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Process Studies and Modeling of Self-Cleaning Capacity of Mountain Creeks for Recreation Planning and Management

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PROCESS STUDIES AND MODELING OF SELF-CLEANING CAPACITY OF MOUNTAIN CREEKS FOR RECREATION PLANNING AND MANAGEMENT

Prepared by

Cheng-lung Chen
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ABSTRACT

Reaeration process studies were conducted on a mountain creek and a large laboratory flume. The method of evaluating the dispersion coefficient, mean velocity, and reaeration coefficient for both creek and flume consisted of finding these values for a deoxygenated portion of the flow containing a conservative tracer (dye). The deoxygenated slug is measured as it moves downstream and the three values are best fit in the analytical solution of the longitudinal dispersion equation which dynamically describes the flow of the dispersing slug in the stream. The best fit was accomplished by using the method of least squares in which the sum of squares of the differences between the dissolved oxygen and dye concentrations calculated from the dispersion equation and those obtained from the actual measurements is minimized.

A reaeration coefficient prediction model of general form was developed. The model is composed of two dimensionless parameters which were identified from the normalized dissolved-oxygen balance equation. A simplified model which has two model parameters was also developed. Both model parameters were evaluated specifically for the mountain creek and laboratory flume. A comparison of this simplified model with existing models revealed that most existing models are incomplete in form. It was found that inclusion of the dispersion coefficient in the reaeration coefficient model improved the prediction accuracy.

The information obtained from this study would aid in determining the oxygen balance of mountain creeks which is essential to the resource management of mountain watersheds.

PREFACE

The present research aimed at the determination of the self-cleaning or reaeration capacity of mountain creeks, a knowledge of which is essential to the planning and management of recreation and land development in mountain watersheds. It was planned that the proposed research would be carried over a period of two years. The first year was devoted largely to the review of literature, selection of sites for field measurements of the reaeration coefficient, and the development of the sampling procedure and technique for dissolved oxygen and dye concentrations measurement. A method for measuring the longitudinal dispersion coefficient of a stream which is essential to the reaeration coefficient modeling was also studied. Although two sites for field measurements were selected from Summit Creek, only one site was thoroughly investigated. Instead, tests on a big laboratory flume were performed so that a good control of flow conditions and hydraulic parameters was secured.

During the first year, the proposed methods for collecting and/or measuring samples by using an oxygen analyzer, a Vacutainer, and/or a spectrophotometer proved somewhat of a problem. A unique sampling and measuring procedure was thus developed in the second year and proved to be valid for creeks studied. During the second year, a general mathematical model of the self-cleaning capacity for a stream and, in particular, for a mountain creek was developed and validated using data collected in this study as well as that obtained from previous investigators' studies.

The research was conducted under the general supervision of Dr. Cheng-lung Chen, Professor of Civil and Environmental Engineering at Utah State University. Dr. Chen, the senior author of this report, primarily developed the theoretical foundation for the computations of the dispersion coefficient, mean velocity, and reaeration coefficient as well as the method for identifying the significant dimensionless parameters involved in the reaeration process. The junior author, Keith D. Davis, Graduate Research Assistant of the Utah Water Research Laboratory, performed most of the experimental work, analysis of data, and computer programming.

Gratitude is due many students who assisted in the field and laboratory reaeration tests for this project. The writers wish to thank Mr. Gilbert Peterson for his assistance in the design and construction of sampling devices used in the measurements. Appreciation is also due Ms. Donna Falkenborg for her editorial assistance and Ms. Annette Brunson for typing this manuscript.

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NOMENCLATURE

Symbol	Definition
a	Coefficient representing the slope in general linear equation, $Y = aX + b$.
A	Cross-sectional area of the stream, in square feet.
b	Y-axis intercept in the general linear equation, $Y = aX + b$.
BOD	Chemical oxygen demand of the carbonaceous material, in milligrams per liter.
C	Dissolved-oxygen concentration, in milligrams per liter.
C_a, C_b	Dissolved-oxygen concentrations at the upstream and downstream ends of the reach, respectively, at the first level of dissolved-oxygen concentration in the disturbed-equilibrium technique, in milligrams per liter.
C_a', C_b'	Dissolved-oxygen concentrations at the upstream and downstream ends of reach, respectively, at the second level of dissolved-oxygen concentration in the disturbed-equilibrium technique, in milligrams per liter.
C_i	Conservative tracer concentration, in parts per billion.
$C_i(x,t)$	Conservative tracer concentration at distance, x, and time, t, from the reference point, in parts per billion.
C_s	Dissolved-oxygen concentration at saturation, in milligrams per liter.
D	Dissolved-oxygen deficit, or the difference between the saturation concentration and the concentration of dissolved-oxygen, in milligrams per liter.
$D(x,t)$	Dissolved-oxygen deficit at distance, x, and time, t, from the reference point, in milligrams per liter.
D'	Transformed dissolved-oxygen deficit being equal to $D \exp(K_2(t-\tau))$, in milligrams per liter.
D_a	Dissolved-oxygen deficit at upstream end of reach, in milligrams per liter.

Symbol	Definition
D_m	Molecular-diffusion coefficient.
D_0	Initial dissolved-oxygen deficit of the stream, in milligrams per liter.
D_x	Longitudinal dispersion coefficient, in square feet per second.
D_B	Parameter that includes the effect of photosynthesis, plant respiration, and removal of dissolved-oxygen by benthal layer; in milligrams per liter per second.
D_*	Dimensionless depth being equal to D/D_0.
DO	Dissolved-oxygen.
E	Rate of energy dissipation per unit mass of liquid, equal to USg for open-channel flow, in square feet per cubic seconds.
E_P	Percent standard error of estimate for the reaeration coefficient, K_2, in percent.
E_S	Standard error of estimate for the reaeration coefficient, k_2, in reciprocal days.
E_{SL}	Standard error of estimate for the reaeration coefficient, k_2, based on common logarithms.
$f(D_x/L_0U)$	Function of the dimensionless parameter D_x/L_0U.
F	Froude number, equal to $U/\sqrt{gH}$.
g	Acceleration of gravity, in feet per second per second.
G	Constant rate of sustained injection, in mass per unit time.
H	Hydraulic depth, equal to the cross-sectional area divided by the surface width of flow.
k_2	Reaeration coefficient or rate constant for oxygen absorption from the atmosphere, common logarithm base, in reciprocal days, or in reciprocal seconds.

$(k_2)_e$ — Reaeration coefficient estimated from a linear regression analysis equation, common logarithm base, in reciprocal days.

$(k_2)_m$ — Reaeration coefficient calculated from the results of an experimental test, common logarithm base, in reciprocal days.

$(k_2)_{20}^{\circ}$ — Reaeration coefficient, common logarithm base, in reciprocal days at 20 degrees centigrade.

K_1 — Deoxygenation coefficient or rate constant for biochemical oxidation of carbonaceous material, natural logarithm base, in reciprocal seconds.

K_2 — Reaeration coefficient or rate constant for oxygen absorption from the atmosphere, natural logarithm base, in reciprocal seconds.

L — Biochemical oxygen demand of carbonaceous material, in milligrams per liter.

L_0 — Hydraulic parameter describing a reference length such as hydraulic radius or hydraulic depth, in feet.

M — Mass of matter injected into stream.

n — An integer.

Q — Discharge in cubic feet per second.

R — Hydraulic radius, equal to the cross-sectional area of flow divided by the wetted perimeter, in feet.

S — Water-surface or channel slope, in feet per foot.

t — Time, in seconds.

t' — Flow time, or the time that must elapse between dissolved-oxygen measurements at the two sampling points in a recirculating flume, in seconds.

T — Temperature in degrees centigrade.

T_* — Dimensionless time equal to tU/L_0.

U — Mean flow velocity in the longitudinal direction, in feet per second.

U_* — Shear velocity, equal to $\sqrt{gHS}$, in feet per second.

W — Width, in feet.

x — Longitudinal coordinate, in feet.

x' — Longitudinal coordinate in a coordinate system moving with the mean flow velocity, equal to $x-Ut$.

x_0 — Initial longitudinal coordinate.

X — Independent variable in the general linear equation, $Y = aX+b$.

X_* — Dimensionless coordinate equal to x/L_0.

Y — Dependent variable in the general linear equation, $Y = aX+b$.

a — Coefficient or parameter in reaeration coefficient prediction equation.

β — Exponent in reaeration coefficient prediction equation.

$\delta(x)$ — Dirac-delta function.

ε_y — Eddy-diffusion coefficient for mass (y subscript indicates vertical direction.)

Θ — Temperature coefficient.

μ — Dynamic viscosity of the liquid.

π — The constant 3.1416. . .

ϱ — Mass density of the liquid.

τ — Second time coordinate having the same reference point as t, in seconds.

$\varnothing(\tau)$ — Distribution function describing a tracer concentration or dissolved-oxygen deficit versus time curve.

INTRODUCTION

The natural purification capacity of a stream or river in terms of assimilating biodegradable wastes is directly related to its ability to replenish the dissolved oxygen in its flow. The oxygen is required for the decomposition of organic material by bacteria. The rate at which this oxygen is replenished from the atmosphere is known as the reaeration rate. With increasing recreation and land-development in mountain and forest areas, the estimation of the self-cleaning capacity of mountain creeks becomes increasingly important because this information would greatly aid in taking protective measures against future pollution of mountain creeks. It would also aid in determining the oxygen balance of mountain creeks by optimizing the need, degree, and costs of waste treatment that may be essential to the long-range planning and management of mountain watersheds.

The primary objective of this study was the determination and modeling of the reaeration rate of mountain creeks. The specific objectives were:

1. To review literature dealing with the reaeration process and methods and techniques used to measure and calculate the reaeration rate.

2. To determine the reaeration rate of a mountain creek at various temperatures and stages of flow by measuring the movement of a slug of artifically deoxygenated water in the creek.

3. To formulate a mathematical model for predicting the reaeration capacity of a mountain creek using parameters such as the longitudinal dispersion coefficient, stream geometry, and flow characteristics.

4. To evaluate the range of error associated with the use of various reaeration coefficient prediction models, including the relationship formulated in this study, on the data collected for this study and available data from previous investigations. A measurement of the reaeration rate in a large laboratory flume at the Utah Water Research Laboratory (UWRL) was also attempted to substantiate the findings.

The uniqueness of this study lies in two major areas. The first area deals with the method of experimentally determining the reaeration rate. In this study the dispersion coefficient, mean velocity, and reaeration coefficient were determined from a deoxygenated slug containing a dye. The three values were optimized in the analytical solution of the longitudinal dispersion equation which dynamically describes the flow of a dispersing slug in the stream. The optimization was achieved by best-fitting calculated dye and dissolved oxygen concentrations to the corresponding measured data. The second area of uniqueness deals with the approach to formulate a reaeration coefficient prediction model. Significant dimensionless parameters involved in the reaeration process were identified from the dissolved-oxygen balance equation in which the reaeration coefficient was used to describe the rate for the absorption of oxygen from the atmosphere. The mathematical model formulated appears to be an improvement over existing models in predicting the reaeration coefficients of mountain creeks under various flow conditions.

1

REVIEW OF LITERATURE

The reaeration process in a stream is the physical absorption of oxygen from the atmosphere by the flowing stream to replenish oxygen consumed in the biodegradation of organic materials. Adeney and Becker (1919, 1920) showed that the amount of reaeration by water is a first order process directly proportional to the saturation deficit, D, which is defined as the difference between the saturated dissolved oxygen (DO) concentration of the water and the actual DO concentration. Mathematically the reaeration process at any point on the water surface may be described by

$$\frac{dD}{dt} = -K_2 D = -K_2(C_s - C) \qquad \ldots \ldots \ldots \ldots (1)$$

where K_2 is the rate coefficient for the absorption of oxygen from the atmosphere commonly referred to as the reaeration coefficient. Many investigators have used common logarithms (base 10) to calculate and model the reaeration coefficient. In this case the coefficient is conventionally designated by k_2 with the conversion equation:

$$K_2 = 2.303\, k_2 \qquad \ldots \ldots \ldots \ldots \ldots \ldots (2)$$

The two primary contributions of this study were the method used to measure the reaeration coefficient and the approach used to formulate a prediction model for the reaeration coefficient. Thus the review of earlier investigations was focused on these two aspects, the measuring technique and model formulation. An extensive review of literature for other aspects such as methods of determining DO concentration and reaeration coefficients, formulations of existing theoretical oxygen-absorption models and empirical and semiempirical reaeration coefficient prediction models, a compilation of the available experimental data, and analyses of errors associated with the use of various models on these experimental data was conducted by Bennett and Rathbun (1972). For simplicity in presentation, however, most of these aspects, except for the two as indicated above, will not be recapitulated herein. Literature reviews of less detail were also provided by Lau (1972a) and Wilson and Macleod (1974).

Reaeration Coefficient Measurements

There are three basic techniques for determining the reaeration coefficient of open-channel flow: the dissolved-oxygen balance technique, the disturbed-equilibrium technique, and the tracer technique.

Dissolved-oxygen balance technique

The starting point for measuring reaeration coefficients by using the dissolved-oxygen balance method is the dissolved oxygen balance equation developed by Streeter and Phelps (1925) and later used by Camp (1963) and Dobbins (1964). It can be written in the form:

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} = D_x \frac{\partial^2 C}{\partial x^2} + K_2(C_s - C)$$
$$- K_1 L - D_B \qquad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3)$$

where C is the DO concentration; C_s is the saturation DO concentration; x is the longitudinal coordinate along the reach; U is the mean stream velocity; D_x is the longitudinal-dispersion coefficient; K_1 is the rate constant for the oxidation of carbonaceous wastes that is assumed to be a first-order process proportional to L; L is the biochemical oxygen demand (BOD) of the carbonaceous material; and D_B is the amount of the removal or addition of DO by diffusion into the benthal layer and the effect of photosynthesis and respiration by plants. The value of D_B can be positive or negative. It is positive by convention (Dobbins, 1964) when the rate of removal of DO exceeds the rate of addition and vice versa. Equation 3 has two basic assumptions among many others: (1) the DO and BOD are uniformly distributed over each cross section and (2) the processes described by K_1 and K_2 are first-order processes.

Reaeration studies involving natural streams or non-recirculating flumes usually assume steady state and nondispersive conditions, thereby letting $\partial C/\partial t$ and D_x of Equation 3 be equal to zero. This leaves Equation 3 in a form which can readily be

solved with two of the K_1, K_2 and D_B values considered as unknowns. For instance, the reaeration coefficient K_2 is found by evaluating the other two values. The value of K_1 is calculated from a standardized 5-day, 20°C BOD test. The parts making up D_B, namely sedimentation, photosynthesis, and respiration, can be measured or estimated by different methods, many of which have been described in detail by Bennett and Rathbun (1972).

Reaeration studies using recirculating flumes usually assume nonsteady-state, nondispersive conditions. Nondispersive conditions assume that the dispersion term is small compared with the convection term, $U \partial C/\partial x$. The sources and sinks of DO can be controlled in such a way that K_1 and D_B of Equation 3 become zero. Thus, Equation 3 reduces to

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} = K_2(C_s - C)$$

which can be alternatively written in terms of the deficit, D, as

$$\frac{\partial D}{\partial t} + U \frac{\partial D}{\partial X} = -K_2 D \quad \dots \dots \dots \dots (4)$$

By making the transformation of the coordinate systems from (x, t) to (x′, t′) using the relations x′ = x - Ut and t′ = t, Equation 4 has the solution

$$D = D_a e^{-K_2 t'} = D_a 10^{-k_2 t'} \quad \dots \dots \dots (5)$$

or

$$k_2 = 1/t' \, (\log D_a - \log D) \quad \dots \dots \dots \dots (6)$$

where D_a is the deficit at the upstream point and D is the deficit at a downstream point. If a deficit is artificially created in the water to be recirculated by the addition of sodium sulfite and a cobalt catalyst, k_2 can easily be calculated by using Equation 6. Equation 6 has been used by Krenkel and Orlob (1962), Negulescu and Rojanski (1969), and Thackston and Krenkel (1969). The DO at two points along the flume are measured at a lapse time of t′ apart which is equal to (x′ - x)/U, where x′ - x is the distance between measuring points. Usually a series of measurements are taken at each point so that when plotting log D versus time, two straight parellel lines are formed from the two sets of data.

Under some special conditions natural streams can be measured with the assumption that K_1 and D_B are negligible such as those reaches chosen by Churchill, Elmore, and Buckingham (1962). Since streams are considered to be uniform within the test reach, Equation 4 can be written as

$$\frac{dD}{dt} = -K_2 D \quad \dots \dots \dots \dots \dots \dots \dots (7)$$

with the solution

$$D = D_a e^{-K_2 t} = D_a 10^{-k_2 t} \quad \dots \dots \dots \dots (8)$$

or

$$k_2 = 1/t \, (\log D_a - \log D) \quad \dots \dots \dots \dots (9)$$

which is identical to Equation 6 except for t′ used in Equation 6. In Equation 9, t is the mean time of flow between sampling stations, but the actual DO measurements do not have to be taken at t time apart as it is necessary with Equation 6 since in Equation 9 conditions are steady state and the DO concentrations at the two stations should remain constant and may be measured at any time.

Disturbed-equilibrium technique

The disturbed-equilibrium technique developed by Gameson and Truesdale (1959) consists of measuring the DO concentration at two points along the stream at two different levels of DO concentration. The downstream sample is taken at a time, t, following the sampling at the upstream point. The time, t, is the mean time of flow between the two points of interest. The second DO concentration level is usually obtained by adding sodium sulfite and a cobalt catalyst, although diurnal variations in DO concentrations due to photosynthesis can also be used. The reaeration coefficient can then be calculated by use of the equation

$$K_2 = \frac{1}{t} \ln \frac{C_a - C_a'}{C_b - C_b'} \quad \dots \dots \dots \dots (10)$$

where C_a and C_a' are the two levels of DO measured at the upstream point and C_b and C_b' are corresponding values at the downstream point. Equation 10 is actually an elaboration of Equations 6 and 9 which can be used when the special conditions required for Equations 6 and 9 do not apply. As long as the rates of all the secondary processes affecting the DO concentration, such as photosynthesis, respiration, sedimentation, etc., are constant during the measurement of both levels of DO concentration, they are more or less cancelled out by use of Equation 10.

For Equation 10 to be valid, values of K_2, U, C_s, photosynthesis and respiration must be either constant during the testing period or in the cases of photosynthesis and respiration negligible. Artificial DO deficits can be created quickly for a second level measurement by use of sodium sulfite and a cobalt catalyst which provide better chances for keeping K_2, C_s, and U values constant. Also, the

relatively large deficit created by use of sulfite justifies the assumption of having the negligible photosynthesis and respiration rates. Hence the disturbed-equilibrium technique is particularly well suited for use in small streams where dosing with sodium sulfite is practical. A modification of Equation 10 is a formula developed by Zogorski and Faust (1973) to use the areas under DO deficit versus time curves at two points on a stream (i.e., the curves formed by measuring DO deficit values of a deoxygenated slug at two points along the stream as it passed by).

Tracer technique

The tracer technique first described by Tsivoglou et al. (1965) is discussed in detail by Tsivoglou (1967). Basically the technique consists of injecting tritiated water along with an inert radioactive tracer gas into a stream and measuring the radioactive gas at two points downstream. The tritiated water acts as a conservative dispersion tracer indicating how much of the radioactive gas has been lost to the atmosphere along the reach or actually the desorption rate of the gas. The most important point of this technique is that the ratio of this desorption rate to the reaeration rate is constant and that it can be measured in the laboratory.

The advantages of the tracer technique are that no assumptions are made as to channel or mixing characteristics and the method can be applied in polluted as well as clean rivers since the ratio of the desorption rate to the reaeration rate is not significantly affected by pollutants, temperature, or turbulence. The major disadvantages are the costs and equipment involved in injecting and monitoring the tracers and the fact that the method is limited to small streams because of the increased injection difficulties of radiation hazards in larger streams.

Prediction Equations and Their Formulations

There have been numerous models developed for the prediction of reaeration coefficients. Theoretical models of the oxygen-absorption process are generally not suited for prediction of the reaeration coefficient in streams because the model parameters have not been adequately related to bulk-flow hydraulic variables. Therefore this literature review will not include such theoretical models. The models extensively reviewed, which seem to be most often cited in literature, belong to semi-empirical and empirical types of equations. These types of equations also most easily lend themselves to the prediction of reaeration rates by using the data collected in this study.

The study by Churchill, Elmore, and Buckingham (1962) is considered by many to have the best and most reliable set of field data on reaeration rate of any investigations into the subject. Their measurements were made on stretches of rivers below dams where the water released from reservoirs was low in DO and BOD because of prolonged storage under thermally stratified conditions. Five hundred and nine values covering 16 different reaches in five rivers were determined using the dissolved-oxygen balance technique. The flow depths varied from about 2 to 11 feet, the mean velocity from 1.8 to 5 fps, and the discharge from 950 to 17,270 cfs.

Churchill, Elmore, and Buckingham (1962) ran many multiple-regression analyses in an attempt to relate their observed reaeration rates with gas and liquid parameters and various stream characteristics such as slope, friction factor, and Reynolds number. Because none of the prediction equations thus obtained proved to be statistically better than any of the others, the authors suggested the simplest one for general usage. This equation was

$$k_2 = 5.026 \, U^{0.969} \, H^{-1.673} \, (1.0241)^{T-20°} \ldots (11)$$

where k_2 is in reciprocal days, U, the velocity, is in feet per sec., H, the depth, is in feet, and T, the temperature, is in degrees Celsius.

Krenkel and Orlob (1962) conducted laboratory measurements of k_2 for flow with depths ranging from 0.08 to 0.20 feet in a 1-foot wide, 60-foot long tilting flume. The water was artificially deoxygenated with sodium sulfite and a cobalt catalyst and the dissolved-oxygen balance technique was used to calculate k_2 values. The longitudinal dispersion coefficient, D_x, was measured by injecting dye into the flow and measuring its change in concentration with time at a point downstream. A regression analysis was run for correlating the measured k_2 values with D_x and the depth of flow, H. The equation obtained was

$$(k_2)_{20°} = 3.659 \, D_x^{1.321} \, H^{-2.32} \ldots \ldots (12)$$

where k_2 is in reciprocal days, D_x is in ft²/sec., H is in feet, and $(k_2)_{20°}$ represents the reaeration coefficient value as it would be measured at 20 degrees centigrade. When measurements are not taken at 20°C, a correction factor of the form Θ^{T-20} is added to the equation. (A value of $\Theta = 1.0241$ has generally been used in previous investigations and has been used throughout this study.)

Krenkel and Orlob (1961) reasoned that the reaeration rate was related to E, the energy dissipated per unit mass of flowing fluid, which is equal to USg (U = velocity, S = slope, and g =

acceleration due to gravity). Another regression analysis was performed but this time relating the k_2 values with the parameters E and H. The new equation formulated was

$$(k_2)_{20^\circ} = 24.66\ E^{0.408}\ H^{-0.66} \quad\ldots\ldots\ldots\ldots(13)$$

where k_2 is in reciprocal days, E is in ft^2/sec^3 and H is in feet.

Adding on to earlier work, Dobbins (1964) developed expressions relating parameters in the film-penetration theory (see, e.g., Bennett and Rathbun, 1972) to ordinary hydraulic variables. In its most convenient form, Dobbins (1965) gave his experimentally derived equation as

$$k_2 = \frac{0.12C_A\ F\ E^{0.375}\coth\left[\dfrac{B\ E^{0.125}}{C_4^{1.5}H}\right]}{C_4^{1.5}H} \quad..(14)$$

The parameters shown in Equation 14 were related to stream characteristics by the following relationships, which also described the variation of k_2 with temperature, T.

$$C_A = 1.0 + F^2$$
$$C_4 = 0.9 + F$$
$$F = 9.68 + 0.054\ (T-20)$$
$$B = 0.976 + 0.0137\ (30 - T)^{1.5}$$
$$E = 30.0\ S\ U$$
$$F = U/\sqrt{gH}$$

In the above equation k_2 is in reciprocal days, U is in feet per second, H is in feet, and S is in feet per 1000 feet.

In the formulation of Equation 14, Dobbins (1965) used data gathered by Krenkel and Orlob (1962), the measurements of Churchill, Elmore, and Buckingham (1962), and data from the Ohio and other rivers previously assembled by O'Connor and Dobbins (1956).

Owens, Edwards, and Gibbs (1964) used the disturbed-equilibrium technique for measuring reaeration coefficients with deoxygenation being achieved by adding sodium sulfite and a cobalt catalyst. They measured 21 different reaches in six English streams and then calculated 32 separate k_2 values. Discharges ranged from 1.50 to 36.2 cfs, depths from 0.34 to 2.44 feet, and mean velocities from 0.13 to 1.83 fps.

Multiple-regression analysis of their data led to the correlation of the k_2 values to the parameters U and H:

$$k_2 = 10.90U^{0.73}\ H^{-1.75}\ (1.0241)^{T-20^\circ} \ \ldots(15)$$

However, after Owens, Edwards, and Gibbs (1964) incorporated the data obtained by Gameson, Truesdale, and Downing (1955) and Churchill, Elmore, and Buckingham (1962) with their own data and performed the same analysis, the resulting equation became

$$k_2 = 9.41\ U^{0.67}\ H^{-1.85}\ (1.0241)^{T-20^\circ} \ \ldots(16)$$

In both Equations 15 and 16 k_2 is in reciprocal days, U is in feet per second, H is in feet, and T is in degrees Celsius.

Langbein and Durum (1967), after combining the river data of O'Connor and Dobbins (1956) and Churchill, Elmore, and Buckingham (1962) with the laboratory flume data of Krenkel and Orlob (1962) and Streeter, Wright, and Kehr (1936), performed a regression analysis to relate k_2 to U and H, and obtained the equation

$$(k_2)_{20^\circ} = 3.3\ U\ H^{-1.33}\ldots\ldots\ldots\ldots\ldots(17)$$

where k_2 is in reciprocal days, U is in feet per second, and H is in feet.

Isaacs and Gaudy (1968) measured reaeration in a circular flume with moving walls intended to simulate streamflow conditions. However, their laboratory flow conditions have been criticized as not being similar to natural flows. Fifty-two k_2 values were obtained under 20 different simulated streamflow conditions with depths ranging from 0.50 to 1.50 feet and simulated mean velocity from 0.55 to 1.63 fps. When a regression analysis was run of their data relating k_2 to U and H, exponent values of 1.0027 and -1.4859 were found for U and H, respectively. The regression analysis was then repeated by holding the exponent values of U and H at 1 and -1.50, respectively. The equation obtained was

$$k_2 = 3.053\ U\ H^{-1.50}\ (1.0241)^{T-20^\circ} \ \ldots\ldots(18)$$

Isaacs and Gaudy (1968) applied the same approach to the data of Churchill, Elmore and Buckingham (1962) and got the equation

$$k_2 = 3.739\ U\ H^{-1.50}\ (1.0241)^{T-20^\circ} \ \ldots\ldots(19)$$

They then used the data of Krenkel and Orlob (1962) to obtain

$$k_2 = 2.44\ UH^{-1.50}\ (1.0241)^{T-20^\circ} \ \ldots\ldots\ldots(20)$$

In Equations 18, 19 and 20, k_2 is in reciprocal days, U is in feet per second, H is in feet, and T is in degrees Celsius.

Cadwallader and McDonnel (1969), after applying a multivariate analysis to the reaeration data of Churchill, Elmore and Buckingham (1962), Owens, Edwards, and Gibbs (1964), and the Water Pollution Research Laboratory channel data (Edwards and Owens, 1962, 1965; Edwards, 1962; and Owens, 1965), found that the rate of energy expenditure was the most significant variable in the reaeration process. Based on this finding, Cadwallader and McDonnell (1969) proposed the equation

$$k_2 = 25.7\, E^{0.5}\, H^{-1.0} \dots\dots\dots\dots\dots (21)$$

where k_2 is in reciprocal days, H is in feet, and E (=USg) is in ft^2/sec^3. Despite the multivariate analysis, it was found that the average error of estimate by using Equation 21 exceeded 50 percent.

Negulescu and Rojanski (1969) used the dissolved-oxygen balance technique to obtain 18 k_2 and D_x measurements from a 66-foot long, 0.66-foot wide recirculating flume. In their experiments, depths ranged from 0.164 to 0.492 feet and velocities from 0.656 to 1.903 fps. Regression analysis relating k_2 to U and H gave

$$k_2 = 4.74\, (U/H)^{0.85} \dots\dots\dots\dots\dots (22)$$

while regression analysis relating k_2 to U, H, and D_x gave

$$k_2 = 14.21\, D_x\, (U/H)^{1.63} \dots\dots\dots\dots (23)$$

In Equations 22 and 23 k_2 is in reciprocal days, U is in feet per second, H is in feet and D_x is in ft^2/sec. Negulescu and Rojanski (1969) compared the measured k_2 values obtained from a river study (name unknown) with those calculated by using Equations 22 and 23 and found that use of Equation 23 was more accurate than that of Equation 22.

Thackston and Krenkel (1969) assumed that k_2 was proportional to the water surface renewal rate which was in turn proportional to the eddy diffusion coefficient of mass at the surface, ε_y and the depth, H. Therefore they proposed the $k_2 \sim \varepsilon_y/H^2$, assuming Reynolds analogy $\varepsilon_y \sim U_* H$, where U_* is the shear velocity. A combination of both resulted in $k_2 \sim U_*/H$. They then performed a regression analysis on reaeration and dispersion experimental data accomplished in a 2-foot wide, 60-foot long tilting flume. Relating k_2 to U_*/H yielded

$$k_2 = 18.58\, U_*/H \dots\dots\dots\dots\dots\dots (24)$$

The experimental data on k_2 were also correlated to D_x/H by

$$k_2 = 1.296\, D_x/H^2 \dots\dots\dots\dots\dots (25)$$

Thackston and Krenkel (1969) found that Equation 24 described quite well the data of O'Connor and Dobbins (1956) and that of Churchill, Elmore, and Buckingham (1962), but found that the fit could be improved by adding a Froude number relation to the regression analysis. The equation obtained was

$$k_2 = 10.8\, (1.0 + F^{0.5})\, U_*/H \dots\dots\dots (26)$$

In Equations 24, 25 and 26, k_2 is in reciprocal days, U_* is in ft/sec and $F = U/\sqrt{gH}$.

Bennett and Rathbun (1972), after making a detailed statistical analysis of the data of Churchill, Elmore and Buckingham (1962) and Owens, Edwards and Gibbs (1964), concluded that the best prediction equation for reaeration of natural streams was

$$k_2 = 46.05\, \frac{U^{0.413}\, S^{0.273}}{H^{1.408}} \dots\dots\dots\dots (27)$$

Performing an analysis on more data, some of which only contained depth and velocity records, they found that the best-fit equation was

$$k_2 = 8.76\, \frac{U^{0.607}}{H^{1.689}} \dots\dots\dots\dots\dots (28)$$

In Equations 27 and 28, k_2 is in reciprocal days, U is in feet per second, H is in feet, and S is the channel slope in feet per foot. The data sources used to obtain Equation 28 were O'Connor and Dobbins (1956), Churchill, Elmore, and Buckingham (1962), Krenkel and Orlob (1962), Owens, Edwards, and Gibbs (1964), Gameson, Truesdale, and Downing (1955), Tsivoglou et al. (1965), Negulescu and Rojanski (1969), and Thackston and Krenkel (1969).

Lau (1972b) used techniques of dimensional analysis to find the functional relationship:

$$\frac{k_2 R}{U} = \varnothing \left(\frac{UR\varrho}{\mu}, \frac{U_*}{U}, \frac{D_m \varrho}{\mu}, \frac{W}{R} \right) \dots\dots (29)$$

where R is the hydraulic radius, $\varnothing$ represents a functional relationship, ϱ is the density, μ is the absolute viscosity, D_m is the coefficient of molecular diffusion in the liqud, W is the stream width, and other parameters are as defined previously. He then examined the data of

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Churchill, Elmore, and Buckingham (1962), Krenkel and Orlob (1962), and Thackston and Krenkel (1969) in an attempt to identify the relative importance of each dimensionless parameter on the right side of Equation 29. He concluded that Equation 29 could be fitted by the foregoing data by reducing it to:

$$k_2H/U = 1089.0\,(U_*/U)^3 \qquad (30a)$$

or

$$k_2 = 1089.0\,U_*{}^3/U^2H \qquad (30b)$$

where k_2 is in reciprocal days, U is in feet per second, H is in feet, and U_* is in feet per second.

Parkhurst and Pomeroy (1972) performed an experimental investigation of the reaeration rates observed in sewers. An important aspect of the study was the fact that the flow velocities tended to be greater than those observed in other studies with the exception of Churchill, Elmore, and Buckingham (1962). A predictive equation for the reaeration coefficient was proposed with the coefficients and exponents being determined by a regression analysis of the experimental data. The best-fit equation was:

$$(k_2)_{20°} = 48.0\,(1 + 0.17\,F^{2.0})\,(SU)^{0.375}$$

$$H^{-1.0} \qquad (31)$$

where k_2 is in reciprocal days, S is in feet per foot, U is in feet per second, H is in feet, and F is equal to $U/\sqrt{gH}$.

THEORETICAL CONSIDERATIONS

A reaeration coefficient measurement technique different from those described in the review of literature must be shown to be theoretically sound before its usage. The method developed in this study is an improvement over the disturbed-equilibrium technique, and is especially useful for k_2 measurements in mountain creeks. The basic equation used in this study is the longitudinal dispersion equation for DO deficit or concentration. It is similar to the dissolved-oxygen balance equation (Equation 3) except for the last two terms describing the oxidation process of the BOD and the DO diffusion process into the benthal layer and the effect of photosynthesis and respiration by plants. Relatively less polluted streams such as mountain creeks were selected so that the last two terms in Equation 3 could be ignored. The remaining portion of Equation 3 with appropriately prescribed initial and boundary conditions was solved analytically through a transformation technique and its closed-form analytical solution applied to determine the k_2 value by means of an optimization technique. Field data collected for a mountain creek under various flowing conditions in different seasons was thus used to compute the k_2 value. The k_2 value was then related to the significant hydraulic parameters of a creek.

A few methods are available for identifying the significant hydraulic parameters of a creek during the reaeration process. One method is dimensional analysis, as conducted by Lau (1972b), and the other method is to normalize the longitudinal dispersion equation for DO deficit or concentration. The latter method was used to define two significant dimensionless parameters which govern the reaeration process of the stream. A prediction model for k_2 was formulated based on these two parameters.

Reaeration Coefficient Measurements

This investigation dealt specifically with the reaeration process in mountain creeks, the flow characteristics of which make the measurement of k_2 extremely difficult by the previously described methods. Generally speaking, mountain creeks are small streams with high gradient and large-scale roughness and water in them flows at small depth and high velocity. The DO concentration in a creek, if not polluted, is maintained at or near saturation. To measure k_2 in such a highly-turbulent, DO-saturated stream is extremely difficult because the water must be first artificially deoxygenated by the addition of sodium sulfite and a cobalt catalyst. However, the construction of an elaborate injection system for the sulfite was reasoned to be impractical. Instead a slug of water was deoxygenated and measured as it traveled downstream. The DO concentration of the slug of deoxygenated water varies with time and space. Hence, neither the k_2 measurement by using the dissolved-oxygen balance technique as represented by Equation 6 or 9 nor that by using the disturbed-equilibrium technique as represented by Equation 10 is valid for the time- and space-varying slug. The following method was primarily developed for measuring k_2 in a mountain creek, but is believed to be generally applicable to any stream where a fully-mixed condition at any cross-section can be readily established.

The reaeration coefficient can be calculated by using the dissolved-oxygen balance equation (Equation 3) by assuming that K_1 and D_B are negligible because the creek is relatively unpolluted and the large artificial DO deficit created by the addition of sulfite makes the two terms, K_1L and D_B, very small in comparison with the reaeration term, $K_2(C_S - C)$. Thus Equation 3 without these two terms is

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = D_x\frac{\partial^2 C}{\partial x^2} + K_2(C_S - C) \ldots (32)$$

or by substituting $C = C_S - D$, where C_S is a constant, into Equation 32 the equation in terms of the DO deficit, D, becomes

$$\frac{\partial D}{\partial t} + U\frac{\partial D}{\partial x} = D_x\frac{\partial^2 D}{\partial x^2} - k_2 D \ldots (33)$$

Equations 32 and 33 are longitudinal dispersion equations for DO concentration and deficit, respectively. The propagation of the slug of deoxygenated water in the stream, if fully mixed over the cross-section, can thus be described by using one of these equations subject to the

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following initial and boundary conditions. If only Equation 33 is considered,

$$D(x,0) = D_0(x) \quad \dots \dots \dots \dots \dots \dots (34)$$

$$D(x_0,t) = \Phi(t) \quad \dots \dots \dots \dots \dots \dots (35)$$

in which $D_0(x)$ is the initial DO deficit distribution expressed in a known function only of the longitudinal coordinate, x, and $\Phi(t)$ is the injected DO deficit time-distribution at $x = x_0$. Even though $D_0(x)$ may be assumed constant (i.e., especially true for a nonpolluted mountain creek where $D_0(x) = 0$), there seems to be no analytical solution available for Equation 33 subject to Equations 34 and 35. However, a rather limited, particular analytical solution which is applicable to the present problem can be obtained by integrating the well-known instantaneous injection solution of Equation 33 through a transformation technique which is demonstrated as follows.

Consider the general longitudinal dispersion equation governing the flow of a conservative tracer of concentration C_i in a stream:

$$\frac{\partial C_i}{\partial t} + U \frac{\partial C_i}{\partial x} = D_x \frac{\partial^2 C_i}{\partial x^2} \quad \dots \dots \dots \dots (36)$$

When an instantaneous injection of a mass, M, with the same density as the flowing water is applied, Equation 36 is subject to the initial and boundary conditions:

$$C_i(x,0) = M/A \, \delta(x-0) \quad \dots \dots \dots \dots (37)$$

$$C_i(\pm\infty, t) = 0 \quad \dots \dots \dots \dots \dots (38)$$

where A is the cross-sectional area of the stream and $\delta(x-0)$ is the Dirac-delta Function at $x = 0$. The solution of Equation 36 through 38 is

$$C_i(x, t) = \frac{M}{A\sqrt{4\pi D_x t}} \exp\left(-\frac{(x-Ut)^2}{4 D_x t}\right) \cdot (39)$$

Next if the tracer is put into the stream by a sustained injection with a constant rate, G, which is equivalent to an increment of injected mass, dM, per unit time, dτ, or

$$G = dM/d\tau \quad \dots \dots \dots \dots \dots \dots (40)$$

where both time coordinates, t and τ, have the same reference point. Since Equation 36 is a linear, partial differential equation of second order, the sustained injection of a tracer in the stream may be conceived of as the summation of the concentration distributions for a series of an instantaneous injection at $t = \tau$, τ varying from the beginning to

the time of interest, t. Therefore, the instantaneous injection solution, Equation 39, can be utilized to formulate the following instantaneous injection solution for t at any τ in the sustained injection.

$$- dC_i(x, t-\tau) = \frac{G}{A\sqrt{4\pi D_x(t-\tau)}} \exp$$

$$\left[-\frac{(x-U(t-\tau))^2}{4 D_x(t-\tau)}\right] d\tau \quad \dots \dots \dots \dots (41)$$

Integrating both sides of Equation 41 from 0 to t yields

$$C_i(x, t) = \int_0^t \frac{G}{A\sqrt{4\pi D_x(t-\tau)}} \exp$$

$$\left[-\frac{(x-U(t-\tau))^2}{4 D_x(t-\tau)}\right] d\tau$$

$$\dots \dots \dots \dots \dots \dots (42)$$

for $C_i(x, 0)$ is zero everywhere except at the point of injection which satisfies the initial condition, Equation 37.

The actual condition at the point of injection ($x = 0$) in practice is not a constant rate of injected mass, but rather a function varying with time, $\emptyset(\tau)Q$, in which Q is the discharge of flow in the stream. The function, $\emptyset(\tau)$, describes the concentration of the tracer, and, if multiplied by the discharge, Q, will result in the same units as used by the rate G. Hence, substituting $G = \emptyset(\tau)Q$ into Equation 42, after letting Q/A be equal to the mean velocity, U, yields

$$C_i(x,t) = \int_0^t \frac{U\emptyset(\tau)}{\sqrt{4\pi D_x(t-\tau)}} \exp$$

$$\left[-\frac{(x-U(t-\tau))^2}{4 D_x(t-\tau)}\right] d\tau \quad \dots \dots \dots \dots (43)$$

Equation 43 is the solution of the tracer concentration at any time, t, and distance, x, downstream from the point of injection where an injected concentration versus time curve, $\emptyset(\tau)$, was measured.

Though Equation 33 has an additional term, $-K_2D$, more than Equation 36, it can be reduced to the same form as Equation 36 by using a transformation technique. Let

$$D = D' e^{-K_2(t-\tau)} \quad \dots \dots \dots \dots \dots (44)$$

where $D = D(x, t-\tau)$ and $D' = D'(x, t-\tau)$ which is the transformed DO deficit defined by Equation

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44. Substituting Equation 44 into Equation 33, and manipulating, yields

$$\frac{\partial D'}{\partial t} + U\frac{\partial D'}{\partial x} = D_x \frac{\partial^2 D'}{\partial x^2} \quad \dots\dots\dots\dots(45)$$

which should have the following solution similar to Equation 41 for an instantaneous injection of deoxygenated water of a known DO deficit distribution, $\varnothing(t)$, at $t = \tau$.

$$-dD'(x, t - \tau) = \frac{U\varnothing(\tau)}{\sqrt{4\pi D_x(t - \tau)}} \exp$$

$$\left[-\frac{(x - U(t - \tau))^2}{4 D_x(t - \tau)}\right] d\tau \quad \dots\dots\dots\dots(46)$$

Transforming D' into D using Equation 44 and then integrating both sides of the results from 0 to t yields

$$D(x, t) = \int_0^t \frac{U\varnothing(\tau)}{\sqrt{4\pi D_x(t - \tau)}} \exp$$

$$\left[-\frac{(x - U(t - \tau))^2}{4 D_x(t - \tau)} - K_2(t - \tau)\right] d\tau \quad \dots\dots\dots\dots(47)$$

Equation 47 can be used to fit the actual test conditions in which $\varnothing(\tau)$ is a function describing the DO deficit versus time curve measured at a point on the stream. Therefore, the DO deficit at any distance, x, below that point at any time, t, can be calculated by means of Equation 47.

The method of measuring the reaeration coefficient developed in this study makes use of both Equations 43 and 47. A slug of deoxygenated water added with a conservative tracer (or dye) is injected into the stream. Both DO and tracer concentrations are measured from a series of samples taken at two points downstream. From this data a DO deficit versus time curve and a tracer concentration versus time curve are constructed for each of the sampling sites. Equation 43 with measured $\varnothing(t)$ at the upstream sampling station is used to calculate tracer concentration values at the downstream sampling station. The mean velocity, U, and the dispersion coefficient, D_x, are then calculated by using an optimization technique to best fit the calculated and measured values of the tracer concentration at the second sampling station. The values of U and D_x so obtained are believed to be the average values at which the deoxygenated slug of water moves from the first to second sampling station. Thus, the same values of U and D_x are next substituted into Equation 47 with measured $\varnothing(\tau)$ which is now a function describing the DO deficit versus time

curve at the upstream station. The K_2 value is then calculated by using again an optimization technique to best fit the measured and calculated values of the DO deficit distribution at the second sampling station. The value of K_2 so obtained is the average rate at which a slug of deoxygenated water absorbs oxygen from the atmosphere as it travels downstream.

Reaeration Coefficient Model

To develop a model for evaluating the reaeration coefficient for a stream requires a knowledge of relevant parameters involved in the reaeration process. There are many factors which may affect the absorption of oxygen in the flowing water (Bennett and Rathbun, 1972). Some of them are significant and others are not. To find significant parameters involved in the reaeration process is a formidable task. In the past, most semi-empirical or empirical models were developed on the basis of the regression analysis that does not truly account for the importance and significance of the parameters involved in the reaeration process. Since the longitudinal dispersion equation for DO deficit, Equation 33, adequately describes the reaeration process in a stream, it can be used to identify the significant dimensionless parameters involved in the process.

Equation 33 is normalized by introducing the following dimensionless variables.

$$D_* = \frac{D}{D_0} \quad \dots\dots\dots\dots(48)$$

$$x_* = \frac{x}{L_0} \quad \dots\dots\dots\dots(49)$$

$$T_* = \frac{tU}{L_0} \quad \dots\dots\dots\dots(50)$$

where the reference magnitudes, D_0, may be either the initial DO deficit of the stream or C_s (the DO concentration at saturation), and L_0 is some hydraulic parameter describing a length. Substituting Equations 48, 49 and 50 into Equation 33, after few manipulations, yields the following:

$$\frac{\partial D_*}{\partial T_*} + \frac{\partial D_*}{\partial X_*} = \left(\frac{D_x}{L_0 U}\right)\frac{\partial^2 D_*}{\partial X_*^2} - \left(\frac{K_2 L_0}{U}\right) D_* \quad \dots\dots(51)$$

Equation 51 is the normalized longitudinal dispersion equation that is independent of the system of units selected since both the $D_x/L_0 U$ and the $K_2 L_0/U$ parameters are dimensionless. If two flow systems under consideration are dynamically similar, both parameters must be identical for both

systems according to the Reynolds analogy. Consequently both parameters are identified as significant ones in the reaeration process with a functional relationship:

$$\frac{K_2 L_0}{U} = f\left(\frac{D_x}{L_0 U}\right) \quad \ldots\ldots\ldots\ldots(52)$$

It is noted that the functional relationship, Equation 29, of several parameters derived by Lau (1972b) through dimensional analysis is similar to Equation 52 except that the dispersion coefficient, D_x, is generally considered as a function of D_m and possibly of other parameters listed on the right hand side of Equation 29 (Chen, 1971). A comparison of Equation 52 with other semi-empirical or empirical models developed by previous investigators also reveals that Equation 52 is the most general of all types in a sense that the other models can be regarded or interpreted as a particular type of Equation 52.

EXPERIMENTAL PROCEDURES

The basic experimental procedure consisted of injecting a slug of deoxygenated water with a conservative tracer into the test stream and measuring the slug of dissolved oxygen and tracer concentrations as it passed two or more points downstream. Initially it had been planned to use oxygen meters to measure the DO of the stream as the slug passed by, but it was later found that the time rate for the oxygen meter to respond to the DO variation was too slow in comparison to the speed at which the deoxygenated slug passed by the sampling station. The alternative used was a time series of individual samples taken from each sampling station of the stream and then analyzed for dissolved oxygen and tracer content.

The means of taking samples also proved to be somewhat of a problem. It was felt that the samples should be taken without having any overly bulky equipment or operator standing in the stream so that the natural flow conditions of the stream would not be altered. The fact that the samples needed to be taken at a pre-set time also required that the samplers would have to be set and loaded in the stream and triggered from outside the stream. Since both time distributions and lateral distributions of the slug were to be measured, the samplers would have to be capable of being fired either individually or simultaneously and of being set up at desired points in the stream. Finally the sample size would have to be large enough to be analyzed for both dissolved oxygen and tracer content. The Winkler method with azide modification as described in "Standard Methods" (1971) was chosen as the means to measure the DO content of the samples because it had been standardized and proven reliable and accurate. Rhodamine B, a fluorescent dye, was selected as the tracer because of its conservative properties, ease of handling, availability, and ease of measuring with a Turner fluorometer which was available at the Utah Water Research Laboratory (UWRL). A preliminary test indicated that a sample of about 400 ml would be sufficient. This is equivalent to 300 ml plus a small amount for overflow for the Winkler test and 10 to 20 ml for the fluorometer reading.

The sampling device finally developed was based on a modification of the Van Dorn sampler.

The samplers and supporting frame are shown in Figure 1. The sample container itself consisted of a section of aluminum pipe, approximately 12 ½ inches in length and 1 ¾ inches in diameter. The container is closed by rubber balls seated against both ends of the pipe. The rubber balls on each end are attached to each other by a piece of latex rubber running through the center of the section of pipe. During assembly the tubing is stretched so that when the sampler is in the closed position, the balls are held against both ends of the pipe tightly enough for it to be watertight. As seen in Figure 1, the section of pipe with rubber balls is attached to a plate which holds the firing mechanism. When loaded, wire loops attached to each ball by a short section of chain (opposite the side to which the latex tubing is attached) are looped over the prongs of a two-pronged pin so that the balls are held out of and behind the pipe ends. When the two-pronged pin is pulled up, the wire loops slide off the prongs and the balls clamp shut over the ends of the pipes trapping inside any water flowing through at that time. The sample container can then be removed from the firing mechanism plate without disturbing the rest of the frame and the sample withdrawn. The frame consists of vertical pipes on which the firing mechanism plates can slide up and down which in turn are connected to a horizontal pipe on which they can be slid back and forth. If the samplers are to be fired individually for a time distribution, the vertical pipes are slid close together, as shown in Figure 1, so that all samples are taken from the central portion of the stream cross-section. If the samplers are to be fired simultaneously for a lateral distribution, the vertical pipes are slid further apart to cover the width of the stream. The pins are pulled, triggering the samplers, by chains and wires strung up to and across the horizontal pipe to an end where they can be pulled by the operator standing on the bank. The horizontal pipe is long enough to run across the stream from one bank to the opposite.

After careful consideration, it was decided to take samples at three stations downstream from the point where the slug is dumped because this gives more flexibility in the analysis. The reaeration coefficient can be evaluated from the first section to the second, from the second to the third, and from the first to the third. Also, if something goes wrong
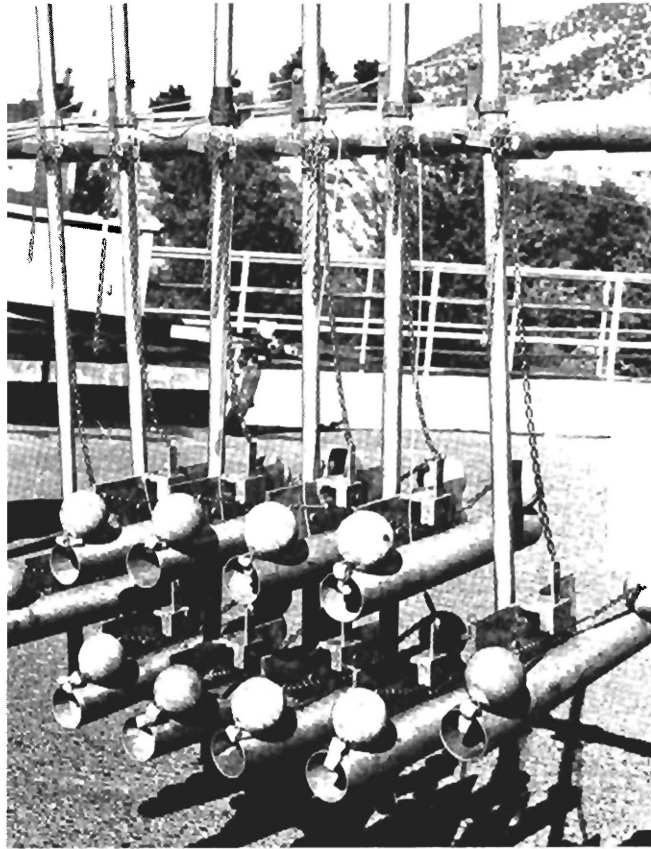
Figure 1. Sampling mechanism in loaded position.

with the data collected at one station, an analysis can still be performed on the remaining two. However, there were only enough samplers built to adequately cover two stations. It was decided to take the third set of samples at the station furthest downstream by hand with large-mouthed, one liter jugs. It has been shown by Gameson, Truesdale and Downing (1955) that a correction of minus 5 percent of the deficit should be included to allow for additional reaeration occurring when sampling is achieved in this manner (water being allowed to flow into a totally immersed bottle). Zogorski and Faust (1973), using this sampling technique, incorporated the correction factor into their calculations and found that the sampling error was within the accuracy of their experiment. It will thus be assumed that the error involved in this type of sampling is also within the accuracy in this investigation. It was decided to sample by hand at the station furthest downstream because a person at this station could sample facing upstream without affecting the stream flow along the test reach.

The foregoing experimental procedure was performed both on a natural stream and on a laboratory flume. In the following these two sample sites and the different sampling procedures used on each are described.

Field Measurements

The field test site was chosen on the basis of its general flow characteristics and its accessibility. The creek tested is called Summit Creek, which has its origin in the mountains east of Smithfield, Utah, flows west down Smithfield Canyon through the town of Smithfield and on into the Bear River. The test section is located on the east side of Smithfield as the creek comes out of the foothills. It consists of a fairly straight stretch of creek approximately 225 feet long with steep banks and relatively uniform flow conditions, as shown in Figure 2. Prior to any testing, the site was surveyed and cross-sections of the stream channel taken every 25 feet along the stretch of interest. This was done in order to make possible the calculation of

Figure 2. A downstream view of the Summit Creek test reach.

important hydraulic parameters for each test. The reference point for the survey was a bridge at the downstream end of the test section as shown in Figure 2. Therefore, the 25-foot lengths were measured and labeled from this point. The first cross-section, being at the bridge, was labeled 0+00, the next, 25 feet upstream, 0+25 and so on upstream to 2+25. The sampling stations were either at these 25-foot stations or measured from them. The station furthest downstream was always at 0+00. The other two upstream stations were either located at 1+50 and 0+75 or at 1+10 (sometimes 1+15) and 0+50. The slug of deoxygenated water was dumped into the stream at station 2+25 or further upstream.

The slug of deoxygenated water was prepared by essentially the same procedure for each test. A large fiberglass drum was partially filled with approximately 100 liters of stream water. A small amount of cobalt chloride was added to act as a catalyst and followed by mixing the sodium sulfite. Both were added while the water was being stirred. The Rhodamine B dye was then added and stirred until the color was uniform throughout. The sodium sulfite and the rhodamine dye were added in premeasured quantities. The quantities chosen were arrived at by trial and error until reasonable dissolved-oxygen and dye concentration curves

were obtained at the measuring stations. Depending upon the stage of the creek, 500 to 900 grams of sodium sulfite and 100 to 200 milligrams of Rhodamine B dye were added to the slug. The amount of sodium sulfite used was many times more than enough to remove the DO in the 100 liter slug, but when dumped into the stream, the quantities cited were required to react in the stream to achieve good DO deficits downstream. As long as no samples taken at the first sampling station were completely void of oxygen, it was assumed that the sulfite was entirely converted to sulfate by the time the slug reached there. This is essential in order to measure the reaeration coefficient between the first and second sampling stations by the method described above. The slug was injected into the stream by simply tipping over the drum and dumping it in.

During the first series of field tests, the samplers at the first two stations were bunched in the center of the stream and fired individually at pre-set times which were again determined, mostly, by trial and error. The third station, located at 0+00, was sampled by hand as usual. In the second series of field tests attempt was made to measure a lateral DO distribution which, if valuable at all, required an accompanying time distribution measurement of the same slug.

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Because there were not enough samplers built to achieve both objectives, it was decided to hand-sample the DO time distribution at the two stations where lateral distributions were also taken. The sampling mechanism was set up so that at a predetermined time when a hand sample was not taken, the mechanism was fired. There was some fear that a person sampling in the middle of the stream might disturb the flow, but in view of the highly turbulent nature of the creek, such fear was superfluous.

Once all the samples were taken, a 300 ml BOD bottle, being especially designed for DO tests, was filled by siphoning from the sampling tube or jugs into the bottle. Each sample was siphoned into a bottle in order to eliminate as much as possible any additional aeration of the sample. A remaining portion of each sample was poured into a small vial for the fluorescence test. Then, while still in the field, the samples were fixed by the addition of chemicals as prescribed in "Standard Methods" (1971). The remaining part of the DO test, consisting of the titration of a 203 ml portion of the fixed sample, was performed at the chemical laboratory of the UWRL. The fluorometer readings of the small samples were also taken in the laboratory.

Additional information required for each field test includes stream temperatures, barometeric pressure, and stream discharge. The stream temperature was taken at each field test with a thermometer. The barometric pressure was read on the day of the test from a barometer located at the UWRL. Usually at the end of each test the stream was gaged at station 0+00, by the bridge, with a current meter enabling the discharge to be calculated. The stream was divided into 2-foot sections across the top width plus one odd size section, if required. The stream depth and velocity were measured at the center of each of these sections and assumed to be the average value therein. The velocity was obtained by using a stream gage to take a reading at a distance of approximately six tenths of the total depth from the bottom. The discharge was then calculated for each of these sections and their sum was the total discharge of the stream. Along with the stream gaging, the distance from the bottom of the bridge to the water surface was measured so that stage versus discharge graphs could be prepared. When stream gaging was not accomplished because of accidents, the distance from the bridge bottom to the water surface was still measured so that the discharge could be estimated from the state versus discharge graph.

Laboratory Flume Measurements

A big fixed flume used for the laboratory tests is located in the UWRL. The horizontal concrete flume, recessed in the floor, is 500 feet long, 8 feet wide and 6 feet deep. The sampling stations were labeled with numbers corresponding to the scheme used in the field. The third sampling station, located furthest downstream was labeled 0+00. The second station 90 feet upstream from 0+00, was labeled 0+90 and the first sampling station, 105 feet closer to the point where the slug was discharged, was labeled 1+95. The slug of deoxygenated water was injected into the throat of a cutthroat flume located at the upstream end of the laboratory flume at 3+45, 150 feet above the first sampling station.

The slug was prepared by using the same procedure as in the field test. However, because of the greatly reduced turbulence in the flume as compared to the creek, only 400 to 600 grams of the sodium sulfite and 100 to 150 milligrams of Rhodamine B dye were added to the slug, of course, depending upon the discharge set.

The basic procedure for sample collection and analysis was the same as described for the field tests. However, lateral distributions were taken with the sampling mechanism at the first two stations during each test while time distributions by hand sampling were carried out at all three stations. Samples were collected at the second station from a platform set just above the water surface, but sampling at the first station was performed with the sampler standing in the water. It is felt that this did not affect flow conditions to any great extent. As usual, the person at the third sampling station, 0+00, stands in the water while sampling, but facing upstream to the test reach.

Stream temperature and barometric pressure were measured and recorded in the same way as for the field tests. The discharge for each test was obtained by simply measuring the water depth in the stilling wells of the cutthroat flume. The cutthroat flume was already calibrated by Skogerboe, et al. (1967). Depth was also measured at each sampling station so that the mean velocity could be calculated and checked with that obtained from a computer program which will be described in the next section.

The water used in the laboratory flume was withdrawn from a reservoir located just above the UWRL. The reservoir is small and shallow with little or no thermal stratification and the water entering the flume was high in dissolved oxygen. The discharge was controlled by adjusting a 48-inch valve at the head of the flume.

EXPERIMENTAL DATA ANALYSES

The data for each test was analyzed to determine the dispersion coefficient, mean velocity, reaeration coefficient and some important hydraulic parameters such as geometric elements of the stream (i.e., top width, cross-section area, hydraulic depth, and hydraulic radius). These values were used in the formulation of a prediction model for the reaeration coefficient and then substituted into previous investigators' models for comparison. The dispersion coefficient, mean velocity, and reaeration coefficient were calculated by using an optimization technique which fits calculated values to the corresponding measured data. Top width, cross-sectional area, hydraulic depth, hydraulic radius and other related geometric elements were calculated simply by using the measured depth and channel geometry. Because of the difficulty in measuring the flow depth accurately at each sampling station due to the tumbling water surface, the cross-sectional area was first computed by the measured discharge and the calculated mean velocity. Formulation of a reaeration coefficient model by using the significant parameters, as identified in Equation 52, was accomplished using a regression analysis.

All the analyses and computations were programmed and executed first on an EAI 590 hybrid computer located at the UWRL and later on the University of Utah UNIVAC 1108 which has a remote terminal at USU. The specific calculations and computer programs in the data analysis are described in detail in the following sections.

Evaluation of the Mean Velocity and Dispersion and Reaeration Coefficients

Each set of calculations, including the determination of the U, D_x, and K_2 values, were obtained from data at two sampling stations. Both a dye concentration versus time curve and a DO deficit versus time curve at an upstream station and a downstream station were necessary. A lateral distribution of the deoxygenated slug was also taken for each test and its cross-sectional average computed so that the time-varying values obtained from samples taken at one point in the stream could be corrected by an averaging factor, thereby, giving the samples the corresponding average cross-sectional values. Despite this correction,

however, when the dye concentration versus time curves were inspected, it was found that the areas under the curves (i.e., total dye mass) at the different stations where the same slug was measured were still not equal for a case in which theoretically they should have been. It was thus necessary to multiply the dye concentrations at each station by a correction factor to make the areas under the curves equal for each run of the dye tests. The measured DO deficit values were then adjusted by multiplying by the same correction factor. The second correction is justified because the reaeration coefficient was calculated from the DO deficit versus time curves at two stations and was solely dependent upon the ratio of the areas under the two curves. The areas under the DO deficit curves were not the same because the reaeration takes place between the two stations, but it is assumed that any loss of dye or change in the areas under the dye concentration versus time curves will manifest itself with the same ratio of loss, or change, in the areas under the DO deficit versus time curves. This correction procedure will enable us to account for the true amount of reaeration. In practice, the dye and DO data at a station having the smallest area were arbitrarily given a correction factor of unity and those at the other two stations given the corresponding correction factors to equalize their areas.

The arbitrary choice of a station having the smallest area under the dye concentration versus time curve to have a correction factor of unity needs to be justified. Theoretically there is no reason why one of the other two stations cannot be selected. Analysis of two complete sets of data were made by varying the correction factors from ten times to one tenth their values obtained as described above. Despite the correction factors ranging in value from 10 to 0.1 times those for the three stations on the same test run, the ratios of their areas under dye concentrations versus time curves were kept the same as originally specified. For each computer analysis, it was found that the values of D_x, U, and K_2 were exactly the same within the specified accuracy for any set of correction factors tried. This can be explained by looking at what happens physically to the curves when the data values making the areas are all multiplied by the correction factors. The dye curves are changed in

magnitude, but the relative spreading and centroid of the curves which determine D_x and U are not changed. Furthermore, because the reaeration process was shown to be a first-order reaction, as described by Equation 1, and the DO deficit values are multiplied by equal-ratio correction factors, the areas under the curves change as if dD/dt and D change at the same rate leaving K_2 as the same value. As a result of this finding, the cross-section-averaging factors derived from the lateral distribution became superfluous as far as the determination of D_x, U, and K_2 by use of Equations 43 and 47 was concerned. Unless an extremely sophisticated instrumentation system can be built to measure the lateral dye and DO concentration distributions continuously, it appears that the present method does not require the correction factors for the lateral distributions, which are already merged in the process of area adjustments under the dye concentration curves. For uniformity in analysis, however, all calculations, regardless of whether or not a lateral distribution was measured, were made for each test run with correction factors included for adjusting areas under the concentration curves to be equal to the smallest of them.

A computer program (see Appendix A) was written to calculate D_x and U from the dye concentration versus time data at two stations. Equation 43 with measured $\emptyset(t)$ at the upstream sampling station, say station A, was used to calculate the dye concentration at the downstream station, say station B. In Equation 43, $\emptyset(t)$ is a function describing the dye concentration versus time curve. By adjusting the values of D_x and U in Equation 43 according to the criteria used in the method of least squares, the integral in Equation 43 was numerically solved by using Simpson's rule and the dye concentrations at station B at specified times were computed to best fit the measured dye concentrations at station B. The time increment taken in the numerical computation was 5 seconds, but can be changed if desired. Simpson's rule is applied to Equation 43 for τ changing from zero to t. For each τ the value of $\emptyset(\tau)$ in Equation 43 is a number fed into the program on data cards. In order to have the dye concentration values every 5 seconds for station A and enough ·data to adequately describe the dye concentration curve at station B, the measured values at each station are plotted, curves drawn through them, and values picked off the curves. This entailed some estimating since the number of samples taken at each station was limited. However, if the measured values did not describe a curve relatively well, the data were not used. The present computer program has the capability to calculate desired points forming a dye concentration versus time curve constructed from the measured data points. The measured and computed dye concentrations at station B were then compared by taking the sum of their difference squared at each point. Next, either D_x or U, one at a time, was varied while the other was held constant until a minimum sum or least squares value was obtained. The roles of D_x and U were then reversed and the least squares value was again found. This procedure was repeated until the final step change for D_x of U did not produce a smaller least squares value and a best fit between the measured and calculated values at station B was attained. It was found from experience that varying the mean velocity, U, first increased the speed of this optimization technique. Figure 3 shows typical dye concentration versus time curves which were calculated from the measured curves using the computer program as described, with the final best fit values of D_x and U. Figure 3 is an example which shows curves drawn from only one complete set of data taken on December 6, 1974. Similar graphs could be drawn from each set of data calculated. There are other complete data sets, summaries of which are listed in Appendix B.

The same computer program (see Appendix A) was used to calculate the reaeration coefficient, K_2, by using Equation 47 in much the same way as D_x and U were calculated. Actually the program was written to use Equation 47 to compute D_x and U by putting K_2 equal to zero. For calculating K_2, $\emptyset(\tau)$ represent the DO deficit versus time curve at station A and the D_x and U calculated from the dye measurements were substituted into Equation 47. The DO deficits at station B were calculated using the same approach as described before and again were calculated for each time, t, at which the measured DO deficit curve was read in. Since D_x and U, which are the terms controlling the shape and overall location of the curve at station B, are held constant, it was found that to equalize the areas under the measured and calculated curves was a better means of best fitting the calculated DO deficit curve to the measured deficit curve. Therefore, the K_2 value was varied until the final step size change for K_2 did not make the areas under both curves closer in value. Figure 4 is similar to Figure 3 except that it shows the measured and calculated DO deficit curves rather than dye concentration. The calculated curves shown were calculated by using the best fit value for K_2, evaluated as previously described, and D_x and U values found in the dye study for the same test run.

The values for D_x, U and K_2 so obtained should be the values which best fit Equations 43 and 47 to the measured dye and DO deficit values. Computer experiments were performed by varying the starting values of all three terms in the optimization computation. The present method seemed to converge to the same values. The final

18
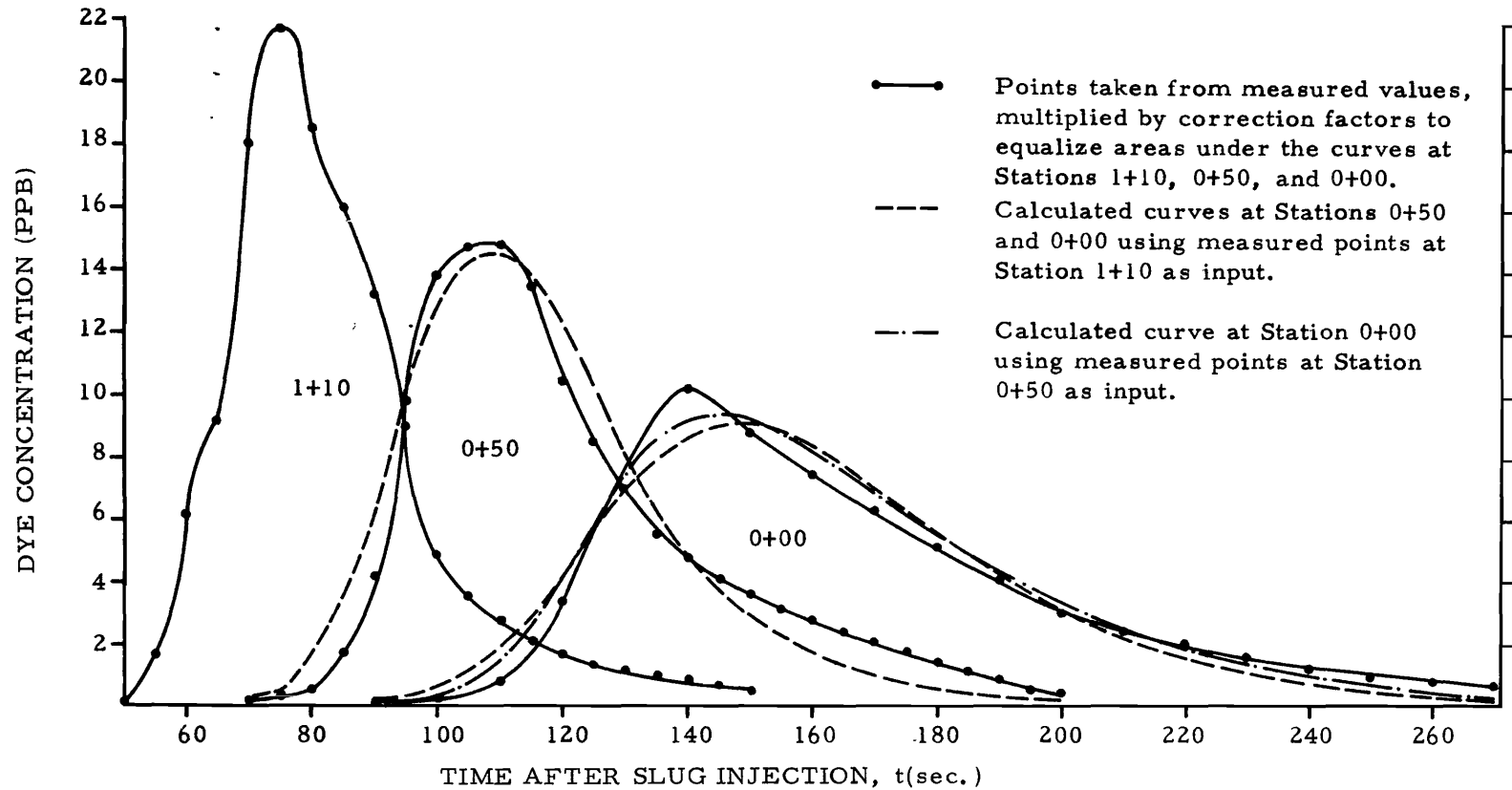
Figure 3. Typical measured and calculated dye distributions at sampling stations 1+10, 0+50, and 0+00 using data from test run on December 6, 1974.

DISSOLVED OXYGEN DEFICIT (PPM)

1+10

0+50

0+00

● ● ● Points taken from measured values multiplied by the same correction factors as used in the dye study

— — — Calculated curves at Sations 0+50 and 0+00 using measured points at Station 1+10 as input

— · — Calculated curve at Station 0+00 using measured points at Station 0+50 as input

TIME AFTER SLUG INJECTION, t (sec.)
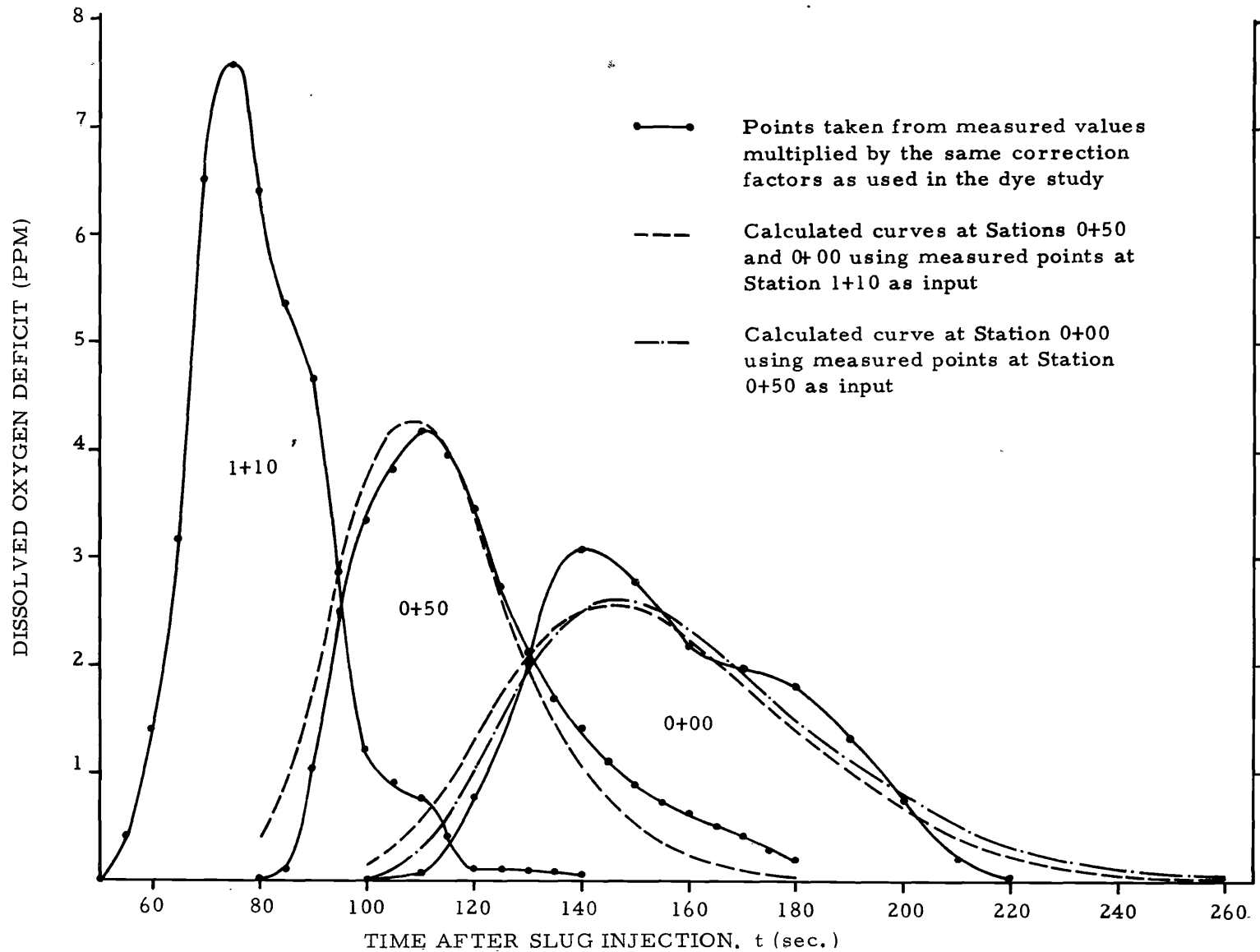
Figure 4. Typical measured and calculated DO deficit distributions at sampling stations 1+10, 0+50, and 0+00 using data from test run on December 6, 1974.

values did change a little, depending upon the starting value and step by which they were changed until the best fitting was achieved. During the optimization, of course, the same values could not always coincide with the new step sizes. It was judged, however, that these differences were within the specified accuracy of the computation. Values for D_x, U, and K_2, so evaluated, along with associated flow conditions are listed in Appendix B.

Evaluation of Hydraulic Parameters

Hydraulic parameters were calculated from the measured discharge, the computed mean velocity, and the stream cross-sectional geometry measured every 25 feet along the stretch of concern. At each 25 foot station within the test reach, the cross-sectional area was found by dividing the discharge by the mean velocity. With the cross-sectional area and configuration known, the maximum flow depth, wetted perimeter, top width, hydraulic depth, and hydraulic radius were easily calculated at each 25 foot station. The mean values of the hydraulic parameters for each test were then the average values of those calculated at the 25 foot stations located within the test reach. A computer program (see Appendix A) was written to perform these calculations. Other values pertaining to each test such as bed slope, stream temperature, and kinematic viscosity were read into the program so that values for shear velocity, Froude number, Reynolds number and energy dissipation could be calculated. Included in the same computer program are those calculated and input data for each test to evaluate the reaeration coefficient, k_2, as predicted by previous investigators' models mentioned in the review of literature. Output from this computer program is listed in Appendix B.

FORMULATION OF THE REAERATION
COEFFICIENT MODEL

The general form of the semi-empirical model for the reaeration coefficient is a functional relationship between the two dimensionless parameters, K_2L_0/U and $D_x/L_0 U$, as expressed by Equation 52. An inspection of the existing semi-empirical models reveals that these two parameters can be related to each other in such a simple form as

$$\frac{K_2L_0}{U} = a\left(\frac{D_x}{L_0U}\right)^\beta \quad \dots\dots\dots\dots(53)$$

in which a is the coefficient and β is the exponent. If the values of a and β can be determined by using field or laboratory data for a stream or flume under study through the regression analysis, Equation 53 is the reaeration coefficient model that comprises a convenient form for use in engineering practice.

The reference length, L_0, in Equation 53 must be the one characterizing the overall hydraulic flow system under investigation. The quantity such as hydraulic radius or hydraulic depth may be used as the reference length, but most of the previous investigators based their analyses on the flow depth that is unfortunately never uniform across the width in a natural stream. Therefore, for modeling the reaeration coefficient, the hydraulic depth, H, is used herein. The definition of the hydraulic depth in the natural stream modeling is in conformity with that of the flow depth in a rectangular channel, thereby eliminating the problem of inconsistency when both are compared. Furthermore, for convenience in comparison, the reaeration coefficient K_2, in Equation 53 is converted to k_2 for this will only make a difference in the a coefficient of Equation 53 since K_2 and k_2 are directly related through Equation 2. With these specific magnitudes assigned, Equation 53 becomes:

$$\frac{k_2H}{U} = a\left(\frac{D_x}{HU}\right)^\beta \quad \dots\dots\dots\dots\dots(54)$$

To evaluate the a and β values in Equation 54, it will be more convenient to express Equation 54 in logarithmic form as

$$\log\left(\frac{k_2H}{U}\right) = \beta \log\left(\frac{D_x}{HU}\right)$$

$$+ \log a \dots\dots\dots\dots\dots\dots\dots(55)$$

where k_2 is in sec^{-1}, H is in feet, U is in ft/sec, and D_x is in feet^2/sec. Equation 55 is now in linear form with log (k_2H/U) and log(D_x/HU) being the Y and X terms, respectively, in general linear equation, Y = aX + b. The slope of the linear equation is represented by β and the Y-axis intercept by log a.

The least-squares regression analysis of the log (k_2H/U) versus log (D_x/HU) data from each test result was conducted to evaluate the slope, β, and the intercept, log a. A computer program (Program III of Appendix A) was written to perform the computations of the a and β values. Also included in the program is the calculation of the standard error of estimate of the present model (Equation 54) and of all the previous investigators' prediction models by using the same standard error of estimate equations as in Bennett and Rathbun's report (1972). The equation used for the standard error of estimate, E_S in units of k_2, is

$$E_S = \sqrt{\frac{\sum\limits_{i=1}^{n}[(k_2)_e - (k_2)_m]_i^2}{n}} \quad \dots\dots(56)$$

where $(k_2)_e$ is the estimated k_2 value obtained from each of the prediction models using the data corresponding to the measured k_2 value, $(k_2)_m$, and n are the number of k_2 values used. Another standard error of estimate equation used is

$$E_{SL} = \sqrt{\frac{\sum\limits_{i=1}^{n}[\log(k_2)_e - \log(k_2)_m]_i^2}{n}} \quad \dots(57)$$

which can be expressed as a percent error, Ep, by the definition

$$E_P = 100\,[1 - 10^{-E_{SL}}] \quad \ldots \ldots \ldots \ldots \ldots (58)$$

E_P is the percent standard error of estimate in terms of $(k_2)_e$ and will always be between 0 and 100 percent.

All sources of existing data were reviewed to find those which could be used in the formulation of the reaeration coefficient model (Equation 54). The parameter which limited the data sets available for use was the dispersion coefficient, D_x. The only sets of data found in available publications containing values for k_2, U, H, and D_x were those of Negulescu and Rojanski (1969) and Thackston and Krenkel (1969). The data from these two investigations are listed in Appendix B. The data of Negulescu and Rojanski (1969), obtained from a 20-cm wide laboratory flume, contained only 8 points, but each was a composite or average of a number of tests. The computer program was written so as to take into consideration the weighted values of points, whenever necessary. The data of Thackston and Krenkel (1969), obtained from a 2-foot wide laboratory flume, consists of 52 separate points or k_2 values. As listed in Appendix B, as well as in the publication by Thackston and Krenkel (1969), the reaeration coefficient values all have been adjusted to 20 degrees centigrade.

The two sets of data in this study, one for a natural stream and the other for a laboratory flume, are also listed in Appendix B. The data points from all four data sets are depicted in Figure 5 by plotting k_2H/U versus D_x/HU for each measured k_2 value on log-log paper.

A least-squares regression analysis was performed on the natural stream data points as shown in Figure 5 by means of Equation 55. An a value of 10.53×10^{-5} and a β value of 0.455 were obtained. Substituting the a and β values into Equation 54 yields the reaeration coefficient prediction model specifically for Summit Creek as follows:

$$(k_2)_{20^\circ} = 10.53 \times 10^{-5}\,D_x^{\,0.455}\,U^{\,0.545}\,H^{\,-1.455} \quad .(59)$$

where k_2 is in sec^{-1}. With k_2 in $days^{-1}$ Equation 59 becomes

$$(k_2)_{20^\circ} = 9.098\,D_x^{\,0.455}\,U^{\,0.545}\,H^{\,-1.455} \ldots (60)$$

where D_x is in ft^2/sec, U is in ft/sec and H is in feet for both Equations 59 and 60. Equation 59 is plotted in Figure 5 amongst the data it represents.

The same type least-squares regression analysis was run on the data points obtained from measurements taken in the laboratory flume for this study. The a and β values obtained were 8.782×10^{-5} and 0.964, respectively. Substituting these values into Equation 54 yields the prediction model for the laboratory flume data:

$$(k_2)_{20^\circ} = 8.782 \times 10^{-5}\,D_x^{\,0.964}\,U^{\,0.036}\,H^{\,-1.964}$$
$$\ldots \ldots \ldots \ldots (61)$$

where k_2 is in sec^{-1}. With k_2 in $days^{-1}$ the equation becomes

$$(k_2)_{20^\circ} = 7.588\,D_x^{\,0.964}\,U^{\,0.036}\,H^{\,-1.964} \ldots (62)$$

where D_x, U, and H are in the same units as for Equations 59 and 60. Equation 61 is plotted in Figure 5 amongst the data it represents.

The least-squares regression analysis of the data of Negulescu and Rojanski (1969) yielded a and β values of 6.713×10^{-5} and 0.285, respectively. Using Equation 54 the prediction model formulated for their data can be written as

$$(k_2)_{20^\circ} = 6.713 \times 10^{-5}\,D_x^{\,0.285}\,U^{\,0.715}\,H^{\,-1.285}$$
$$\ldots \ldots \ldots \ldots (63)$$

where k_2 is in sec^{-1}. With k_2 in $days^{-1}$ Equation 63 becomes

$$(k_2)_{20^\circ} = 5.80\,D_x^{\,0.285}\,U^{\,0.715}\,H^{\,-1.285} \ldots (64)$$

where D_x, U and H are in the same units as for Equations 59 and 60. Equation 63 is plotted in Figure 5 amongst the data it represents.

The least-squares regression analysis of the data of Thackston and Krenkel (1969) yielded a and β values of 2.313×10^{-5} and 0.407, respectively. Using Equation 54 the prediction model formulated for their set of data can be written as

$$(k_2)_{20^\circ} = 2.313 \times 10^{-5}\,D_x^{\,0.407}\,U^{\,0.593}\,H^{\,-1.407}$$
$$\ldots \ldots \ldots \ldots (65)$$

where k_2 is in sec^{-1}. With k_2 in $days^{-1}$ Equation 65 becomes

$$(k_2)_{20^\circ} = 1.999\,D_x^{\,0.407}\,U^{\,0.593}\,H^{\,-1.407} \ldots (66)$$

where D_x, U and H are in the same units as for Equations 59 and 60. Equation 65 is plotted in Figure 5 amongst the data it represents. Also plotted amongst this set of data is Equation 25 which was formulated by Thackston and Krenkel (1969). Equation 25 can easily be put into the form of Equation 54 with a and β being 1.296 and 1.0,
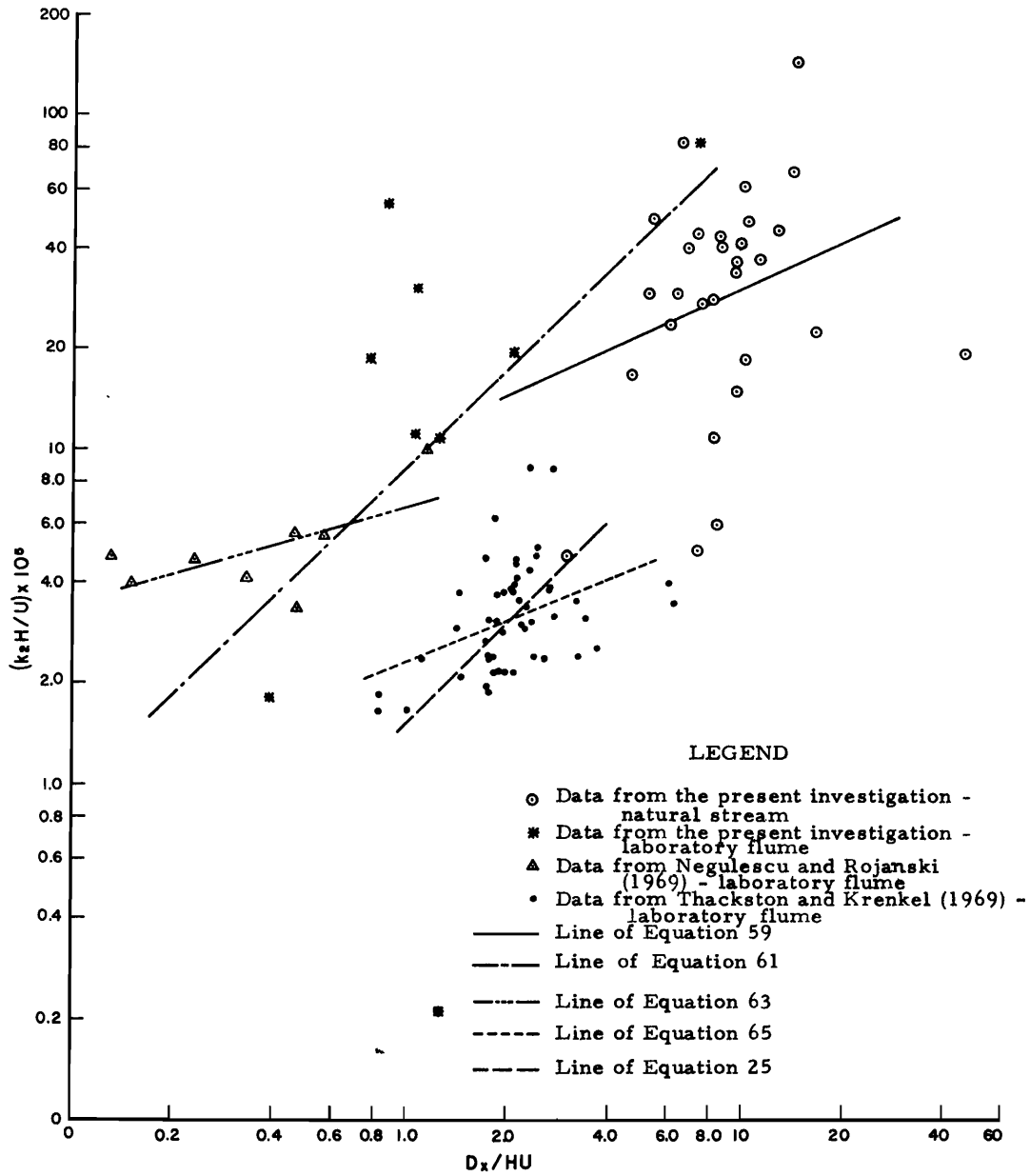
Figure 5. The $k_2 H/U$ versus D_x/HU relationships.

respectively, where k_2 is in days^{-1}, and hence can be plotted in Figure 5 for comparison.

Significance levels were calculated for the slopes of the lines in Figure 5 representing Equations 59, 61, 63, and 65. It was found that the slopes or β values of Equations 59, 63, and 65 had confidence levels greater than 99.9 percent of their being different from zero. A slope of zero is a horizontal line at the mean value. The β value of Equation 61 was found to be different from zero with a confidence level of approximately 83 percent.

Table 1 lists the standard error of estimate, as calculated from Equations 56, 57, and 58, involved in using the various reaeration coefficient prediction equations on each data set, including the equation newly formulated from the particular data set. In data set three of Table 1, there are blanks opposite many of the prediction equations (excluding Equations 60, 62 and 66). These are the equations which require either a bed slope or energy grade line in their calculation and Negulescu and Rojanski (1969) gave neither slope values among their data. Table 1 will be discussed further in the following section.

Table 1. Performance of k^2 prediction equations, evaluated by the standard error of estimate.

Equation Number	Data Set One E_S (days^{-1})	One E_P (%)	Two E_S (days^{-1})	Two E_P (%)	Three E_S (days^{-1})	Three E_P (%)	Four E_S (days^{-1})	Four E_P (%)
11	89.1	85.9	57.4	85.1	23.2	57.4	296.7	90.2
12	1154.5	77.8	36.4	81.2	9.0	75.6	92.9	73.6
13	75.3	66.3	59.0	87.3	-	-	34.4	54.2
14	97.8	98.3	63.5	98.9	-	-	25.9	82.6
15	81.9	76.1	52.4	82.7	96.1	80.5	851.7	96.3
16	84.3	79.3	53.7	83.0	99.2	79.8	975.1	96.6
17	90.6	87.8	58.4	86.3	6.7	33.1	54.3	65.0
18	92.9	91.2	59.9	88.6	10.0	36.3	98.8	76.3
19	91.6	89.3	59.1	87.3	13.8	39.6	127.0	80.5
20	94.1	92.8	60.7	90.1	7.2	38.9	73.7	70.7
21	78.6	70.5	60.9	90.6	-	-	117.8	78.2
22	92.0	89.3	59.1	87.4	2.9	25.7	14.7	37.5
23	5470.9	92.4	94.8	84.8	3.2	24.7	208.7	81.5
24	89.6	84.9	62.3	94.5	-	-	6.9	21.3
25	86.8	73.5	59.2	92.6	11.0	88.0	14.6	33.2
26	89.9	85.5	62.4	94.9	-	-	7.2	21.7
27	82.0	75.3	58.5	86.6	-	-	356.5	91.9
28	86.4	81.6	54.9	83.4	67.7	74.5	583.8	94.7
30	83.9	68.4	63.4	98.2	-	-	68.5	53.8
31	84.3	77.9	60.6	90.0	-	-	57.1	65.1
60	72.2	53.4						
62			34.4	79.3				
64					3.3	18.7		
66							10.6	28.7

Key: Data set one: The present investigation - natural stream
Data set two: The present investigation - laboratory flume
Data set three: Negulescu and Rojanski (1969) - laboratory flume
Data set four: Thackston and Krenkel (1969) - laboratory flume
E_S: standard error of estimate (days^{-1}) as calculated from Equation 56

E_P: standard error of estimate (percent) as calculated from Equations 57 and 58

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DISCUSSIONS

The method of finding the dispersion coefficient, mean velocity, and reaeration coefficient proposed and used in this study should provide good average values of D_x, U, and k_2 for flow in a creek or flume within the test reach. The curves formed by the calculated dye concentrations fit quite well to those curves formed by the corresponding measured dye concentrations as shown in the sample data plotted in Figure 3. This indicates Equation 47 and the method used to evaluate them are working properly. Originally a routing equation proposed by Fischer (1966) similar to Equation 47 (no reaeration term was involved) was used in computing the dye concentrations and best-fitting the dispersion coefficient and velocity values. Fischer's equation which only approximates the actual flow and dispersing situation was abandoned when it was found that completely different, best-fit values of D_x and U produced almost identical calculated concentration curves. By using Fischer's routing equation, the best-fit values for D_x and U, if any, were dependent on the starting values used in the numerical iteration of the optimization process and the final values thus obtained could be entirely different. No such problem has been experienced by use of Equation 47. A few computer experimental runs by varying the starting values of D_x and U showed that Equation 47 and its associated computer program always approached the same best-fit values of D_x and U within the specified computation accuracy.

There are, however, several perplexities among the D_x, U, and k_2 values obtained in this study. An inspection of Table 2 in Appendix B indicates that there are several instances where a value for D_x, U or k_2 obtained from data at the two stations furthest apart, say stations 1 and 3, does not fit within the range of the values obtained from stations 1 and 2 and from stations 2 and 3 (station 2 lying between stations 1 and 3) on the same test. This discrepency could possibly be due to the errors in the measurements or the accuracy of the computation by which calculated and measured values are compared in the best-fitting process, or both. Minimizing the sum of the squares of the differences between measured and calculated values may not be the best way to fit limited number of data points describing a nonlinear function of the concentration distribution. Other questionable results are the data obtained from the test on the laboratory flume. The values, listed in

Table 3 of Appendix B, do not show all of the flume test results. Several runs which resulted in a zero reaeration coefficient are not listed. Bad results might be caused by the overdose of the sodium sulfite which continued to react within the test reach. However, it was observed in some instances that the k_2 did have a value in the first section, but went to zero in the second section, immediately downstream from the first. In other instances this trend was reversed. These inconsistencies in the results may be attributed to the poor mixing of the deoxygenated slug in the flume. During several tests the slug was seen to be moving from side to side as it traveled downstream. The problems associated with the flume could also be due to sampling errors, but the flume tests should not be subjected to more sampling errors than the natural stream tests in which such problems did not occur noticeably. At any rate, the data obtained from the flume tests are highly scattered and somewhat questionable. On the other hand, data from the stream test show a better grouping when they are compared in terms of Equation 54, despite some irregularities. Although there is a fair amount of scatter in stream data points as shown in Figure 5, this is generally expected from any experimental measurement and a majority of the points seem to be grouped in a general area. Of course, more data points than those shown in Figure 5 are needed in order to draw a more conclusive relationship between the parameters considered.

The functional relationship proposed by Equation 52 is theoretically sound and is believed to be the general form of the semi-empirical model for the reaeration coefficient. The form of the relationship proposed in Equation 54 is a simplified version of Equation 52. Equation 54 was assumed simply because a majority of those prediction models discussed in the Review of Literature are of a similar form. The a and β values in Equation 54 may themselves be functions of various hydraulic parameters which should be investigated further.

A comparison of the other reaeration coefficient prediction models with Equation 54 may verify the validity of its proposed relationship. For comparison, Equation 54 is rewritten as

$$k_2 = a\, D_x{}^\beta U^{1-\beta} H^{-(1+\beta)} \dots\dots\dots\dots(67)$$

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The prediction model formulated by Churchill, Elmore and Buckingham (1962), Equation 11, has only U and H terms with exponents of less than one and less than minus one, respectively. These exponents in Equation 11 related the U and H terms in a similar fashion to those shown in Equation 67. The fact that the exponents of U and H in Equation 11 are not close to $1 - \beta$ and $-(1 + \beta)$, respectively, in Equation 67 could be the consequence of the exclusion of the dispersion coefficient, D_x from Equation 11. From Table 1 it can readily be seen that Equation 11 fits average the measured data points for data sets one, two and three, but it fits poorly for data set four. When this result is coupled with the fact that, according to Bennett and Rathbun (1972), Equation 11 fit the published data of Churchill, Elmore and Buckingham (1962) with an E_S of 0.52 days^{-1} and an E_p of 28 percent, it can easily be assumed that Equation 11 lacks some factor or factors which would make it more applicable to a wider range of flow conditions.

Krenkel and Orlob (1962) ran a regression analysis to correlate their measured k_2 values with the dispersion coefficient, D_x, and the depth of flow, H and obtained Equation 12. The D_x and H exponents in Equation 12 are related exactly to each other as proposed by Equation 67 with β being 1.32. However, there is no velocity term in Equation 12, which by Equation 67 should be present with an exponent of -0.32. If the difference in the test velocities were small, the velocity term with this exponent would remain reasonably constant which could easily be absorbed in the a coefficient. Krenkel and Orlob (1962) also ran a regression analysis to correlate their measured k_2 values with the energy dissipated per unit mass of flowing fluid, $E (= USg)$, and H. Equation 13 was obtained from their analysis. If USg is substituted into Equation 13 for the E term it can be seen that U and H have the exponents 0.408 and -0.66, respectively, which are not related to each other as proposed in Equation 67. However, for turbulent flow, for instance, the slope, S, is proportional to U^2/R from the Darcy-Weisbach or Chezy formula. If the hydraulic radius, R, is assumed to be approximately equal to the hydraulic depth, H, the exponents of U and H become 1.224 and -1.068, respectively. These exponents, though still not in the relationship as proposed in Equation 67, become a little closer to the theoretical values. Considering other flow regimes in which S is not proportional to U^2/R, but rather U^m/R^n where m and n are the exponents of U and R, respectively, with the values ranging from 1 to 2, one may justify the relationships among the exponents in Equation 67. Neither Equations 12 or 13 showed significant correlation with the data sets of Table 1, except that Equation 13 with the data

set one did show a lower than average, although still quite high, standard error of estimate. Bennett and Rathbun (1972) reported Equation 13 fitted the data from which it was derived with an E_S of 7.51 days^{-1} and an E_p of 15 percent. Again, the indication is that Equations 12 and 13 are not suitable for a wide range of flow conditions and some factor or factors may have been missing from their formulations.

Equation 14, developed by Dobbins (1965), cannot readily be arranged into a form of Equation 67. Despite its very complicated form, use of Equation 14 to fit the data sets involved in this study is below average in comparison with the other prediction equations as shown in Table 1.

Owens, Edwards, and Gibbs (1964) formulated Equations 15 and 16 by means of multiple regression analyses. Both equations consist only of U and H as variables. Their exponents are in a range suggested by Equation 67 but still not very close. Again, the difference could be due to the fact that D_x was left out of the regression analyses. Table 1 shows that both Equations 15 and 16 fit the data sets one and two with average standard error of estimates, but fit the data sets three and four with higher than average standard error of estimates. Equation 17 developed by Langbein and Durum (1967) is of the same form as Equations 15 and 16, having U and H as the only variables. Equation 17 has an average fit to the data sets one, two, and four, but has a better-than-average fit to the data set three.

Isaacs and Gaudy (1968) formulated Equations 18, 19, 20 from regression analyses of separate groups of data, holding the exponents on U and H (the only variables in the equations) at 1.0 and -1.5, respectively. Equations 18, 19, and 20 do not fit any of the data sets one, two, and four of Table 1 better than average, indicating they are not general enough and lack some factor or factors. Equation 21 developed by Cadwallader and McDonnell (1969) has E and H as the only variables with exponents of 0.5 and -1.0, respectively. If USg is substituted for E and S is assumed to be proportional to U^2/H as done before, the variables become U and H with exponents 1.5 and -1.5, respectively. This leaves Equation 21 in a form similar to Equations 18, 19, and 20 (except for exponent of U being 1.0 instead) with about the same results as to fitting the data sets of Table 1.

Negulescu and Rojanski (1969) formulated two equations, Equations 22 and 23, from their data which is represented by data set three in Table 1. The value U/H was held as a single variable in the regression analysis for each equation, thereby

giving H the negative exponent of U. This restriction partially prevented either Equation 22 or 23 from being in the form of Equation 67, despite that the dispersion coefficient, D_x, was added as a variable in the regression analysis for Equation 23. As shown in Table 1, Equation 22 fits data sets one and two in an average fashion and data set four better than average, while Equation 23 fits data sets one and four poorly and data set two with an average standard error of estimate. Of course, both equations fit data set three, from which they were derived, quite well with Equation 23 having the lower Ep. However, Equation 64 which was also derived from data set three but has the form of Equation 67, has the same number of variables as Equation 23 with an even lower Ep.

Equations 24, 25, and 26 were obtained through regression analyses by Thackston and Krenkel (1969) of their experimental data representing the data set four in Table 1. Equations 24 and 26 have the term U_*/H as the major variable with an exponent of 1.0 in both equations. Equation 25, formulated by using the dispersion coefficient, D_x, and H as variables, is indeed identical to the form of Equation 67 with a being 1.296 and β being 1.0. Although Equations 24 and 26 have a little better fit to the data from which they were derived than Equation 25, as can be seen in Table 1, Equation 25 is a better fit to both data sets one and two than Equations 24 and 26. This may imply that Equation 25 is a more general prediction model than the other two and is applicable to a wider range of flow conditions. Equation 66, formulated from the same data set as was Equation 25, has a slightly lower standard error of estimate than Equation 25. The reasons for this result may be two-fold: (1) The velocity term was intentionally left out of the regression analysis for Equation 25 and (2) the exponents of D_x and H in Equation 25 were probably either held constant at 1.0 and -2.0, respectively, or rounded off to these values. For comparison, both Equations 25 and 66 (an alternative form thereof, Equation 65) are plotted in Figure 5. Although the two equations have comparable standard error of estimates, the slopes of the lines representing both equations are different.

The equations of Bennett and Rathbun (1972), Equations 27 and 28, were both developed from regression analyses of more than one set of data. Both equations have U and H exponents similar in relationship to those proposed in Equation 67. The dispersion coefficient was not included in their analysis because D_x values were not measured in the investigations from which the pertinent data sets were obtained. It is felt,

however, that if D_x values had been available in the data, a fit of Equation 67 to that data would have been a better fit than provided by either Equations 27 or 28.

Lau (1972b) applied a dimensional-analysis approach and derived a reaeration coefficient prediction model which is very similar to the one used in this study. The model he arrived at has the same form as Equation 54 except that in his model (Equation 30) the role the D_x/UH term is playing in Equation 54 is characterized by U_*/U. Lau (1972b) fit his model to several sets of data including that of Thackston and Krenkel (1969) and found a and β values (assuming Equation 30 is equivalent to the form of Equation 54) of 1089 and 3, respectively, with k_2 in days^{-1} in Equation 30. This equation has a reasonably good fit to both data sets one and four of Table 1 in comparison to other prediction equations.

Table 1 shows that Equation 31 devloped by Parkhurst and Pomeroy (1972) fit all the data sets with an average standard error of estimate, but it cannot be arranged into a form similar to that of Equation 67. It may be due to the fact that Equation 31, along with Equations 14 and 26, which have a term of the order of A + f(F), is approximating the relationship of Equation 52 better than as a power function. Note that the term A is a coefficient, usually 1.0, and f (F) represents a function of the Froude number.

It is seen that most reaeration coefficient prediction models thus developed are incomplete in form, if compared with the relationship described in Equation 52, especially Equation 54 or 67. A few of the prediction models such as Equations 24 and 30, despite no dispersion coefficient, D_x, appearing in their formulations, do contain a shear velocity, U_*, term as a variable. It is understood that since the shear velocity plays a significant role in the evaluation of the dispersion coefficient, as demonstrated by Taylor (1954) and Elder (1959) in their formulations of the dispersion coefficient, the shear velocity may be judged as a good alternative of D_x in the formulation of the k_2 model.

Equations 60, 62, 64, and 66 were formed from the regression analysis of individual data sets one, two, three, and four of Table 1, respectively, and are not expected to fit other sets of data with low error. The same reasoning also applies to the other models in view of their poor standard errors of estimate as shown in Table 1. All the previous models have been developed for particular streams or flumes of their own interest. None of them including Equations 60, 62, 64, and 66, is general

enough to be applicable to all streams or flumes. However, a general model, such as Equation 54 or 67, has its wide applicability if the a and β parameters in Equation 54 or 67 can be expressed in terms of the relevant hydraulic parameters which can readily be measured. It is noticed that the β values in the models (Equations 59, 63, and 65) formulated from three of the data sets as depicted in Figure 5 show an interesting trend of being similar in value. This can be seen by comparing the slopes of the lines of Equations 59, 63, and 65 in Figure 5. The slopes appear much the same. The model (Equation 61) formulated from data set two, the UWRL flume data, shows a completely different slope, but this data set is the most questionable of those analyzed. Whether the similar values of β in the three models mentioned indicate an actual trend or are just a coincidence with the very limited number of data sets can only be verified by formulating similar models for different streams or flumes under a wide range of flow conditions.

There is no question about the validity of Equation 54 or 67. It has been derived theoretically and estimates the value of the reaeration coefficient as well as, if not better than, the equations of different forms which were developed by the previous investigators using the same set of data. Although some of the existing models were obtained from more rigorous regression analyses involving the evaluation of more than one coefficient and one exponent as is only required for Equation 54 or 67, use of Equation 54 or 67 with experimentally determined a and β values yields a comparable estimate of the k_2 value. The independent variables in Equation 54 or 67 are restricted to having specific relationships to each other. Prediction models formulated with no such restrictions do not fit actual data better than Equation 54 or 67. This means the relationship proposed by Equation 54 or 67 is not only theoretically valid but models the reaeration coefficient better than other equations even though they may be more complicated.

SUMMARY AND CONCLUSIONS

For evaluating the dispersion coefficient, mean velocity, and reaeration coefficient of a mountain creek, the dissolved oxygen and dye concentrations of an artificially deoxygenated portion of the flow containing a tracer dye were measured at two stations downstream from the point of a slug injection. At each station samples were taken with regular time intervals, one at a time, and then measured for dissolved oxygen content and dye concentration. Because Equations 43 and 47, when evaluated by using Simpson's rule of numerical integration, describe very well the movement of the slug as it travels downstream in terms of dye concentration and dissolved oxygen deficit, respectively, the dispersion coefficient and mean velocity in Equation 43 can be evaluated by best-fitting dye concentration values computed from that equation to corresponding measured values while the reaeration coefficient in Equation 47 can be evaluated in a similar fashion by using the dissolved oxygen deficit values. The present optimization technique used in the evaluation of the dispersion coefficient, mean velocity, and reaeration coefficient is believed to be most suitable for a stream, especially a mountain creek, which can readily be mixed fully over the entire cross-section upon the injection of a deoxygenated slug.

The general reaeration coefficient prediction model developed in the present study (Equation 52) is a functional relationship of the two dimenionless parameters, $k_2 L_0/U$ and $D_x/L_0 U$, in which the reference length L_0 can be either hydraulic radius, R, or hydraulic depth, H. A particular form of Equation 52 may be expressed in terms of a power function of a type as shown in Equation 54. In order for Equation 54 to be universally applicable as a prediction model of the reaeration coefficient for a stream, it is essential that the model parameters, a and β be accurately evaluated for the stream under investigation. The general determination of the a and β values is a formidable task. The a and β values that appear to vary with some hydraulic parameters cannot readily be determined for universal use in k_2 prediction for any stream unless relationships of factors interacting between them can be established. No attempt was made to find such relationships. Instead, the a and β values for any streams and flumes with available

or existing data on k_2, H, U, and D_x have been determined. Those determined include the two sets of data collected for this study, namely the Summit Creek data and the UWRL flume data, along with two additional data sets obtained from publications involving previous investigations.

Inclusion of D_x in the reaeration coefficient prediction model is justified from both theoretical and practical points of view. It is well known that the value of the dispersion coefficient for a stream is a quantitative measure of the dispersivity of any tracing fluid or substance including DO and dye in the flowing body of the stream. The D_x is also an indicator of the combined effects of many hydraulic parameters which affect the flow characteristics. Unfortunately, not too many previous prediction models include the D_x variable in their model formulations, partly because of its difficulty in the measurement and partly because of the empirical manner in which their models have been developed. A comparison of the results obtained from the various existing models by using available data indicates that the inclusion of the D_x, or a significant part thereof, such as the shear velocity, U_*, in the prediction model tends to yield the better estimate of the k_2. This conclusion, however, does not eliminate the possibility that there are some instances in which existing models predict the k_2 value better than those which contain the D_x, or specifically U_*. The fact that most existing models, which were developed separately for particular streams or flumes, could not be arranged into the form of Equation 54 or 67, as well as not having values of D_x, has prevented further investigation of the role which D_x plays in the accuracy of the k_2 measurement or computation. Such an investigation, though very difficult, can be undertaken in the future when more data on D_x for various streams become available.

The establishment of a functional relationship (Equation 52) between the two dimensionless parameters, $k_2 L_0/U$ and $D_x/L_0 U$, or a simplified form thereof (Equation 54 or 67), may lead to the conclusion that the k_2 measurement for a given stream or flume is not much more difficult than the D_x measurement. Given a stream, the D_x value may only vary with the depth of flow because other

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Variable	Definition	Variable	Definition

ITESTB

Integer code having any value greater or equal to one indicating the coefficient at station B is to be optimized.

ITESTD

Integer code having any value greater or equal to one indicating the dispersion coefficient is to be optimized.

ITESTV

Integer code having any value greater or equal to one indicating the velocity is to be optimized.

ITESTK

Integer code having any value greater or equal to one indicating the reaeration coefficient is to be optimized.

JAIR

The subscript on the variable AIRCOF which increments by one each time the reaeration coefficient completes an optimization cycle.

JCOA

The subscript on the variable ACOEF which increments by one each time the coefficient at station A completes an optimization cycle.

JCOB

The subscript on the variable BCOEF which increments by one each time the coefficient at station B completes an optimization cycle.

JDIS

The subscript on the variable DISCOF which increments by one each time the dispersion coefficient completes an optimization cycle.

JVEL

The subscript on the variable VELCTY which increments by one each time the velocity completes an optimization cycle.

KAYJAY

If equal to two the program will proceed to the optimization of the coefficients at stations A and B before that of velocity and dispersion coefficient.

KBCVAL (L)

The final calculated DO deficit value (mg/l - real value) at time level L after the reaeration coefficient has been optimized.

KEY

Term used in subroutines BSTFIT and SUBADD to indicate the end of an optimization cycle when equal to two.

KLSD

The least-squares deviation value (real) arrived at after optimizing the reaeration coefficient.

KORDER

If value input is two then velocity is optimized initially before the dispersion coefficient.

KSIZE

Value (real) read into the program indicating by what size step the reaeration coefficient is changed during its optimization.

LSDVAL (1 or 2)

Specific least-squares deviation values (real) in subroutine SUBADD for comparison purposes in finding a best fit value during the last steps of an optimization cycle.

MAREA

Area under the measured DO deficit versus time curve at station B between two successive measured points.

MUMBLE

Variable indicating the path of the optimization procedure, if equal to one velocity and dispersion are optimized then the coefficients at stations A and B are optimized; if equal to two the steps at one are completed then an additional velocity and dispersion optimization is carried out with the new values of A and B coefficients; if equal to three, steps of two are completed with an additional optimization of coefficients at stations A and B, and so on.

NAIR

Subscript on the variable RAIR which increments by one for each successive reaeration coefficient evaluated during its optimization cycle.

NCOA

Subscript on the variable RACOEF which increments by one for each successive value for the coefficient at station A during its optimization cycle.

NCOB

Subscript on the variable RBCOEF which increments by one for each successive value for the coefficient at station B during its optimization cycle.

NCOR

Subscript on the variable SUMDEV which increments for each least-squares deviation value during an optimization cycle and is given values in the subroutine SUBADD which trigger operations in the optimization procedure.

Variable	Definition	Variable	Definition

NDIS

Subscript on the variable RDSCOF which increments by one for each successive value for the dispersion coefficient during its optimization cycle.

NOPT

Subscript on the variable OPTION which increments by one in the subroutine SUBADD for each successive value of the term being optimized as transferred from subroutine BSTFIT.

NSTEPS

The number of three-point intervals involved in numerically integrating over the concentration versus time curve at station A.

NUMA

The number of time and corresponding concentration values describing the measured data at station A as transferred to the subroutine BSTFIT.

NUMAD

The number of time and corresponding dye concentration values read in to describe the measured dye data at station A.

NUMAO

The number of time and corresponding DO concentration values read in to describe the measured DO data at station A.

NUMB

The number of time and corresponding concentration values describing the measured data at station B as transferred to the subroutine BSTFIT.

NUMBD

The number of time and corresponding dye concentration values read in to describe the measured dye data at station A.

NUMBLE

Value incremented by one after velocity and dispersion optimization and again after A and B coefficients optimization in order to reach the read in value of MUMBLE.

NUMBO

The number of time and corresponding DO concentration values read in to describe the measured DO data at station B.

NVEL

Subscript on the variable RVLCTY which increments by one for each successive value for the velocity during its optimization cycle.

OPTION (NOPT)

The value of the variable being optimized as specified in the transfer to subroutine SUBADD at NOPT. NOPT increments as the value is changed and tested during optimization.

RACOEF (NCOA)

Value number NCOA in the successive values of the coefficient at station A during its optimization cycle.

RAIR (NAIR)

Value number of NAIR in the successive values of the reaeration coefficient during its optimization cycle ($sec^{-1} \times 10^5$).

RBCOEF (NCOB)

Value number NCOB in the successive values of the coefficient at station B during its optimization cycle.

RDSCOF (NDIS)

Value number NDIS in the successive values of the dispersion coefficient during its optimization cycle (ft^2/sec).

RNUMA

Real value of the integer NUMA.

RNUMAD

Real value of the integer NUMAD.

RNUMAO

Real value of the integer NUMAO.

RNUMB

Real value of the integer NUMB.

RNUMBD

Real value of the integer NUMBD.

RNUMBO

Real value of the integer NUMBO.

RVLCTY (N VEL)

Value number NVEL in the successive values of the velocity during its optimization cycle (ft/sec).

S(15)

Dimensional heading with 15 spaces for the testing stations.

Variable	Definition	Variable	Definition

SIZE

The step size used for optimizing a value as transferred to the subroutine SUBADD.

SSIZE

Either positive or negative SIZE depending upon the direction required to optimize the value, OPTION.

STEPS

Real value of the integer NSTEPS.

SUMDEV (NCOR)

Value number NCOR in the successive least-squares deviation values for the term being optimized as specified in the transfer to the subroutine SUBADD.

SUMEVE

Sum of the values obtained from using Equation 47 on each of the even numbered points of the numerical integration over the measured concentration versus time curve at station A.

SUMODD

Sum of the values obtained from using Equation 47 on each of the odd numbered points (excluding the first and the last) of the numerical integration over the measured concentration versus time curve at station A.

TCAREA

Total area under the calculated DO deficit versus time curve at station B (sum of the CAREA terms).

TMAREA

Total area under the measured DO deficit versus time curve at station B (sum of the MAREA terms).

VALUE (1 or 2)

Specific values of the term being optimized for use in calculating the size and direction of the final steps in the optimization cycle.

VBCVAL (L)

The final calculated dye concentration value (ppb) at time level L at station B after the velocity has completed an optimization cycle.

VELCTY (JVEL)

The velocity (ft/sec) after optimization cycle number JVEL-1. JVEL = 1 is the initial velocity read into the computer program.

VLSD

The least-squares deviation value arrived at after the velocity has been put through an optimization cycle.

VSIZE

Value read into the program indicating by what size step the velocity is changed during its optimization.

```
C      PROGRAM I - VARIABLE OPTIMIZATION
C      FORTRAN
       REAL KSIZE
       REAL KLSD
       REAL KBCVAL
       DIMENSION ADTIME(99) , ACONC(99), AOTIME(99 ), ADO(99) , BDTIME(45),
      2BCONC(45), BOTIME(45), BDO(45), D(20), S(15)
       COMMON/BLK1/ JDIS,JVEL,JCOA,JCOB,JAIR,DLSD,VLSD,ALSD,BLSD,KLSD,DIS
      2T,AMTIME,BMTIME,DOSAT,DSIZE,VSIZE,ASIZE,BSIZE,KSIZE,DELTA
       COMMON/BLK2/ DISCOF(15), VELCTY(15),ACOEF(15),BCOEF(15),AIRCOF(5),D
      2BCVAL(45),VBCVAL(45),ABCVAL(45),BBCVAL(45),KBCVAL(45),BMSAL(45)
       COMMON/BLK3/ NCOR,SUMDEV(45),KEY,VALUE(2),LSDVAL(2)
C      IREP IS THE NUMBER OF SEPERATE TESTS IN DATA DECK
       READ(5,70)IREP
 70    FORMAT(I5)
       DO 411 IJ=1,IREP
C      D(I) IS 20 SPACES FOR THE DATE OF TESTING AND S(I) IS 15 SPACES FOR
C      THE STATIONS INVOLVED
       READ(5,80)(D(I), I=1,20)
 80    FORMAT(20A1)
       READ(5,90)(S(I), I=1,15)
 90    FORMAT(15A1)
C      READ IN VALUES TO BE BEST FIT - A VALUE OF ONE OR MORE FOR EACH VARIABLE
C      BELOW WILL CAUSE DISPERSION, VELOCITY, A AND B COEF., OR REAERATION TO
C      BE OPTIMIZED
       READ(5,92)ITESTD,ITESTV,ITESTA,ITESTB,ITESTK
 92    FORMAT(5I1)
C      READ IN THE INITIAL VALUES FOR DISPERSION, VELOCITY, A COEF., BCOEF.,
C      AND REAERATION COEF.
       READ(5,94)DISCOF(1),VELCTY(1),ACOEF(1),BCOEF(1),AIRCOF(1)
 94    FORMAT(5F10.5)
C      READ IN STEP SIZE FOR BEST FITTING DISPERSION COEF., VELOCITY, A COEF. ,
C      B COEF., AND REAERATION COEF.
       READ(5,36)DSIZE,VSIZE,ASIZE,BSIZE,KSIZE
 96    FORMAT(5F10.5)
C      READ IN NUMBER OF POINTS ON DYE AND DO DEFICIT CURVES AND THE DISTANCE
C      BETWEEN STATIONS AND THE DO SATURATION VALUE
       READ(5,98)AMTIME,BMTIME,NUMAD,RNUMAD,NUMBD,RNUMBD,NUMAO,RNUMAO,NUM
      2BO,RNUMBO,DIST,DOSAT
 98    FORMAT(F10.4,F10.4,I5,F5.1,I5,F5.1,I5,F5.1,I5,F5.1,F10.4,F10.4)
C      READ IN CONTROL VARIABLES CONTROLING THE ORDER OF OPTIMIZATION
       READ(5,100)MUMBLE,KORDER,KAYJAY
100    FORMAT(I5,I5,I5)
C      DELTA IS THE TIME INTERVAL FOR WHICH DATA IS READ IN AT STATION A
       READ(5,101)DELTA
101    FORMAT(F10.4)
C      READ IN TIMES AND CONCENTRATIONS OF DYE AND DO AT STATIONS A AND B
       READ(5,105)(ADTIME(J),ACONC(J), J=1,NUMAD)
105    FORMAT((3(F5.1,F5.1)))
       READ(5,105)(BDTIME(I),BCONC(I), I=1,NUMBD)
       READ(5,115)(AOTIME(J),ADO(J), J=1,NUMAO)
115    FORMAT((3(F5.1,F5.2)))
       READ(5,115)(BOTIME(I),BDO(I), I=1,NUMBO)
       JDIS=1
       JVEL=1
       JCOA=1
       JCOB=1
       JAIR=1
       DLSD=0.0
       VLSD=0.0
       ALSD=0.0
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          BLSD=0.0
          KLSD=0.0
          NUMBLE=0
          IF (ITESTD.LT.1.AND.ITESTV.LT.1) GO TO 219
          IF (KAYJAY.EQ.2) GO TO 197
120       WRITE(6,121)(D(I), I=1,20)
121       FORMAT(17H1 TEST DATE      =,20A1)
          WRITE(6,122)(S(I), I=1,15)
122       FORMAT(17H  TEST STATIONS =,15A1)
          WRITE(6,123)
123       FORMAT(30H  SUSTAINED INJECTION EQUATION)
          WRITE(6,124)
124       FORMAT(38H0 BEST FITTING DISPERSION AND VELOCITY)
          WRITE(6,125)ACOEF(JCOA),BCOEF(JCOB)
125       FORMAT(28H  CONCENTRATION COEF. AT A =,F8.5 6X,27H CONCENTRATION C
     20EF. AT B =,F8.5)
          IF (ITESTD.LT.1) GO TO 140
          IF (KORDER.EQ.2) GO TO 140
C    CALL BSTFIT TO OPTIMIZE DISPERSION COEFFICIENT (INDEX=1)
130       CALL BSTFIT(1,ADTIME,ACONC,BDTIME,BCONC,NUMAD,RNUMAD,NUMBD,RNUMBD,
     20.0)
          IF (DISCOF(JDIS).LE.0.0) GO TO 401
          IF (ITESTV.LT.1) GO TO 180
          IF (JVEL.EQ.1) GO TO 140
          IF (DLSD.GE.VLSD) GO TO 180
C    CALL BSTFIT TO OPTIMIZE VELOCITY (INDEX=2)
140       CALL BSTFIT(2,ADTIME,ACONC,BDTIME,BCONC,NUMAD,RNUMAD,NUMBD,RNUMBD,
     20.0)
          IF (ITESTD.LT.1) GO TO 180
          IF (JDIS.EQ.1) GO TO 130
          IF (VLSD.GE.DLSD) GO TO 180
C    AS A PRECAUTION THE NUMBER OF VELOCITY OPTIMIZATION CYCLES IS LIMITED IN
C    THE DISPERSION-VELOCITY BEST FITTING PROCEDURE
          IF (JVEL.GT.14) GO TO 180
          GO TO 130
180       CONTINUE
          WRITE(6,182)
182       FORMAT(65H0 TIME       CALC. 2ND SECTION CONC.      MEASURED 2ND SECT
     2ION CONC.)
          IF (ITESTD.LT.1) GO TO 185
          IF (ITESTV.LT.1) GO TO 184
          IF (DLSD.LE.VLSD) GO TO 184
          IF (VLSD.LT.DLSD) GO TO 185
184       WRITE(6,186)(BDTIME(L),DBCVAL(L),BMSAL(L), L=1,NUMBD)
          WRITE(6,187)DLSD
          GO TO 189
185       WRITE(6,186)(BDTIME(L),VBCVAL(L),BMSAL(L), L=1,NUMBD)
          WRITE(6,187)VLSD
186       FORMAT(1X,F5.1,3X,F10.3,21X,F10.3)
187       FORMAT(26H0LEAST SQUARES DEVIATION =,F8.4)
189       WRITE(6,190)DISCOF(JDIS)
190       FORMAT(28H0BEST FIT DISPERSION COEF. =,F6.3)
          WRITE(6,192)VELCTY(JVEL)
192       FORMAT(28H BEST FIT VELOCITY         =,F6.3)
          WRITE(6,194)DSIZE
194       FORMAT(30H0INCREMENT FOR DISPERSION FIT=,F6.3)
          WRITE(6,196)VSIZE
196       FORMAT(30H INCREMENT FOR VELOCITY FIT  =,F6.3)
197       NUMBLE=NUMBLE+1
          IF ((NUMBLE-1).EQ.MUMBLE) GO TO 319
219       IF (ITESTA.LT.1.AND.ITESTB.LT.1) GO TO 319
220       WRITE(6,121)(D(I), I=1,20)
          WRITE(6,122)(S(I), I=1,15)
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           WRITE(6,123)
           WRITE(6,224)
224        FORMAT(62HO BEST FITTING CONCENTRATION COEF. A AND CONCENTRATION C
          2OEF. B)
           WRITE(6,225)DISCOF(JDIS),VELCTY(JVEL)
225        FORMAT(20H   DISPERSION COEF. =,F6.3,6X,11H VELOCITY =,F6.3)
           IF (ITESTA.LT.1) GO TO 240
C    CALL BSTFIT TO OPTIMIZE STATION A COEFFICIENT (INDEX=3)
230        CALL BSTFIT(3,ADTIME,ACONC,BDTIME,BCONC,NUMAD,RNUMAD,NUMBD,RNUMBD,
          2O.0)
           IF (ITESTB.LT.1) GO TO 284
           IF (JCOB.EQ.1) GO TO 240
           IF (ALSD.GE.BLSD) GO TO 280
C    CALL BSTFIT TO OPTIMIZE STATION B COEFFICIENT (INDEX=4)
240        CALL BSTFIT(4,ADTIME,ACONC,BDTIME,BCONC,NUMAD,RNUMAD,NUMBD,RNUMBD,
          2O.0)
           IF (ITESTA.LT.1) GO TO 285
           IF (JCOA.EQ.1) GO TO 230
           IF (BLSD.GE.ALSD) GO TO 280
           IF (JCOB.GT.4) GO TO 280
           GO TO 230
280        CONTINUE
           WRITE(6,182)
           IF (ALSD.LE.BLSD) GO TO 284
           IF (BLSD.LT.ALSD) GO TO 285
284        WRITE(6,182)
           WRITE(6,186)(BDTIME(L),ABCVAL(L),BMSAL(L), L=1,NUMBD)
           WRITE(6,187)ALSD
           GO TO 289
285        WRITE(6,182)
           WRITE(6,186)(BDTIME(L),BBCVAL(L),BMSAL(L), L=1,NUMBD)
           WRITE(6,187)BLSD
289        WRITE(6,290)ACOEF(JCOA)
290        FORMAT(37HO BEST FIT CONCENTRATION COEF. AT A =,F8.5)
           WRITE(6,292)BCOEF(JCOB)
292        FORMAT(37H  BEST FIT CONCENTRATION COEF. AT B =,F8.5)
           WRITE(6,294)ASIZE
294        FORMAT(29HO INCREMENT FOR COEF. A FIT =,F6.3)
           WRITE(6,296)BSIZE
296        FORMAT(29H  INCREMENT FOR COEF. B FIT =,F6.3)
           NUMBLE=NUMBLE+1
           IF ((NUMBLE-1).EQ.MUMBLE) GO TO 319
           GO TO 120
319        IF (ITESTK.LT.1) GO TO 400
           WRITE(6,121)(D(I), I=1,20)
           WRITE(6,122)(S(I), I=1,15)
           WRITE(6,123)
           WRITE(6,324)
324        FORMAT(60HO BEST FITTING REAERATION COEF. WITH OTHER VARIBLES CONS
          2TANT)
C    CALL BSTFIT TO OPTIMIZE REAERATION COEFFICIENT (INDEX=5)
           CALL BSTFIT(5,AOTIME,ADO,BOTIME,BDO,NUMAO,RNUMAO,NUMBO,RNUMBO,AIRC
          2OF(1))
           WRITE(6,382)
382        FORMAT(74HO TIME      CALCULATED D.O. DEFICIT (MG/L)       MEASURED D
          2.O. DEFICIT (MG/L))
           WRITE(6,384)(BOTIME(L),KBCVAL(L),BMSAL(L), L=1,NUMBO)
384        FORMAT(1X,F5.1,12X,F10.3,25X,F10.3)
           WRITE(6,386)KSIZE
386        FORMAT(31HO INCREMENT SIZE FOR BEST FIT =,F6.3)
400        IF (ITESTD.LT.1.AND.ITESTV.LT.1.AND.ITESTA.LT.1.AND.ITESTB.LT.1.AN
          2D.ITESTK.LT.1) GO TO 900
           GO TO 401
```

48

```
900    WRITE(6,121)(D(I), I=1,20)
       WRITE(6,122)(S(I), I=1,15)
       WRITE(6,123)
       WRITE(6,902)
902    FORMAT(40HO CALCULATIONS USING ONLY INITIAL VALUES)
C    CALL BSTFIT TO CALCULATE DYE CONCENTRATIONS AT STATION B LEAVING ALL
C    VARIABLES CONSTANT (INDEX=6)
       CALL BSTFIT(6,ADTIME,ACONC,BDTIME,BCONC,NUMAD,RNUMAD,NUMBD, RNUMBD
      2,0.0)
       WRITE(6,182)
       WRITE(6,186)(BDTIME(L),DBCVAL(L),BMSAL(L), L=1,NUMBD)
401    WRITE(6,402)DISCOF(JDIS)
402    FORMAT(38H- FINAL DISPERSION COEF. (FTXFT/SEC) =,F6.3)
       WRITE(6,404)VELCTY(JVEL)
404    FORMAT(38H  FINAL VELOCITY            (FT/SEC)  =,F6.3)
       WRITE(6,406)ACOEF(JCOA)
406    FORMAT(34HO FINAL CONCENTRATION COEF. AT A = ,F8.5)
       WRITE(6,408)BCOEF(JCOB)
408    FORMAT(34H  FINAL CONCENTRATION COEF. AT B = ,F8.5)
       WRITE(6,409)DELTA
409    FORMAT(49HO INTERVAL SIZE FOR NUMERICAL INTEGRATION (SEC) =,F7.4)
       WRITE(6,410)AIRCOF(JAIR)
410    FORMAT(47HO FINAL REAERATION COEF. - K2X10**5 (PER SEC) =,F8.4)
411    CONTINUE
       STOP
       END
```

```
      SUBROUTINE BSTFIT(INDEX,ATIME,AMVAL,BTIME,BMVAL,NUMA,RNUMA,NUMB,RN
     2UMB,AIRVAL)
      REAL KSIZE
      REAL KLSD
      REAL KBCVAL
      REAL MAREA
      DIMENSION RDSCOF(45), RVLCTY(45), RACOFF(45), RBCOEF(45), RAIR(45)
     2, FUNC(99), BCVAL(45), ATIME(99), AMVAL(99), BTIME(45), BMVAL(45),
     3 AMSAL(99)
      COMMON/BLK1/ JDIS,JVEL,JCOA,JCOB,JAIR,DLSD,VLSD,ALSD,BLSD,KLSD,DIS
     2T,AMTIME,BMTIME,DOSAT,DSIZE,VSIZE,ASIZE,BSIZE,KSIZE,DELTA
      COMMON/BLK2/ DISCOF(15),VELCTY(15),ACOEF(15),BCOEF(15),AIRCOF(5),D
     2BCVAL(45),VBCVAL(45),ABCVAL(45),BBCVAL(45),KBCVAL(45),BMSAL(45)
      COMMON/BLK3/ NCOR,SUMDEV(45),KEY,VALUE(2),LSDVAL(2)
      NCOR=1
      NDIS=1
      NVEL=1
      NCOA=1
      NCOB=1
      NAIR=1
      KEY=1
C     SET INITIAL VALUES FOR OPTIMIZATION TO THE LATEST VALUES IN MAIN PROGRAM
      RDSCOF(1)=DISCOF(JDIS)
      RVLCTY(1)=VELCTY(JVEL)
      RACOEF(1)=ACOEF(JCOA)
      RBCOEF(1)=BCOEF(JCOB)
      RAIR(1)=AIRVAL
      IF (DELTA.LE.2.5) GO TO 425
      NSTEPS=(NUMA-1)/2
      STEPS=(RNUMA-1.0)/2.0
      IODD=NSTEPS-1
      IEVEN=NSTEPS*2-1
425   CONTINUE
440   IF (NDIS.GT.1.OR.NVEL.GT.1.OR.NAIR.GT.1) GO TO 542
C     MULTIPLY MEASURED DYE VALUES AT STATION A BY COEFFICIENT A
C     CALCULATE DO DEFICITS FROM MEASURED DO VALUES AT STATION A AND MULTIPLY
C     BY COEFFICIENT A
      DO 441 L=1,NUMA
      IF (INDEX.NE.5) AMSAL(L)=RACOEF(NCOA)*AMVAL(L)
      IF (INDEX.EQ.5) AMSAL(L)=(DOSAT-AMVAL(L))*RACOEF(NCOA)
      IF (AMSAL(L).LT.0.0) AMSAL(L)=0.0
441   CONTINUE
C     MULTIPLY MEASURED DYE VALUES AT STATION B BY COEFFICIENT B
C     CALCULATE DO DEFICITS FROM MEASURED DO VALUES AT STATION B AND MULTIPLY
C     BY COEFFICIENT B
      DO 442 L=1,NUMB
      IF (INDEX.NE.5) BMSAL(L)=RBCOEF(NCOB)*BMVAL(L)
      IF (INDEX.EQ.5) BMSAL(L)=(DOSAT-BMVAL(L))*RBCOEF(NCOB)
      IF (BMSAL(L).LT.0.0) BMSAL(L)=0.0
442   CONTINUE
C     CALCULATE AREA UNDER MEASURED CONCENTRATION VERSUS TIME CURVE AT STATION B
      TMAREA=0.0
      NEB=NUMB-1
      DO 541 L=1,NEB
      MAREA=0.5*(BMSAL(L+1)+BMSAL(L))*(BTIME(L+1)-BTIME(L))
      TMAREA=TMAREA+MAREA
541   CONTINUE
542   IF (RAIR(NAIR).LT.0.0) NCOR=29
      IF (RDSCOF(NDIS).LE.0.0) GO TO 490
C     CALCULATE A CONCENTRATION VALUE AT STATION B FOR EACH TIME FOR WHICH A
C     VALUE WAS READ IN FOR STATION B
      DO 466 M=1,NUMB
C     EVALUATE EQUATION 47 (INSIDE INTEGRAL SIGN) AT EACH TIME A CONCENTRATION
```

```
C     VALUE IS READ IN FOR STATION A
      DO 450 K=1,NUMA
      IF (ATIME(K).GE.BTIME(M)) GO TO 448
      EXFUNC=-(DIST-RVLCTY(NVEL)*(BTIME(M)-ATIME(K)))**2/(4.0*RDSCOF(NDI
     2S)*(BTIME(M)-ATIME(K)))-(RAIR(NAIR)/(10.0**5))*(BTIME(M)-ATIME(K))
      IF (EXFUNC.LT.-88.0) GO TO 448
      FUNC(K)=(AMSAL(K)/SQRT(4.0*3.14*RDSCOF(NDIS)*(BTIME(M)-ATIME(K))))
     2*EXP(EXFUNC)*RVLCTY(NVEL)
      GO TO 450
448   FUNC(K)=0.0
      EXFUNC=0.0
450   CONTINUE                      .
      SUMODD=0.0
      DO 455 I=1,IODD
      NODD=1+I*(NUMA-1)/NSTEPS
      SUMODD=SUMODD+FUNC(NODD)
455   CONTINUE
      SUMEVE=0.0
      DO 460 I=1,IEVEN,2
      NEVEN=1+I*(NUMA-1)/(2*NSTEPS)
      SUMEVE=SUMEVE+FUNC(NEVEN)
460   CONTINUE
C     OBTAIN A CALCULATED VALUE AT STATION B BY USING SIMPSONS NUMERICAL
C     INTEGRATION
      BCVAL(M)=((ATIME(NUMA)-ATIME(1))/(6.0*STEPS))*(FUNC(1)+FUNC(NUMA)+
     22.0*SUMODD+4.0*SUMEVE)
466   CONTINUE
      IF (INDEX.NE.5) GO TO 469
C     OBTAIN AREA UNDER CALCULATED DO DEFICIT VERSUS TIME CURVE AT STATION B
      TCAREA=0.0
      DO 467 L=1,NEB
      CAREA=0.5*(BCVAL(L+1)+BCVAL(L))*(BTIME(L+1)-BTIME(L))
      TCAREA=TCAREA+CAREA
467   CONTINUE
      SUMDEV(NCOR)=(TMAREA-TCAREA)**2
      GO TO 570
469   SUMDEV(NCOR)=0.0
C     COMPARE CALCULATED TO MEASURED CURVE BY MAGNITUDE OF THE SUM OF THE
C     DIFFERENCES SQUARED AT EACH POINT
      DO 470 N=1,NUMB
      DEV=(BCVAL(N)-BMSAL(N))**2
      SUMDEV(NCOR)=SUMDEV(NCOR)+DEV
470   CONTINUE
570   IF (NCOR.GT.1) GO TO 474
      IF (INDEX.EQ.1) GO TO 471
      IF (INDEX.EQ.2) GO TO 571
      IF (INDEX.EQ.3) GO TO 671
      IF (INDEX.EQ.4) GO TO 771
      IF (INDEX.EQ.5) GO TO 871
      IF (INDEX.EQ.6) GO TO 502
471   WRITE(6,472)VELCTY(JVEL)
472   FORMAT(50H0 BEST FITTING DISPERSION WITH VELOCITY (FT/SEC) = ,F8.4)
      WRITE(6,473)
473   FORMAT(69H0 DISPERSION COEF.    NUMBER          LEAST SQUARES DEVIAT
     2ION    NUMBER)
      GO TO 474
571   WRITE(6,572)DISCOF(JDIS)
572   FORMAT(53H0 BEST FITTING VELOCITY WITH DISPERSION (FTXFT/SEC) =,F8
     2.4)
      WRITE(6,573)
573   FORMAT(69H0      VELOCITY          NUMBER          LEAST SQUARES DEVIAT
     2ION    NUMBER)
      GO TO 474
671   WRITE(6,672)BCOEF(JCOB)
```

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```
672    FORMAT(47H0 BEST FITTING A COEFICIENT WITH B COEFICIENT =,F8.4)
       WRITE(5,673)
673    FORMAT(69H0    A COEFICIENT        NUMBER          LEAST SQUARES DEVIAT
      2ION    NUMBER)
       GO TO 474
771    WRITE(6,772)ACOEF(JCOA)
772    FORMAT(47H0 BEST FITTING B COEFICIENT WITH A COEFICIENT =,F8.4)
       WRITE(6,773)
773    FORMAT(69H0    B COEFICIENT        NUMBER          LEAST SQUARES DEVIAT
      2ION    NUMBER)
       GO TO 474
871    WRITE(6,873)              .
873    FORMAT(69H0K2X10**5 (PER SEC)    NUMBER          LEAST SQUARES DEVIAT
      2ION    NUMBER)
C   CALL SUBADD TO MAKE INCREMENTS ON VARIABLE BEING OPTIMIZED
474    IF (INDEX.EQ.1) CALL SUBADD(RDSCOF,NDIS,DSIZE)
       IF (INDEX.EQ.2) CALL SUBADD(RVLCTY,NVEL,VSIZE)
       IF (INDEX.EQ.3) CALL SUBADD(RACOEF,NCOA,ASIZE)
       IF (INDEX.EQ.4) CALL SUBADD(RBCOEF,NCOB,BSIZE)
       IF (INDEX.EQ.5) CALL SUBADD(RAIR,NAIR,KSIZE)
       IF (KEY.EQ.2) GO TO 490
       GO TO 425
490    CONTINUE
C   EQUATE CALCULATED TERMS TO VARIOUS VARIBLES SO THAT MAIN PROGRAM WILL
C   KNOW WHAT VARIBLE HAS BEEN OPTIMIZED
       IF (INDEX.EQ.1) GO TO 491
       IF (INDEX.EQ.2) GO TO 493
       IF (INDEX.EQ.3) GO TO 495
       IF (INDEX.EQ.4) GO TO 497
       IF (INDEX.EQ.5) GO TO 499
491    JDIS=JDIS+1
       DISCOF(JDIS)=RDSCOF(NDIS)
       DO 492 L=1,NUMB
       DBCVAL(L)=BCVAL(L)
492    CONTINUE
       DLSD=SUMDEV(NCOR)
       RETURN
493    JVEL=JVEL+1
       VELCTY(JVEL)=RVLCTY(NVEL)
       DO 494 L=1,NUMB
       VBCVAL(L)=BCVAL(L)
494    CONTINUE
       VLSD=SUMDEV(NCOR)
       RETURN
495    JCOA=JCOA+1
       ACOEF(JCOA)=RACOEF(NCOA)
       DO 496 L=1,NUMB
       ABCVAL(L)=BCVAL(L)
496    CONTINUE
       ALSD=SUMDEV(NCOR)
       RETURN
497    JCOB=JCOB+1
       BCOEF(JCOB)=RBCOEF(NCOB)
       DO 498 L=1,NUMB
       BBCVAL(L)=BCVAL(L)
498    CONTINUE
       BLSD=SUMDEV(NCOR)
       RETURN
499    JAIR=JAIR+1
       AIRCOF(JAIR)=RAIR(NAIR)
       DO 500 L=1,NUMB
       KBCVAL(L)=BCVAL(L)
500    CONTINUE
```

```fortran
           KLSD=SUMDEV(NCOR)
           WRITE(6,501)TCAREA
501        FORMAT(39HO AREA UNDER CALCULATED DEFICIT CURVE =,F10.4)
           WRITE(6,601)TMAREA
601        FORMAT(39H  AREA UNDER MEASURED DEFICIT CURVE   =,F10.4)
           RETURN
502        WRITE(6,503)SUMDEV(NCOR)
503        FORMAT(27HO LEAST SQUARES DEVIATION =,F10.4)
           DO 504 L=1,NUMB
           DBCVAL(L)=BCVAL(L)
504        CONTINUE
           RETURN
           END




           SUBROUTINE SUBADD(OPTION,NOPT,SIZE)
           REAL SIZE
           REAL LSDVAL
           DIMENSION OPTION(45)
           COMMON/BLK3/ NCOR,SUMDEV(45),KEY,VALUE(2),LSDVAL(2)
           WRITE(6,475)OPTION(NOPT),NOPT,SUMDEV(NCOR),NCOR
475        FORMAT(4X,F9.4,11X,I2,13X,F12.4,14X,I2)
           IF (NCOR.EQ.1) GO TO 480
           IF (NCOR.EQ.2) GO TO 481
           IF (NCOR.EQ.3) GO TO 482
           IF (NCOR.EQ.29) GO TO 488
           IF (NCOR.EQ.45) GO TO 488
           IF (NCOR.EQ.33) GO TO 485
           IF (NCOR.EQ.34) GO TO 486
           IF (NCOR.EQ.35) GO TO 488
           IF (SUMDEV(NCOR).LT.SUMDEV(NCOR-1)) GO TO 483
           IF (SUMDEV(NCOR).GE.SUMDEV(NCOR-1)) GO TO 484
C      FROM NCOR=2 AND NCOR=3 THE DIRECTION OF INCREMENT IS OBTAINED (PLUS
C      OR MINUS)
480        NCOR=2
           NOPT=2
           OPTION(NOPT)=OPTION(1)+0.01
           RETURN
481        NCOR=3
           NOPT=3
           OPTION(NOPT)=OPTION(1)-0.01
           RETURN
482        IF (SUMDEV(2).GT.SUMDEV(3)) SSIZE=-(SIZE)
           IF (SUMDEV(2).LT.SUMDEV(3)) SSIZE=SIZE
           IF (SUMDEV(3).GT.SUMDEV(1).AND.SUMDEV(2).GT.SUMDEV(1)) GO TO 487
483        NCOR=NCOR+1
           NOPT=NOPT+1
           OPTION(NOPT)=OPTION(NOPT-1)+SSIZE
           IF (NCOR.GT.35) OPTION(NOPT)=OPTION(NOPT-1)/2.0
           IF (OPTION(NOPT).LE.0.0) GO TO 587
           RETURN
C      LEAST SQUARES HAS PASSED MINIMUM VALUE - GO BACK TO FIND
484        SUMDEV(30)=SUMDEV(NCOR-2)
           SUMDEV(31)=SUMDEV(NCOR-1)
           SUMDEV(32)=SUMDEV(NCOR)
           IF (NCOR.EQ.4.AND.SUMDEV(3).LT.SUMDEV(2)) SUMDEV(30)=SUMDEV(1)
           NCOR=33
           NOPT=NOPT+1
           IF (SUMDEV(30).LT.SUMDEV(32)) GO TO 584
           IF (SUMDEV(30).GE.SUMDEV(32)) GO TO 684
584        VALUE(1)=OPTION(NOPT-3)
           LSDVAL(1)=SUMDEV(30)
```

```
      IF (NOPT.EQ.5.AND.SUMDEV(3).LT.SUMDEV(2)) VALUE(1)=OPTION(1)
      GO TO 784
684   VALUE(1)=OPTION(NOPT-1)
      LSDVAL(1)=SUMDEV(32)
784   VALUE(2)=OPTION(NOPT-2)
      LSDVAL(2)=SUMDEV(31)
      OPTION(NOPT)=(VALUE(1)+VALUE(2))/2.0
      RETURN
485   NCOR=NCOR+1
      NOPT=NOPT+1
      IF (LSDVAL(2).LE.SUMDEV(33)) OPTION(NOPT)=(OPTION(NOPT-1)+VALUE(2)
     2)/2.0
      IF (LSDVAL(2).GT.SUMDEV(33)) OPTION(NOPT)=(OPTION(NOPT-1)+VALUE(1)
     2)/2.0
      RETURN
C     FIND OPTION VALUE WITH SMALLEST LEAST SQUARES OF THOSE CALCULATED
486   NCOR=NCOR+1
      NOPT=NOPT+1
      IF (SUMDEV(34).LE.SUMDEV(33).AND.SUMDEV(34).LE.LSDVAL(2)) OPTION(N
     2OPT)=OPTION(NOPT-1)
      IF (SUMDEV(33).LE.SUMDEV(34).AND.SUMDEV(33).LE.LSDVAL(2)) OPTION(N
     2OPT)=OPTION(NOPT-2)
      IF (LSDVAL(2).LE.SUMDEV(34).AND.LSDVAL(2).LE.SUMDEV(33)) OPTION(NO
     2PT)=VALUE(2)
      RETURN
487   NCOR=29
      NOPT=NOPT+1
      OPTION(NOPT)=OPTION(1)
      RETURN
C     IF OPTION MINUS SIZE IS LESS THAN ZERO, TAKE NEW OPTION AS HALF OF OLD
C     OPTION VALUE
587   SUMDEV(37)=SUMDEV(NCOR-1)
      NCOR=38
      OPTION(NOPT)=OPTION(NOPT-1)/2.0
      RETURN
488   KEY=KEY+1
      RETURN
      END
```

PROGRAM II: EVALUATION OF HYDRAULIC PARAMETERS

The major purpose of this program is the calculation of hydraulic parameters and comparison of the measured reaeration coefficient to those predicted by use of other models described in the literature review. The ability to calculate the hydraulic parameters for the laboratory flume was also built into the program. If variable IX, (N, K) has K equal to 8, the parameters are calculated from the uniform cross-sectional geometry of the flume rather than the stream geometry. The predicted reaeration coefficients as listed in the program printout along with the measured reaeration coefficient have all been corrected to 20 degrees centigrade and are in terms of per day with the base 10 logarithm form (k_2). The measured reaeration coefficient read into the program is in terms of per second times 10^5 with the natural logarithm form (K_2). The program with a data deck consisting of 3 tests (with data from all three to be printed on the same page) requires approximately 0.7 seconds of execution time with 8 to 10 pages of printout (excluding a program listing).

Function of subroutine CROSS. CROSS calculated the cross-sectional area at each cross-sectioned station by dividing the measured discharge by the velocity calculated from Program I. It then calculates maximum depth, surface width, and wetted perimeter for each cross-sectioned station within the test reach.

Program II data deck -

	Variable	Format	Example	Comments
Card 1	BASE(I) I=1,7	7F10.4	0.0, 0.0, 3.67, 3.50, 0.0, 0.0, 0.0	Any horizontal width at the bottom of the stream bed at the cross-section stations within any test reaches, that is 0+00 to 1+50, every 25 feet.
Cards 2, 3, & 4	STEP(1,J), LEFT (1,J) RIGHT(1,J) J=1,6	6F10.4	0.6, 0.432, 0.381, 0.30, 0.432, 0.551, etc.	Channel geometry at first station, 0+00. The depth between levels (STEP) and the side slopes between levels (LEFT and RIGHT) are read in. The levels being formed by a horizontal plane moving up from the bottom and stopping at each change of channel geometry.
Cards 5, 6, & 7	STEP(2,J), LEFT(2,J) RIGHT(2,J) J=1,6	6F10.4	length, slope, slope, length, slope, slope..	Channel geometry at second station, 0+25.
Cards 8, 9, & 10	STEP(3,J), etc.	"	"	Channel geometry at third station, 0+50.
Cards 11, 12, & 13	STEP(4,J), etc.	"	"	" " " fourth station, 0+75.
Cards 14, 15, & 16	STEP(5,J), etc.	"	"	" " " fifth station, 1+00.
Cards 17, 18, & 19	STEP(6,J), etc.	"	"	" " " sixth station, 1+25.
Cards 20, 21, & 22	STEP (7,J), etc.	"	"	" " " seventh station, 1+50.
Card 23	NUMRUN	I5	----1	Number of times cards 24 through 28 are repeated for different tests.
Card 24	NUMTST	I5	----1	Number of tests to be listed on the same page (1 to 3).
Card 25	DA(N,I), I=1,15 & ST(N,J) J=1,15	15A1, 15A1	12/6/74 1+10 to 0+50	First 15 spaces for test data and second 15 spaces for stations involved, both for test N (1 to 3).
Card 26	IX(N,K) K=1,8	8I1	--345---	Any value indicates the 25 foot cross-sectioned stations within the test reach, IX(N,1) is 0+00, IX(N,2) is 0+25, etc.
Card 27	VEL(N,J) J=1,8	8F10.4	-,-,2.05, 2.05, 2.05, -,-,-	Velocities at 25-foot stations within test reach.
Card 28	Q(N), T(N), DX(N), S(N) VIS(N), K2(N)	F10.4, F10.4, F10.4 F10.6, F10.4, F10.4	12.3, 4.5, 8.90, 0.017, 1.661, 525.0	Discharge, temperature, dispersion coef., slope, kinematic viscosity $(x10^5)$, reaeration coef. $(sec^{-1} \times 10^5)$ for test N.

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VARIABLE DEFINITIONS FOR PROGRAM II.

Variable   **Definition**

A (J)
Average cross-section area (ft^2) of the test reach for test J (J = 1, NUMTST).

ADJ
Temperature adjustment factor added to prediction equations. Has the form $(1.0241)^{(T-20)}$.

AREA (INDEX 3, INDEX)
Cross-sectional area (ft^2) at 25-foot station INDEX for test INDEX 3.

AVEMD (J)
Average maximum depth (ft) of the test reach for test J.

BASE (I)
Width (ft) of a horizontal section at the bottom of 25-foot station I (if no horizontal width then equal to zero).

D (K)
Average hydraulic depth (ft) for the test reach for test K.

DA (N, 15)
Dimensional heading with 15 spaces for the date of test N.

DX (N)
Average dispersion coefficient (ft^2/sec) for the test reach of test N.

E (K)
Average energy dissipated per unit mass (ft^2/sec^3) for the test reach of test K.

EQUA (N, K)
Reaeration coefficient as predicted by equation N (not the same numbers as in literature review) with data from test K.

F (K)
Average Froude Number for the test reach of test K.

H (INDEX 3, INDEX)
Maximum depth (ft) at 25-foot station INDEX for test INDEX 3.

INDEX
Integer coding for 25-foot stations. 1 is 0+00, 2 is 0+25, 3 is 0+50, 4 is 0+75, 5 is 1+00, 6 is 1+25 and 7 is 1+50.

Variable   **Definition**

INDEX 3
Integer coding for the test number. There can be 1, 2 or 3 tests per program loop, number designated by NUMTST.

IX (N, K)
Integer coding with any value above zero indicating 25-foot station K (numbers related to station the same as described by INDEX) is within the reach of test N. Value for IX (N, 8) indicates test is from laboratory flume.

K2 (N)
Measured reaeration coefficient (real) for test N in terms of the natural logarithm (sec^{-1} x 10^5).

LEFT (K, L)
Slope of north side of the stream at 25-foot station K between level L and the level below it.

NUMRUN
The number of sets, of 1 to 3 tests each, in the data deck.

NUMTST
The number of tests within the data set.

P (J)
Average wetted perimeter (ft) of the test reach of test J.

PERT (J)
The number of 25-foot stations within the test reach for test J.

Q (N)
The measured discharge for test N.

R (K)
Average hydraulic radius (ft) for the test reach of test K.

RIGHT (K,L)
Slope of the south side of the stream at 25-foot station K between level L and the level below it.

RN (K)
Average Reynolds Number for the test reach of test K.

S (N)
Measured slope (ft/ft) for test N.

ST (N)
Measured slope (ft/ft) for test N.

Variable	Definition	Variable	Definition

ST (N, 15)

Dimensional heading with 15 spaces for the end stations of test n.

STEP (K, L)

Distance (ft) between level L and the level below it at 25-foot station K.

SU (K)

Average shear velocity (ft/sec) for the test reach of test K.

T (N)

Measured temperature (°C) of stream for test N.

TA (N)

Cross-sectional area between level N and the level below it at 25-foot station INDEX as transferred to subroutine CROSS.

TOP (INDEX 3, INDEX)

Surface width of stream (ft) at 25-foot station INDEX during test INDEX 3.

TT (N)

Stream width (ft) at level N at 25-foot station INDEX as transferred to subroutine CROSS.

TWP (N)

Wetted perimeter (ft) of stream below level N at 25-foot station INDEX as transferred to subroutine CROSS.

VEL (N, J)

Velocity at 25-foot station J for test N.

VIS (N)

Kinematic viscosity (ft^2/sec x 10^5) of stream during test N

U (J)

Average velocity (ft/sec) for the test reach of test J.

W (J)

Average surface width (ft) for the test reach of test J.

WP (INDEX 3, INDEX)

Wetted perimeter (ft) at 25-foot station INDEX during test INDEX 3.

Program II listing.

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C    PROGRAM II - EVALUATION OF HYDRAULIC PARAMETERS
C    FORTRAN          .
     REAL LEFT
     REAL K2
     DIMENSION DA(3,15), ST(3,15), IX(3,10), T(3), DX(3), S(3), VIS(3),
    2K2(3), A(3), AVEMD(3), U(3), W(3), P(3), PERT(3), R(3), D(3), SU(3
    3), F(3), RN(3), E(3), EQUA(25,3)
     COMMON BASE(10),STEP(10,10),LEFT(10,10),RIGHT(10,10),AREA(3,10),Q(
    23),VEL(3,10),H(3,10),TOP(3,10),WP(3,10)
C    READ IN CHANNEL GEOMETRY 'AT STATIONS 1 THROUGH 7
     READ(5,50)(BASE(I), I=1,7)
 50  FORMAT(7F10.4)
     READ(5,52)(STEP(1,J),LEFT(1,J),RIGHT(1,J), J=1,6)
 52  FORMAT(6F10.4)
     READ(5,52)(STEP(2,K),LEFT(2,K),RIGHT(2,K), K=1,6)
     READ(5,52)(STEP(3,L),LEFT(3,L),RIGHT(3,L), L=1,6)
     READ(5,52)(STEP(4,M),LEFT(4,M),RIGHT(4,M), M=1,6)
     READ(5,52)(STEP(5,N),LEFT(5,N),RIGHT(5,N), N=1,6)
     READ(5,52)(STEP(6,I),LEFT(6,I),RIGHT(6,I), I=1,6)
     READ(5,52)(STEP(7,J),LEFT(7,J),RIGHT(7,J), J=1,6)
C    READ IN THE NUMBER OF SETS, OF 1 TO 3 TESTS EACH, IN DATA DECK
     READ(5,59)NUMRUN
 59  FORMAT(I5)
     DO 400 JACK=1,NUMRUN
C    SET INITIAL VALUES EQUAL TO ZERO
     DO 58 IJ=1,3
     DO 58 LM=1,8
     IX(IJ,LM)=0
     VEL(IJ,LM)=0.0
     AREA(IJ,LM)=0.0
     H(IJ,LM)=0.0
     TOP(IJ,LM)=0.0
     WP(IJ,LM)=0.0
 58  CONTINUE
C    READ IN THE NUMBER OF TESTS IN THE SET (1, 2, OR 3)
     READ(5,59)NUMTST
     DO 70 N=1,NUMTST
C    READ IN TEST DATE AND STATIONS INVOLVED FOR EACH TEST N
     READ(5,60)(DA(N,I), I=1,15),(ST(N,J), J=1,15)
 60  FORMAT(15A1,15A1)
C    READ IN NUMBERS OF CROSS-SECTIONS WITHIN TEST REACH FOR EACH TEST N
     READ(5,62)(IX(N,K), K=1,8)
 62  FORMAT(8I1)
C    READ IN VELOCITIES AT CROSS-SECTIONS WITHIN TEST REACH FOR EACH TEST N
     READ(5,64)(VEL(N,J), J=1,8)
 64  FORMAT(8F10.4)
C    READ IN DISCHARGE, TEMPERATURE, DISPERSION COEF., SLOPE, KINEMATIC
C    VISCOSITY, AND REAERATION COEF. FOR EACH TEST N
     READ(5,66)Q(N),T(N),DX(N),S(N),VIS(N),K2(N)
 66  FORMAT(F10.4,F10.4,F10.4,F10.6,F10.4,F10.4)
 70  CONTINUE
C    CALL CROSS FOR EACH CROSS-SECTION, LM, DURING EACH TEST, IJ
     DO 75 IJ=1,NUMTST
     DO 75 LM=1,8
     IF (IX(IJ,LM).GT.0) CALL CROSS(IJ,LM)
 75  CONTINUE
C    SET INITIAL PARAMETER VALUES TO BE ZERO
     DO 76 N=1,3
     A(N)=0.0
     AVEMD(N)=0.0
     U(N)=0.0
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               W(N)=0.0
               P(N)=0.0
               PERT(N)=0.0
   76     CONTINUE
C    MAKE FIRST GROUP OF PARAMETERS THE SUM OF THE SAME PARAMETERS AT EACH
C    CROSS-SECTION WITHIN THE TEST REACH
               DO 85  J=1,NUMTST
               DO 85  M=1,8
               IF (IX(J,M).EQ.0) GO TO 85
               A(J)=A(J)+AREA(J,M)
               U(J)=U(J)+VEL(J,M)
               AVEMD(J)=AVEMD(J)+H(J,M)
               W(J)=W(J)+TOP(J,M)
               P(J)=P(J)+WP(J,M)
               PERT(J)=PERT(J)+1.0
   85     CONTINUE
C    MAKE FIRST GROUP OF PARAMETERS THE AVERAGE OF THE CROSS-SECTIONAL
C    VALUES BY DIVIDING BY THE NUMBER OF CROSS-SECTIONS WITHIN THE TEST
C    REACH
               DO 87  J=1,NUMTST
               A(J)=A(J)/PERT(J)
               U(J)=U(J)/PERT(J)
               AVEMD(J)=AVEMD(J)/PERT(J)
               W(J)=W(J)/PERT(J)
               P(J)=P(J)/PERT(J)
   87     CONTINUE
C    EVALUATE SECOND GROUP OF PARAMETERS USING FIRST GROUP AND ADDITIONAL
C    DATA READ IN
               DO 90  K=1,NUMTST
               R(K)=A(K)/P(K)
               D(K)=A(K)/W(K)
               SU(K)=SQRT(32.2*R(K)*S(K))
               F(K)=U(K)/SQRT(32.2*D(K))
               RN(K)=U(K)*R(K)/VIS(K)
               E(K)=U(K)*S(K)*32.2
C    LIST HYDRAULIC PARAMETER VALUES
   90     CONTINUE
               IF (NUMTST.EQ.2) GO TO 100
               IF (NUMTST.EQ.3) GO TO 105
               WRITE(6,95)(DA(1,I), I=1,15)
   95     FORMAT(1H1,31X,15A1)
               WRITE(6,97)(ST(1,I), I=1,15)
   97     FORMAT(4X,'PARAMETER',17X,15A1)
               GO TO 110
  100     WRITE(6,101)(DA(1,I), I=1,15),(DA(2,J), J=1,15)
  101     FORMAT(1H1,31X,15A1,1X,15A1)
               WRITE(6,103)(ST(1,I), I=1,15),(ST(2,J), J=1,15)
  103     FORMAT(4X,'PARAMETER',17X,15A1,1X,15A1)
               GO TO 110
  105     WRITE(6,106)(DA(1,I), I=1,15),(DA(2,J), J=1,15),(DA(3,K), K=1,15)
  106     FORMAT(1H1,31X,15A1,1X,15A1,1X,15A1)
               WRITE(6,108)(ST(1,I), I=1,15),(ST(2,J), J=1,15),(ST(3,K), K=1,15)
  108     FORMAT(4X,'PARAMETER',17X,15A1,1X,15A1,1X,15A1)
  110     WRITE(6,111)
  111     FORMAT(2X,'**********************************************************
          2*********************')
               WRITE(6,114)(T(I), I=1,NUMTST)
  114     FORMAT(1H0,1X,'STREAM  TEMPERATURE',11X,F10.4,6X,F10.4,6X,F10.4)
               WRITE(6,115)
  115     FORMAT(4X,'(T=DEGREES CENTIGRADE)')
               WRITE(6,118)(VIS(J), J=1,NUMTST)
  118     FORMAT(1H0,1X,'KINEMATIC VISCOSITY X 10**5',2X,F10.4,6X,F10.4,6X,F
          210.4)
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      WRITE(6,119)
119   FORMAT(4X,'(VIS=FTXFT/SEC)')
      WRITE(6,122)(S(K), K=1,NUMTST)
122   FORMAT(1H0,1X,'SLOPE',24X,F10.4,6X,F10.4,6X,F10.4)
      WRITE(6,123)
123   FORMAT(4X,'(S=FT/FT)')
      WRITE(6,126)(W(L), L=1,NUMTST)
126   FORMAT(1H0,1X,'SURFACE WIDTH',16X,F10.4,6X,F10.4,6X,F10.4)
      WRITE(6,127)
127   FORMAT(4X,'(W=FT)')
      WRITE(6,128)(P(L), L=1,NUMTST)
128   FORMAT(1H0,1X,'WETTED PERIMETER',13X,F10.4,6X,F10.4,6X,F10.4)
      WRITE(6,129)
129   FORMAT(4X,'(P=FT)')
      WRITE(6,130)(A(M), M=1,NUMTST)
130   FORMAT(1H0,1X,'CROSS-SECTIONAL AREA',9X,F10.4,6X,F10.4,6X,F10.4)
      WRITE(6,131)
131   FORMAT(4X,'(A=FTXFT)')
      WRITE(6,134)(D(N), N=1,NUMTST)
134   FORMAT(1H0,1X,'HYDRAULIC DEPTH',14X,F10.4,6X,F10.4,6X,F10.4)
      WRITE(6,135)
135   FORMAT(4X,'(D=FT)')
      WRITE(6,138)(R(I), I=1,NUMTST)
138   FORMAT(1H0,1X,'HYDRAULIC RADUIS',13X,F10.4,6X,F10.4,6X,F10.4)
      WRITE(6,139)
139   FORMAT(4X,'(R=FT)')
      WRITE(6,142)(DX(J), J=1,NUMTST)
142   FORMAT(1H0,1X,'DISPERSION COEFFICIENT',7X,F10.4,6X,F10.4,6X,F10.4)
      WRITE(6,143)
143   FORMAT(4X,'(DX=FTXFT/SEC)')
      WRITE(6,144)(Q(M), M=1,NUMTST)
144   FORMAT(1H0,1X,'DISCHARGE',20X,F10.4,6X,F10.4,6X,F10.4)
      WRITE(6,145)
145   FORMAT(4X,'(Q=FT**3/SEC)')
      WRITE(6,146)(U(K), K=1,NUMTST)
146   FORMAT(1H0,1X,'VELOCITY',21X,F10.4,6X,F10.4,6X,F10.4)
      WRITE(6,147)
147   FORMAT(4X,'(U=FT/SEC)')
      WRITE(6,150)(SU(L), L=1,NUMTST)
150   FORMAT(1H0,1X,'SHEAR VELOCITY',15X,F10.4,6X,F10.4,6X,F10.4)
      WRITE(6,151)
151   FORMAT(4X,'(U*=FT/SEC)')
      WRITE(6,154)(F(M), M=1,NUMTST)
154   FORMAT(1H0,1X,'FROUDE NUMBER (U/SQRT(32.2H)',1X,F10.4,6X,F10.4,6X,
     2F10.4)
      WRITE(6,155)
155   FORMAT(4X,'(F=DIMENSIONLESS)')
      WRITE(6,158)(RN(N), N=1,NUMTST)
158   FORMAT(1H0,1X,'REYNOLDS NUMBER/10**5',8X,F10.4,6X,F10.4,6X,F10.4)
      WRITE(6,159)
159   FORMAT(4X,'(RN=DIMENSIONLESS)')
      WRITE(6,162)(E(I), I=1,NUMTST)
162   FORMAT(1H0,1X,'ENERGY DISSIPATED/UNIT MASS',2X,F10.4,6X,F10.4,6X,F
     210.4)
      WRITE(6,163)
163   FORMAT(4X,'(E=FTXFT/SEC**3)')
C     CALCULATE REAERATION COEFFICIENT FROM VARIOUS PREDICTION MODELS
      DO 310 K=1,NUMTST
      ADJ=1.0241**(T(K)-20)
      EQUA(1,K)=(5.026*U(K)**0.969/D(K)**1.673)*ADJ
      EQUA(2,K)=(24.66*E(K)**0.408/D(K)**0.66)
      EQUA(3,K)=(3.659*DX(K)**1.321/D(K)**2.32)
      EQUA(4,K)=(10.90*U(K)**0.73/D(K)**1.75)*ADJ
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      EQUA(5,K)=(9.41*U(K)**0.67/D(K)**1.85)*ADJ
      EQUA(6,K)=(3.3*U(K)/D(K)**1.33)
      EQUA(7,K)=(3.739*U(K)/D(K)**1.50)*ADJ
      EQUA(8,K)=(2.44*U(K)/D(K)**1.50)*ADJ
      EQUA(9,K)=(4.74*(U(K)/D(K))**0.85)*ADJ
      EQUA(10,K)=(14.21*DX(K)*(U(K)/D(K))**1.63)*ADJ
      EQUA(11,K)=(1.296*DX(K)/D(K)**2)*ADJ
      EQUA(12,K)=(18.58*SU(K)/D(K))*ADJ
      EQUA(13,K)=(10.80*(1.0+F(K)**0.5)*SU(K)/D(K))*ADJ
      EQUA(14,K)=(8.76*U(K)**0.607/D(K)**1.689)*ADJ
      EQUA(15,K)=(46.05*U(K)**0.413*S(K)**0.273/D(K)**1.408)*ADJ
      EQUA(16,K)=(1089.0*SU(K)**3/(U(K)**2*D(K)))*ADJ
      EQUA(17,K)=(48.0*(1.0+0.17*F(K)**2)*(S(K)*U(K))**0.375/D(K))
      EQUA(18,K)=((K2(K)*60.0*60.0*24.0)/(2.303*10.0**5))*ADJ
      CA=1.0+F(K)**2
      C4=0.9+F(K)
      FD=9.68
      B=0.976+0.0137*(30-20)**1.5
      ED=30.0*S(K)*1000.0*U(K)
      X=(B*ED**0.125)/(D(K)*C4**1.5)
      EQUA(19,K)=0.12*CA*FD*(ED**0.375)*(COSH(X)/SINH(X))/(D(K)*C4**1.5)
      EQUA(20,K)=(25.7*E(K)**0.5/D(K))*ADJ
310   CONTINUE
C   LIST VARIOUS PREDICTION MODELS AND VALUES OBTAINED USING THEM
      WRITE(6,320)
320   FORMAT(1H1,30X,'REAERATION COEFFICIENTS - K2 (/DAY - BASE 10)')
      WRITE(6,322)
322   FORMAT(41X,'ADJUSTED TO 20 DEG. CENT.')
      WRITE(6,324)
324   FORMAT(30X,'*********************************************')
      IF (NUMTST.EQ.2) GO TO 330
      IF (NUMTST.EQ.3) GO TO 335
      WRITE(6,326)(DA(1,I), I=1,15)
326   FORMAT(4X,'REFERENCE AND ',15X,15A1)
      WRITE(6,327)(ST(1,I), I=1,15)
327   FORMAT(6X,'EQUATION',16X,15A1)
      GO TO 340
330   WRITE(6,331)(DA(1,I), I=1,15),(DA(2,J), J=1,15)
331   FORMAT(4X,'REFERENCE AND ',15X,15A1,1X,15A1)
      WRITE(6,332)(ST(1,I), I=1,15),(ST(2,J), J=1,15)
332   FORMAT(6X,'EQUATION',16X,15A1,1X,15A1)
      GO TO 340
335   WRITE(6,336)(DA(1,I), I=1,15),(DA(2,J), J=1,15),(DA(3,K), K=1,15)
336   FORMAT(4X,'REFERENCE AND ',15X,15A1,1X,15A1,1X,15A1)
      WRITE(6,337)(ST(1,I), I=1,15),(ST(2,J), J=1,15),(ST(3,K), K=1,15)
337   FORMAT(6X,'EQUATION',16X,15A1,1X,15A1,1X,15A1)
340   WRITE(6,341)
341   FORMAT(1X,'*******************',6X,'*************',4X,'******
     2*******',4X,'*************')
      WRITE(6,345)
345   FORMAT(1X,'CHURCHILL AND OTHERS (1962)')
      WRITE(6,346)(EQUA(1,I), I=1,NUMTST)
346   FORMAT(3X,'5.026(U**0.969)/(H**1.673)',1X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,347)
347   FORMAT(1H0,'DOBBINS (1963, 1964, 1965)')
      WRITE(6,348)(EQUA(19,J), J=1,NUMTST)
348   FORMAT(2X,'SEE LIT. REVIEW FOR EQUATION',F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,349)
349   FORMAT(1H0,'KRENKEL AND ORLOB (1963)')
      WRITE(6,350)(EQUA(2,J), J=1,NUMTST)
350   FORMAT(3X,'24.66(E**0.408)/(H**0.66) ',2X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,351)(EQUA(3,K), K=1,NUMTST)
351   FORMAT(3X,'3.659(DX**1.321)/(H**2.32)',1X,F10.2,6X,F10.2,6X,F10.2)
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      WRITE(6,354)
  354 FORMAT(1H0,'OWENS AND OTHERS (1964)')
      WRITE(6,355)(EQUA(4,L), L=1,NUMTST)
  355 FORMAT(3X,'10.90(U**0.73)/(H**1.75)',3X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,356)(EQUA(5,M), M=1,NUMTST)
  356 FORMAT(3X,'9.41(U**0.67)/(H**1.85)',4X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,359)
  359 FORMAT(1H0,'LANGBEIN AND DURUM (1967)')
      WRITE(6,360)(EQUA(6,N), N=1,NUMTST)
  360 FORMAT(3X,'3.3U/(H**1.33)',13X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,363)
  363 FORMAT(1H0,'ISAACS AND GAUDY (1968)')
      WRITE(6,364)(EQUA(7,I), I=1,NUMTST)
  364 FORMAT(3X,'3.739U/(H**1.5)',12X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,365)(EQUA(8,J), J=1,NUMTST)
  365 FORMAT(3X,'2.440U/(H**1.5)',12X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,366)
  366 FORMAT(1H0,'CADWALLADER AND MCDONNELL (1969)')
      WRITE(6,367)(EQUA(20,K), K=1,NUMTST)
  367 FORMAT(3X,'25.7(E**0.5)/H',13X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,368)
  368 FORMAT(1H0,'NEGULESCU AND ROJANSKI (1969)')
      WRITE(6,369)(EQUA(9,K), K=1,NUMTST)
  369 FORMAT(3X,'4.74(U/H)**0.85',12X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,370)(EQUA(10,L), L=1,NUMTST)
  370 FORMAT(3X,'1.423DX(U/H)**1.63',9X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,373)
  373 FORMAT(1H0,'THACKSTON AND KRENKEL (1969)')
      WRITE(6,374)(EQUA(11,M), M=1,NUMTST)
  374 FORMAT(3X,'1.296DX/(H**2)',13X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,375)(EQUA(12,N), N=1,NUMTST)
  375 FORMAT(3X,'18.58U*/H',18X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,376)(EQUA(13,I), I=1,NUMTST)
  376 FORMAT(3X,'10.8(1+F**0.5)(U*/H)',7X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,379)
  379 FORMAT(1H0,'BENNETT AND RATHBUN (1972)')
      WRITE(6,380)(EQUA(14,J), J=1,NUMTST)
  380 FORMAT(3X,'8.76(U**0.607)/(H**1.689)',2X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,381)(EQUA(15,K), K=1,NUMTST)
  381 FORMAT(1X,'46.05(U**.413)(S**.273)/H**1.40 8',F7.2,6X,F10.2,6X,F10.
     22)
      WRITE(6,384)
  384 FORMAT(1H0,'LAU (1972)')
      WRITE(6,385)(EQUA(16,L), L=1,NUMTST)
  385 FORMAT(3X,'1089(U*)**3/((U**2)(H))',4X,F10.2,6X,F10.2,6X,F10.2)
      WRITE(6,388)
  388 FORMAT(1H0,'PARKHURST AND POMEROY (1972)')
      WRITE(6,389)(EQUA(17,M), M=1,NUMTST)
  389 FORMAT(3X,'48(1+0.17F**2)((SU)**.375)/H',F9.2,6X,F10.2,6X,F10.2)
      WRITE(6,392)
  392 FORMAT(1H0,'METHOD FORMULATED IN')
      WRITE(6,393)(EQUA(18,N), N=1,NUMTST)
  393 FORMAT(3X,'THIS RESEARCH PROJECT',6X,F10.2,6X,F10.2,6X,F10.2)
  400 CONTINUE
      STOP
      END
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      SUBROUTINE CROSS(INDEX3,INDEX)
      REAL LEFT
      DIMENSION TT(10), TWP(10), TA(10)
      COMMON BASE(10),STEP(10,10),LEFT(10,10),RIGHT(10,10),AREA(3,10),Q(
     23),VEL(3,10),H(3,10),TOP(3,10),WP(3,10)
      IF (INDEX.EQ.8) GO TO 208
      AREA(INDEX3,INDEX)=Q(INDEX3)/VEL(INDEX3,INDEX)
C     CALCULATE WIDTH, WETTED PERIMETER, AND AREA AT EACH LEVEL CREATED
C     BY A HORIZONTAL PLANE MOVING UP FROM THE STREAM FLOOR AND STOPPING
C     AT EACH CHANGE OF CHANNEL GEOMETRY
      TT(1)=BASE(INDEX)+STEP(INDEX,1)/LEFT(INDEX,1)+STEP(INDEX,1)/RIGHT(
     2INDEX,1)
      TWP(1)=BASE(INDEX)+SQRT((STEP(INDEX,1)/LEFT(INDEX,1))**2+STEP(INDE
     2X,1)**2)+SQRT((STEP(INDEX,1)/RIGHT(INDEX,1))**2+STEP(INDEX,1)**2)
      TA(1)=0.5*STEP(INDEX,1)*(BASE(INDEX)+TT(1))
      DO 200 N=2,5
      TT(N)=TT(N-1)+STEP(INDEX,N)/LEFT(INDEX,N)+STEP(INDEX,N)/RIGHT(INDE
     2X,N)
      TWP(N)=TWP(N-1)+SQRT((STEP(INDEX,N)/LEFT(INDEX,N))**2+STEP(INDEX,N
     2)**2)+SQRT((STEP(INDEX,N)/RIGHT(INDEX,N))**2+STEP(INDEX,N)**2)
      TA(N)=TA(N-1)+0.5*STEP(INDEX,N)*(TT(N-1)+TT(N))
200   CONTINUE
C     PLACE ACTUAL AREA AT EACH CROSS-SECTION IN BETWEEN THE LEVELS ABOVE
C     AND BELOW ITS VALUE
      IF (AREA(INDEX3,INDEX).LE.TA(1)) GO TO 210
      IF (AREA(INDEX3,INDEX).LE.TA(2)) GO TO 212
      IF (AREA(INDEX3,INDEX).LE.TA(3)) GO TO 214
      IF (AREA(INDEX3,INDEX).LE.TA(4)) GO TO 216
      IF (AREA(INDEX3,INDEX).LE.TA(5)) GO TO 218
      IF (AREA(INDEX3,INDEX).GT.TA(5)) GO TO 220
C     CALCULATE AREA, DEPTH, WIDTH AND WETTED PERIMETER IF TEST IS FROM
C     LABORATORY FLUME WITH UNIFORM CHANNEL GEOMETRY
208   AREA(INDEX3,INDEX)=Q(INDEX3)/VEL(INDEX3,INDEX)
      H(INDEX3,INDEX)=AREA(INDEX3,INDEX)/8.0
      TOP(INDEX3,INDEX)=8.0
      WP(INDEX3,INDEX)=2.0*H(INDEX3,INDEX)+8.0
      RETURN
C     CALCULATE MAXIMUM DEPTH, WIDTH AND WETTED PERIMETER IF ACTUAL AREA
C     LIES LOWER THAN THE FIRST LEVEL AREA OF CROSS-SECTION INDEX
210   AA=1.0/LEFT(INDEX,1)+1.0/RIGHT(INDEX,1)
      B=2.0*BASE(INDEX)
      C=-2.0*AREA(INDEX3,INDEX)
      H(INDEX3,INDEX)=(-B+SQRT(B**2-4.0*AA*C))/(2.0*AA)
      TOP(INDEX3,INDEX)=BASE(INDEX)+H(INDEX3,INDEX)*AA
      WP(INDEX3,INDEX)=BASE(INDEX)+SQRT((H(INDEX3,INDEX)/LEFT(INDEX,1))*
     2*2+H(INDEX3,INDEX)**2)+SQRT((H(INDEX3,INDEX)/RIGHT(INDEX,1))**2+H(
     3INDEX3,INDEX)**2)
      RETURN
C     IF ACTUAL AREA IS ABOVE FIRST LEVEL AND BELOW SECOND GO TO 212
212   NO=2
      SUM=STEP(INDEX,1)
      GO TO 222
C     IF ACTUAL AREA IS ABOVE SECOND LEVEL AND BELOW THIRD GO TO 214
214   NO=3
      SUM=STEP(INDEX,1)+STEP(INDEX,2)
      GO TO 222
C     IF ACTUAL AREA IS ABOVE THIRD LEVEL AND BELOW FOURTH GO TO 216
216   NO=4
      SUM=STEP(INDEX,1)+STEP(INDEX,2)+STEP(INDEX,3)
      GO TO 222
C     IF ACTUAL AREA IS ABOVE FOURTH LEVEL AND BELOW FIFTH GO TO 218
218   NO=5
      SUM=STEP(INDEX,1)+STEP(INDEX,2)+STEP(INDEX,3)+STEP(INDEX,4)
```

```
      GO TO 222
C   IF ACTUAL AREA IS ABOVE FIFTH LEVEL GO TO 220
220   NO=6
      SUM=STEP(INDEX,1)+STEP(INDEX,2)+STEP(INDEX,3)+STEP(INDEX,4)+STEP(I
     2NDEX,5)
222   AA=1.0/LEFT(INDEX,NO)+1.0/RIGHT(INDEX,NO)
      B=2.0*TT(NO-1)
      C=-2.0*(AREA(INDEX3,INDEX)-TA(NO-1))
      SH=(-B+SQRT(B**2-4.0*AA*C))/(2.0*AA)
      H(INDEX3,INDEX)=SH+SUM
      TOP(INDEX3,INDEX)=TT(NO-1)+SH*AA
      WP(INDEX3,INDEX)=TWP(NO-1)+SQRT((SH/LEFT(INDEX,NO))**2+SH**2)+SQRT
     2((SH/RIGHT(INDEX,NO))**2+SH**2)
      RETURN
      END
```

PROGRAM III: MODEL FORMULATION AND EVALUATION

The major function of this program is to estimate the a and β values in Equation 54 by the least-squares regression analysis of the logarithm of Equation 54 using measured k_2 values and associated hydraulic parameters. The measured k_2 values read in are in terms of per day and are already adjusted to 20°C using the adjustment factor.

$$(k_2)_{20°} = (k_2)_{T°} (1.0241)^{(T-20°)}$$

Also included in the program is the evaluation of the standard error of estimate in terms of per day and in percent, by using Equations 56, 57 and 58. These standard errors are calculated using each of the prediction equations cited in the literature review and Equation 54. To run four sets of data consisting of 29, 9, 8, and 52 k_2 values, respectively, required about 1.8 seconds of execution time with 12 pages of printout including a program listing.

Program III data deck

	Variable	Format	Example	Comments
Card 1	NUMSET	I5	----1	The number of data sets included in the data deck. One model is formulated from each data set. (The remaining cards in this example deck consist of one data set).
Card 2	HEAD1(N) N=1,70	70A1	AUTHORS DATA (1975)	Seventy spaces for heading describing data set.
Card 3	HEAD2(M) M=1,70	70A1	NATURAL STREAM	Seventy additional spaces for heading.
Card 4	NUM, RNUM, IQUE, NOSLOP	I5, F5.1, 2I5	---26-26.0, -, -,	NUM is the number of data points in the set.RNUM in the real value of integer NUM. IQUE is an index number. If it has any value the points in the set have weighted values. If it does not have a value the weight of each point is one. NOSLOP is an index number. If it has any value greater than zero the data points have no slope value given.
Cards 5 to 30 (26 cards)	MK2(J), DX(J), U(J), H(J), S(J), T(J), WEIGHT(J), J=1, NUM	4F10.5, F 10.7, 2F 10.5	281.5, 36.5, 2.345, 1.04, 0.0173, 9.0, --	Measured reaeration coef., dispersion coef., velocity, depth, slope, temperature and weight of value are read in for each data point in set. If WEIGHT and IQUE values are left blank, weight of each point is assumed to be one.

Variable	Definition

ADJ
Temperature adjustment factor added to prediction equations. It has the form of $(1.0241)^{(T-20)}$.

ALPHA
The Y-intercept in the linearized form $(Y = \beta X + \log a)$ of the logarithm of the k_2 prediction model with k_2 being in terms of per second.

ALPHA 2
The Y-intercept in the linearized form $(Y = \beta X + \log a)$ of the logarithm of the k_2 prediction model with k_2 being in terms of per day.

BETA
The slope in the linearized form $(Y = \beta X + \log a)$ of the logarithm of the k_2 prediction model (same for per day or per second).

CK2 (J, K)
Calculated reaeration coefficient, k_2 (day^{-1}), for data point K using equation J (not the same number as described in review of literature).

DIFK (J)
The sum of the squares of the differences between the measured reaeration coefficient and the corresponding reaeration coefficient calculated from equation J.

DIFLK (J)
The sum of the squares of the differences between the log of the measured reaeration coefficient and the log of the corresponding reaeration coefficient calculated from equation J.

DX (N)
The dispersion coefficient (ft^2/sec) of data point N.

E (N)
The energy dissipation per unit mass (ft^2/sec^3) calculated from data of point N.

ESUBP (J)
The standard error of estimate in terms of percent for the reaeration coefficients predicted using equation J.

ESUBS (J)
The standard error of estimate in terms of days^{-1} for the reaeration coefficients predicted using equation J.

ESUBSL (J)
The standard error of estimate using reaeration coefficients predicted using equation J which is converted to ESUBP (M) by an equation.

FN (N)
The Froude Number as calculated from data of point N.

H (N)
The depth (ft) measurement for data point N.

HEAD 1 (70)
Dimensional heading with 70 spaces to describe the data set.

HEAD 2 (70)
Dimensional heading allowing 70 additional spaces and a second line to describe the data set.

IQUE
Integer coding with any value above zero indicating that the data points have weighted values other than one.

MK 2 (K)
The measured reaeration coefficient, k_2, in terms of days^{-1} and adjusted to 20° centigrade, of data point K.

NOSLOP
Integer coding with any value above zero indicating that the data points have no slope values given.

NUM
The number of data points contained in the data set.

NUMSET
The number of data sets in the data deck.

PARAX (K)
The value of D_x/HU using data of point K.

PARAY (K)
The value of k_2H/U using data of point K.

PARAY 2 (J)
The PARAY value at point J multiplied by 10^5 for printout purposes.

RNUM
Real value of the integer NUM.

S (J)
The slope (ft/ft) of data point J.

SU (N)
The shear velocity (ft/sec) calculated from data of point N.

SUMX
Sum of the X(N) values as N goes from 1 to NUM.

Variables	Definition	Variables	Definition

SUMXX

Sum of the $X(N)^2$ values as N goes from 1 to NUM.

SUMXY

Sum of the X(N) times Y(N) values as N goes from 1 to NUM.

SUMY

Sum of the Y(N) values as N goes from 1 to NUM.

T (J)

The temperature (degrees centigrade) of data point J.

U (J)

The velocity (ft/sec) of data point J.

WEIGHT (J)

The weighted value of data point J (i.e. data point N is actually a composite of 8 tests). If IQUE has no value the WEIGHT (N) is assumed to be one, a value does not have to be read in.

WNUM

The sum of the the WEIGHT (N) values as N goes from 1 to NUM.

X (K)

The value of log (D_X/HU) using data of point K.

Y (K)

The value of log (k_2H/U) using data of point K.

```
C     PROGRAM III - MODEL FORMULATION AND EVALUATION
C     FORTRAN
      REAL MK2
      DIMENSION HEAD1(70), HEAD2(70), MK2(100), DX(100), U(100), H(100),
     2S(100), T(100), PARAY(100), PARAX(100), Y(100), X(100), FN(100),
     3E(100), CK2(30,100), DIFK(30), DIFLK(30), FSUBS(30), ESUBSL(30),
     4ESUBP(30), WEIGHT(100), SU(100), PARAY2(100)
C     READ IN THE NUMBER OF DATA SETS TO BE SEPERATELY EVALUATED
      READ(5,90)NUMSET          .
 90   FORMAT(I5)
      DO 400 IJ=1,NUMSET
C     TWO CARDS, 70 SPACES EACH FOR HEADINGS
      READ(5,105)(HEAD1(N), N=1,70)
      READ(5,105)(HEAD2(M), M=1,70)
105   FORMAT(70A1)
C     READ IN NUMBER OF POINTS IN DATA SET (INTERGER AND REAL). A VALUE FOR
C     IQUE INDICATES SOME POINTS ARE WEIGHTED.  A VALUE FOR NOSLOP INDICATES
C     THE DATA SET HAS NO SLOPE VALUES
      READ(5,100)NUM,RNUM,IQUE,NOSLOP
100   FORMAT(I5,F5.1,2I5)
C     READ IN MEASURED REAERATION COEF., DISPERSION COEF., VELOCITY, DEPTH,
C     SLOPE, TEMPERATURE AND WEIGHTED VALUE OF POINT (IF IQUE=0 THEN WEIGHT
C     ASSUMED TO BE ONE, NOT NECESSARY TO READ IN)
      READ(5,110)(MK2(J),DX(J),U(J),H(J),S(J),T(J),WEIGHT(J), J=1,NUM)
110   FORMAT(4F10.5,F10.7,2F10.5)
      SUMX=0.0
      SUMY=0.0
      SUMXY=0.0
      SUMXX=0.0
C     EVALUATE KH/U AND DX/HU PARAMETERS AND FIND COMBINATION SUM VALUES
      DO 150 K=1,NUM
      PARAY(K)=(MK2(K)/86400.0)*H(K)/U(K)
      PARAX(K)=DX(K)/(U(K)*H(K))
      Y(K)=ALOG10(PARAY(K))
      X(K)=ALOG10(PARAX(K))
      SUMX=SUMX+X(K)
      SUMY=SUMY+Y(K)
      SUMXY=SUMXY+X(K)*Y(K)
      SUMXX=SUMXX+X(K)**2
150   CONTINUE
C     EVALUATE ALPHA AND BETA FROM LOG(KH/U)=ALPHA+BETA*(LOG(DX/HU))
      BETA=(RNUM*SUMXY-SUMX*SUMY)/(RNUM*SUMXX-SUMX**2)
      ALPHA=10.0**((SUMY/RNUM)-BETA*SUMX/RNUM)
      DO 200 N=1,NUM
C     CALCULATE ADDITIONAL REQUIRED PARAMETERS - SHEAR VELOCITY, FROUDE NUMB R
C     AND ENERGY
      SU(N)=SQRT(32.2*H(N)*S(N))
      FN(N)=U(N)/SQRT(32.2*H(N))
      E(N)=U(N)*S(N)*32.2
C     CALCULATE TEMPERATURE ADJUSTMENT FACTOR
      ADJ=1.0241**(T(N)-20.0)
C     CALCULATE THE PREDICTED REAERATION COEF. FOR EACH POINT USING THOSE
C     EQUATIONS DISCUSSED IN LITERATURE REVIEW SECTION
      CK2(1,N)=(5.026*(U(N)**0.969)/(H(N)**1.673))*ADJ
      CK2(2,N)=(3.659*(DX(N)**1.321)/(H(N)**2.32))
      CK2(3,N)=(24.66*(E(N)**0.408)/(H(N)**0.66))
      IF (NOSLOP.GE.1) CK2(4,N)=0.0
      IF (NOSLOP.GE.1) GO TO 160
      CA=1.0+FN(N)**2
      C4=0.9+FN(N)
      F=9.68+0.054*(T(N)-20.0)
```

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          B=0.976+0.0137*(30.0-T(N))**1.5
          ED=30.0*S(N)*U(N)
          PART=(B*ED**0.125)/(H(N)*C4**1.5)
          CK2(4,N)=0.12*CA*F*(ED**0.375)*(COSH(PART)/SINH(PART))/(H(N)*C4**1
         2.5)
160       CK2(5,N)=(10.90*(U(N)**0.73)/(H(N)**1.75))*ADJ
          CK2(6,N)=(9.41*(U(N)**0.67)/(H(N)**1.85))*ADJ
          CK2(7,N)=(3.3*U(N)/(H(N)**1.33))
          CK2(8,N)=(3.053*U(N)/(H(N)**1.5))*ADJ
          CK2(9,N)=(3.739*U(N)/(H(N)**1.5))*ADJ
          CK2(10,N)=(2.44*U(N)/(H(N)**1.5))*ADJ
          CK2(11,N)=(25.7*(E(N)**0.5)/(H(N)**1.0))*ADJ
          CK2(12,N)=4.74*((U(N)/H(N))**0.85)*ADJ
          CK2(13,N)=14.21*DX(N)*((U(N)/H(N))**1.63)*ADJ
          CK2(14,N)=(18.58*SU(N)/H(N))*ADJ
          CK2(15,N)=1.296*(DX(N)/H(N)**2)*ADJ
          CK2(16,N)=10.8*(1.0+FN(N)**0.5)*(SU(N)/H(N))*ADJ
          CK2(17,N)=(46.05*(U(N)**0.413)*(S(N)**0.273)/(H(N)**1.408))*ADJ
          CK2(18,N)=(8.76*(U(N)**0.607)/(H(N)**1.689))*ADJ
          CK2(19,N)=(1089.0*(SU(N)**3.0)/((U(N)**2.0)*(H(N)**1.0)))*ADJ
          CK2(20,N)=(48.0*(1.0+0.17*FN(N)**2.0)*((S(N)*U(N))**0.375)/H(N))
C     PREDICT REAERATION COEF. USING EQUATION DEVELOPED FROM FINDING ALPHA
C     AND BETA
          CK2(21,N)=86400.0*ALPHA*(DX(N)**BETA)*(U(N)**(1.0-BETA))/H(N)**(1.
         20+BETA)
200       CONTINUE
          DO 220 J=1,21
          DIFK(J)=0.0
          DIFLK(J)=0.0
          WNUM=0.0
C     EVALUATE THE SUM OF THE SQUARE OF THE DIFFERENCES AND THE SUM OF THE
C     SQUARE OF THE LOG DIFFERENCES, WITHOUT OR WITH WEIGHTED POINTS
          IF (IQUE.GT.1) GO TO 211
          DO 210 K=1,NUM
          IF (CK2(J,K).LE.0.0) GO TO 210
          DIFK(J)=DIFK(J)+(CK2(J,K)-MK2(K))**2
          DIFLK(J)=DIFLK(J)+(ALOG10(CK2(J,K))-ALOG10(MK2(K)))**2
210       CONTINUE
          GO TO 213
211       DO 212 K=1,NUM
          IF (CK2(J,K).LE.0.0) GO TO 311
          DIFK(J)=DIFK(J)+WEIGHT(K)*(CK2(J,K)-MK2(K))**2
          DIFLK(J)=DIFLK(J)+WEIGHT(K)*(ALOG10(CK2(J,K))-ALOG10(MK2(K)))**2
311       WNUM=WNUM+WEIGHT(K)
212       CONTINUE
          IF (IQUE.GT.1) RNUM=WNUM
C     CALCULATE THE STANDARD ERROR OF ESTIMATE PER DAY AND PER CENT
213       ESUBS(J)=SQRT(DIFK(J)/RNUM)
          ESUBSL(J)=SQRT(DIFLK(J)/RNUM)
          ESUBP(J)=100.0*(1.0-1.0/10.0**ESUBSL(J))
270       CONTINUE
240       CONTINUE
C     LIST RESULTS
          DO 285 J=1,NUM
          PARAY2(J)=PARAY(J)*10**5
285       CONTINUE
          WRITE(6,290)
290       FORMAT(1H1,10X,'NUMBER',5X,'KH/U X 10**5 ',5X,'DX/HU')
          WRITE(6,295)(I,PARAY2(I),PARAX(I), I=1,NUM)
295       FORMAT(10X,I5,10X,F6.2,7X,F6.2)
          WRITE(6,300)(HEAD1(I), I=1,70)
300       FORMAT(1H1,9X,70A1)
          WRITE(6,302)(HEAD2(J), J=1,70)
```

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302       FORMAT(10X,70A1)
          WRITE(6,304)
304       FORMAT(8X,'*************** ** ** ** ** ** ** ************ ** ************** **********
     2***************')
          WRITE(6,306)
306       FORMAT(10X,'EQUATION FROM REVIEW',5X,'STANDARD ERROR OF',6X,'STAND
     2ARD ERROR OF')
          WRITE(6,308)
308       FORMAT(13X,'OF LITERATURE',9X,'ESTIMATE (PER DAY)',5X,'ESTIMATE (P
     2ERCENT)')
          WRITE(6,310)
310       FORMAT(10X,'*************** ** ** ** ** ** ** ',5X,'******************** ',5X,'*****
     2***************')
          DO 330 M=1,20
          IF (M.LE.18) KEQ=M+10
          IF (M.GT.18) KEQ=M+11
          WRITE(6,320)KEQ,ESUBS(M),ESUBP(M)
320       FORMAT(1H0,13X,'EQUATION',1X,I2,14X,F7.2,18X,F7.2)
330       CONTINUE
          WRITE(6,335)ESUBS(21),ESUBP(21)
335       FORMAT(1H0,13X,'EQUATION  X',14X,F7.2,16X,F7.2)
          WRITE(6,340)
340       FORMAT(1H0,19X,'EQUATION X CAN BE WRITTEN  (KH/U)=ALPHA(DX/HU)**BE
     2TA')
          WRITE(6,342)
342       FORMAT(47X,'K AS PER SECOND')
          WRITE(6,345)ALPHA
345       FORMAT(47X,'WHERE ALPHA =',E10.4)
          WRITE(6,350)BETA
350       FORMAT(49X,'AND   BETA =',E10.4)
C     CONVERT ALPHA BACK TO TERMS OF K2 AS PER DAY
          ALPHA2=86400.0*ALPHA
          WRITE(6,352)
352       FORMAT(1H0,46X,'OR WITH K AS PER DAY')
          WRITE(6,354)ALPHA2
354       FORMAT(53X,'ALPHA =',E10.4)
400       CONTINUE
          STOP
          END
```

Appendix B

Experimental Data

This appendix contains the experimental data used to formulate reaeration coefficient prediction models and to evaluate existing models. The data include those obtained from the investigation of a natural stream (Table 2), a laboratory flume (Table 3), and previous investigations (Tables 4 and 5) which have lent themselves to the model formulation in this study. The data from previous investigations, which had dispersion coefficient measurements, are only those of Negulescu and Rojanski (1969) and Thackston and Krenkel (1969).

Table 2. Summary of field data from tests on Summit Creek (k_2 values adjusted to 20° C.).

Test Date	Stations	k_2 (day^{-1})	D_x (ft^2/sec)	U (ft/sec)	H (ft)	S (ft/ft)	T (°C)	W (ft)	Q (ft^3/sec)
9/23/74	0+75 to 0+00	281.5	36.50	2.35	1.040	0.0173	9.0	12.31	30.0
9/26/74	0+75 to 0+00	118.0	22.84	1.79	0.886	0.0173	11.5	11.10	17.6
10/ 1/74	1+50 to 0+00	18.5	10.72	1.62	0.787	0.0173	8.5	11.20	14.3
10/17/74	1+10 to 0+50	7.48	2.87	1.37	0.691	0.017	10.5	11.51	10.9
ditto	0+50 to 0+00	52.4	12.19	1.30	0.824	0.018	10.5	10.22	10.9
ditto	1+10 to 0+00	32.2	5.99	1.24	0.777	0.0175	10.5	11.32	10.9
10/23/74	0+50 to 0+00	168.0	10.73	1.92	0.825	0.018	9.5	10.23	16.2
10/25/74	1+10 to 0+00	70.7	10.89	2.18	0.773	0.0175	9.0	11.30	19.0
10/30/74	0+50 to 0+00	73.6	16.44	1.93	0.868	0.018	7.0	10.45	17.5
11/ 1/74	1+15 to 0+50	20.7	12.24	2.82	0.586	0.017	7.0	10.59	17.5
ditto	0+50 to 0+00	50.9	12.99	1.90	0.879	0.018	7.0	10.51	17.5
ditto	1+15 to 0+00	36.5	15.28	2.11	0.745	0.0175	7.0	11.16	17.5
11/ 4/74	1+50 to 0+00	83.0	12.22	1.92	0.807	0.0173	6.5	11.30	17.5
11/ 5/74	0+75 to 0+00	37.2	15.49	1.88	0.803	0.0173	7.0	10.60	16.0
11/ 8/74	1+50 to 0+75	39.2	7.09	2.01	0.738	0.0173	5.5	11.80	17.5
ditto	0+75 to 0+00	134.1	16.85	2.05	0.806	0.0173	5.5	10.62	17.5
ditto	1+50 to 0+00	85.7	13.27	1.87	0.824	0.0173	5.5	11.39	17.5

Table 2. Continued.

Test Date	Stations	k_2 (day^{-1})	D_x (ft^2/sec)	U (ft/sec)	H (ft)	S (ft/ft)	T ($^\circ$C)	W (ft)	Q (ft^3/sec)
11/20/74	1+10 to 0+50	147.0	10.50	2.52	0.596	0.017	5.0	10.67	16.0
ditto	0+50 to 0+00	58.4	13.22	1.95	0.810	0.018	5.0	10.15	16.0
ditto	1+10 to 0+00	98.4	14.58	2.01	0.721	0.0175	5.0	11.04	16.0
12/ 6/74	1+10 to 0+50	136.2	8.90	2.05	0.573	0.017	4.5	10.47	12.3
ditto	0+50 to 0+00	9.7	10.46	1.52	0.801	0.018	4.5	10.11	12.3
ditto	1+10 to 0+00	66.1	10.88	1.59	0.708	0.0175	4.5	10.96	12.3
4/24/75	1+10 to 0+50	108.5	11.56	3.05	0.708	0.017	8.0	11.58	25.0
ditto	0+50 to 0+00	108.5	128.12	4.19	0.641	0.018	8.0	9.32	25.0
ditto	1+10 to 0+00	70.5	37.62	2.83	0.781	0.0175	8.0	11.34	25.0
4/30/75	1+10 to 0+50	98.7	14.03	2.34	1.043	0.017	8.0	13.37	33.8
ditto	0+50 to 0+00	70.5	34.35	2.18	1.198	0.018	8.0	12.98	33.8
ditto	1+10 to 0+00	80.3	25.44	1.96	1.222	0.0175	8.0	14.11	33.8

Table 3. Summary of data from tests on UWRL flume. (k_2 values adjusted to 20° C.).

Test Date	Stations	k_2 (day^{-1})	D_x (ft^2/sec)	U (ft/sec)	H (ft)	S (ft/ft)	T (^{o}C)	W (ft)	Q (ft^3/sec)
3/11/75	0+90 to 0+00	95.4	1.48	1.86	0.911	0.00046	6.0	8.0	13.52
4/10/75	0+90 to 0+00	140.4	12.93	1.83	0.934	0.00046	7.0	8.0	13.68
4/16/75	1+95 to 0+90	36.5	1.03	1.73	0.773	0.00046	7.0	8.0	10.70
ditto	0+90 to 0+00	0.3	1.67	1.59	0.841	0.00046	7.0	8.0	10.70
ditto	1+95 to 0+00	18.6	1.64	1.64	0.815	0.00046	7.0	8.0	10.70
4/22/75	1+95 to 0+90	62.8	0.74	1.29	0.525	0.00059	8.5	8.0	5.42
4/29/75	1+95 to 0+90	3.4	0.46	1.59	0.733	0.00059	6.0	8.0	9.29
ditto	0+90 to 0+00	41.0	2.43	1.68	0.691	0.00069	6.0	8.0	9.29
ditto	1+95 to 0+00	21.5	1.25	1.62	0.717	0.00064	6.0	8.0	9.29

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Table 4. Summary of laboratory data from Negulescu and Rojanski (1969).

Number of experiments	k_2 (day^{-1})	D_x (ft^2/sec)	U (ft/sec)	H (ft)	T* ($^{\circ}$C)
5	15.90	0.121	1.90	0.49	20.0
8	11.50	0.118	1.64	0.49	20.0
12	17.25	0.071	0.95	0.16	20.0
16	18.70	0.045	0.85	0.16	20.0
27	8.64	0.116	1.05	0.49	20.0
34	11.50	0.373	0.66	0.49	20.0
40	8.64	0.247	0.89	0.49	20.0
41	14.45	0.180	1.08	0.36	20.0

* Temperatures were assumed to be 20° C

Table 5. Summary of laboratory data from Thackston and Krenkel (1969).

Run	k_2 (day^{-1})	D_x (ft^2/sec)	U (ft/sec)	H (ft)	S (ft/ft)	T ($^\circ$C)
A-2	19.70	0.300	1.323	0.125	0.00596	22.1
A-3	22.12	0.273	1.449	0.109	0.00894	22.7
A-4	27.22	0.335	1.878	0.124	0.01161	22.8
A-5	37.32	0.276	1.707	0.092	0.01446	22.8
A-7	43.46	0.312	1.858	0.098	0.01623	23.0
A-8	47.78	0.396	2.320	0.122	0.01785	22.5
A-9	55.12	0.340	2.029	0.117	0.01418	22.0
A-10	24.97	0.388	2.079	0.170	0.00843	21.0
A-11	20.39	0.218	1.225	0.112	0.00557	21.2
A-12	19.44	0.186	0.994	0.104	0.00430	21.8
A-13	22.64	0.182	0.921	0.107	0.00380	22.3
A-14	16.50	0.148	0.802	0.094	0.00236	23.0
A-15	17.88	0.175	0.940	0.107	0.00423	23.0
A-16	11.06	0.154	0.773	0.113	0.00245	24.3
A-17	14.52	0.141	0.776	0.099	0.00282	24.0
A-18	28.77	0.110	0.754	0.065	0.00571	23.9
A-19	43.47	0.161	1.116	0.065	0.01124	23.8
A-20	44.41	0.137	0.961	0.063	0.00894	23.8
A-21	47.95	0.164	1.096	0.072	0.00897	23.7
A-22	35.60	0.097	0.678	0.052	0.00674	24.6
A-23	43.98	0.139	0.848	0.078	0.00524	24.5
A-24	18.23	0.136	0.802	0.082	0.00438	23.6
B-1	18.75	0.075	0.365	0.085	0.00315	25.8
B-2	41.39	0.063	0.389	0.070	0.00600	25.3
B-3	16.68	0.144	0.645	0.101	0.00659	25.5
B-4	24.88	0.214	0.865	0.119	0.00794	25.3
B-5	29.20	0.338	1.100	0.146	0.00791	25.1
B-6	27.30	0.111	0.636	0.082	0.01076	25.0
B-7	27.99	0.292	1.110	0.129	0.01074	25.2
B-8	72.92	0.086	0.558	0.057	0.02012	25.4

Table 5. Continued.

Run	k_2 (day^{-1})	D_x (ft^2/sec)	U (ft/sec)	H (ft)	S (ft/ft)	T ($^\circ$C)
B-9	12.44	0.156	0.476	0.141	0.00144	25.6
B-10	13.13	0.328	0.955	0.175	0.00426	26.0
B-11	6.65	0.217	0.484	0.232	0.00065	25.9
B-12	51.41	0.413	1.471	0.154	0.01332	26.0
B-13	36.37	0.482	1.743	0.150	0.02038	26.0
B-14	44.84	0.653	2.015	0.189	0.01688	25.5
S-2	21.18	0.096	1.333	0.089	0.00241	24.0
S-3	20.74	0.065	0.978	0.067	0.00174	24.9
S-4	32.31	0.056	0.592	0.037	0.00182	25.2
S-5	16.68	0.055	0.843	0.080	0.00103	25.7
G-1	31.45	0.158	0.762	0.078	0.00441	23.7
G-2	21.00	0.235	0.900	0.111	0.00324	24.0
G-3	24.36	0.159	0.764	0.096	0.00324	23.4
G-4	26.18	0.314	0.968	0.121	0.00324	23.1
G-5	24.97	0.208	0.738	0.088	0.00324	22.8
G-6	17.88	0.210	0.688	0.083	0.00324	22.6
G-7	24.02	0.274	0.595	0.073	0.00324	22.9
G-8	34.39	0.330	0.666	0.082	0.00324	22.9
G-9	20.74	0.234	0.855	0.085	0.00465	22.9
G-10	20.13	0.212	0.782	0.080	0.00406	22.8
G-11	27.91	0.185	0.753	0.072	0.00406	23.7
G-12	20.39	0.263	0.741	0.148	0.00309	23.0