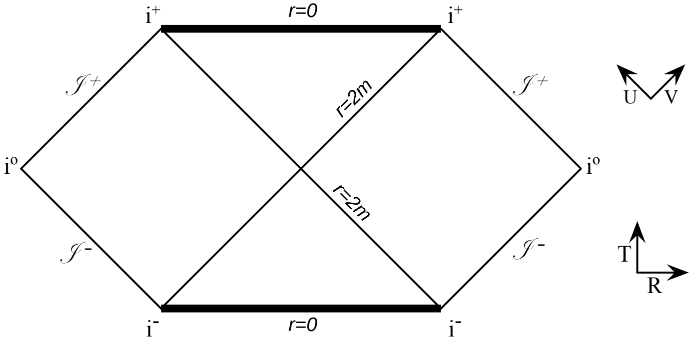
13 July 2009 / Marcel Grossman Meeting

Joining Schwarzschild spacetimes Along energy shell boundaries



Penrose Diagram for Schwarzschild spacetime



Asymptotic Coordinate Transformation (eg. Hartle [2003])

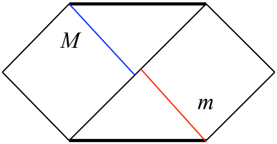
$$U = \tan^{-1}(u) \quad , \quad V = \tan^{-1}(v)$$

Dray and 't Hooft Shells

Metric and Stress-Energy Tensor

- Dray and 't Hooft [1985]
  - Gravitational shock wave in Schwarzschild spacetime.

$$ds^2 = -\frac{32m^3}{r} e^{-\frac{r}{2m}} du(dv + \Theta(u) df) + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$



$$\Theta(u) = \text{step function}$$
$$f = f(\theta, \phi)$$
$$r = r(u, v + \Theta f)$$

Dray and 't Hooft Shells

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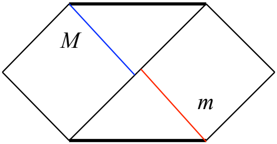

$$\Theta(u) = \text{step function}$$
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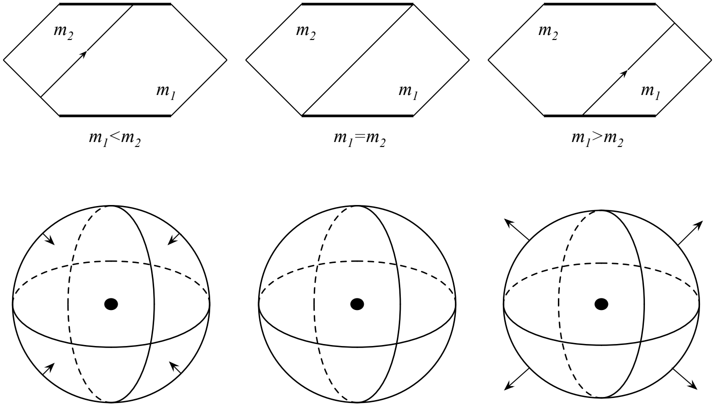
$$r = r(u, v + \Theta f)$$

  - Spherically Symmetric Shell at the horizon

$$T_{uu} = \frac{\kappa}{4\pi e}\delta(u) \Rightarrow f = \kappa$$

Dray and ’t Hooft Shells

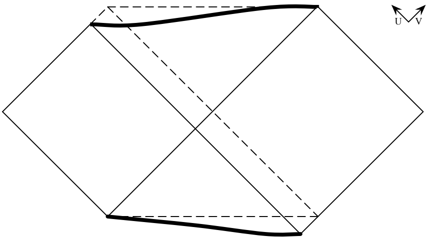


Equal Mass Diagrams

Shifted Diagrams

The shell shifts the null Kruskal-Szekeres v_2 before the transformation

$$U = \tan^{-1}(u_1) \quad , \quad V = \tan^{-1}(\underbrace{v_1 + \kappa}_{v_2})$$



Equal Mass Diagrams

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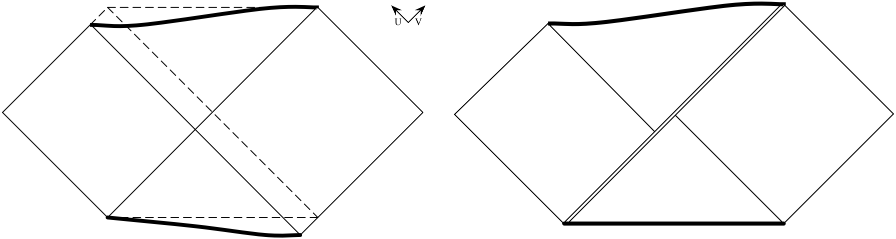

Unequal Mass Diagrams

Boundary Condition for $m_1 \neq m_2$

- Boundary condition

$$\frac{dv_2}{dv_1} = \frac{m_1^3}{u_2'(\alpha) m_2^3} e^{-\frac{r}{2m_1} + \frac{r}{2m_2}} |_{u_1 = \alpha}$$

Unequal Mass Diagrams

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- Assuming $u_2 = u_1 + \alpha\left(\frac{m_2}{m_1} - 1\right) \Rightarrow \quad u_2'(u_1) = \frac{du_2}{du_1} = 1$

Unequal Mass Diagrams

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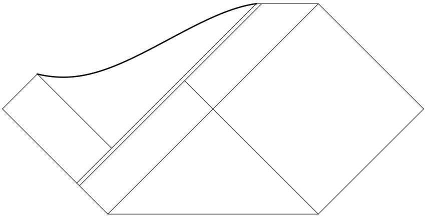
- Assuming $u_2 = u_1 + \alpha \left(\frac{m_2}{m_1} - 1 \right) \Rightarrow \quad u_2'(u_1) = \frac{du_2}{du_1} = 1$
- Solution involving the Lambert W function ($\mathscr{W}$),

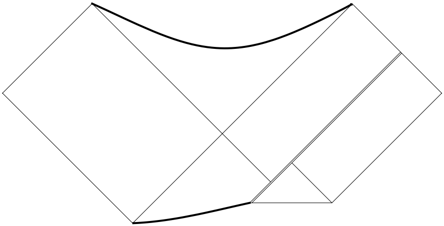
$$v_2(v_1) = \left(m_1 \, \mathscr{W}\!\left(-\frac{v_1 \alpha}{e} \right) + 2\,(m_1 - m_2) \right) \frac{m_1}{\alpha m_2^2 u_2'(\alpha)} e^{\frac{m_1}{m_2} \left(\mathscr{W}\left(-\frac{v_1 \alpha}{e} \right) \right)}$$

Unequal Mass Diagrams

Positive Shells



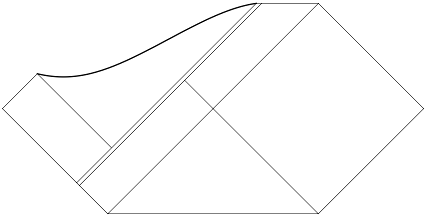
$m_2 = \frac{10}{8} m_1$, $\alpha = \frac{1}{2}$



$m_2 = \frac{8}{10} m_1$, $\alpha = -\frac{3}{2}$

Unequal Mass Diagrams

Positive Shells



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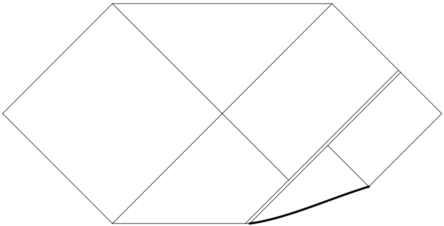
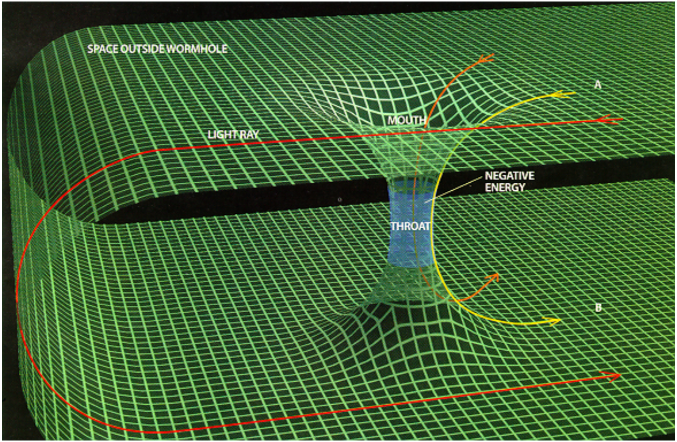


$m_2 = \frac{8}{10} m_1$, $\alpha = -\frac{3}{2}$

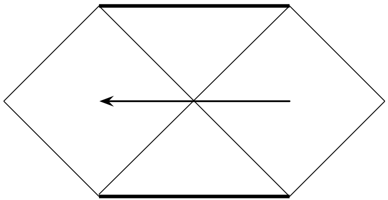
Negative Energy

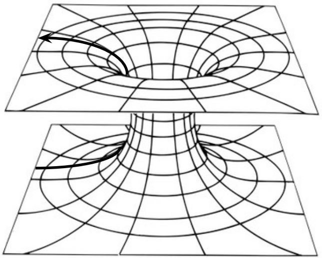


Bondi [1957], Morris and Thorne [1988], Visser [1989]

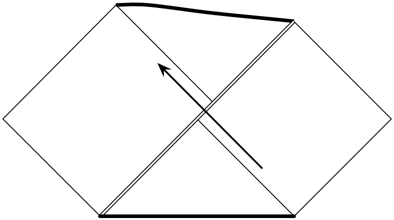
http://en.wikipedia.org/wiki/Wormhole

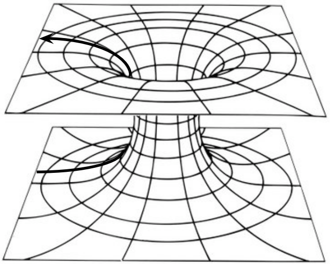
Einstein-Rosen Bridge



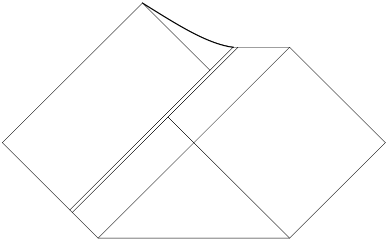


Equal Mass Wormhole

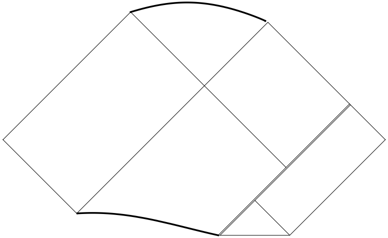




Unequal Mass Wormhole



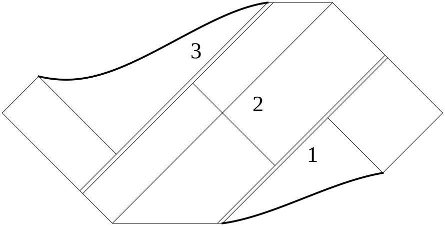
$m_2 = \frac{1}{2} m_1$, $\alpha = \frac{1}{2}$

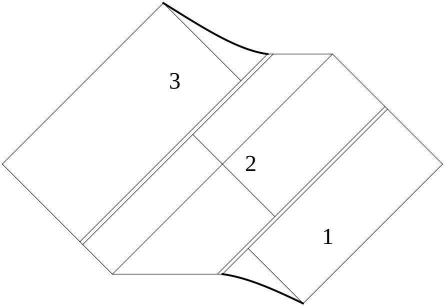


$m_2 = \frac{5}{4} m_1$, $\alpha = -\frac{3}{2}$

Successive Shells

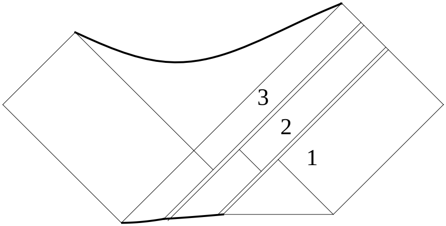
u_2 and v_2 coordinates





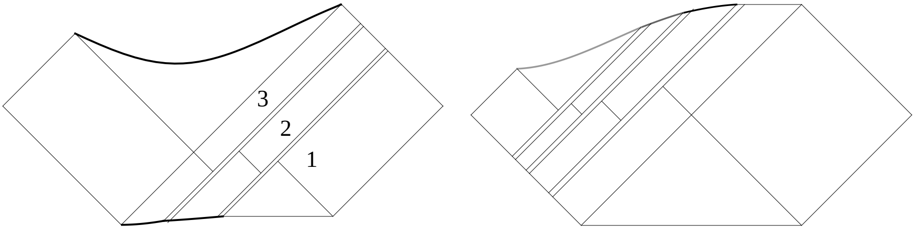
Successive Shells

u_1 and v_1 coordinates



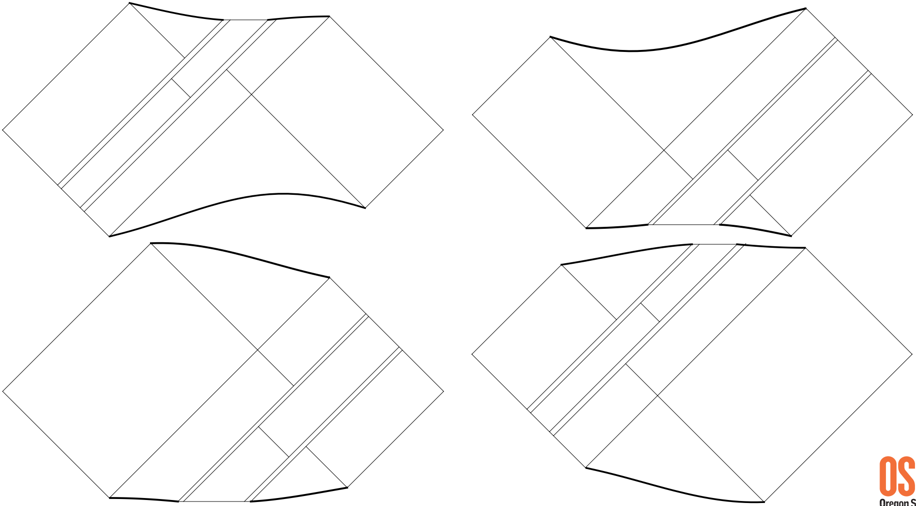
Successive Shells

u_1 and v_1 coordinates



Successive Shells

Equal Mass Shells: Opposite Sign



Successive Shells

Equal Mass Shells: Opposite Sign

$$t = 2m_2 \log\left(-\frac{v_2}{u_2}\right)$$

Successive Shells

Equal Mass Shells: Opposite Sign

$$t = 2m_2 \log\left(-\frac{v_2}{u_2}\right)$$
$$\Delta t = 2m_2 \log\left(\frac{\alpha_1}{\alpha_2}\right)$$

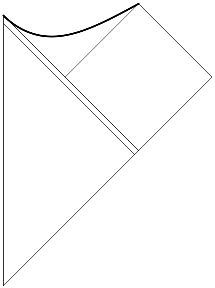
Next Steps: Limiting Cases

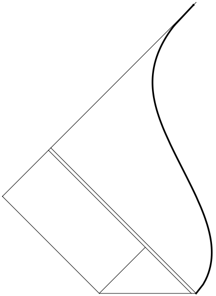
- Mass Limits

$$\lim_{m_2 \to 0} \Rightarrow \mathbb{M}^4 \text{ in region 2}$$

$$\lim_{m_1 \to 0} \Rightarrow \mathbb{M}^4 \text{ in region 1}$$

Next Steps: Limiting Cases





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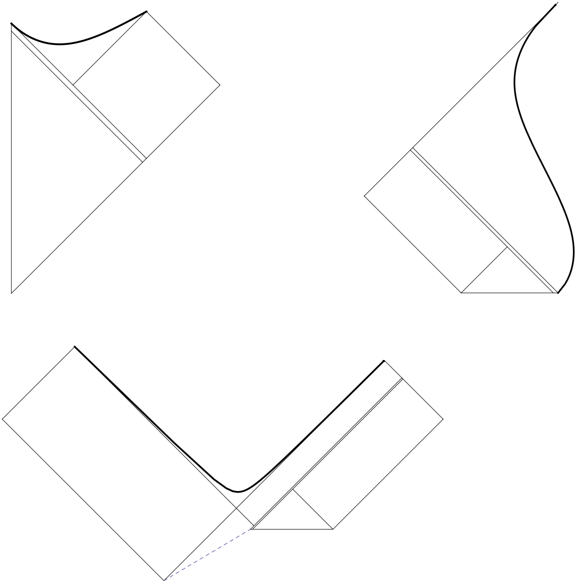

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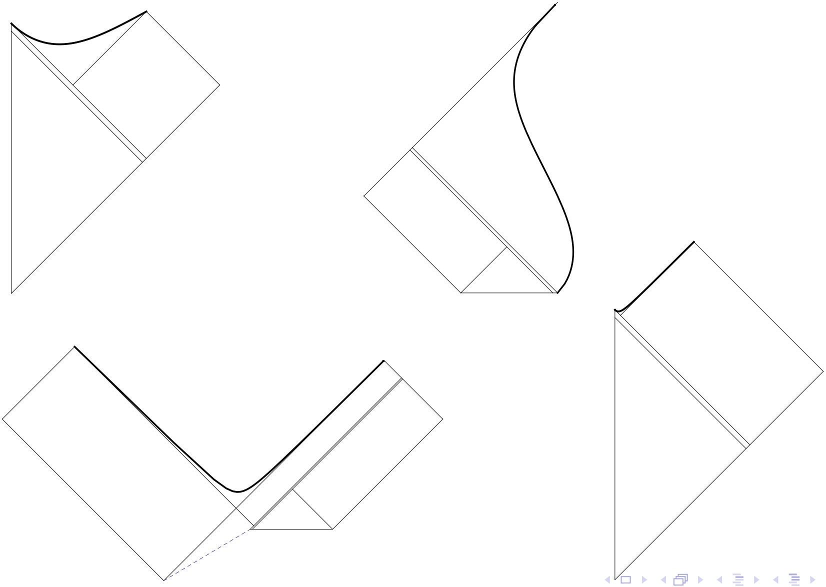
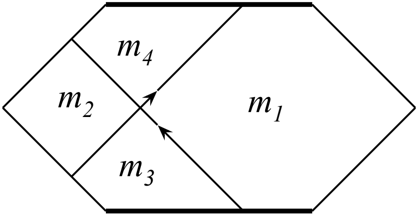
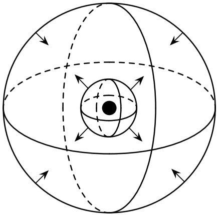


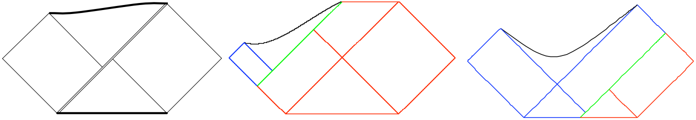
Next Steps: Crossing Shells





Summary

- Shifted Penrose Diagrams
  - Equal Mass Diagrams
  - Unequal Mass Diagrams



- Negative Energy Shells
  - Traversable Wormholes