January 1965

Submerged Parshall Flumes of Small Size

Gaylord V. Skogerboe

M. Leon Hyatt

J. Raymond Johnson

Joe D. England

SUBMERGED PARSHALL FLUMES

OF SMALL SIZE

Prepared by

Gaylord V. Skogerboe
M. Leon Hyatt
J. Raymond Johnson
Joe D. England

Utah Water Research Laboratory
College of Engineering
Utah State University
Logan, Utah

July 1965 Report PR-WR6-1

ACKNOWLEDGEMENTS

The authors wish to acknowledge that the information used in this report was taken from technical bulletin 61 of the Colorado Agricultural Experiment Station, "Parshall Measuring Flumes of Small Sizes" which was prepared by A. R. Robinson. The information contained in this bulletin has been very useful for demonstrating that the method of submerged flow analysis developed at Utah State University for trapezoidal and rectangular flat-bottomed measuring flumes is also valid for Parshall flumes. The authors have discussed the submerged open channel flow measurement program being conducted by the Utah Water Research Laboratory with Mr. Robinson in the past, and have always been grateful for his contributions and cooperation.

Gaylord V. Skogerboe
M. Leon Hyatt
J. Raymond Johnson
Joe D. England

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

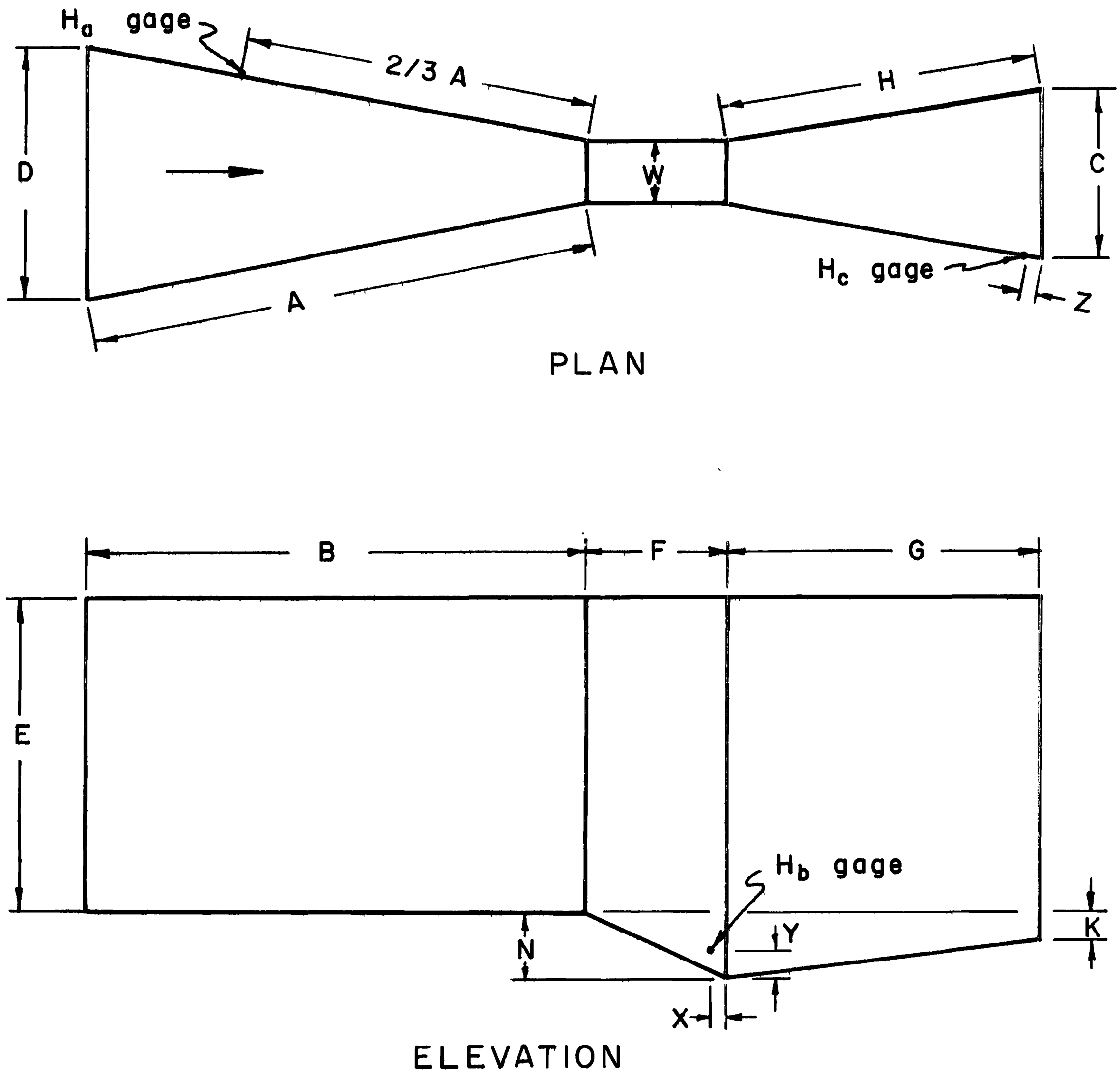
Symbol	Definition
A	Cross-sectional area of flow, $ft.^2$
F_{max}	Maximum Froude number in the flume, dimensionless
g	Acceleration due to gravity, 32.2 $ft./sec.^2$
H_a	Depth of flow in a Parshall flume located two-thirds of the length of the converging entrance section upstream from the throat crest, ft.
H_b	Depth of flow in a Parshall flume measured at a particular referenced point in the throat, ft.
h_1	Depth of flow upstream from the flume, ft.
h_4	Depth of flow downstream from the flume, ft.
h_m	Minimum depth of flow in the throat, ft.
Q	Actual discharge, cfs.
V	Average velocity, fps.

SUBMERGED PARSHALL FLUMES OF SMALL SIZE

INTRODUCTION

The calibration of small Parshall flumes for measuring flows ranging in magnitude from 0.01 to 1.1 cubic feet per second (cfs) was accomplished by A. R. Robinson (1960) at Colorado State University. The purpose of Robinson's investigation was to accurately calibrate and standardize the design of small Parshall measuring flumes. The rated flumes were constructed of galvanized sheet metal. Data was collected for Parshall flumes having throat widths of 1-, 2-, and 3-inches. Calibration tables or curves were prepared for both free and submerged flow. The dimensions of the Parshall flumes rated by Robinson are shown in Figure 1.

The study reported herein was made to illustrate that the analysis of submergence developed at Utah State University (Hyatt, 1965) for trapezoidal flumes is applicable to small Parshall flumes. The data reported by Robinson (1960) is analyzed by the submergence parameters reported by Hyatt (1965) and the resulting equations and calibration curves are listed in this report.



W	A	2/3A	B	C	D	E	F	G	H	K	N	X	Y	Z
in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
1	14 9/32	9 17/32	14	3 21/32	6 19/32	6- 9	3	8	8 1/8	3/4	1 1/8	5/16	1/2	1/8
2	16 5/16	10 7/8	16	5 5/16	8 13/32	6-10	4 1/2	10	10 1/8	7/8	1 11/16	5/8	1	1/4
3	18 3/8	12 1/4	18	7	10 3/16	12-18	6	12	12 5/32	1	2 1/4	1	1 1/2	1/2

Figure 1 Dimensions of 1-, 2-, and 3-inch Parshall measuring flumes.

FLOW ANALYSIS

Free Flow Analysis

Provided critical depth occurs in the throat of a Parshall flume, the discharge is dependent upon only H_a, the upstream depth. The free flow ratings reported by Robinson (1960) for the 1-, 2-, and 3-inch flumes are found in Tables 1, 2, and 3. The only stipulation placed on their use is that throat dimensions are within the tolerances shown in Figure 1. The free flow equations have been listed below because they are of value later in this report.

For a 1-inch Parshall flume, the free flow discharge equation is:

$$Q = 0.338 H_a^{1.55} \quad \text{. . . . . 1}$$

For the 2-inch flume, the equation reported by Robinson (1960) is:

$$Q = 0.676 H_a^{1.55} \quad \text{. . . . . 2}$$

The free flow discharge equation for the 3-inch Parshall flume is:

$$Q = 0.992 H_a^{1.55} \quad \text{. . . . . 3}$$

Submerged Flow Analysis

Often it is impractical to install a measuring flume in such a manner that free flow conditions always exist. Submerged conditions are said to exist in small Parshall measuring flumes when the depth of flow at H_b exceeds, approximately, 50 percent of the depth of flow at H_a. Calibration curves for free flow conditions cannot be used in

Table 1 -- Free flow discharge through 1-inch Parshall measuring flume.

Upper head H_a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Feet	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs
0.00						0.0032	0.0043	0.0055	0.0068	0.0081
0.10	0.0095	0.0110	0.0126	0.0142	0.0160	.0179	.0196	.0216	.0237	.0257
0.20	.028	.030	.032	.035	.037	.039	.042	.045	.047	.050
0.30	.052	.055	.058	.061	.064	.066	.069	.072	.075	.078
0.40	.082	.085	.088	.091	.095	.098	.101	.105	.108	.112
0.50	.115	.119	.123	.126	.130	.134	.138	.141	.145	.149
0.60	.153	.157	.161	.165	.169	.173	.177	.182	.186	.190

Table 2 -- Free flow discharge through 2-inch Parshall measuring flume.

Upper head H_a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Feet	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs
0.00						0.0065	0.0087	0.0109	0.0135	0.0162
0.10	0.0191	0.0221	0.0251	0.0284	0.0321	.0358	.0392	.0433	.0473	.0513
0.20	.055	.060	.065	.070	.074	.079	.084	.089	.094	.099
0.30	.105	.110	.116	.121	.127	.132	.139	.145	.151	.157
0.40	.163	.170	.176	.182	.189	.196	.203	.210	.217	.224
0.50	.230	.238	.245	.253	.260	.268	.275	.283	.290	.298
0.60	.306	.314	.322	.330	.338	.347	.355	.363	.372	.381
0.70	.389	.397	.406	.415	.424	.433	.442	.451	.459	.469

Table 3 -- Free flow discharge through 3-inch Parshall measuring flume.

Upper head H_a	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Feet	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs	Q cfs
0.10	0.028	0.033	0.037	0.042	0.047	0.053	0.058	0.064	0.070	0.076
0.20	.082	.089	.095	.102	.109	.117	.124	.131	.138	.146
0.30	.154	.162	.170	.179	.187	.196	.205	.213	.222	.231
0.40	.241	.250	.260	.269	.279	.289	.299	.309	.319	.329
0.50	.339	.350	.361	.371	.382	.393	.404	.415	.427	.438
0.60	.450	.462	.474	.485	.497	.509	.522	.534	.546	.558
0.70	.571	.584	.597	.610	.623	.636	.649	.662	.675	.689
0.80	.702	.716	.730	.744	.757	.771	.786	.800	.814	.828
0.90	.843	.858	.872	.887	.902	.916	.931	.946	.961	.977
1.00	.992	1.007	1.023	1.038	1.054	1.070	1.086	1.102	1.118	1.134

determining the discharge when the ratio H_b/H_a exceeds 50 percent, but rather a submerged flow calibration curve must be used.

The method employed by Robinson (1960) in presenting the submerged calibration curves is shown in Figures 2, 3, and 4. As an illustrative example, for a 2-inch standard Parshall flume with a H_a gage reading of 0.030 feet and a H_b gage reading of 0.255 feet--both referenced to the elevation of the crest of the throat--a submergence of 85 percent is obtained. From Figure 3, with a submergence value of 85 percent and the H_a gage reading of 0.30 feet, a discharge of 0.080 cfs is obtained, which compares to a discharge of 0.105 cfs (Table 2) for free flow conditions.

A different approach to the submerged flow problem was taken by Hyatt (1965). The parameters involved were developed for trapezoidal measuring flumes, and were later verified for the rectangular measuring flume by Skogerboe, Walker, and Robinson (1965). The parameters are submergence, h_4/h_1, where h_4 is the flow depth downstream from the flume and h_1 the flow depth upstream from the flume; the maximum Froude number occurring in the flume, F_{max}; and an energy loss parameter relating conditions at three important cross sections as defined by $(h_1-h_4)/h_m$, where h_m is the minimum flow depth occurring in the throat.

The parameters involved in submerged flow can be obtained from dimensional analysis as follows:

$$V = f(g, h_1, h_4, h_m) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4$$

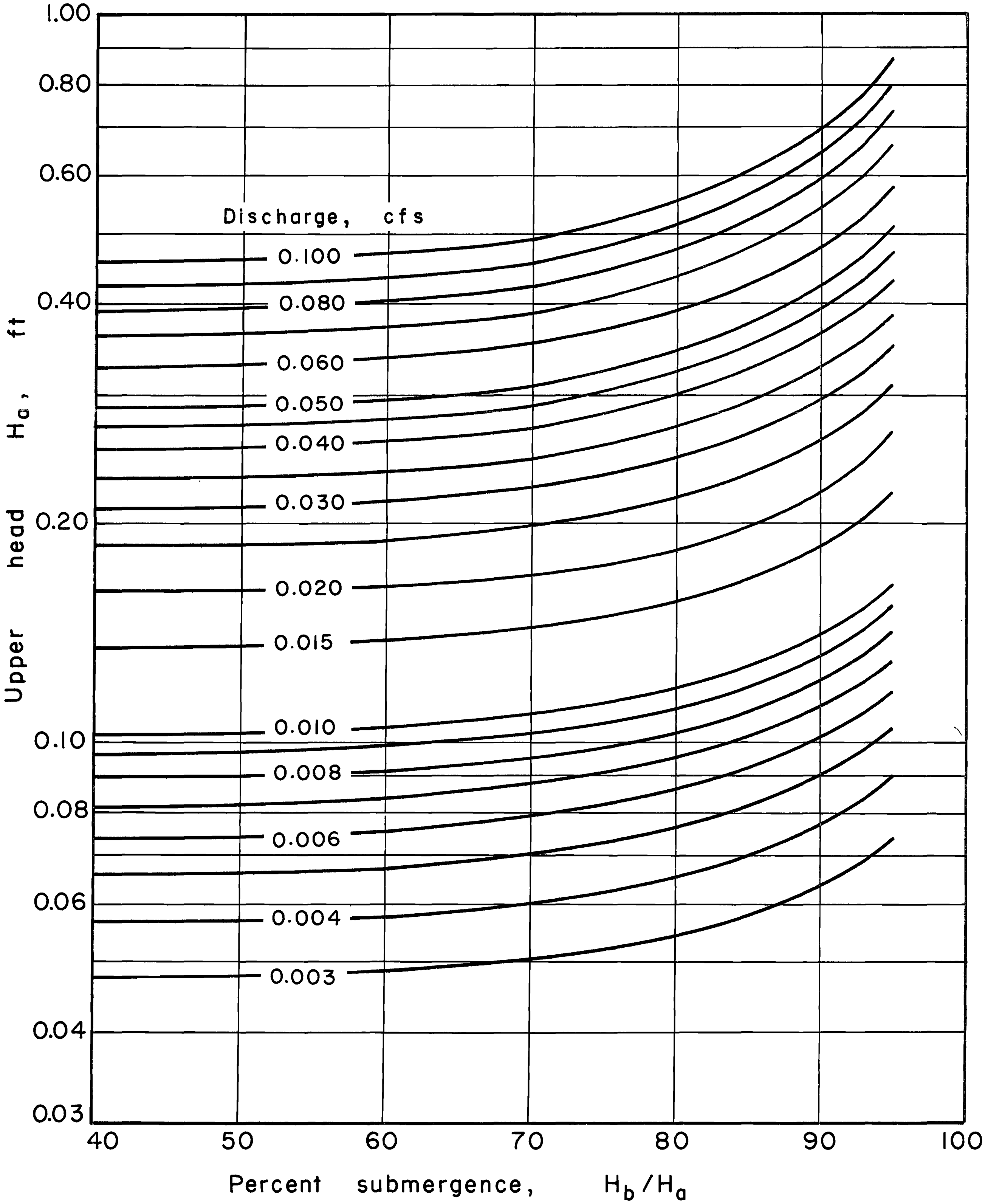


Figure 2 Rate of submerged flow through a 1-inch Parshall measuring flume.

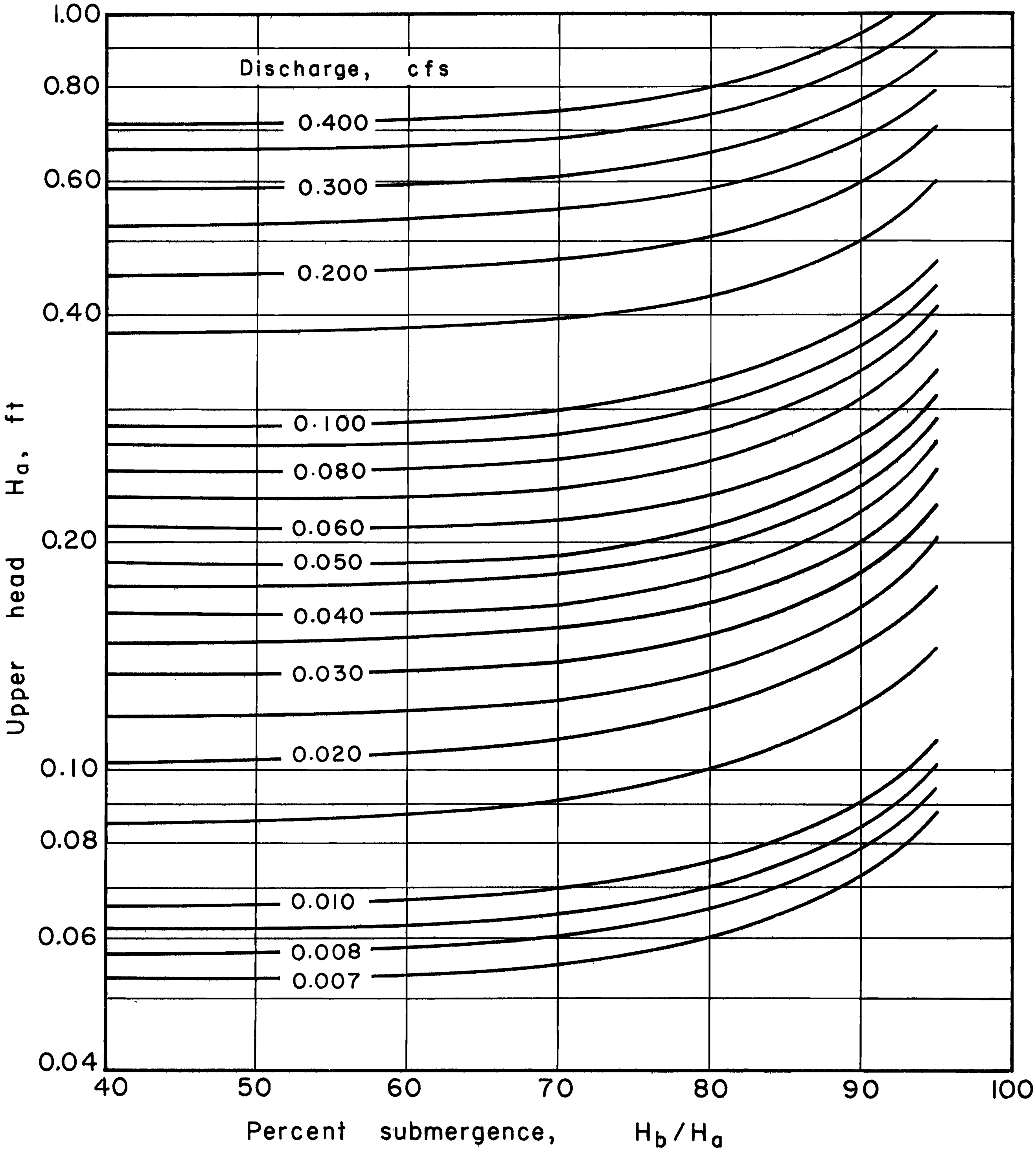


Figure 3 Rate of submerged flow through a 2-inch Parshall measuring flume.

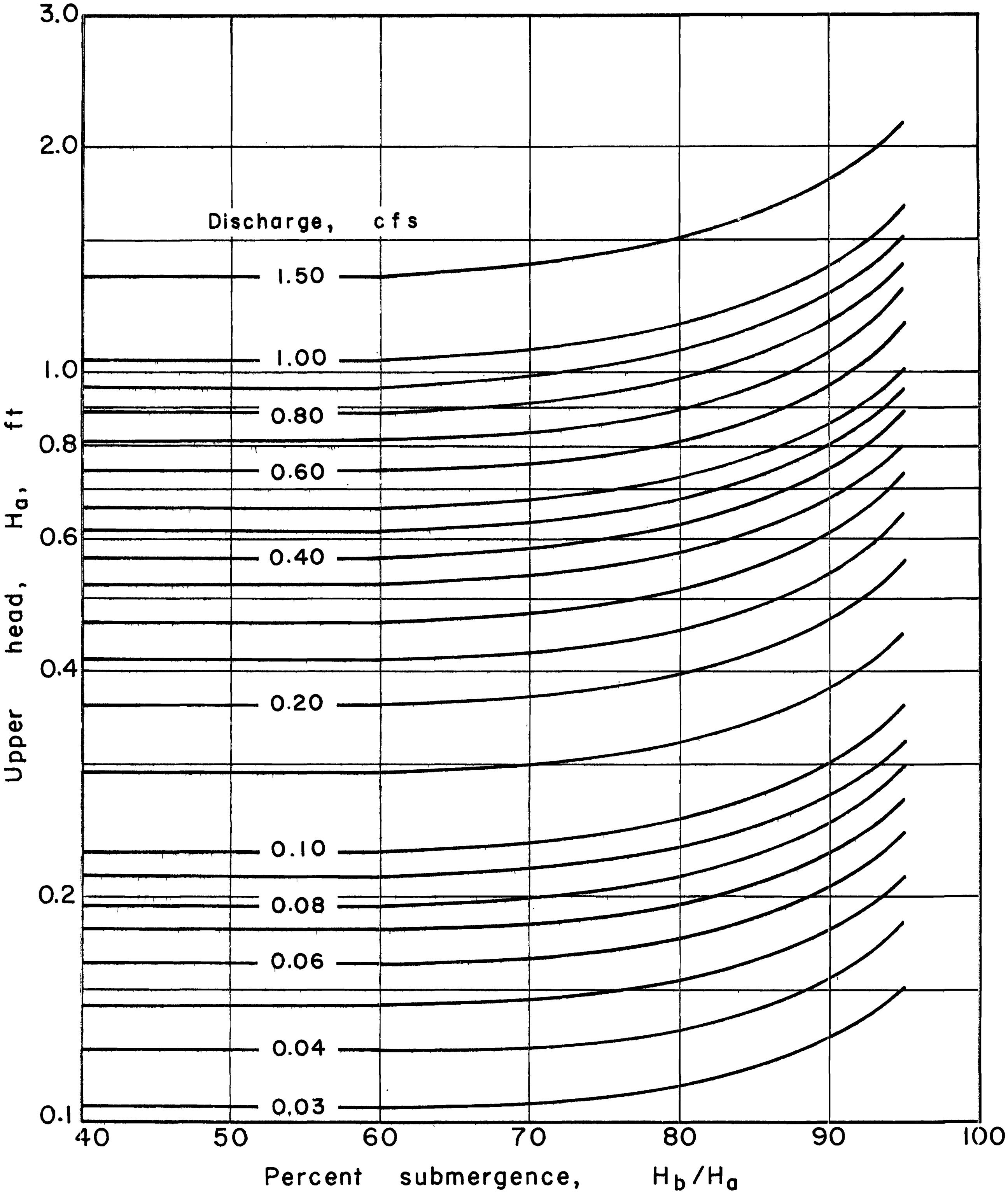


Figure 4 Rate of submerged flow through a 3-inch Parshall measuring flume.

With five independent quantities and two dimensions, three pi-terms are derived.

$$\pi_1 = \frac{V}{\sqrt{gh_m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5$$

$$\pi_2 = \frac{h_4}{h_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6$$

$$\pi_3 = \frac{h_1 - h_4}{h_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 7$$

Equation 5 can be modified by replacing V with Q/A so that

$$\pi_1 = \frac{Q}{A\sqrt{gh_m}} = F_{max} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 8$$

The relationship of the three pi-terms F_{max}, h_4/h_1, and $(h_1-h_4)/h_m$ as developed by Hyatt (1965) for a particular trapezoidal flume is illustrated in Figure 5. The energy loss parameter was plotted on the log scale as the ordinate. Plotted on the abscissa are submergence on a cartesian scale and maximum Froude number on a log scale. The two resulting equations relating these pi-terms are:

$$\frac{h_1 - h_4}{h_m} = 0.300 \quad F_{max}^{2.38} \quad . \quad . \quad . \quad . \quad . \quad . \quad 9$$

and

$$\frac{h_4}{h_1} = \frac{0.99}{10^{0.34(h_1 - h_4)/h_m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad 10$$

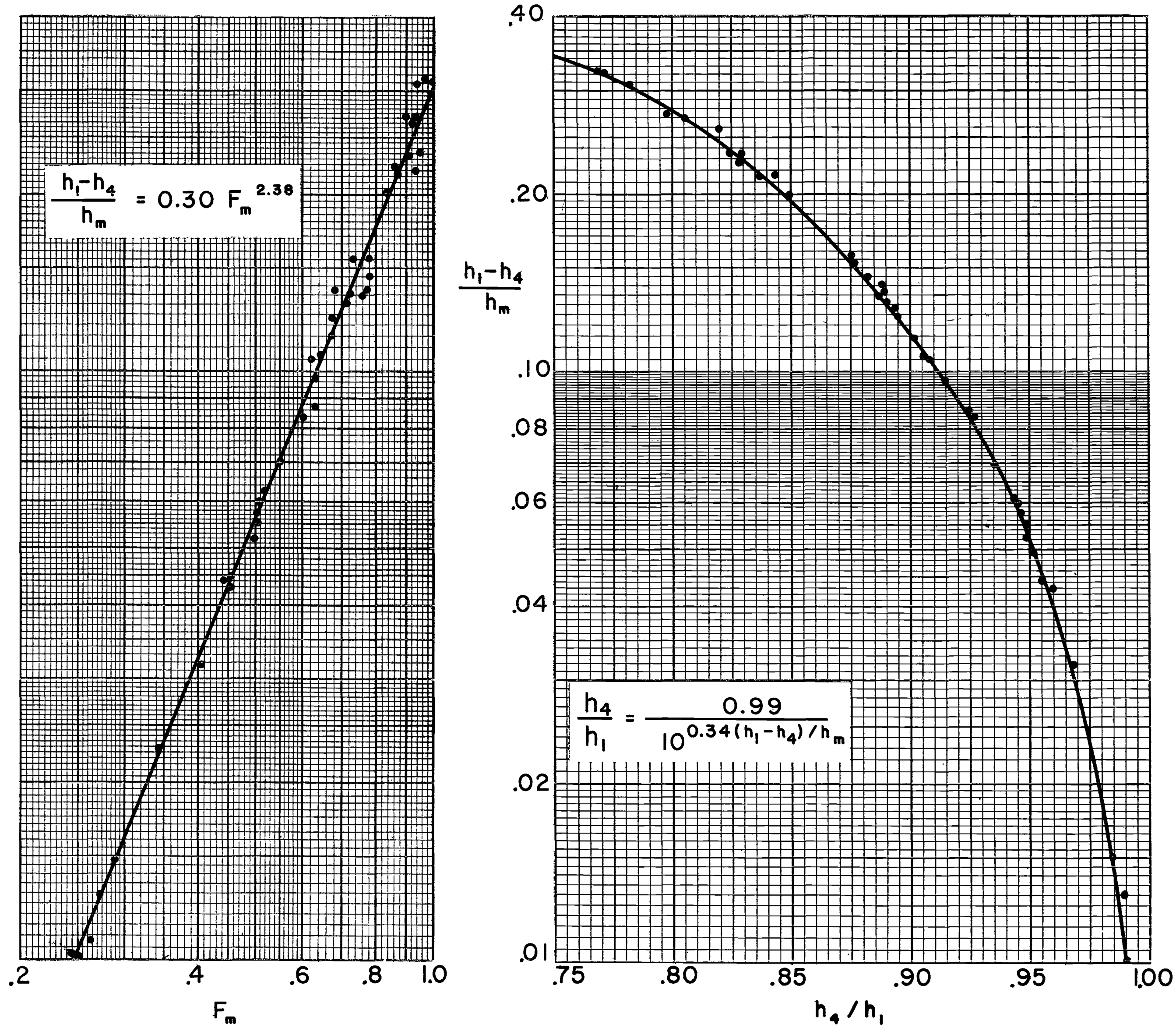


Figure 5 Relationship between pi-terms.

When three-dimensional log-log plots of h_m, F_{max}, and Q are prepared, it is possible to relate minimum depth and the maximum Froude number to the discharge, resulting in the equation,

$$Q = 34.7\ F_{max}\ h_m^{1.74} \quad . \quad . \quad . \quad . \quad . \quad . \quad 11$$

By combining Equations 9, 10, and 11, the relationship describing submerged flow for a particular trapezoidal flume is obtained.

$$Q = \frac{-13.83\ (h_1 - h_4)^{1.74}}{\left(\log \frac{h_4}{h_1} + 0.0044\right)^{1.32}} \quad . \quad . \quad . \quad . \quad . \quad . \quad 12$$

The equation developed by Skogerboe, Walker, and Robinson (1965) for a particular rectangular flume was

$$Q = \frac{-46.6\ (h_1 - h_4)^{1.53}}{\left(\log \frac{h_4}{h_1} + 0.0044\right)^{1.02}} \quad . \quad . \quad . \quad . \quad . \quad . \quad 13$$

Although equations 12 and 13 are only valid for the particular flumes studied, they do show that only the upstream and downstream depths need to be measured to determine the discharge under submerged flow conditions.

Using the submerged flow calibration curves developed by Robinson (1960), the same approach to submergence described above was applied to small Parshall flumes. The submerged flow calibration curves developed from Robinson's curves for 1-, 2-, and 3-inch flumes are shown in Figure 6, 7, and 8. The equations for these curves are of the form exemplified by Equations 12 and 13 with only the coefficient varying for each size of flume.

For a 1-inch standard Parshall measuring flume, the submerged-flow discharge equation which best fits the plots from Robinson's (1960) investigation is

$$Q = \frac{-0.295\,(H_a - H_b)^{1.55}}{\log \frac{H_b}{H_a} + 0.0044} \quad \ldots\ldots\ldots \quad 14$$

For the 2-inch flume, the equation becomes

$$Q = \frac{-0.614\,(H_a - H_b)^{1.55}}{\log \frac{H_b}{H_a} + 0.0044} \quad \ldots\ldots\ldots \quad 15$$

The equation for the 3-inch flume is

$$Q = \frac{-0.953\,(H_a - H_b)^{1.55}}{\log \frac{H_b}{H_a} + 0.0044} \quad \ldots\ldots\ldots \quad 16$$

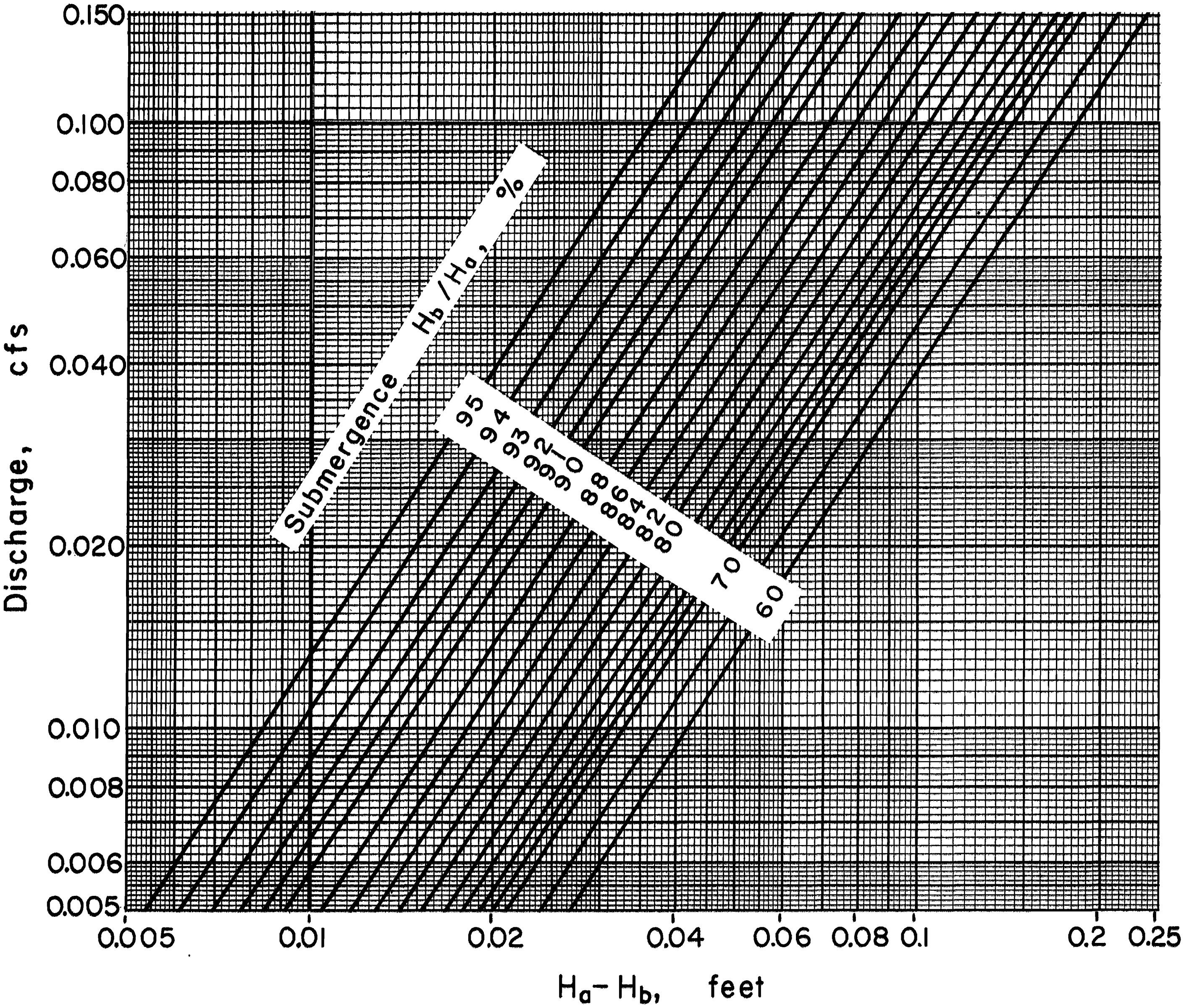


Figure 6 Calibration curves for submerged Parshall flume with 1-inch throat.

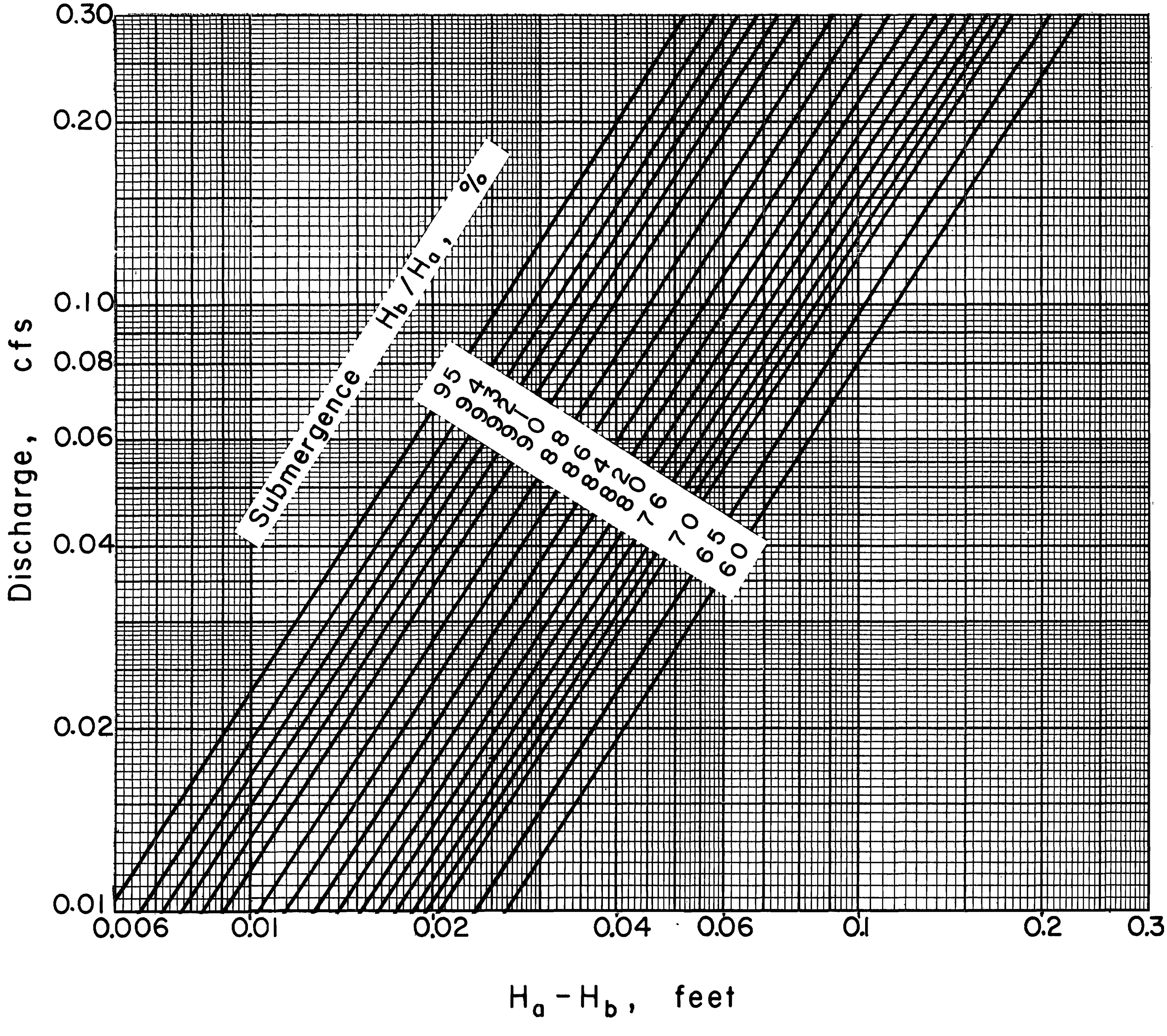


Figure 7 Calibration curves for submerged Parshall flume with 2-inch throat.

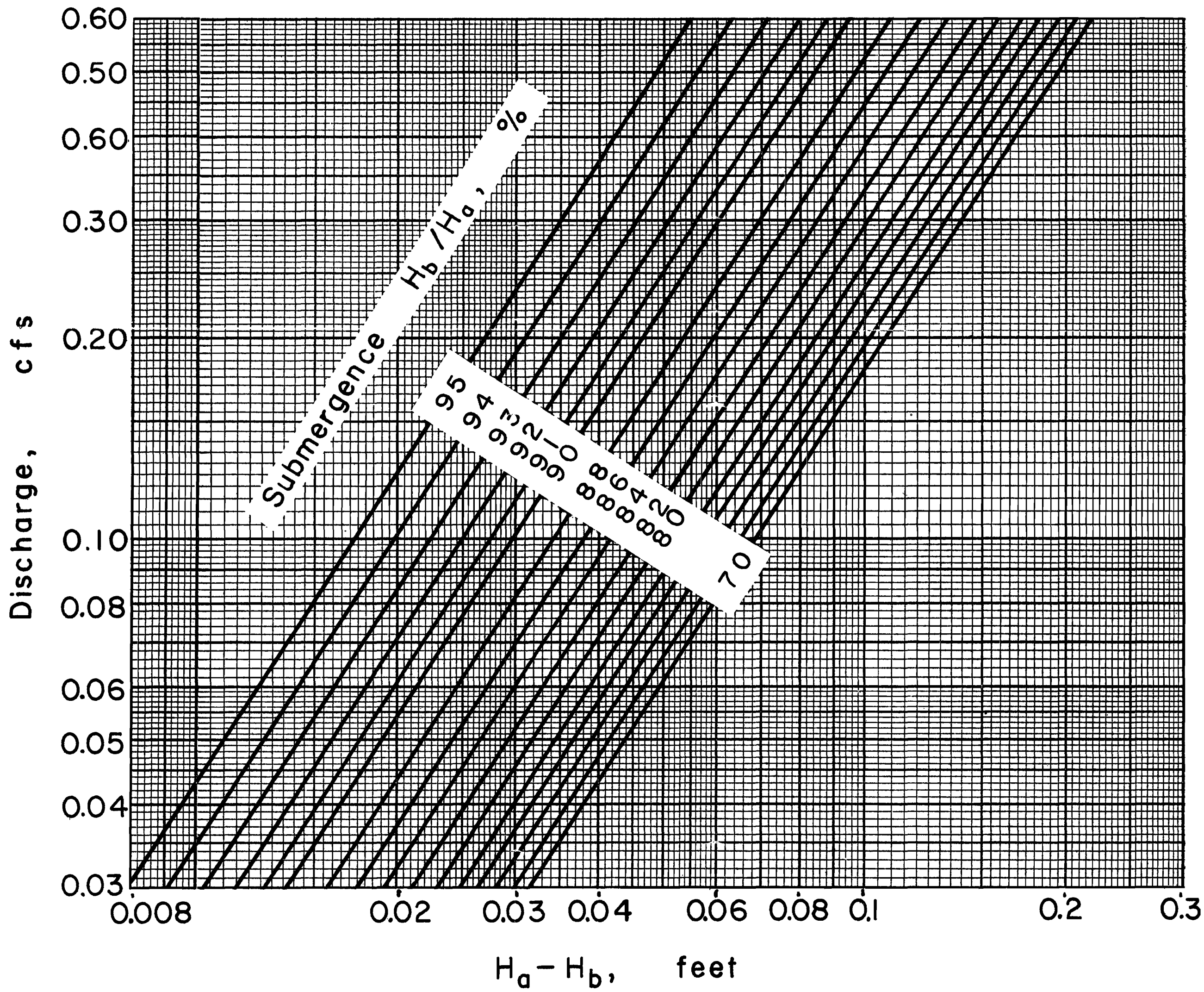


Figure 8 Calibration curves for submerged Parshall flume with 3-inch throat.

TRANSITION FROM FREE TO SUBMERGED FLOW

The form of the equation for both free and submerged flow provide a unique solution for submergence, H_b/H_a, at which the transition from free to submerged flow occurs. As an example, the equations for the 1-inch Parshall flume (Equations 1 and 11) can be set equal to one another.

$$0.338\, H_a^{1.55} = \frac{-0.295\, (H_a - H_b)^{1.55}}{\log \frac{H_b}{H_a} + 0.0044}$$

$$\log \frac{H_b}{H_a} + 0.0044 = \frac{-0.295}{0.338} \left(\frac{H_a - H_b}{H_a}\right)^{1.55}$$

$$\log \frac{H_b}{H_a} + 0.0044 = -0.873 \left(1 - \frac{H_b}{H_a}\right)^{1.55}$$

A solution is obtained by trail and error.

$$H_b / H_a = 0.52$$

Thus, free flow exists in a 1-inch Parshall flume when the submergence is less than 52 percent and for submergences greater than 52 percent, submerged flow occurs. The solution of the free and submerged flow equations for the 2-inch and 3-inch Parshall flumes result in a critical submergence of 61 percent for the 2-inch flume and 69 percent for the 3-inch flume.

An evaluation of the transition submergences for the three Parshall flumes can be ascertained from Robinson's plots (Figures 2, 3, and 4). In each case, it can be seen that the transition submergence values are higher than would be indicated by Figures 2, 3, and 4. The discrepancy between computed transition submergences and the transition values indicated by the plots could be attributed to inaccuracies in the free flow and submerged flow equations. The computation of the transition submergence is very sensitive to changes in the coefficients or powers of the discharge equations. Also, Hyatt (1965), in his studies of a trapezoidal measuring flume, found that under submerged flow conditions, the degree of error in computing discharge was less at high values of submergence as compared to submergences just slightly above the transition submergence. The greater inaccuracy of the submerged flow equation near the transition from free to submerged flow, plus the sensitivity of the computed transition submergence to changes in the coefficients or powers of the discharge equations, accounts for the discrepancy between the actual and computed transition submergence.

SUMMARY

Small standard Parshall flumes have been found to give accurate measurements of discharge for both free flow and submerged flow conditions (Robinson, 1960). The usable range of capacity ranges from about 0.005 to 0.2 cfs for the 1-inch flume; 0.01 to 0.5 cfs for the 2-inch flume; and 0.03 to 1.1 cfs for the 3-inch flume. Care must be taken that the tolerances of construction listed by Robinson (1960) are considered.

The submerged flow analysis reported by Hyatt (1965), has been found to be valid for small Parshall measuring flumes. Calibration curves and equations for submerged flow have been developed for Parshall Flumes having throat widths of 1-, 2-, and 3-inches.

REFERENCES

Hyatt, M. Leon. 1965. Design, calibration, and evaluation of a trapezoidal measuring flume by model study. M. S. Thesis, Utah State University, Logan, Utah. March.

Robinson, A. R. 1960. Parshall measuring flumes of small sizes, Technical Bulletin 61. Agricultural Experiment Station, Colorado State University, Fort Collins, Colorado. August.

Skogerboe, G. V., W. R. Walker, and L. R. Robinson. 1965. Design, operation, and calibration of the Canal "A" submerged rectangular measuring flume. Report PR-WG24-3, Utah Water Research Laboratory, Utah State University, Logan, Utah. March.