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Subcritical Flow Over Various Weir Shapes

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NOMENCLATURE

Symbol	Definition
A	Area
b	Bottom width of flume at section where t is measured
B	Contraction ratio, or the ratio of b to the width of the flume where h is measured
C	Coefficient in free flow equation
C_1	Coefficient in numerator of submerged flow equation
C_2	Coefficient in denominator of submerged flow equation
F	Force
F_m	Maximum Froude number
g	Acceleration due to gravity
h	Upstream depth of flow measured from the elevation of the crest of structure
h_d	Design head
h_m	Minimum depth of flow in flume throat
H_1	Total energy head at upstream section measured from the crest of the structure
L	Total width of the roadway (pavement plus two shoulders)
L_p	Pavement width of embankment
L_s	Shoulder width of embankment
n_1	Exponent in the free flow equation and numerator of the submerged flow equation
n_2	Exponent in the denominator of the submerged flow equation

NOMENCLATURE (Continued)

Symbol	Definition
P	Height of weir or embankment
$P + E$	Height of ogee spillway
q	Discharge per foot of length of weir
Q	Total flow rate, or discharge
S	Submergence, which is the ratio of a downstream depth to an upstream depth with both depths referenced to a common elevation
S_e	Embankment slope
S_p	Pavement cross slope
S_s	Shoulder slope
S_t	Transition submergence
t	Downstream depth of flow measured from the crest of the structure
V	Average velocity
V_1	Average velocity at section 1
V_2	Average velocity at section 2
y	Flow depth
y_1	Flow depth at section 1
y_2	Flow depth at section 2
y_o	Flow depth at crown line
β	Momentum correction coefficient
π_1	Maximum Froude number occurring in the flume, $V/(gh_m)^{1/2}$
π_2	Submergence, t/h
π_3	Energy loss parameter, $(h - t)/h_m$

NOMENCLATURE (Continued)

Symbol	Definition
γ	Specific weight of fluid
λ	Embankment slope
ρ	Density of fluid

INTRODUCTION

Submerged flow exists for any given structure when a change in flow depth downstream from the structure causes a change in flow depth upstream from the structure for any given constant value of discharge. The two flow depths, normally measured when submerged flow exists, consist of a depth upstream from the structure, which is used also for free flow conditions, and a depth of flow located any place downstream from the structure.

The initial studies in which the submerged flow analysis was developed were made on flat-bottomed flumes (Hyatt, 1965; and Skogerboe, Walker, and Robinson, 1965). Later studies verified the method of analysis for Parshall flumes (Skogerboe, Hyatt, England, and Johnson, 1965; and Hyatt, Skogerboe, and Eggleston, 1966), as well as for highway embankments (Skogerboe, Hyatt, and Austin, 1966). Because of previous findings, it was felt this method of analyzing submerged flow could be applied to various types and kinds of weirs.

Original development of the parameters and relationships which describe submerged flow came from a combination of dimensional analysis and empiricism. Further verification of the parameters developed in this manner are obtained by employing momentum relationships. Both approaches to the submerged flow problem are discussed in this report.

Considerable effort and study has been expended on free and submerged flow weirs by other authors in previous years. For this reason the authors of this report went to the literature as a source of data.

Various studies typifying a particular type of weir structure were investigated and the data selected from these studies were subjected to the submerged flow analysis developed by the authors. The data from these studies provide further verification of validity of the approach to the submerged flow problem made by the authors.

Acknowledgment is given and appreciation expressed to those authors whose studies provided the data used in the analysis presented in this report.

Although no investigation was made of a contracted weir, the authors feel that the submerged flow analysis as explained in this report would be just as valid for this type of structure.

DEFINITION OF FLOW REGIMES

The two most significant flow regimes or flow conditions are free flow and submerged flow. The distinguishing difference between the two is that critical depth occurs, usually near the crest of the weir, for the free flow condition. This critical-flow control requires only the measurement of a depth upstream from the point of critical depth for determination of the free flow discharge. When the downstream or tailwater depth is raised sufficiently, the flow depths at every point through the structure become greater than critical depth, and submerged flow conditions exist. In the submerged flow regime a change in the tailwater depth also affects the upstream depth and a rating for the weir requires that two flow depths be measured, one upstream and one downstream from the structure.

The flow condition at which the regime changes from free flow to submerged flow is a transition state that is unstable and difficult to produce in the laboratory. The value of submergence at which this condition occurs is often referred at as the transition submergence, symbolized by S_t. This change from supercritical flow to subcritical flow (transition submergence) signifies that the Froude number is equal to 1 at a single flow cross-section, and for every other cross-section the Froude number is less than 1. At the transition from free flow to submerged flow, the discharge equations for the two flow conditions should be equal. Consequently, if the discharge equations are known, the transition submergence can be obtained by setting the free and submerged flow equations equal to one another.

THEORETICAL ASPECTS OF SUBMERGED FLOW

Application of momentum theory can result in the development of submerged flow discharge equations for weirs, flumes, or other structures. Such equations are beneficial and instructive for comparison with the empirical equations developed from dimensional analysis. An embankment-shaped weir will be selected to illustrate the application of momentum theory. A control volume of fluid will be used which is bounded by the vertical sections at 1 and 2, the water surface, and the surface of the embankment, as shown in Fig. 1.

A solution for the horizontal component of the form resistance force, F_{e_x}, due to the embankment will be developed. A generalized diagram of the force of the embankment acting on the fluid is shown in Fig. 2.

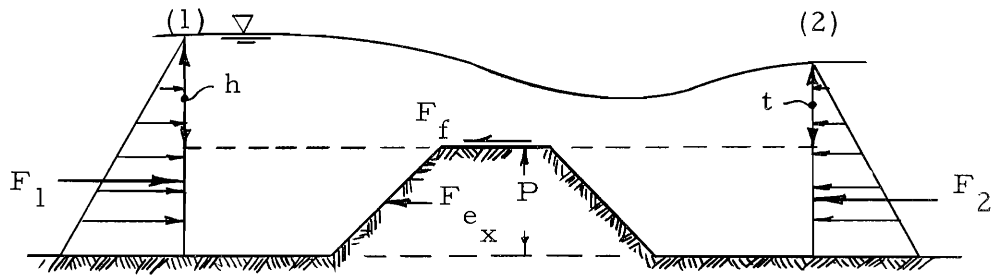

Fig. 1. Control volume for analysis of embankment-shaped weir.

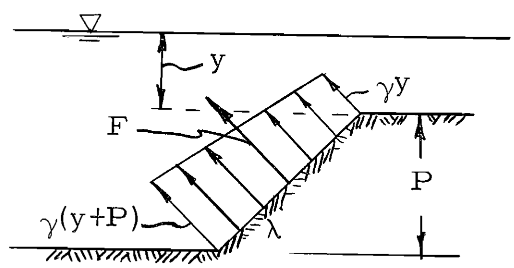

Fig. 2. Definition sketch for force acting on the fluid due to embankment.

$$F\ (\text{lb/ft}) = \left[\gamma y + \frac{\gamma(y + P) - \gamma y}{2}\right] \frac{P}{\sin \lambda}$$

$$= \frac{\gamma P}{\sin \lambda} \left(y + \frac{P}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1$$

$$F_x(\text{lb/ft}) = \frac{\gamma P}{\sin \lambda} \left(y + \frac{P}{2}\right) \sin \lambda$$

$$= \gamma P \left(y + \frac{P}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 2$$

The force of the embankment on the fluid will be designated as F_u for the upstream slope and F_d for the downstream slope. The assumption is made that the pressures acting on both the upstream and downstream slopes of the embankment are hydrostatic. Assuming the pressure on the upstream slope is due to the water surface elevation at section 1, and the pressure on the downstream slope is due to the water surface elevation at section 2, the horizontal components of F_u and F_d can be developed from similarity with the equation for F_x (Eq. 2).

$$F_{u_x} = \gamma P (h + P/2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 3$$

$$F_{d_x} = \gamma P (t + P/2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4$$

$$F_{e_x} = \gamma P (h + P/2) - \gamma P (t + P/2)$$

$$= \gamma P (h - t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5$$

The forces acting on the control volume at sections 1 and 2 (Fig. 1) can be determined by assuming hydrostatic pressure distributions.

$$F_1 = \gamma (h + P)^2/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6$$

$$F_2 = \gamma (t + P)^2/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 7$$

If friction losses are neglected ($F_f = 0$), the summation of forces in the horizontal direction can be evaluated.

$$\Sigma F_x = F_1 - F_2 - F_{e_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 8$$

$$\Sigma F_x = \gamma (h + P)^2/2 - \gamma (t + P)^2/2 - \gamma P (h - t)$$

$$= \gamma (h^2 - t^2)/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 9$$

Assuming uniform velocity distributions at sections 1 and 2, the following momentum equation can be written.

$$\Sigma F_x = q\rho \; (V_2 - V_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 10$$

The summation of horizontal forces is given by Eq. 9.

$$\gamma (h^2 - t^2)/2 = q\rho \; (V_2 - V_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 11$$

Assuming steady flow, the continuity equation, $q = Vy$, can be employed.

$$q = V_1 \; (h + P) = V_2 \; (t + P) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 12$$

The continuity equation can be substituted into Eq. 11.

$$\frac{\gamma(h^2 - t^2)}{2} = q \frac{\gamma}{g} \left[\frac{q}{t + P} - \frac{q}{h + P} \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 13$$

$$q = (g/2)^{1/2} \sqrt{(h + t) \; (t + P) \; (h + P)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 14$$

Manipulation of Eq. 14 yields

$$q = \frac{(g/2)^{1/2} \; (h - t)^{3/2}}{\sqrt{\dfrac{(1 - S)^3}{(1 + S) \; (S + P/h) \; (1 + P/h)}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 15$$

where S is the submergence, t/h.

Application of momentum theory to a flume with a similar development will produce an equation almost identical to Eq. 15. The resulting equation is

$$Q = \frac{(g/2)^{1/2}\ b\ (h - t)^{3/2}}{\sqrt{\dfrac{(1 - BS)\ (1 - S)^2}{S\ (1 + S)}}} \quad . . . . \quad 16$$

where b is the width of the flume at the section where t is measured, and B is the constriction ratio defined by the ratio of b to the width at the section where h is measured.

Although the assumptions made in the development of the theoretical submerged flow equations are not entirely valid, the equations do contain certain characteristics which are similar to and which supplement the submerged flow equation developed from a dimensional analysis approach to submerged flow.

One additional concept for discussion is the theoretical equation which could be developed for a contracted weir. This equation would be a combination of Eq. 15 and Eq. 16. The discharge would be a function of B, P, h, and t, but for a particular weir the values of B and P would become constant leaving the discharge as a function of only h and t.

DIMENSIONAL ANALYSIS APPROACH TO SUBMERGED FLOW

Dimensional analysis was first applied to a trapezoidal flume (Hyatt, 1965) to develop the dimensionless parameters which describe submerged flow. For any particular flume geometry, the variables involved can be written as follows:

$$V = f\ (g,\ h,\ h_m,\ t) \quad . . . . \quad 17$$

With five independent quantities and two dimensions, three pi-terms are necessary. The parameters or π terms which were found to describe submerged flow were:

1. The maximum Froude number occurring in the flume (which corresponds with the point of minimum depth of flow, h_m, in the flume throat) as expressed by

$$\pi_1 = F_m = \frac{V}{(g\ h_m)^{1/2}} \quad . \ . \ . \ . \ . \ . \ . \quad 18$$

2. The second π term is submergence, defined at the ratio of the tailwater depth to the upstream depth of flow, and is expressed by

$$\pi_2 = S = t/h \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad 19$$

3. An energy loss parameter defined as the difference between the upstream depth and tailwater depth divided by the minimum depth of flow in the flume throat is the third π term and is written

$$\pi_3 = (h - t)/h_m \quad . \ . \ . \ . \ . \ . \ . \ . \quad 20$$

Plotting of the three parameters provides a unique relationship for any particular flume geometry. Most significant of the plots made is the plot of π_2, t/h, as the ordinate, and π_3, $(h - t)/h_m$, as the abscissa (Fig. 3). The curve must pass through the point 0, 0, for as the submergence approaches 100 percent ($\log S = 0$), the difference in water surface elevation, $h - t$, will approach zero. The relationship which exists between π_2 and π_3 can be approximated by a straight line as shown in Fig. 3 over a large range of submergence values with some sacrifice in the accuracy of the submerged flow calibration plot. When the equation resulting from the straight line approximation is manipulated with the other equations relating the dimensionless parameters, a submerged flow discharge equation results which is

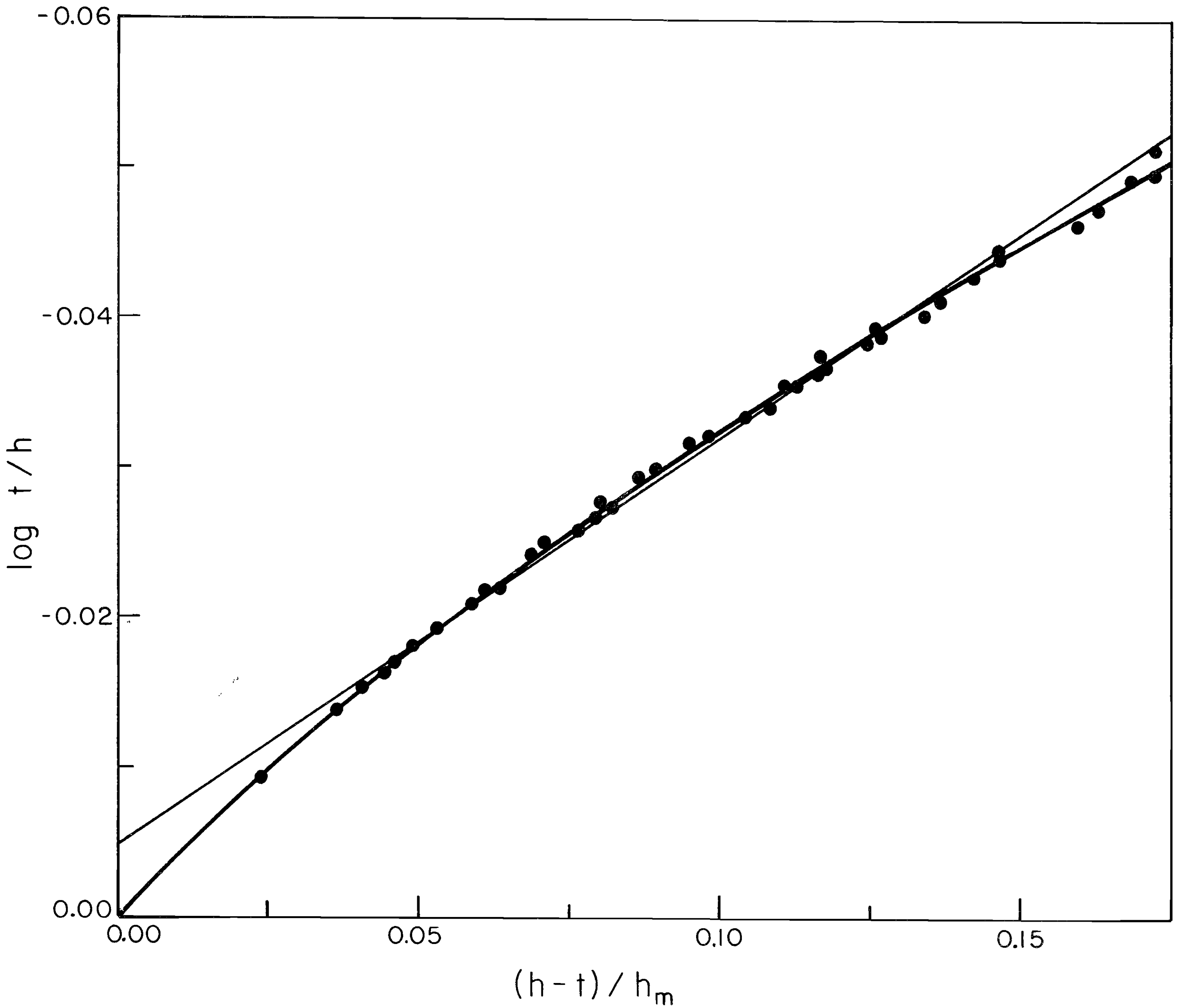

Fig. 3. Relationship between π_2 and π_3.

dependent upon only the upstream and downstream flow depths. The general form of the submerged flow equation can be expressed as

$$Q = \frac{C_1 (h - t)^{n_1}}{[-(\log t/h + C_2)]^{n_2}} \quad \ldots\ldots\ldots\ldots\ldots \quad 21$$

where h and t are flow depths measured upstream and downstream from the point of minimum flow depth, C_1 and C_2 are coefficients which depend upon the geometry of the structure, and n_1 and n_2 are exponents which are also related to the structure geometry.

Equation characteristics

The real value in developing a submerged flow equation from the theoretical viewpoint as well as employing dimensional analysis and empiricism is the verification of the equation format each gives the other. Evaluation of Eq. 16 reveals that for any particular flume geometry both B and b are constants as is P for any particular weir in Eq. 15. Consequently, the discharge in the theoretical submerged flow equations (Eqs. 15 and 16) becomes a function of $(h - t)^{3/2}$ and the submergence, S. Therefore, the theoretical equations are similar in format to the empirical equation derived from dimensional analysis (Eq. 21) where the discharge is a function of $(h - t)^{n_1}$ and the submergence, S.

Attention is called to the coefficient C_2 in Eq. 21. This coefficient is the intercept on the ordinate resulting from the straight line approximation of the plot drawn in Fig. 3. Although the coefficient C_2 actually varies as shown by the curvature of the plot in Fig. 3, the variation is of major importance in only the 96 to 99.9 percent submergence range. The value of C_2 approaches zero as the submergence approaches 100 percent. Because C_2 varies so greatly in this higher submergence range and since submergence values in this range are not practical, the authors will not write submerged flow equations utilizing a constant value of C_2 which cover these higher submergence values in the following sections.

As is shown in the section on embankment-shaped weirs, the empirically developed submerged flow equation for an embankment-shaped weir differs from Eq. 21 in that the C_2 coefficient is absent. Explanation for this is found in examination of the flow conditions. The flow over embankment-shaped weirs is usually considered as two-dimensional and given in terms of discharge, q, per foot of length whereas the flow in flumes is a three-dimensional flow condition. It would thus appear that the absence or presence of a C_2 coefficient would indicate the absence or presence of side effects exerted by a structure on the flow regime. Common reasoning would indicate that flow over an embankment of great length (such as the example presented in the embankment section) is a two-dimensional flow condition and that side effects would be nonexistent. In flumes and suppressed weirs of short lengths, the sides do exert an effect on the flow and the presence of a C_2 coefficient in the submerged flow equations for these structures verifies this effect.

The real value provided by approaching the submerged flow problem with dimensional analysis and the application of momentum theory is the fact that both approaches reveal only the upstream depth of flow, h, and the downstream depth of flow, t, are all that is required for the determination of the discharge.

Application principles

The calibration curves which depict the relationships given by Eqs. 15, 16, and 21 are obtained by plotting three-dimensionally the discharge, Q, as the ordinate, difference between upstream and downstream depths of flow, h - t, as the abscissa, and submergence, t/h, as the varying parameter. A family of lines of constant submergence result from this plotting as illustrated by Figs. 5, 7, and 10. Hence, measurement of the upstream and downstream depths of flow provides the necessary information for obtaining the discharge for any particular structure. Once these depths of flow are measured for a structure, the discharge for a given flow condition may be obtained from calibration curves for that structure by the intersection of the h - t value and the t/h value.

The free flow equation for flow measuring flumes can be expressed by

$$Q = C\,h^{n_1} \qquad \text{. . . . .} \qquad 22$$

A noteworthy factor discovered from the flume studies (Skogerboe, Hyatt, England, and Johnson, 1965; and Skogerboe, Hyatt, Johnson, and England, 1965) is the fact that the exponent on the h - t term in the submerged flow equation (Eq. 21) is identical to the exponent on the h term in the free flow equation (Eq. 22) for any given flume. Hence, the value of submergence at which the transition from free flow to submerged flow occurs can be estimated by equating the free flow equation and the approximate submerged flow equation, Eqs. 21 and 22, for a particular structure.

The form of Eq. 21, which describes submerged flow through flumes, is then found to be valid as the equation form which describes submerged flow over various types of weir structures. Those types of weirs discussed in the following sections are (1) embankment-shaped weirs, (2) suppressed-sharpcrested weirs, (3) suppressed-ogee crest weirs, and (4) the triangular-shaped weir of Crump. Since no data on contracted weirs were readily accessible to the authors the submerged flow analysis was not applied to this type of weir structure. However, the authors feel the analysis would be just as valid for the contracted weir as it is shown to be for suppressed weir structures.

EMBANKMENT-SHAPED WEIRS

Kindsvater (1964) placed considerable effort into a study to evaluate the discharge characteristics of flows over highway embankments. A highway embankment is a form of broad-crested weir when overtopped by flood waters. The submerged flow data collected by Kindsvater (1964) has been previously analyzed by the authors, Skogerboe, Hyatt, and Austin (1966). The results of this application are reiterated in this section as additional illustration of the submerged flow method of analysis as explained in the previous section.

The basic embankment design used in the study conducted by

Kindsvater is illustrated in Fig. 4. The basic model was constructed at a 1:9 scale. Corresponding to the 1:9 construction scale, the unit-discharge, q, in the model is 1/27 of the discharge for the prototype embankment. In the original model, the intersections of the shoulder, embankment, and pavement surfaces were sharp and precise. Subsequent use and polishing rounded these intersections, but the results of Kindsvater (1964) indicated no significant effects due to the rounding.

The principal variables used to describe flow over an embankment are illustrated by Fig. 4. The laboratory facilities were such that the discharge and degree of submergence could be controlled. The upstream flow depth, h, was measured at a distance of approximately 52 feet 2 inches upstream from the crown line, while the downstream depth, t, was measured at a distance of 81 feet downstream from the crown line. Throughout the study, scale-model tests were made on 17 variations of the basic embankment design. These tests were made by varying the hydraulic parameters illustrated in Fig. 4 as well as testing various roughness elements.

A plot of the free flow data for the basic model embankment design resulted in the following free flow equation,

$$q = 3.19\, h^{1.53} \qquad \text{. . . . .} \qquad 23$$

The data generated by Kindsvater (1964) for the basic embankment design were plotted three-dimensionally with the discharge per foot of embankment length, q, as the ordinate, the difference in upstream and downstream flow depths, h - t, as the abscissa, and the submergence,

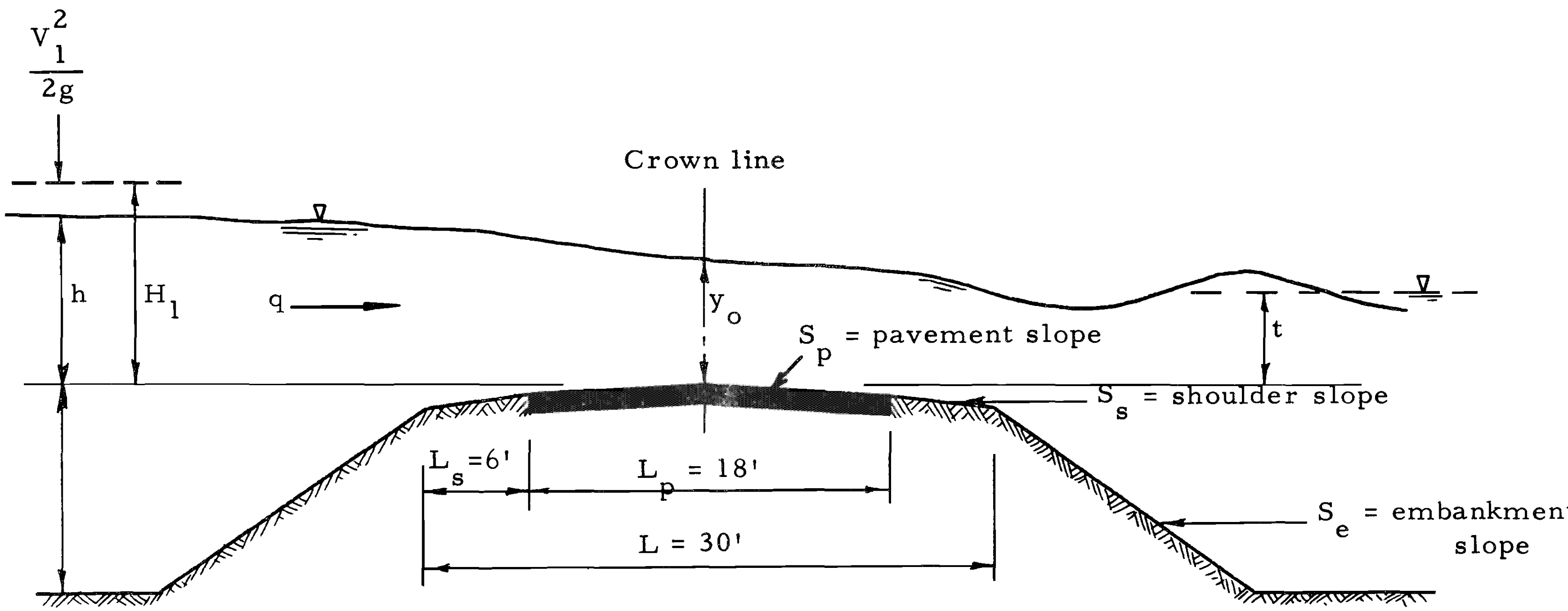

Fig. 4. Principal parameters describing flow over an embankment. (Prototype dimensions)

t/h, as the varying parameter (Fig. 5). Essentially, Fig. 5 is the graphical presentation of the submerged flow equation (Eq. 21).

Lines of constant submergence which best fit the data are drawn with a slope corresponding to the exponent of h in the free flow equation for the basic embankment model. For example, the constant submergence lines of 89.0, 93.7, 95.4, 96.4, 97.5, and 98.5 percent have been drawn in Fig. 5. The slope of these lines of constant submergence is 1.53, which corresponds with the exponent of h in the free flow equation (Eq. 23).

The submerged flow equation which fits the plotted data of Fig. 5 is

$$q = \frac{2.41\ (h - t)^{1.53}}{(-\log t/h)^{1.20}} \qquad 24$$

The basic embankment model data have been converted to prototype data from which an equation was developed and then plotted in Fig. 6 as submerged flow calibration curves for the prototype structure. Thus, Fig. 6 becomes the field rating curves for a highway embankment similar in form to the basic model structure studied by Kindsvater (1964).

As previously discussed, the value of submergence at which the transition from free flow to submerged flow occurs can be estimated by equating the free flow and submerged flow equations. To illustrate the solution for the transition submergence, S_t, the free flow and submerged flow discharge equations for the basic embankment model will be equated.

$$\frac{2.41\ (h - t)^{1.53}}{(-\log t/h)^{1.20}} = 3.19\ h^{1.53} \qquad 25$$

$$0.755\ (1 - t/h)^{1.53} = (-\log t/h)^{1.20}$$

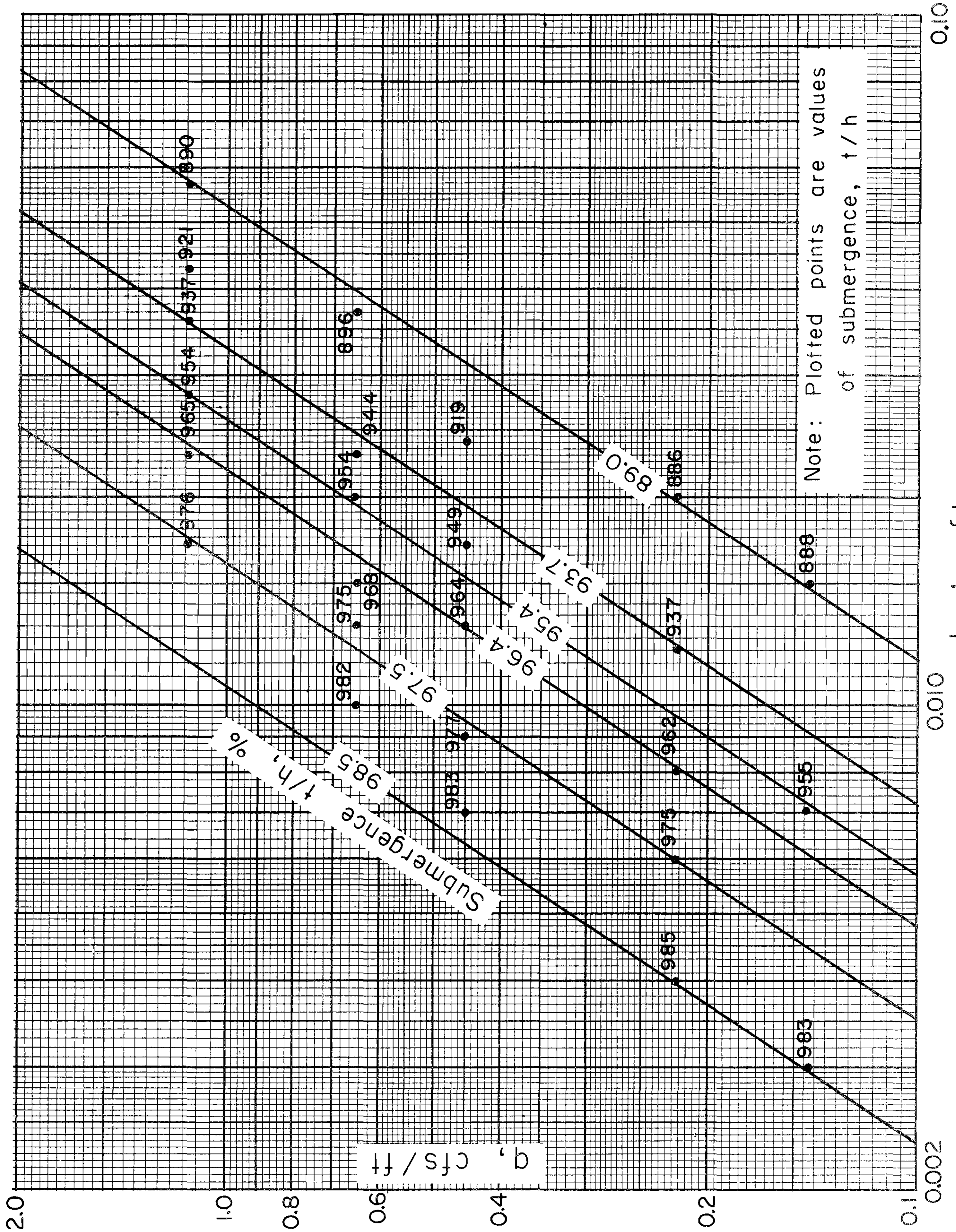

Fig. 5. Plot of submerged flow data for the basic embankment model.

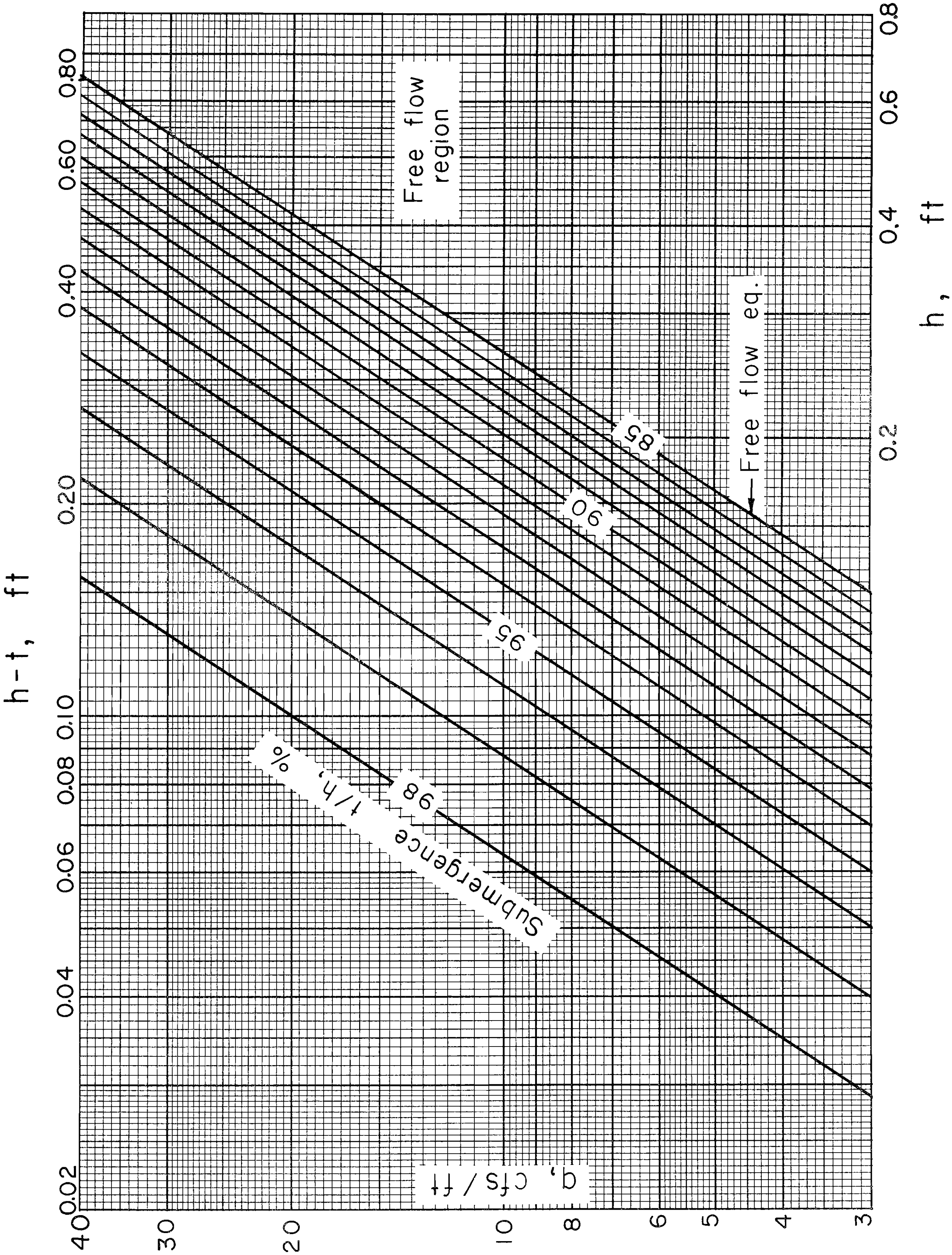

Fig. 6. Submerged flow calibration curves for basic prototype embankment design.

A solution is obtained by trial and error.

$t/h = S_t = 0.849 = 84.9\%$

Kindsvater (1964) states that the value of the transition submergence determined from his study is about 84 percent. The transition submergence given by Kindsvater is based on the downstream flow depth, t, divided by the total upstream head, H_1 (upstream flow depth plus velocity head), whereas the value cited by the authors (85%) is based on the ratio of the downstream flow depth divided by the upstream flow depth, (t/h). The transition submergence will naturally be greater when computed from the ratio, t/h, as compared with the ratio, t/H_1.

The constant submergence line of 85 percent drawn on Fig. 6 is the transition submergence line dividing the free and submerged flow conditions. Hence, Fig. 6 can be used for either free or submerged flow conditions. To obtain a free flow discharge, enter the curve from below with the measured value of h and move vertically upward until the transition submergence line (85%) or free flow line is intersected and then move horizontally to the left to obtain the discharge, q. The submerged discharge is obtained by moving vertically downward with the (h - t) value until the line of constant submergence (t/h) is intersected, then move horizontally to the left to obtain the submerged flow discharge.

Utilizing the data from Kindsvater's (1964) study the other 17 modifications of the basic embankment model were analyzed by Skogerboe, Hyatt, and Austin (1966) in a similar manner. All modifications gave comparable results to those described in this section as to verifying the validity of the method of analyzing submergence presented by the authors.

SUPPRESSED WEIRS

Sharpcrested

Several sharpcrested weirs of varying height were investigated by Cox (1928). The weirs ranged in height from 1.14 to 5.93 feet high. Cox also included in his report data collected on other sharpcrested weirs by Bazin (1888), Fteley and Stearns (1883), and Francis (1871). The submerged flow data reported by Cox on the studies conducted by Bazin (1888), Fteley and Stearns (1883), and Francis (1871) were analyzed utilizing the previously discussed concepts of submerged flow and found to be valid although the results of the analysis are not included in this report. For this report and purposes of illustration only the data on the sharpcrested weirs of height 2.00 and 5.93 feet collected by Cox (1928) are subjected to the submerged flow analysis.

All sharpcrested weirs used in Cox's study were constructed to conform to standard methods at that time with the weir blades cut at a 45 degree angle and a 1/16 inch thickness at the weir crest.

The 2.00 foot high weir was tested in a channel 87 feet long, 1.96 feet wide, and 4.5 feet deep. The discharge for the 2.00 foot high weir was measured by an 8" x 4" Venturi Meter.

The 5.93 foot high weir was tested in a different channel which was 66.5 feet long, 2 feet wide and 8.5 feet deep. The discharge for the 5.93 foot high weir was measured by means of a standard rectangular weir with a notch 2 feet long.

Cox (1928) investigated the relationship between the weir height and the location for measurement of the tailwater depth. He concluded that the tailwater depth should be measured at a distance downstream from the weir of 2.54 times the weir height. In his studies on the sharpcrested weirs the tailwater depth was measured at this point. The upstream depth of flow was measured at a distance of 6.0 feet upstream from the weir. Both the upstream and tailwater depths were measured with point gages. The submergence was varied during the study by adjusting a vertical lift gate placed downstream from the weir.

A plot of the free flow data of the two weirs resulted in the following equations:

for the sharpcrested weir 2.00 feet high,

$$Q = 6.85\ h^{1.55} \qquad (26)$$

and for the sharpcrested weir 5.93 feet high,

$$Q = 6.86\ h^{1.52} \qquad (27)$$

Figs. 7 and 8 result from plotting three-dimensionally the submerged flow data generated from Cox's study (1928). The lines of constant submergence were drawn to best fit the data at the same slope as that which results from the free flow plot, 1.55 in Fig. 7 for the 2.00 foot high weir, and 1.52 in Fig. 8 for the 5.93 foot high weir. Also shown are lines of constant submergence ranging in value from approximately 0.0 to 99.9 percent submergence with a few values that are negative or less than zero.

Cox (1928) states the negative submergence values result from the

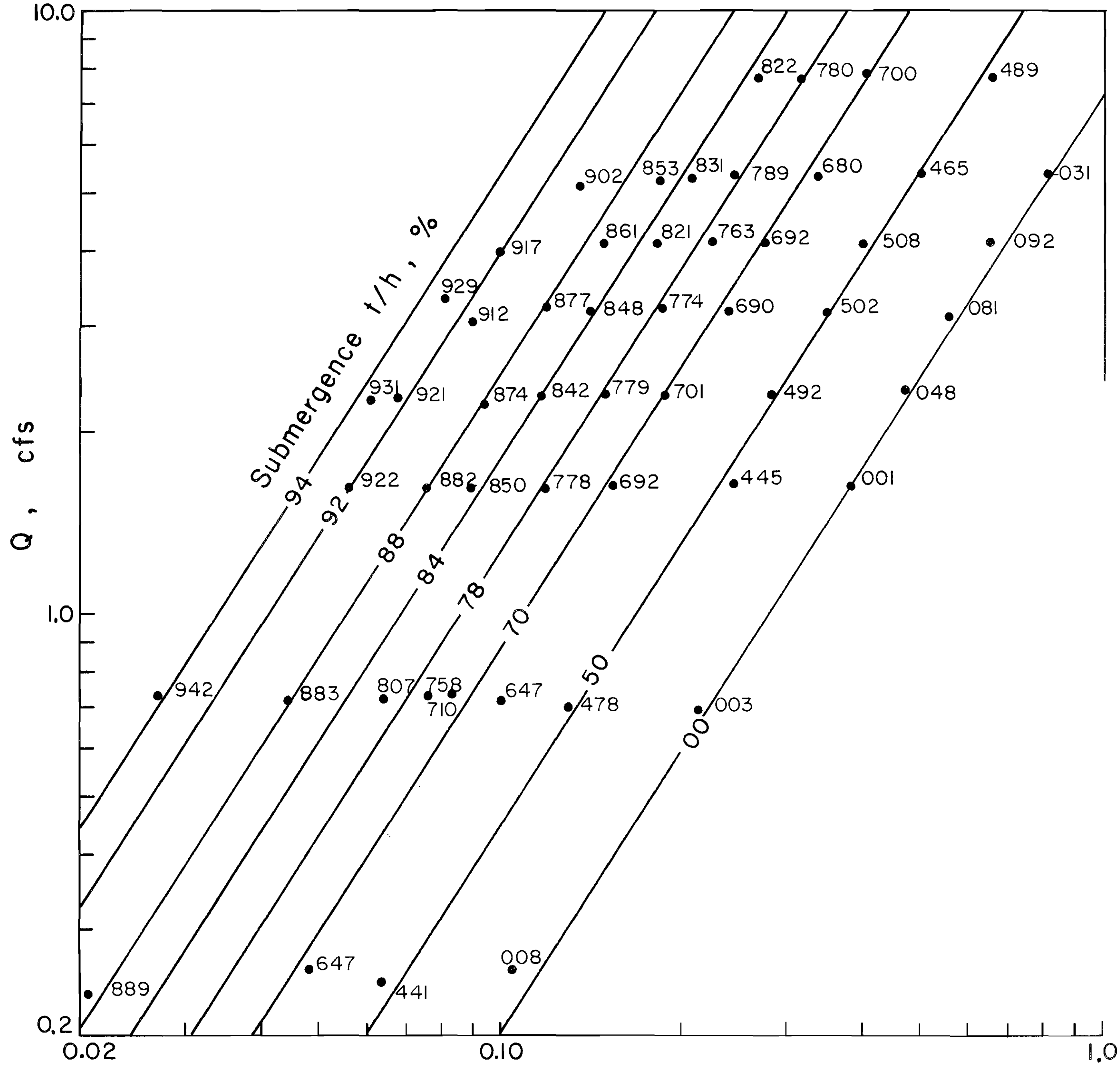

Fig. 7. Plot of submerged flow data for 2.00 foot high sharpcrested weir.

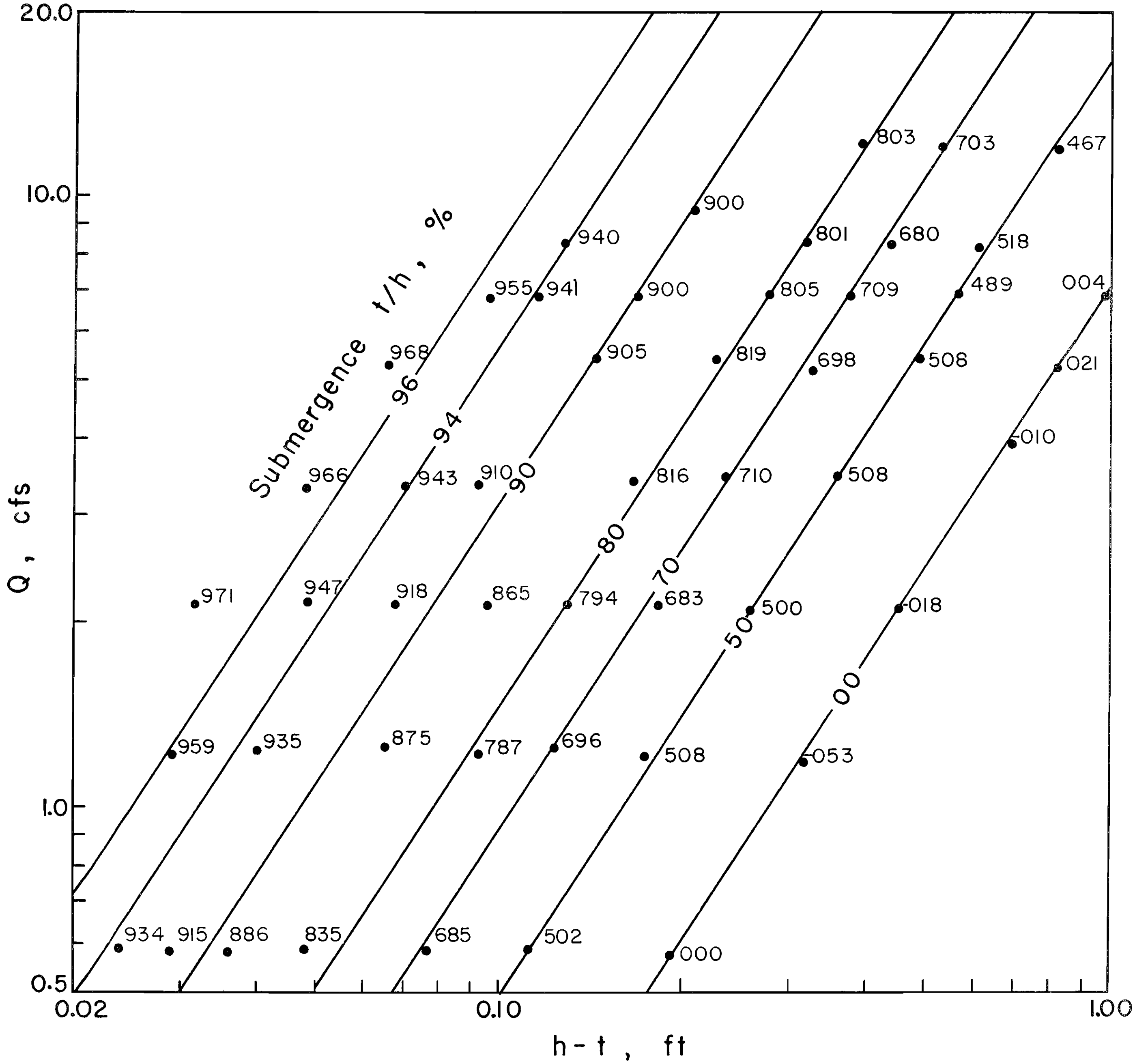

Fig. 8. Plot of submerged flow data for 5.93 foot high sharpcrested weir.

nappe over the weir plunging below the surface. Cox found that the flow condition was changed, depending on which of the two nappe condition existed. At the lower values of submergence it was possible for the nappe to plunge below the surface and remain there for a considerable distance from the weir before it came to the surface. At the point the nappe jet reached the water surface the surface velocity moved both upstream towards the weir and downstream towards the outlet. The other nappe condition occurred as the tailwater level rose to the point where the nappe no longer plunged below the surface but remained on the surface with the velocity directed downstream. Both nappe conditions affected the discharge in a different manner, but there existed a considerable overlap for each flow condition whether the nappe plunged below or on the surface. Cox recommended that the submerged sharpcrested weir be operated with the nappe remaining on the surface.

Crump (1952) found the same condition (the nappe plunging under the surface at low submergences and negative submergence values) when he analyzed the submerged flow data of Francis (1871) and Fteley and Stearns (1883) for the sharpcrested weir. The explanation offered by Crump for this condition is that the transition submergence is reached well before the tailwater depth rises to the crest level. Before the transition point is reached, however, the nappe is discharging into free air and the air pocket is maintained at atmospheric pressure from the vents in the side walls. Located below the air pocket is a mass of turbulent water. Then as the tailwater depth rises the air pocket is replaced by water at less than atmospheric pressure increasing the curvature and velocities of the nappe at the control section. This actually carries a greater flow through the control section for the same given upstream depth. This condition exists until the degree of submergence rises, which increases the pressure on the under side of the nappe to the point where the air pocket is completely filled with water and drowns out the effect.

The authors can only speculate as to the effect the nappe (either plunging below the surface or remaining on the surface) has on various flow conditions without making additional studies themselves. Current plans involve investigation later this year to give further study and answer questions on concepts dealing with submerged weirs. Based on the data thus far studied, the authors feel explanation offered by Crump (1952) is correct except for the fact that the flow regime still passes through critical depth, though the value and location of the depth probably changes, until the transition submergence is reached somewhere between the submergence value of 30.0 and 60.0 percent.

Because of the difficulties encountered in analyzing submerged flow conditions at the lower submergence values, a submerged flow equation was not written to include submergence values from 0 to 50 percent. Neither was an equation written to include submergence values from 96 to 99.9 percent because of the changing coefficient, C_2, in this range. The lines of constant submergence drawn in Figs. 7 and 8 can be described by the following equations for a range of submergence between 50 and 96 percent:

for the sharpcrested weir 2.00 feet high,

$$Q = \frac{4.83\ (h - t)^{1.55}}{-\ (\log t/h + 0.0015)} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad 28$$

and for the sharpcrested weir 5.93 feet high,

$$Q = \frac{4.66\ (h - t)^{1.52}}{-\ (\log t/h + 0.0015)} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad 29$$

Noted in Eqs. 28 and 29 is the presence of the coeffient, C_2, with a value of 0.0015 which shows that the sides of the suppressed weirs exert an effect on the flow conditions.

Although Eqs. 28 and 29 are limited because they describe only the submerged flow regime from 50 to 96 percent submergence they merit value because of the form they take. Both equations are of the equation form previously described (Eq. 21).

OGEE CREST WEIRS

2 to 1 vertical face

Cox (1928) conducted an investigation on submerged flow over three ogee weirs with a 2 to 1 upstream face which had heights, P, of 1.24, 2.13, and 6.11 feet. Although the submerged flow analysis of the authors was applied to the data collected by Cox for all three weirs only the weir with a height of 2.13 feet is discussed in this report. However, the data from all three of the 2 to 1 upstream faced ogee weirs when analyzed portray the validity of the submerged flow analysis.

Fig. 9 shows a plan view of the typical 2 to 1 upstream faced ogee

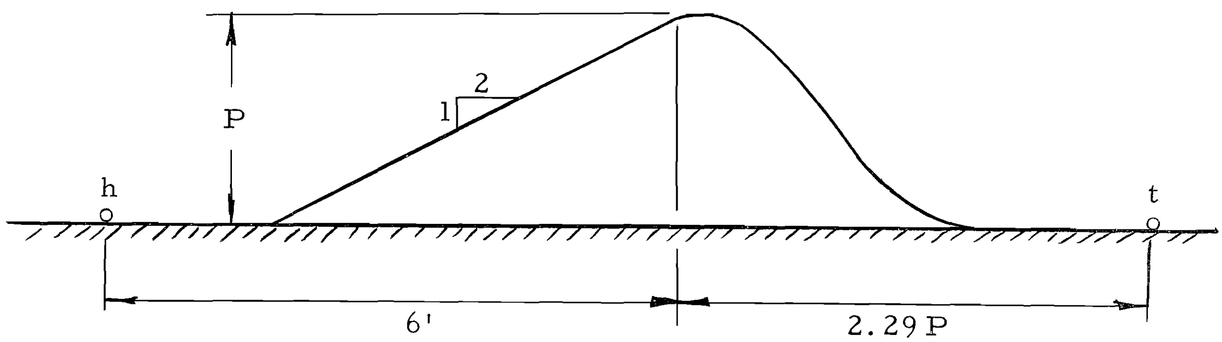

Fig. 9. Plan view of a 2 to 1 upstream faced ogee crest weir.

weir study by Cox (1928). The point Cox selected for measurement of the downstream depth was 2.29 times the weir height, P, and the upstream depth was measured at a distance of 6 feet upstream from the face of the dam (Fig. 9).

The 2.13 foot high weir was tested in the same channel as the 2.00 foot high sharpcrested weir and in a similar manner with the discharge measured by a 8" x 4" Venturi Meter, flow depths measured with a point gage, and the submergence varied by adjusting a vertical lift gate placed downstream.

The free flow equation which described the 2.13 foot high 2 to 1 vertical faced ogee crest weir is

$$Q = 7.28\ h^{1.55} \qquad \qquad 30$$

The submerged flow data collected by Cox (1928) for the 2.13 foot high weir were plotted as shown in Fig. 10. Further noted in Fig. 10 are the lines of constant submergence which range in value from approximately 0.0 to 99.9 percent submergence. The explanation for this is dealt with in the section on sharpcrested weirs.

The submerged flow equation which describes the data plotted in Fig. 10 for the weir 2.13 feet high is

$$Q = \frac{4.35\ (h - t)^{1.55}}{[\ -(\log t/h + 0.0015)]^{1.21}} \qquad \qquad 31$$

Eq. 31 is written to cover only submergence values from 50 to 96 percent as was previously explained. The power on the h - t term in Eq. 31 is the same as that on the h term in Eq. 30 and is the slope of the lines of constant submergence in Fig. 10.

The constant C_2, which equals 0.0015 in Eq. 31, indicates the side effects on the discharge. Eq. 31 is similar in format to the basic submerged flow equation (Eq. 21) and further shows the validity of submerged flow analysis presented by the authors.

Cox (1928) states that an ogee weir that has been silted up will operate as though no silting had occurred as long as the nappe flows under, but when the nappe flows on the surface the weir will operate differently. Hence, Cox recommends that the ogee weirs be operated with the nappe flowing under.

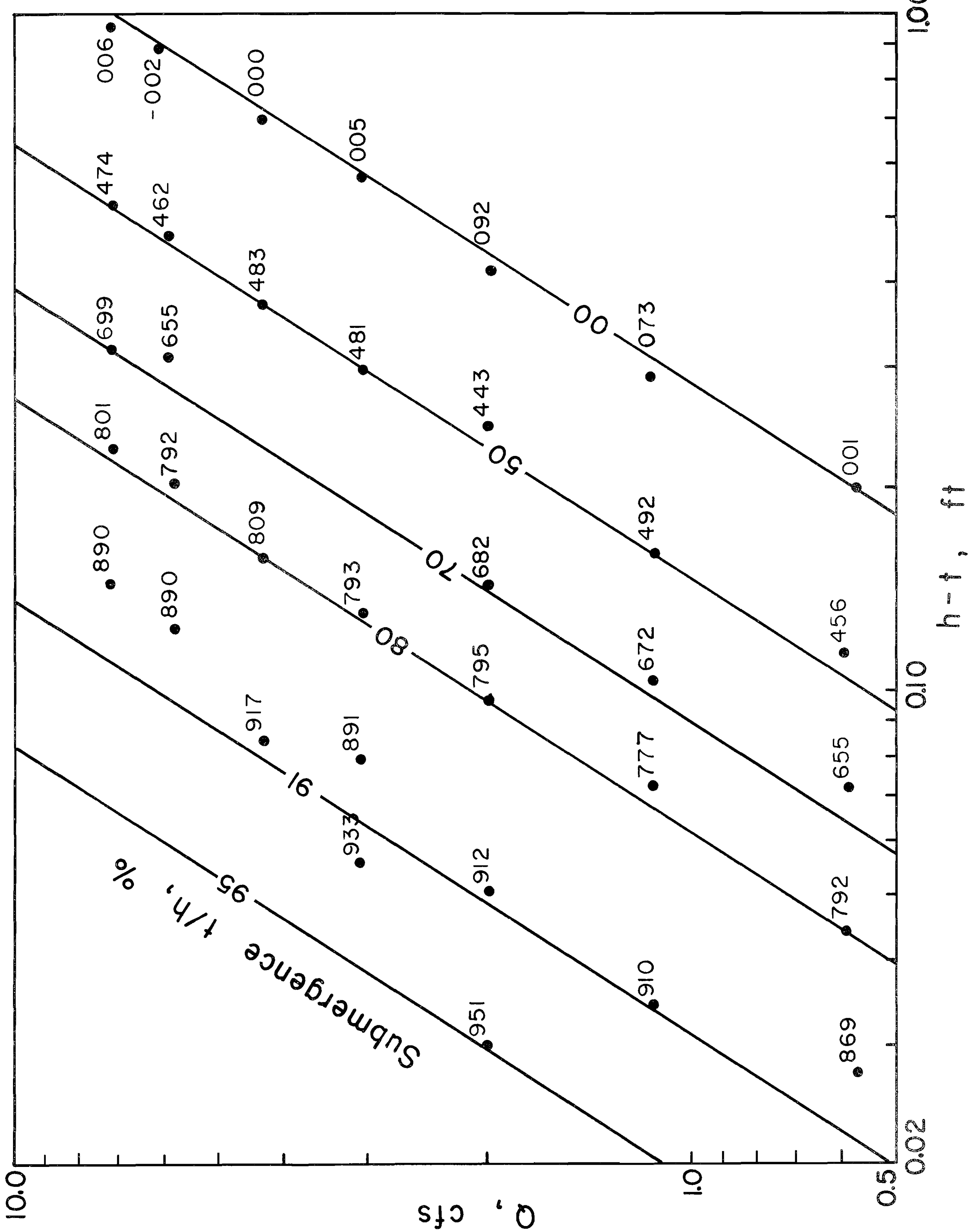

Fig. 10. Plot of submerged flow data for 2.13 foot high 2 to 1 upstream faced ogee crest weir.

Vertical upstream face

Suppressed ogee weirs with a vertical upstream face were another type of weir investigated by Cox (1928) in his study. These weirs had heights of 1.24, 2.13, and 6.11 feet. Fig. 11 shows the shape of weir studied and the location of the points where measurements were taken.

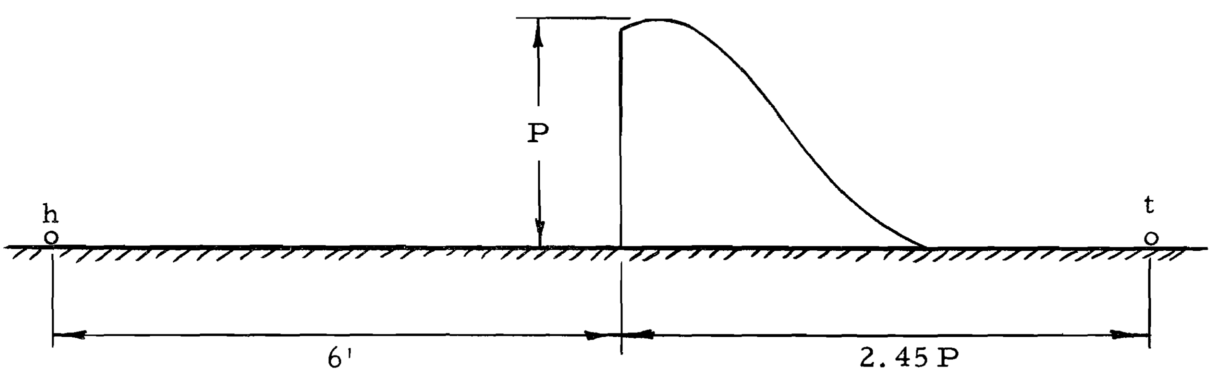

Fig. 11. Plan view of vertical upstream faced ogee crest weir.

Analysis of Cox's submerged flow data indicates some discrepancies and questions in the data of the vertical faced weirs. Plotting the free flow data for the 1.24, 2.13, and 6.11 foot high weirs results in the same free flow equation for all three weir heights, which does not appear logical to the authors. The free flow equation which results for these three weirs is

$$Q = 7.25\, h^{1.58} \quad \ldots \quad (32)$$

When the submerged flow data is plotted as previously described (Fig. 12), the slope of the lines of constant submergence for any one of the three weirs is not consistent or uniform. For example, the data of the vertical upstream faced weir 6.11 feet high with a crest width of 2.03 feet is plotted in Fig. 12. The slope of the lines of constant submergence which best fit the data through the submergence range of 0.0 to 80.0 percent is 1.58. Thereafter the slope of the constant submergence lines decreases as the value of submergence increases until at 96 percent submergence the slope has a value of about 1.36. This same trend exists for the submerged flow plots of the 1.24 and 2.13 feet high weirs.

Koloseus (1951) investigated an ogee crest spillway which was almost identical to the structure studied by Cox (1928). In the ogee crest spill-say section the data of Koloseus plots in a manner consistent with that proposed by the authors for a submerged flow plot. Hence, the only explanation offered by the authors for the changing slope in the submerged flow plot of Cox's data (Fig. 12) is that some of the data are in error.

The authors have included Fig. 12 in this report because it shows that a trend exists similar to that proposed by the authors and verifies to a certain extent that the submerged flow analysis presented in this report is valid for another weir shape.

Due to the plotting of the data for vertical upstream faced weirs (Fig. 12), no attempt was made to write a submerged flow equation describing these weirs. The lines of constant submergence in Fig. 12 show only the trend of the data.

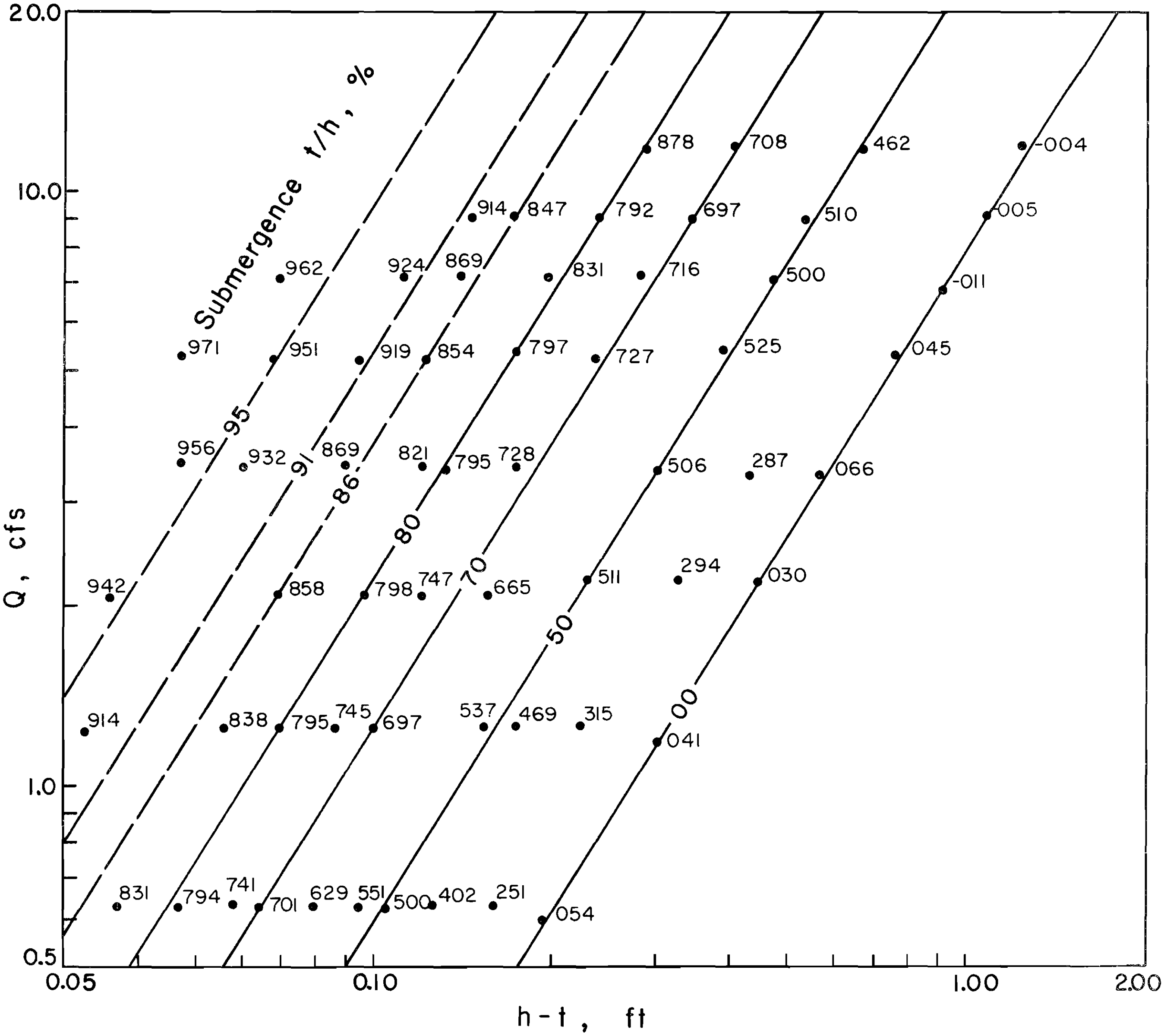

Fig. 12. Plot of submerged flow data for vertical upstream faced ogee crest weir 6.11 feet high.

Further mention is also made of the recommendation given by Cox (1928) that better operation occurs when ogee weirs are used with the nappe plunging below the surface.

Ogee crest spillways

Koloseus (1951) conducted model studies on several small diversion dams or ogee crest spillways to obtain the design critera for these structures. The submerged spillways had an ogee section which conformed to the profile of the lower nappe from a ventilated sharp crested weir. The upstream face of the crest was vertical. Typical of the spillways studied by Koloseus is that shown in Fig. 13.

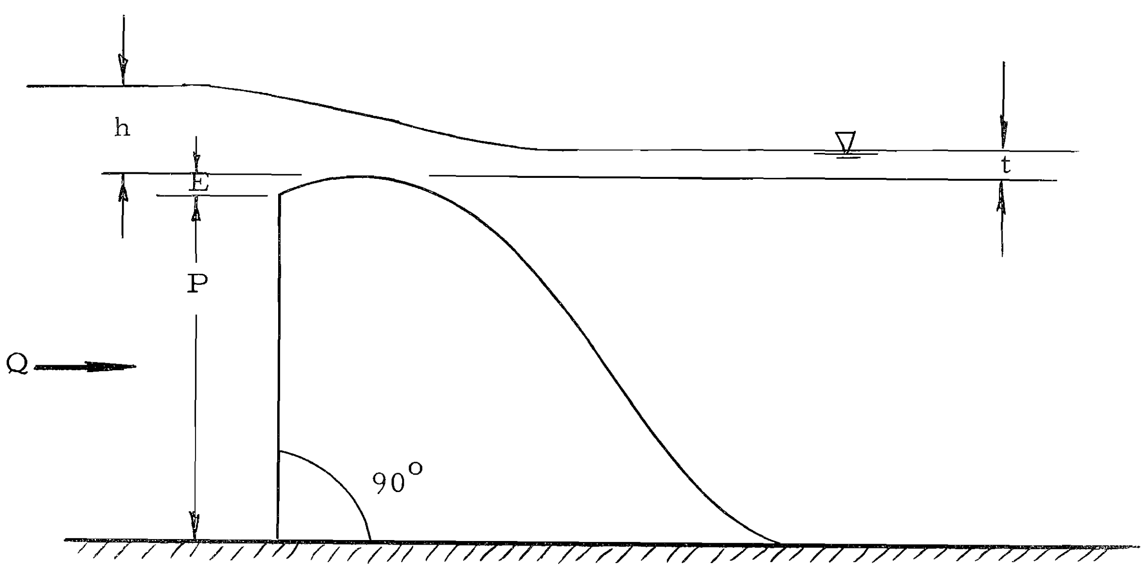

Fig. 13. Plan view of typical ogee crest spillway studied by Koloseus.

The spillways studied had heights of 0.5, 1.0, 1.5, and 2.0 feet with $h_d/(P + E)$ ratios varying from 1/4 to 1. The testing was conducted in a glass-walled flume 18 feet long, 2 feet wide, and 3 feet high. The spillway models were placed 6.3 feet downstream from the flume entrance. Measurement of the flow depths, h and t, and the water surface profile was made with a traversing point gage. The value used for the flow depths was taken as the average of three measurements taken downstream from the flume entrance, at distances of 1, 2, and 3 feet for the upstream depth of flow, and at 14, 15, and 16 feet for the downstream depth of flow. Measurements of the discharge were made by the use of a 5 inch orifice for the lower discharges and a 10 1/2 inch orifice for the higher discharges.

The data from two of the model spillways were selected from the thesis of Koloseus (1951) to illustrate the submerged flow analysis. One spillway had a height, P + E, of 1.0 foot with a $h_d/(P + E)$ ratio of 1/2 and the other spillway had a height of 1.5 feet with a $h_d/(P + E)$ ratio of 1/4.

A plot of the free flow data of the two spillways analyzed resulted in the following equations:

for $h_d/(P + E) = 1/2$,

$$q = 4.69\, h^{1.69} \qquad (33)$$

and for $h_d/(P + E) = 1/4$,

$$q = 4.79\, h^{1.66} \qquad (34)$$

The data generated by Koloseus (1951) for the two forementioned spillways were plotted three dimensionally with Q as the ordinate, h-t as the abscissa, and t/h as the varying parameter (Figs. 14 and 15). In

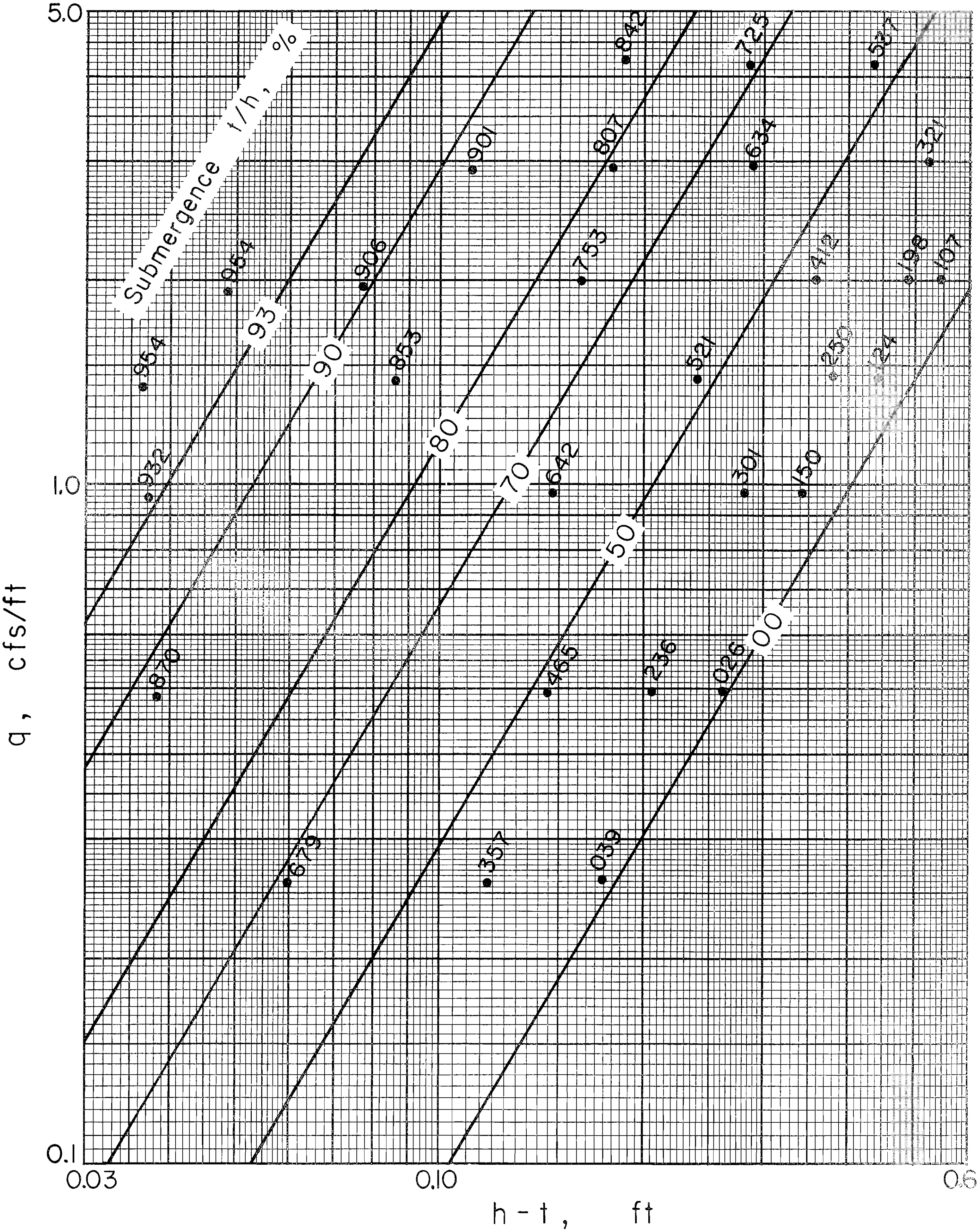

Fig. 14. Plot of submerged flow data for ogee crest spillway with $h_d/(P + E)$ ratio of 1/2.

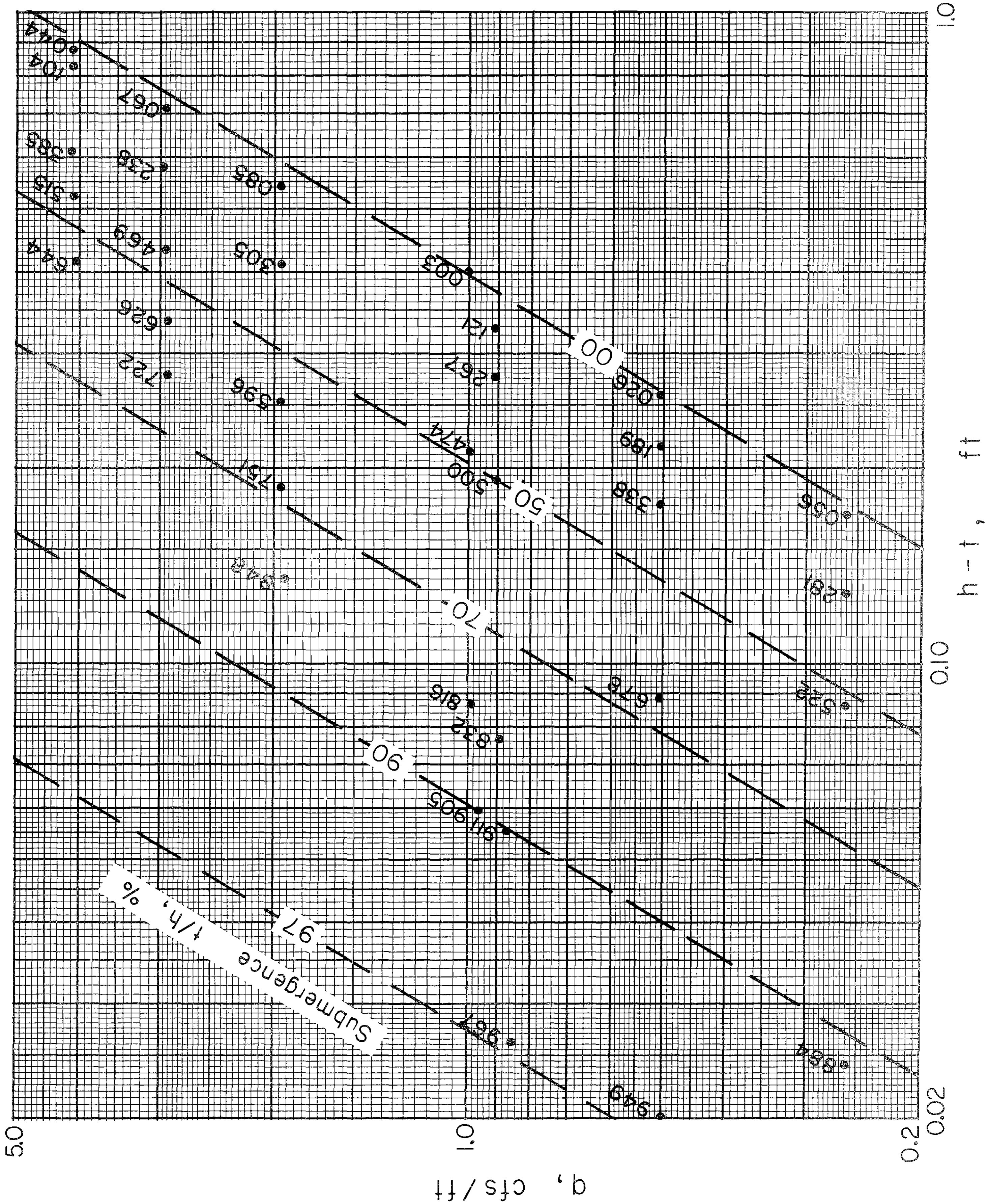

Fig. 15. Plot of submerged flow data for ogee crest spillway with $h_d/(P + E)$ ratio of 1/4.

Figs. 14 and 15 the lines of constant submergence were drawn to best fit the data. The slope of these lines of constant submergence is the same as that which resulted from the plot of the free flow data, 1.69 in Fig. 14 for the $h_d/(P + E)$ ratio of 1/2 and 1.66 in Fig. 15 for $h_d/(P + E)$ ratio of 1/4. The dashed lines of constant submergence drawn in Fig. 15 are so drawn because of the scarcity of the data provided by Koloseus for that particular spillway. The slope of these lines of constant submergence is that indicated by the free flow plot. However, the data does not fit this slope as well as desired, probably due to some error in the data.

Figs. 14 and 15 show that the lines of constant submergence range from approximately 0.0 to 99.9 percent submergence (previously explained in the sharpcrested weir section). However, study of Koloseus' data (1951) by the authors indicated that the upstream depth of flow for any given discharge did not change until the downstream depth reached 50 to 60 percent of the upstream depth (transition submergence). This occurred even though Koloseus took the submergence ratio as zero when the downstream depth was at the spillway crest level.

The submerged flow equation written to fit the lines of constant submergence from 50 to 95 percent in Fig. 14 for the $h_d/(P + E)$ ratio of 1/2 is

$$q = \frac{3.44\,(h - t)^{1.69}}{[\,-(\log t/h + 0.0025)]^{1.20}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 35$$

Mention is made that the coefficient C_2, 0.0025, appears in Eq. 35. No equation was written for the data in Fig. 15 because of the scarcity of data.

The U. S. Bureau of Reclamation (1948) conducted model studies of submerged flow over overfall dams which were very similar in structure to those studied by Koloseus (1951). Although not included in this report, the data collected in the Boulder Canyon project (U. S. Bureau of Reclamation, 1948) was subjected to the submerged flow analysis presented by the authors and found to be valid. The Bureau of Reclamation's study was limited in the range of discharge employed but the data did indicate that the submerged flow analysis reported herein was applicable.

TRIANGULAR- SHAPED WEIR OF CRUMP

The data collected by Crump (1952) for a submerged weir of triangular profile was also subjected to analysis. As shown by Fig. 16, the weir-block had an upstream face sloped at 2 to 1 and a downstream face sloped at 5 to 1—producing a crest-angle of 142 degrees 7 minutes—both of which were truncated to give two vertical faces. The weir-block had a length of 14 inches and a crest height above the steel flume floor of 3.063 inches. Measurement of the upstream and downstream depths of flow, at the points shown in Fig. 16, were made using manometers and the discharge was obtained from a standard V-notch weir placed upstream from the model. The discharge was kept constant for each run and the tailwater varied by a gate at the downstream end of the 22 inch wide rectangular flume.

The equation which describes the free flow condition over the model studied by Crump is

$$Q = 8.33\,h^{1.75} \qquad \text{. . . . .} \qquad 36$$

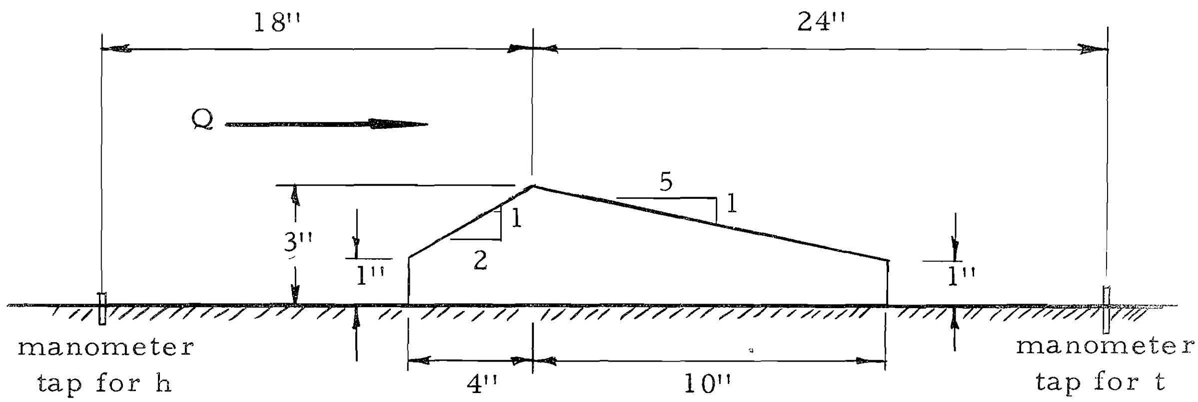

Fig. 16. Plan view of the test model triangular-shaped weir of Crump.

The submerged flow data which resulted from this study were plotted three dimensionally as shown by Fig. 17. The slope of the lines of constant submergence which best fit the data is 1.75, the same slope given by Eq. 36. Though the slope of 1.75 was selected for use in this analysis, some flexibility exists. The data from Crump (1952) furnished only four free flow discharge values and hence only four points on the free flow plot. A slope of 1.75 fits these points very well but the slope could easily be varied by $\pm$ 0.02.

Fig. 17 shows several lines of constant submergence drawn at a slope of 1.75 which range in value from 75.0 to 97.7 percent. The submerged flow discharge equation written to describe these lines over this range is

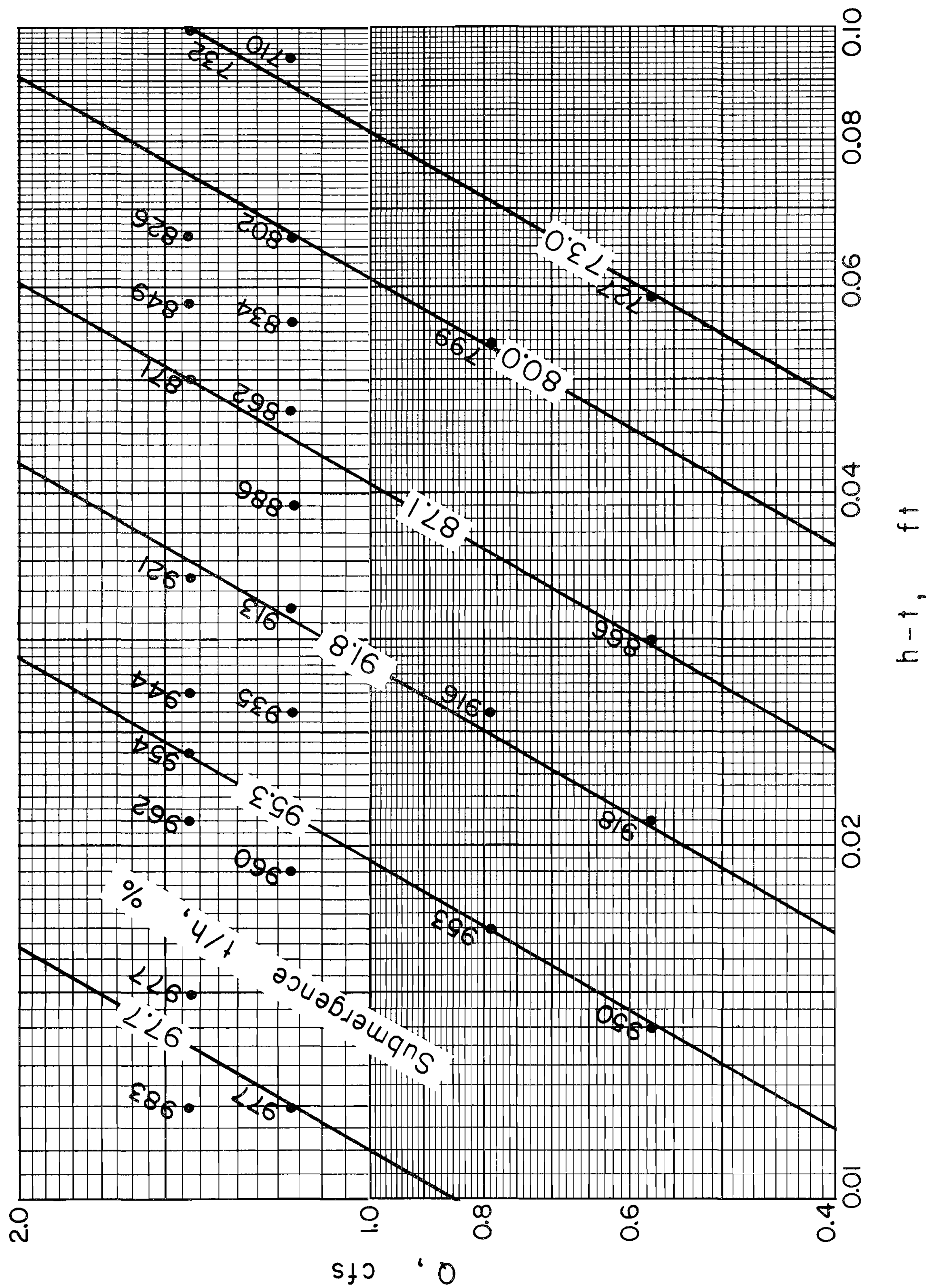

Fig. 17. Plot of Crump's triangular shaped weir submerged flow data.

$$Q = \frac{5.71\ (h - t)^{1.75}}{[\ -\ (\log t/h)]^{1.36}} \quad \ldots \quad 37$$

Utilizing Eq. 37, the calibration curves for this particular triangular-shaped weir are drawn as shown by Fig. 18.

Further understanding can be gained from Fig. 18 by equating Eqs. 36 and 37 for the solution of the transition submergence, S_t. The transition submergence solves to be 77 percent and is drawn on Fig. 18 as the dividing point between free and submerged flow conditions. Justification for the transition submergence obtained by the authors may be made by quoting from Crump (1952): "It is seen that the model has a high modular limit (transition submergence) of 70 percent, and that with a submergence ratio of 80 percent the departure from modularity is less than 1 percent."

Fig. 18 can be used as the calibration curve for either free flow or submerged flow conditions. The free flow discharge is obtained by entering from below with the measured value of h and moving vertically upward until the 77 percent submergence line or free flow line is intersected and then moving horizontally to the left to obtain the discharge, Q. The submerged flow discharge is obtained by moving vertically downward with the measured h - t value until the line of constant submergence t/h is intersected and then moving horizontally to the left.

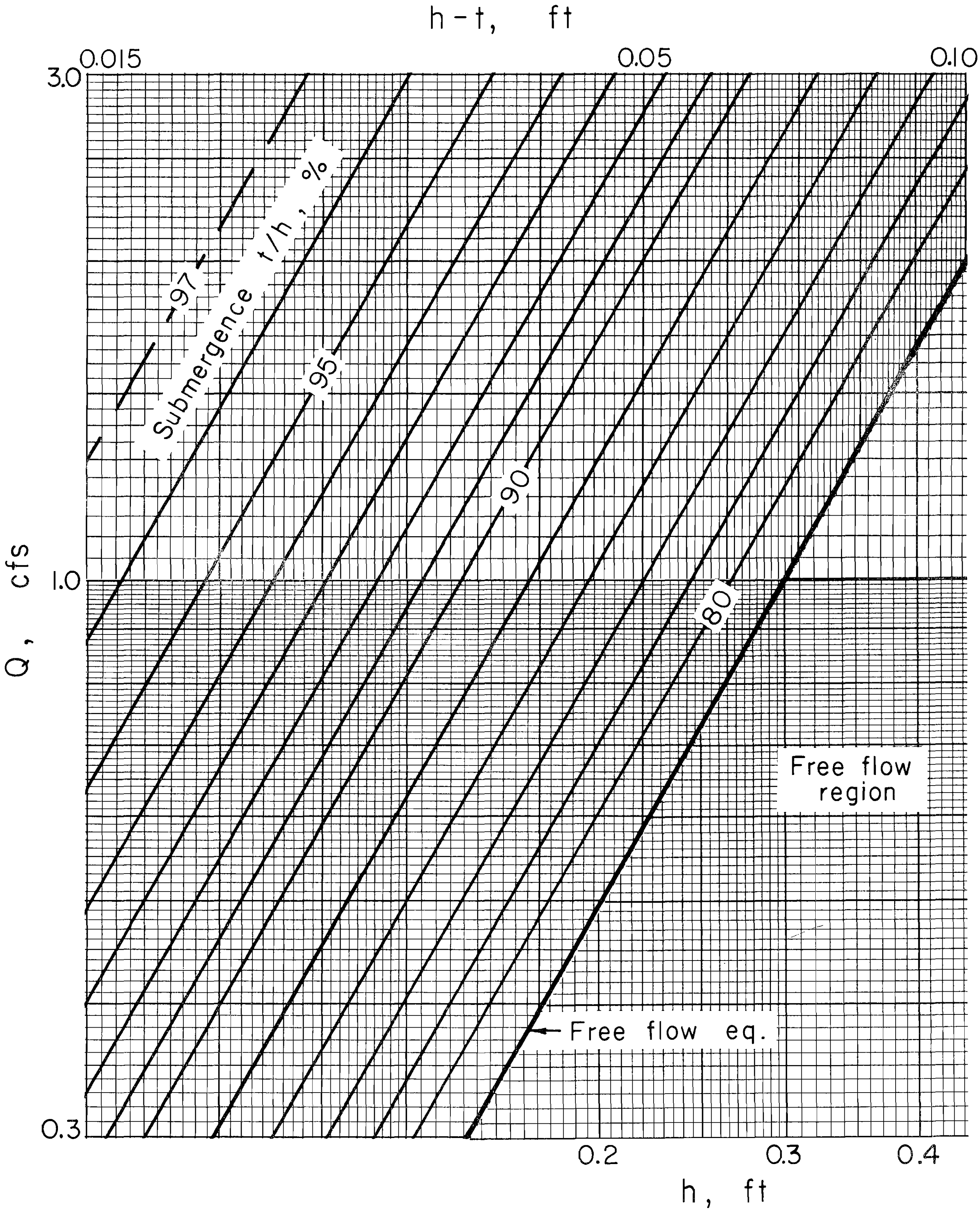

Fig. 18. Submerged flow and free flow calibration curves for triangular shaped weir of Crump.

SUMMARY

The parameters which describe submerged flow in measuring flumes are developed by theoretical momentum relationships and supplemented with dimensional analysis. These submerged flow parameters and method of analysis are shown to be valid for many types of weir structures, such as sharpcrested, ogee, embankment-shaped, and triangular. The data for analysis was taken from selected submerged flow studies found in the literature. The plots and examples found in the report are illustrative of the compatability and validity of the submerged flow analysis developed by the authors.

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