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An Economic Appraisal of Reuse Concepts in Regional Water Supply Planning

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ABSTRACT

Using a conceptual model of a water supply firm, the necessary conditions for production and market efficiency are derived when renovated wastewater is considered as a potential water resource. The nature and extent of the supply augmentation due to recycled reuse is demonstrated using classical optimization techniques. Three stages of short-run supply corresponding to no recycling, partial recycled reuse and complete recycling of all reclaimable water are identified through appropriate Lagrangian Multipliers as well as graphical techniques.

A mathematical programming model is structured to determine the optimal water resource allocation and pricing policy for Salt Lake County. By maximizing the sum of consumer and producer surplus (the difference between total willingness-to-pay and total cost) economically efficient equilibria are derived. The feasibility of recycled reuse for municipal purposes is examined in a planning context. The impact of higher water quality discharge standards on the attractiveness of water recycling option is studied. To ensure social acceptability of renovated wastewater for culinary purposes, blending restrictions are imposed, which stipulate that the amount of water for reuse be less than a fixed percentage of the water from other sources. The effect of such a constraint on the prices and water allocation are delineated.

The hydrologic uncertainty in water supply is treated using stochastic programming techniques. Application of the concepts of single and joint chance-constrained programming are illustrated. The resulting changes in pricing and allocation policies are discussed.

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CHAPTER I

INTRODUCTION

Along with population growth and economic activity, questions relating to the allocation of water resources, pricing policies, wastewater disposal, and environmental degradation have become crucial in the management of water supply and quality. Belonging to the class of natural monopolies, water supply utilities are subject to government regulation in formulating pricing and allocation policies. The quality standards for wastewater discharges are dictated by federal and state ordinances. Due to the absence of competitive elements in the market for water, automatic achievement of economic efficiency cannot be realized, and therefore, planning is essential to aid decision-making. Many planning models (Lynn, 1966; Dracup, 1966; Lofting, 1968; Clyde et al., 1971; Hughes, 1972) have been developed to supply a specified "target" quantity of water at minimum cost. The results of these analyses, however, might not reflect market efficiency since demand for water was not explicitly introduced in the models. The allocation model proposed by Clausen (1970) does incorporate the effect of demand factors, but his profit maximizing objective leads to monopolistic solution and hence a welfare loss. The present study attempts to devise a planning methodology to arrive at policies consistent with competitive equilibrium. This analysis also takes into account social, economic, and legal considerations and their influences on pricing and allocation of water resources.

An important prelude to accomplishing these objectives is to examine alternative sources and costs of supplying water. Technological advances have made available increased resource alternatives in the past decade. One such alternative that has received considerable attention, both from technological and economic points-of-view is the water recycling option. Several planning models incorporating this option have been built (Dracup, 1966; Young et al., 1970; Clausen, 1970; Bishop and Hendricks, 1971) within the context of a mathematical programming framework. But, there has been no attempt to explicitly analyze the nature of supply augmentation by recycling and contrast it with increases in supply achieved through more traditional means such as acquiring water rights for a well or importing water by reaching out further in distance. This research makes use of the tools available in microeconomic theory to examine the effect of the recycling alternative on the supply of water and derive conclusions on the nature and magnitude of supply augmentation due to this alternative.

The total cost of providing water to any user is not only the cost of supplying the intake water, but also the cost of removing, effectively, the wastewater that is generated. Otherwise, the user is likely to impose a social cost on other members of the society. When such negative externalities result, an efficient compromise between the parties involved could theoretically lead to Pareto optimality (Meade, 1952). Due to certain simplifying assumptions used in the theory, high transaction costs and some important technical reasons such as nonseparability, nonmeasurability, and stochasticity of the damage functions (Kneese and Bower, 1968), practical implementation of these theoretical compromises is extremely difficult. One possibility of achieving a practical solution is through social choice. Legislation requiring that effluent discharges meet certain quality standards can be regarded as a collective solution. The local sanitary districts provide this service to the community by collecting the wastewater and treating it to the required standard before discharge. This water can be reclaimed for further reuse in the system. Thus, constraints on the constituents of wastewater have significant impact on water management in three ways. First, when reviewed from the perspective of the community, the demand for water is dependent not only on the price of water, but also on the price of wastewater disposal. Second, the supply conditions are affected since the cost of wastewater treatment will have to be included. Third, the more stringent the water quality standards are, the more attractive will be the recycling option.

Thus, the recycling alternative and water quality standards make it more appropriate to consider the concept of integrated management of water supply and water quality. This study will use this concept in building a planning model to arrive at economically efficient water supply management strategies. The model will be amenable to the incorporation of institutional constraints such as higher water quality standards, social constraints arising out of psychological effects of using recycled water such as stipulating a blending restriction on the renovated water. Furthermore, uncertainties (more technically known as "risks") common in water management due to stochastic hydrology and demand fluctuations (seasonal and random) could be analyzed through this model.

CHAPTER II

REVIEW OF LITERATURE

Many of the studies in the area of water and wastewater management are based on the philosophy of the "requirement approach." Water needs are estimated based on the population and the level of per capita consumption and wastewater facilities required to serve this estimated usage are calculated. Then using systems analysis techniques, policies with regard to water allocation, investment, and waste management strategies are derived. Lack of a good data base to generate demand curves and the additional complexity due to the introduction of demand curves in the model analysis have popularized the use of this approach. Arguments in defense of this method can be found in Harl et al. (1971). The deficiencies underlying this methodology prompted several studies to be undertaken on the demand for water. Since the purpose of this research is to explicitly incorporate demand curves in a mathematical model where reclaimable water will be a potential source of water supply, and obtain pricing and allocation policies consistent with economic efficiency, a brief survey of the existing literature on the demand for water and the mathematical models used in water and wastewater management is in order.

A Survey of Residential Water Demand Studies

Metcalf (1926) presented the relationship between variations in per capita consumption corresponding to a given percentage change in the price of water for 30 cities. Siedel and Baumann (1957) examined the correlation between price and quantity of water consumed in the residential area for 400 cities through a cross-sectional study. Both of these studies failed to compute the demand elasticity. Gottlieb (1963), in the study of the Kansas waterworks, reports the price-quantity relationships in the context of cross sectional as well as time series analyses. His regression of cross-sectional price and income data on the annual water consumption reports elasticities from -1.24 to -0.65.

Gardner and Schick (1964) conducted a cross-sectional demand study for Northern Utah. In this analysis of 43 systems, price, median income, value of homes, per capita lot size, percentage of homes with complete plumbing units, precipitation and temperature were regressed, on per capita daily consumption of water. Only the price and lot size per capita were found to be statistically significant. These two variables were regressed on quantity consumed. Both linear and hyperbolic relationships were hypothesized. A constant elasticity demand curve showed a coefficient of -0.77.

Howe and Linaweaver (1967) separated the residential use of water into domestic component and sprinkling component of water demand. The domestic water demand elasticity is about -0.23 at the mean and the sprinkling demand elasticity ranged from -1.12 to -1.57. Their methodology did remove the bias due to data aggregation. Howe et al. (1971) present a comprehensive analysis of the demand for water in urban, industrial, and agricultural sectors. They focus on the impacts of market trends, public policy and changes in technology on present and future water use patterns.

A Survey of Mathematical Programming Techniques in Water and Wastewater Management

Scientific management of water and wastewater systems has received considerable attention for over a decade. In a pioneering work, Lynn et al. (1962) used a special case of the generalized network model for the sewage treatment plants to design the optimal combination of unit processes to remove a given amount of BOD at minimum cost. Optimization techniques were again used by Lynn (1964) to solve the capacity expansion problem of waste treatment facilities subject to the availability of funds, level of treatment required, quantity of waste, etc.

Sobel (1965) has shown that a desired water quality improvement program (dissolved oxygen) can be arrived at by maximizing the benefit-cost ratio within the framework of a standard linear programming model. Thomann (1965) demonstrates how optimal control of dissolved oxygen can be achieved in the Delaware Estuary through a linear programming (LP) approach using equations to describe the time and space variability.

Gradually, the application of systems analysis techniques came to be more widely used in water supply. Using a quadratic cost function, Lynn (1966) set up a programming model to supply well water at minimum cost. Dracup (1966) proposed that a transportation model be used to supply a given amount of water to each user at minimum cost. This model, which included the water recycling option, is decomposable and was solved using the LP technique.

Dynamic programming, a sequential decision approach developed by Bellman (1962), was used by Liebman et al. (1966) to minimize the cost of providing waste treatment to meet a specified DO standard along a stream.

It was reported by Clausen (1970) that McLaughlin (1967) used an LP technique to maximize the net benefits in a water supply analysis of the South American river basins. A similar approach was taken by Heaney (1968) to model part of the Colorado River Basin water supply system. Loucks et al. (1967) presented two LP models to determine the amount of wastewater treatment required to achieve, at minimum cost, any particular set of stream dissolved oxygen standards within a river basin using the Streeter-Phelps equation for DO profile. A better pollution control scheme using an LP approach to achieve a specified load allocation, in contrast to the uniform removal scheme, was proposed by Johnson (1967) to establish the optimal effluent charge. Stochasticity and time considerations entered the linear dynamic decomposition programming approach used by Shailendra et al. (1967) to optimize the Northern California Water Resource System.

Revelle et al. (1968) applied a linear programming technique for water quality management in a river basin, primarily aimed at selecting the treatment plant efficiencies such that a specific DO standard can be achieved at minimum cost. Using an input-output framework for statewide water resources modeling, Lofting et al. (1968) applied a linear programming technique to optimize allocation of water over time. A nonlinear programming model was proposed by DeVries (1968) to supply water for the municipal sector. The problem was cast in a separable programming framework to represent the nonlinearities in terms of piecewise linear functions. Alternatives in regional waste treatment policies were evaluated by Anderson et al. (1968) for the Miami River Basin using the linear programming method. The system cost was minimized establishing optimal levels of BOD reduction for all treatment plants within their operating efficiencies and the required standard along the river. The "Balas" algorithm was used to solve an integer programming formulation developed by Liebman (1968) to evaluate the effectiveness of the three approaches that have been adopted to achieve water quality goals, viz., the cost minimization approach, uniform treatment approach, and zoned uniform treatment approach.

Dynamic programming was employed to solve the two dimensional multistage allocation problems by Evenson (1969) to arrive at cost minimizing design, to remove a given amount of BOD and total dissolved solids. An optimal investment scheme in water supply projects in response to growing demand conditions was proposed by Butcher et al. (1969) using a dynamic programming approach. Milligan (1969) used a linear programming model for optimum conjunctive use of groundwater and surface water.

Shih and DeFilippi (1970) employed dynamic programming to design a multistage waste treatment plant which would meet given specifications at minimum cost. The optimal solution establishes the combination of the unit process and their efficiencies, thus obtaining the optimal design of the integrated system.

A nonlinear programming model with a water recycling possibility was set up by Young et al. (1970) and solved using a long-step gradient method based on the method of feasible directions. The cost functions reflected economies of scale. Changes in demand over time and changes in cost conditions due to technology were given consideration. The work is important in that it placed the water resources problem in a wider perspective, but failed to study the theoretical aspects of the model. Clausen (1970) recognized the demand for water is not completely inelastic and used a profit maximization approach to solve the water allocation problem. The model takes into account the water reuse alternatives. The model uses quadratic cost functions and was decomposed into subproblems. The problem was solved using the decomposition principle.

Harl et al. (1971) employed a linear programming model for optimal water allocation. A river quality simulation model was used in conjunction with the LP model such that the two models interact. The LP problem is solved using a set of parameters generated by the quality model and the solution is fed back to the simulation subroutine. This process is repeated until changes in the parameters and changes in the LP solution cease. Bishop et al. (1971) evaluated the reuse alternative in water supply using a transshipment model. Clyde et al. (1971) developed an LP approach to state-wide water resource planning. Haimes (1971) employed the multilevel approach to nonlinear optimization for pollution control. The same technique is again used (Haimes et al., 1972) in determining the optimal taxation that will achieve the required quality. Hinomoto (1972) made use of dynamic programming in planning capacity expansion of water treatment systems. A concave cost function reflecting economies of scale was minimized over the solution space to yield to optimal time and size of plant capacities. Hughes (1972) proposed the use of mixed integer programming to water supply planning.

Wanielista (1972) and Converse (1972) optimized the size and location of treatment plants using a dynamic programming approach. Uri Regev et al. (1973) take up the problem of simultaneous optimization of investment and allocation of water. A discrete time control theory is applied in which interaction of regional and seasonal consideration play a crucial role. The cost functions reflecting increasing returns to scale were treated as integer variables, so that theoretically a global optimum is guaranteed. Mulvihill et al. (1974) constructed a mathematical model with a water reuse option to determine optimal timing and sizing of water and waste treatment facilities. The co.' functions were concave, reflecting economies of scale. Relative optimum was arrived at using a multilevel approach.

The literature survey of the demand for residential water will aid in selecting an appropriate study to be used in this research. The state-of-the-art summary of various systems analysis techniques applied to water and wastewater management will establish a suitable framework of analysis for this proposed study.

CHAPTER III

A MICROECONOMIC THEORY OF WATER RECYCLING PROCESS

In this section, an integrated approach to water supply and quality management utilizing the tools available in microeconomic theory, will be proposed. In the examination of the factors determining the demand for water, the price of collecting and treating the resulting wastewater will be shown importantly to enter in individuals' decision-making. A theory of water supply will be described with water recycling option to illustrate the nature and magnitude of supply augmentation that this alternative could provide. Then, market equilibrium conditions consistent with economic efficiency will be derived and later the same technique will be extended to the analysis of multiuser problems.

The Theory of Demand

Since the following analysis is pertinent only to the municipal sector, it will be assumed that water is an economic good and individuals behave as though they maximize utility. It will further be assumed that the wastewater discharged by a consumer is a constant fraction k of the gross intake of water. Let

$$u_i = u_i\,(AOG_i, W_i)$$

in which u_i, the utility derived by the i^{th} individual, is a function of all other goods AOG_i and the quantity of intake water W_i consumed by the i^{th} individual. If P_{AOG} is the price of "all other goods," P_w is the price of intake water and P_s is the price paid for sewage services, then the income constraint can be written as

$$P_{AOG} \cdot AOG_i + P_w W_i + P_s S_i = I_i$$

in which S_i is the quantity of sewage discharged by the i^{th} individual and I_i is his income. Since $S_i = kW_i$ the budget constraint of the i^{th} individual becomes

$$P_{AOG} \cdot AOG_i + P_w W_i + P_s kW_i = I_i$$

To maximize the utility function subject to the above constraint the Lagrange function can be written as

$$L = U_i\,(AOG_i, W_i) - \lambda(P_{AOG}\ AOG_i + P_w W_i + P_s kW_i - I_i)$$

The first order conditions are

$$\frac{\partial L}{\partial AOG_i} = \frac{\partial U_i}{\partial AOG_i} - \lambda P_{AOG} = 0$$

$$\frac{\partial L}{\partial W_i} = \frac{\partial U_i}{\partial W_i} - (P_w + kP_s)\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = P_{AOG} \cdot AOG_i + P_w W_i + kP_s W_i - I_i = 0$$

The marginal conditions derived from manipulating the first order conditions are

$$\frac{MU_{\text{all other goods}}}{P_{\text{all other goods}}} = \frac{MU_{\text{water}}}{P_{\text{water}} + kP_{\text{sewage}}}$$

That is, the ratio of marginal utility of all other goods to the price of all other goods should equal the marginal utility of water to the price of water and the price of treating the effluent from that unit of water to the required quality.

The implication of these necessary conditions for utility maximization is that when estimating the demand for water statistically, the price of sewage services (if any) will have to be introduced as an independent variable. This analysis presumes that sewage services do not directly yield any utility, or in other words, do not appear as an argument in the utility function.

Similarly, for the production sector, it can be shown that the cost minimization assumption yields the necessary condition that the ratio of the marginal product of "all factors" to the "price of all factors" should equal the ratio of the marginal product of water to the sum of the price of a unit of water and the price of cleaning up the effluent resulting from that unit of water to the prescribed quality.

By varying the price of water and treatment services, it is possible to generate the equilibrium quantities of water intake (and therefore the sewage discharged) consistent with utility maximizing. This is nothing more than the individual's demand curve for the services of water. By aggregating this individual demand curve over all individuals, market demand curves can be obtained.

The Theory of Supply

A flow diagram of a simple one-sector water supply model is shown in Figure 1.

Water can be supplied to the user from primary or recycled sources. A primary source is defined here as all other sources of water excepting the recycled source, for instance, groundwater, surface water and import water. The wastewater discharged by the user will be subjected to secondary treatment to meet the quality standard before entering the system outflow. The effluent can be, at this point, transported to a recycling plant for advanced tertiary treatment to be put back into the system for reuse. Let q_p be the quantity of water from primary sources and q_r be the recycled water so that the total quantity supplied to the user is

$$Q = q_p + q_r$$

Since a fraction k of the total water supplied represents the quantity of sewage, the total sewage is

$$q_s = kQ$$

After this amount is treated to meet the discharge quality requirement, it can be disposed into the system outflow or transported to the recycling plant. Therefore,

$$q_r \leqslant q_s$$

Let $C_p(q_p)$, $C_r(q_r)$, and $C_s(q_s)$ represent the total cost functions $C_p(q_p)$ the cost of collecting, treating, and transporting primary sources of water, $C_r(q_r)$ the cost of reclamation and renovation of secondary treated water, and $C_s(q_s)$ the cost of collecting and treating wastewater to a specified water quality (secondary treatment) standard for discharge. The Lagrange function for minimization of total cost then becomes,

$$L = C_p(q_p) + C_r(q_r) + C_s(q_s) - \lambda_1(q_p + q_r - Q)$$
$$- \lambda_2(q_s - kq_p - kq_r) - \lambda_3(q_r + q_o - q_s)$$

in which q_o is the quantity discharged in the outflow. The first order conditions are derived for three cases.

Case 1: $q_r = 0$ implies no recycling.

$$\frac{\partial L}{\partial q_p} = C_p' - \lambda_1 + k\lambda_2 = 0$$

$$\frac{\partial L}{\partial q_s} = C_s' - \lambda_2 + \lambda_3 = 0$$

$$\frac{\partial L}{\partial q_o} = - \lambda_3 = 0$$

$$\frac{\partial L}{\partial \lambda_1} = q_p + q_r - Q = 0$$

$$\frac{\partial L}{\partial \lambda_L} = q_s - kq_p - kq_r = 0$$

$$\frac{\partial L}{\partial \lambda_3} = q_r + q_o - q_s = 0$$

Solving the first three conditions,

$$\lambda_1 = C_p' + kC_s' \text{ and } \lambda_2 = C_s'$$

It can be shown that the derivative of total cost with respect to quantity is

$$C'(Q) = \frac{\partial TC}{\partial Q^*} = \lambda_1^* = C_p' + kC_s' \qquad \text{(For proof, Hadley, 1964)}$$

in which $TC = C_p(q_p) + C_r(q_r) + C_s(q_s)$. The optimal values are indicated by *. The implication of the above equation is that marginal cost consistent with cost minimization when there is no recycling is the sum of the cost of supplying an additional unit of water from the primary source and the additional cost of treating the resulting wastewater from that one unit to a prescribed level.

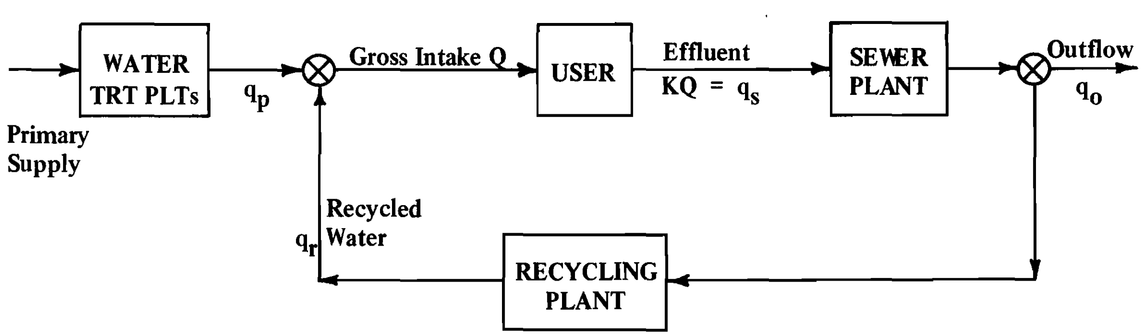

Figure 1. Simple water supply model.

Case 2: $q_r > 0$ $q_o > 0$ implies recycling is practiced, but not all the potentially reclaimable water is used. The marginal conditions are

$$\frac{\partial L}{\partial q_p} = C_p' - \lambda_1 + k\lambda_2 = 0$$

$$\frac{\partial L}{\partial q_s} = C_s' - \lambda_2 + \lambda_3 = 0$$

$$\frac{\partial L}{\partial q_r} = C_r' - \lambda_1 + k\lambda_2 - \lambda_3 = 0$$

$$\frac{\partial L}{\partial q_o} = -\lambda_3 = 0$$

Solving these conditions,

$$\lambda_1 = C_p' + kC_s' = C_r' + kC_s' \text{ and } \lambda_2 = C_s'$$

Therefore,

$$C_p' = C_r' \text{ and } C'(Q) = \frac{\partial TC}{\partial Q^*} = \lambda_1^*$$

$$= C_p' + kC_s' = C_r' + kC_s'$$

The necessary conditions indicate that the water should be supplied to the user from primary and recycled sources on an equi-marginal cost principle, and that the cost of an additional unit of water will be the sum of the marginal cost of either of the above sources and the cost of treating the sewage resulting from that one unit to a prescribed level.

Case 3: $q_o = 0$ implies all the reclaimable water is recycled. The first order conditions are

$$\frac{\partial L}{\partial q_p} = C_p' - \lambda_1 + k\lambda_2 = 0$$

$$\frac{\partial L}{\partial q_r} = C_r' - \lambda_1 + k\lambda_2 - \lambda_3 = 0$$

$$\frac{\partial L}{\partial q_s} = C_s' - \lambda_2 + \lambda_3 = 0$$

Solving these equations,

$$\lambda_1 = (1 - k)\, C_p' + kC_r' + kC_s'$$

and therefore,

$$C'(Q) = \frac{\partial TC}{\partial Q^*} = \lambda_1^* = (1 - k)\, C_p' + kC_r' + kC_s'$$

When all the water potentially available for recycling is used up, an additional unit of water consistent with cost minimization is supplied to the user by taking 1 - k units from primary source and k units from the recycled source. Therefore, the marginal cost of supplying that unit will be 1 - k times the marginal cost of acquiring one unit from the primary source plus k times the marginal cost of acquiring one unit from the recycled source and the cost of treating the resulting effluent from that one unit of intake water, to a prescribed level.

The preceding analysis can be supplemented with a graphical exposition. In Figure 2, line AB represents the marginal cost of supplying water from primary sources; DE the marginal cost for recycled water. For the sake of simplicity, it will be assumed that the marginal cost of treating the sewage to the stipulated water quality level is constant for any amount of effluent. The length KA represents the additional cost of treating k units of sewage to the prescribed level. Since the marginal cost of supplying recycled water is greater than that of supplying water from primary sources, for quantities less than Q_1, recycled water will not be used and all the water supplied will be from the primary source. Line segment AP is the relevant marginal cost of intake water and line KL is the marginal cost of the services of water that includes the cost of intake water and sewer services. At Q_1, the cost of an additional unit, either from the recycled source or from primary sources is the same since points P and D have the same ordinate. Line PQ is drawn as a horizontal

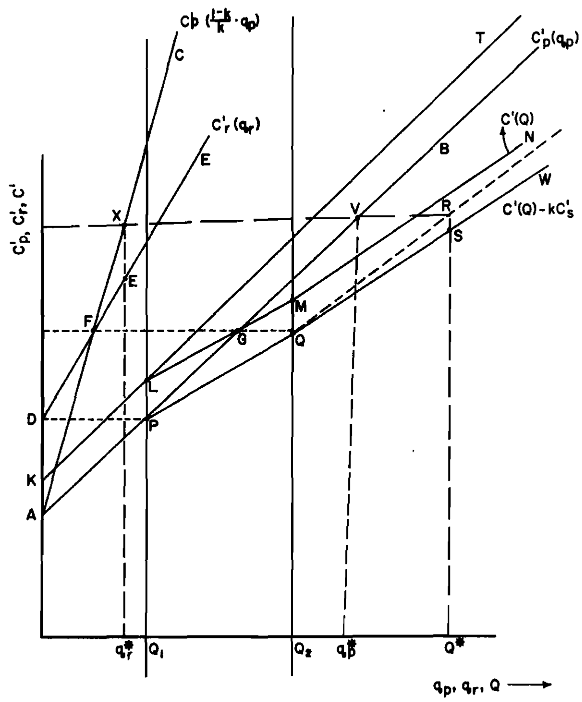

Figure 2. Graphical analysis of water supply water recycling option.

summation of lines DE and PB. Therefore, PQ represents the marginal cost of intake water, when the quantities allocated from the recycled source and primary sources are on an equi-marginal cost basis. LM is the marginal cost of the services of water, which includes LP, the cost of treating k units of sewage to the specified water quality level.

Line AC is drawn such that at any given cost, the corresponding quantity of water is k/1-k times the quantity of water represented by AB, the marginal cost curve for supplying water from the primary sources. Since

$$q_p + q_r = Q$$

and $$q_r \leqslant kQ$$

then $$q_r \leqslant k(q_p + q_r)$$

Therefore, $$q_r \leqslant \frac{k}{1-k} q_p$$

That is, the maximum amount of water available for recycling will be k/1-k times the amount of water used in the system from the primary source. The line AC thus serves as a boundary indicating the amount of potential water available for recycling. Since this line intersects DE at F, the allocation of water as primary and recycled sources cannot be maintained at an equi-marginal cost level for total quantities in excess of Q_2. In other words, movement along FE is not possible because the potential water availability for recycling is represented by FC beyond point F. Therefore, any increase in recycled water should be along FC. The marginal cost curve QW at quantities in excess of Q_2 is drawn such that the marginal cost corresponding to any quantity is equal to 1-k times the marginal cost of obtaining 1-k of the total quantity from the primary sources plus k times the marginal cost of obtaining k of the total quantity from the recycled source. A graphical method of constructing QW can be shown. Draw QR as a horizontal summation of FC and GB. To arrive at the marginal cost of supplying a total quantity Q*, first find the amount of water from primary source q_p^* and the amount of water from recycled source q_r^* as shown in the figure. The marginal cost of supplying q_p^* is the ordinate Vq_p^* (which is equal to Xq_r^*) and the marginal cost of supplying q_r^* is the length Eq_r^*. Therefore the marginal cost of intake water at Q* is

$$C'(Q^*) - kC'_s = (1-k)\,C'_p(q_p^*) + kC'_r(q_r^*)$$

$$= (1-k)\ \ Xq_r^* + kEq_r^*$$

$$= Xq_r^* - k(Xq_r^* - Eq_r^*)$$

$$C'(Q^*) - kC'_s = Xq_r^* - k(EX)$$

From the above equation, choose the point S below R such that RS is k times XE. Therefore S is a point on the marginal cost curve. It can be easily shown that QW will have a greater slope than PQ. Since QW represents the marginal cost of intake water, if MQ is added to it, the line segment MN represents the marginal cost of the services of water. Thus, the line KLMN represents the marginal cost curve for the services of water consistent with cost minimization. If the recycling option is not introduced into the model the marginal cost curve for the services of water will merely be the vertical summation of the marginal cost of supplying an additional unit of water from primary sources and the cost of treating the resulting effluent from that unit to the prescribed water quality level. This is indicated by line KLT.

Variations in the shapes of the cost curves and the parameter k will conceivably affect the shape of the marginal cost curve for the services of water. For instance, if the value of the parameter k is such that the line AC does not intersect DE, then the C'(Q) curve will not have a kink at M. It will just be an extension of the line LM. The cost curves could be rising in discrete steps, in which case, the marginal cost curve for the services of water with the recycling option may coincide with the marginal cost curve without the recycling option in some ranges. Yet, the results of the model are fairly general and need only slight modification before applying to specific instances.

Market Equilibrium—Derivation and Implications

Due to the absence of competitive forces in the market for water, an efficient solution is not automatically achieved. In fact, the water supply utility can set a price and decide to meet the quantity demanded. By supplying the quantity where marginal cost equals marginal revenue, this sector can exploit its natural monopoly power. Other reasons why the authorities may adopt pricing policies that are not consistent with competitive equilibrium could be responses to political and administrative pressures or consideration of distributional aspects.

If the price is set above the marginal cost, the society's marginal valuation is greater than the cost. Therefore, more resources will have to be transferred to this sector to increase the output level. Similarly, prices set below marginal cost imply excessive resource utilization in the industry with the marginal valuation less than the cost. In any case, non-marginal cost pricing leads to resource misallocation. Therefore, economic efficiency in the Pareto sense requires that price be equal to marginal cost.

Another way to look at the equilibrium is in terms of consumer surplus and producer surplus. Let P(Q) be the demand curve for water, then the consumer surplus is

$$CS = \int_0^{Q^*} P(Q)\, dQ - P^*Q^* \quad \text{in which}$$

P^* and Q^* are the equilibrium price and quantity.

If $C'(Q)$ is the marginal cost curve, the producer surplus is

$$PS = P^*Q^* - \int_0^{Q^*} C'(Q)\, dQ$$

Lemma: Maximum total surplus implies marginal cost pricing.
Proof: Total surplus, TS, will be defined as the sum of consumer and producer surplus:

$$TS = CS + PS$$

$$= \int_0^{Q^*} P(Q)\, dQ - \int_0^{Q^*} C'(Q)\, dQ$$

Differentiating TS with respect to Q and setting it equal to zero,

$$\frac{dTS}{dQ} = P(Q^*) - C'(Q^*) = 0$$

Therefore, $P(Q^*) = C'(Q^*)$.

If total surplus is considered an index of social welfare, and if the second order conditions are satisfied, maximum welfare occurs at the point where price equals marginal cost. The term $\int P(Q)\, dQ$ is sometimes referred to as "total willingness to pay" and the term $\int C'(Q)\, dQ$ is the total cost function. Total surplus for any quantity is the area in between the demand curve and the marginal cost curve. (This area in Figure 3 is a maximum at Q^*, the quantity where the two curves intersect.) Beyond Q^*, this area decreases since the marginal cost is greater than the marginal valuation.

The results of this lemma can now be extended to the supply model of this study. The total cost of supplying water is given by

$$TC = C_p(q_p) + C_r(q_r) + C_s(q_s)$$

If P(Q) is the demand curve for water, then is

$$TS = \int P(Q)\, dQ - C_p(q_p) - C_r(q_r) - C_s(q_s)$$

Theorem: The maximum of the total surplus subject to the following constraints.

$$q_p + q_r = Q$$

$$q_s = kQ = k(q_r + q_p)$$

and

$$q_r + q_o = q_s$$

implies marginal cost pricing.
Proof: The Lagrangian function for maximizing TS subject to the given constraints is

$$L = \int P(Q)\, dQ - C_p(q_p) - C_r(q_r) - C_s(q_s)$$
$$- \lambda_1 (q_p + q_r - Q) - \lambda_2 (q_s - kq_p - kq_r)$$
$$- \lambda_3 (q_r + q_o - q_s)$$

Again, the first order conditions for the three different cases considered in the previous section can be derived. The results will be presented below.

Case 1: $q_r = 0$

$P(Q) = C_p' + kC_s'$

Case 2: $q_o > 0, q_r > 0$

$P(Q) = C_p' + kC_s' = C_r' + kC_s'$

Case 3: $q_o = 0$

$P(Q) = (1 - k) C_p' + kC_r' + kC_s'$

The right hand sides of the above three equations already have been shown to be the marginal cost for the respective cases. Therefore, maximum total surplus implies marginal cost pricing. If the second order conditions are satisfied, price equals marginal cost implies maximum total surplus.

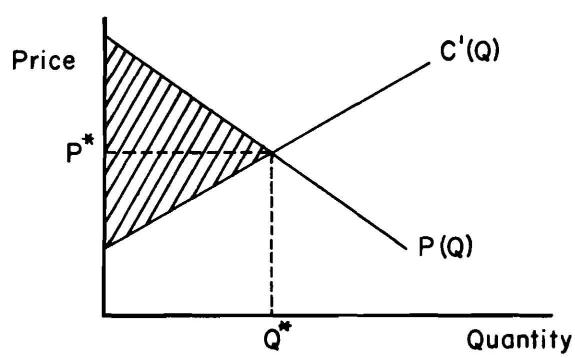

Figure 3. Total surplus analysis.

CHAPTER IV

GEOGRAPHICAL DESCRIPTION AND HYDROLOGICAL ATTRIBUTES OF THE STUDY AREA

The model developed in the preceding sections will be applied to a small region in the State of Utah, to illustrate water supply planning concepts. The area of study encompasses Salt Lake County, located in the North Central region of Utah. Enclosed by the Wasatch Mountains on the east, the Oquirrh Range on the west, Traverse Mountains on the south and the Great Salt Lake on the north, this county forms a closed system and was found ideal for this study.

For convenience of analysis, the water and sewer districts serving the study area were lumped into five major subdivisions. Region 1 consists of mainly West Jordan, Midvale, Sandy City, South Jordan, and Riverton. Region 2 includes only Murray City and lies north of Region 1. Region 3 comprises Kearns, Taylorsville, and Granger on the east side of the Jordan River and South Salt Lake on the west. Salt Lake City constitutes Region 4, and Region 5 integrates the northwest part of the county, comprising mainly the Magna area. The water supplied for municipal use (M) to these regions will be denoted by M_1 through M_5.

The regional subdivisions of Salt Lake County are shown in the accompanying map (Figure 4).

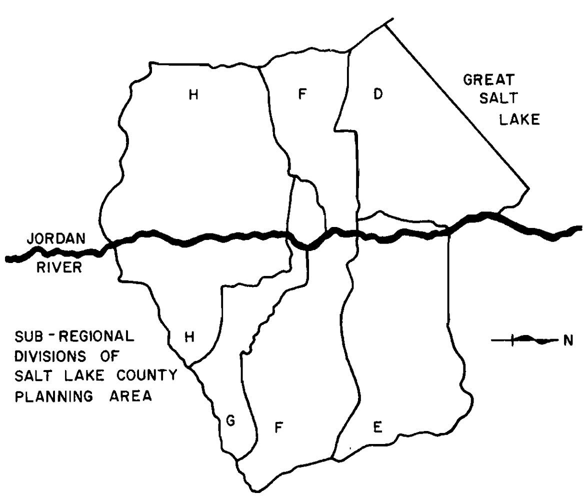

Figure 4. Subregion delineation for the Salt Lake County case study area.

There is a wide variety of water resources in this area that makes the study particularly interesting. Broadly speaking, surface water of high quality from the mountain streams and low quality water from the Jordan River, groundwater sources of various qualities, and import water are presently being used to supply water for culinary, industrial, and agricultural purposes in this county. The high quality surface water sources (C) constitute the six creeks (i.e., flowing into the county from the east) from City, Mill, Big Cottonwood, and Littte Cottonwood Creeks. The first four of these creeks are lumped into a single source (C_1) since they all flow into Region 4. Mill Creek and the Big Cottonwood are the major creeks flowing into Region 3; they comprise the second surface water source (C_2). Little Cottonwood Creek is treated as a separate source (C_3), and it flows into Region 1.

The Jordan River which flows through the county, cutting the area into east and west sections, is a fairly big source of poor quality surface water which is mainly used for industrial and agricultural purposes. The surface runoffs and municipal and industrial effluent discharges are currently being carried by this river. The present quality condition justifies dispensing with this river as a potential municipal water resource for this study.

Past studies of groundwater conditions in the county provide estimates of well water availabilities for municipal purposes. Groundwater (G) in each region will be considered as a source and will be denoted by G_1 through G_5 for the five regions.

In addition, there are two import sources (I) of water for municipal purposes: The Salt Lake City aqueduct (I_2), which delivers about 14,500 acre feet of water per year from Deer Creek Reservoir, and the Central Utah Project (I_1), which is expected to deliver up to 70,000 acre feet by the year 1985.

There are four water treatment plants (W): City (W_1), Mountain Dell (W_2), Big Cottonwood (W_3), and Little Cottonwood (W_4). There are nine major wastewater treatment plants (S) in operation in this county. These plants are lumped into five treatment facilities corresponding to each region and are designated by S_1 through S_5. The estimated average annual capacities of these plants are available for use in the model. Excess wastewater will be allowed to go into a proposed additional treatment facility denoted by S_6.

The nonconsumed effluents (E) from each region, designated E_1 through E_5, constitute a potential source of water for recycled supply. It is estimated that 50 percent of the gross intake water is being consumptively used. The other 50 percent is discharged into the sewer system. This wastewater is collected and transported to the waste treatment facilities serving these regions, where it is treated to meet the discharge quality. This water can be either discharged into receiving stream (0) or can be recycled after tertiary treatment. It will be assumed for modeling purposes that a recycling plant (R) exists in each region (represented by R_1 through R_5) with some finite capacity.

CHAPTER V

MODEL FORMULATION FOR THE STUDY AREA

A mathematical model will be built in this section to analyze the pricing and allocation of water resources consistent with competition. Questions regarding imposition of higher water quality requirements on the effluents, restrictions on blending reclaimed water with primary or import water, and randomness in some of the sources will be considered within the model. The basic framework of the analysis will draw heavily on mathematical programming tools, particularly nonlinear and linear programming.

The aggregate demand curves for each of the five regions under consideration were derived from the study made by Gardner and Schick (1964), in which the per capita quantity of water demanded for household purposes is estimated as a function of price and per capita lot size. Since the charges for sewer services is a flat rate, the effective price is zero and, hence, does not affect the consumers' marginal decision to consume water. The loglinear demand curve fitted in the Gardner and Schick study was, therefore, used as the demand for the services of water. Data on lot size per capita and population were used to arrive at a constant elasticity demand curve for each of the five regions in the county. Let these aggregate demand curves for the r^{th} region be

$$P_r = K_r Q_r^{1/\eta}, \quad r = 1, 2, ..., 5$$

in which P_r is the price of water services in the r^{th} region, η is the elasticity of demand, Q_r is the quantity demanded and K_r is the antilog of the intercept term of the loglinear form of the equation. The "total willingness to pay" for the r^{th} region is then given by

$$TWP_r = \int K_r Q_r^{1/\eta} \, dQ_r$$

$$= \frac{K_r}{\frac{1}{\eta} + 1} Q_r^{1/\eta + 1}$$

The discussion will now turn to the cost side of the mathematical model. Water from several sources, varying in quality characteristics and located at different geographical points, will have to be transported to the consuming communities either directly via the distribution system or indirectly through water treatment plants. The nonconsumed effluent of each region will be available for reuse in the system. The effluents are collected and transported to the sewer plants serving the region and treated to comply with discharge quality standards. This water can then be reclaimed for further reuse in the system through a tertiary treatment process or allowed to be discharged in the system outflow. The water treatment plants, the existing wastewater treatment plants, and the tertiary treatment plants are all to be considered as intermediate points in the transportation system. That is, they are depicted as both sources and destinations.

If C_{ij} is the unit cost of delivering water—including necessary treatment expenses—from the i^{th} origin to the j^{th} destination, then the total cost of water delivery is

$$TC = \sum_i \sum_j C_{ij} X_{ij}$$

in which X_{ij} is the quantity of water transported from the i^{th} source to the j^{th} destination. The total surplus is then expressed as

$$TS = \Sigma TWP_r - TC$$

$$= \Sigma \frac{K_r}{\frac{1}{\eta} + 1} Q_r^{1/\eta + 1} - \sum_i \sum_j C_{ij} X_{ij}$$

By maximizing this surplus subject to a set of constraints[1] on the system, a competitive equilibrium for each region can be obtained such that

$$P_r = \frac{\partial TC}{\partial Q_r}$$

A general flow diagram of the possible alternatives of water allocation is shown in Figure 5.

Water available from a secondary or effluent source (E) in a given region will be allowed to go into

[1]The specific constraints applicable to this model will be explained in the following paragraphs.

DESTINATIONS / SOURCES	M	O	W	S	R
E				*	
C			*		
G	*		*		
I	*		*		
W	*				
S		*			*
R	*				

* Possible water allocation alternatives.

Figure 5. Water allocation possibilities.

the waste treatment facility (S) located in that region where it will be treated to secondary levels. The outflow of the sewer plants can go to any one of the proposed tertiary plants (R) or to the system outflow (O). The water fed into the recycling plant will go for municipal use (M). Creek water (C) will be transported to the water treatment plants (W). The outflows of these treatment plants will supply the municipal sectors of the various regions through their distribution systems. Groundwater (G) and import water (I) will be allowed to enter a distribution system either directly or through treatment plants depending on the quality of these sources.

The unit cost matrix with all possible water allocation schemes are shown in Figure 6, where all source categories and regional subdivisions are shown. The costs of transport (which includes collection, pumping, pipeline, treatment, and distribution costs appropriate to the individual variables) are shown inside the matrix in dollars per acre foot. The variables corresponding to blank entries are not feasible alternatives and can, therefore, be left out of consideration by placing a high cost in the objective function or by simply deleting the variables from the problem. To carry out the optimization procedure, the following additional constraints will have to be introduced.

For notational compactness, none of the variables will be deleted; instead, corresponding to an infeasible alternative, a high cost will be assigned in the objective function, and vectors will be arranged so that the order shown in the general schematic is preserved.

Let N be the number of communities to be served,
Let V be the number of primary and import sources,
Let L be the number of intermediate nodes (water treatment, wastewater treatment and tertiary treatment plants), and k be the fraction of nonconsumed effluent per unit of gross intake water.

Effluent availability constraint:

$$\sum_{j=1}^{N+L} X_{ij} - kQ_i = 0 \qquad i = 1, 2, ..., N \quad \cdots \quad (1)$$

The quantity of effluent from the i^{th} region transported to "all destinations" will be equal to k times the quantity Q_i demanded and supplied to the i^{th} region. The allowed destinations are the sewer plants in various regions. Variables corresponding to other destinations are eliminated by a high cost in the objective function.

Primary and import source availability constraints:

$$\sum_{j=1}^{N+L} X_{ij} \leqslant Q_i \qquad i = N+1, ..., N+V \quad \cdots \quad (2)$$

The quantity of water shipped from the i^{th} primary or import source to "all destinations" should be less than or equal to the expected quantity of water available in the origin. The allowed destinations are water treatment plants and the municipal distribution system. Variables corresponding to other destinations are eliminated by a high cost in the objective function.

Intermediate node constraints:

$$\sum_{j=1}^{N+L} X_{ij} + X_{io} \geqslant 0 \qquad i = N+V+1, ..., N+V+L \quad (3)$$

The amount of water from the i^{th} plant going to "all destinations" should be greater than or equal to zero. From a water treatment plant, the destination is the municipal distribution system; from a waste treatment facility, the destinations are tertiary plants and system outflow; and from a recycling plant, the destination is the municipal distribution system. By placing a high transport cost corresponding to other destinations, alternatives are prevented from entering into the solution.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1	13	13																			
	C_2			13																		
	C_3				13																	
Import Water	I_1					85			85			85			85			85				
	I_2				13																	
Water Treatment Plants	W_1					47			46			46			45			45				
	W_2					47			46			46			45			45				
	W_3					46			45			45			46			46				
	W_4					45			45			45			46			46				
Region 1	G_1				9	41			55			79			84			89				
	E_1						104			130			140			148			156		125	
	S_1							79			88			93			111			119		X
	R_1					30																
Region 2	G_2					62			41			66			72			89				
	E_2						120			104			120			131			147		125	
	S_2							108			79			95			101			115		X
	R_2								30													
Region 3	G_3			9	9	77			63			41			69			66				
	E_3						175			118			104			124			142		125	
	S_3							117			94			79			96			106		X
	R_3											30										
Region 4	G_4	9				90			69			63			41			82				
	E_4						147			131			122			104			142		125	
	S_4							122			101			98			79			108		X
	R_4														30							
Region 5	G_5					93			83			66			79			41				
	E_5						150			145			140			133			104		125	
	S_5							131			120			116			118			79		X
	R_5																	30				
*	S_6							79			79			79			79			79		

* Proposed sub-regional wastewater facilities
⊕ System outflow
X Cost varies with effluent quality requirements

Figure 6. Unit cost matrix.

Municipal water supply contraint:

$$\sum_{i=1}^{N+V+L} X_{ij} - Q_j = 0 \qquad j = 1, 2, \ldots, N \cdot \cdot \cdot \cdot (4)$$

The amount of water allocated from all sources to the j^{th} municipal distribution system should be equal to the quantity demanded by the j^{th} region. All sources refer to groundwater, import water, water treatment plants and tertiary treatment plants.

Capacity constraints for the treatment plants:

$$\sum_{i=1}^{N+V+L} X_{ij} \leqslant C_j \qquad j = N+1, \ldots, N+L \cdot \cdot (5)$$

The total amount of water fed into the j^{th} treatment plant from all sources should be less than or equal to its capacity. For a water treatment plant, all sources refer to groundwater, surface water, and import sources; for a waste treatment facility, it refers to the effluent sources from each region; and for a tertiary treatment plant, it refers to the sewer plants.

Flow balance equations:

$$\sum_{i=1}^{N+V+L} X_{i,N+\ell} - \sum_{j=1}^{N+L} X_{N+V+\ell,j} = 0 \quad = 1, 2, \ldots, L \quad (6)$$

Assuming water losses in treatment facilities are negligible, the amount of water entering any treatment plant should equal the amount flowing out of the treatment plant.

The objective is then to maximize total surplus

$$TS = \sum_{j=1}^{N} \frac{K_j}{\frac{1}{\eta}+1} Q_j^{1/\eta+1} - \sum_{i=1}^{N+V+L} \sum_{j}^{N+L} C_{ij} X_{ij} \quad . (7)$$

subject to the six constraints stated above.

This problem involves nonlinear programming due to the term

$$\sum_{j=1}^{N} \frac{K_j}{\frac{1}{\eta}+1} Q_j^{1/\eta+1}$$

in the objective function. To show that any relative maximum will be an absolute maximum, it must be proved that

$$f(Q_j) = \frac{K_j}{\frac{1}{\eta}+1} Q^{1/\eta+1}$$

is a concave function over a closed convex set. If it is so, then the sum $\sum_j f(Q_j)$ will be a concave function. Note that K_j is always positive and η is negative.

Theorem: $f(Q_j)$ is a concave function. (For proof, see Appendix A.)

Constraints (1) through (6) are all linear, any relative maximum of the concave objective function over a convex set will be an absolute maximum. Also, it can be shown that if the global maximum occurs at two different points, then there is an infinite number of points where the global maximum will be taken on.

There are many ways to solve a nonlinear programming problem of this type. The technique that will be adopted in this study is the separable programming method. The nonlinear function is approximated by several linear segments. The problem is then solved strictly as a linear programming problem. Since it is already known that any relative optimum will be a global one, it is not necessary to explicitly use what is known as the restricted basis entry procedure. For details of this procedure, reference can be made to Hadley (1964).

The modified structure of the objective function and the constraints will now be delineated.

Recall that the objective is to maximize total surplus

$$TS = \sum_{j=1}^{N} \frac{K_j}{\frac{1}{\eta}+1} Q_j^{1/\eta+1} - \sum_{i=1}^{N+V+L} \sum_{j}^{N+L} C_{ij} X_{ij}$$

$$= - \sum_{j=1}^{N} \phi_j(Q_j) - \sum_{i=1}^{N+V+L} \sum_{j}^{N+L} C_{ij} X_{ij}$$

in which $\phi_j(Q_j) = - \dfrac{K_j Q_j^{1/\eta+1}}{\frac{1}{\eta}+1}$ for all j.

This is equivalent to minimizing

$$- TS = \sum_{i=1}^{N+V+L} \sum_{j=1}^{N+L} C_{ij} X_{ij} + \sum_{j=1}^{N} \phi_j (Q_j)$$

This objective function can be stated in separable form as follows: (the "Lambda formulation" is used)

Minimize,

$$\sum_{i=1} \sum_{j=1} C_{ij}X_{ij} + \sum_{j=1}^{N} \sum_{s=1}^{r_j} \lambda_{sj} \phi_j (Q_{sj})$$

in which λ_{sj} is the s^{th} variable for the j^{th} separable set. r_j is the total number of grid points chosen for the j^{th} variable and ϕ_j (Q_{sj}) is the value of the function

$$\frac{-K_j Q_j^{1/\eta + 1}}{1 + 1/\eta}$$

evaluated at the point Q_{sj}.

Constraint Equations 1 and 4 will have to be modified since the nonlinear variables Q_j appear in these constraints. Therefore, Q_j will have to be written in separable form as shown in Equations 1a and 4a.

$$\sum_{j=1}^{N+L} X_{ij} - \sum_{s=1}^{r_j} \lambda_{ij} \quad (k \cdot Q_{si}) = 0 \quad i = 1, 2, ..., N \quad . (1a)$$

$$\sum_{i=1}^{N+V+L} X_{ij} - \sum_{s=1}^{r_j} \lambda_{sj} \quad (k \cdot Q_{sj}) = 0 \quad j = 1, 2, ..., N \quad (4a)$$

In addition to these changes, the following constraint will be imposed on the lambdas.

$$\sum_{s=1}^{r_j} \lambda_{sj} = 1 \quad j = 1, 2, ..., N \quad . \ . \ . \ . \ . \ (8)$$

The complete set of equations can now be written:

Minimize

$$\sum_{i=1}^{N+V+L} \sum_{j=1}^{N+L} C_{ij}X_{ij} + \sum_{j=1}^{N} \sum_{s=1}^{r_j} \lambda_{sj} \phi_j (Q_{sj}) \quad . \ (9)$$

Subject to

$$\sum_{j=1}^{N+L} X_{ij} - \sum_{s=1}^{r_j} \lambda_{si} \ (k \cdot Q_{si}) = 0 \quad i = 1, 2, ..., N \ . \ . (10)$$

$$\sum_{j=1}^{N+L} X_{ij} \leqslant a_i \quad i = N + 1 \ ... \ N + V \quad . \ . \ . \ . (11)$$

$$\sum_{j=1}^{N+L} X_{ij} + X_{io} \geqslant 0 \quad i = N + V + 1 \ ... \ N + V + L \ (12)$$

$$\sum_{i=1}^{N+V+L} X_{ij} - \sum_{s=1}^{r_j} \lambda_{sj} \ (k \cdot Q_{sj}) = 0 \quad j = 1,2, ..., N \ . \ . (13)$$

$$\sum_{i=1}^{N+V+L} X_{ij} \leqslant C_j \quad j = N + 1 \ ... \ N + 1 \quad . \ . \ . \ . (14)$$

$$\sum_{i=1}^{N+V+L} X_{i,N+\ell} - \sum_{j=1}^{N+L} X_{N+V+\ell,j} = 0 \quad \ell = 1,2, ...L \ . (15)$$

$$\sum_{s=1}^{r_j} \lambda_{sj} = 1 \quad j = 1,2, ..., N \quad . \ . \ . \ . \ . \ . \ . (16)$$

As stated previously, this programming problem will be solved strictly as a linear programming problem. The solution will consist of all X_{ij} s, the quantities of water allocated from i^{th} origin to the j^{th} destination, and λ_{sj} s. The equilibrium quantity for each region can be obtained from

$$Q_j = \lambda_{rj}^* \ Q_{rj} + \lambda_{rj}^* \ Q_{kj} \quad \text{in which } r = k \pm 1$$

It is imperative to remember that the separable programming technique is only an approximation to the original nonlinear problem. As a result the solution arrived at through this procedure are solutions to the approximating problem. The accuracy of the solution depends on the selection of the grid points. The finer the specification of the grid, the closer will be the solution of the approximating problem to that of the original problem.

CHAPTER VI

APPLICATION OF THE MODEL AND RESULTS

The technique described in the preceding section was applied to Salt Lake County for present and future planning of water and waste management strategies. The process enables the determination of how water supply sources will be utilized in terms of allocation for municipal use and how pricing policies consistent can be arrived at with economic efficiency. In addition, particulars regarding the economic feasibility of using reclaimed water in the system for domestic purposes is examined. The model was used to test the implications of policy alternatives such as the implementation of higher water quality requirements on the effluent discharges or the stipulation of a blending restriction on the reclaimed water for reuse.

For planning purposes equilibrium prices and quantities were found for both present and future time periods. Since the analysis prescribed in this study is static, this was accomplished by obtaining solutions at discrete points in time. The five specific years chosen for analysis were 1975, 1980, 1985, 2000, and 2020. The solutions corresponding to these years span approximately a period of half a century. The loglinear demand relationships were derived from the study made by Gardner and Schick (1964) for each of the five regions and for each of the five years are shown in Table 1. The demand for water was found to be significantly dependent upon price and lot size per capita. For this study, the lot size per capita was estimated from land use and population projections for each region in the county. Substituting these estimated values of per capita lot size for the present and future years into the equations and changing the units of measurement, the aggregate demand relationships between the quantity of water in acre feet and price per acre foot were obtained. The "total willingness to pay" curves, necessary for the objective function, were calculated by integrating these demand curves.

The unit cost matrix (Figure 6) was derived from the study by Bishop et al. (1974). The estimated water availabilities and treatment plant capacities are indicated in Tables 2 and 3. These were obtained from several studies (Hely et al., 1971; Templeton, Linke, and Alsup Consulting Engineers, 1974; Caldwell et al., 1971; Bishop et al., 1974).

The equilibrium quantities for each region at a given time (which are decision variables represented by Q_j) were approximated 0 and 100,000 acre feet. There were, therefore, 200 "lambda" variables corresponding to each Q_j, and for five regions, a total of 1000 "lambda" variables were present in the model. On the cost side, there were 630 X_{ij} variables associated with 30 origins and 21 destinations. There was a total of 71 constraints, of which 51 correspond to the origin availability and destination constraints (Equations 10 through 14). Another 15 were associated with the flow balance equations, and the remaining five were convexity rows corresponding to the separable variables.

Thus, with 1630 variables and 71 constraints, the problem was solved using the mathematical programming system, TEMPO, available with the Burroughs B6700 computer facility. The cost of a run

Table 1. Loglinear demand relationships.[a]

Regions	Years: 1975	1980	1985	2000	2020
Region One	12173968	19030343	22962038	36033795	59148504
Region Two	23387590	28877066	34232658	56994573	98332354
Region Three	50088761	57377066	66129441	92973454	131879881
Region Four	60456120	63725968	69075828	77802524	83106806
Region Five	1580805	1847148	2169093	3056439	4497362

[a]η = -0.7662. The quantities are in acre feet. The prices are in \$/acre foot.
Coefficients K_j of the constant elasticity demand equations ($P_j = K_j Q_j^{1/\eta}$).

averaged about \$12. The CPU time was about 60 seconds per run.

Equilibrium With Existing Water Quality Requirements

Subsequent to the 1972 Amendments to the Federal Water Pollution Control Act (PL 92-500), the Utah State Water Pollution Committee and the Board of Health issued an order that by December 31, 1978, all discharges must be altered, as necessary, to meet the Class 'C' standards. The requirements of these standards were described in terms of the limitations on water quality parameters. As an interim measure, the order requires that all dischargers must provide effective secondary treatment or the equivalent by December 31, 1974.

In the model, the treatment plants S_1 through S_6 provide just the secondary treatment. The costs of treatment were computed on the basis of these quality requirements. The secondary treated water from S_1 through S_6 is discharged into the outflow (o) with zero cost. The X symbols in the unit cost matrix were replaced by zeros and the problem was solved for each of the target years.

Table 2. Water availability in Salt Lake County.

Sources		Quantity in Acre Feet/Year
Surface water	C_1	39,200
	C_2	63,400
	C_3	49,100
Groundwater	G_1	3,200
	G_2	8,300
	G_3	6,700
	G_4	3,600
	G_5	24,200
Import water	I_1	a
	I_2	14,500

[a]Anticipated availability: 3,000 acre feet in year 1975, 36,500 in 1980, and 70,000 from 1985 onwards.

Table 3. Treatment plant capacities.

Plant		Average Annual Capacity in Acre Feet/Year
Water treatment plants	W_1	19,100
	W_2	35,900
	W_3	42,580
	W_4	112,100
Existing wastewater treatment plants	S_1	8,400
	S_2	5,600
	S_3	32,000
	S_4	50,400
	S_5	1,500
	S_6	a

[a]Proposed treatment plant with very large capacity.

Note: R_1 through R_5 are proposed recycling facilities. These tertiary treatment plants will have an assumed capacity of 17,900 acre feet/year.

The optimal allocation of water for each of the target years is shown in Tables 8 - 12 in Appendix C. Water is supplied mainly from groundwater and surface water availability. The Salt Lake aqueduct is the only import source in use; the Jordan aqueduct does not enter the solution until the year 2020. The wastewater treatment plants have sufficient capacity to meet the demand until the year 2000. (Thereafter construction of additional treatment facilities will be required for Regions 1, 2, and 5.) The water treatment plants have excess capacity until year 2020. The water recycling alternative does not enter the solution for another half century.

Figures 7 - 11 in Appendix C show the demand curves for the five target years and the quilibrium prices and quantities associated with each time period for the five regions. In each diagram, the line joining the equilibrium points represents what might be termed a quasi supply curve (Bishop et al., 1975). The five supply curves are more or less horizontal, implying that the additional cost of supplying one more unit of water remains fairly constant. The equilibrium quantities and the corresponding prices for each region in each of the target years are shown in Table 4.

Although a direct comparison of the prices arrived at through this study with the prevailing prices is difficult due to the disparity in these two rate structures, a cursory examination will reveal some interesting conclusions. For instance, the Salt Lake City Water Department charges 16 cents per 100 cubic feet with a minimum of \$5.25 for three months. With a per capita consumption of 200 gallons per day, Salt Lake City (Region 4) with a population of about 208,000 is supplied 41.68 MGD which is equivalent to 46,679 acre feet a year. The results of this study suggest a price of \$110 per acre foot as opposed to the prevailing rate of about \$70 per acre foot. This proposed increase of 58 percent in price of water would reduce the consumption of water by 45 percent to a total of 25,000 acre feet per year.

In general, the results of the model indicate an equilibrium quantity of water less than and price greater than the actual price and quantity observed in these regions. The magnitude of the proposed increase in price ranges from about 20 percent to 60 percent for various regions. Recalling that the estimated price elasticity of demand is -0.77, a price increase will lead to an increase in total revenue combined with a decrease in total costs due to the reduction in the quantity of water service demanded. Water utilities would, therefore, experience an increase in the profits. Although the assessment of a fixed minimum charge on households creates a disparity in the existing and proposed price structure, actual comparison of the two is justifiable on the grounds that an average household with four persons per dwelling unit consumes a quantity in the first block of the marginal rates. This study does not raise any objection to the declining marginal rates, since the economies of scale could permit a lower price of delivery for a consumer, whose intake quantities are higher than an average household.

Equilibrium With Higher Water Quality Requirements

A recent decision of the Utah State Water Pollution Committee and the Board of Health prescribes that the quality of any point discharge meet a set of standards (Level 2) by 1980 and a still higher set of standards (Level 1) by 1985. The standards corresponding to these years have been provided in terms of water quality parameters. The unit cost of achieving Level 2 has been computed to be $21 an acre foot and $71 an acre foot for Level 1 beyond secondary treatment. These costs replaced the zero cost of discharge from any sewer plants to the overflow used in the previous section, to take the higher quality requirement into consideration in the model.

The legal requirement of higher water quality on the discharges engenders interesting water allocation patterns. Groundwater sources are fully utilized at each point in time. Surface water use is high at first since in 1975, the quality requirement is only secondary treatment on the wastewater. In 1980, the use increases since the quality requirement is only Level 2. But in 1985, Level 1 requirement makes the recycled source of water so much more attractive that the use of surface water declines. Thereafter, the water from the Wasatch creeks C_1, C_2, and C_3 are used only to sustain the recycled supply. The Central Utah Project water does not become economically feasible even under year 2020 demand conditions. The allocation for years 1980, 1985, 2000, and 2020 are shown in Tables 13 - 16 in Appendix D. The 1975 allocation is the same as in Table 8 of Appendix C.

The demand curves for each region corresponding to each target year are shown in Figures 12 - 16 in Appendix D. The locus of equilibrium points over time for each region is shown on these figures and these are compared with the quasi-supply resulting from existing water quality requirement. With higher quality restrictions imposed on the effluent discharges, the marginal costs shift to the left and, therefore, the locus of the equilibrium points moves to the left. This implies a higher equilibrium price beyond year 1980 for the services of water. There is a general increase of 20 percent in the price of water with higher water quality restrictions. Correspondingly, the cost of providing the services also goes up. This study concludes that water recycling is economically feasible in the year 1985 when Level 1 treatment is required on all effluent discharges. The solution prices and quantities are shown in Table 5.

Table 4. Equilibrium quantities with secondary treatment.

Year	Region:	R1	R2	R3	R4	R5
1975	Quantity	7,500	11,500	21,500	25,000	2,000
	Price/acre ft.	107	117	110	110.0	78.0
1980	Quantity	10,500	13,500	24,000	26,000	2,000
	Price/acre ft.	108	118	110	110.0	91.0
1985	Quantity	12,000	15,500	27,000	27,500	2,000
	Price/acre ft.	109	116	109	111	107.0
2000	Quantity	16,800	22,500	35,000	30,500	3,000
	Price/acre ft.	110	119	109	109	89.0
2020	Quantity	23,000	34,000	44,500	32,000	3,500
	Price/acre ft.	120	120	113	110	107.0

Social Response to the Use of Recycled Water

There have been a number of survey studies on the issue of assessing the social desirability of recycling water for culinary purposes. To ensure complete safety, reduce the impact failures in treatment processes and to provide water with the same physical characteristics as that of the primary sources, certain restrictions on the use of renovated water for domestic purposes can be considered.

This study examines the impact of stipulating a blending restriction such that the quantity of water from recycled source cannot exceed a stipulated percentage of the water derived from all other sources. For illustrative purposes, a 25 percent blending restriction was used in the model. For every region, additional constraints stating that the water from recycled source minus 25 percent of the water from all other sources be less than or equal to zero, were incorporated in the model and it was solved for the five target years.

Although recycling was an attractive alternative beyond 1985, due to higher water quality requirements, the amount of recycled water was less than without the blending restriction. Increased usage of surface water can be noticed. The marginal costs went up due to the blending restraints. The optimal allocations and the demand supply relationships are shown in Tables 17 - 20, and Figures 17 - 21 respectively in Appendix E. The supply of water with blending restriction is compared with the quasi-supply derived without the blending restrictions. Since the marginal costs are higher for the former case, the consumer will be paying a higher price for water, with blending restrictions. The equilibrium prices and quantities for each of the target years and for each time period is shown in Table 6.

Stochastic Considerations

The parameters used in the model have to this point been assumed to be deterministic. Yet, in reality many such coefficients are random in nature. The quantity of water available from surface and groundwater sources depends upon such factors as precipitation, temperature, etc., which are characterized by random variations. The demand for water can also be regarded random. The cost coefficients used in the model may not represent the actual cost that will be incurred in the implementation of the activities. If these random variables can be characterized statistically, there are techniques available to incorporate their stochastic nature in the model.

Although the model presented in this study would allow a wide variety of stochastic programming structures, only limited illustration of its capability will be presented. The water availability from creeks C_1, C_2, and C_3 will be treated as stochastic and all other parameters will be assumed deterministic.

Stochasticity in one or more of the "right-hand-side" elements such as surface water availability can be analyzed within the framework of chance constrained programming (Charnes and Cooper, 1959). The deterministic constraint set

$$\sum_{j=1}^{N+L} X_{ij} \leqslant \overline{C}_i \qquad i = 1,2, \text{and } 3$$

in which $\overline{C}_i$ is the expected value of water availability in the i^{th} creek, can be replaced by

Table 5. Equilibrium quantities with recycling.

Year	Region:	R1	R2	R3	R4	R5
1975	Q	7,500	11,500	21,500	25,000	2,000
	P	107	117	110	110	78
1980	Q	9,500	12,500	22;500	24,500	2,000
	P	123	130	120	119	91
1985	Q	10,500	13,900	23,700	24,500	2,000
	P	130	134	129	129	107
2000	Q	14,500	19,500	29,500	26,000	2,500
	P	134	144	136	135	112.0
2020	Q	20,000	29,000	37,000	27,000	3,000
	P	144	148	144	137	130

Q – Quantity in acre feet.
P – Price per acre foot.

$$\sum_{j=1}^{N+L} X_{ij} \leqslant \bar{C}_i - K_{\alpha_i}\sigma_i \quad i = 1, 2, \text{and } 3$$

in which σ_i is the standard deviation of the water availability from the i^{th} creek and $k_{\alpha_i} = F^{-1}(C_i)$, in which F is the cumulative distribution of C_i and α_i is the specified probability level for the i^{th} constraint to hold.

It has to be noted that the random right-hand-side elements are assumed to be statistically independent.

The model described in Section B was tested to analyze the effects of stochasticity in the creek flows C_1, C_2, and C_3. The allocation that resulted from these modified constraints was identically the same as the one in Section B. Recalling that surface water was used only to sustain the recycling process as observed by introducing stochasticity of these sources in the model.

To further illustrate the capability of the model to treat yet another stochasticity concept the joint change constrained programming (JCCP) is introduced (Miller and Wagner, 1965; Jagannathan and Rao, 1973; Jagannathan, 1973). The model described in Section A is used for application of this concept. Suppose that C_1, C_2, and C_3 quantities of water available are stochastic variables with means $\bar{C}_1$, $\bar{C}_2$, and $\bar{C}_3$ and variance σ_1^2, σ_2^2, and σ_3^2. If the joint probability of the three availability constraints

$$\sum_{j=1}^{N+L} X_{ij} \leqslant \bar{C}_i \quad i = 1, 2, \text{and } 3$$

simultaneously holding should be set greater than or equal to β%.

$$P_r\,[\{\Sigma X_{1j} \leqslant C_1\}\,\Omega\,\{\Sigma X_{2j} \leqslant C_2\}\,\Omega\{\Sigma X_{3j} \leqslant C_3\}] \geqslant \beta$$

or

$$P_r\,[\{C_1 \geqslant \Sigma X_{1j}\}\,\Omega\,\{C_2 \geqslant \Sigma X_{2j}\}\,\Omega\{C_3 \geqslant X_{3j}\}] \geqslant \beta$$

or

$$P_r\,[\{C_1 - \bar{C}_1 \geqslant \Sigma X_{ij} - \bar{C}_1\}\,\Omega\,\{C_2 - \bar{C}_2 \geqslant \Sigma X_{2j} - \bar{C}_2\}\Omega\{C_3 - \bar{C}_3 \geqslant \Sigma X_{3j} - \bar{C}_3\}] \geqslant \beta$$

or

$$P_r\{C_1 - \bar{C}_1 \geqslant -y_1\} \cdot P_r\{C_2 - \bar{C}_2 \geqslant -y_2\} \cdot P_r\{C_3 - \bar{C}_3 \geqslant -y_3\} \geqslant \beta$$

in which

$$Y_i = \bar{C}_i - \Sigma X_{ij} \quad i = 1,2, \text{and } 3 \quad . \; . \; . \; . \; (17)$$

It can be shown (Feller, 1971, p. 151) that

$$P_r\,[x \geqslant -t] \geqslant \frac{t^2}{\sigma^2 + t^2}\,,\; t > 0,$$

for any random variable x with mean zero and variance σ^2. Therefore,

$$\frac{y_1^2}{\sigma_1^2 + y_1^2}\;\frac{y_2^2}{\sigma_2^2 + y_2^2}\;\frac{y_3^2}{\sigma_3^2 + y_3^2} \geqslant \beta$$

$$\prod_{i=1}^{3} \ln \frac{y_i^2}{\sigma_i^2 + y_i^2} \leqslant \ln \beta$$

$$-\prod_{i=1}^{3} \ln \frac{y_i^2}{\sigma_i^2 + y_i^2} \leqslant -\ln \beta \; . \; . \; . \; . \; . \; . \; . \; (18)$$

Table 6. Equilibrium quantities with 25% blending.

Year	Region:	R1	R2	R3	R4	R5
1975	Q	7,500	11,500	21,500	25,000	2,000
	P	107	117	110	110	78
1980	Q	9,500	12,500	22,500	24,500	2,000
	P	123	130	120	119	91
1985	Q	10,000	13,500	22,000	23,000	2,000
	P	138	139	142	140	107
2000	Q	14,000	20,000	28,500	25,000	2,500
	P	140	139	143	142	112
2020	Q	19,500	28,500	37,500	26,500	3,000
	P	149	151	141	140	130

Q – Quantity in acre feet.
P – Price per acre foot.

The three deterministic constraints are thus replaced by Equations 17 and 18 which are

$$\Sigma X_{ij} + y_i = \bar{C}_i \quad i = 1, 2, 3$$

and

$$-\sum_{i=1}^{3} \ln \frac{y_i^2}{\sigma_i^2 + y_i^2} \leqslant - \ln \beta$$

Note that although the last equation is nonlinear, it is convex. Under these conditions the model in Section A with these revised equations will still yield a global optimum. The proof that the last equation is convex is provided in Appendix B. In separable form these four equations can be written as

$$\Sigma X_{ij} + \sum_{k=0}^{P_i} \gamma_{ik}\, y_{ik} = \bar{C}_i \qquad i = 1,2,3$$

and

$$\sum_{i=1}^{3} \sum_{k=0}^{P_i} \gamma_{ik}\, \phi_i\, (y_{ik}) \leqslant - \ln \beta$$

in which γ_{ik} are the variables,

$$\phi_i = - \ln \frac{y_i^2}{\sigma_i^2 + y_i^2}$$

and P_i are the number of grid points chosen for the i^{th} separable set. β was chosen to be 70 percent.

The resulting structure was solved for the model in Section A. The optimal allocation and equilibrium quantities and the corresponding prices are shown in Tables 21 - 25 and Figures 22 - 26 in Appendix F. The supply curve to the left corresponds to the joint chance constrained model and the curve to the right corresponds to the model of Section A without the joint chance constraint. The marginal costs are significantly higher since water supply will have to be derived from more dependable sources. The equilibrium prices and quantities for various regions corresponding to each of the target years is shown in Table 7. Note that the stochastic variables are assumed to be distribution-free and no specific mention of the symmetry of distribution is made in the problem structure.

Table 7. Equilibrium quantities for joint CCP model.

Year	Region:	R1	R2	R3	R4	R5
1975	Q	7,500	11,500	21,500	25,000	2,000
	P	107	117	110	110	78
1980	Q	10,500	13,500	24,000	26,000	2,000
	P	108	118	110	110	91
1985	Q	12,000	15,500	26,500	27,500	2,000
	P	109	116	112	111	107
2000	Q	16,000	21,500	33,000	28,500	3,000
	P	117	126	118	119	89
2020	Q	19,500	29,500	38,500	27,000	3,000
	P	149	144	137	137	130

Q – Quantity in acre feet.
P –Price per acre foot.

CHAPTER VII

SUMMARY OF THE RESEARCH

The concept of water recycling in water supply management is introduced and its potential to augment the water supply is analyzed within the framework of microeconomic theory. The specific nature and magnitude of increases in water supply associated with this alternative are analyzed. Based on the theory, an operational model for a multiregion municipal water supply system was built as a nonlinear programming problem. Using separable programming methods, this problem was solved to yield results consistent with competitive equilibrium. The optimal solution indicates how each of the water resources will have to be allocated such that the total cost of supplying water to all the regions is a minimum. At the optimal quantities, the marginal cost of water is equal to its marginal valuation. The analysis takes into acount the cost of providing secondary treatment of wastewater before discharge.

Proposed changes in effluent standards are examined by incorporating the appropriate costs to improve the quality of the effluent to meet such standards.

To ensure social acceptability in the use of recycled water for culinary purposes, blending restrictions were introduced. This was done by stipulating that the amount of reclaimed water for reuse should be less than a fixed percentage of the water derived from all other sources. The resulting marginal costs were higher than the marginal costs without these constraints.

Stochasticity in surface water sources was analyzed through chance constrained programming. Single chance constraints were specified for each of the stochastic creek flows. The optimal allocations did not show any change over the planning horizon. To examine the effect of specifying that the constraints pertinent to the surface water should all be satisfied jointly at a given probability level, the Joint Chance Constrained Programming technique was used. At a 70 percent probability level, significant increases in marginal costs were noticed.

The model used in the study has certain limitations which deserve mention. The assumption that cost is a linear function of the activity is restrictive. The use of average annual water quantities in the model gives rise to pessimistic estimates of the utilization of water and wastewater treatment facilities. The model is suitable to analyze only the short-run situation since scale of operation was not introduced. The assumption of constant returns to scale is questionable over a very wide range of quantities. The separable programming technique yields quantities within ± 250 acre feet of the optimal solution. Although a more accurate solution can be obtained by decreasing the interval between grid points, it is expensive and more time consuming. Some of these problems are currently being investigated.

This research did not explicitly take water rights into account since such constraints were deemed inappropriate in the context of allocative efficiency. Nevertheless, water rights constraints can be incorporated in the model. From the results of the analysis, policy decisions regarding pricing of water, allocation, water quality decision and even changes in the existing water rights can be made with economic efficiency as a central theme.

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APPENDICES

Appendix A

A Proof for the Concavity of the Objective Function

To show that $f(Q_j)$ is a concave function.

Proof:

A function is said to be concave over a closed convex set x if, for any two points X_1 and X_2, $X_1 \neq X_2$ in X and for all λ, $0 < \lambda < 1$

$$f[\lambda X_2 + (1 - \lambda) X_1] \geqslant f(X_2) + (1 - \lambda) f(X_1)$$

$$f(X) = \frac{k_j}{1 + \frac{1}{\eta}} X^{1 + 1/\eta}$$

For simplicity let $X_1 = 0$

$$f[\lambda X_2 + (1 - \lambda) X_1] = \frac{k_j}{1 + \frac{1}{\eta}} (\lambda X_2)^{1 + 1/\eta} \qquad \text{(A-1)}$$

$$\lambda f(X_2) + (1 - \lambda) f(X_1) = \frac{k_j}{1 + \frac{1}{\eta}} [\lambda X_2^{1 + 1/\eta} + (1 - \lambda) 0^{1 + 1/\eta}] \qquad \text{(A-2)}$$

For $\eta = 0$, $\eta = -1$, and $\eta = -\infty$, Equations A-1 and A-2 are equal. For any other η, Equation A-1 is greater than Equation A-2. Therefore, $f(Q_j)$ is a concave function.

Appendix B

A Proof for the Convexity of the Joint Constraint

To show that $f(Y_1) = -\ln \dfrac{y_1^2}{\sigma_1^2 + y_1^2}$ is convex.

Proof:

$$f(y) = -\ln \frac{y^2}{\sigma^2 + y^2}$$

$$= -\ln (y^2) + \ln (\sigma^2 + y^2)$$

$$f^1(y) = -\frac{1}{y^2} \cdot 2y + \frac{L}{\sigma^2 + y^2} \cdot 2y$$

$$= -2\sigma^2 / y(\sigma^2 + y^2) \leqslant 0$$

$$f^{11}(y) = \frac{-2 \cdot -1}{y^2} + \frac{(\sigma^2 + y^2)\, 2 - 2y \cdot 2y}{(\sigma^2 + y^2)^2}$$

$$= \frac{2}{y^2} + \frac{2\sigma^2 + 2y^2 - 4y^2}{(\sigma^2 + y^2)^2}$$

$$= \frac{2}{y^2} + \frac{2\sigma^2 - sy^2}{(\sigma^2 + y^2)^2}$$

$$= \frac{2(\sigma^4 + y^4 + 2\sigma^2 y^2) + 2\sigma^2 y^2 - 2y^4}{(\sigma^2 + y^2)^2\, y^2}$$

$$= 2\sigma^4 + 2y^4 + 4\sigma^2 y^2 + 2\sigma^2 y^2 - 2y^4$$

$$= \frac{2\sigma^4 + 6\sigma^2 y^2}{(\sigma^2 + y^2)^2\, y^2} \geqslant 0$$

Since the slope decreases in absolute value, $f(y_1)$ is a convex function.

Appendix C

Optimal Water Allocation and Pricing Policies for Existing Water Quality Standards

Table 8. Allocation for 1975.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1		21.4																			17.8
	C_2																					63.4
	C_3				7.8																	41.3
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2														21.4							
	W_3																					
	W_4					4.3			3.2			14.8										
Region 1	G_1					3.2																
	E_1						3.75															
	S_1																					3.75
	R_1																					
Region 2	G_2								8.3													
	E_2									5.6			0.15									
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												10.75									
	S_3																					10.9
	R_3																					
Region 4	G_4														3.6							
	E_4															12.5						
	S_4																					12.5
	R_4																					
Region 5	G_5																	2.0				
	E_5																		1.0			
	S_5																					1.0
	R_5																					
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 9. Allocation for 1980.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1		22.4																			16.8
	C_2																					63.4
	C_3				15.3																	33.8
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2														22.4							
	W_3																					
	W_4					7.3			5.2			17.3										
Region 1	G_1					3.2																
	E_1						5.25															
	S_1																					5.25
	R_1																					
Region 2	G_2								8.3													
	E_2									5.6			1.15									
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												12.0									
	S_3																					13.15
	R_3																					
Region 4	G_4														3.6							
	E_4															13.0						
	S_4																					13.0
	R_4																					
Region 5	G_5																	2.0				
	E_5																			1.0		
	S_5																					1.0
	R_5																					
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 10. Allocation for 1985.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1		23.9																			15.3
	C_2																					63.4
	C_3				21.8																	27.3
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2														23.9							
	W_3																					
	W_4					8.8			7.2			20.3										
Region 1	G_1					3.2																
	E_1						6.0															
	S_1																					6.0
	R_1																					
Region 2	G_2								8.3													
	E_2									5.6			2.15									
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												13.5									
	S_3																					15.65
	R_3																					
Region 4	G_4														3.6							
	E_4															13.75						
	S_4																					13.75
	R_4																					
Region 5	G_5																	2.0				
	E_5																		1.0			
	S_5																					1.0
	R_5																					
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 11. Allocation for 2000.

Sources	Destination	Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1	19.1	7.8																			12.3
	C_2																					63.4
	C_3				41.6																	7.5
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1														19.1							
	W_2														7.8							
	W_3																					
	W_4					13.6			14.2			28.3										
Region 1	G_1					3.2																
	E_1						8.4															
	S_1																					8.4
	R_1																					
Region 2	G_2								8.3													
	E_2									5.6			5.7									
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												17.5									
	S_3																					23.2
	R_3																					
Region 4	G_4														3.6							
	E_4															15.3						
	S_4																					15.3
	R_4																					
Region 5	G_5																	3.0				
	E_5																		1.5			
	S_5																					1.5
	R_5																					
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 12. Allocation for 2020.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1	19.1	9.3																			10.8
	C_2			19.7																		43.7
	C_3				49.1																	
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1														19.1							
	W_2														9.3							
	W_3								19.7													
	W_4					19.8			6.0			37.8										
Region 1	G_1					3.2																
	E_1						8.4														3.1	
	S_1																					8.4
	R_1																					
Region 2	G_2								8.3													
	E_2									5.6			9.8								1.7	
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												22.3									
	S_3																					32.0
	R_3																					
Region 4	G_4														3.6							
	E_4															16.0						
	S_4																					16.0
	R_4																					
Region 5	G_5																	3.5				
	E_5																		1.5		0.25	
	S_5																					1.5
	R_5																					
*	S_6																					5.0

* Proposed sub-regional wastewater facilities
⊕ System outflow

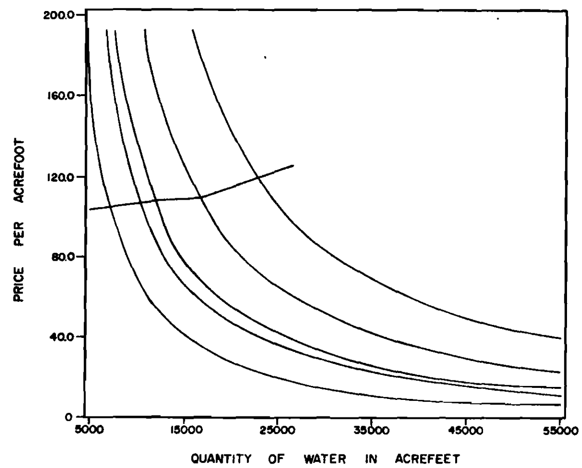

Figure 7. Demand and supply curves for area 1.

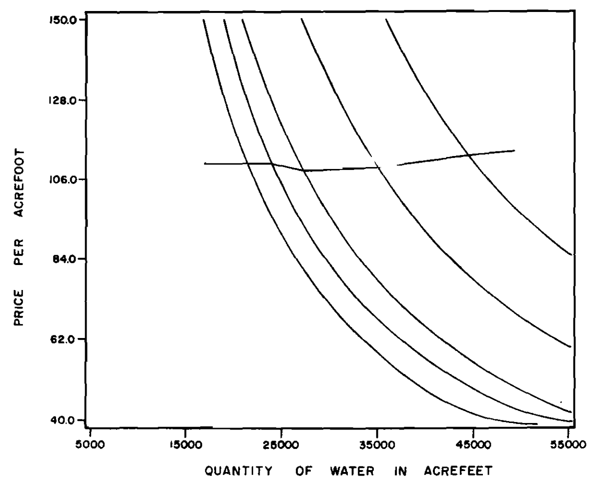

Figure 9. Demand and supply curves for area 3.

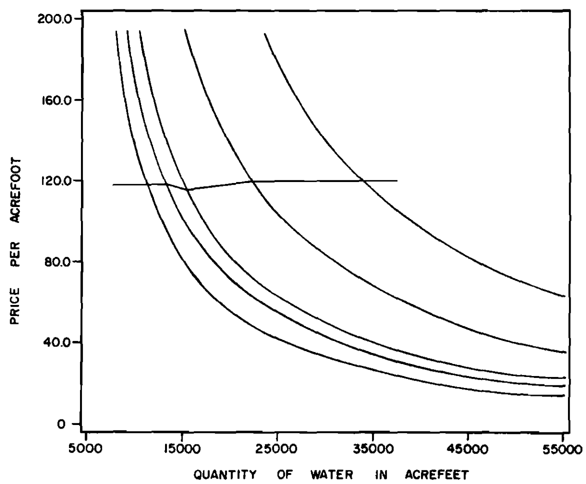

Figure 8. Demand and supply curves for area 2.

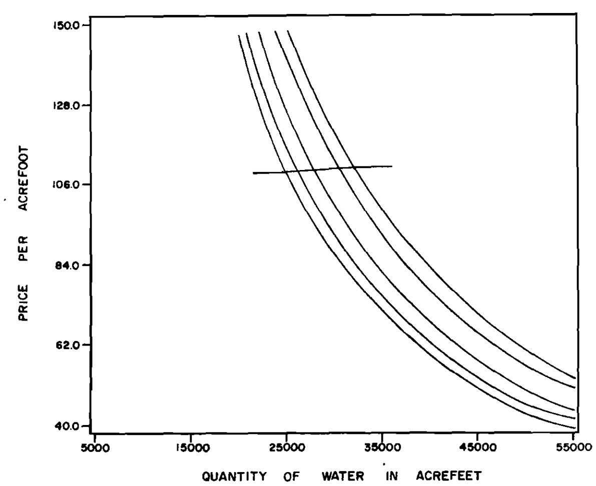

Figure 10. Demand and supply cruves for area 4.

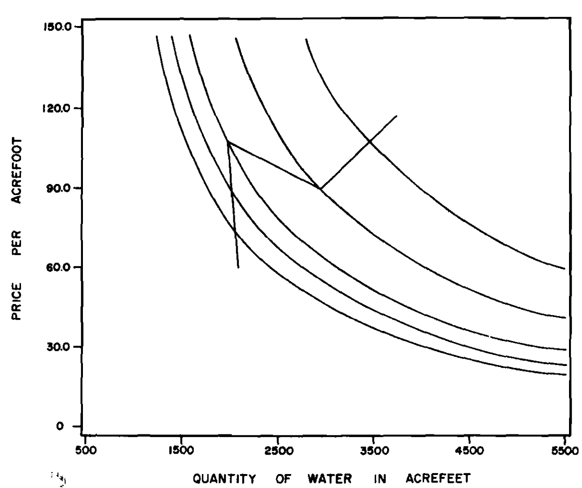

Figure 11. Demand and supply cruves for area 5.

Appendix D

Optimal Water Allocation and Pricing Policies for Higher Water Quality Constraints

Table 13. Allocation for 1980.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1		20.9																			18.3
	C_2																					63.4
	C_3				11.8																	37.3
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2														20.9							
	W_3																					
	W_4					6.3			4.2			15.8										
Region 1	G_1					3.2																
	E_1						4.75															
	S_1																					4.75
	R_1																					
Region 2	G_2								8.3													
	E_2									5.6			0.65									
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												11.25									
	S_3																					11.9
	R_3																					
Region 4	G_4														3.6							
	E_4															12.25						
	S_4																					12.25
	R_4																					
Region 5	G_5																	2.0				
	E_5																		1.0			
	S_5																					1.0
	R_5																					
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 14. Allocation for 1985.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1																					39.2
	C_2																					63.4
	C_3																					49.1
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2																					
	W_3																					
	W_4					2.05						3.8			8.65							
Region 1	G_1					3.2																
	E_1						5.25															
	S_1							5.25														
	R_1					5.25																
Region 2	G_2								8.3													
	E_2									5.6			1.35									
	S_2										5.6											
	R_2								5.6													
Region 3	G_3											6.7										
	E_3												11.85									
	S_3													13.2								
	R_3											13.2										
Region 4	G_4														3.6							
	E_4															12.25						
	S_4																12.25					
	R_4														12.25							
Region 5	G_5																	1.0				
	E_5																		1.0			
	S_5																			1.0		
	R_5																	1.0				
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 15. Allocation for 2000.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1	8.45																				30.75
	C_2																					63.4
	C_3																					49.1
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1														8.45							
	W_2																					
	W_3																					
	W_4					3.05			5.6			4.9			0.95							
Region 1	G_1					3.2																
	E_1						7.25															
	S_1							8.25														
	R_1					8.25																
Region 2	G_2								8.3													
	E_2						1.0			5.6			3.15									
	S_2										5.6											
	R_2								5.6													
Region 3	G_3											6.7										
	E_3												14.75									
	S_3													17.9								
	R_3											17.9										
Region 4	G_4														3.6							
	E_4															13.0						
	S_4																13.0					
	R_4														13.0							
Region 5	G_5																	1.25				
	E_5																		1.25			
	S_5																			1.25		
	R_5																	1.25				
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 16. Allocation for 2020.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1	5.5																				33.7
	C_2			12.4																		51.0
	C_3				2.3																	46.8
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1														5.5							
	W_2																					
	W_3											12.4										
	W_4					8.4			8.4													
Region 1	G_1					3.2																
	E_1						8.4														1.6	
	S_1							8.4														
	R_1					8.4																
Region 2	G_2								8.3													
	E_2									5.6											8.9	
	S_2										5.6											
	R_2								12.3													
Region 3	G_3											6.7										
	E_3												18.5									
	S_3										0.6			17.9								
	R_3											17.9										
Region 4	G_4														3.6							
	E_4															13.5						
	S_4																13.5					
	R_4														17.9							
Region 5	G_5																	1.5				
	E_5																		1.5			
	S_5																			1.5		
	R_5																	1.5				
*	S_6										6.1						4.4					

* Proposed sub-regional wastewater facilities
⊕ System outflow

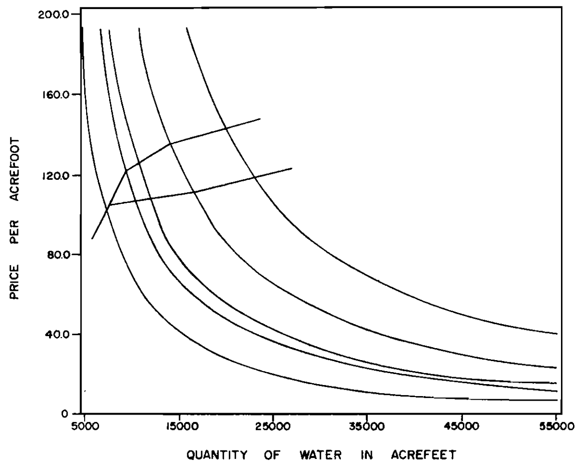

Figure 12. Demand and supply curves for area 1.

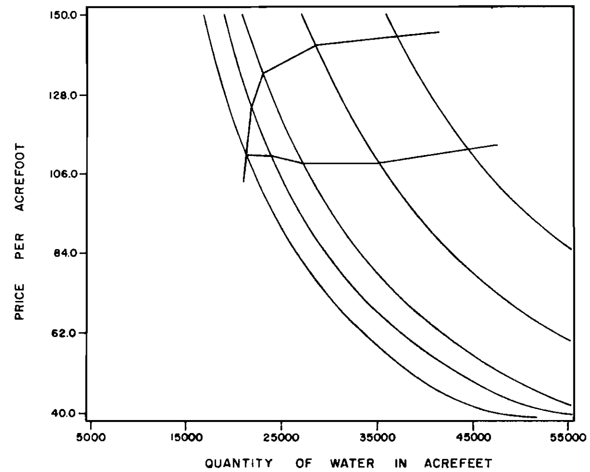

Figure 14. Demand and supply curves for area 3.

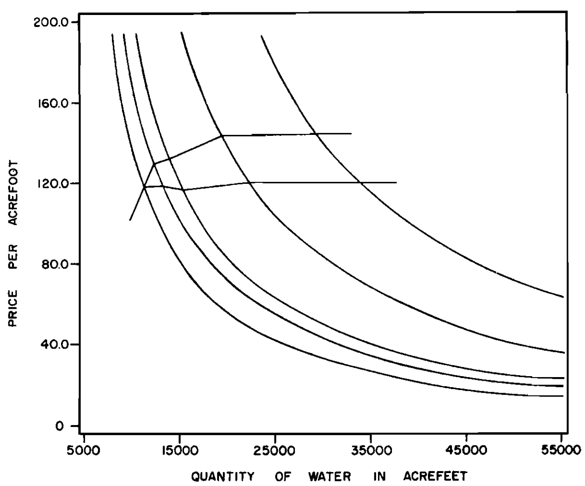

Figure 13. Demand and supply curves for area 2.

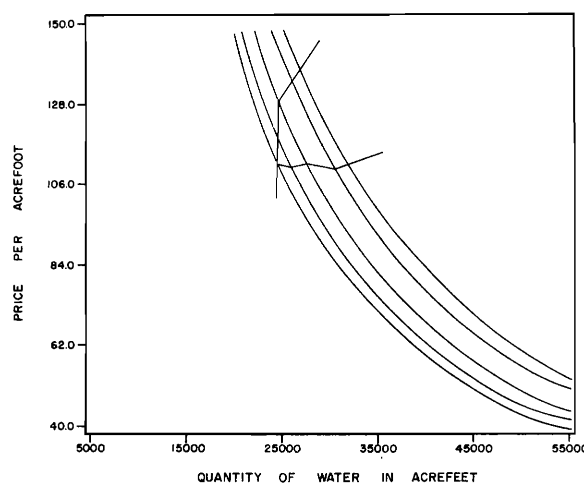

Figure 15. Demand and supply curves for area 4.

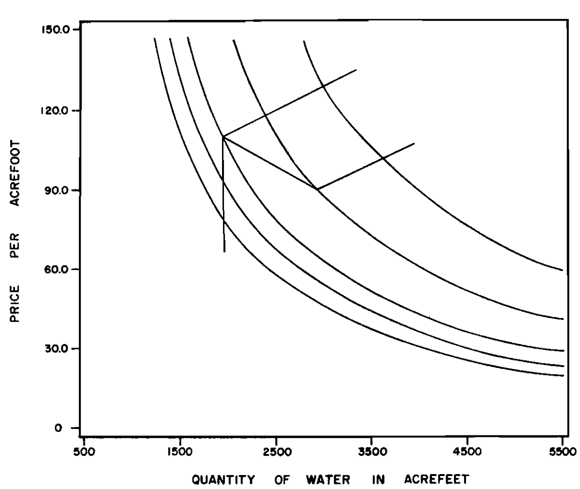

Figure 16. Demand and supply curves for area 5.

Appendix E

Optimal Water Allocation and Pricing Policies With Blending Restrictions (At Higher Quality Standards)

Table 17. Allocation for 1980.

Sources	Destination	Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1		20.9																			18.3
	C_2																					59.2
	C_3				7.6																	41.5
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2														20.9							
	W_3								4.2													
	W_4					6.3						15.8										
Region 1	G_1					3.2																
	E_1						4.75															
	S_1																					5.4
	R_1																					
Region 2	G_2								8.3													
	E_2						0.65			5.6												
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												11.25									
	S_3																					11.25
	R_3																					
Region 4	G_4														3.6							
	E_4															12.25						
	S_4																					12.25
	R_4																					
Region 5	G_5																	2.0				
	E_5																		1.0			
	S_5																					1.0
	R_5																					
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 18. Allocation for 1985.

Sources	Destination	Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1		14.8																			24.4
	C_2																					63.4
	C_3				1.2																	47.9
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2														14.8							
	W_3																					
	W_4					4.8						10.9										
Region 1	G_1					3.2																
	E_1						5.0															
	S_1							2.0														4.15
	R_1					2.0																
Region 2	G_2								7.9													
	E_2						1.15			5.6												
	S_2										5.6											
	R_2								5.6													
Region 3	G_3											6.7										
	E_3												11									
	S_3													4.4								6.6
	R_3											4.4										
Region 4	G_4														3.6							
	E_4															11.5						
	S_4																4.6					6.9
	R_4														4.6							
Region 5	G_5																	1.6				
	E_5																		1.0			
	S_5																			0.4		0.6
	R_5																	0.4				
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 19. Allocation for 2000.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1		16.4																			22.8
	C_2																					63.4
	C_3				9.6																	39.5
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2														16.4							
	W_3																					
	W_4					8.0						16.1										
Region 1	G_1					3.2																
	E_1						7.0															
	S_1							2.8			1.7											2.5
	R_1					2.8																
Region 2	G_2								8.3													
	E_2									5.6											4.4	
	S_2										5.6											
	R_2								11.7													
Region 3	G_3											6.7										
	E_3												14.25									
	S_3													5.7								8.55
	R_3											5.7										
Region 4	G_4														3.6							
	E_4															12.5						
	S_4																5.0					7.5
	R_4														5.0							
Region 5	G_5																	2.0				
	E_5																		1.25			
	S_5																			0.5		0.75
	R_5																	0.5				
*	S_6										4.4											

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 20. Allocation for 2020.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1		17.6																			21.6
	C_2			2.3																		61.1
	C_3				21.2																	27.9
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2														17.6							
	W_3								2.3													
	W_4					12.4						23.3										
Region 1	G_1					3.2																
	E_1						8.4														1.35	
	S_1							3.9			2.3											2.2
	R_1					3.9																
Region 2	G_2								8.3													
	E_2									5.6											8.65	
	S_2										5.6											
	R_2								17.9													
Region 3	G_3											6.7										
	E_3												18.75									
	S_3													7.5								11.25
	R_3											7.5										
Region 4	G_4														3.6							
	E_4															13.25						
	S_4																5.3					7.95
	R_4														5.3							
Region 5	G_5																	2.4				
	E_5																		1.5			
	S_5																			0.6		0.9
	R_5																	0.6				
*	S_6										10.0											

* Proposed sub-regional wastewater facilities
⊕ System outflow

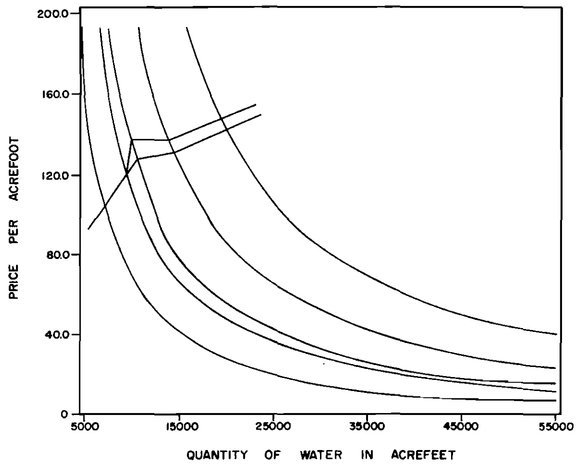

Figure 17. Demand and supply curves for area 1.

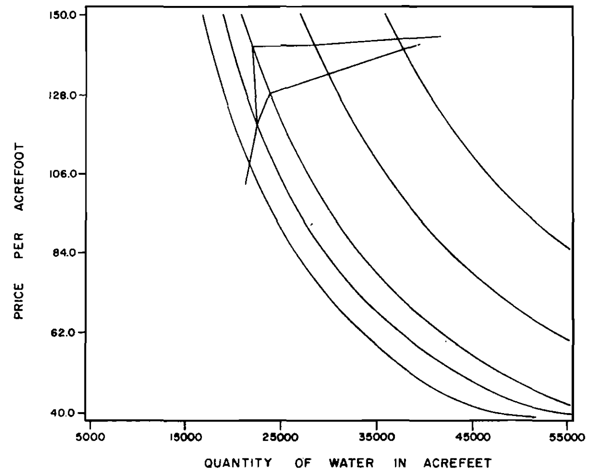

Figure 19. Demand and supply curves for area 3.

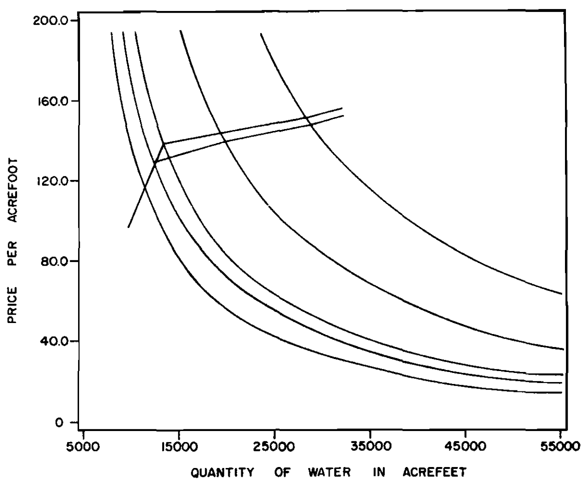

Figure 18. Demand and supply curves for area 2.

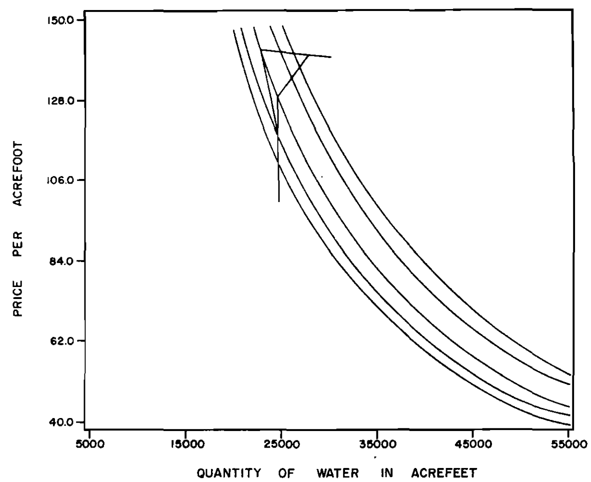

Figure 20. Demand and supply curves for area 4.

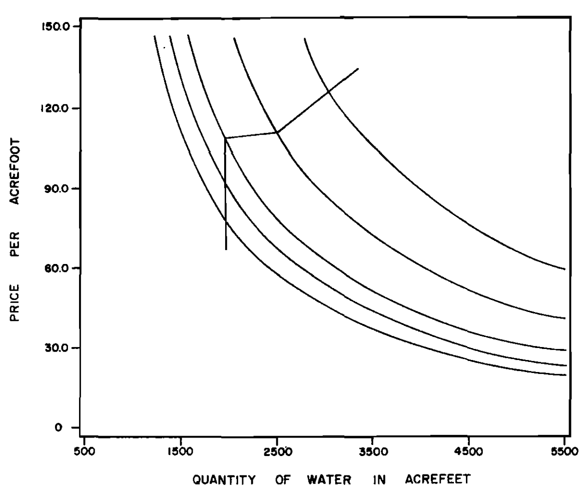

Figure 21. Demand and supply curves for area 5.

Appendix F

Optimal Water Allocation and Pricing Policies With Joint Constraints

Table 21. Allocation for 1975.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1		17.8																			
	C_2			7.6																		
	C_3				3.8																	
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2														17.8							
	W_3								3.2			0.8			3.6							
	W_4					4.3						14.0										
Region 1	G_1					3.2																
	E_1						3.75															
	S_1																					3.9
	R_1																					
Region 2	G_2								8.3													
	E_2						0.15			5.6												
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												10.75									
	S_3																					10.75
	R_3																					
Region 4	G_4														3.6							
	E_4															12.5						
	S_4																					12.5
	R_4																					
Region 5	G_5																	2.0				
	E_5																		1.0			
	S_5																					1.0
	R_5																					
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 22. Allocation for 1980.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1		16.6																			
	C_2			13.4																		
	C_3				7.7																	
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2														16.6							
	W_3								5.2			2.4			5.8							
	W_4					7.3						15.0										
Region 1	G_1					3.2																
	E_1						5.3															
	S_1																					6.4
	R_1																					
Region 2	G_2								8.3													
	E_2						1.2			5.6												
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												12.0									
	S_3																					12.0
	R_3																					
Region 4	G_4														3.6							
	E_4															13.0						
	S_4																					13.0
	R_4																					
Region 5	G_5																	2.0				
	E_5																		1.0			
	S_5																					1.0
	R_5																					
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 23. Allocation for 1985.

Destination / Sources		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1		14.6																			
	C_2			19.0																		
	C_3				11.8																	
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1																					
	W_2														14.6							
	W_3								7.2			2.4			9.4							
	W_4					8.8						17.5										
Region 1	G_1					3.2																
	E_1						6.0															
	S_1																					8.15
	R_1																					
Region 2	G_2								8.3													
	E_2						2.15			5.6												
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												13.25									
	S_3																					13.25
	R_3																					
Region 4	G_4														3.6							
	E_4															13.75						
	S_4																					13.75
	R_4																					
Region 5	G_5																	2.0				
	E_5																		1.0			
	S_5																					1.0
	R_5																					
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 24. Allocation for 2000.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1	8.9																				
	C_2			25.6																		
	C_3				16.2																	
Import Water	I_1																					
	I_2				14.5																	
Water Treatment Plants	W_1														8.9							
	W_2																					
	W_3								13.2			12.4										
	W_4					12.8						2.0			16.0							
Region 1	G_1					3.2																
	E_1						8.0															
	S_1																					8.0
	R_1																					
Region 2	G_2								8.3													
	E_2									5.6			5.2									
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												16.5									
	S_3																					21.7
	R_3																					
Region 4	G_4														3.6							
	E_4															14.3						
	S_4																					14.3
	R_4																					
Region 5	G_5											11.9						3.0				
	E_5																		1.5			
	S_5																					1.5
	R_5																					
*	S_6																					

* Proposed sub-regional wastewater facilities
⊕ System outflow

Table 25. Allocation for 2020.

Sources \ Destination		Water Treatment Plants				Region 1			Region 2			Region 3			Region 4			Region 5			*	⊕
		W_1	W_2	W_3	W_4	M_1	S_1	R_1	M_2	S_2	R_2	M_3	S_3	R_3	M_4	S_4	R_4	M_5	S_5	R_5	S_6	O
Creeks	C_1	8.2																				
	C_2			26.1																		
	C_3				16.6																	
Import Water	I_1														6.2							
	I_2				14.5																	
Water Treatment Plants	W_1														8.2							
	W_2																					
	W_3								21.2			4.9										
	W_4					16.3						5.7			9.1							
Region 1	G_1					3.2																
	E_1						8.4														1.35	
	S_1																					8.4
	R_1																					
Region 2	G_2								8.3													
	E_2									5.6			9.2									
	S_2																					5.6
	R_2																					
Region 3	G_3											6.7										
	E_3												19.3									
	S_3																					28.4
	R_3																					
Region 4	G_4														3.6							
	E_4															13.5						
	S_4																					13.5
	R_4																					
Region 5	G_5											21.2						3.0				
	E_5																		1.5			
	S_5																					1.5
	R_5																					
*	S_6																					1.4

* Proposed sub-regional wastewater facilities
⊕ System outflow

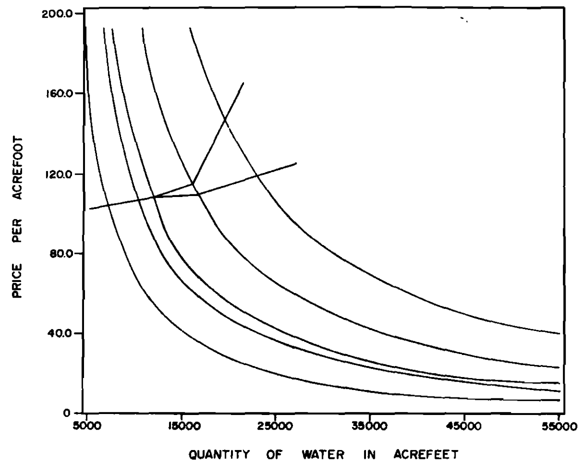

Figure 22. Demand and supply curves for area 1.

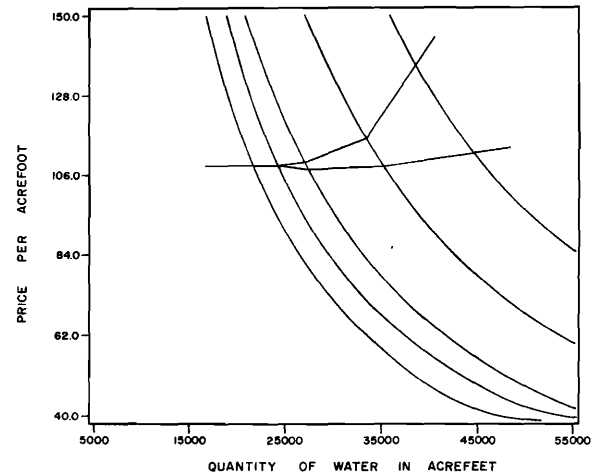

Figure 24. Demand and supply curves for area 3.

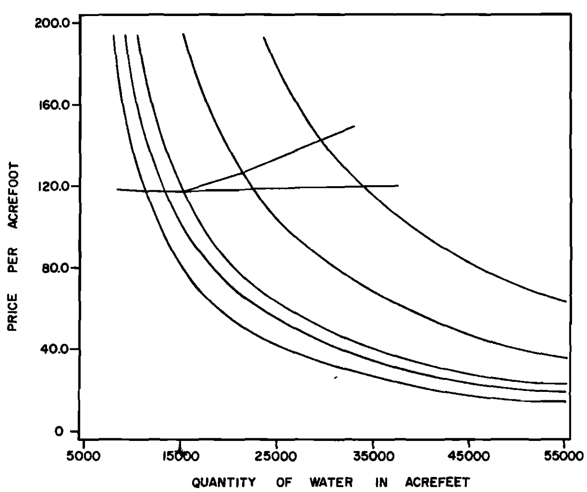

Figure 23. Demand and supply curves for area 2.

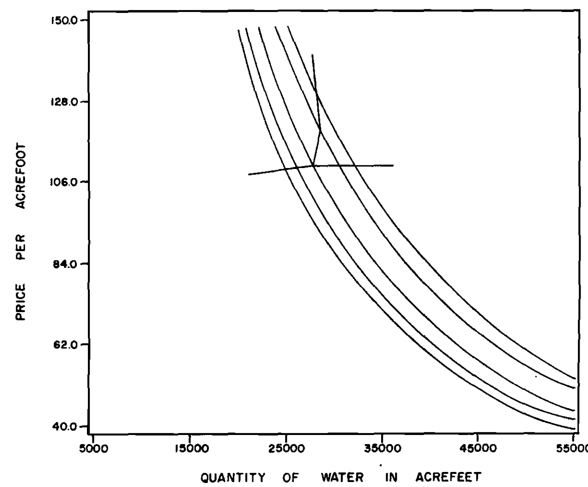

Figure 25. Demand and supply curves for area 4.

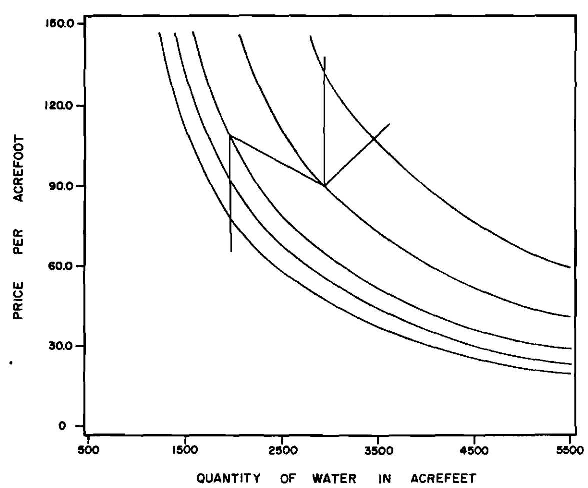

Figure 26. Demand and supply curves for area 5.