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Optimizing Conjunctive Use of Groundwater and Surface Water

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OPTIMIZING CONJUNCTIVE USE OF
GROUNDWATER AND SURFACE WATER

by

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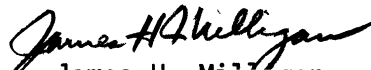
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James H. Milligan

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CHAPTER I

INTRODUCTION

The quantity and quality of available water resources have long been recognized as limiting factors in the development of most arid and semi-arid regions. Recent experiences have shown that these limiting factors may also apply in the more humid areas previously thought to be immune to water shortage problems. The optimal utilization of existing water resources is therefore of ever increasing importance.

While water supply is replenished in a general recurring seasonal and annual pattern, it is not yet within man's power to significantly increase the over-all supply. The best that can be done is to conserve the recurring supply and bring it under control, to preserve the quality, and to better serve the more vital uses. The planning and execution of the best possible programs for the conservation and control of water should be recognized as one of the nation's most important natural resource problems--especially in arid regions.

To attain this objective of conservation and control of the water resource, water must be stored at times when the supply exceeds the demands. The use of surface reservoirs to attain the objectives of water supply and flood control and for better conservation and the demands. The use of surface reservoirs to attain the objectives of water supply and flood control and for better conservation and management of the water resource is a well established practice. Groundwater aquifers have also been long recognized as important sources of water. However, in the past, subsurface reservoirs have

been used with almost complete disregard of surface storage and the interrelationships that exist between surface and groundwater supplies. Only recently have attempts been made to understand the interaction between surface and groundwater and to establish a rational basis for the development and use of subsurface storage in water resource development.

As more information is gathered concerning groundwater hydrology and as water demands increase, the requirement for an optimal development and use policy for groundwater and surface water resources is brought into sharper focus. It is both appropriate and necessary to develop a methodology for optimizing conjunctive use of these resources. In fact, some experts in the water resources field believe that high efficiency and maximum development can be attained only by conjunctive use. Accordingly, the objectives of the research reported herein are aimed at developing guidelines and procedures for designing conjunctive use systems in an optimal manner.

While conjunctive use is a relatively new idea in water resources development, some applications of this concept are already being made in water development projects. However, the idea of optimizing the quantity of, or the economic return from, integrated use of both surface and groundwaters is still a new concept and one which is further developed in this report. The determination of optimal allocations of surface water and groundwater resources that will accomplish the objective of economic efficiency as measured by maximizing net benefits is the basic objective of the several models developed and described in this report. The mathematical procedure used for optimizing the water resource allocations in this fashion

makes use of several algorithms of linear programming as available on a Univac 1108 digital computer. Each of the solutions derived from the models presented is analyzed in order to evaluate the optimization procedure.

Models are formulated for one hypothetical hydrologic unit or basin and for two real river basins. The real river basins were used to test the viability of the modeling procedure and to evolve a practical methodology. The particular basins considered in this study were chosen based on their simplicity for modeling. Each of the real basins was chosen to satisfy the requirement of location over an alluvial basin known to contain a groundwater basin, and the requirement that principal water use in the basin be for satisfying agricultural water use needs. The basins were also chosen on the basis of availability of data. Realistic data were assumed for the hypothetical basin, but extensive and detailed data had to be available for the two real basins chosen. Extensive information is required concerning costs of facilities for developing water supplies from alternative surface and underground sources. Information describing the physical characteristics of the surface water supply and distribution systems and of the underground aquifers is also necessary in order to develop the mathematical models.

Despite the detail and extent of the data required for developing the mathematical models, it must be kept in mind that any mathematical modeling of existing real and complex systems requires simplifying assumptions in order that the final representation of the system be tractable and practical to use. Solutions derived from such representations can only be regarded with the simplifying assumptions in mind.

The effect of modeling different degrees of reality is one question which is examined in this study.

The basic structure of the mathematical models presented in this study is such that water from local surface water sources, from groundwater sources and from imported sources is allocated for utilization by agricultural demand or for groundwater recharge. The allocation is to be an optimal allocation as measured by the objective function. Once such optimal allocation quantities are known to the planners and designers, they can be used for optimal sizing of facilities such as storage reservoirs, distribution canals, artificial recharge works, and wells.

The consideration in the model of engineering aspects such as recharge and groundwater, canals and reservoirs, and hydrologic inputs along with economic aspects such as benefits and costs associated with the various water uses and activities places the allocation problem at the interface between economics and engineering. Many smaller problems in each of these areas must be solved before the allocation problem can be solved.

In spite of the development of mathematical tools to aid planners in the determination of optimal allocation of water resources, the optimal development of water resources will not come of its own accord. It will not be achieved by economic forces; it can be brought about only by deliberate public policy. This accomplishment will require conscious, systematic, and comprehensive planning. It is hoped that this report will serve as a stepping stone in assisting those responsible for this type of planning in water resource systems.

CHAPTER II

REVIEW OF THE LITERATURE

Literature concerning the applications of systems analysis and optimization techniques to water resource problems has appeared only since 1960 and most of this literature deals with concepts and simple examples rather than with actual examples. Literature dealing with the concepts of conjunctive use of groundwater reservoirs and surface water facilities is more extensive and earlier. However, most of the literature dealing with conjunctive use has been of a qualitative nature and has dealt primarily with problems of a local nature. Literature dealing with the management of groundwater supplies has been concerned primarily with the problems of groundwater depletion. Groundwater supplies in California, for example, were depleted in the 1930's as a result of a long-term decrease in precipitation and a large increase in pumping rates. The management decision suggested at that time was that education of groundwater pumpers would be the most economical method to prevent continued depletion of storage. Groundwater management should extend beyond the questions of what to do when the supply runs short; management should begin as soon as possible to achieve efficiency of operation in conjunction with the surface water resources.

The complexities of the problem of conjunctive operation of ground and surface water facilities were explored by some early writers who recognized that the two resources were really a single system and that economic advantages could be had by operating the

system as a complete unit (Banks, 1953, and Kazmann, 1951). Although these early writers have discussed the benefits of joint utilization of groundwater and surface water, only recently have investigators begun to apply optimization methods to the problems of allocating groundwaters and surface waters.

Engineering considerations

Authors who have dealt with the problems of conjunctive use of groundwater and surface water systems such as Clendenen (1954), Thomas (1957), Macksoud (1961), and others, have discussed the economic advantages of such a combination and have pointed out its effectiveness in the conservation of sizeable volumes of water. When these authors have dealt with the problems of economic optimization, the methods of analysis are based upon investigation of a limited number of alternatives and the selection of the best one according to the benefit-cost ratio during the economic life of the project. The work of these authors, however, has been concerned mainly with the engineering problems in the design and operation of the conjunctive-use system.

Fowler (1964) has suggested that solving the engineering problems associated with the development of a conjunctive-use system requires a thorough understanding and investigations of the geology of the groundwater basin, of the hydrology of surface and groundwaters, of the existing surface and groundwater facilities including storage and transmission characteristics, and of existing and expected water demands and the economics associated with meeting those demands. Fowler states that when groundwater basins can be operated in a fully integrated fashion with surface water supplies, then optimum use

of water resources can be achieved. However, in order to achieve this integrated operation, new methods and institutions must be devised to coordinate and manage the operation.

Saunders (1967) states that in order to assess the value of planned conjunctive use in relation to a particular area or basin, it is necessary to look at the economic, hydrologic, and legal system as a whole. A planning procedure is then presented to enable a planning agency to determine, at minimum cost, the feasibility of planned conjunctive use. The procedure consists of determining system characteristics and is discussed in terms of systems analysis and linear programming.

Tyson and Weber (1964) use a computer simulation approach to formulate a "most economical plan" for operating groundwater basins in conjunction with surface facilities. The computational procedure involves two phases: 1) development and verification of the model; and 2) use of the model in predicting basin behavior under imposed conditions. An electronic differential analyzer, or analog computer, is used for the first phase and a digital computer is used in the second phase. In order to develop the mathematical model of the groundwater system, the groundwater complex is replaced by a simplified model divided into small polygonal zones. Assumptions used in deriving the model are that the aquifer is unconfined, that there is no vertical variation in aquifer properties, and that the aquifer thickness is small in comparison to its lateral dimensions. Flow in the aquifer is defined by a single linear equation derived by combining the continuity equation with the Darcy equation. The time dependent flow rate in the aquifer is the algebraic sum of several extraction and replenishment flows.

For modeling on the analog computer the flow equation is transformed to an equivalent system of difference-differential equations. The system is solved simultaneously on the analog computer to give the groundwater level at the node points of the polygonal zones. However, the solution of a system of difference-differential equations on the analog computer is subject to inherent instability which is difficult to overcome.

Once the model on the analog computer is verified by comparing computed water levels with historical data, the equations are modeled on the digital computer for operational studies of the basin. Alternative schemes for operation of the basin are studied by successive iterations using different inputs for aquifer replenishment and withdrawals. The system is gradually improved by choosing the best alternative tried on the model. Simulation of this type provides great detail concerning system operation but does not necessarily provide the optimum alternative.

Economic approaches

A common procedure for identifying the most economical and feasible plan for integrated operation of groundwater and surface water systems has been to choose a number of alternative solutions or plans, which engineering and economic judgment indicate should be desirable, and then compare the costs and benefits of the alternatives. In this approach, "most economical" is usually loosely defined as "least cost," which may not be an appropriate measure of the best solution in all cases.

Chun, Mitchell, and Mido (1964) present an approach of this nature for studying the conjunctive operation of groundwater basins

with surface supplies. Their approach is applied to a regional water supply system supplying the Los Angeles basin. In this study alternative plans were formulated representing use of the groundwater basin in coordination with surface facilities in order to meet imposed demands on the system. Each alternative plan which was studied was presented in terms of groundwater basin operation. Each alternative plan of operation was a combination of four decision variables:

- 1) the areal pattern of groundwater extractions, 2) the methods of prevention of sea-water intrusion, 3) a schedule of spreading artificial recharge water in given locations, and 4) the pumping schedule for fixed locations.

The design is based on the use of existing facilities and on a limited number of possible recharging areas. From the vast number of alternatives, the relatively few having practical importance were selected in a preliminary examination. For each practical alternative, analyses were carried out separately for the subsurface and surface systems. The subsurface system was simulated on an analog computer in order to develop the mathematical model of the subsurface system. Operational studies of the subsurface system were then carried out on a digital computer. In the analysis of the surface system, future water demands in the region were taken into account. The most economical subsurface and surface facilities were selected on the basis of the operation studies. The final optimum alternative combination of subsurface and surface facilities was selected according to the criterion of minimizing the total annual costs. Economic comparisons of alternative plans of operation are made on the basis of converting these annual costs into total present worth. The plan chosen as the most economical one is the alternative

having the least total present worth. The authors state that, "Because all plans were formulated to satisfy identical physical requirements, the plan with the least total present worth has the greatest benefit/cost ratio."

Despite the wide scope and detailed analysis characterizing this work, no modern techniques of mathematical programming for solving the problem of economical optimization were used. This approach is actually a "trial and error" approach. Some have classified the approach as a steepest descent method of cost minimization. The final result is supposed to be the most economic approach to the problem. However, there is no way of determining whether the final solution is the "lowest point of the bowl" or just a low point on the side of the bowl. In other words, the result may be a "local" minimum cost, but it is not necessarily the global optimum value. Also, a cost minimizing procedure is not necessarily the "most economical" approach nor the proper measure of objectives for all situations.

Renshaw (1963) presents the argument that decisions regarding the use of groundwater resources should be based on the value of the groundwater resource. The basis of the argument is that water left in storage has economic worth. The economic returns from water left in the ground can be estimated by two methods presented by the author. In the first method the returns are based on reduced pumping costs due to reduced mining of groundwater. The second method is based on the economic returns on the capitalized value of water left in storage. Renshaw's arguments emphasize the value of not pumping groundwater.

Koenig (1963) presents the opposite view regarding the economics of groundwater development and use. Koenig's thesis is that

the attitudes and practices of groundwater development in the nation as a whole are far too conservative, and he recommends a much greater use of groundwater resources. Koenig argues that extractions from groundwater reserves should be viewed in the same manner as extractions from other resource reserves such as oil or coal or natural gas. Without consideration of any further replenishment of groundwater reserves, the life of the current reserve of groundwater is more than 18 times greater than the corresponding life of any other nonreplenishable resource with the exception of bituminous coal. According to Koenig, if the present rate of depletion of groundwater storage is continued, the reserve life would be 7800 years. Alternatives to local storages of groundwater are reducing the level of the economy in the local area or importing water to the water-short areas from areas of abundance. The conservative attitude toward groundwater development cannot be justified economically, according to Koenig.

Domenico, Anderson, and Case (1968) present a mathematical expression relating the economic worth of groundwater mining to the remaining worth of a basin after it has been partially depleted. This expression permits the establishment of an optimal, one-time storage reserve that may justifiably be exploited. In this argument, sustained yields are taken as use rates determined by and limited to natural replenishment; and mining yields are volumes of nonrenewable water in storage independent of the rate of mining. The volume of mining yield may be mined rapidly or slowly, but the volume extracted is limited. Maximization of present worth is taken as the conventional management objective. Optimality is determined by conventional calculus methods.

Optimization techniques applied

The concept of optimization almost always implies either maximizing or minimizing some objective function. The objective function might be maximization of net benefit, for instance, if the objective is economic efficiency. Other objectives might be income redistribution or regional development, provided functional relationships can be written to describe these objectives. In the application of optimization techniques to water resource problems, the guiding principle in selecting the objective function is almost always the allocation of scarce resources. There are many constraints or limits on the allocation of water resources, so the problem becomes one of maximizing or minimizing some objective function subject to several constraints. In other words the problem is a constrained optimization problem. Several such problems are described in the literature. The more pertinent examples are described below.

Hall and Howell (1963) point out a general method for the determination of the optimum size of a single purpose reservoir designed for multi-seasonal storage. The criterion for optimization is the maximization of the expected present value of the net income derived from water during a certain period. The return functions are given for each time period. Likewise the discount factor and the salvage value of the water at the end of the economic life are taken into consideration. In this case it is proposed to solve the design problem indirectly by studying the system operation. The optimum operation is determined by the numerical solution of the recursion equations of dynamic programming. Since it is assumed that a serial correlation exists between inflows in successive seasons,

it is suggested to use samples taken from a long synthetic series for the flow data. By repeating the calculations for several reservoir sizes and using a number of samples in each case, one obtains not only the average value of the maximum expected benefit but also the distribution of the benefit-cost ratio, which can be used as an estimate of the risk involved in the project. On the basis of these results, and after repeated computations for various reservoir sizes, the optimum capacity is determined.

Fiering (1961) deals with the optimum design of a single reservoir impounding water for three purposes: irrigation, power generation, and flood control. He assumes that monthly inflows obey a truncated normal distribution, with a given serial correlation between successive months. As an example, he presents a certain model and defines the operating procedure using concepts from queuing theory. In order to solve the correlation problem, he employs a synthetic series of inflows. There are three decision variables in his model: the size of the reservoir, the parameter representing the operating procedure, and the level of development which appears as a parameter in the benefit functions of the various water uses. In order to obtain the probability distribution of the water releases, the behavior of a system is simulated with various combinations of values of the decision variables. On the basis of these results, the value of the objective function is computed for each case. The alternatives are then compared in order to obtain the optimal combination.

Dorfman (Maass et al., 1962, Chapter 13) shows solutions through linear programming for a number of models of simplified water resource systems. Possible solutions are shown for cases where the

benefit and cost functions, and even some of the constraints (when they depend on only one or two separable variables) are not linear but lend themselves to piecewise linearization. In these cases, the calculations become more cumbersome because of the iterative computations necessary for finding the optimum. It should be emphasized here that in problems of systems engineering, which are to be solved by means of mathematical programming, it is not enough to define the model and point out the solution method; it is necessary as well to evaluate the efficiency of the computational procedure and to ascertain its feasibility by means of existing computers.

The first models considered by Dorfman are deterministic. When he takes into account the stochastic nature of the hydrology, approximating the probabilistic distribution of the inflows by means of discrete values, he limits the model to a multipurpose single reservoir.

An analysis of a more complex system, where stochastic hydrology was taken into account in order to determine the components of the optimum design, was carried out by the Harvard Water Program (Maass et al., 1962, Chapters 9 and 10) using simulation. The operating rules of the system were assumed fixed, while the design variables included 12 characteristic values of the system's units and its target outputs. Combinations of different discrete values of decision variables were investigated by simulating on a digital computer the operation of the system during the specified project life. The value of the objective function was calculated for each case. A sample taken from a long synthetically constructed series was used as inflow data. In order to facilitate finding the optimum in the vast number

of possible combinations, an attempt was made to investigate systematically the multi-dimensional response surface which is the geometric expression of the relation between the value of the objective function and the decision variables. On the basis of such an investigation, it was determined how to proceed in the selection of the samples and how closely the optimum point had been approached. Both systematic and random sampling methods were used.

Burt (1964) derives decision rules for use of the groundwater resource from a dynamic programming formulation of a more general resource-use problem in which the resources being managed or used are either fixed in supply or only partially renewable at a point in time. In the model proposed by Burt, the operating decisions are related to the volume of water pumped in each season. This volume is based on the storage available at the beginning of that season. The volume of the net natural seasonal recharge is a random variable with a given probability density function. The criterion for determining the optimum operating policy is the maximum present value of the sum of net benefits. The approach to this analysis is that of a sequential decision process under stationary conditions. The function equation, solved by dynamic programming, is

$$U^*(S) = \text{Max}_P [U(P,S) + q \int_0^{\infty} U^*(S + N - P) h(N,S) dN]$$

where

$U^*(S)$ = maximum benefits over the economic life period

S = storage available at the beginning of the season

P = volume of water pumped in each season

N = net natural seasonal recharge (random variable)

$U(P,S)$ = expected seasonal net benefit

$h(N,S)$ = probability density function of natural recharge

q = discount factor

A direct analytical solution of this equation is impossible in most cases. Burt analyzed the case where $U^*(S)$ is approximated by the sum of the first few terms of a Taylor series expansion (first and second approximations). When it is assumed that the volume of water pumped is the expected value of the net natural seasonal recharge, the results yielding the optimum value of the storage, i.e., the long range equilibrium storage, are identical for both the first and second approximations. Burt refers to the pumpage at the optimum storage level as an "optimal safe yield." The increase in marginal pumping costs above the economic limit prevents the lowering of the equilibrium water table below the optimum storage level. This model does not consider outflows through aquifer boundaries explicitly, but they are included in the net natural recharge term. Burt gives a numerical example in which there is also a surface reservoir.

Castle and Lindeborg (1961) define optimal allocation of water resources on the basis of maximizing beneficial use as determined by a linear programming mode. Water is allocated from surface water and groundwater sources to two agricultural areas. A simplifying assumption is made regarding the production function for water-- that water users in the two agricultural areas would expand their inputs of other production factors in proportion to increases in the amounts of available water. This assumption allows the model to be formulated in the linear fashion required by the linear programming approach. Post-optimal analysis of the optimal solution is presented to indicate the stability of the solution to the allocation problem. The results of the study are used to argue for modification of the

institutional arrangements governing water resource allocation.

Buras (1963) applies advanced analytical methods to the analysis of the conjunctive operation of reservoirs and aquifers where the water released from the two storage sources is used for irrigation in two agricultural areas. For a given set of hydrological data, the optimization of the operation of a conjunctive-use system involves the solution of three problems: 1) the determination of design criteria for the surface facilities including recharge facilities, 2) the determination of the extent of the system service area, and 3) the determination of the operating policy specifying reservoir releases and aquifer pumpage. The problem of optimizing the conjunctive use of surface and groundwater is solved by considering a system made up of a surface reservoir of capacity Q_M , an aquifer of capacity S_M , and recharge facilities of capacity R_M . The continuous probability distribution function of the inflows into the surface reservoir is approximated by a discrete distribution, allocating probabilities p_j to various magnitudes of inflow X_j . At the same time, the natural replenishment of the aquifer is considered as a deterministic value N . The operating policy is developed for a number of identical seasons or years. The water is used to irrigate two areas, each having a different benefit function. It is assumed that the water pumped from the aquifer, P_i , is used only to irrigate one of the areas, while the other area is irrigated by releases from the reservoir, Y_i . The solution is achieved through the application of dynamic programming. The state of the system at any state i is described by a three-dimensional vector (W_i, S_i, T_i) , representing the quantities of water in the surface reservoir, in the aquifer, and in transit from the recharge facility to the aquifer, respectively.

There are also three decision variables: P_i , Y_i , and R_i , where R_i is the amount of water released from the surface reservoir for groundwater recharge. The functional equation which expresses the maximum present value of the expected net benefits from the remaining n periods of operation is

$$U_n^*(W,S,T) = \text{Max}_{Y,P,R} [U(Y,P) + q \sum_j p_j U_{n-1}^*(W+X_j - R - Y, S+T-P, R+N)]$$

subject to the constraints

$$0 \leq Y + R \leq W \quad \text{and} \quad 0 \leq P \leq S$$

where $U(Y,P)$ is the net benefit from one season, and q is the discount factor. The numerical solution of the equation leads to a constant operating policy when n reaches a certain value. This indicates the attainment of a "steady state" unaffected by decisions far removed into the future. Repeated combinations in which the size of the system components and the target outputs are varied yield optimal values for these parameters.

Dracup (1966) formulates a mathematical model for a groundwater and surface water system which is solved using parametric linear programming. The parametric analysis includes the variation of both the objective function cost coefficient, C_j , and the right-hand-side terms, b_j . The model is formulated to represent the San Gabriel Valley in southern California. The unit cost of water importation, treatment, storage, pumpage, boostage, and artificial recharge to aquifers is determined by economic analysis. Five sources of water are utilized to optimally satisfy three water requirements. The analysis extends over a 30-year period. Three possible decision rules which may be implemented by a planning agent are analyzed to determine an optimum operating procedure. A sensitivity analysis on the cost coefficients and the significance of the shadow or imputed prices is included.

CHAPTER III
THE GENERAL CONJUNCTIVE-USE MODEL APPLIED
TO HYPOTHETICAL BASINS

In this chapter the basic physical features of a water resource system which are included in the mathematical model are defined and discussed. Concepts and definitions are given in the context of modeling a synthetic or hypothetical water resource system. The basic reasoning and ideas which apply to the hypothetical basin are extended in later chapters to applications in real river basins.

The mathematical model representing the water resource system is formulated as an allocation problem in which various water sources are to be allocated to the various water uses. The mathematical model is formulated in this manner so that the methods of linear programming can be applied to obtain an optimal solution to the water-use problem. The main advantage of the linear programming technique is that after setting up the model, standardized and easily computerized computations can be used to determine optimal decisions even under complicated conditions. Because of the simplification required in order to represent a physical system as complex as a water resource system with a mathematical model, the results obtained should be considered as a first approximation to the solution. These results may serve as a useful starting point for more elaborate and detailed methods of analysis such as a refined systems simulation.

Physical features modeled

The models described in this chapter are formulated to represent realistic hydrology of a hypothetical water resource system but are not construed to represent any actual river basins. Hence, the physical features modeled are general in nature and might be found in any real basin.

The size of the area represented by the models is not fixed but would probably be classed as a relatively small area. Perhaps the size could be estimated from the average annual runoff used in the model.

For the models of the hypothetical system, it is assumed that the main aquifer is unconfined and consists of unconsolidated sediments with the water table at approximately 125 feet below the ground surface. The thickness of the water bearing materials is assumed to be approximately 500 feet with a total usable storage capacity of about 200,000 acre-feet. The usable storage capacity is assumed to be within the limits of the economic pumping lift. It is also assumed that part of the groundwater storage will be carried over from season to season. The actual amount of the carryover storage is a decision variable in the model.

The surface water features of the assumed basin consist of a single major stream. The natural surface inflow to the valley area may be either from surrounding mountains or from an adjacent basin. However, the annual flows are known to the extent that a probability distribution of the net inflows (inflows corrected for evapotranspiration losses in the stream system) can be derived.

For the hypothetical system, natural surface-water inflows are

assumed to consist of a single stream with a mean annual runoff equal to 60,400 acre-feet per year. The probability characteristics of this surface inflow are described as follows. The surface inflow is labeled SFIN and P_i is the probability that the random variable SFIN is less than or equal to $SFIN_i$. SFIN is assumed to be normally distributed, and $SFIN_i$ is chosen arbitrarily within the range of the random variable SFIN. Probability distributions other than the normal distribution may describe the hydrologic parameters better. The normal distribution is used here since the technology for using other distributions in this type of model has not yet been developed. The probability density function

$$f(SFIN) = \frac{1}{\sqrt{2\pi} \sigma_{SFIN}} e^{-1/2 \left(\frac{SFIN - \mu_{SFIN}}{\sigma_{SFIN}} \right)^2}$$

describes the probability characteristics of SFIN. From observed data $\hat{\mu}_{SFIN}$ and $\hat{\sigma}_{SFIN}$ are found to be 60,400 acre-feet per year and 19,660 acre-feet per year respectively. The probability that the random variable SFIN is less than or equal to $SFIN_i$ is found by evaluating the integral

$$\int_{-\infty}^{SFIN_i} \frac{1}{\sqrt{2\pi} (19,660)} e^{-1/2 \left(\frac{SFIN - 60,400}{19,660} \right)^2} d SFIN$$

If the lower bound of $SFIN_i$ is 20,000 acre-feet per year then the above integration can be written as follows:

$$\int_{20,000}^{SFIN_i} \frac{1}{\sqrt{2\pi} (19,660)} e^{-1/2 \left(\frac{SFIN - 60,400}{19,660} \right)^2} d SFIN$$

This integration is standardized as the functional

$$\Phi \left(\frac{SFIN_i - 60,400}{19,660} \right)$$

and the value of $\Phi (\cdot)$ can then be found from the standard normal tables in any statistics textbook.

If the streamflows for the hypothetical basin are described by the following discrete points:

$$SFIN_1 = 35,000 \text{ acre-feet per year}$$

$$SFIN_2 = 50,000 \text{ acre-feet per year}$$

$$SFIN_3 = 65,000 \text{ acre-feet per year}$$

$$SFIN_4 = 80,000 \text{ acre-feet per year}$$

$$SFIN_5 = 95,000 \text{ acre-feet per year}$$

then the probabilities that SFIN is equal to or less than $SFIN_i$ and greater than $SFIN_{i-1}$ are:

$$1. \quad p_1 = \text{prob} (20,000 < SFIN \leq SFIN_1 = 35,000)$$

$$= \int_{20,000}^{35,000} \frac{1}{\sqrt{2\pi} (19,660)} e^{-1/2 \left(\frac{SFIN - 60,400}{19,660} \right)^2} d SFIN$$

$$= \int_{-\infty}^{35,000} f(SFIN) d SFIN - \int_{-\infty}^{20,000} f(SFIN) d SFIN$$

$$= \Phi\left(\frac{35,000 - 60,400}{19,660}\right) - \Phi\left(\frac{20,000 - 60,400}{19,660}\right)$$

$$= 0.0783 \approx 0.08.$$

$$2. \quad p_2 = \text{prob} (35,000 < \text{SFIN} \leq 50,000)$$

$$= \Phi\left(\frac{50,000 - 60,400}{19,660}\right) - \Phi\left(\frac{20,000 - 60,400}{19,660}\right)$$

$$= 0.19996 \approx 0.20$$

similarly

$$3. \quad p_3 = \text{prob} (50,000 < \text{SFIN} \leq 65,000) = 0.32$$

$$4. \quad p_4 = \text{prob} (65,000 < \text{SFIN} \leq 80,000) = 0.26$$

$$5. \quad p_5 = \text{prob} (80,000 < \text{SFIN} \leq 95,000) = 0.13.$$

Table 3-1 summarizes the streamflow characteristics used in the model of the hypothetical system.

In the actual mathematical model of the system a downstream requirement of 30,000 acre-feet per year to satisfy downstream water rights reduces each value of SFIN_i by 30,000.

For the hypothetical system a single surface reservoir site exists in the system on the main stream. It is assumed that the physical storage capacity of the surface storage facility is sufficient to store approximately 80 percent of the net average annual runoff.

Table 3-1. Probability distribution of surface-water inflow for the hypothetical system

i	SFIN _i	P _i
1	35,000	0.08
2	50,000	0.20
3	65,000	0.32
4	80,000	0.26
5	95,000	0.13
$\hat{\mu}_{\text{SFIN}} = 60,400$		$\hat{\sigma}_{\text{SFIN}} = 19,660$

This figure can be changed easily in the mathematical model. Other surface features include a canal system for conveyance of water from the surface reservoir to the agricultural use area and an artificial recharge facility for putting surface waters artificially into groundwater storage. The type of recharge facility is not important in the model except as it might affect unit recharge costs and provided the facility can provide the capacity determined in the optimization model.

Natural recharge to the groundwater basin consists of quantities of water in the hydrologic cycle that enter and leave the groundwater system that are beyond the control of the operators of the system. Consequently, for the synthetic models natural recharge is assumed to be the net of the following inflow and outflow components:

Inflow components

1. Subsurface inflow

2. Percolation of precipitation
3. Percolation from streambeds

Outflow components

1. Subsurface outflow
2. Base flow to surface streams
3. Extraction by evapotranspiration

The natural recharge in the hypothetical basin is assumed to average 18,600 acre-feet per year and is composed primarily of components from precipitation and from percolation from natural stream channels. In order to derive its distribution, natural recharge is defined as

$$\text{NATRE} = K(X + Y)$$

where

NATRE = natural recharge

X = streamflow component

Y = annual precipitation component

K = a scale factor

It is assumed that the distributions of annual streamflow and of annual precipitation are known from observed data and that they follow a normal distribution. Then the distribution of natural recharge will also be normal with mean $K(\mu_X + \mu_Y)$ and variance $K^2(\sigma_X^2 + \sigma_Y^2 + 2\sigma_{X,Y})$.

Estimators of μ and σ are found from the sample data, and the scale factor can be found from water budget studies for a real basin. For the hypothetical basin the scale factor is found to be 0.072. The

following information is assumed for the hypothetical basin:

$$\hat{\mu}_{\text{NATRE}} = 18,600$$

$$\hat{\sigma}_{\text{NATRE}}^2 = 6,386,987$$

Discrete points in the distribution of NATRE are chosen in a manner similar to that of the streamflow discussed earlier. The discrete points chosen are:

$$\text{NATRE}_1 = 14,870$$

$$\text{NATRE}_2 = 16,790$$

$$\text{NATRE}_3 = 18,711$$

$$\text{NATRE}_4 = 20,632$$

$$\text{NATRE}_5 = 22,553$$

By the same procedure as discussed earlier the probabilities that the NATRE is less than or equal to NATRE_i and greater than NATRE_{i-1} can be obtained as follows:

$$(1) \text{ Prob } (14,000 < \text{NATRE} \leq 14,870)$$

$$= 0.1401 \doteq 0.14$$

$$(2) \text{ Prob } (14,870 < \text{NATRE} \leq 16,790)$$

$$= 0.2149 \doteq 0.22$$

$$(3) \text{ Prob } (16,790 < \text{NATRE} \leq 18,711)$$

$$= 0.2955 \doteq 0.30$$

$$(4) \text{ Prob } (18,711 < \text{NATRE} \leq 20,632)$$

$$= 0.2344 \doteq 0.23$$

$$(5) \text{ Prob } (20,632 < \text{NATRE} \leq 22,553) \\ = 0.1072 \doteq 0.11.$$

The following table summarizes the natural recharge characteristics used in the hypothetical basin.

Table 3-2. Probability distribution of natural recharge to groundwater for the hypothetical basin

i	NATRE _i	P _i
1	14,870	0.14
2	16,790	0.22
3	18,711	0.30
4	20,632	0.23
5	22,553	0.11
$\overline{\text{NATRE}} = 18,600$		

Features in the models which describe interconnections between surface water and groundwater include the natural recharge, artificial recharge, net extractions from groundwater (pumpage), and conveyance losses from the surface distribution system (canal losses).

Three models are formulated and solved for the hypothetical system. The basic features of the models are:

Model 1. A two season model including a wet season and a dry season

Model 2. A single season model in which all inflows (surface

inflows and natural recharge to groundwater) are deterministic

Model 3. A single season model in which all inflows (surface inflows and natural recharge to groundwater) are probabilistic.

All three models are formulated with the same average annual inflows and downstream requirements so that the results of each approach can be compared with the others. The only differences in the three models are in the nature of formulation (seasonal vs deterministic vs probabilistic) and in the optimal levels of the decision variables.

All of the models of this study have been structured as supply models rather than as demand models. They are structured so that the amount of water which can be supplied to the various allocations is one of the decision variables. Since the models are supply models, constraints imposed by water demands do not occur.

The objective function--economic characteristics

The economic characteristics of the system are formulated in an objective function. The purpose of the optimization is to obtain an optimal allocation of the water resource. The allocation must consider the alternative uses of the water and the alternative sources of the water. The economic characteristics of the system must reflect the benefits attributable to the particular water uses as well as the costs associated with providing water to the particular uses from the alternative sources.

Irrigation benefits. In the models of the hypothetical basin

irrigation is the only use to which a direct benefit is attached. Water may be allocated to irrigation or to groundwater recharge to be used eventually for irrigation. In the hypothetical basin only one irrigated area is considered, so there is only one benefit term in the objective function. The net worth of water is difficult to determine because there is not an active market in water rights. Estimates of relative values of water in alternative uses have been made based on distributions of market prices paid for water at points of use and using distributions of actual costs incurred in putting water to work. Comparisons of gross values are often inaccurate since an arbitrary decision to include or exclude a given cost component could have a significant effect on the total costs.

Renshaw (1958) published a table of values of water in the United States based on 1950 prices. In this table the maximum value of irrigation water is reported as \$27.04 per acre-foot with a mean value of \$1.67 per acre-foot. It should be noted that Renshaw's "values" are based on prices paid and are not necessarily at all related to actual values of irrigation water. Wollman of the University of New Mexico and his colleagues have shown that the average value added to the economy of the Southwest through the use of irrigation water is \$44.00 to \$51.00 an acre-foot (Todd, 1965). Studies in Colorado have shown that the marginal value of irrigation water to the individual farmer ranges from about \$9.00 per acre-foot to about \$70.00 per acre-foot, based on 1960 prices (Hartman and Whittelsey, 1960). Studies for Sevier County, Utah, have indicated that the marginal benefits from irrigation water range from \$10.50 per acre-foot to \$25.50 per acre-foot (Davis, 1965).

In the models of the hypothetical basin benefits of irrigation water are assumed to be \$45.00 per acre-foot. These benefits are assumed to be benefits within the basin, so some secondary benefits would be included. The irrigation benefit is not meant to be the direct benefit to the farmer nor is it meant to reflect all of the secondary benefits that might be added by interactions outside of the basin. The value of \$45.00 per acre-foot is used as an average value of irrigation benefits rather than a marginal value.

Water supply costs. Cost terms in the objective function are the costs of making the water available to the irrigation use. Cost considerations in the system relating to this irrigation use are listed below and discussed in more detail in the pages that follow:

1. Costs directly attributable to pumping water from groundwater at the assumed average depth.
2. Costs associated with conveyance of water from the natural channel or surface storage to the place of use. These costs include all costs of diversion and conveyance.
3. Costs in connection with surface storage facilities and their operation.
4. Costs associated with water artificially recharged into the groundwater aquifers.
5. In the models which include stochastic or uncertain inflows, costs associated with shortages sustained when actual deliveries are less than the guaranteed deliveries for irrigation.

Since empirical pumping cost data are scarce in Utah, some guidelines for estimating pumping costs as suggested in a recent

publication are used for this study (Nuzman, 1967). Following these guidelines, the pumping costs are considered in two basic categories. Fixed costs, including exploration and development, are all capital expenditures and are usually made prior to the use of the water. Variable costs include all operational costs which are necessary to maintain water production. The pumping cost curve shown in Figure 3-1 is based on the guidelines presented by Nuzman with the following assumptions:

Interest rate = 7%

Life of well, pump and motor = 20 yrs.

Average power costs = 1.12¢/kwh

Efficiency of pumping plant = 0.52 g

Average pumping rate ranges 1000 gpm to 4500 gpm

Pumping season = 100 days

Also shown on the curve are two points representing actual data from pumping experiences in Utah. The two points agree favorably with the theoretical cost curve. This cost curve is used throughout the report as a guide in estimating pumping costs for the various models presented. For the hypothetical basin an average pumping lift of 200 feet was assumed with a corresponding total pumping cost of about \$5.00 per acre-foot. This cost includes fixed and variable costs of extracting groundwater.

Conveyance and diversion costs are presumed to reflect as charges against benefits all of the cost items which depend upon the quantity of water diverted for beneficial uses or that depend upon the direct use of that diverted quantity. The conveyance costs do not

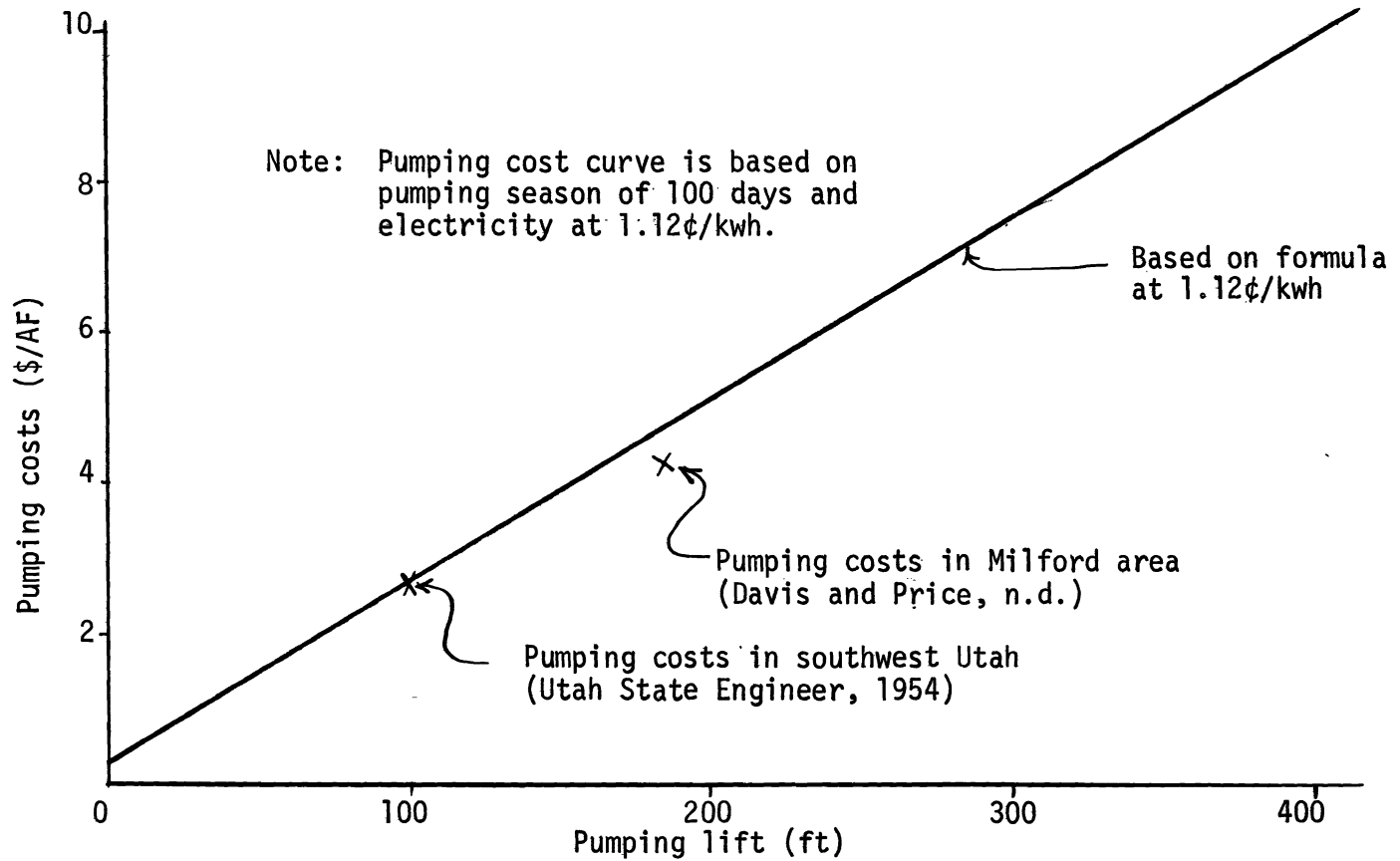


Figure 3-1. Pumping costs vs pumping lift

include the costs of the storage reservoir which cannot be related solely to the allocation of diverted waters. Empirical data on conveyance costs have been difficult to determine for conveyance systems in Utah. Several inquiries were sent to canal companies and to various water agencies in the state. None of the inquiries were answered, presumably because the information is simply not available.

A recent California publication indicates that unit annual costs of conveyance systems, including annualized capital costs and fixed and variable operation and maintenance costs total about \$20.00 per acre-foot of water (State of California, 1966). In this case the conveyance systems are pipelines transporting water over large distances into the Los Angeles area. It is expected that conveyance costs in open canals carrying irrigation water much shorter distances would be less than half the quoted costs.

Analysis of a few of the canal companies listed in an Agricultural Experiment Station Report at Utah State University yielded an estimate of annual conveyance costs of irrigation water in canals which totaled about \$8.00 to \$10.00 per acre-foot of water delivered (Richards, Davis, and Griffin, 1964). It must be realized, however, that this estimate is a crude one based on few available data and that many canal companies actually charge much less than \$8.00 per acre-foot annually.

Based on the estimate, a conveyance cost of \$8.20 per acre-foot was used for the hypothetical basin. Further discussion concerning the validity of using such a crude estimate is included in Chapter V in a discussion of sensitivity analysis of cost coefficients.

The costs of reservoirs constructed for storage of surface water vary and are dependent upon factors such as the size class of the reservoir, the storage capacity of the reservoir in comparison with the mean annual runoff of the stream, the assumed life of the reservoir, the purposes to be served by the reservoir, and the interest rate chosen for discounting, along with several other factors. Average annual unit costs of surface storage capacity range from \$1.07 per acre-foot to \$8.65 per acre-foot with the larger value for smaller reservoirs (Löf and Hardison, 1966). Values for individual reservoirs may vary widely from these costs depending upon individual site conditions. A value of \$8.20 per acre-foot was chosen for the hypothetical basin models since most new reservoir sites available are in the smaller size classes.

The unit cost of artificial recharge varies over a wide range. This variation is dependent upon the quantities of water recharged, intake characteristics of the soils in the recharge area, land values, method of artificial recharging, and quality of the recharge water. Todd (1965) reported that recharge costs in California varied from \$0.43 per acre-foot to \$48.50 per acre-foot. These include the costs of all of the components of an artificial recharge project including costs of diversion, conveyance, and operation and maintenance, as well as costs of land, site development, landscaping, and fencing. Artificial recharge costs reportedly average approximately \$8.00 per acre-foot (Frankel, 1967). For the purposes of the hypothetical basin models an artificial recharge cost of \$15.00 per acre-foot has been assumed.

For the purposes of this study, a shortage is defined as the difference between a guaranteed (full supply) quantity of water to be delivered and an amount of water than can actually be delivered. In a mathematical programming model, constraints on decisions appear in the form of linear inequalities such as an inequality stating that irrigation deliveries must be less than or equal to the amount of water available for delivery either from storage or from natural streamflow. When water availabilities are uncertain, the strict inequalities falsify the actual problem. A firm water-delivery commitment does not really mean that the water must be supplied in the most adverse conceivable circumstances. Some risk of nonfulfillment must be admitted. The amount of the nonfulfillment is termed the shortage for this study. There is a variety of courses open to the manager of a water supply system in the face of a shortage. For example, he may restrict demand (by administrative fiat curtailing certain uses); or he may move to increase available supplies (by tapping emergency supplies or by tapping supplies outside the basin); or he may choose a combination of measures. Whatever is done to meet the shortage will imply some cost to the water users.

Little work has been done in the way of actually evaluating such shortage costs. It is clear that the cost of shortage depends upon the water use. Some crops can stand shortages at a cost of decreased yield whereas a shortage may completely destroy other crops. In addition, the value of the water depends upon the complement of other resources that are used in conjunction with the water. Assuming that shortage costs for a model dealing only with deliveries of water

for irrigation can be approximated by marginal values of irrigation water, then such shortage costs would range between \$9.00 to \$70.00 per acre-foot of shortage (Hartman and Whittlesey, 1960). The variation in costs depends upon the available resources, the amount of the shortage, the timing of the shortage, and many other factors.

For the particular hypothetical models in question, a shortage cost of \$60.00 per acre-foot is assumed. In other words, during any year in which deliveries are less than the guaranteed amount, a loss of \$60.00 per acre-foot of shortage is sustained. Shortage costs as used in this model are based on the marginal values of irrigation water and are, therefore, considerably higher than the values used for irrigation benefits which were based upon average values of irrigation water. The shortage situation is assumed to be a marginal condition rather than an average condition.

Water surpluses may be implied by the same conditions which would cause a shortage. In the stochastic model, surpluses can be used to satisfy downstream requirements; but no benefits or costs are attached to those surpluses in the objective function.

The constraints on the system

The system constraints describe the physical and hydrologic relationships of the basin. The feasible solution space is defined by the constraint system.

Six groups of constraints have been developed for the conjunctive use models of this study. Each group may contain one or several constraints depending upon the particular model. The constraint that none of the decision variables be negative is implied in the linear

programming approach and is not listed as one of the groups of constraints. The constraint groups are stated briefly below and are described in more detail in the pages that follow:

1. Flows in all reaches of the system must be nonnegative.
2. Releases from storage must be less than or equal to the sum of the expected inflows plus initial or carry-over storage.
3. Storage contents at any time cannot exceed the storage capacity.
4. Aspired carry-over storage can be reattained each year-- these are the constraints describing probabilities of uncertain flows.
5. Constraints are required which define shortages.
6. Other constraints, which are appropriate for the particular system being modeled, may be necessary. For example, some of these may be constraints defining maximum physical capacities of sites and structures.

Nonnegative flows. The first group of constraints requires that the flows in all reaches of the system must be nonnegative. This requirement must be satisfied at every point in the system where withdrawals are made, whether these withdrawals are for storage or for diversions. In some cases this requirement can be satisfied for more than one location in the system by only one constraint and the addition of other constraints would be redundant. This set of constraints also implies that diversions for irrigation will not be greater than the natural flow plus releases from storage. Flow constraints for meeting downstream requirements also fit in this constraint group.

Storage releases. This second group of constraints requires

that releases from storage must be less than or equal to the sum of the expected inflows plus initial storage. Inflows to the surface reservoirs are assumed to be adjusted for natural evapotranspiration losses along the channels and for evaporation losses on the reservoir itself. Inflows to the surface reservoirs may consist of natural streamflow, return flows from irrigation, and base flows from groundwater. Inflows to the groundwater reservoirs include natural recharge, artificial recharge, deep percolation losses from irrigation, and conveyance losses from the surface distribution system. Natural recharge is made up of quantities of water entering and leaving the groundwater system that are beyond the control of the operators of the system. Percolation losses from irrigation and conveyance losses are not fully controllable but are not considered here as part of the natural recharge. Natural recharge is the net of subsurface inflow, percolation from precipitation, percolation from streambeds, subsurface outflow, and extractions by evapotranspiration. A safe yield requirement is not maintained by this group of constraints alone, since this constraint would allow the groundwater reservoir to be completely emptied if it were economical to do so. The groundwater reservoirs are operated on a safe-yield basis. This is assured by the group of constraints labeled "carry-over storage."

Storage contents. This group of constraints requires that the storage contents at any time cannot exceed the storage capacity. In other words, the initial storage content plus inflows minus outflows must be less than or equal to the storage capacity. This constraint applies, of course, to each storage unit in the system whether it is

surface storage or underground storage. This constraint can also be used to prevent groundwater levels from causing waterlogging in any areas by specifying the groundwater storage capacity at a value which would fix the maximum water table elevation.

Carry-over storage. At this point a simplifying assumption is made which causes an approximation in the solution of the mathematical model. The carry-over storage or initial storage is regarded as a decision variable along with storage capacities, storage releases, and guaranteed supply levels. This assumption causes no particular problem in the nonstochastic models; but when inflows are uncertain, the carry-over storage can no longer be a fixed value. Thus the concept must be changed from a consideration of actual storage at the end of any year to an aspired level of storage at the end of any year. Then the carry-over storage can still be a decision variable and can be chosen at any level, provided that it does not exceed the mathematical expectation of the quantity of water in storage at the end of the year. Defining carry-over storage in this manner is realistic in that on the average once this level of carry-over storage has been attained, it can be expected to be reattained at the end of the year so that the same storage releases can be expected year after year. The constraints which satisfy the above conditions assure the safe-yield operation both of the surface reservoirs and of the groundwater reservoirs. This group of constraints introduces the uncertainty considerations into the mathematical model.

Shortages. Shortages have been defined for this study as the difference between a guaranteed quantity of water to be delivered and an amount of water that can actually be delivered. The actual

deliveries are based upon the available inflows, whereas the guaranteed deliveries are based upon average conditions and corrected in consideration of the shortage costs. A series of constraints defining the shortage variables is required for each area to which guaranteed and actual deliveries are allocated.

Other constraints. This group of constraints is made up of the constraints appropriate to a particular model. For example, several constraints may be required to define maximum physical storage capacities. Some constraints due to water rights considerations might also be included in this group.

CHAPTER IV
FORMULATION AND SOLUTION OF THE
MATHEMATICAL MODELS

The basic problem of water resources systems planning is the allocation of water from various sources to competing uses. Broadly speaking, mathematical programming problems deal with determining optimal allocations of limited resources to meet desired objectives. These problems are characterized by the large number of solutions which satisfy the basic conditions of each problem. The selection of a particular solution as the best solution depends on some over-all objective implied in the statement of the problem. Thus the problem is a two-sided one concerned not only with the allocation of limited resources among those uses competing for them, but also with the influence that these allocations will exert upon the objective.

In this study the limited resources are the quantities of water available in the groundwater reservoirs, the imported water supplies, and the local surface water. The surface water resource is to be managed optimally in conjunction with the groundwater resource to maximize the returns from irrigation.

Linear programming

A linear programming problem differs from the general mathematical programming problem in that the mathematical model or description of the problem can be stated using relationships that are "straight-line" or linear. Mathematically these relationships are of

the form

$$a_1x_1 + a_2x_2 + \dots + a_jx_j + \dots + a_nx_n = b_1$$

where the a_j 's are known coefficients, the b is the resource availability, and the x_j 's are decision variables. The complete mathematical statement of the linear programming problem includes a set of simultaneous linear equations which represent the conditions of the problem and a linear function which describes the objective of the problem. The mathematical statement of a general form of the linear programming problem is the following. Find x_1, x_2, \dots, x_n which maximize the linear objective function

$$Z = c_1x_1 + c_2x_2 + \dots + c_nx_n \tag{4-1}$$

subject to the constraints,

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq, =, \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq, =, \geq b_2 \\ &\vdots \\ &\vdots \\ &\vdots \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq, =, \geq b_m \end{aligned} \tag{4-2}$$

and

$$x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0,$$

where the a_{ij} , b_i , and c_j are given constants. The x_j 's are the decision variables. Written in matrix notation the problem statement becomes: Find X to maximize the objective function

$$Z = CX \quad (4-3)$$

subject to the constraints

$$AX \{ \leq, =, \geq \} B \quad (4-4)$$

and

$$X \geq 0$$

where $A = \{a_{ij}\}$; $X = \{x_j\}$; $B = \{b_i\}$; and

$C = \{c_j\}$, and where

$i = 1, 2, \dots, m$, and $j = 1, 2, \dots, n$.

In linear programming terminology any set of x_j 's which satisfies the constraints is called a *solution* to the linear programming problem. A solution which also satisfies the non-negativity conditions is called a *feasible solution*. A feasible solution which optimizes the value of the objective function is called an *optimal feasible solution* (Hadley, 1962).

The linear constraints represent a set of hyperplanes dividing the space into a series of half spaces, the intersection of which forms a convex set. Only points in this set satisfy the constraints and become feasible solutions to the linear programming problem. The

extreme points of this convex set of solutions are *basic feasible solutions* and if an optimal solution exists, at least one basic feasible solution will be optimal. If the optimal solution is not unique, points other than extreme points are also optimal.

All techniques actually used in obtaining an optimal solution to a linear programming problem are iterative. No method has been devised yet which will yield the optimal solution in a single step. The best known and most efficient method for solving linear programming problems is called the simplex method. This method is an algebraic iterative procedure or algorithm which will solve, exactly, any linear programming problem, properly formulated in a finite number of steps.

Briefly, the simplex algorithm can be described as a method which proceeds in systematic steps from an initial basic feasible solution to adjacent basic feasible solutions and finally in a finite number of steps to an optimal basic feasible solution. The value of the objective function at each step (iteration) is better (or at least not worse) than at the preceding step. Because the value of the objective function is improved (or at least not worsened) at each step, the number of iterations needed before an optimal solution is arrived at is, in general, small relative to the total number of existing basic solutions. In linear programming the basic feasible solutions are "corners" on the boundaries of the convex set. If there is an optimal solution, one of the extreme points is optimal. Thus, in common terms, the simplex method involves moving along the edge of the region of feasible solutions from one corner to an adjacent one in such a manner that each step gives the maximum increase (or

decrease) in the value of the objective function. At each corner the simplex method indicates whether the corner is optimal and if not which extreme point will be the next one examined in the iterative procedure.

If at any stage the simplex method comes to an extreme point which has an edge leading to infinity (unbounded convex set) and if the value of the objective function can be increased (or decreased) by moving along that line, an unbounded solution is indicated.

In formulating a linear programming problem for the simplex method of solution, slack variables are used to change the inequalities to equalities. Thus the problem is treated as a system of linear equations. The slack variables take on physical meaning in an applied problem, and their values represent the amount of the resource redundant to the optimal activities of the final solution.

For a more detailed discussion of the theory of linear programming solutions, see Gass (1964), Hadley (1962), and Hillier and Lieberman (1967). The key to the successful application of linear programming is the ability to recognize when a problem can be solved by linear programming and to formulate the corresponding model.

Shadow prices and the dual

According to Dorfman, Samuelson, and Solow (1958) resource allocation and pricing are two aspects of the same problem, and since linear programming solves the allocation problem it also solves the pricing problem. This is the essence of the dualism property of linear programming in an economic interpretation.

The formulation of a typical linear programming problem is shown in Equations 4-1 and 4-2. This formulation is known as the primal of the linear programming problem. The dual of the linear

programming problem is formulated from the primal formulation as follows:

1. Transpose the rows and columns of the constraint coefficients.
2. Transpose the objective function coefficients and the right-hand side values of the constraints.
3. Reverse the inequalities of the constraints
4. Minimize instead of maximize.

Analytically, then, the statement of the dual problem is to find $w_i \geq 0$ ($i = 1, 2, \dots, m$) in order to minimize

$$Z' = b_1 w_1 + b_2 w_2 + \dots + b_m w_m$$

subject to the constraints,

$$a_{11} w_1 + a_{21} w_2 + \dots + a_{m1} w_m \geq c_1$$

$$a_{12} w_1 + a_{22} w_2 + \dots + a_{m2} w_m \geq c_2$$

⋮

$$a_{1n} w_1 + a_{2n} w_2 + \dots + a_{mn} w_m \geq c_n$$

In the formulation seen above, the coefficients of the j^{th} constraint of the dual formulation are the coefficients of x_j in the primal constraints, and vice versa. Also, the right-hand side of the j^{th} dual constraint is the coefficient of x_j in the primal objective function, and vice versa. Hence, there is one dual constraint for each primal variable and one dual variable for each primal constraint.

The relationship between the primal problem and its dual are summarized as follows (Dorfman, Samuelson, and Solow, 1958):

1. The dual has one variable for each constraint in the original problem.

2. The dual has as many constraints as there are variables in the original problem.
3. The dual of a maximizing problem is a minimizing problem, and vice versa.
4. The coefficients of the objective function of the original problem appears as the constant terms of the constraints of the dual, and the constant terms of the original constraints are the coefficients of the objective function of the dual.
5. The coefficients of a single variable in the original constraints become the coefficients of a single constraint in the dual. Stated visually, each column of coefficients in the constraints of the original problem becomes a row of coefficients in the dual.
6. The sense of the inequalities in the dual is the reverse of the sense of the inequalities in the original problem, except that the inequalities restricting the variables to be nonnegative have the same sense in the direct problem and the dual.

The optimal dual problem provides a very useful economic interpretation of the primal problem. To illustrate this point, let w_i^* denote the optimal value of the i^{th} dual variable w_i ($i = 1, 2, \dots, m$), and recall the corresponding i^{th} constraint in the primal problem, $a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \leq b_i$. The value b_i is interpreted as the amount of resource i available, whereas the optimal value of the objective function might be interpreted as the total net benefits obtained by using the optimal solution. In this case the w_i^* indicates the rate at which benefits increase (decrease) if the amount of resource i available were increased (decreased) over a certain range. (This range is the

range of b_i over which the original optimal basis is not changed.) Thus, w_i^* may be interpreted as the "marginal value" of resource i . For example, if one more unit of resource i were made available, the resulting increase of benefits would be w_i^* (assuming that the optimal basis remains the same).

The economic interpretation of the dual problem can be understood further by examining the dimensional units of the variables and their coefficients. Dracup (1966, p. 71) describes the dimensional relationships as follows:

The dimensions in the primal problem of c_j are dollars per unit of good j , i.e. dollars per acre-foot of water. The physical dimensions of the variables x_j are the units of some good produced for some given time period, i.e. acre-feet of water per year. The dimensions of b_i are units of resource i available in a given time period, i.e. acre-feet of water per year. The a_{ij} then have units of resource i per unit of good j . In the problem under investigation the a_{ij} terms are therefore dimensionless.

The dimensions of the dual problem are now considered. The units of $a_{ij}w_i$ have dimensions of dollars per unit good j , i.e. dollars per acre-foot of water. Since the a_{ij} terms in this problem are dimensionless, then the dimensions of w_i must be dollars per unit of resource i , i.e. dollars per acre-foot of water.

Thus the w_i are prices or values associated with units of resource i . These dual variables, w_i , are referred to as imputed values or shadow prices since they merely reflect the worth of the resource within the context of the model and in no way should they be construed to be the actual costs of the resource.

Sensitivity analysis

Practical problems that are formulated as linear programming problems are seldom completely "solved" as soon as the simplex algorithm identifies an optimal solution for the model. The coefficients

of the model (c_j, a_{ij}, b_i) are seldom known with complete certainty or to the desired degree of precision. Therefore, it is usually desirable to perform a sensitivity analysis to establish the effect on the optimal solution of changing particular coefficients to other possible values. If the sensitivity analysis indicates that the optimal value of the objective function is relatively sensitive to changes in certain coefficients, special care should be taken in estimating these coefficients. If errors and omissions are discovered or if new information so indicates, the estimates of the coefficients should be revised.

It is not necessary to re-solve the problem from the beginning each time a minor change is to be made in the model. Given the previous optimal solution and the corresponding set of equations, it is usually possible to determine whether the same basis is optimal and, if not, to use it as the starting point to solve quickly for the new optimal solution.

Formulation of the linear programming problem

The hypothetical conjunctive use system described in Chapter III is now formulated as a linear programming problem. Three mathematical models are formulated and solved for the hypothetical system. Basically the three models are:

Model 1. A two-season model including a wet season and a dry season.

Model 2. A single-season model in which all inflows (surface inflows and natural recharge) are deterministic.

Model 3. A single-season model in which all inflows (surface inflows and natural recharge) are probabilistic.

Model 1. The hypothetical basin modeled in Models 1, 2, and 3 is depicted schematically in Figure 4-1. The physical description of the basin was presented in Chapter III. The two-season model of the hypothetical basin is shown in a flow diagram in Figures 4-2a and 4-2b. The constraints and limitations on the two-season conjunctive use system are formulated from the following:

1. Surface supply during the dry season.
2. Surface supply during the wet season.
3. Water demand during the dry season.
4. Water demand during the wet season.
5. Hydraulic continuity of the groundwater system.
6. Groundwater storage capacities at various groundwater levels.
7. Downstream water requirements.
8. Recharge to the groundwater aquifer.

The objective function for this particular model expresses the total net benefits to be derived from the hypothetical water resources system. A cost or benefit in terms of dollars per acre-foot is assigned to each variable which appears in the objective function as well as the constraint system. The sign of each coefficient determines cost or benefit. A matrix map representing the mathematical model is shown in Figure 4-3. The matrix map shows the form of the matrix in coded pictorial form. It is readily seen from the matrix map that the matrix is a sparse matrix. The elements of the matrix map are code symbols

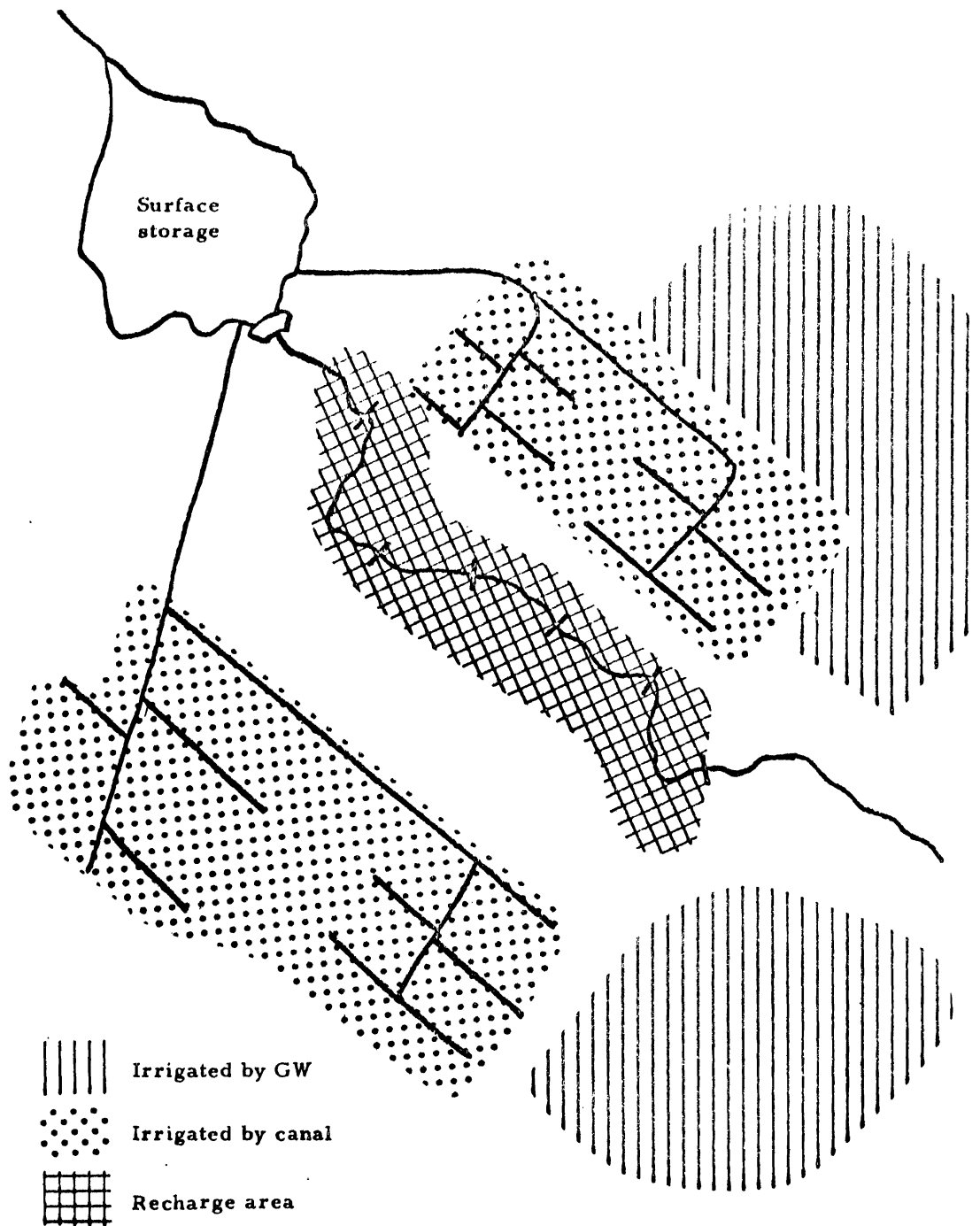


Figure 4-1. Schematic representation of the hydrologic model

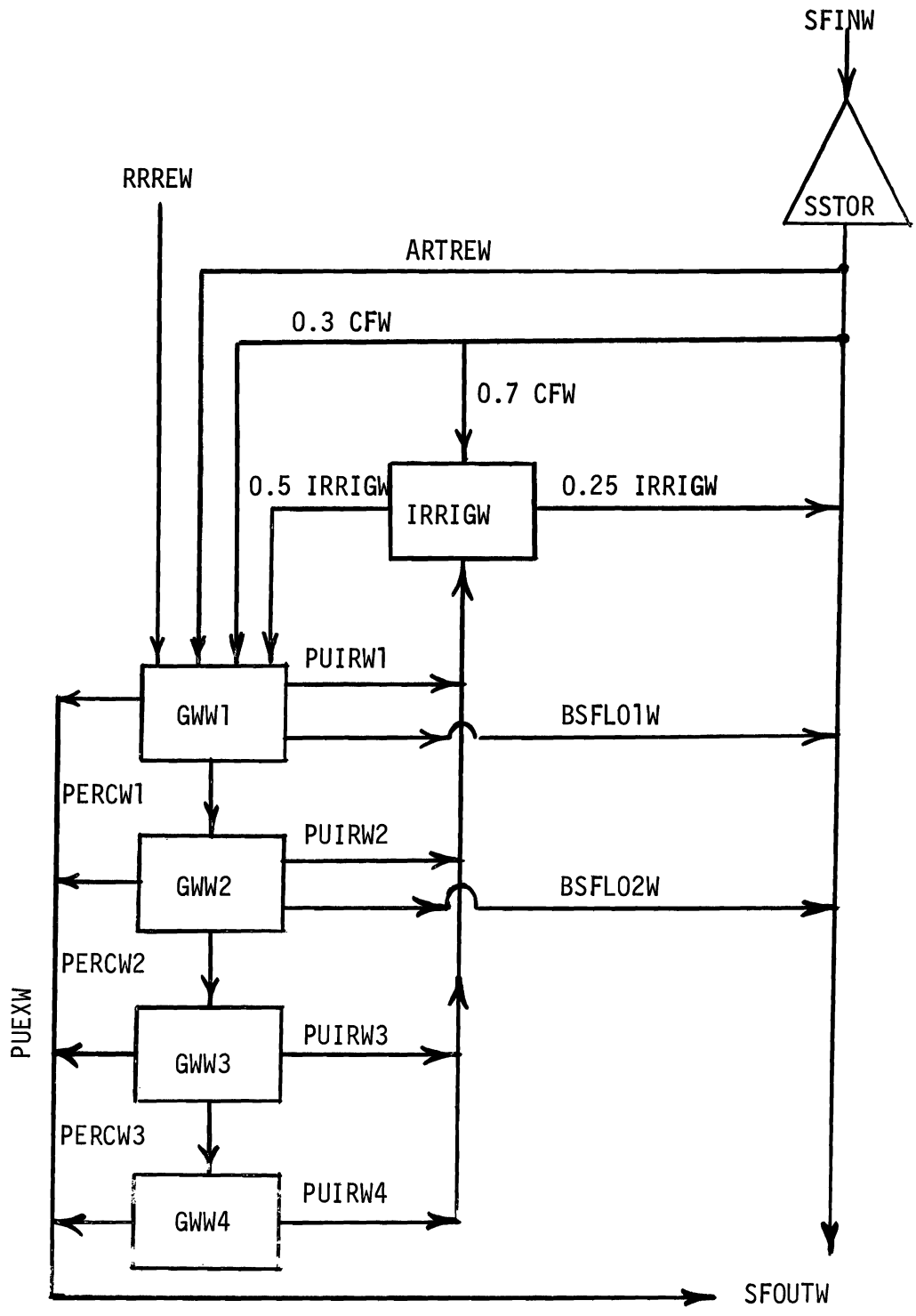


Figure 4-2a. Flow diagram of the hydrologic model (wet season)

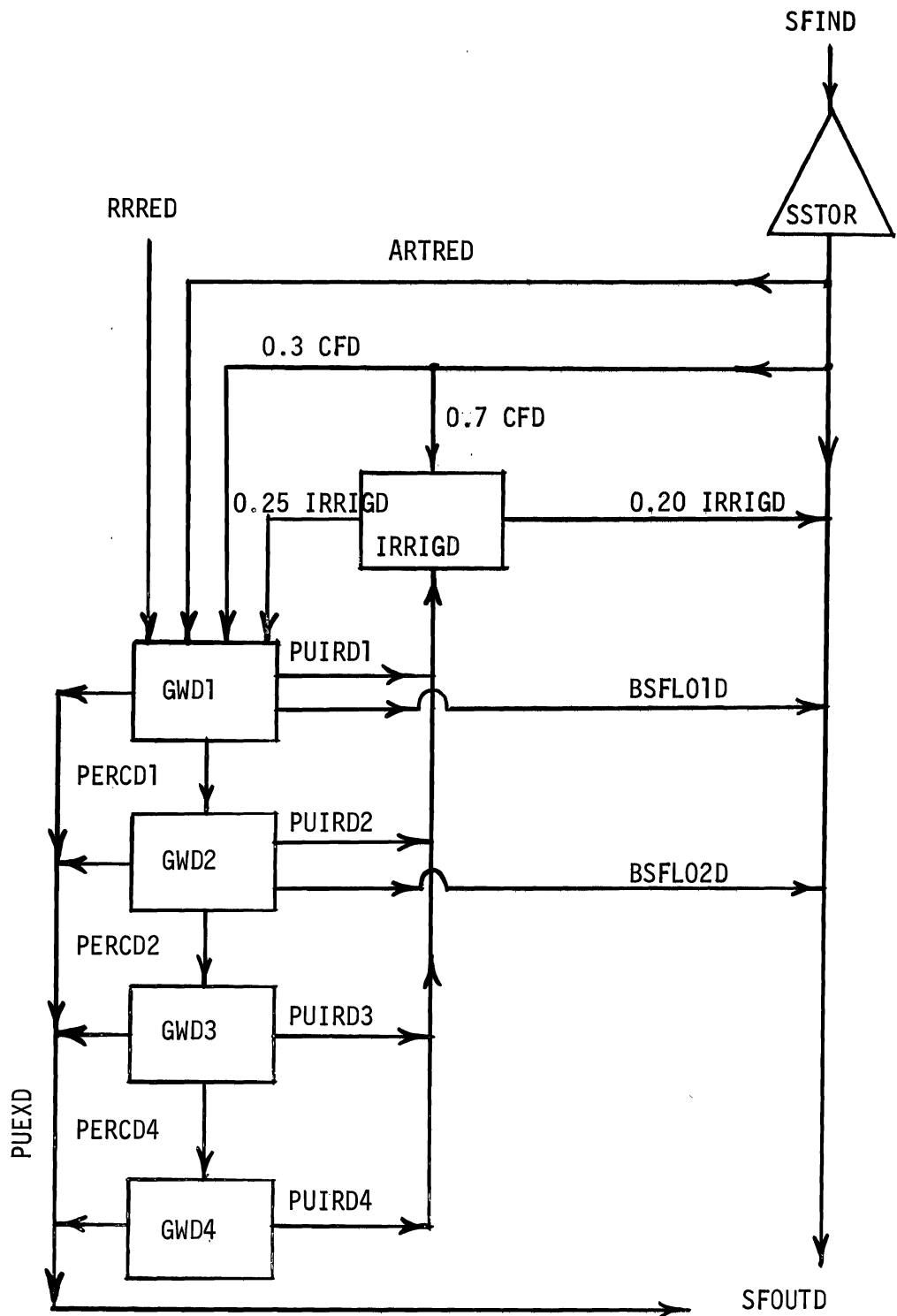


Figure 4-2b. Flow diagram of the hydrologic model (dry season)

representing the magnitude of the elements of the actual matrix which is the mathematical expression of the objective function and the constraint system. The code representation for the matrix map is given below.

The matrix elements are represented as follows:

<u>Code</u>	<u>Magnitude of elements</u>		<u>Occurrences</u>
	Greater than ~	Equal to or less than	
0	0.0	0.0001	0
2	0.0001	0.001	0
3	0.001	0.01	0
4	0.01	0.1	10
5	0.1	0.9999	10
1		=1.0	88
6	1.0	10.0	56
7	10.0	100.0	34
8	100.0	1000.0	6
9	1000.0		0

The row labels given in the matrix tableau are coded to the objective function and the constraints of the mathematical model. Row label BEN1 represents the objective function. All other row labels represent the various constraints. The column labels represent the variables included in the model with the exception of column labels COST, B-VEC, *B1, *B2, and *B3. The label COST has no meaning for this model. The labels B-VEC, *B1, *B2, and *B3 represent the right-hand side vector in the model. The remaining column labels are defined as follows:

Definition of variables:

GW - groundwater storage;	D or W following refers to dry or wet season, number following refers to level
PUIR - pumping for irrigation;	same as above
PUEX - pumping for export	same as above
PERC - percolation	same as above
IRRIG - irrigation	
SSTOR - surface storage	
CF - canal flow; D refers to dry season, W refers to wet season	
ARTRE - artificial recharge	same as above

The complete mathematical expression of the objective function and the constraint system is given in Figure A-1 of the Appendix, Matrix Tableau, which occupies several pages. The row and column labels in the Matrix Tableau are the same as those in the matrix map.

In the formulated problem, it is necessary to find the value of the variables which will satisfy the constraints and maximize the objective function (BEN1). From examination of the objective function, it is seen that the problem is to optimally allocate water from five sources (four groundwater storage reservoirs and one surface storage reservoir) to three uses (irrigation, artificial recharge, and downstream requirements); and each source and use is represented in both seasons.

Water allocated to irrigation yields a net benefit, but costs associated with making that water available are also incurred. These costs are the costs of storing the water in surface storage, of conveyance by canals, and of pumpage. Artificial recharge costs are incurred when water is allocated to that use. No costs are incurred in meeting downstream requirements except loss of benefits from irrigation and possibly pumping for export of water downstream. The determination of the actual values of these cost and benefit

coefficients was discussed in Chapter III.

Model 2. The single-season model in which all inflows are deterministic is depicted in the flow diagram shown in Figure 4-4. This model is a much simpler model than the two-season model. In this case the objective function to be maximized is:

$$\begin{aligned} Z_{\max} = & 45.00 \text{ IRRIG} - 8.20 \text{ CF} - 15.00 \text{ ARTRE} - 5.00 \text{ PUIR} \\ & - 3.50 \text{ STCAP} \end{aligned}$$

subject to the following constraints:

$$\begin{aligned} \text{CF} + \text{ARTRE} - \text{SSTOR} & \leq 27.8 \\ -0.6 \text{ CF} - \text{ARTRE} + \text{PUIR} - \text{GWST} & \leq 18.6 \\ -\text{CF} - \text{ARTRE} - \text{STCAP} & \geq -27.8 \\ 0.6 \text{ CF} + \text{ARTRE} - \text{PUIR} - \text{GWCAP} & \geq -18.6 \\ -0.4 \text{ CF} - \text{PUIR} + \text{IRRIG} & \leq 0 \\ \text{GWCAP} & \leq 55.0 \\ \text{GWST} - \text{GWCAP} & \leq 0 \\ \text{SSTOR} - \text{STCAP} & \leq 0 \end{aligned}$$

In the objective function the coefficient for IRRIG is a benefit coefficient indicating a net benefit of \$45.00 per acre-foot of applied irrigation water. Conveyance and distribution (CF) cost is \$8.20 per acre-foot of water diverted. The cost of artificial recharge (ARTRE) is \$15.00 per acre-foot of water recharged to groundwater. Pumpage from groundwater for irrigation (PUIR) costs \$5.00 per acre-foot of water extracted. Surface storage capacity (STCAP) is assumed to cost

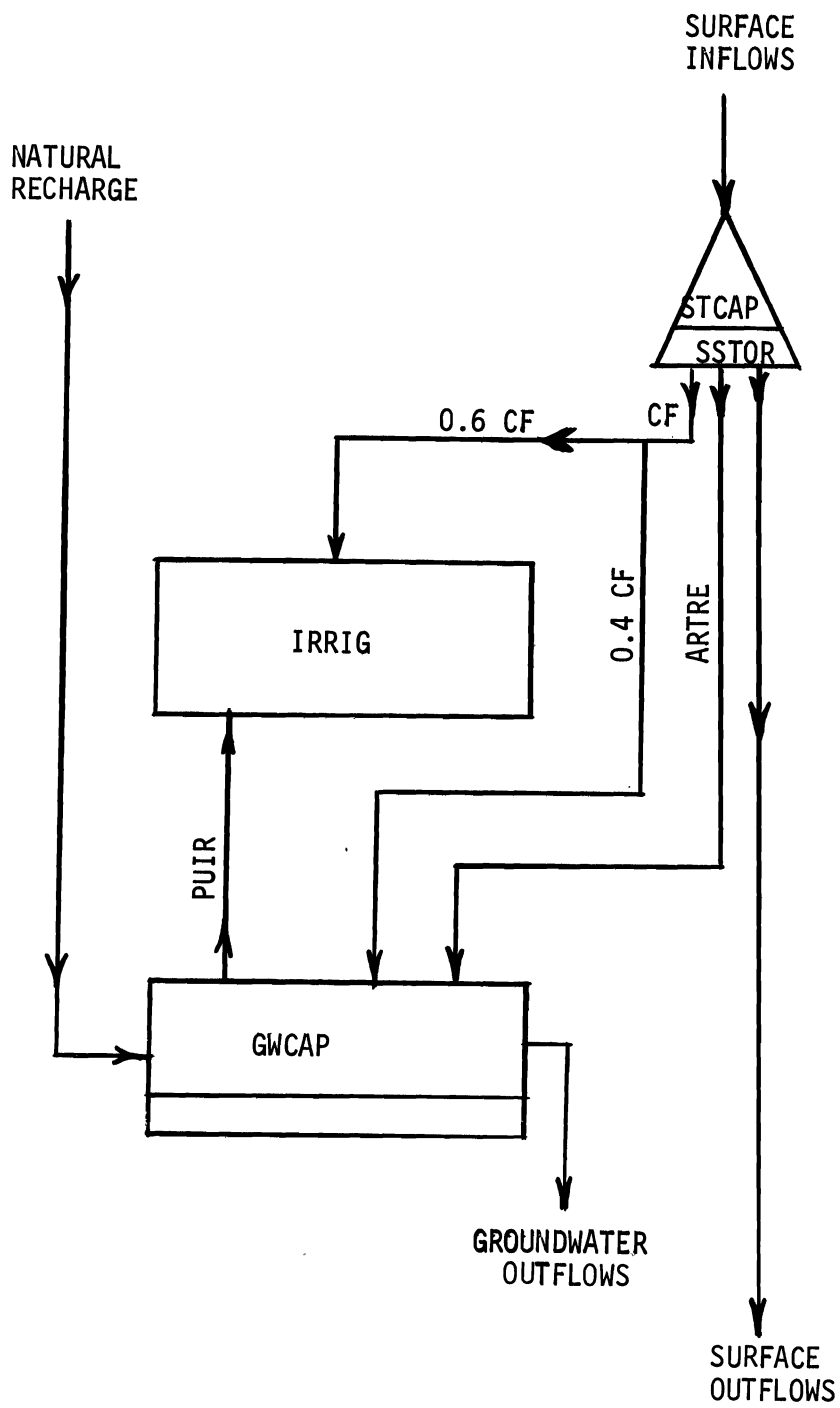


Figure 4-4. Flow diagram of Model 2--simplified conjunctive use model with deterministic inflows

\$3.50 per acre-foot of storage capacity.

The remaining variables in the constraints are defined as follows:

SSTOR = amount of carry-over storage in the surface reservoir.

GWST = amount of carry-over storage in the groundwater reservoir.

GWCAP = the capacity needed in the groundwater reservoir.

There are no direct benefits or costs associated with these variables. Again in this model the problem is to optimally allocate water from a surface-water source and from a groundwater source to the competing uses which are irrigation, artificial recharge, and downstream requirements.

The first two constraints describe the requirement that releases from storage must be less than the sum of the initial storage plus inflows. The next two constraints formulate the requirement that at any time the streamflows below the reservoir must be nonnegative. The requirement that the delivery to irrigation must not exceed the possible extractions from storage is formulated in the fifth constraint. The sixth constraint indicates that the maximum physical capacity of the groundwater reservoir is 55,000 acre-feet. Finally, the last two constraints simply say that the initial storage, or carry-over storage, cannot exceed the storage capacity. The resource availabilities adjusted for evaporation losses are reflected in the right-hand sides of the constraints. In this model the average annual inflow to the surface reservoir is 27,800 acre-feet per year. The average annual natural recharge to groundwater storage is 18,600 acre-feet per year,

and the maximum available groundwater storage capacity is 55,000 acre-feet.

Model 3. The single-season model in which all of the inflows are uncertain or probabilistic is depicted in Figure 4-5. This model is based upon the same assumptions and physical model as the single-season model with deterministic inflows, with the addition of constraints and variables necessary to represent the uncertain inflows. The matrix form of the model is much larger (27 rows) because of the addition of constraints and variables used to represent the uncertain inflows. The matrix form of Model 3 is shown in Table A-2 of the Appendix. In the Matrix Tableau, the columns beginning with CF1 are the decision variables in the model. The column labeled COST is a column showing costs of slack variables which are all zero costs. The column labeled B-VEC is the right-hand side elements of the constraints and is repeated at the end of the tableau under the label *B1. The column labels are interpreted as follows:

CF_i = diversions to conveyance network supplying irrigation deliveries.

$ARTRE_i$ = diversions to artificial recharge of groundwater.

SSTOR = initial or carry-over storage capacity in surface reservoirs.

$PUIR_i$ = pumpages from groundwater supplying irrigation deliveries.

STCAP = surface storage capacity.

GWST = initial or carry-over storage in groundwater reservoirs.

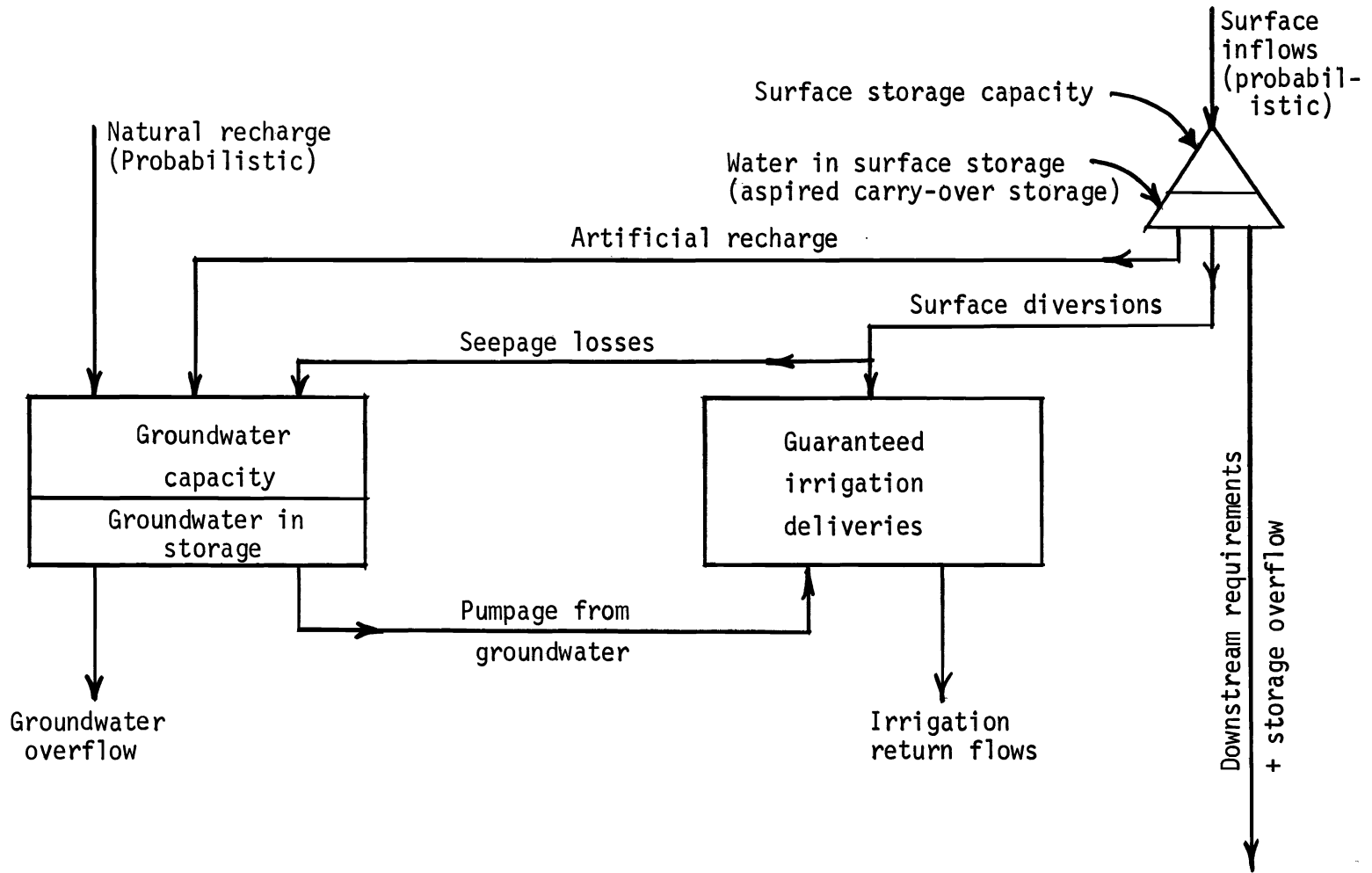


Figure 4-5. Conjunctive use model with uncertainty in inflows

GWCAP = required groundwater storage capacity.

SHORT_j = annual amounts of shortage.

IRRIG = guaranteed annual delivery to irrigation.

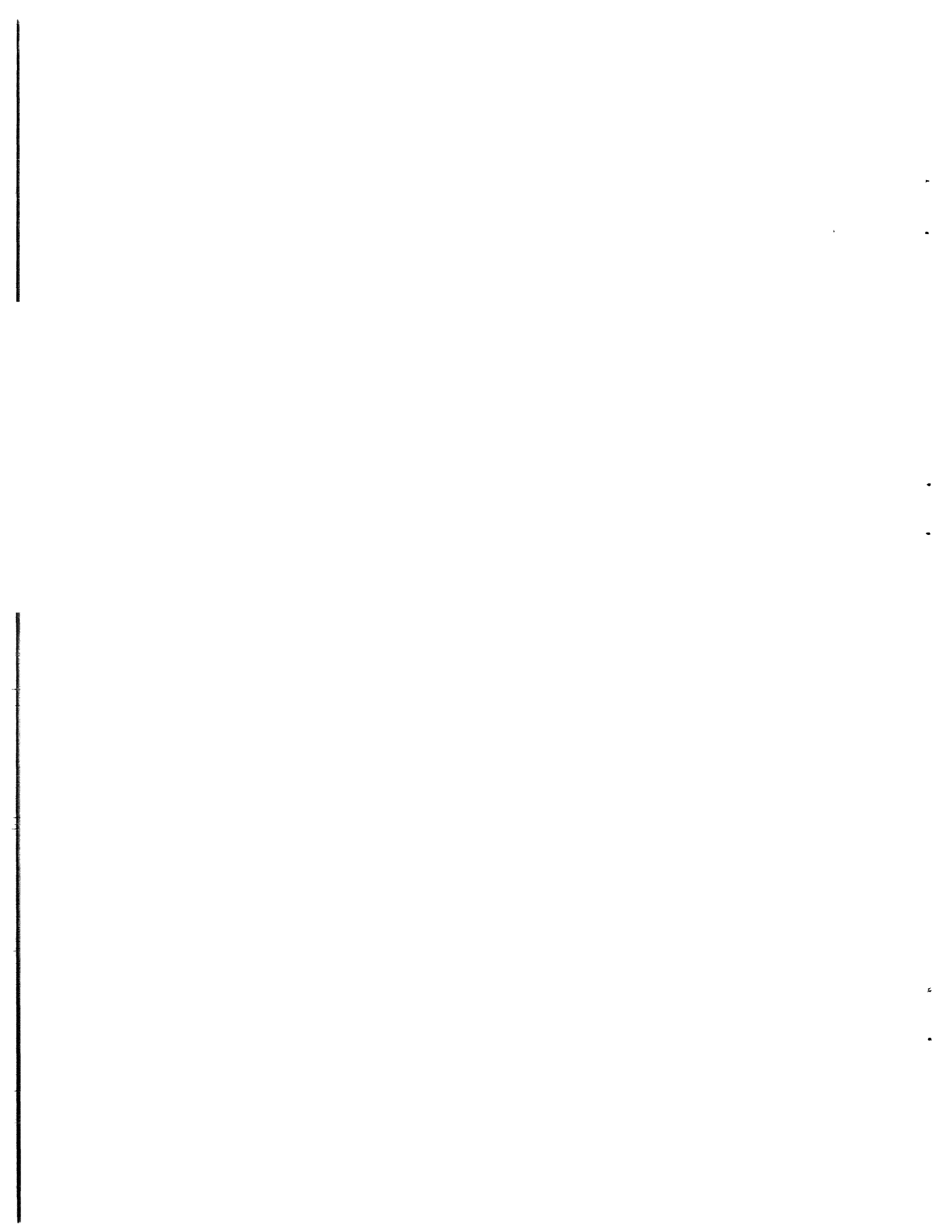
In this model of probabilistic inflows CF_j is the amount of water diverted when a surface inflow volume of $SFIN_j$ is realized. Similarly $ARTRE_j$ is the volume of water allocated to artificial recharge when a surface inflow volume of $SFIN_j$ is available and $PUIR_j$ is the volume of water to be pumped when a natural recharge volume $NATRE_j$ is available. The values used for available surface inflows, $SFIN_j$, and natural recharge volumes, $NATRE_j$, are the available resources included in the right-hand side values of the constraints.

Referring again to the Matrix Tableau, the rows labeled COST and BEN1 are the same and are the objective function of the model. Rows 2 through 6 are constraints describing the condition that releases from surface storage must not exceed the sum of initial storage and inflows. Rows 7 through 11 describe the same condition for groundwater storage. Rows 12 through 21 formulate the constraint that at any time the storage contents must not exceed the storage capacity for both surface storage and groundwater storage. Rows 22 and 23 define the expectations of surface inflows, of natural recharge, and of carry-over storages. Rows 24 through 28 define the shortages and the requirement that the guaranteed deliveries to irrigation are made up of pumpages and deliveries through canals. Conveyance losses and percolation losses are accounted for in all constraints. The average annual resource availabilities are the same in Model 3 as in Model 2 of the hypothetical basin.

The computer program

The linear programming problems formulated above were solved using the Univac 1108 computer and an advanced large scale linear programming system provided by Univac. The Univac linear programming system employs a modified simplex method in which the inverse is maintained in product form. The system provides capacity for solving models up to 4094 rows and 99,000 columns in size.

The actual solutions of the linear programming problems formulated in this chapter will be presented and discussed in the next section.



CHAPTER V

RESULTS OF THE HYPOTHETICAL BASIN STUDIES

Results of the hypothetical basin studies are discussed in this chapter with reference to the three models formulated in the preceding chapter. The three models are: (1) Model 1--a two-season model consisting of a wet season and a dry season; (2) Model 2--a single-season model in which the system inflows are deterministic; and (3) Model 3--a single-season model in which the system inflows are probabilistic. The objective of the analysis was to optimize the allocation of water resources available in the hypothetical basin by use of linear programming. The results are discussed in terms of the linear programming optimization.

Results--Model 1

Model 1 was constructed such that 80 percent of the irrigation requirement had to be met during the normal irrigation season (the dry season), and 20 percent of the requirement could be met during the winter season by pre-irrigation methods. The solution of this model yielded the interesting result that all of the irrigation requirement in the normal irrigation season would be supplied by pumping groundwater. Pumping would be continued during the winter irrigation season, but at a much lower level; and a considerable amount of water would be distributed through canals for winter irrigation. Artificial recharge did not enter the solution, even when the cost of artificially recharged water was reduced from \$15.00 per acre-foot to \$5.00 per

acre-foot. Changing the irrigation benefits from \$20.00 per acre-foot to \$45.00 per acre-foot did not change the original optimal activities, viz., the pumping and canal flow quantities as well as all other quantities remained the same over this range. Sensitivity analysis on this model indicates that the optimal conjunctive use pattern is fairly insensitive to changes in the objective function coefficients. The analysis indicates that surface-water diversions during the dry season would enter the solution if the costs of surface-water distribution could be reduced from \$8.20 to \$6.65 per acre-foot. This cost coefficient is the most sensitive to change. The reduction in costs to \$6.65 per acre-foot seems to be well within the practical range of values to be expected. Many actual operations suggest that this value should be below \$6.65 for surface distribution systems.

The groundwater basin is operated on a safe-yield basis. The total of dry season and wet season pumpage is equal to the total of all inflows to the groundwater basin. The inflows to the groundwater basin consist of natural recharge which is independent of the decision variables and of deep percolation losses from the surface distribution system and from irrigation. In this particular model the losses to groundwater from irrigation were more than twice the total of all conveyance losses to groundwater and natural recharge. Pumping for irrigation would not be optimal if the natural recharge were reduced from 9,000 acre-feet per year to 4,000 acre-feet per year.

This model included a downstream requirement of 50,000 acre-feet per year. The actual releases to downstream requirements for the optimum level were 50,100 acre-feet per year.

In summary, the optimal allocations indicated for this model

satisfy all of the model requirements, and the results appear to be realistic. It is interesting to note the high level of groundwater development for irrigation, while surface water distribution is at a fairly low level.

Results--Model 2

Model 2 is somewhat simpler than Model 1 since it is a single-season model with deterministic inflows. That is, average annual inflows are used, and the single season is simply the average year. The purpose for using this model was to facilitate comparison of results from a deterministic model with results of a stochastic model.

Optimal water resource development in this model includes both surface-water development and groundwater development. The amounts of water developed from each source are nearly equal with groundwater development at 29,700 acre-feet per year and surface-water development at 27,800 acre-feet per year. Activities which are not part of the optimal solution include carry-over surface storage and artificial recharge to groundwater storage. However, artificial recharge would enter the solution if the costs of artificial recharge were reduced from \$15.00 per acre-foot to \$5.20 per acre-foot, or if pumping costs were increased from \$5.00 per acre-foot to \$11.34 per acre-foot, or if surface-water distribution costs increased from \$8.20 per acre foot to \$18.00 per acre-foot. Surface storage is needed only to the extent of providing seasonal storage capacity. About 60 percent of the groundwater storage capacity is used.

Post-optimal analysis on this model shows that as available

water supplies decrease, the amounts of water allocated through canal flow and pumpage naturally decrease; but it is also interesting to note that canal flows eventually exceed pumpage, which was not the case at the original optimum solution. When available inflows (surface and groundwater) have been decreased by about 30 percent, canal flow is 18,800 acre-feet per year, while pumpage is 17,120 acre-feet per year. The optimal water development pattern does not change drastically for decreasing or increasing water availability.

The optimal water development pattern was also investigated for changes in some of the objective function coefficients. Irrigation benefits would have to be reduced to \$10.00 per acre-foot before the development pattern would change. The effects of changing costs of artificial recharge are discussed in a previous paragraph.

Again, in this model the groundwater basin is operated on a safe-yield basis with annual pumping equal to the average annual inflow to the groundwater basin. About 30 percent of the inflow to the groundwater basin comes from deep percolation losses from the surface-water distribution system. If natural recharge to groundwater were reduced by about 40 percent it would no longer be optimal to develop groundwater for irrigation by pumping at a pumping cost of \$5.00 per acre-foot.

In summary, the optimal allocations for this model appear to be realistic, and the hydrologic requirements of the basin are satisfied. The linear programming solution provides useful information other than the optimal activity levels of the decision variables.

Results--Model 3

The single-season model in which all of the inflows are uncertain or probabilistic is represented as Model 3. Because of uncertainties in the inflows, shortages must also be allowed for in this model with appropriate shortage costs appearing in the objective function.

The major difference in results of Model 2 and Model 3 is that in Model 3 storage variables (surface storage and groundwater storage) enter the optimal solution at much higher levels. This result is as would be expected. Uncertainty in the inflows along with high shortage costs would naturally place greater emphasis on storage of water to avoid shortages. Because of this greater emphasis on storage, the emphasis is also shifted more toward groundwater development with its cheaper and larger storage capacity. In fact, in this model the optimal activity levels show no canal flows or surface distribution. Several modifications could cause canal flows to enter the optimal solution. The cost coefficients are the model parameters which can most readily be changed. Reducing the costs of surface distribution systems by about 60 percent would allow canal flows to enter the optimal solution. Increases in pumping costs and in costs of artificially recharged water would also allow canal flows to enter the optimal solution. This change in solution is much more sensitive to increases in these latter costs than to reductions in the surface distribution system costs.

For this model, the effects of changing the irrigation benefit coefficient was also investigated. It was found that when irrigation benefits were reduced to \$15.00 per acre-foot of water delivered for

irrigation the optimal solution changed. The changes are mostly in water storages and are summarized as follows:

1. The optimal amount of surface storage is reduced.
2. The optimal amount of groundwater storage is increased.
3. The amount of carry-over storage remains the same for surface storage but decreases slightly for groundwater storage.
4. The guaranteed level of irrigation delivery is reduced.
5. The amount of surface water wasted downstream is increased.

It is evident that the reduction of irrigation benefits to \$15.00 per acre-foot causes a major change in the optimal solution, but the change occurs only after a large amount of change in the irrigation benefit.

The effect of reducing the losses from the surface-water conveyance system was also investigated for this model. It was found that the effect of lining all canals (seepage losses to groundwater are essentially zero) is generally to increase the importance of the surface distribution system. With this modification in the model, canal flows were found in the optimal solution, while pumpage and groundwater storage were reduced. Surface storage is increased, and artificial recharge is greatly reduced.

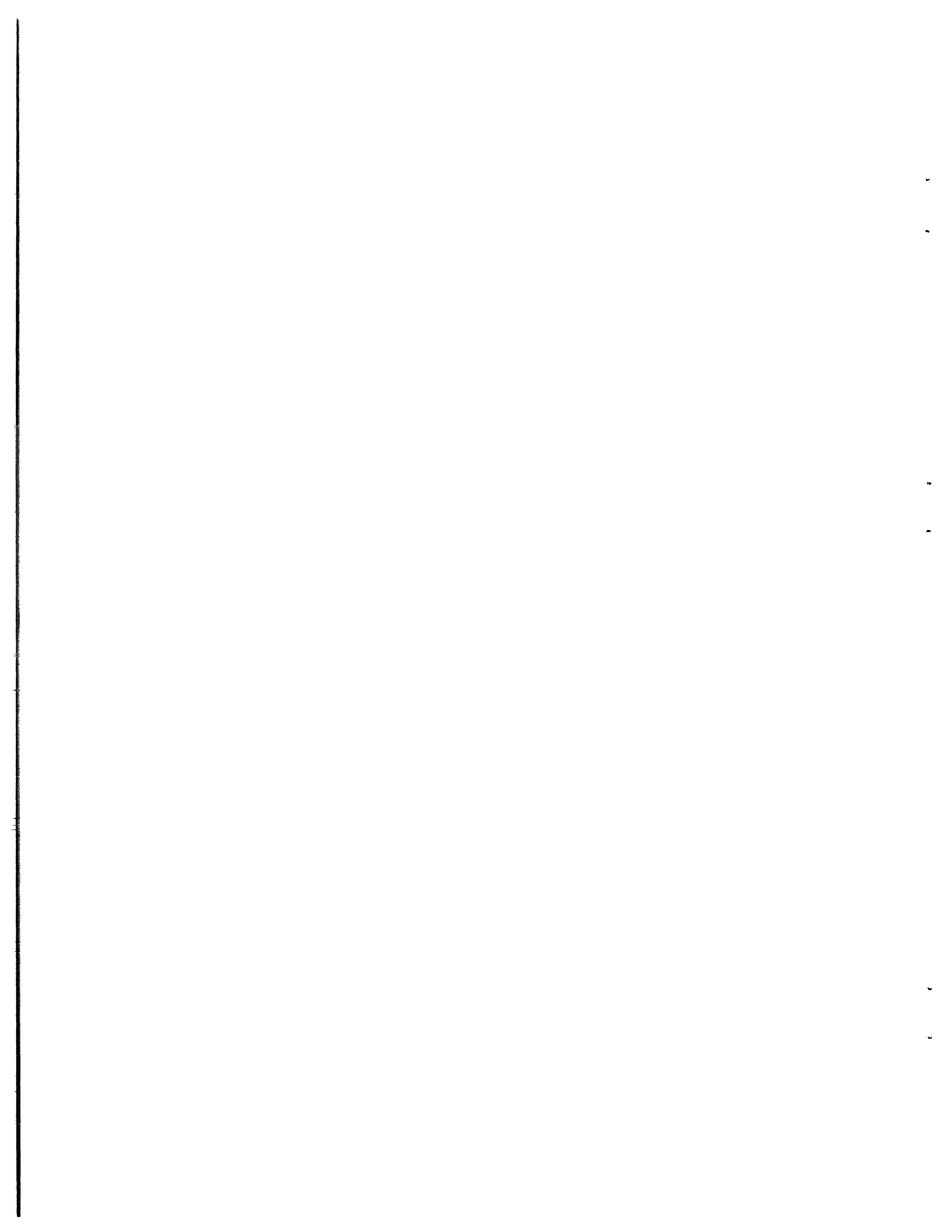
The model is very insensitive to changes in the probability distributions of natural inflows as long as the mean values of the inflows remain nearly the same.

In general, the solution to Model 3 yields more information than solutions from the other two models. However, Model 3 required considerably more data and effort for formulation and more computer time and expense for its solution. In actual practice the value of

the additional information from the stochastic model would have to be weighed against the increased costs of getting the solution.

General results--hypothetical
basin studies

The primary function of the hypothetical basin studies was to study the methodology of applying linear programming techniques to the solution of the conjunctive use problem and to determine the kinds of information that could be obtained from the various types of models. With the completion of the hypothetical basin studies, the stage was set for applying the technology to real basin studies. This is the topic of the next two chapters.



CHAPTER VI
APPLICATION TO A RIVER BASIN--LITTLE
LOST RIVER BASIN, IDAHO

The Little Lost River Basin in Idaho was chosen as a simple river basin to model as a real river system in order to test the methodology developed in the hypothetical basin studies. This is one of several such basins along the northwest flank of the Snake River plain that has no surface outlet to the Snake River. The general location of the basin is shown in the map, Figure 6-1. The economy of the area depends almost entirely upon agriculture; and with a very small population in the valley, almost all of the water demand is for agricultural use. The average annual precipitation on the valley floor is about 10 inches, so irrigation is required for production of cultivated crops.

Prior to about 1954, the source of irrigation water was almost entirely surface water. Substantial groundwater development began in about 1954, and by about 1960, approximately 40 percent of the water supply for irrigation came from groundwater. Total water use for irrigation in 1960 was about 93,000 acre-feet.

The physical system

The Little Lost River Basin is roughly rectangular, about 50 miles long and 15 to 25 miles wide, and encloses slightly over 900 square miles of drainage area. The basin is flanked by high mountain

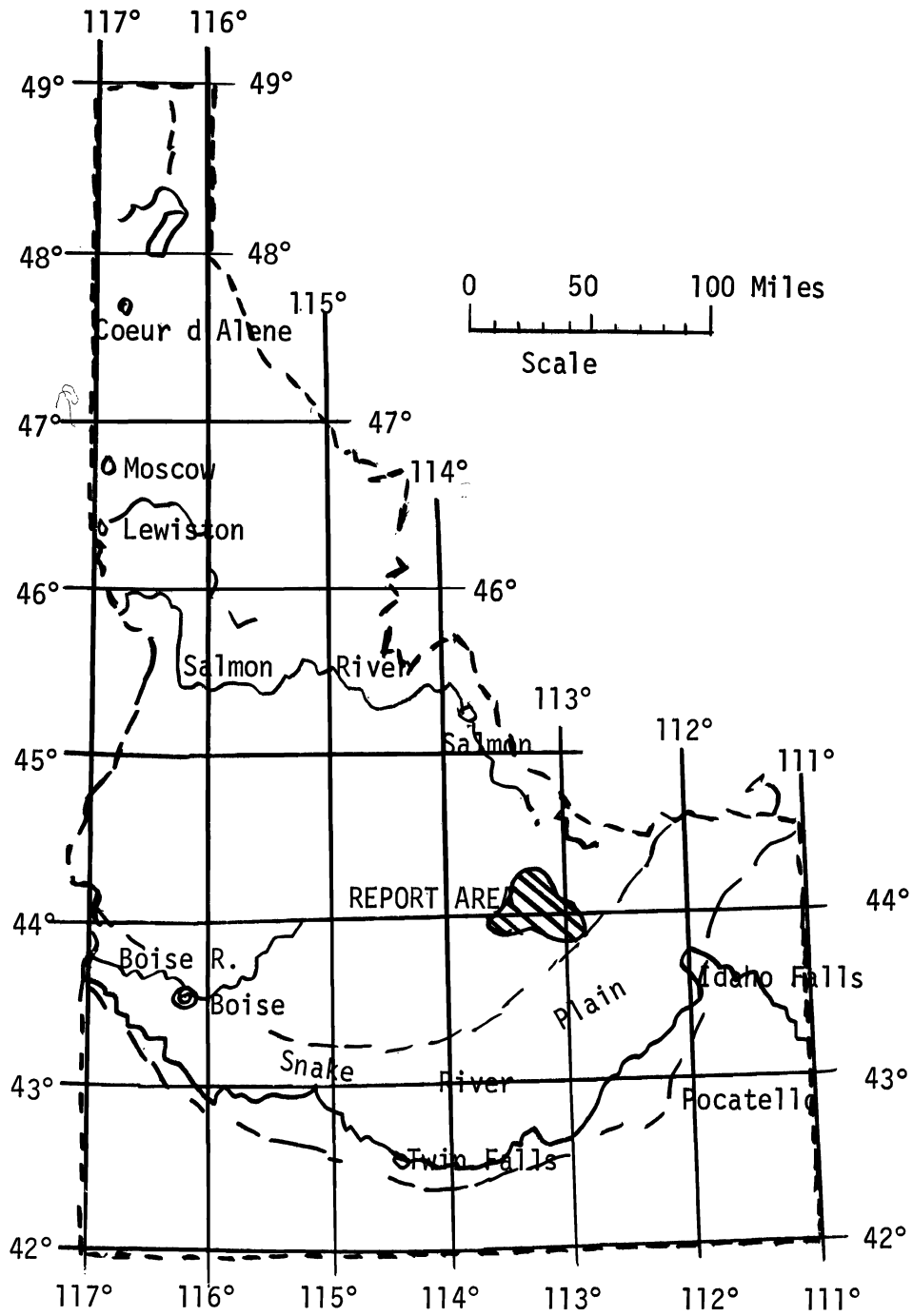


Figure 6-1. Location map of Little Lost River basin, Idaho

ranges with the Lost River Range on the southwest and the Lemhi River on the northeast. The average height of the ridge crests is probably about 10,000 feet above sea level. These high mountain ranges receive moderately large amounts of precipitation. The runoff from these flanking mountains percolates into the porous and highly permeable alluvium that forms the valley floor. Several large alluvial fans have been formed by streams from the flanking mountains, and in places these fans extend more than half way across the valley floor.

The valley of the Little Lost River was formed by block faulting of the type characteristic of the Basin and Range Physiographic Province. Previous studies (Baldwin, 1951) show a normal fault along the southwest base of the Lemhi Range throughout the length of the valley. Several other faults within the valley further complicate the geologic structure, which is the primary factor in controlling the movement and occurrence of groundwater within the basin. The single most prominent geologic feature affecting the occurrence of groundwater in the valley is a low bedrock ridge about 11 miles upvalley from the town of Howe. This ridge projects from the Lemhi Range about half way across the valley and seems to cause a groundwater barrier completely across the valley.

The depth of the alluvial material in the block-faulted valley is not known. The slopes of the flanking mountain ranges suggest that the valley fill might be about 3,000 feet thick. The fan material at the edges of the valley floor has been reworked and stratified by the Little Lost River thus giving the alluvial material near the river much higher permeabilities than the poorly sorted alluvial fan deposits.

Near the mouth of the valley, southeast of the town of Howe,

the alluvium is composed almost entirely of fine sandy silt. In this area, basalt is exposed at the surface; and drillers logs of wells in the area indicate that basalt is interbedded with the alluvium in this area.

The U. S. Geological Survey (Mundorf, Broom, and Kilburn, 1963) measured most of the tributary streams in the valley as well as the Little Lost River at several locations. In this same study an inventory of the wells in the valley was also made. Using this information, the average annual water yield of the basin was estimated at about 200,000 acre-feet. Current consumptive use by irrigated agriculture in the basin was estimated at about 25,000 acre-feet per year.

Formulation of the mathematical model

In developing the linear programming model of the Little Lost River basin, the valley was further subdivided into two parts: an upper basin and the Howe basin. The boundary between the upper basin and the Howe basin is formed by the bedrock ridge which extends across the valley forming a groundwater barrier. This bedrock ridge is near the area referred to as Fallert.

Flow diagram. Figure 6-2 is a flow diagram of the system formulated in the mathematical model. Shown on the diagram are the upper basin and the Howe basin with the associated irrigated areas, tributary inflows and diversions for irrigation, and artificial recharge. The two groundwater basins are also shown with associated inflows from natural recharge, percolation from conveyance and irrigation losses, and artificial recharge. Interpretation of the symbols used in the diagram and in the mathematical model is as follows:

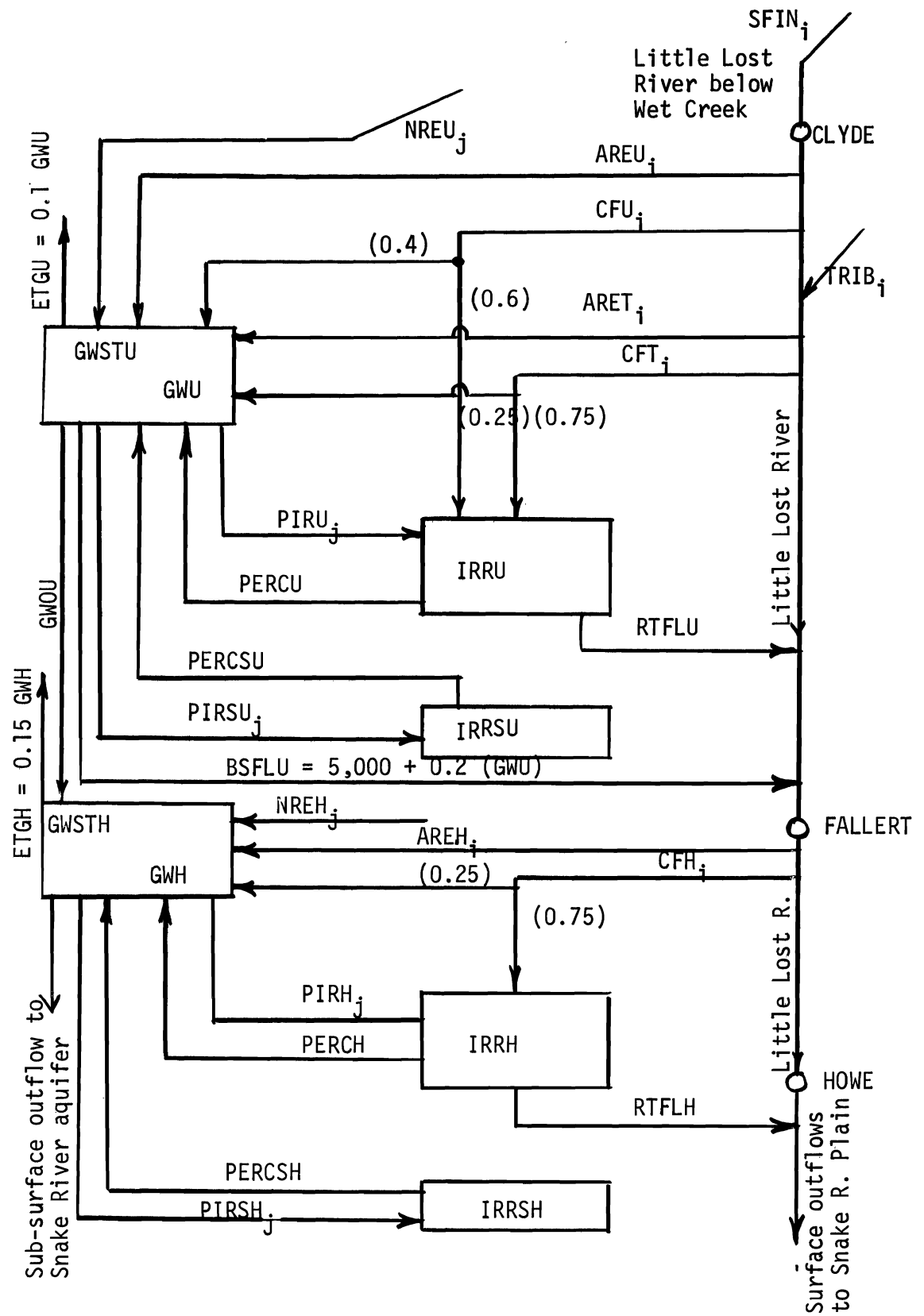


Figure 6-2. Flow diagram for Little Lost River basin

- SFIN_i = surface inflow from streams above Clyde in the upper basin (probabilistic inflows)
- TRIB_i = tributary inflows to the main river in the upper basin
- SREU_g = natural recharge from streams and precipitation in the upper basin (probabilistic input)
- AREU_i = artificial recharge diversions in the upper basin above lower tributaries
- CFU_i = diversions to canals in the upper basin above lower tributary inflows
- ARET_i = artificial recharge diversions in the upper basin below lower tributary inflows
- CFT_i = diversions to canals in the upper basin below lower tributary inflows
- ETGU = evapotranspiration from groundwater in storage-- taken as 10 percent of carry-over groundwater storage
- GWSTU = groundwater storage capacity used in the upper basin
- GWU = groundwater carry-over storage in the upper basin
- PIRU_j = pumpage from groundwater storage in the upper basin
- PERCU = percolation losses from irrigation in the upper basin--taken as 35 percent of the water applied
- RTFLU = return flow from irrigation in the upper basin to the Little Lost River--taken as 20 percent

of the water applied

- IRRU = guaranteed annual quantity of irrigation water for use in the upper basin
- SHU_i = shortage of supply for irrigation in the upper basin = guaranteed level minus actual deliveries
- IRRSU = guaranteed annual quantity of irrigation water for new sprinkler irrigation development on upper northeast bench in the upper basin
- PIRSU_j = pumpage from groundwater to the new sprinkler irrigation development area
- SHSU_i = shortage of supply for irrigation in the new sprinkler irrigation development
- PERCSU = percolation losses from irrigation in the new sprinkler irrigation development--taken as 30 percent of the water applied
- GWOU = groundwater outflow from the upper groundwater basin to the lower groundwater basin
- BSFLU = base flow from the upper groundwater basin to the Little Lost River

The notation for the Howe basin is similar:

- NREH_j = natural recharge from streams and precipitation in the Howe basin (probabilistic input)
- AREH_i = artificial recharge diversions in the Howe basin
- CFH_i = diversions to canals in the Howe basin
- ETGH = evapotranspiration losses from groundwater storage

in the Howe basin--taken as 15 percent of the
carry-over groundwater storage

- SWSTH = groundwater storage capacity used in the Howe
basin
- GWH = groundwater carry-over storage in the Howe basin
- PIRH_j = pumpage from groundwater storage in the Howe basin
- PERCH = persolation losses to groundwater storage from
irrigation in the Howe basin--taken as 40 per-
cent of the water applied
- RETFLH = return flows to the Little Lost River from
irrigation--taken as 20 percent of the water
applied
- IRRH = guaranteed annual quantity of irrigation water
for use in the Howe basin
- SHH_j = shortage of supply for irrigation in the Howe
basin = guaranteed delivery quantity minus
actual delivery quantity
- IRRSB = guaranteed annual quantity of irrigation water
for new sprinkler development on the northeast
bench in the Howe basin
- PIRSB_j = pumpage from groundwater to the new sprinkler
irrigation development area
- SHSB_j = shortage of supply for irrigation in the new
sprinkler irrigation development in the Howe
basin
- PERCSB = percolation losses to groundwater from irrigation
in the sprinkler irrigation development area

There is no base flow from the groundwater basin in the Howe area. The Little Lost River sinks into the ground in the area south-east of Howe so that all of the outflow from the basin is in the form of groundwater outflows.

It should be noted that the diversions from the surface water inflows are interpreted as the quantity of water to be diverted (CFU_i) if inflow $SFIN_i$ is realized. Similarly $AREU_i$ is the amount of water to be diverted for artificial recharge if inflow $SFIN_i$ is realized. The same logic is applied to the operation of the groundwater basin so that $PIRU_j$ is the amount of water to be pumped from groundwater storage when inflows $NREU_j$ and $AREU_i$ are realized.

Economic characteristics. Benefits from the irrigation water, shortage costs, and the costs associated with making the water available for irrigation comprise the economic characteristics of the system.

Irrigation benefits for the Little Lost River system were difficult to assess. During the interviews conducted in the basin, the water users themselves and Soil Conservation Service (SCS) personnel seemed to have no idea of the amount of benefits derived from irrigation water. The basic farm and ranch operation in the upper part of the basin is based on growing livestock, alfalfa, pasture, and small grains. In the lower part of the basin, operations are more diversified with the addition of some cultivated crops in the operation. Based on these general types of operation, numerical values were assigned for irrigation benefits with reference to the work previously mentioned by Hartman and Whittelsey (1960).

The benefit value for irrigation in the upper part of the basin

was assumed to be \$14.50 per acre-foot. For the lower part of the basin the value assumed is \$16.50. The values assumed for new sprinkler irrigation development are \$17.50 and \$20.00 for the upper and lower parts of the basin, respectively. These higher values are based on higher-valued crops expected in a sprinkler irrigation development.

The water costs used in the Little Lost River model are based on information obtained by verbal communication with ranch operators and water users of the system and partly on extrapolation of data from other areas to the Little Lost River basin. The system costs are:

1. water diverted through surface distribution systems,
2. water artificially recharged into groundwater storage,
3. water pumped from groundwater storage for irrigation, and
4. shortages in actual irrigation deliveries.

For the Little Lost River basin, the average cost of water diverted through surface distribution systems is assumed to be in the range of \$3.00 to \$4.00 per acre-foot, depending upon the extent of the diversion works required. The costs used for CFU, CFT, and CFH were \$3.85, \$4.00, and \$3.00 per acre-foot, respectively. These figures are based on a verbal communication with the local Soil Conservation Service personnel and with canal company officials.

A small volume of water is recharged artificially under the present system in the basin. The recharge operation occurs late in the fall after the normal irrigation season and before hard freezing occurs. The operation uses the normal surface water distribution works, and the water is simply spread on some of the more highly permeable

fields to percolate to groundwater storage. Under this type of low-volume operation, the costs are very low--estimated to be around \$0.50 to \$1.00 per acre-foot. However, if the artificial recharge program were carried out on a larger scale such that new facilities would have to be purchased and such that land for spreading grounds would have to be purchased, the costs would probably rise to about \$7.00 to \$10.00 per acre-foot with the higher costs being in the area of higher land values. For this model, it was assumed that the artificial recharge operations would be on a much larger scale than present; and accordingly the costs used were \$7.19, \$8.16, and \$9.50 per acre-foot for AREU, ARET, and AREh respectively. Because of the high degree of uncertainty regarding these particular cost coefficients, the sensitivity of the optimal solution to them should certainly be examined as part of the model solution. Artificial recharge costs include costs of diversion, conveyance, land, and operation and maintenance costs. A report by Todd (1965) was used as a guide in establishing the artificial recharge costs.

Pumping costs for the Little Lost River basin are based on the average pumping costs reported by several groundwater users in the area and checked against curves developed from pumping cost formulas reported by Nuzman (1967). The pumping costs are a direct function of the pumping head or lift which depends upon the extent of the groundwater development. The various pumping areas, pumping heads, and pumping costs were estimated as follows:

<u>Pumping Area</u>	<u>Pumping Head</u>	<u>Pumping Cost</u>
Upper Basin		
PIRU	100 ft	\$2.20/AF
PIRSU	300 ft	\$5.00/AF
Howe Basin		
PIRH	250 ft	\$4.30/AF
PIRSH	450 ft	\$7.15/AF

The above costs for pumping from groundwater are in close agreement with costs reported by groundwater users which ranged from \$2.00 per acre-foot to \$10.00 per acre-foot.

Costs associated with shortages of irrigation supply were more difficult to estimate for several reasons. It appears that little work has been done in trying to evaluate shortage costs in irrigated agriculture. Shortage costs can depend upon many factors including the crop, fertility level, stage of growth at which shortage occurs, and the actual amount of the shortage. For this study, it is assumed that shortage costs are related to marginal values of irrigation water in different types of farm operation. A report by Hartman and Whittelsey (1960) on marginal values of irrigation water was used as a guide in establishing shortage costs for the Little Lost River basin. Shortage costs used are summarized as follows:

Upper Basin	
SHU	\$21.00 per AF
SHSU	\$18.50 per AF

Howe Basin

SHH	\$24.00 per AF
SHSH	\$21.50 per AF

Again, the sensitivity of the optimal solution to shortage costs should be investigated.

System constraints. Constraints on the system define the allocations of groundwater and surface water to the various uses in the Little Lost River Basin. The constraints also define the hydrologic budget considerations, surface water-groundwater interrelationships as well as any downstream requirements that might be imposed upon the system.

The first set of constraints which form the mathematical model define the condition that flows in all reaches of the Little Lost River and tributaries below diversions must be nonnegative. The constraints are:

1. $AREU_i - CFU_i \leq SFIN_i \quad i = 1, 2, \dots, 5$
2. $ARET_i - CFT_i \leq TRIB_i \quad i = 1, 2, \dots, 5$
3. $5CFH_i + 5AREH_i - GWU - PIRU_j - 0.6CFU_i - 0.75CFT_i$
 $\leq 25,000 \quad i = 1, 2, \dots, 5$

The inflows $SFIN_i$ and $TRIB_i$ are random inflows with probability distributions as described in Table 6-1. The variables $AREU_i$, $ARET_i$, CFU_i , and CFT_i are decision variables which are to be determined by the optimizing algorithm for the given model.

The next set of constraints describe the condition that releases from storage must be less than or equal to the sum of inflow and initial

Table 6-1. Probability distributions of surface water inflows to the Little Lost River system

INFLOW SFIN _i (acre-feet)	Probability p _i	INFLOW TRIB _i (acre-feet)	Probability p _i
34,200	0.13	7,500	0.12
39,400	0.27	8,640	0.26
44,600	0.30	9,760	0.30
49,800	0.19	10,880	0.20
55,000	0.07	12,000	0.08

storage. Since there are no surface storage sites in the basin, this set of constraints applies only to the two groundwater storage basins where storage releases are pumpages from groundwater storage. The constraints are:

$$\begin{aligned}
 1. \quad & 0.65 \text{PIRU}_j + 0.7\text{PIRSU}_j + \text{GWOU} - 0.7\text{GWU} - \text{AREU}_j \\
 & - \text{ARET}_j - 0.6\text{CFU}_j - 0.51\text{CFT}_j = \text{NREU}_j - 5,000 \\
 & j = 1, 2, \dots, 5
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & 0.60\text{PIRH}_j + 0.7\text{PIRSH}_j - 0.85\text{GWH} - \text{AREH}_j - 0.55\text{CFH}_j \\
 & - \text{GWOU} \leq \text{HREH}_j \quad j = 1, 2, \dots, 5
 \end{aligned}$$

The stochastic inflows NREU_j and HREH_j are defined by the distributions given in Table 6-2.

The variables PIRU_j , PIRH_j , PIRSU_j , PIRSH_j , GWOU , and GWU are the new decision variables appearing in this set of constraints.

The third set of constraints in the mathematical model define

Table 6-2. Distributions of natural recharge

Natural recharge $NREU_j$ (acre-feet)	Probability q_j	Natural recharge $NRET_j$	Probability q_j
74,700	0.12	16,000	0.13
85,400	0.23	30,500	0.24
96,100	0.31	45,000	0.29
106,800	0.24	59,500	0.23
117,500	0.10	74,000	0.11

the condition that storage contents at the end of the season cannot exceed storage capacity. Again, since surface storage is not an alternative in the system, this set of constraints applies only to the groundwater basin. Groundwater storage capacity in this case, however, refers to the storage capacity required for optimal operation of the system and is not to be considered as the total physical capacity of the groundwater basin. The constraints for this group are:

$$1. \quad 0.61 CFU_j + 0.51 CFT_j + AREU_j + ARET_j - 0.65 PIRU_j - 0.7 PIRSU_j + 0.7 GWU - GWSTU - GWOU \leq 5,000 - NREU_j$$

$$j = 1, 2, \dots, 5$$

$$2. \quad 0.55 CFH_j - 0.6 PIRH_j - 0.7 PIRSH_j + AREH_j + 0.9 GWH - GWSTH + GWOU \leq - NREH_j \quad j = 1, 2, \dots, 5$$

In this set of constraints the new decision variables are GWSTU and GWSTH.

The fourth set of constraints describes the condition that the aspiration level for initial storage at the beginning of each year is reattainable each year. The aspiration level for initial storage may be thought of as an aspired carry-over storage. This level of storage must not exceed the mathematical expectation of the quantity of water in storage at the end of each year. This group of constraints is imposed on the system to assure that the carry-over storage is realistic in the sense that once it has been attained it can be expected to be reattained at the end of each year for carryover into the succeeding year. The constraints defining this condition for the Little Lost River system are:

$$1. \quad \sum_{i=1}^5 p_i \text{CFU}_i + \sum_{i=1}^5 p_i \text{AREU}_i \leq \overline{\text{SFIN}}$$

$$2. \quad \sum_{i=1}^5 p_i \text{CFT}_i + \sum_{i=1}^5 p_i \text{ARET}_i \leq \overline{\text{TRIB}}$$

$$3. \quad \sum_{i=1}^5 p_i \text{CFH}_i + \sum_{i=1}^5 p_i \text{AREH}_i \leq 5,000 + 0.2 \text{GWU} + 0.2 \sum_{j=1}^5 q_j$$

$$\text{PIRU}_j + 0.2 \sum_{i=1}^5 p_i (0.6 \text{CFU}_i + 0.75 \text{CFT}_i)$$

$$4. \quad 0.65 \sum_{j=1}^5 q_j \text{PIRU}_j + 0.70 \sum_{j=1}^5 q_j \text{PIRSU}_j - 0.61 \sum_{i=1}^5 p_i \text{CFU}_i$$

$$- 0.51 \sum_{i=1}^5 p_i \text{CFT}_i - \sum_{i=1}^5 p_i \text{AREU}_i - \sum_{i=1}^5 p_i \text{ARET}_i + \text{GWOU}$$

$$+ 0.3 \text{ GWU} \leq \overline{\text{NREU}} - 5,000$$

$$5. \quad 0.60 \sum_{j=1}^5 q_j \text{PIRH}_j + 0.70 \sum_{j=1}^5 q_j \text{PIRSH}_j - 0.55 \sum_{i=1}^5 p_i$$

$$\text{CFH}_i - \sum_{i=1}^5 p_i \text{AREH}_i - \text{GWOU} + 0.15 \text{GWH} \leq \overline{\text{NREH}}$$

In each of the above constraints, the parameters $\overline{\text{SFIN}}$, $\overline{\text{TRIB}}$, $\overline{\text{NREU}}$, and $\overline{\text{NREH}}$ are the average annual inflows. In the case of groundwater storage, these constraints require that the groundwater reservoir be operated on a safe-yield basis.

The last group of constraints for this model define the source of deliveries of irrigation water as well as define the shortages in supply of irrigation water. The amount of shortage is defined as

$$\text{Shortage} \geq \text{Guaranteed deliveries} - \text{Actual deliveries}$$

where the actual deliveries are controlled by the stochastic inflows. Shortages are defined for each of the irrigated areas in the model as follows:

$$1. \quad \text{SHU}_{ij} \geq \text{IRRU} - 0.6 \text{CFU}_i - 0.75 \text{CFT}_i - \text{PIRU}_j$$

$$2. \quad \text{SHSU}_j \geq \text{IRRSU} - \text{PIRSU}_j$$

$$3. \quad \text{SHH}_{ij} \geq \text{IRRH} - 0.75 \text{CFH}_i - \text{PIRH}_j$$

$$4. \quad \text{SHSH}_j \geq \text{IRRSH} - \text{PIRSH}_j$$

The amounts of shortages are decision variables for which optimal values will be determined by the optimizing algorithm.

Solution of the linear programming model

The solution of a linear programming problem consists of finding values of all of the decision variables which will optimize (in this case maximize) the value of the objective function and at the same time satisfy all of the constraints. The linear programming model for the Little Lost River system consists of 79 decision variables and 60 constraints. There are an infinite number of solutions to this model which will satisfy the constraints, but only one solution satisfies the constraints and also maximizes the net benefits. This solution was obtained using the Univac 1108 computer and a modification of the simplex algorithm. The size of the model when fully expanded precludes its inclusion in this report. The computer output of the solution also consists of several pages. A summary of the output regarding some of the more pertinent decision variables is shown in Table 6-3.

Discussion of results--Little Lost River Model

Optimal water resource development for the Little Lost River Basin includes both groundwater and surface water development with most of the water developed for irrigation coming from groundwater. For optimal development in the upper basin, surface water supplies about 10 percent of the water used for irrigation while pumping from groundwater supplies about 90 percent of the water. Optimal development in the lower basin shows about 27 percent surface water and 73 percent groundwater. Just under 18 percent of the groundwater developed by pumping is put into groundwater storage by artificial recharge with most of the artificial recharge occurring in the Howe basin.

Table 6-3. Optimal values of pertinent decision variables

Decision variable	Optimal level (10 ³ acre-feet)	Objective value (\$/AF)
IRRU	190.068	14.50
CFU	34.342	-3.85
CFT	0	-4.00
PIRU	169.463	-2.20
IRRSU	209.758	17.50
PIRSU	209.758	-5.00
IRRH	119.175	16.50
CFH	43.015	-3.00
PIRH	86.915	-4.30
IRRSR	0	20.00
AREU	5.058	-7.19
ARET	8.640	-8.16
AREH	31.400	-9.50
GWH	14.696	0

Note: The decision variable symbols are defined on pages 79 to 81.

In comparison with already existing development in the Little Lost River valley, the optimal surface water development is almost the same. Present surface water diversions to irrigated lands is about 55,000 acre-feet annually whereas total optimal surface water diversions are 52,800 acre-feet annually. However, present diversions

from surface streams to artificially recharged groundwater are only about 10 percent of the optimal quantities.

Present groundwater development is much less than the optimal development level, being about 14 percent of the optimal. The optimal groundwater development level exceeds the water yield of the basin, but very high percolation losses from streams, conveyance structures, and from irrigation make possible a high degree of re-cycling of water which accounts for the high level of groundwater development.

The optimal level of artificial recharge activity would require careful planning in order to be able to get the indicated quantity of water into the ground. With the highly permeable valley fill materials existing in the basin, these quantities should not be impossible. Using suggested design factors for artificial recharge basins (Edward E. Johnson, Inc., 1966) and operating the basins during the non-irrigating season, the total quantity of water to be recharged could be accomplished using about 160 acres for recharge basins. Injection wells and modification of existing stream channels could be used as alternative methods to reduce the acreage required if this acreage could not be acquired.

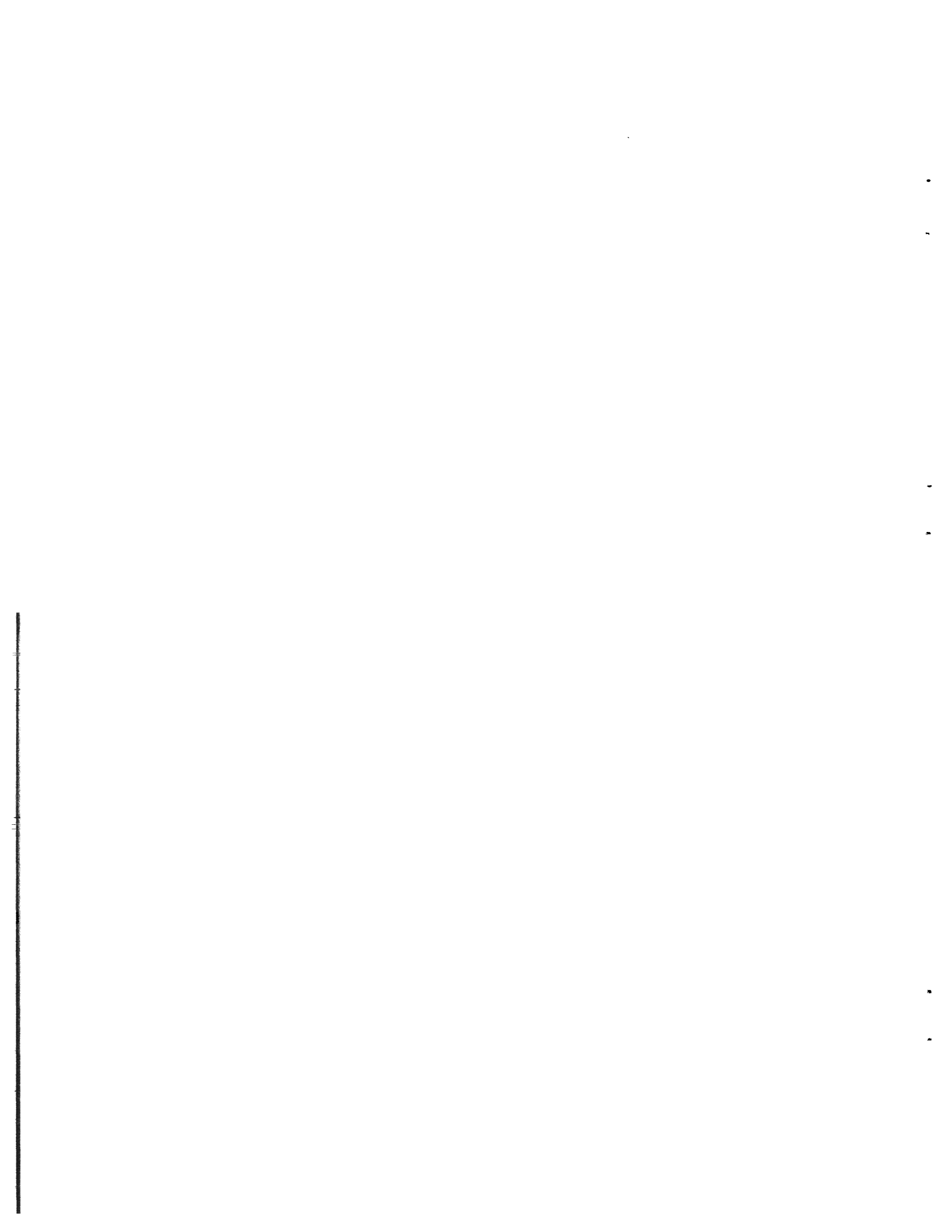
Further analysis of the optimal solution shows that small increases in tributary inflows in the upper basin would greatly increase the value of the objective function. The increase would be even larger for the same amount of increase in the natural recharge in the upper basin. This last conclusion would indicate that canal lining is not a desirable practice if pumping costs remain at the level used (about \$5.00 per acre-foot). Increase in water availability in the upper basin would cause a greater increase in the value of the

objective function than would the same amount of increase in water availability in the Howe basin.

Sprinkler irrigation development in the upper basin appears desirable and optimal at a high level of development (about 50,000 acres if available). However, similar development in the lower basin did not enter the solution. A reduction in pumping costs of about 30 percent would allow this activity for the lower basin to enter the optimal solution.

Sensitivity analysis on the objective coefficients used in this model shows that small increases in irrigation benefits would allow shortages to be part of the optimal solution. Otherwise the optimal solution seems to be highly insensitive to changes in the objective coefficients. The model is especially insensitive to changes in costs of artificial recharge.

In summary, analysis of this model shows that conjunctive use of groundwater and surface water is highly desirable in the Little Lost River Basin, with groundwater development far exceeding surface water development. This pattern of development might have been expected since water storage is desirable, but surface storage sites are non-existent in the basin. Results of the analysis show the desirability of an optimizing procedure for planning



CHAPTER VII

APPLICATION TO A RIVER BASIN--THE SANPETE BASIN, UTAH

In this chapter the physical and hydrologic characteristics of the Sanpete Basin are described in sufficient detail for the reader to gain some feeling for the nature of the basin being modelled. The mathematical model representing the hydrologic-economic system is then defined and described. Finally, a linear-programming solution to the mathematical model is given and discussed.

A previous report by Ballif (1968) was prepared in connection with this research project and Ballif worked with the writer in preparing the mathematical model for the linear programming solution. Therefore, most of the description of the physical system and of the mathematical model in this chapter are taken directly from Ballif.

The Sanpete valley is a part of the San Pitch River Watershed located in central Utah, a part of the Great Basin drainage. The drainage area of the basin is approximately 714 square miles. The Sanpete valley is situated at the border between the Basin and Range Province and the Colorado Plateau Province in south-central Utah. The valley is bounded on the east by the Gunnison Plateau and on the west by the San Pitch Mountains. It is drained by the San Pitch River which empties into the Sevier River.

A variety of crops is grown in the valley, and livestock and poultry raising are also important industries.

The climate is semi-arid. Irrigation is necessary for the production of crops. Canal systems are supplied by San Pitch River flow. The mountain streams are tapped by ditches near the mouths of

the canyons, but this supply is insufficient. Consequently, pumping from groundwater is used to supplement the supply (Richardson, 1907). A map outlining the Sanpete valley boundaries is shown as Figure 7-1.

Previous studies

Richardson (1907) described the topography and geology of the Sanpete and Central Sevier valleys in Utah. The description of the physical system is mostly from this work by Richardson and is not further referenced.

Robinson (1964, 1965, 1966) studied the Sanpete valley in conjunction with Utah State University and the Utah Water and Power Board. He summarized annual pumping rates, groundwater fluctuations, and descriptions of the Sanpete valley.

The U. S. Bureau of Reclamation (1965) made a reconnaissance study of the Sanpete area and available data in conjunction with the Central Utah Project.

The Soil Conservation Service (U. S. Department of Agriculture, 1963) has a study in progress that includes the Sanpete valley. Available data include water budgets, consumptive use estimates for delineated irrigation areas, and possible reservoir sites.

The U. S. Geological Survey made an extensive study of selected wells and springs in the area including data on discharge transmissibility, drawdown, specific electrical conductance, total dissolved solids, sodium adsorption ratio, percent sodium, geologic formations, pervious depths, and well or spring locations.

The physical system

According to Richardson, the Sanpete valley is a structural trough filled with wash derived from the adjacent highlands. The

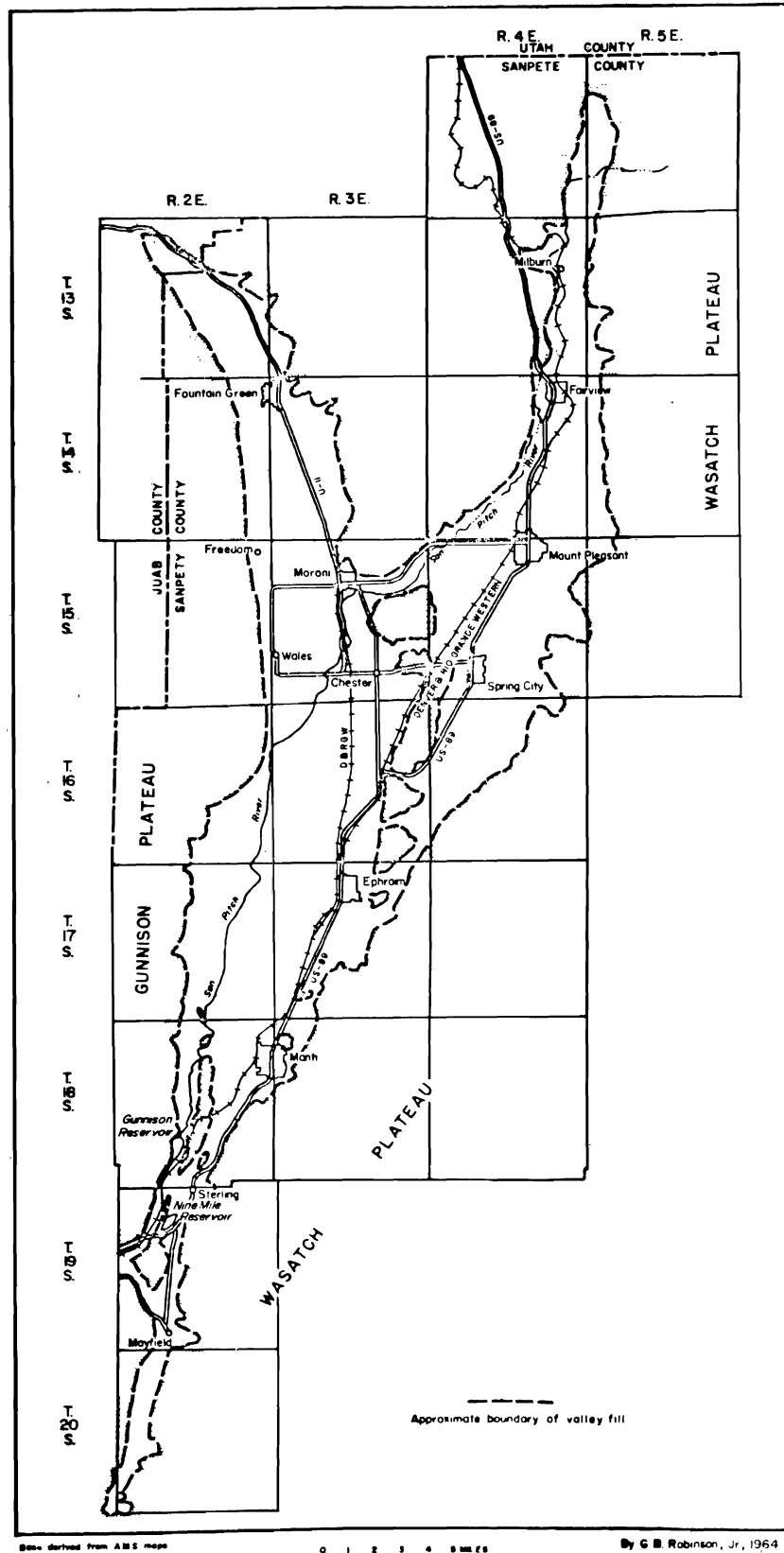


Figure 7-1. Map of Sanpete valley

valley trends northeast-southwest, and it contains numerous relatively small streams. The valley is about 45 miles in length and averages 6 miles in width. The main stream, the San Pitch River, has a number of tributaries, the most important of which flow from the eastern plateaus, where the precipitation is greater than on the relatively low and narrow western highlands. At the mouths of the canyons the discharge is largely diverted into irrigation canals. The lower stream courses in the broad lowlands are generally dry except during floods. The chief tributaries of the San Pitch River are Cottonwood, Pleasant, Cedar, Oak, Canal, Ephraim, Willow, Manti, Sixmile, and Twelvemile Creeks, all of which have small drainage basins on the Wasatch Plateau.

The geology of the Sanpete valley is favorable for groundwater development. The valley fill consists of permeable material capable of receiving and transmitting water. Groundwater occurs both in confined and unconfined conditions. Certain of the underlying consolidated formations are also capable of receiving and transmitting water.

Most of the water yield occurs through natural avenues as springs and seeps, while a lesser amount has been developed through the installation of pumped wells (U. S. Bureau of Reclamation, 1965).

There is no evidence available to suggest any loss of groundwater by subterranean routes to points outside the basin. Development and consumptive use of groundwater thus deplete the flow of the San Pitch River. (U.S. Bureau of Reclamation, 1965, p. 84)

Figure 7-2 shows a structural section of the Sanpete valley at the extreme southern end. The broad central floor of Sanpete valley is composed of fine-textured soils, chiefly sand and clay loam; but toward the highlands, the material becomes coarser. The mountains are flanked by alluvial fans and slopes consisting of sand and gravel

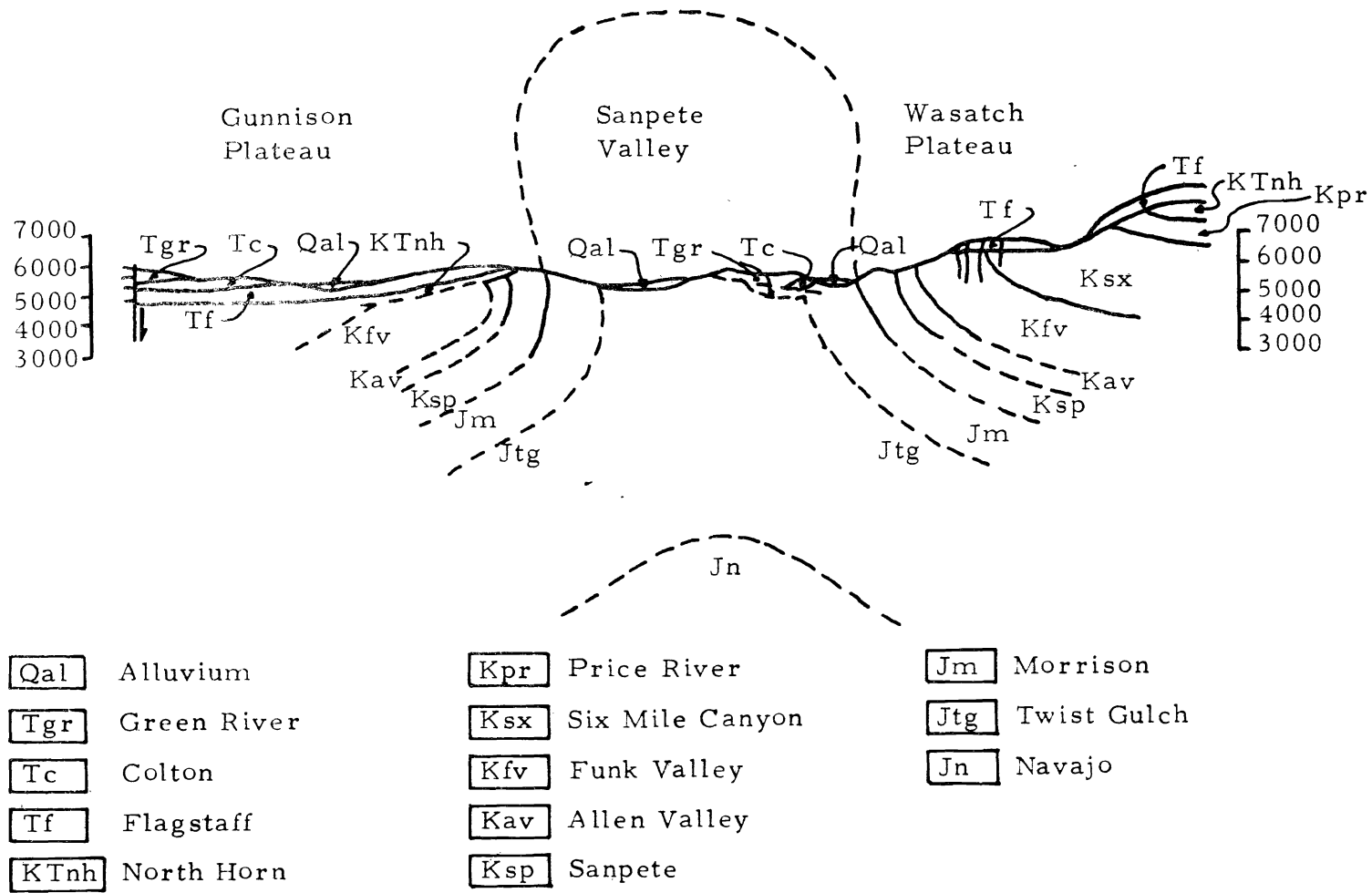


Figure 7-2. Structural section of Sanpete valley (Spieker, 1949)

with subordinate clay. The coarser material preponderates near the mountains. These deposits are derived from the disintegration of the adjacent highlands and transported to the valley by streams. In their mountain courses the volume and velocity of the creeks are considerable, especially during floods; and their carrying power is proportionately large. Upon entering the valley, both the volume and velocity of flow decrease. The result is that the coarser materials carried by the streams are dropped near the base of the highlands while the finer debris are carried farther into the lowlands. Alluvial fans are thus formed about the mouths of the canyons. Alluvial slopes accumulate along the base of the mountains between the creeks, chiefly as the result of torrential storms. These alluvial areas are good recharge sites. The deposits beneath the surface of the broad valleys consist of gravel, sand, and clay, the thickness of which is considerable but unknown. Minimum depths in the main part of the valley are about 650 feet in the Sanpete valley, as shown by wells, in which consolidated rock was not found. Alternating beds of gravel, sand, and clay, from a few inches to many feet in thickness, are encountered in drilling wells. These deposits are in large part loose, porous, and saturated with water and constitute the most important underground reservoirs of the region.

There are about 106,000 acres irrigated in the San Pitch River drainage during an average year. Pumping from groundwater augments the main supply from small streams and springs. About 64,000 acres of this irrigated land have favorable drainage conditions, and about 42,000 acres have drainage deficiencies of varying degrees. The poorly drained lands are located on the low area along the valley bottom. These lands tend to be saline with salinity increasing toward the south end of the valley (U. S. Bureau of Reclamation, 1965).

The conveyance system consists mostly of earth ditches constructed through porous soils, resulting in high water losses. These water losses may vary from about 30 to 80 percent of the flow, depending on the stream size, time of year, and location.

Major surface storage in the Sanpete valley consists of Wales Reservoir (1,480 acre-feet), Loggers Fork Reservoir (1,600 acre-feet), Patten Reservoir (130 acre-feet), Funks Lake Reservoir (700 acre-feet), and Gunnison Reservoir (20,000 acre-feet). Loggers Fork, Patten, and Funks Lake Reservoirs are controls for Manti Creek.

Some possible future reservoir sites and pertinent data are listed in Table 7-1.

Site	Capacity (acre-feet)	Surface area (acres)	Estimated cost (1967)
Black Hills	120	-	\$ -
Canal Creek	67	-	118,000
Cottonwood	86	-	56,500
Freeman Allred	291	-	139,000
Moroni	8,000	480	940,000
Jensen	800	36	375,000
Johnson	430	21	195,000
New Canyon	160	-	129,000
Willow Creek	450	18	203,000

The only sources of water in the Sanpete basin are precipitation on the drainage areas tributary to the valley and transmountain diversions.

The direction of groundwater movement in Sanpete valley is shown by contours in Figure 7-3. The groundwater moves in the same

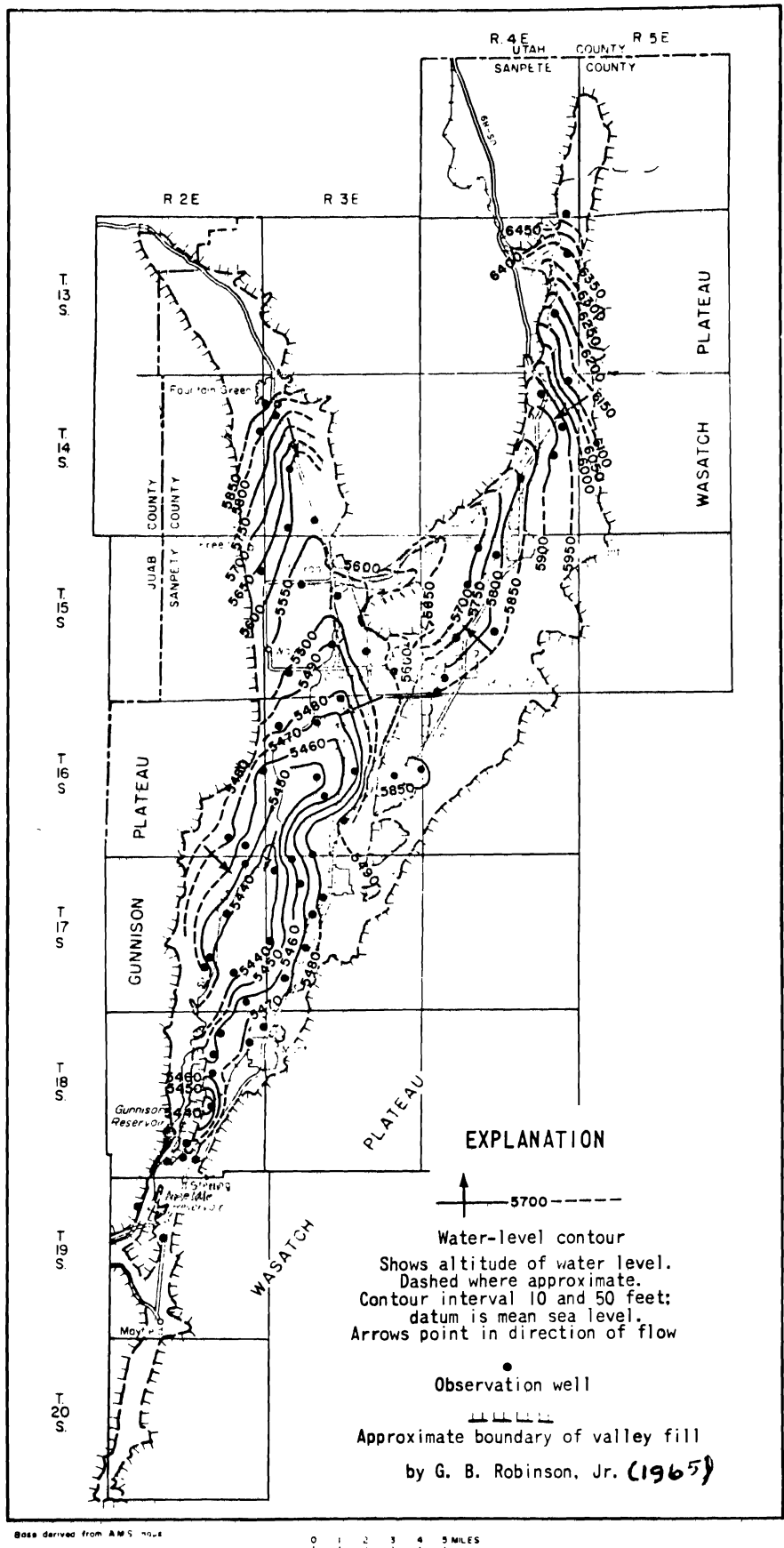


Figure 7-3. Map of Sanpete valley showing water-level contours

general direction as the surface streams, toward the Gunnison Reservoir in the lowest and southernmost part of the main valley.

The general pattern of the contours indicates that recharge to the west arm of the valley is mostly from the Gunnison Plateau. Recharge to the east arm is mostly from the Wasatch Plateau. Recharge to the main part of the valley is mostly from the Wasatch Plateau and groundwater inflow from the two arms. The water-level gradient in the two arms of the valley ranges from about 10 to 200 feet per mile. In the main valley the gradient ranges from about 2 to 30 feet per mile (Robinson, 1965).

Although data are lacking for estimating the quantity of water available for replenishing the underground storage from the flow of streams, the available data indicate that the amount is considerable. Infiltration from stream beds is the chief source of underground water in the Sanpete valley. Ephraim Creek on August 30, 1905, flowing 8.2 cfs near the mouth of its canyon in a course of 0.6 mile over a gravelly bed, lost 0.8 cfs, or 16 percent, per mile. Oak Creek on September 18, 1905, flowing 4.88 cfs at a point 3 miles southeast of Spring City in a course of 2.5 miles, lost 0.46 cfs, or 3.7 percent, per mile. Twin Creek on September 19, 1905, flowing 8.1 cfs at a point 3.5 miles southeast of Mount Pleasant in a course of about 2.75 miles, lost 3.1 cfs, or 13.8 percent, per mile. These figures clearly indicate the manner in which the underground supply of the Sanpete valley is maintained (Richardson, 1907).

The underground water supply of Sanpete valley is also augmented by the underflow from the bedrock and by the flow of springs from bedrock. A number of springs that issue along fault lines convey water to the valley from a distant source in bedrock. The total

discharge of these fault springs amounts to a constant flow of about 95 cfs, and absorption of a part of the flow adds an appreciable amount to the underground waters.

In the practice of irrigation, part of the water applied to the fields is absorbed by the soil, percolates below the reach of roots and beyond the sphere of capillary action, and joins the underground supply. The amount thus transmitted varies considerably from place to place, depending on the porosity of the soil and the quantity of water applied to the fields in excess of the irrigation need.

Robinson (1964, 1965) noted that more than 1,500 wells have been constructed in the Sanpete valley, most of which are concentrated along the lower parts of the valley between Ephraim and Manti and between Ephraim and Moroni. Most of the large-diameter irrigation wells, which have the greatest discharge, are concentrated near Manti, Ephraim, south of Moroni, south of Fountain Green, or between Spring City and Mount Pleasant.

During 1964, wells in the Sanpete valley discharged about 16,000 acre-feet of water as follows (Robinson, 1965, p. 61):

Irrigation	11,600 AF
Pumped wells (equipped with large turbine pumps)	8,000
Flowing wells (and wells equipped with small pumps)	3,600
Public supply (pumped wells)	500
Industry (pumped wells)	400

Domestic, stock, and some irrigation (flowing wells equipped with small pumps)	3,500
TOTAL	16,000 AF

Large seasonal water-level changes occur in the Sanpete valley, particularly between early spring and late summer.

Under existing conditions a considerable groundwater yield is available within the valley. Most of the present yield occurs through natural avenues such as springs and seeps while a lesser amount has been developed through the installation of artesian and pumped wells.

The U. S. Bureau of Reclamation (1965) has estimated the total groundwater yield for an average year to be 50,000 acre-feet, of which about 16,000 acre-feet is developed from wells.

The following 30-year average (1931-1960) water budget is from a Soil Conservation Service unpublished report (U. S. Department of Agriculture, 1963):

<u>Items of Supply</u>	
Streams Inflows (Including Transmountain Diversions)	170,100 AF
Precipitation	
Cropland	50,320 AF
Wetlands	40,640 AF
TOTAL SUPPLY:	<u>261,060 AF</u>
<u>Items of Disposal</u>	
Streams Outflows	33,510 AF
Consumptive Use	
Cropland	109,750 AF
Wetlands	115,990 AF
Increase in Groundwater Storage	<u>1,810 AF</u>
TOTAL DISPOSAL:	<u>261,060 AF</u>

Estimated pumpage of groundwater for the same period in the Sanpete valley is around 16,000 acre-feet. Noting the increase in groundwater storage in the above water budget gives an estimated safe yield of 17,800 acre-feet.

Using data collected in the Robinson reports (1964, 1965, 1966) and plotting by the Hill method gives an estimated groundwater safe yield of 18,500 acre-feet with the present pattern of cropland and wetlands (Figure 7-4).

These values compare favorably and suggest that a modest groundwater development is feasible even with no change in agricultural pattern. By drying up nonbeneficial or marginal value wetlands, more groundwater would be available for development. The safe yield thus could be 20 to 80,000 acre-feet, depending on the amount salvaged.

Further details on the Sanpete system may be found in Ballif (1968).

Formulation of the mathematical model

In order to describe the Sanpete basin in terms of a linear programming model, the Sanpete valley was further subdivided into four subdivisions, A-1, A-2, A-3, and A-4 as shown in Figure 7-5. Work being done in the Sanpete basin by the Soil Conservation Service served as a guide in outlining the four subdivisions (U. S. Department of Agriculture, 1963). The groundwater basin is subdivided into two sub-basins. Groundwater sub-basin A lies in subdivision A-1, while groundwater sub-basin B lies in the other three subdivisions.

Flow diagram. Figure 7-6 is a flow diagram of the system formulated in the mathematical model. Shown on the flow diagram are

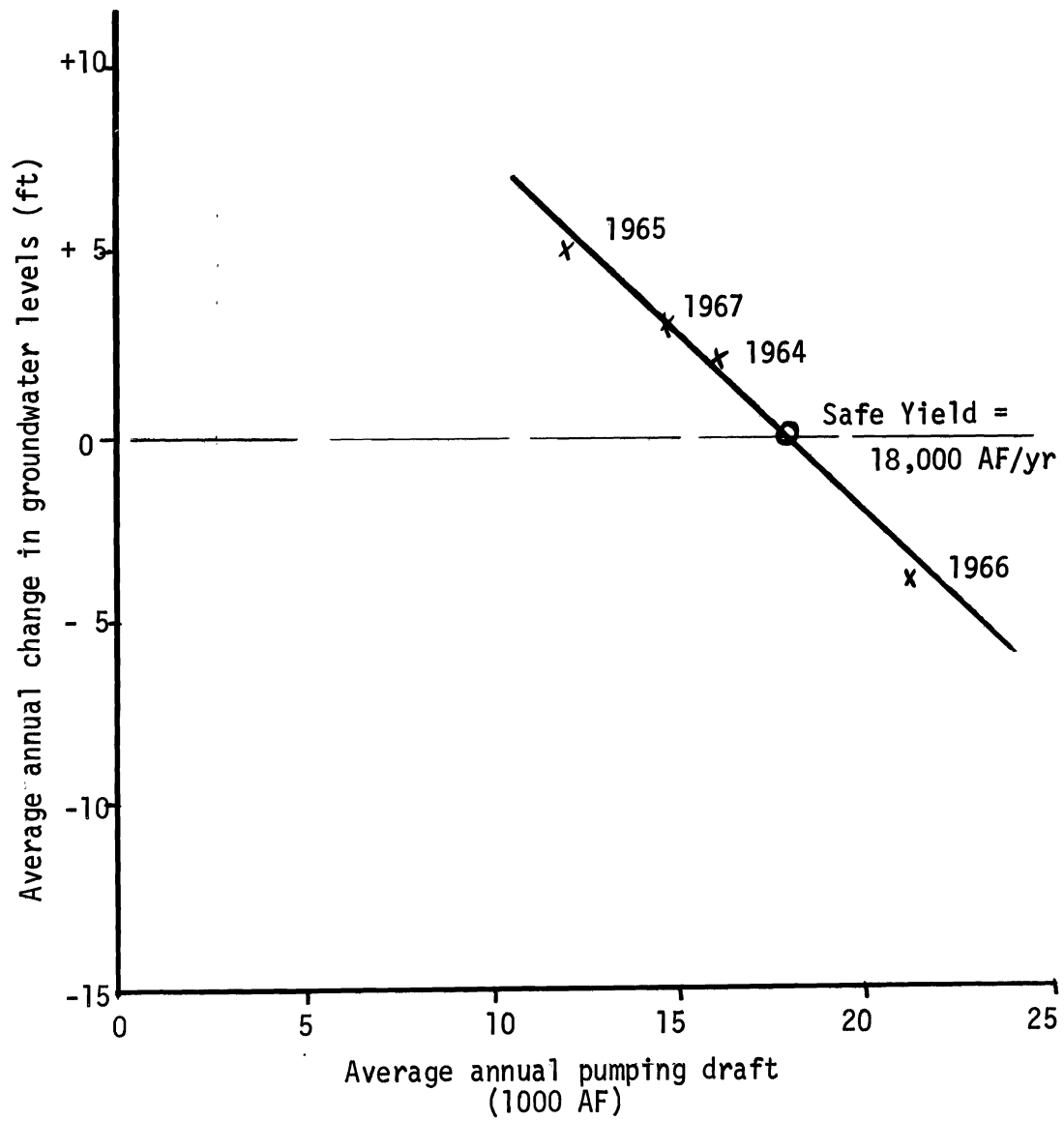


Figure 7-4. Safe yield in Sanpete basin (Hill method)

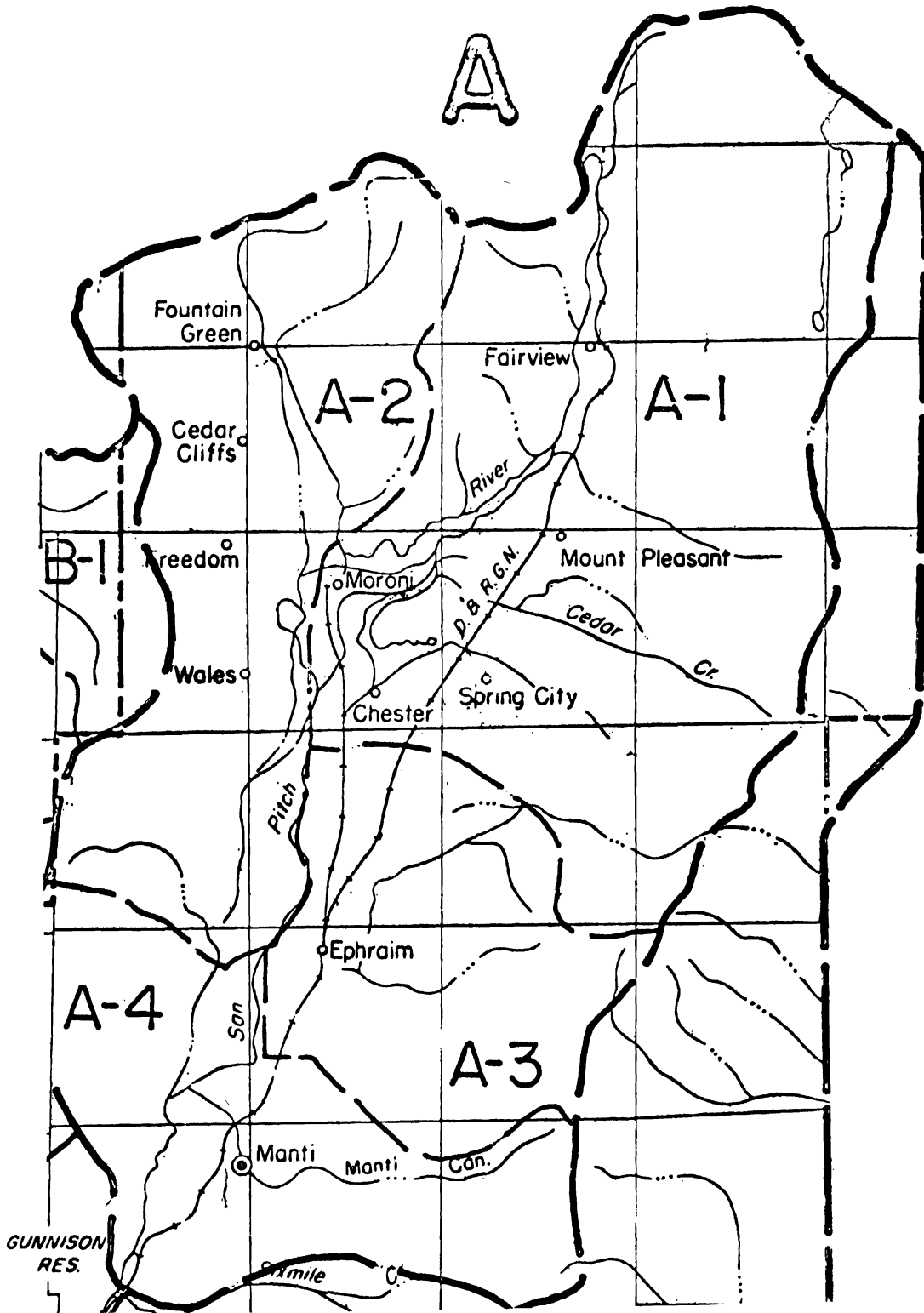


Figure 7-5. Map of San Pitch River basin showing subareas

SANPETE MODEL
SCHEMATIC FLOW DIAGRAM

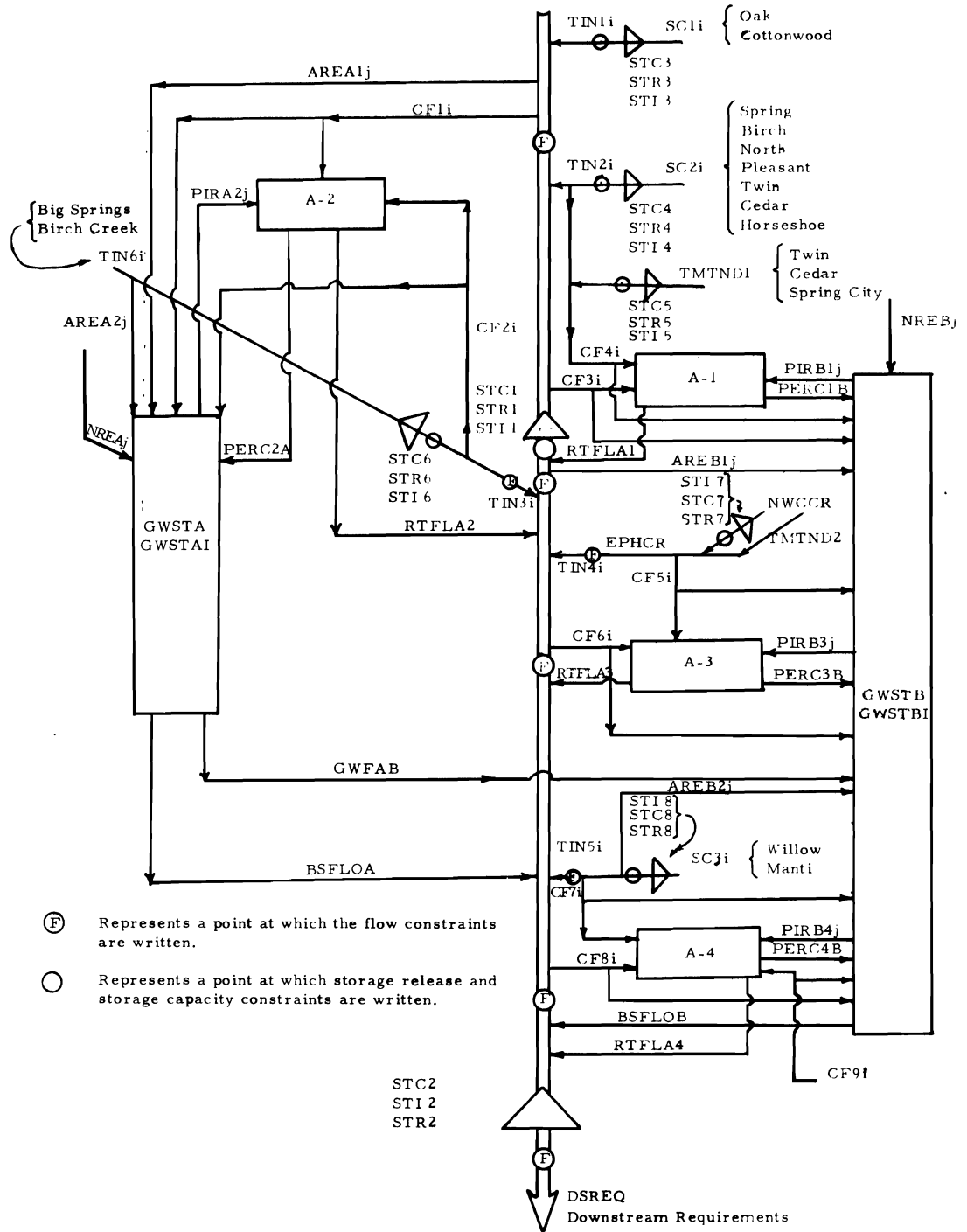


Figure 7-6. Flow diagram of Sanpete basin

the four area subdivisions with all of the related tributary inflows, diversions for irrigation, surface storage sites, groundwater basins, and other elements of the system. Tributary inflows shown in the figure are natural inflows with storage effects, if any. The San Pitch River runs vertically through the center of the diagram. Interpretation of the symbols used in the diagram and in the mathematical model is included as Table 7-2. Several of the smaller reservoirs and creeks are considered together as a unit. These are indicated in Table 7-2.

Economic characteristics of the system. Economic characteristics of the system are necessary in order to define the objective function of the linear programming model. The essential economic characteristics of the system relate to the costs of:

1. The water pumped from groundwater storage for irrigation use.
2. The water artificially recharged into groundwater storage.
3. The storage of surface water.
4. The water diverted through surface distribution systems.
5. The shortages of supply for irrigation.

The economic characteristics of the system must also include the benefit for irrigation water.

Nuzman (1967) developed some economic evaluations for pumping which have been used to evaluate pumping costs. Costs are broken down into two basic categories: fixed costs and variable costs. Fixed costs include exploration and development and all capital expenditures usually made prior to the use of water. Variable costs are all operational costs needed to maintain water production.

Table 7-2. Description of schematic items

Feature	Description
A-1	Irrigation in Subarea A-1
A-2	Irrigation in Subarea A-2
A-3	Irrigation in Subarea A-3
A-4	Irrigation in Subarea A-4
STC1	Storage Capacity Reservoir 1 (Moroni Reservoir)
STC2	Storage Capacity Reservoir 2 (Gunnison Reservoir)
STC3	Storage Capacity Reservoir 3 (Cottonwood Reservoir)
STC4	Storage Capacity Reservoir 4 (Black Hills and Johnson Reservoirs)
STC5	Storage Capacity Reservoir 5 (Canal Creek, Freeman Allred, and Jensen Reservoirs)
STC6	Storage Capacity Reservoir 6 (Wales Reservoir)
STC7	Storage Capacity Reservoir 7 (New Canyon and Willow Creek Reservoirs)
STC8	Storage Capacity Reservoir 8 (Loggers Fork, Patten, and Funks Lake Reservoirs)
STI1	Initial Storage Reservoir 1
STI2	Initial Storage Reservoir 2
STI3	Initial Storage Reservoir 3
STI4	Initial Storage Reservoir 4
STI5	Initial Storage Reservoir 5
STI6	Initial Storage Reservoir 6
STI7	Initial Storage Reservoir 7
STI8	Initial Storage Reservoir 8
STR1	Storage Release Reservoir 1
STR2	Storage Release Reservoir 2
STR3	Storage Release Reservoir 3
STR4	Storage Release Reservoir 4
STR5	Storage Release Reservoir 5
STR6	Storage Release Reservoir 6
STR7	Storage Release Reservoir 7
STR8	Storage Release Reservoir 8
AREA1 _j	Artificial Recharge to GWSTA #1
AREA2 _j	Artificial Recharge to GWSTA #2
AREB1 _j	Artificial Recharge to GWSTB #1
AREB2 _j	Artificial Recharge to GWSTB #2
GWSTA	Groundwater Storage Basin A
GWSTB	Groundwater Storage Basin B
GWSTAI	Initial Storage in Groundwater Basin A
GWSTBI	Initial Storage in Groundwater Basin B

Table 7-2. Continued

Feature	Description
CF1 _i	Canal Flow 1
CF2 _i	Canal Flow 2
CF3 _i	Canal Flow 3
CF4 _i	Canal Flow 4
CF5 _i	Canal Flow 5
CF6 _i	Canal Flow 6
CF7 _i	Canal Flow 7
CF8 _i	Canal Flow 8
CF9 _i	Canal Flow 9
SC1 _i	Sum of Creeks (Oak Creek and Cottonwood Creek)
SC2 _i	Sum of Creeks (Creeks listed on Figure 7-6)
SC3 _i	Sum of Creeks (Willow Creek and Manti Creek)
TIN1 _i	Tributary Inflow 1
TIN2 _i	Tributary Inflow 2
TIN3 _i	Tributary Inflow 3
TIN4 _i	Tributary Inflow 4
TIN5 _i	Tributary Inflow 5
TIN6 _i	Tributary Inflow 6 (Combined flows of Big Springs and Birch Creek)
TMTND1	Transmountain Diversion 1 (Twin Creek, Cedar Creek, and Spring City tunnels)
TMTND2	Transmountain Diversion 2 (Ephraim, Larsen, and Horseshoe tunnels)
M6DIV	Six Mile Diversions
NREA _i	Natural Recharge to Groundwater Basin A
NREB _i	Natural Recharge to Groundwater Basin B
RTFLA1	Return Flow from A-1
RTFLA2	Return Flow from A-2
RTFLA3	Return Flow from A-3
RTFLA4	Return Flow from A-4
PIRA2 _j	Pumping for Irrigation from GWSTA to A-2
PERC2A	Percolation from A-2 to GWSTA
GWFAB	Groundwater Flow from GWSTA to GWSTB
BSFLOA	Base Flow from GWSTA

Table 7-2. Continued

Feature	Description
PIRB1 _j	Pumping for Irrigation from GWSTB to A-1
PERC1B	Percolation from A-1 to GWSTB
PIRB3 _j	Pumping for Irrigation from GWSTB to A-3
PERC3B	Percolation from A-3 to GWSTB
PIRB4 _j	Pumping for Irrigation from GWSTB to A-4
PERC4B	Percolation from A-4 to GWSTB
BSFLOB	Base Flow from GWSTB
NWCCR	New Canyon Creek

Annual fixed costs are given by:

$$FC = \Sigma[(CRF)(I_w) + (CRF)(I_p) + (CRF)(I_m)] + 0.02 \Sigma[I_w + I_p + I_m]$$

where

CRF = capital recovery factor

FC = annual fixed costs in dollars

I_w = investment cost of well = 19.25 (depth)

I_p = investment cost of pump = 173.3 x (X_p) - 866.6

X_p = size index

Q = discharge in gallons per minute

H = total head in feet

I_m = investment cost of electric motor = 341.30 + 23.29 (WHP)

WHP = required water horsepower = $QH/3956$

Q = discharge in gallons per minute

H = total head in feet

The first term in the annual fixed cost equation represents the annual investment cost, and the second term represents annual tax assessments and insurance costs.

Annual variable costs are given by:

$$VC = (1.886 \times 10^{-6} C_k \times Q \times H \times T_h) / E_f + 0.0607 \times Q^{-.47} \\ \times H^{0.26} \times T_h^{0.34} + 0.0475 \times Q^{0.84} \times H^{0.40}$$

where

VC = annual variable costs

C_k = cost of electric power in cents per kilowatt hour

Q = pump discharge in gallons per minute

H = total head in feet

T_h = season operating time in hours

E_f = over-all efficiency of conversion

The first term in the annual variable costs equation represents energy costs, and the second and third terms represent operation and maintenance.

Total annual costs are given by:

$$TC = VC + FC$$

where

TC = total costs (annual in dollars)

VC = total variable costs (annual in dollars)

FC = total fixed costs (annual in dollars)

Cost evaluations were made using the following values for variables:

Interest Rate = 7%

Life of Well, Pump, and Electric Motor = 20 years

Depth = 200 feet

Ck = 0.6¢/kwh and 1.12¢/kwh

Th = 2000 hours

Ef = 0.529

H = varies between 20 - 450 feet

Q = varies between 1000 - 4500 gpm

Pumping Season = 100 days

Figure 7-7 shows how pumping costs vary with pumping lift for 0.6¢/kwh and 1.12¢/kwh.

Artificial recharge is defined as the process of replenishment of the water retained in the groundwater storage through works provided primarily for that purpose. Artificial recharge costs vary greatly depending upon geologic, hydrologic, and cultural conditions at the selected site. One of the more important factors governing project operation is the infiltration rate at potential sites.

Franke1 (1967) estimates that groundwater recharge costs average approximately \$8.00/acre-foot. This value is assumed as a representative estimate of artificial recharge costs in the Sanpete valley. This cost includes land, landscaping, site development, fencing, and hydraulic control works.

The Utah State Engineer (1938) and Brown (1968) have estimated the costs of several possible reservoir sites in the Sanpete valley.

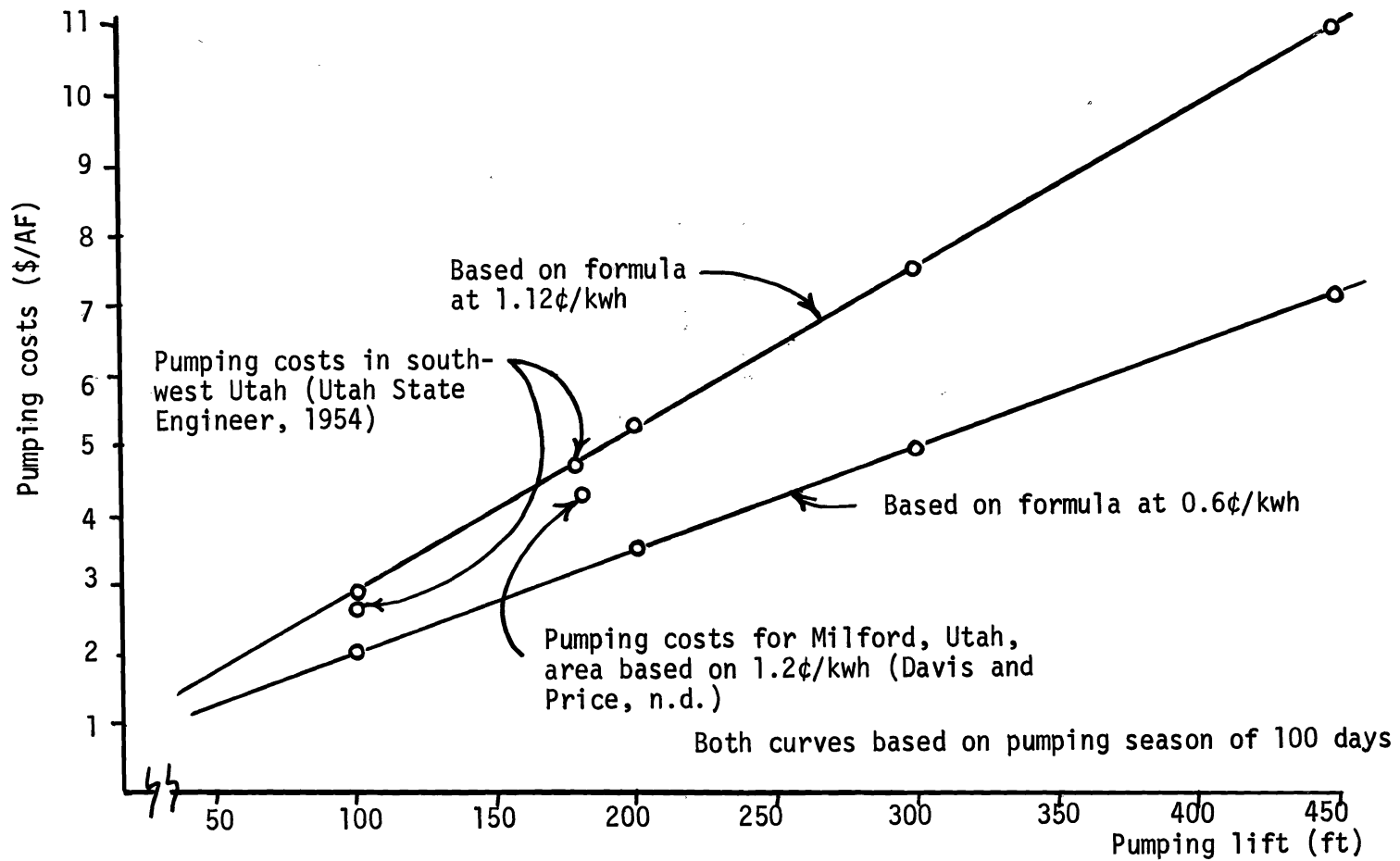


Figure 7-7. Pumping costs vs pumping lift

Values in the State Engineer's report were updated to 1967 by the U. S. Bureau of Reclamation index for earth dams, which was begun in 1949. This index rose approximately 0.3 from 1949 to 1967. Estimating the rise from 1938 to 1949 to be 0.2 gives a ratio of 1.5 to multiply 1938 costs by to get 1967 costs. These values were amortized over a 50-year life at a 3-1/2 percent interest rate.

Table 7-3 lists pertinent data for possible future surface storage.

Table 7-3. Costs of possible surface storage sites

Reservoir site	Capacity (ac-ft)	Estimated cost (\$)	Annual cost (\$)	Annual cost (\$/ac-ft stor.)
Black Hills	120	-	-	-
Canal Creek	67	118,000	5,040	75.10
Cottonwood	86	56,500	2,415	28.10
Freeman Allred	291	139,000	5,940	20.40
Moroni	8,000	940,000	40,000	5.00
Jensen	800	375,000	16,000	20.00
Johnson	430	195,000	8,330	19.40
New Canyon	160	129,000	5,500	34.40
Willow Creek	450	203,000	8,660	19.20

The average costs of water diverted through the various surface distribution systems in the Sanpete basin were established by verbal

communication with irrigation company officials in the area and with Soil Conservation Service employees working in the area. The costs of water diverted through surface distribution systems is assumed to be in the range from \$3.75 per acre-foot to \$4.50 per acre-foot.

Again for this model the costs associated with shortages of irrigation water supply are difficult to establish because of a lack of work in establishing such costs generally, and again it is assumed that shortage costs are related to marginal values of irrigation water. Based on work done by Hartman and Whittelsey (1960) and by Davis (1965), it was assumed that shortage costs for the Sanpete basin would be in the range from \$35.00 to \$38.00 per acre-foot.

Estimates of irrigation benefits were established using Davis (1965) as a guide. It is assumed that irrigation benefits for the Sanpete basin lie in the range of \$16.00 to \$20.00 per acre-foot. These benefit coefficients are based on average values of irrigation water and are, therefore, lower than the shortage costs previously stated.

With the economic characteristics of the system outlined above, the objective function to be maximized is:

$$\begin{aligned} \text{NETBEN} = & 16.00 (\text{IRRA1}) + 16.00 (\text{IRRA2}) + 18.00 (\text{IRRA3}) \\ & + 20.00 (\text{IRRA4}) - 38.00 (\text{SHA1}) - 37.00 (\text{SHA2}) - 36.00 (\text{SHA3}) \\ & - 35.00 (\text{SHA4}) - 4.00 \text{ CF1} - 4.00 \text{ CF2} - 4.00 \text{ CF3} - 4.00 \text{ CF4} \\ & - 4.00 \text{ CF5} - 4.00 \text{ CF6} - 4.00 \text{ CF7} - 4.00 \text{ CF8} - 4.00 \text{ CF9} \\ & - 8.00 \text{ AREA1} - 8.00 \text{ AREA2} - 8.00 \text{ AREB1} - 8.00 \text{ AREB2} \\ & - 2.30 \text{ PIRA2} - 2.30 \text{ PIRB1} - 2.90 \text{ PIRB3} - 2.90 \text{ PIRB4} \end{aligned}$$

- (0.00) ST6 - 87.60 ST3 - 20.00 ST4 - 20.00 ST5
- 5.00 ST1 - 107.00 ST7 - 19.20 ST8 - 0.00 ST2

Note: The coefficients are given in \$/AF and variables are in AF.

The system constraints. Allocations of groundwater and surface water to the various use areas in the Sanpete basin are described by the constraint system. The constraints also define other relationships and considerations within the system. The Sanpete model constraints are as follows:

Sanpete model--constraint system

I. Flows in all reaches must be nonnegative

1. $TIN1_i - AREA1_j - CF1_i \geq 0$
2. $TIN2_i - .94 CF3_i + STI1 - AREB1_j + .04 CF4_i + 0.1 PIRB1_j \geq 0$
3. $AREA2_j \leq TIN6_i$
4. $-.075 CF1_i + 0.91 CF2_i - 0.25 PIRA2_j + AREA2_j - STI6 - STI7 + CF5_i + CF6_i \leq (TMTND2 + TIN6_i + EPHCR_i + WWCCR_i)$
5. $AREB2_j + CF7_i + CF8_i - 0.06 CF5_i - 0.09 CF6_i - 0.1 PIRB3_j - STI8 - 0.1 GWSTAI \leq SC3_i$
6. $0.15 GWSTBI + STI2 + 0.06 CF7_i + 0.09 CF8_i + 0.07 CF9_i + 0.1 PIRB4_j \geq DSREQ$

II. Releases from storage less than or equal to sum of inflows and initial storage

1. $TIN1_i - STI3 \leq SC1_i$
2. $TIN2_i + CF4_i - STI4 - STI5 \leq SC2_i + TMTND1$
3. $CF1_i + CF3_i + CF6_i + CF8_i + AREA1_j + AREB1_j - STI1 - TIN1_i - TIN2_i \leq 0$
4. $CF2_i + AREA2_j - STI6 + TIN3_i \leq TIN6_i$
5. $CF5_i - STI7 + TIN4_i \leq NWCCR_i + EPHCR_i + TMTND2$
6. $CF7_i + AREB2_j - STI8 + TIN5_i \leq SC3_i$
7. $STI1 + STI2 + TIN1_i + TIN2_i + TIN3_i + TIN4_i + TIN5_i - AREA1_j - AREB1_j + 0.1 GWSTAI + 0.15 GWSTBI + 0.1 PIRB1_j + 0.25 PIRA2_j + 0.1 PIRB3_j + 0.1 PIRB4_j - 0.93 CF1_i + 0.09 CF2_i - .94 CF3_i + .04 CF4_i + .06 CF5_i - 0.91 CF6_i + 0.06 CF7_i - 0.91 CF8_i + 0.07 CF9_i \geq DSREQ = 22,000$
8. $CF9_i \leq M6DIV_i$

Groundwater storage

1. $0.65 PIRA2_j - 0.675 CF1_i - 0.61 CF2_i - AREA1_j - AREA2_j + GWFAB - 0.9 GWSTAI \leq NREA_j$

$$\begin{aligned}
2. \quad & 0.5 \text{ PIRB1}_j + 0.5 \text{ PIRB3}_j + 0.5 \text{ PIRB4}_j - \text{AREB1}_j - \text{AREB2}_j \\
& - .64 \text{ CF3}_i - .76 \text{ CF4}_i - .7 \text{ CF5}_i - .55 \text{ CF6}_i - .7 \text{ CF7}_i \\
& - .55 \text{ CF8}_i - .625 \text{ CF9}_i - .85 \text{ GWSTBI} \leq \text{NREB}_j
\end{aligned}$$

III. Contents of reservoir at end of season cannot exceed capacity
(Initial storage + inflow - outflow \leq capacity)

Surface reservoirs

1. $\text{STI3} - \text{STC3} - \text{TIN1}_i \leq -\text{SC1}_i$
2. $\text{STI4} + \text{STI5} - \text{STC4} - \text{TIN2}_i - \text{CF4}_i \leq -(\text{SC2}_i + \text{TMTND1})$
3. $\text{STC5} \leq 10,000$
4. $\text{STI1} - \text{STC1} + \text{TIN1}_i + \text{TIN2}_i - \text{AREA1}_j - \text{AREB1}_j - \text{CF1}_i$
 $- \text{CF3}_i - \text{CF6}_i - \text{CF8}_i \leq 0$
5. $\text{STI6} - \text{STC6} - \text{AREA2}_j - \text{CF2}_i - \text{TIN3}_i \leq -\text{TIN6}_i$
6. $\text{STI7} - \text{STC7} - \text{TIN4}_i - \text{CF5}_i \leq -(\text{EPHCR}_i + \text{NWCCR}_i + \text{TMTND2})$
7. $\text{STI8} - \text{STC8} - \text{AREB2}_j - \text{TIN5}_i - \text{CF7}_i \leq -\text{SC3}_i$
8. $\text{STI2} + \text{STI1} - \text{STC1} + \text{TIN1}_i + \text{TIN2}_i + \text{TIN3}_i + \text{TIN4}_i$
 $+ \text{TIN5}_i - \text{AREA1}_j - \text{AREB1}_j - .93 \text{ CF1}_i + .09 \text{ CF2}_i$
 $- .94 \text{ CF3}_i + .04 \text{ CF4}_i + .06 \text{ CF5}_i - .91 \text{ CF6}_i + .06 \text{ CF7}_i$
 $- .91 \text{ CF8}_i + .07 \text{ CF9}_i + .01 \text{ GWSTAI} = .15 \text{ GWSTBI}$

$$\leq \text{DSREQ} = 22,000$$

Groundwater reservoirs

1. $0.9 \text{ GWSTAI} - \text{GWSTA} - \text{GWFAB} + \text{AREA2}_j + \text{AREA1}_j + .67 \text{ CF1}_i$
 $+ .61 \text{ CF2}_i - .65 \text{ PIRA2}_j \leq -\text{NREA}_j$
2. $0.85 \text{ GWSTBI} - \text{GWSTB} + \text{GWFAB} + \text{AREB1}_j + \text{AREB2}_j + .64 \text{ CF3}_i$
 $+ .76 \text{ CF4}_i + .7 \text{ CF5}_i + .55 \text{ CF6}_i + .7 \text{ CF7}_i + .55 \text{ CF8}_i$
 $+ .62 \text{ CF9}_i - .5 \text{ PIRB1}_j - .5 \text{ PIRB3}_j - .5 \text{ PIRB4}_j \leq -\text{NREB}_j$

IV. Aspired level for initial storage reattainable each year

Surface storage

1. $\sum p_i \text{ TIN1}_i \leq \overline{\text{SC1}} = 13,890$
2. $\sum p_i \text{ TIN2}_i + \sum p_i \text{ CF4}_i \leq \overline{\text{SC2}} + \text{TMTND1} = 68,940$
3. $\sum p_i \text{ CF1}_i + \sum p_i \text{ CF3}_i + \sum p_i \text{ CF6}_i + \sum p_i \text{ CF8}_i + \sum q_j \text{ AREA1}_j$
 $+ \sum q_j \text{ AREB1}_j - \sum p_i \text{ TIN1}_i - \sum p_i \text{ TIN2}_i \leq 0$
4. $\sum q_j \text{ AREA2}_j + \sum p_i \text{ CF2}_i + \sum p_i \text{ TIN3}_i \leq \overline{\text{TIN6}}$
5. $\sum p_i \text{ TIN4}_i + \sum p_i \text{ CF5}_i \leq \overline{\text{NWCCR}} + \overline{\text{EPHCR}} + \text{TMTND2}$
6. $\sum p_i \text{ TIN5}_i + \sum p_i \text{ CF7}_i + \sum q_j \text{ AREB2}_j \leq \overline{\text{SC3}}$
7. $\sum p_i \text{ TIN1}_i + \sum p_i \text{ TIN2}_i + \sum p_i \text{ TIN3}_i + \sum p_i \text{ TIN4}_i + \sum p_i \text{ TIN5}_i$
 $- \sum q_j \text{ AREA1}_j - \sum q_j \text{ AREB1}_j + 0.1 \text{ GWSTAI} + 0.15 \text{ GWSTBI}$

$$\begin{aligned}
& - \sum p_i .93 CF1_i + \sum p_i .09 CF2_i - \sum p_i .94 CF3_i \\
& + \sum p_i .04 CF4_i + \sum p_i .06 CF5_i - \sum p_i .91 CF6_i \\
& + \sum p_i .06 CF7_i - \sum p_i .91 CF8_i + \sum p_i .07 CF9_i \\
& + \sum q_j 0.1 PIRB1_j + \sum q_j 0.25 PIRA2_j + \sum q_j 0.10 PIRB3_j \\
& + \sum q_j 0.1 PIRB4_j \geq DSREQ
\end{aligned}$$

$$8. \sum p_i CF9_i \leq \overline{M6DIV} = 4160$$

Groundwater storage

$$\begin{aligned}
1. \quad & \sum q_j .65 PIRA2_j - \sum q_j AREA1_j - \sum q_j AREA2_j - \sum p_i .67 CF1_i \\
& - \sum p_i .61 CF2_i + GWFAB + .1 GWSTAI \leq \overline{NREA}
\end{aligned}$$

$$\begin{aligned}
2. \quad & \sum q_j 0.5 PIRB1_j + \sum q_j 0.5 PIRB3_j + \sum q_j 0.5 PIRB4_j \\
& + 0.15 GWSTBI - GWFAB - \sum q_j AREB1_j - \sum q_j AREB2_j \\
& - \sum p_i .64 CF3_i - \sum p_i .76 CF4_i - \sum p_i .7 CF5_i - \sum p_i .55 CF6_i \\
& - \sum p_i .7 CF7_i - \sum p_i .55 CF8_i - \sum p_i .62 CF9_i \leq \overline{NREB}
\end{aligned}$$

V. Constraints describing shortage

$$1. \quad IRRA1 - 0.4 CF4_i - 0.6 CF3_i - PIRB1_j \leq SHA1_{ij}$$

$$2. \quad IRRA2 - 0.5 CF1_i - 0.6 CF2_i - PIRB2_j \leq SHA2_{ij}$$

$$3. \quad IRRA3 - 0.4 CF5_i - 0.6 CF6_i - PIRB3_j \leq SHA3_{ij}$$

$$4. \quad IRRA4 - 0.4 CF7_i - 0.6 CF8_i - 0.5 CF9_i - PIRB4_j \leq SHA4_{ij}$$

Variables on the left side of the equation are decision variables that are to be solved for in the solution of the model. Variables on the right side of the equation are probabilistic inputs.

In reality, stream flows and natural recharge are probabilistic variables (parameters). Other deterministic variables depend directly on these probabilistic inflows. Therefore it is necessary to describe probabilistic variates and their corresponding flow in the constraint equations in order to optimize the objective function.

Downstream water requirements are regarded as deterministic quantities and are reflected in the right-hand side values.

Probability density coefficients

Kim (1968) developed a method of obtaining probability density coefficients from annual stream flow data. His method is used to describe the flow level probability.

The method consists of deriving from the annual stream flow data six discrete points. The points are chosen in the following manner. The minimum annual flow is chosen as the first discrete point. The succeeding discrete points are obtained by adding to the prior discrete point the quotient of the difference of the maximum annual stream flow minus the minimum annual stream flow divided by five. The last and sixth discrete point is the maximum annual stream flow.

A probability density coefficient is obtained for each interval between discrete points by the following equation:

$$\begin{aligned} \text{Probability Density Coefficient (i)} &= \Phi \frac{(X_{i+1} - \bar{X})}{S} - \Phi \frac{(X_i - \bar{X})}{S} \\ &= \Phi(z) \end{aligned}$$

where

$i = 1, 2, \dots, 6$

X_i = discrete point

\bar{X} = average of annual stream flow data

S = standard deviation of annual stream flow

Φ = functional relation

Now from cumulative standard normal tables for values of $\Phi(z)$ (corresponding to the "Z" column in the tables), look up corresponding values of $G(z)$ in the tables which are the probability density coefficients. There is a set of five probability density coefficients for each probabilistic input.

Figure 7-8 shows an illustrative plot of probability density coefficient vs corresponding flow. The bar graph approximates the curve shown by the dashed lines. Bar columns are divided by the discrete point intervals. If the period of record for annual flow were infinite, the curve would be a normal distribution. Since the actual length of record is limited, the curve usually is not normal and usually skewed. If the data were infinite, the probability density coefficients would add up to 1.0. In actual limited data this is reduced by the amount in the upper and lower tails of the curve.

Probability density coefficients were derived for Twin Creek using both estimated and recorded data. Recorded data on Twin Creek began in 1955. Runoff data for Twin Creek were estimated from 1949 to 1955 by correlation with Ephraim Creek. The year 1949 is thought by some to be the beginning of a new cycle of hydrologic conditions. Foliage on the range land gives some evidence of being more constant

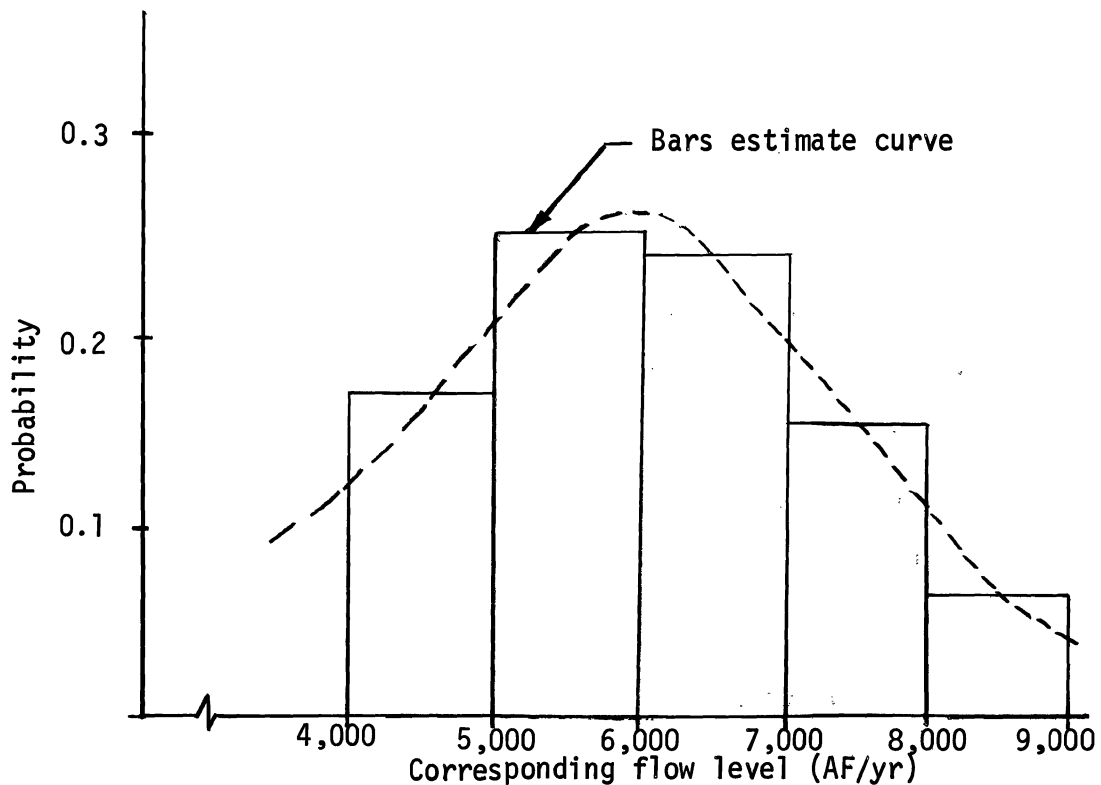


Figure 7-8. Probability density coefficient vs corresponding flow level

from 1949 to the present. Thus, runoff patterns would be similar for this time base.

Table 7-4 lists probability density coefficients derived from the runoff data along with corresponding flows.

Probability density coefficients for Pleasant Creek were derived from data from the base period 1949 to 1965. Table 7-5 gives the probability density coefficients and corresponding flows.

Probability density coefficients for Ephraim Creek were derived from actual data for 1949 to 1963. Table 7-6 lists the

Table 7-4. Twin Creek probability density coefficients

Discrete point interval	Probability density coefficient	Corresponding flow
3,540 - 4,588	.163	4,064
4,588 - 5,636	.234	5,112
5,636 - 6,684	.232	6,160
6,684 - 7,732	.160	7,208
7,732 - 8,780	.075	8,256

Table 7-5. Pleasant Creek probability density coefficients

Discrete point interval	Probability density coefficient	Corresponding flow
7,900 - 10,360	.175	9,130
10,360 - 12,820	.273	11,590
12,820 - 15,280	.256	14,050
15,280 - 17,740	.145	16,510
17,740 - 20,200	.050	18,970

probability density coefficient and corresponding flow.

Probability density coefficients were derived for Big Springs using estimated data. Data were estimated from 1949 to 1955, and from 1963 to 1966. Actual records were available on Big Springs from 1955

Table 7-6. Ephraim Creek probability density coefficients

Discrete point interval	Probability density coefficient	Corresponding flow
8,796 - 12,716	.160	10,756
12,716 - 16,636	.234	14,676
16,636 - 20,556	.235	18,586
20,556 - 24,476	.260	22,516
24,476 - 28,396	.077	26,436

through 1962. This gave a base period of from 1949 to 1966.

Table 7-7 follows listing probability density coefficients and corresponding flow level.

Table 7-7. Big Springs probability density coefficients

Discrete point interval	Probability density coefficient	Corresponding flow
3,431 - 4,555	.142	3,993
4,555 - 5,679	.256	5,117
5,679 - 6,893	.275	6,241
6,893 - 7,927	.196	7,365
7,927 - 9,050	.042	8,489

In order to arrive at probability density coefficients for natural recharge to groundwater basin "A" (NREA), it was necessary to develop an equation describing NREA. The equation estimates annual recharge to the area.

Natural recharge depends directly upon stream flow and precipitation on the area. Thus, the following equation relating NREA to stream flow and runoff was developed:

$$\text{NREA} = 1.11 (\text{stream flow}) + 1.06 (\text{precipitation at Moroni})$$

where values are given in acre-feet.

Adequate stream flow records have not been kept in the area of groundwater basin "A," so stream flow values were estimated using the following equation:

$$\text{Stream flow} = 0.135 (\text{Pleasant Creek}) + 0.865 (\text{Big Springs})$$

where values are given in acre-feet.

Table 7-8 shows components of stream flow data. Table 7-9 follows listing NREA and its component parts, along with its discrete points and statistics of the annual data.

Probability density coefficients for natural recharge to groundwater area "A," (NREA), are listed in Table 7-10.

As with NREA, it is necessary to estimate the natural recharge to groundwater area "B" (NREB) on an annual basis. A base period needed to be established before probability density coefficients could be derived. The following equation was developed relating NREB with

Table 7-8. Stream flow for NREA in acre-feet

Year	Pleasant Creek	Big Springs	2.5/18.5 (Pleasant)	(Big Springs)	Stream flow
1955	11,210	4,260	1,520	3,680	5,200
1956	10,020	5,548	1,350	4,800	6,150
1957	16,030	7,446	2,170	6,430	8,600
1958	16,230	8,760	2,200	7,580	9,780
1959	8,830	5,329	1,192	4,600	5,792
1960	10,330	4,453	1,400	3,860	5,260
1961	7,900	3,431	1,070	2,960	4,030
1962	15,450	6,205	2,090	5,360	7,450

Table 7-9. NREA and its components

Year	Stream flow	1.11 (Stream flow)	Moroni (Precip.)	1.06 (Precip.)	NREA
1955	5,200	6,780	10,540	11,200	17,980
1956	6,150	6,840	7,120	7,550	14,390
1957	8,600	9,550	13,120	13,900	23,450
1958	9,780	10,850	6,400	6,790	17,640
1959	5,792	6,440	8,450	8,950	15,390
1960	5,260	5,850	10,000	10,600	16,450
1961	4,030	4,480	12,390	13,100	17,580
1962	7,450	8,280	9,250	9,800	18,080

S = 2,950 Discrete points:

\bar{X} = 17,610

14,390
16,202
18,014
19,826
21,636
23,450

Values checked closely with corresponding items of an unpublished S.C.S. water budget for the area.

Table 7-10. NREA probability density coefficients

Discrete point interval	Probability density coefficients	Corresponding flow
14,390 - 16,202	.180	15,296
16,202 - 18,014	.237	17,108
18,014 - 19,826	.219	18,920
19,826 - 21,638	.141	20,732
21,638 - 23,450	.062	22,544

stream flow and precipitation:

$$\text{NREB} = 0.218 (\text{stream flow}) + \text{precipitation (average of Manti and Moroni)}$$

where values are given in acre-feet.

Stream flow was distributed by the following ratio:

$$\frac{\text{Ephraim stream flow}}{\text{Av. Ephraim stream flow}} = \frac{\text{Stream flow}}{\text{Av. stream flow}}$$

where:

Av. stream flow = 81,570 acre-feet

Av. Ephraim stream flow (1949-1963) = 16,670 acre-feet.

Table 7-11 lists NREB and its component parts.

Table 7-11. NREB and its components

Year	Ephraim	Ratio	Stream flow	0.218 (Stream flow)	Precip.	NREB
1949	18,217	1.1	89,600	19,500	26,000	45,500
1950	13,592	.816	66,600	14,500	23,750	38,250
1951	13,342	.803	65,500	14,270	31,600	45,870
1952	27,054	1.63	133,000	29,000	27,300	56,300
1953	17,621	1.06	86,500	18,820	31,500	50,320
1954	16,780	1.01	82,500	18,000	31,750	49,750
1955	14,586	.875	71,500	15,590	27,400	42,990
1956	12,417	.748	61,000	13,300	23,100	36,400
1957	25,466	1.53	125,000	27,200	44,200	71,400
1958	-	-	-	-	19,530	-
1959	8,796	.529	43,100	9,400	26,850	36,250
1960	13,738	.826	67,500	14,700	28,400	43,100
1961	10,936	.658	53,600	11,700	41,200	52,900
1962	28,397	1.71	139,500	33,000	28,000	61,000
1963	12,204	.735	60,000	13,080	33,100	46,180
				S = 10,220	Discrete points:	
				\bar{X} = 48,400	36,250	
					43,280	
					50,310	
					57,340	
					64,370	
					71,400	

The following table lists the probability density coefficients for natural recharge to groundwater area "B," (NREB), with corresponding flow levels.

Probability density coefficients were needed for each probabilistic input. See Figure 7-5, the schematic flow diagram, for locations of probabilistic inputs. Table 7-2 lists descriptions of the abbreviated components of the schematic flow diagram.

Table 7-12: NREB probability density coefficients

Discrete point interval	Probability density coefficient	Corresponding flow
36,250 - 43,280	.192	39,765
43,280 - 50,310	.265	46,795
50,310 - 57,340	.234	53,825
57,340 - 64,370	.133	60,855
64,370 - 71,400	.047	67,885

The probabilistic inputs consist of $NREA_i$, $NREB_i$, $SC1_i$, $SC2_i$, $SC3_i$, $NWCCR_i$, $EPHCR_i$, and $TIN6_i$. Transmountain diversions are relatively constant year after year and are not described by probability density coefficients. The flow and storage levels of the other variables will be solved for in the solution to the linear programming model.

$NREA_i$ and $NREB_i$ are described by the probability density coefficients derived for them. $SC1_i$ and $SC2_i$ are represented by the average of Twin and Pleasant Creeks probability density coefficients.

EPHCR, NMCCR_i, and SC3_i are described by the probability density coefficients derived for Ephraim Creek. TIN6_i is described by the coefficients derived for Big Springs.

Table 7-13 lists the probabilistic inputs and the corresponding sets of probability density coefficients for these variables.

Solution of the linear programming model

The solution of the Sanpete linear programming model consists of finding the values of the water resource allocations (the decision variables) which will maximize the objective function and at the same time satisfy the constraint system. The linear programming model for the Sanpete basin is made up of 155 decision variables and 156 constraints. Associated with the 156 constraints are 156 slack variables bringing the total number of variables to 311. However, the matrix is a sparse matrix with a density of 5.223. The solution to this linear programming problem was obtained on the Univac 1108 computer using a modification of the simplex algorithm. A computer listing of the model equations including the objective function is 23 pages long which indicates the size of the model and why it cannot be included here. A summary of the optimal activity levels for the more important decision variables is shown in Table 7-14.

Discussion of results--Sanpete basin model

In the framework of the linear programming model, conjunctive use of groundwater and surface water is the optimal pattern for water resources use in the four subdivisions of the Sanpete basin. The

Table 7-13. Probability density coefficients for stochastic inputs

Stochastic input	Probability density coefficients	flow
NREA _i	.180	15,296
	.237	17,108
	.219	18,920
	.141	20,732
	.062	22,544
NREB _i	.192	39,765
	.265	46,795
	.234	53,825
	.133	60,855
	.047	67,885
SC1 _i	.169	9,720
	.254	12,200
	.244	14,630
	.253	17,180
	.063	19,700
SC2 _i	.169	45,900
	.254	52,200
	.244	69,100
	.253	81,000
	.063	92,800
SC3 _i	.160	18,700
	.234	25,500
	.235	32,300
	.260	39,200
	.077	46,100
EPHCR _i	.160	10,756
	.234	14,676
	.235	18,586
	.260	22,516
	.077	26,436
NWCCR _i	.160	(5,270)
	.234	(7,190)
	.235	(9,100)
	.260	(11,010)
	.077	(13,000)

Table 7-13. Continued

Stochastic input	Probability density coefficients	flow
TIN6 _i	.142	7,050
	.256	9,050
	.275	11,000
	.196	13,020
	.042	15,000
M6DIV _i	.169	2,926
	.254	3,529
	.244	4,132
	.253	4,735
	.063	5,338

Table entries in parentheses estimated by the equation

$$NWCCR_i = \frac{\bar{X}[NCCR (EST)]}{\bar{X} (EPHRAIM)} \times EPHCR_i$$

$$NWCCR_i = .49 \times EPHCR_i$$

optimal development pattern for the entire basin shows 78 percent of the irrigation water supply coming from groundwater and 22 percent coming from surface water. In the west arm of the basin, where there is less opportunity for water to enter groundwater storage, the surface water use exceeds groundwater use. In this sub-basin (subdivision A-2), only 13 percent of the irrigation water supply in the optimal solution comes from groundwater. For the other subdivisions, however, groundwater volumes in the optimal solution greatly exceed surface water as a source of irrigation water supply. The percentages of irrigation water supplied from groundwater for subdivisions A-1, A-3, and A-4 are 86 percent, 94 percent, and 61 percent, respectively.

Table 7-14. Summary of optimal activity levels--major variables

Subdivision	Surface diversions (ac-ft/yr)	Pumping (ac-ft/yr)	Irrigation (ac-ft/yr)	Artificial recharge (ac-ft/yr)	Surface storage (ac-ft)	Groundwater storage used (ac-ft)
A-1	CF3 = 7,743 CF4 = 0	PIRB1 = 29,174	IRRA1 = 33,820	AREA1 = 0 AREA2 = 0	STC1 = 0 STC6 = 21,500	GWSTA = 24,875
A-2	CF1 = 7,742 CF2 = 9,970	PIRA2 = 1,440	IRRA2 = 11,294	N.A.	STC3 = 9,307 STC4 = 4,000 STC5 = 0	GWSTB = 151,754
A-3	CF5 = 28,867 CF6 = 0	PIRB3 = 175,083	IRRA3 = 186,630	AREB1 = 93,000	STC7 = 0	
A-4	CF7 = 32,257 CF8 = 3,550 CF9 = 0	PIRB4 = 89,384	IRRA4 = 146,514	AREB2 = 22,000	STC2 = 35,930 STC8 = 42	
Totals:	90,129	294,080	378,280	115,000	50,779	176,628

In order to compare existing development in the Sanpete basin with the optimal development pattern found by solution of the linear programming model, reference is made to the S.C.S. water budget presented earlier in this chapter. The water budget shows an average annual cropland consumptive use of 109,750 acre-feet, and an average annual wetlands consumptive use of 115,990 acre-feet. In comparison, using the linear programming model, the consumptive use for cropland under optimal development would be about 173,850 acre-feet per year, or 64,100 acre-feet per year more than the present development. The same reference (U. S. Department of Agriculture, 1963) reports a potential cropland consumptive use of 127,740 acre-feet per year which is only about 46,000 acre-feet less than the optimal. It seems reasonable that the conversion of wetland acreage to cropland acreage by groundwater pumpage could be accomplished. This conversion would involve about 25,000 acres or about half of the existing wetland acreage in the basin.

Continuing the comparison, the optimal development pattern shows much more groundwater development than the present pattern. The present-day trend, however, is toward more groundwater use in spite of the strong impedance imposed by Utah court decisions regarding groundwater development. Historically, water resource development has favored surface-water use because of the difficulty of extracting the groundwater and because groundwater is an "invisible" resource. With the development of better wells and pumps and with increased knowledge of the groundwater supplies, water resource development patterns trend toward more extensive use of groundwater. This is the case in the

Sanpete basin where several additional large irrigation wells are put into service each year. This trend is accelerated during dry years when the additional storage provided by groundwater aquifers is also needed.

The optimal development pattern calls for a total volume of 115,000 acre-feet per year to be artificially recharged to groundwater storage. Careful planning of artificial recharge facilities would be required in order to get this volume of water into the ground each year. If water could be diverted to artificial recharge during six months of the year, a total stream of 320 cfs would be required in order to get 115,000 acre-feet per year into groundwater storage. This would be possible, but several separate facilities would be required along the valley boundaries in highly permeable alluvial materials. The information from Richardson (1907) shows that a single creek at the valley margin was capable of recharging over 1.3 cubic feet per second in a one-mile reach of the stream. This is equivalent to nearly 1000 acre-feet per year. This indicates that lengthening the stream channel by relocation across highly permeable zones would be one means of putting large volumes of water into the ground artificially. Also, reworking the streambeds periodically could maintain even higher recharge rates. An additional 30 miles of stream channel in the highly permeable zones (15 streams with an additional 2 miles each) could put over 25 percent of the optimal quantity of artificial recharge water into the ground.

Another feasible means of artificially recharging large quantities of water would be the use of recharge basins. Using a design factor for such basins suggested by the Edward E. Johnson Company (1966), the full amount of optimal artificial recharge water

(115,000 acre-feet) could be put into the ground in six months' time using basins covering about 420 acres. This would amount to about 28 acres of basins near the mouth of each of the 15 streams.

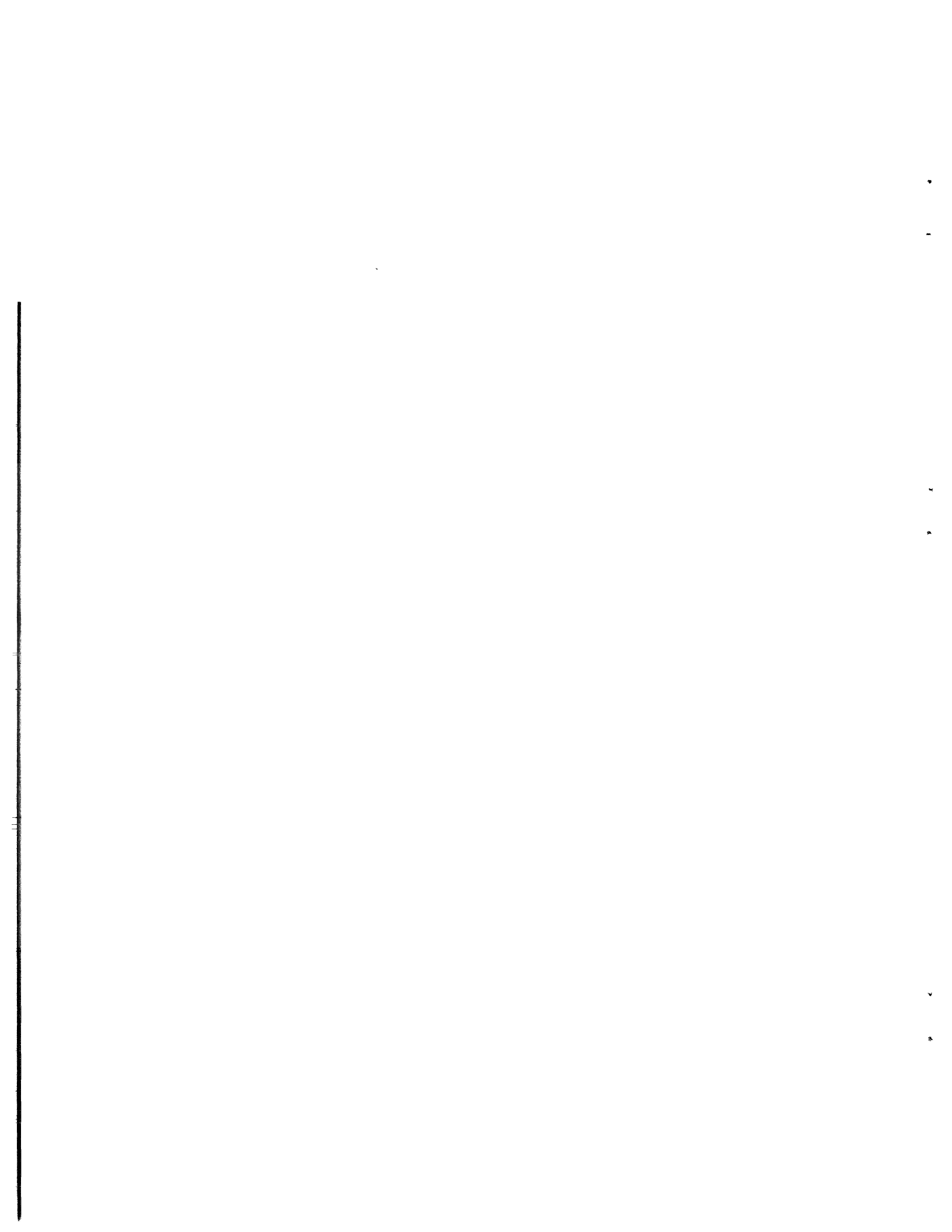
A still further alternative for artificial recharge would be the use of injection wells. Undoubtedly the best procedure would be a combination of all three methods and perhaps others as well. The point is that the optimal quantity of recharge could feasibly be moved into the ground.

The effect on the water table of putting this much water into the ground artificially is another question that should be noted. The Sanpete basin contains both water table and artesian aquifers. The detailed extent of each is unknown, so the combined storage coefficient is difficult to estimate. However, assuming a combined storage coefficient of 0.05 to 0.10, and assuming that no water would be extracted from the 115,000 acre-feet during the time of recharge and transmission, then the water table would be raised 10 to 20 feet. This amount would not cause difficult problems once the water table were lowered somewhat by development of the groundwater. Artificial recharge facilities would be developed only after the water table was lowered to such a point as to salvage the water now wasted by phreatophytes.

Post-optimal analysis of the solution shows that the optimal value of the objective function could be increased significantly for small increases in water availabilities at SC2, TMTND1, TMTND2, NWCCR, and EPHCR. These are most of the small streams on the east side of the valley together with transmountain diversions. Watershed management practices which would produce these increases should be pursued.

Sensitivity analysis on the cost coefficients used in this model shows that the optimal solution is insensitive to changes in the cost coefficients. However, small increases or decreases in the irrigation benefits would cause changes in the optimal solution. This points out the need for additional research in defining irrigation benefits.

In summary, the linear programming analysis of the Sanpete basin shows that conjunctive use of surface water and groundwater is the optimal pattern of development with much greater emphasis on groundwater development than in the present development pattern. In view of uncertain hydrologic inflows and accompanying desirability for storage, the emphasis on groundwater storage and groundwater development is highly logical. This shift in emphasis in the optimal pattern points out the desirability of using such an optimizing procedure in water resources planning.



CHAPTER VIII

CONCLUSIONS

The purpose of the research reported herein has been to develop a procedure or methodology for planning conjunctive use of groundwater and surface water. The results have been presented in sections dealing first with the development of mathematical models of a hypothetical river basin, then with the development of similar models for two real river basins. Conclusions regarding each of the models are presented at the end of each section dealing with the respective models. It is felt that in this final chapter some general conclusions should be presented regarding the mathematical programming approach used in this research. Hence, some of the advantages and disadvantages of the linear programming approach to water resources allocation in the context of conjunctive use will be presented. Some other methods of mathematical programming are currently being researched for their application to water resources planning, hence the linear programming approach will be compared with other methods of analysis. Finally, future research possibilities stemming from this work will be presented in this final chapter.

Critique of methodology of linear programming analysis

Linear programming is an iterative optimization technique developed for computer solution. The technique guarantees that an optimal solution will be found after a finite number of iterations provided the linear programming model forms a convex set. In the study presented, linear programming applies to the task of optimally

allocating the resources within a water resource system--particularly a system in which groundwater resources and surface water resources are used conjunctively.

The linear programming application is advantageous in that the solution gives the optimal quantitative values for each of the decision variables in the model while simultaneously satisfying all of the constraints on the system. The constraints take account of the hydrologic, engineering, and economic considerations.

If the assumptions necessary for formulating the linear model are felt to be too restrictive, the solution can at least serve as a useful starting point for decision making by planners. Nonlinear relationships can be handled provided they are separable, although this procedure has not been used in this study.

Models describing much more elaborate and complex systems can still be handled within the computational framework of the algorithm. For practical purposes, the size of the model in terms of the number of constraints and variables is limited only by the computer capacity, the computer time available, and by the patience and time of the researcher.

Alternative decisions are easily studied using the linear programming approach, and the effect of changes in resource availabilities and in benefits and costs on the optimal solution are easily studied. The simple and readily available sensitivity analysis and parametric analysis are strong advantages of the linear programming approach.

Constraints and considerations which are not readily formulated as part of the mathematical model can be evaluated quantitatively by imposing these conditions as constraints on the model. Solution of

the model with and without the constraint gives an imputed effect of the condition in the context of the model.

Proponents of other planning techniques such as dynamic programming and simulation are sure to point out the disadvantages of the linear programming approach. Among the disadvantages are:

1. Cost and benefit coefficients must be considered as average values over the period of analysis. There is no means for describing in the model the differential value of money with time.
2. Water table fluctuations can be dealt with only by repeated solutions of the problem. This problem can be overcome partially by making use of some of the newer linear programming codes which permit changes in objective coefficients and right-hand side values simultaneously.
3. In order to build a reliable model of a conjunctive-use system, accurate and detailed information concerning costs of the various activities and benefits of the water uses must be formulated. This information seems to be very scarce in the form required and it is difficult to obtain for most real river basins.
4. Many of the relationships defined as linear for this method of analysis are in reality not so. Linear approximations can be made, but this in turn reduces the accuracy of the results from the nonlinear situation as well as increases the size of the problem.
5. Actual application of the results may be difficult or impossible in an actual river basin because of social and

political consideration which have not been formulated in the analytical model. The optimum results may also lead to changes in the political and social climate. Such changes are difficult, if not impossible, to forecast.

When all of the outside considerations are presented at the conclusion of the linear programming analysis, the question might well be presented: "Is there an optimal pattern of water resource allocation for a study area?" Conventional methods and approaches are not capable of specifying the social optimum resource use in terms that are practical and operational. Mathematical methods of analysis, such as the linear programming approach, find difficulty in dealing with concepts that are not sufficiently advanced to offer a satisfactory conceptual and analytical framework. The stage has not yet been reached where the social and political behavior of man can be made susceptible to optimizing techniques. The social and political optimum in resource use can at best be stated only partially and incompletely.

Other considerations which might cause questions concerning any mathematical optimum are those concerning the crucial issues of the selection and formulation of objectives and criteria. The criterion of economic efficiency does not always reflect the best public policy. Perhaps what is needed in place of an optimal resource allocation methodology is an optimizing approach to public policy making.

The final problem to be mentioned in this respect is the problem of actual implementation of the results of the mathematical optimization. In formulating the mathematical model the planning area or river basin is considered as a whole unit. Subdivisions and

component parts are considered as parts of the whole and not as separate entities. In actuality, there may be several agencies and individual companies involved in the allocation and use of the water resources of the basin. For example, in the Sanpete basin there are 64 individual irrigation and canal companies. Generally, one would not expect any degree of general agreement on the objectives of planning and development of the water resources of the basin. There would be even less agreement with regard to implementation of the results of the mathematical optimization. In order to carry out the optimal activities, some strong central organization and authority would be necessary. So the question remains: "Is there a practical optimal pattern of water resource allocation for any given real basin?"

Some comparisons with other methods of analysis

Two primary objections to the use of linear programming are the requirements of linearity in the model and of convexity of the feasibility region. Dynamic programming, which was developed primarily for dealing with sequential decision problems, is not restricted by the linearity and convexity limitations of linear programming. However, linear programming analysis does have some distinct advantages over dynamic programming analysis in that it is capable of dealing with a large number of decision variables simultaneously, whereas dynamic programming analysis is severely limited in this regard. Three or four variables are considered to be an upper limit on the number of state variables that can be handled with present computer capabilities. On the other hand, linear programming analysis with present computer capabilities can deal effectively with several hundred variables on most machines, with several thousand variables on

a few larger machines, and with up to 99,000 variables on certain machines equipped with large drum storage capacities. In resource allocation problems where sequential decisions are not of the essence, the capability of dealing with a large number of variables simultaneously becomes a large advantage.

The simulation approach has the advantage of detailed modeling of even complex systems. This capability not only gives a closer representation of the real system, but also gives the analyst a better understanding of the system. However, the advantages of detail may not be so important in the planning stages of a development. In planning optimal resource development, simulation has the disadvantage of not yielding a direct optimal solution. Each solution obtained by simulation depends upon the particular set of system variables chosen by the analyst. Repeated solutions, using different activity levels of the system variables chosen by the user, can lead to successively better or worse solutions. The best answer obtained after repeated solutions cannot be guaranteed to be the optimum even within the framework of the model chosen.

Extensions for future research

This study has emphasized the need for future research in establishing more reliable values for the objective coefficients. Additional information is especially needed in regards to benefits attached to various water uses and to various levels of water use. Further information is also needed in establishing shortage costs associated with not meeting demands.

In this study, one particular method of handling stochastic inflows has been presented and used. However, this approach places

some limitations on the interpretation of the sensitivity analysis and would certainly make parametric analysis much more difficult. It is believed that additional research effort is needed in order to develop a better method of dealing with these stochastic inflows.

For the models used in this study, actual parametric analysis was not carried out because of problems in the computer routine. Further research should be carried out on the models used in this study to complete a parametric analysis and interpretation.

Multiple use of water resource facilities can be dealt with only indirectly in the models formulated in this study. Further research effort could expand the model formulation to include multiple-use alternatives, especially in surface reservoir operation. The resulting models would be much larger than the ones presented in this study and could describe some basins more accurately.

Since linear programming analysis leaves no means of validating the models, except by examining the reasonableness of the results, some future research work is warranted in this direction. Perhaps linear programming results from several river basins could be modeled for analog or digital simulation in order to test the validity of the several assumptions involved in making the linear programming models.

The results of the models presented herein emphasize groundwater development. Additional effort is needed to investigate the institutional requirements and changes that might be necessary in order to implement the optimal water resources development pattern. Perhaps some institutional changes (such as the establishment of a basin-wide groundwater district) could be brought about which would allow ground-

water development to by-pass the limitations caused by rulings of the courts. Undoubtedly, many other facets regarding institutional requirements and arrangements need to be investigated before an attempt is made to implement the optimal development plan.

Summary

The objective of this study was not to develop new theory in the optimization field but rather to develop a working tool and method of analysis for the water resources planner. The method of analysis is outlined and illustrated with five different model formulations. The approach presented here can aid the water resources planner in at least three areas: (1) actual formulation of optimal resource allocation, (2) proper emphasis in the fact-finding and data acquisition phase of a study, and (3) guidance and direction when time and means do not permit gathering of all the information needed in a study.

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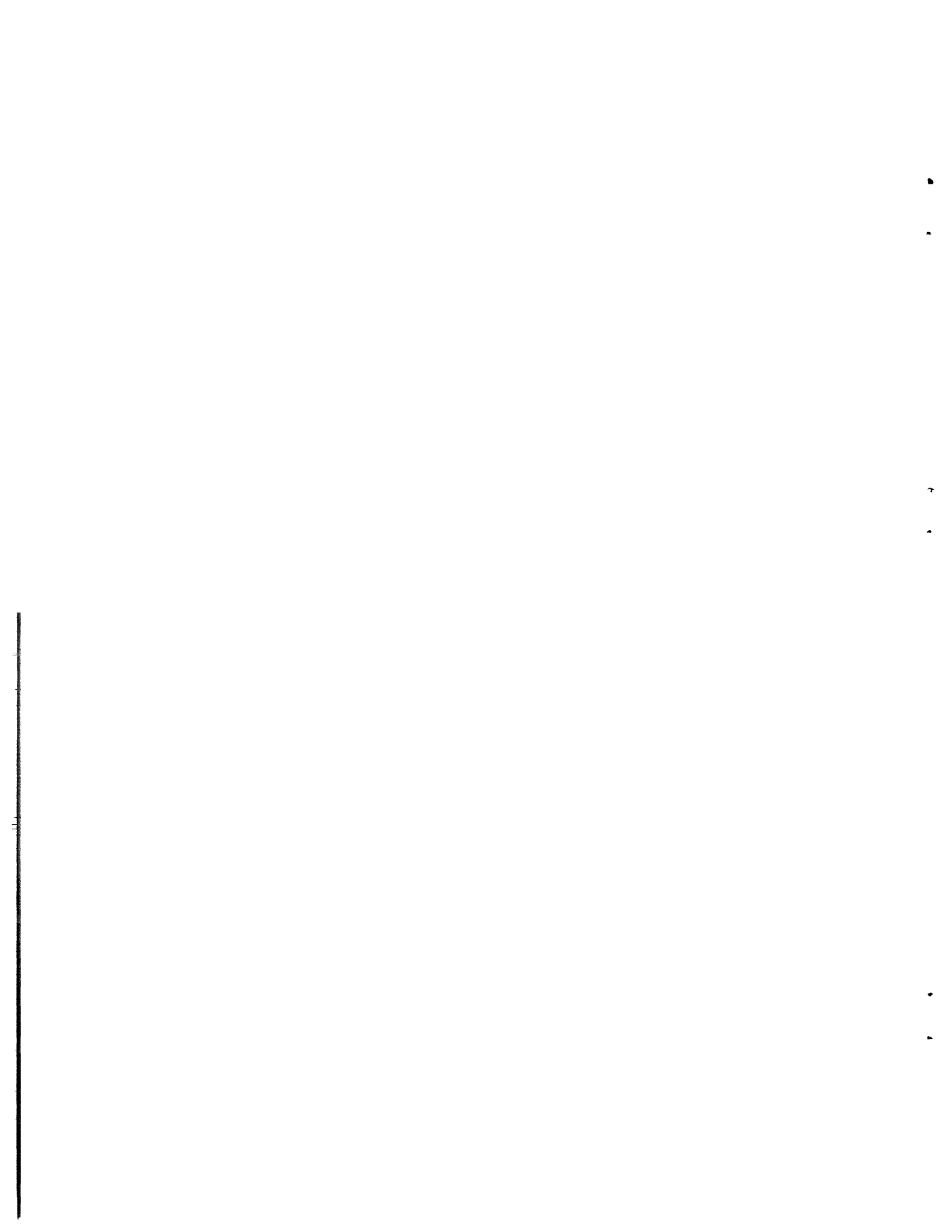
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APPENDIX

Label Cost	ARTRED	SSTOR	CFW	ARTREW	GW1	GW2	GW3	GW4	PU1RW1	PU1RW2
15.000000	-3.500000	-7.000000	-14.000000	.000000	2.510000	5.020000	7.530000	-6.000000	-8.000000	
Row Label										
1 BEN1	15.000000	-3.500000	-7.000000	-14.000000	.000000	2.510000	5.020000	7.530000	-6.000000	-8.000000
2 GWS11	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
3 SS1D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
4 GWS21	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
5 SS2D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
6 GWS31	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
7 SS3D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
8 GWS41	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
9 SS4D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
10 DIVD	1.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
11 DIVW	.000000	1.000000	1.000000	1.000000	.000000	.000000	.000000	.000000	.000000	.000000
12 IRRD	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
13 IRRW	.000000	.000000	.700000	.000000	.000000	.000000	.000000	.000000	1.000000	1.000000
14 GWS1W	.000000	.000000	-3.000000	-1.000000	1.065000	.000000	.000000	.000000	1.000000	.000000
15 SS1W	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000	.000000
16 GWS2W	.000000	.000000	.000000	.000000	.000000	1.036000	.000000	.000000	.000000	1.000000
17 SS2W	.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000
18 GWS3W	.000000	.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000
19 SS3W	.000000	.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000
20 GWS4W	.000000	.000000	.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000
21 SS4W	.000000	.000000	.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000
22 CONT	1.000000	-1.000000	1.000000	1.000000	-.085000	-.036000	.000000	.000000	.000000	.000000
23 CONMA	1.000000	-1.000000	-1.000000	-1.000000	.065000	.036000	.000000	.000000	.000000	.000000

Label Cost	PU1RW3	PU1RW4	PUEXW1	PUEXW2	PUEXW3	PUEXW4	PERCW1	PERCW2	PERCW3	*B1
-12.000000	-15.000000	-3.000000	-4.000000	-5.000000	-6.000000	.000000	.000000	.000000	.000000	.000000
Row Label										
1 BEN1	12.000000	-15.000000	-3.000000	-4.000000	-5.000000	-6.000000	.000000	.000000	.000000	.000000
2 GWS1D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	34.500000
3 SS1D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	59.999999
4 GWS2D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	59.999999
5 SS2D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	59.999999
6 GWS3D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	59.999999
7 SS3D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	59.999999
8 GWS4D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	180.000000
9 SS4D	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	180.000000
10 DIVD	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	14.400000
11 DIVW	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	35.699999
12 IRRD	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
13 IRRW	1.000000	1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
14 GWS1W	.000000	.000000	1.000000	.000000	.000000	.000000	1.000000	.000000	.000000	4.500000
15 SS1W	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	59.999999
16 GWS2W	.000000	.000000	.000000	1.000000	.000000	.000000	-1.000000	1.000000	.000000	.000000
17 SS2W	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	59.999999
18 GWS3W	1.000000	.000000	.000000	.000000	1.000000	.000000	.000000	-1.000000	1.000000	.000000
19 SS3W	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	59.999999
20 GWS4W	.000000	1.000000	.000000	.000000	.000000	1.000000	.000000	.000000	-1.000000	.000000
21 SS4W	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	180.000000
22 CONT	.000000	.000000	-1.000000	-1.000000	1.000000	-1.000000	.000000	.000000	.000000	10.000000
23 CONMA	.000000	.000000	1.000000	1.000000	1.000000	1.000000	.000000	.000000	.000000	-9.500000

Figure A-1. Continued

Label COST RC# LABEL	R-VEC .000000	CF1 -1.660000	CF2 -1.720000	CF3 -2.620000	CF4 -2.130000	CF5 -1.070000	ARTPE1 -1.200000	ARTPE2 -3.150000	ARTPE3 -4.800000	ARTPE4 -3.900000	ARTPE5 -1.950000	SSTOR .000000	PU11 -.700000	PU12 -1.100000
1 REL1	.000000	-1.660000	-1.720000	-2.620000	-2.130000	-1.070000	-1.200000	-3.150000	-4.800000	-3.900000	-1.950000	.000000	-.700000	-1.100000
2 RELS01	5.000000	1.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000
3 RELS02	20.000000	.000000	1.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000	.000000	.000000
4 RELS03	34.999999	.000000	.000000	1.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000	.000000
5 RELS04	50.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000
6 RELS05	65.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000	1.000000	-1.000000	.000000	.000000
7 RELS06	14.870000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	1.000000	.000000
8 RELS07	16.790999	.000000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	1.000000
9 RELS08	18.711000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	1.000000
10 RELS09	20.632000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000
11 RELS10	22.553000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000
12 CAPA01	-5.000000	-1.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000
13 CAPA02	-20.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	1.000000	.000000
14 CAPA03	-34.999999	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	1.000000
15 CAPA04	-50.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	1.000000
16 CAPA05	-65.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	1.000000
17 CAPA06	-14.870000	.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000
18 CAPA07	-16.790999	.000000	.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000	.000000	-1.000000
19 CAPA08	-18.711000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000	.000000
20 CAPA09	-20.632000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000	.000000
21 CAPA10	-22.553000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	1.000000	.000000	.000000	.000000
22 PRCS01	27.000000	.000000	.000000	21.600000	.000000	2.000000	.000000	13.000000	.000000	21.000000	32.000000	2.000000	.000000	.000000
23 PRCS02	18.614999	.000000	.000000	.000000	.000000	.000000	-0.000000	-21.000000	-13.000000	-32.000000	-2.000000	.000000	1.000000	.000000
24 SUBS11	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000
25 SUBS12	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-1.000000
26 SUBS13	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
27 SUBS14	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
28 SUBS15	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000

Label COST RC# LABEL	PU13 -1.500000	PU14 -1.150000	PU15 -1.550000	STCAP -3.500000	GWST .000000	GWAP .000000	SHORT1 -6.600000	SHORT2 -14.400000	SHORT3 -19.199999	SHORT4 -13.200000	SHORT5 -6.600000	IPRIG 15.000000	*PI .000000
1 REL1	-1.500000	-1.150000	-1.550000	-3.500000	.000000	.000000	-6.600000	-14.400000	-19.199999	-13.200000	-6.600000	15.000000	.000000
2 RELS01	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	5.000000
3 RELS02	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	20.000000
4 RELS03	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	34.999999
5 RELS04	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	50.000000
6 RELS05	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	65.000000
7 RELS06	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	14.870000
8 RELS07	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	16.790999
9 RELS08	1.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	18.711000
10 RELS09	.000000	1.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	20.632000
11 RELS10	.000000	.000000	1.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	22.553000
12 CAPA01	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-5.000000
13 CAPA02	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-20.000000
14 CAPA03	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-34.999999
15 CAPA04	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-50.000000
16 CAPA05	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-65.000000
17 CAPA06	.000000	.000000	.000000	.000000	1.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	-14.870000
18 CAPA07	.000000	.000000	.000000	.000000	1.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	-16.790999
19 CAPA08	-1.000000	.000000	.000000	.000000	1.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	-18.711000
20 CAPA09	.000000	-1.000000	.000000	.000000	1.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	-20.632000
21 CAPA10	.000000	.000000	-1.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	-22.553000
22 PRCS01	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	27.000000
23 PRCS02	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	18.614999
24 SUBS11	.000000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000
25 SUBS12	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000
26 SUBS13	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000
27 SUBS14	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000	.000000
28 SUBS15	.000000	.000000	-1.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	-1.000000	.000000	.000000

Figure A-2. Matrix representation of the stochastic model