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The Effect of Negative Energy Shells on Schwarzschild Spacetime and their Penrose Diagrams

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The effect of negative energy shells on Schwarzschild spacetime and their Penrose diagrams

Jeffrey Hazboun¹ and Tevian Dray²

¹Department of Physics
Oregon State University

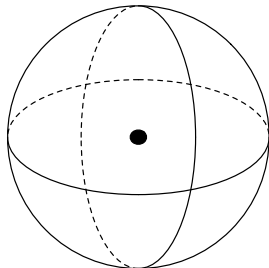
²Department of Mathematics
Oregon State University

13 July 2009 / Marcel Grossman Meeting

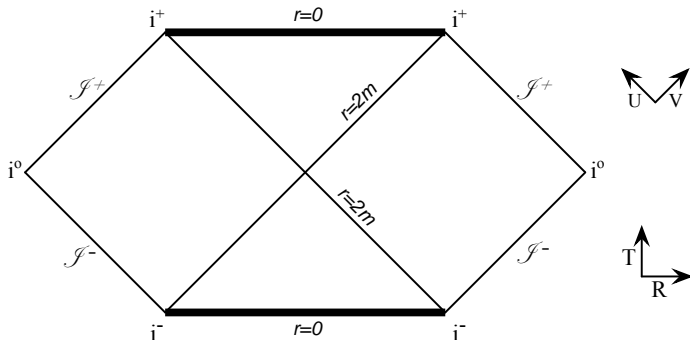
Joining Schwarzschild spacetimes

Along energy shell boundaries

- 1 Previous Work
 - Penrose Diagrams
 - Spherically Symmetric Shells
- 2 Positive Energy Shells
 - Equal Mass Penrose Diagrams
 - Unequal Mass Penrose Diagrams
- 3 Negative Energy Shells
 - Negative Energy
 - Wormholes
- 4 Successive Shells
- 5 Next Steps



Penrose Diagram for Schwarzschild spacetime



Asymptotic Coordinate Transformation (eg. Hartle [2003])

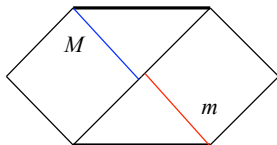
$$U = \tan^{-1}(u) \quad , \quad V = \tan^{-1}(v)$$

Dray and 't Hooft Shells

Metric and Stress-Energy Tensor

- Dray and 't Hooft [1985]
 - Gravitational shock wave in Schwarzschild spacetime.

$$ds^2 = -\frac{32m^3}{r} e^{-\frac{r}{2m}} du(dv + \Theta(u)df) + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$



$\Theta(u) = \text{step function}$

$f = f(\theta, \phi)$

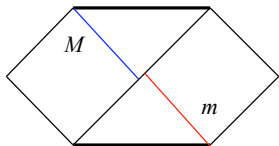
$r = r(u, v + \Theta f)$

Dray and 't Hooft Shells

Metric and Stress-Energy Tensor

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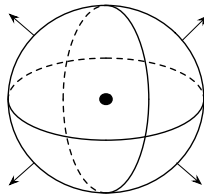
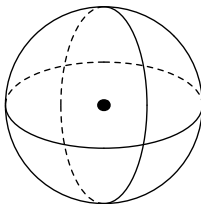
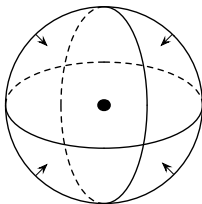
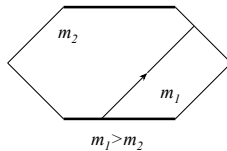
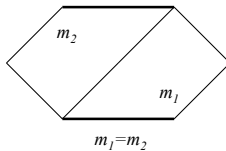
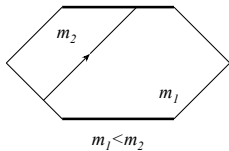
$f = f(\theta, \phi)$

$r = r(u, v + \Theta f)$

- Spherically Symmetric Shell at the horizon

$$T_{uu} = \frac{\kappa}{4\pi e} \delta(u) \Rightarrow f = \kappa$$

Dray and 't Hooft Shells

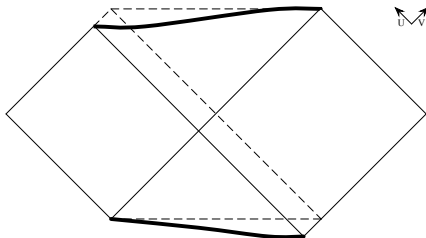


Equal Mass Diagrams

Shifted Diagrams

The shell shifts the null Kruskal-Szekeres
 v_2 before the transformation

$$U = \tan^{-1}(u_1) \quad , \quad V = \tan^{-1}(\underbrace{v_1 + \kappa}_{v_2})$$

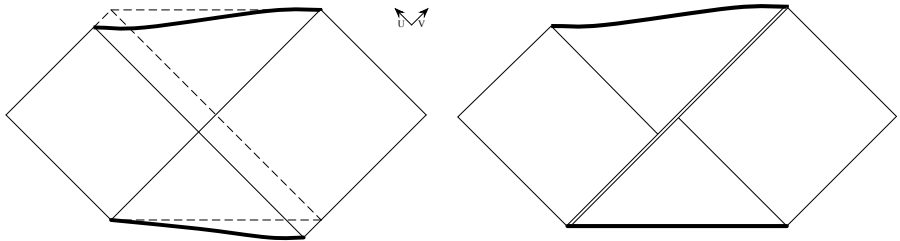


Equal Mass Diagrams

Shifted Diagrams

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Unequal Mass Diagrams

Boundary Condition for $m_1 \neq m_2$

- Boundary condition

$$\frac{dv_2}{dv_1} = \frac{m_1^3}{u_2'(\alpha)m_2^3} e^{-\frac{r}{2m_1} + \frac{r}{2m_2}} \Big|_{u_1=\alpha}$$

Unequal Mass Diagrams

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- Assuming $u_2 = u_1 + \alpha \left(\frac{m_2}{m_1} - 1 \right) \Rightarrow u_2'(u_1) = \frac{du_2}{du_1} = 1$

Unequal Mass Diagrams

Boundary Condition for $m_1 \neq m_2$

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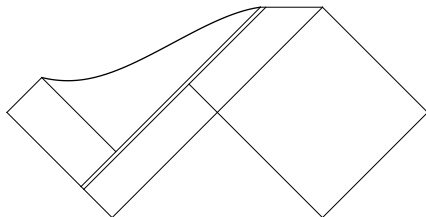
$$\frac{dv_2}{dv_1} = \frac{m_1^3}{u_2'(\alpha)m_2^3} e^{-\frac{r}{2m_1} + \frac{r}{2m_2}} \Big|_{u_1=\alpha}$$

- Assuming $u_2 = u_1 + \alpha \left(\frac{m_2}{m_1} - 1 \right) \Rightarrow u_2'(u_1) = \frac{du_2}{du_1} = 1$
- Solution involving the Lambert W function (\mathcal{W}),

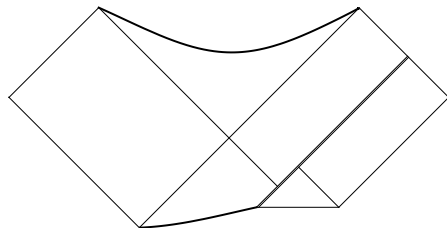
$$v_2(v_1) = \left(m_1 \mathcal{W}\left(-\frac{v_1 \alpha}{e}\right) + 2(m_1 - m_2) \right) \frac{m_1}{\alpha m_2^2 u_2'(\alpha)} e^{\frac{m_1}{m_2} \left(\mathcal{W}\left(-\frac{v_1 \alpha}{e}\right) \right)}$$

Unequal Mass Diagrams

Positive Shells



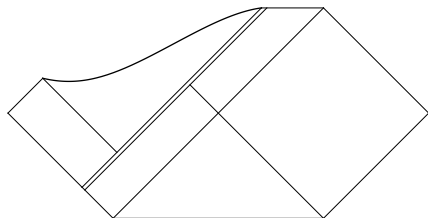
$$m_2 = \frac{10}{8} m_1, \alpha = \frac{1}{2}$$



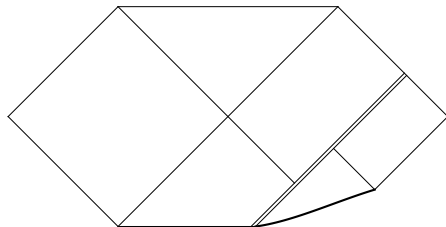
$$m_2 = \frac{8}{10} m_1, \alpha = -\frac{3}{2}$$

Unequal Mass Diagrams

Positive Shells

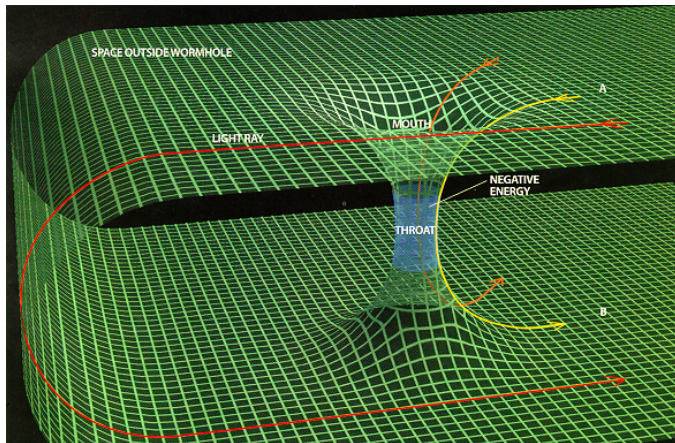


$$m_2 = \frac{10}{8} m_1, \alpha = \frac{1}{2}$$



$$m_2 = \frac{8}{10} m_1, \alpha = -\frac{3}{2}$$

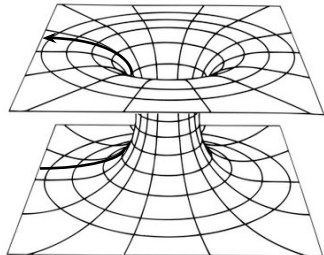
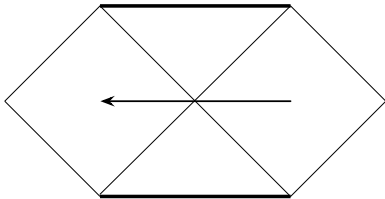
Negative Energy



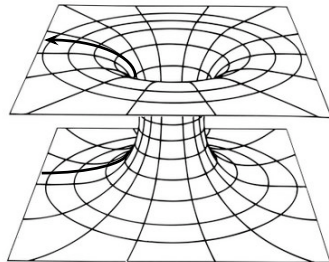
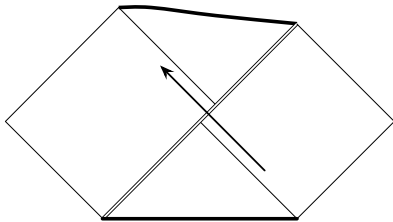
Bondi [1957], Morris and Thorne [1988], Visser [1989]

<http://en.wikipedia.org/wiki/Wormhole>

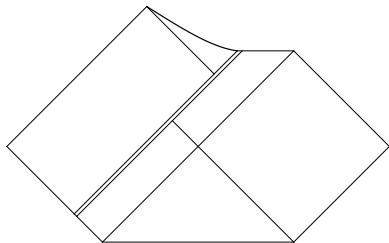
Einstein-Rosen Bridge



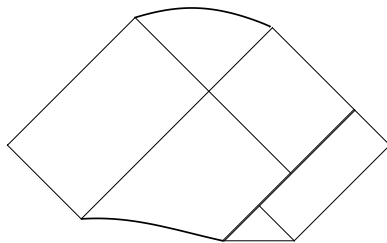
Equal Mass Wormhole



Unequal Mass Wormhole



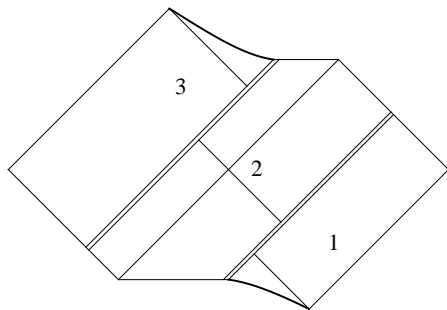
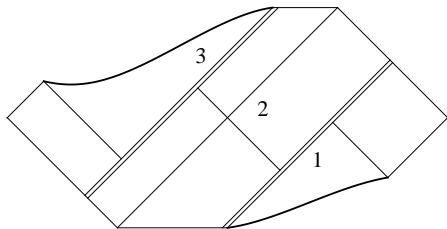
$$m_2 = \frac{1}{2} m_1, \alpha = \frac{1}{2}$$



$$m_2 = \frac{5}{4} m_1, \alpha = -\frac{3}{2}$$

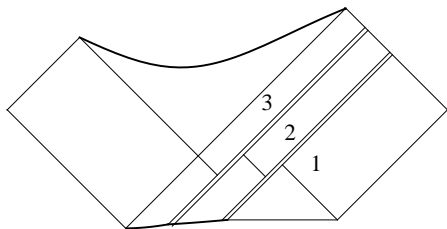
Successive Shells

u_2 and v_2 coordinates



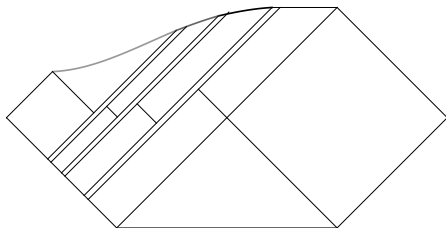
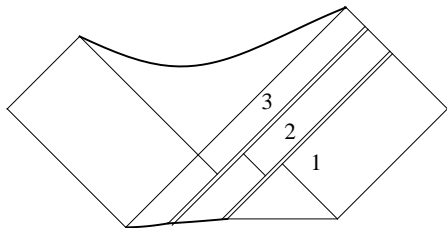
Successive Shells

u_1 and v_1 coordinates



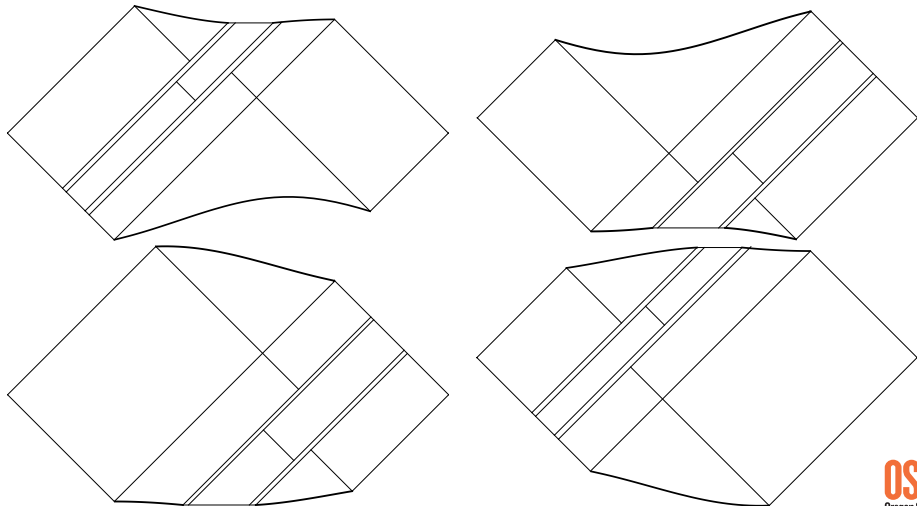
Successive Shells

u_1 and v_1 coordinates



Successive Shells

Equal Mass Shells: Opposite Sign



Successive Shells

Equal Mass Shells: Opposite Sign

$$t = 2m_2 \log \left(-\frac{v_2}{u_2} \right)$$

Successive Shells

Equal Mass Shells: Opposite Sign

$$t = 2m_2 \log \left(-\frac{v_2}{u_2} \right)$$

$$\Delta t = 2m_2 \log \left(\frac{\alpha_1}{\alpha_2} \right)$$

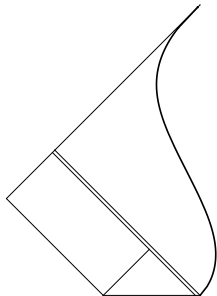
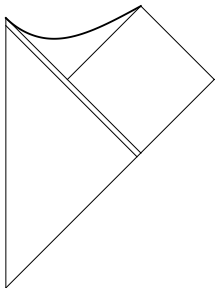
Next Steps: Limiting Cases

- Mass Limits

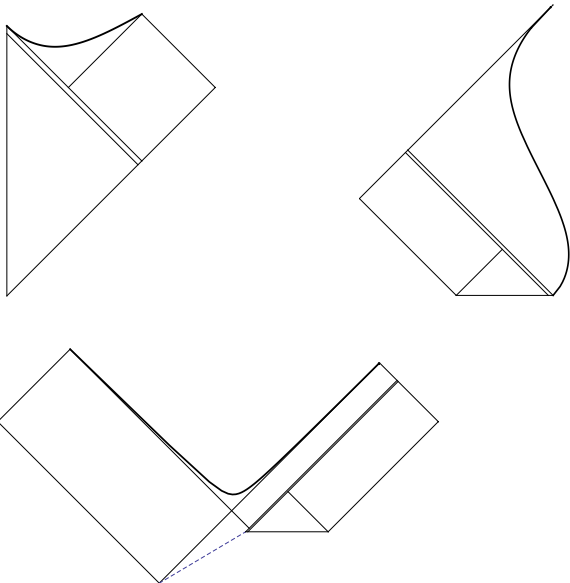
$$\lim_{m_2 \rightarrow 0} \Rightarrow M^4 \text{ in region 2}$$

$$\lim_{m_1 \rightarrow 0} \Rightarrow M^4 \text{ in region 1}$$

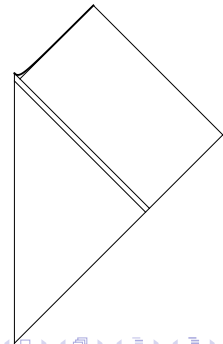
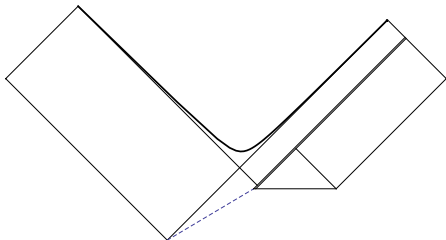
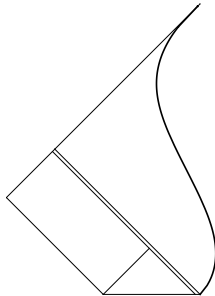
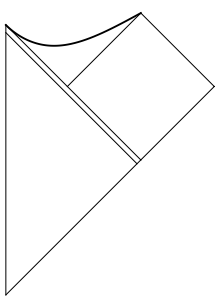
Next Steps: Limiting Cases



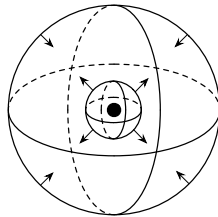
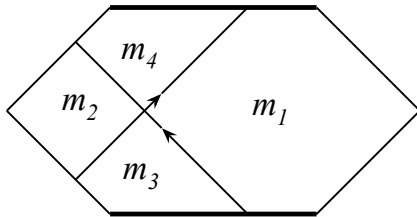
Next Steps: Limiting Cases



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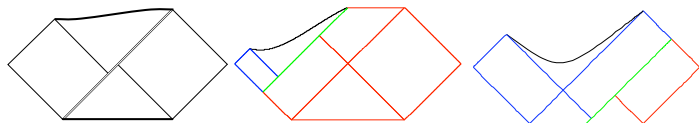


Next Steps: Crossing Shells



Summary

- Shifted Penrose Diagrams
 - Equal Mass Diagrams
 - Unequal Mass Diagrams



- Negative Energy Shells
 - Traversable Wormholes

