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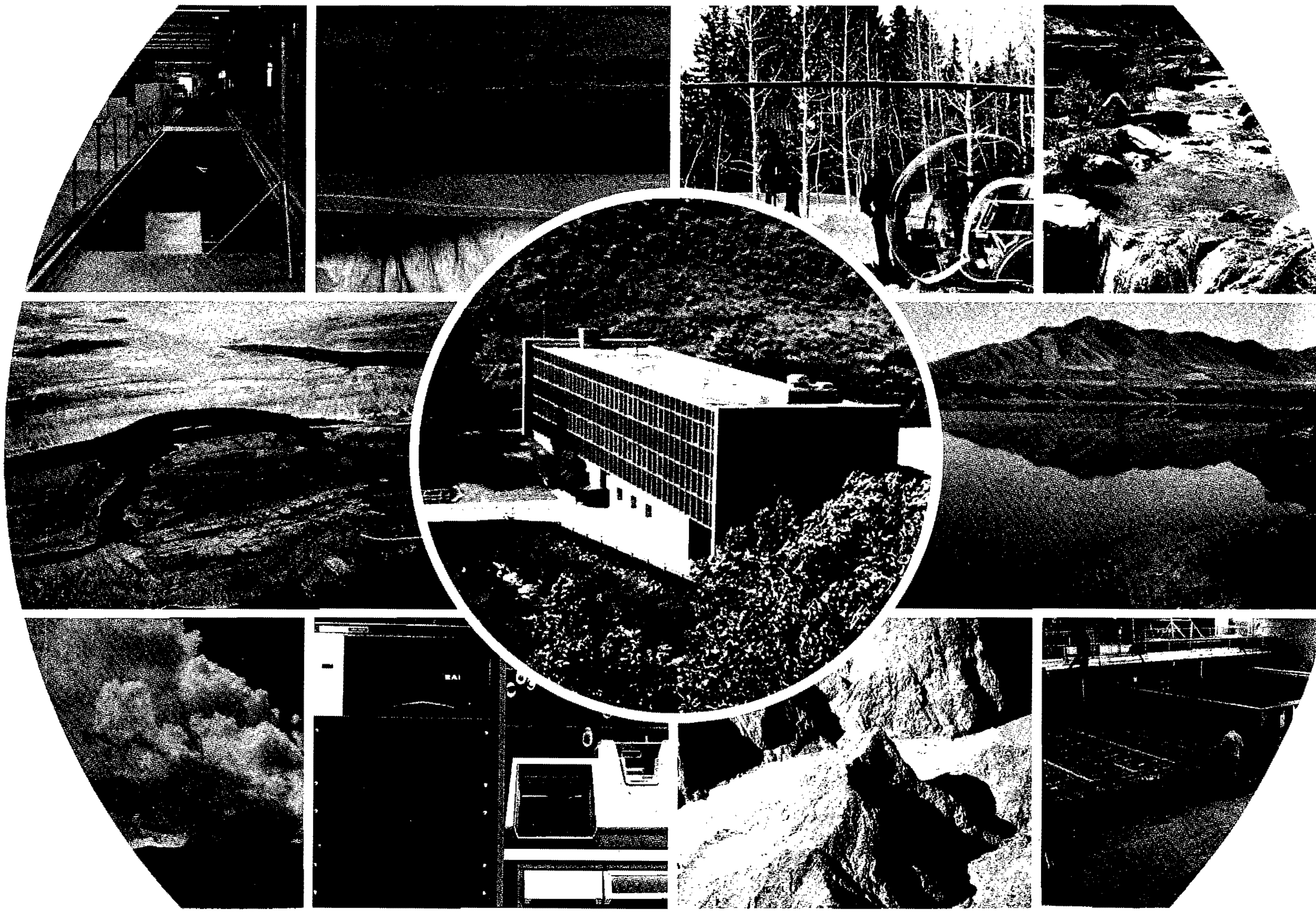

Utah Water Research Laboratory
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WATER QUALITY SERIES
UWRL/Q-79/01

STOCHASTIC ANALYSIS OF WATER QUALITY

by

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ABSTRACT

This report demonstrates the feasibility of applying stochastic techniques to linear water quality models. The Monte Carlo, First Order, and Generation of Moment Equation techniques are applied to a long term phosphorus model of Lake Washington. The effect of uncertainty of the phosphorus loading term on simulated phosphorus levels is analyzed. All three stochastic techniques produced the same results. The simulated concentrations of phosphorus in the water column are very responsive to uncertainty in annual phosphorus loading, the sediment concentrations relatively insensitive. The Monte Carlo technique is shown to require the most computation time of the three stochastic techniques applied. The First Order and Generation of Moment Equation techniques are shown to be precise and efficient methods of stochastic analysis. In this application they required less than one thousandth the computation time of the Monte Carlo technique.

The Generation of Moment Equations technique is also applied to a steady state salinity model of the Colorado River system. Two sources of uncertainty are considered: 1) the estimation of "steady state" values of salinity loading from a limited historic data base and 2) the estimation of salinity loading from irrigated land by a semi-empirical approach. Six stochastic simulations of the Colorado River system are presented. Coefficients of variations of simulated salinities at Imperial Dam are shown to vary from 5.7 to 10.3 percent. The major source of uncertainty in all simulations is the estimation of the steady state salinity loading with the agricultural loading term becoming important in some simulated management alternatives.

ACKNOWLEDGMENTS

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INTRODUCTION

Water quality models simulate the interactions among physical, chemical, and biological parameters of natural water systems. They are valuable tools in our efforts to understand these systems. They permit the scientist or engineer to analyze mathematically a large number of reactions simultaneously, thus increasing our understanding of the complex interactions which occur in these natural bodies of water.

Water quality models are being utilized increasingly by water resource managers to assess the effect of various development alternatives upon our rivers and lakes and consequently have a direct impact upon the decision making process. Water quality models presently in common use are deterministic, that is, a single set of output variables (effects) results from a set of input variables (causes) (Burges and Lettenmaier, 1975). However, the natural systems they are intended to represent are frequently characterized by stochastic variations and uncertainty.

Natural systems are highly variable with uncertainty stemming from several sources. The very assessment of the state of a system depends upon the measurement of selected parameters. These measurements are subject to uncertainty due to variations in sampling and analysis techniques, instrumentation precision, etc. For example, coefficients of variation from interlaboratory precision tests for measurements of nitrate have ranged from 5.5 to 96.4 percent depending upon technique and sample concentration (APHA, 1965). Additional uncertainty results from representation of large systems by observations from only a few sample points in time and space.

Perhaps more importantly, waters are subject to random fluctuations by man related and natural phenomena. Variations in light, temperature, river flow, diversions, and industrial and domestic waste discharges all contribute to the nature of uncertainty associated with water quality. Edmondson (1970) presented information describing the variability of five day biochemical oxygen demand (BOD_5) values in secondary effluents from eight sewage treatment plants discharging to Lake Washington. The mean coefficient of variation of these plants was nearly 85 percent. Measured nitrite and nitrate nitrogen and phosphate in the Cedar River, at the inlet to Lake Washington, reflected relative standard deviations in the ranges of 64 to 81 and 54 to 64 percent respectively over the period of the study.

Beyond these problems of measurement of water quality parameters are the uncertainties in model construction and the assignment of values to model coefficients (e.g. rate constants, flows, loading rates) used for water quality prediction. Many of the chemical and biological interactions that occur in natural waters are poorly understood. Those that are understood qualitatively may not be understood well enough to permit accurate quantitative representation. The results of any water quality modeling effort are therefore subject to some degree of uncertainty.

Deterministic models are limited in that they provide the user with only a single set of output values for each proposed management alternative, when in fact the output values would be more accurately represented by a distribution in probability. In the past, an attempt has been made to satisfy these needs by applying sensitivity analysis. Selected parameters were varied and repetitious deterministic simulations performed, thus providing the user with a range of results for a range of parameter values.

Sensitivity analysis does not, however, indicate the likelihood of a particular result occurring. Stochastic modeling provides a more practical and theoretically sound approach to assessing uncertainty in projections. Stochastic models treat selected variables as random variables having distributions in probability and provide a theoretically sound framework for propagation of input uncertainty into uncertainty of results. Outputs from stochastic models provide both a range and likelihood of occurrence through calculations of means and variances associated with output values.

Three methods of uncertainty propagation are considered in this report. The most familiar of these methods is the Monte Carlo technique. This method involves statistical analysis of artificially generated occurrences. The other two methods, First Order and Generation of Moment Equations techniques, provide precise uncertainty propagation for analytical solutions and differential equations respectively.

Two systems were selected to test the applicability of stochastic techniques to the water quality issues. The deterministic long-term phosphorus model previously applied to Lake Washington by Lorenzen et al., (1976) was modified to reflect the uncertainty associated with phosphorus loadings to the lake. The uncertainty in this case reflects uncertainty associated with estimating historic inflows as well as natural variability of phosphorus loadings. The system was analyzed using all three of the stochastic techniques mentioned above. This permitted both verification of results and a comparison of techniques. The best of these techniques was then applied to the second case study.

In the second case study, uncertainty associated with salinity modeling of the Colorado River system was assessed. Estimation and modeling uncertainties associated with natural and agricultural salinity sources were defined through application of the Generation of Moment Equations technique. This procedure effectively defined the limits of reliability for the salinity modeling effort resulting from these uncertainties.

The objectives of this study were: 1) to demonstrate the feasibility of applying the selected stochastic techniques to questions of water quality, and 2) to assess uncertainties associated with salinity modeling of the Colorado River system. The two case studies illustrate the applicability and value of stochastic techniques in water quality modeling and provide examples of stochastic output used to evaluate risk associated with water quality decisions.

DEVELOPMENT OF THEORY

The term "stochastic model" will be used in this report to describe the basic equations defining the relationship between input variables and output variables where some of the variables are thought of as having a distribution in probability (Clarke, 1973). Although such random variables may be completely defined by their density functions, we have limited ourselves to consideration of their means and variances. The terms "stochastic method" or "stochastic technique" will be used to describe the methodology used to evaluate the stochastic model. The following sections, describe the stochastic model and the three methods of evaluation (Monte Carlo, First Order, and Generation of Moment Equations) applied in the Lake Washington case study.

Description of Stochastic Model

Stochastic models can be developed to evaluate the effects of a wide variety of uncertainties. This Lake Washington analysis is limited to evaluation of the effects of inputs and initial conditions that are stochastic in nature. Other sources of uncertainty such as variation in coefficients, model uncertainty or measurement errors are not considered.

The linear stochastic model used here is

$$\dot{\tilde{X}}(t) = F(t)\,\tilde{X}(t) + H(t) + G(t)\,\tilde{w}(t) \quad . \quad . \quad . \quad . \quad (1)$$

in which

$\dot{\tilde{X}}(t)$ = first derivative of $\tilde{X}(t)$ at time t

$\tilde{X}(t)$ = stochastic state variable at time t

$H(t)$ = deterministic input variable(s) at time t

$\tilde{w}(t)$ = white noise disturbance on input at time t

$F(t)$, $G(t)$ = coefficients

This model is a generalized form of a model presented by Schweppe (1973). It is a state space structure, white process model characterized by a known stochastic white input disturbance and known initial conditions.

The stochastic white noise process is defined (Schweppe, 1973) by:

$$E[\tilde{w}(t)] = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$E[\tilde{w}(t_1)\,\tilde{w}^T(t_2)] = \delta(t_1 - t_2)\,Q(t) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which

$\delta(t_1 - t_2)$ = dirac delta function; unity at $t_1 = t_2$. zero elsewhere

$Q(t)$ = variance of $\tilde{w}(t)$

Equation 2 states that the expected value or mean of the white noise process is always zero. The second condition (Equation 3) can be summarized in the general statement: A white process has no time structure. The value of a white noise process at one instant of time provides no knowledge of its value at any other time. These definitions make no assumption about the density function of $\tilde{w}$. The white noise process used here is not necessarily normally distributed. Examples of application of white noise functions to represent random disturbances or variations are given by Chiu (1968), Chiu and Lee (1972), Moore (1973), and Moore and Schweppe (1973).

Monte Carlo Method

The Monte Carlo method has been described by Freeze (1975) as repetitive simulations using a mathematical model coupled with a statistical analysis of results. Yevjevich (1972) noted that this method is as old as the theory of probability itself, since in concept it differs little from tossing coins to define probabilities. The use of high speed computers coupled with deterministic models has contributed much to the popularization and sophistication of this technique.

The Monte Carlo simulation method utilized in this study was described by Hahn and Shapiro (1967) (Figure 1). Random number generation is used to select values of input variables from assumed density functions. These random values define the systems response through a deterministic model. This process is repeated a large number of times to obtain many individual deterministic simulations (sample traces). The accumulated samples can then be analyzed to define the statistical characteristics of the systems response.

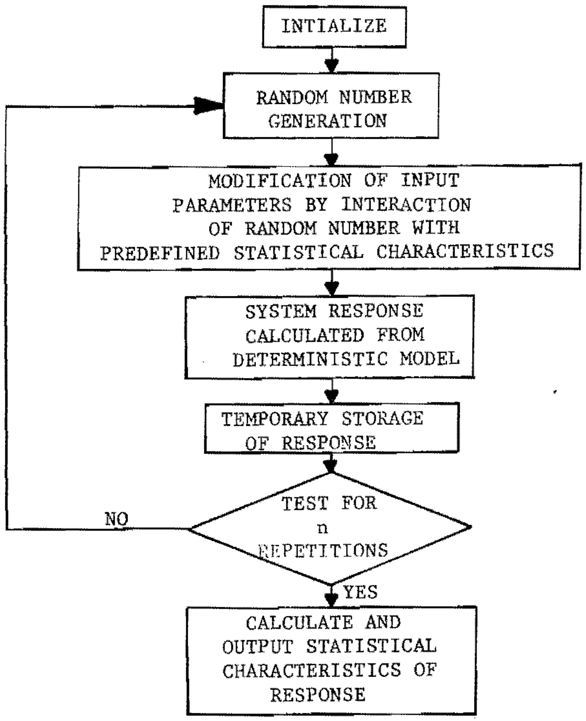

Figure 1. Flow chart of procedural steps followed in the Monte Carlo technique (after Hahn and Shapiro, 1967).

The Monte Carlo technique has been used extensively in the study of stochastic hydraulic and hydrologic phenomenon. Stochastic analysis of the effect of irregular channel characteristics upon velocity profiles in open channels (Chiu, 1968) and natural stream beds (Chiu and Lee, 1972) have been performed. Freeze (1975) utilized Monte Carlo simulation to show the effects of random parameters on one dimensional porous media flow problems. Benson (1952), Nash and Amorocho (1966), and Ott and Linsley (1972) employed Monte Carlo techniques to investigate the effects of short term streamflow records upon the prediction of flood peaks. Similarly, Matalas and Wallis (1972) investigated the implications of assumed frequency distributions of floods upon reservoir design.

Recently the Monte Carlo technique has been utilized as a standard with which to compare other stochastic methods. For example, Burges and Lettenmaier (1975) used it to check the accuracy of their First Order Analysis of dissolved oxygen and biochemical oxygen demand in streams. Simulations by both techniques were used to determine the importance of uncertainty in travel time and the BOD decay constant upon predictions of dissolved oxygen deficits.

Two major problems must be considered when the Monte Carlo technique is used. These are accurate generation of random numbers and estimation of required sample size. It must be recognized that the so called "random number generators" actually produce pseudo-random numbers. Yevjevich (1972) describes the requirements which a program written to compute pseudo-random numbers must satisfy; 1) the program must generate numbers with extremely weak autocorrelation, 2) the distribution function of pseudo-random numbers must approximate the uniform distribution, 3) the program must be stable in producing a stationary series, and 4) the generated sample must not reproduce the same sequence. Similarly recognizing the limititations of the digital computer, Matalas and Slack (1974) emphasize the importance of analyzing pseudo-random number algorithms.

The second consideration deals with the question of how many sample simulations to generate. The Monte Carlo simulation method depends upon statistical analysis of large amounts of generated data to define the stochastic characteristics of the output variables. Confidence in this analysis, therefore, varies with the number of samples generated. Two methods have been proposed to determine the adequate number of samples. One method utilized by Burges and Lettenmaier (1975) consists of iterating the sample generating program with increasingly greater sample sizes until the convergence rate of the desired statistic can be determined. They found their Monte Carlo model of dissolved oxygen stabilized with respect to mean, variance, and skewness beyond 2,000 generated samples. The other approach directly calculates the required number of simulations. Hahn and Shapiro (1967) gave equations that estimated the number of Monte Carlo samples that would be required to define the mean within desired error bounds. They indicated that most of the usual statistical methods for obtaining a desired degree of precision are directly applicable to Monte Carlo analysis. These methods are at best approximate because typically the variance of the output function must be estimated for the calculation.

The principal advantages of the Monte Carlo technique are that it is easy to understand and very flexible. Since it can be readily understood it is more likely to be accepted by those not familiar sample sizes until the convergence rate of the desired statistic can be determined. They found their Monte Carlo model of dissolved oxygen stabilized with respect to mean, variance, and skewness beyond 2,000 generated samples. The other approach directly calculates the required number of simulations. Hahn and Shapiro (1967) gave equations that estimated the number of Monte Carlo samples that would be required to define the mean within desired error bounds. They indicated that most of the usual statistical methods for obtaining a desired degree of precision are directly applicable to Monte Carlo analysis. These methods are at best approximate because typically the variance of the output function must be estimated for the calculation.

The principal advantages of the Monte Carlo technique are that it is easy to understand and very flexible. Since it can be readily understood it is more likely to be accepted by those not familiar with stochastic techniques. This undoubtedly explains in part, its wide application and acceptance. The flexibility comes in the ease with which the Monte Carlo method can be superimposed upon any deterministic model. Input and output density functions can be defined to essentially any degree of completeness desired. Empirical or any of the standard probability density functions can be used to describe the stochastic nature of input parameters. Another advantage that is the accuracy of the method is limited only by the accuracy of the deterministic model and the number of samples generated.

Unfortunately the Monte Carlo method uses large computer programs which require a great deal of computer time to generate the required number of samples. Furthermore, Hahn and Shapiro (1967) noted that there is frequently no way of determining whether any of the variables are more dominant or more important than others without repeating the entire set of simulations.

First Order Analysis Technique

Cornell (1972) characterized First Order uncertainty analysis by its two major features: 1) random functions are defined solely by their mean and covariance functions, and 2) first order analysis is used to determine functional relationships among variables. Thus defined, First Order analysis reflects a truncated application of the technique of generation of system moments (also referred to as the "statistical error propagation" or "delta method"). As described by Hahn and Shapiro (1967) the method of generation of system moments is based upon a Taylor series expansion of functional or system relationships about the expected values of the state variables. The accuracy of this technique is determined by the number of central moments analyzed and the number of terms retained in the individual Taylor series expansions. Hahn and Shapiro (1967) provide general derivations of the expressions for determining the mean, variance, skew, and kurtosis.

First Order uncertainty analysis reflects a truncated application of the method of generation of system moments in that only the mean and second central moments are analyzed and only linear components of the Taylor series expansion are retained. For example, Cornell (1972) states the function y

$$y = f(x) \quad \ldots \quad (4)$$

can be approximated by Equation 5:

$$y \doteq f(\mu_x) + \dot{f}(\mu_x)\,(x-\mu_x) \quad \ldots \quad (5)$$

in which

μ_x = mean of the random variable x

$f(\mu_x)$ = the function f(x) evaluated at μ_x

$\dot{f}(\mu_x)$ = first derivative of f(x) evaluated at μ_x

The symbol of equality used in Equation 5, "$\doteq$", is used here to denote equal in the first order sense. In the more general matrix notation Equation 5 becomes:

$$Y = g(X) \doteq g(\mu_x) + b^T\,(X-\mu_x) \quad \ldots \quad (6)$$

in which

X = a column vector of random variables

μ_X = a column vector of the mean of the random variable in X

b^T = the transpose of a column vector of partial derivative b_i as given by Equation 5

$$b_i = \frac{\partial g(X)}{\partial X_i} \quad \ldots \quad (7)$$

Equation 6 leads to the mean (μ_y) and variance (σ_y) equations of

$$\mu_Y \doteq g(\mu_X) \quad \ldots \quad (8)$$

$$\sigma_Y^2 \doteq b^T Q_X b \quad \ldots \quad (9)$$

in which

Q_X = covariance matrix of the vector of variables X

First order uncertainty analysis is considered an approximate technique since two criteria must be met for it to be complete and exact. First, the functional relationships must be linear. Secondly, the resulting probability distribution of Y must be completely described by the mean and variance, i.e., be a normal distribution. When applied to nonlinear systems or nonnormal distributions the method becomes approximate. Cornell (1972) justifies the use of such an approximate technique by noting that 1) in actual engineering applications it is rare that sufficient data exist to establish the full probability law of a variable, 2) the analysis is frequently approximate anyway because of modeling uncertainty, and 3) design parameters are not often sensitive to moments higher than the mean and variance.

Example applications of first order analysis include its application to hydraulic design (Tang and Yen, 1972; Yen and Tang, 1976), and water quality management (Thomann, 1967; DiToro and O'Conner, 1968; Chamberlain et al., 1974; and Burges and Lettenmaier, 1975). The advantages of this technique lie primarily in its ease of application. Direct calculation of the mean and variance is much less costly than sample generating techniques such as the Monte Carlo method. Limiting the analysis to linear operators and the first two moments is significantly less complicated than using full probability distribution functions.

The most apparent disadvantage of First Order analysis stems from its approximate nature. The method is limited to those applications that possess a functional relationship between the dependent and independent variables. Further, that relationship must be differentiable.

Generation of Moment Equations Technique

The theory of this approach was described by Schweppe (1973). The application made in this study was a specialized case, in that only linear systems with white noise disturbances are considered. The system of differential equations (Equation 1) can be represented in discrete form as follows:

$$X(n\Delta + \Delta) = \Phi(n\Delta)X(n\Delta) + \Delta M(n\Delta) + \Delta G(n\Delta)W(n\Delta) \quad . \quad . \quad (10)$$

in which

$\Phi(n\Delta)$ = state transition matrix $(I+\Delta F(n\Delta))$

I = identity matrix

Δ = time step

The discrete equation can then be manipulated to permit direct computation of the mean and variance of the state variables. For example, the covariance matrix, $\Gamma(n\Delta+\Delta)$ can be computed directly from the expansion of:

$$\Gamma(n\Delta+\Delta) = E\{[X(n\Delta+\Delta) - E(X(n\Delta+\Delta))]$$
$$[X(n\Delta+\Delta)-E(X(n\Delta+\Delta))]^T\} \quad . \quad . \quad (11)$$

The resulting discrete equations are evaluated by numerical techniques. Many systems, for example, permit the direct iterative solution of Equations 10 and 11.

This stochastic method has not had wide application. A similar technique was used by Moore, Dandy, and DeLucia (1976) in their analysis of uncertainty associated with water quality sampling programs. This application, as with Moore and Schweppe (1973), differed from applications in this study in that the stochastic model required linearization before propagation of the means and variances.

The primary advantage of this technique is that it results in direct computation of the mean and variance of state variables. For linear systems these results are theoretically exact. An analytical solution of the systems relationships is not required. Application of this method is relatively inexpensive and uncomplicated.

The accuracy of the method is limited only by numerical errors in the propagation routine for the linear white noise case. However, the method becomes approximate when linearization is required. It must be assumed that the state variables at given time are independent of the input noise at that time.

STOCHASTIC ANALYSIS OF LAKE WASHINGTON

Lake Washington lies adjacent to Seattle and has been studied intensively (Edmondson, 1968, 1969, 1970, 1975). The two major inflows to the lake, Cedar River and Sammamish River display low nutrient levels. Prior to influence by man the lake was in an oligotrophic state. In the early 1900s the lake received raw sewage from a population of up to 50,000. This early pollution was alleviated in 1936 by diversion of the sewage to Puget Sound.

In 1941 a second episode of nutrient enrichment began as a series of 10 secondary wastewater treatment plants were constructed with outfalls to Lake Washington. Additional nutrient loading resulted from septic tanks within the basin and from Seattle's combined sewer system. The lake during this period was phosphorus limited (Edmondson, 1970) with nearly 75 percent of the phosphorus loading coming from sewage sources. Prompted by the increasingly eutrophic conditions, the effluents of the six activated sludge plants and four trickling filter plants were diverted away from the lake during the period 1963 to 1968. The lake displayed significant improvement in terms of both phosphorus and phytoplankton levels.

The recovery of Lake Washington has made it nearly a classical example of the lake reclamation potential of wastewater diversion. Unfortunately, not all lakes respond so well. For example, Lake Sammamish which lies 4 miles (6.4 km) east of Lake Washington has undergone a similar diversion program with less favorable results (Emery, Moon, and Welch, 1973). The high cost of diversion programs as well as other reclamation actions make it imperative that the probability of improvement be well understood before substantial investments are made. This requires an understanding of the phenomenon contributing to the lake's eutrophic state and an accurate assessment of the uncertainties associated with predicting a future response.

A long-term phosphorus model has been developed for Lake Washington (Lorenzen, Smith, and Kimmel, 1976). This deterministic model considers external (inflow) and internal (sediment release) phosphorus loadings in its projections of water column and sediment phosphorus concentrations.

Such external loading rates are inherently variable and difficult to estimate accurately. Assessment of the uncertainty associated with this parameter provides a more realistic basis for interpreting the projected response of the system. In this section a comparison is made of the usefulness of the three methods for assessing the effect of uncertainty in external loading rates upon the projected lake phosphorus levels is compared.

It was not the intent here to assess the actual levels of uncertainty associated with Lorenzen's projections, but rather to demonstrate the applicability of stochastic methods for making such assessments and to provide a framework for selecting a stochastic method most appropriate for other water quality systems.

<u>Long-term Phosphorus Model</u>

Lorenzen, Smith, and Kimmel (1976) proposed the following coupled differential equations to describe the cycling of phosphorus in Lake Washington.

$$\dot{C}_c(t) = \frac{M}{V} + \frac{K_2 A C_s(t)}{V} - \frac{K_1 A C_c(t)}{V} - \frac{C_c(t) Q}{V} \quad . \quad . \quad . \quad (12)$$

$$\dot{C}_s(t) = \frac{K_1 A C_c(t)}{V_s} - \frac{K_2 A C_s(t)}{V_s} - \frac{K_1 K_3 A C_c(t)}{V_s} \quad . \quad . \quad . \quad (13)$$

in which

- $C_c(t)$ = average annual total phosphorus concentration in water column at time t (g/m^3)
- $\dot{C}_c(t)$ = first derivative of $C_c(t)$ with respect to time at time t
- $C_s(t)$ = total exchangeable sediment phosphorus at time t (g/m^3)
- $\dot{C}_s(t)$ = first derivative of C_s with respect to time at time t
- M = total annual phosphorus loading (g/yr)
- V = lake water column volume (m^3)
- V_s = sediment volume (m^3)
- A = or sediment surface area (m^2)
- D = annual outflow (m^3/yr)
- K_1 = specific rate of phosphorus transfer to the sediments (m/yr)
- K_2 = specific rate of phosphorus transfer from the sediments (m/yr)
- K_3 = fraction of total phosphorus input to the sediment that is

unavailable for the exchange process (dimensionless)

The model approximates the long term exchange processes that occur between sediments and the water column.

The model has a number of characteristics that make it particularly useful as a test case in our study:

(1) It is a linear model with a differentiable analytical solution that will allow the calculation of theoretically exact means and variances with both the First Order and the Generation of Moment Equation methods.

(2) Equations 12 and 13 are readily solved by simple numerical methods (Euler's technique was used with Monte Carlo and Generation of Moment Equations methods).

(3) Desirable if not necessary assumptions about independence of state variables and input noise are reasonable.

(4) Phosphorus loading, physical characteristics, coefficients, and verification data for its application to Lake Washington are readily available.

Stochastic Phosphorus Model

Equation 12 can be modified as follows to represent random variations in the input loading, M, by addition of a white noise term w(t). Equation 14 results:

$$\dot{\tilde{C}}_w(t) = \frac{M+\tilde{w}(t)}{V} + \frac{K_2A\tilde{C}_s(t)}{V} - \frac{K_1A\tilde{C}_c(t)}{V} - \frac{\tilde{C}_c(t)D}{V} \quad . \quad (14)$$

Equation 15 expresses the coupling of Equations 13 and 14 in a vector format.

$$\dot{\tilde{C}}(t) = F\tilde{C}(t) + M(t) + \tilde{W}(t) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (15)$$

in which

$\tilde{C}(t)$ = column vector of stochastic state variables C_c and C_s (mg/l)

F = matrix of constant coefficients

M(t) = column vector of phosphorus loadings (g/m3-yr)

$\tilde{W}(t)$ = column vector of white noise variations in loading

The white noise term was assumed to have the following characteristics:

$$E[W(t)] = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (16)$$

$$E[\tilde{W}(t_1)\tilde{W}^T(t_2)] = \delta(t_1 - t_2)\, Q(t_1) \quad . \; . \; . \; . \quad (17)$$

$$Q(t_1) = [PF * M(t_1)]^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

in which

PF = factor of proportionality for standard deviation of white noise term

Equation 18 implies the standard deviation of the white noise term, $\tilde{w}(t)$, is proportional to the magnitude of the phosphorus loading to Lake Washington. The actual relationship defining the variance Q(t1) would depend upon what type of uncertainties were lumped into the white noise term. This study has not identified the type of uncertainty considered in the white noise term since our objective was to test the stochastic techniques. The proportional relationship was selected as representative of the type of relationship that could be used with likely sources of uncertainty such as measurement errors, variations in the phosphorus discharge of the wastewater treatment plants, estimation errors where data is lacking, or natural variability of diffuse sources. In all these cases one would expect the loading uncertainty to increase with increases in the magnitude of the phosphorus loading. The proportional relationship reflects this expectation. Any deterministic relationship could have been used without affecting the validity or structure of the stochastic model. The results would be sensitive to this selection but not the techniques of analysis. The variance of the white noise could also be empirically determined and input as a constant for a selected time interval if such information was available.

An assumption of independence between the state variables and the noise term completes the definition of the continuous stochastic model:

$$E[\tilde{C}(t)\tilde{W}^T(t)] = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (19)$$

Application of Generation of Moment Equations Technique

In order to implement this method the differential equation is rewritten in discrete form:

$$C(n\Delta + \Delta) = \Phi C(n\Delta) + \Delta M(n\Delta) + \Delta W(n\Delta) \quad . \; . \; . \; . \quad (20)$$

$$E[W(n\Delta)] = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (21)$$

$$E[W(n_1\Delta)\, W^T(n_2\Delta)] = \begin{cases} 0 & n_1 \neq n_2 \\ Q(n\Delta)\Delta^{-1} = [PF * M(n\Delta)]^2\Delta^{-1} & n_1 = n_2 \end{cases} \quad . \quad (22)$$

$$E[C(0)W^T(n\Delta)] = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (23)$$

in which

Φ = state transition matrix $(I + \Delta F)$

The definition of these terms parallels those of the continuous model where "Δ" is the discrete time step (1/time) and "n" the number of time steps to time "$n\Delta$." Initial values and variances of the state variables are defined by Equations 24 and 25:

$$E[C(0)] = C_o \quad . . . . (24)$$

$$E[C(0)C^T(0)] = \Psi \quad . . . . (25)$$

The equations for the mean and variances of the state variables, $C(n\Delta)$, are readily generated from this discrete phosphorus model. Considering Equations 20 through 25, it can be seen that the expected value of $C(n\Delta + \Delta)$ is simply:

$$E[C(n\Delta + \Delta)] = \Phi C(n\Delta) + \Delta M(n\Delta) \quad . . . . . (26)$$

Equation 28 for the covariance matrix, $\Gamma(n\Delta)$, results from expansion of Equation 27 (see Equation 11).

$$\Gamma(n\Delta+\Delta) = E\{[C(n\Delta) - E\{C(n\Delta)\}] \quad . . . . (27)$$

$$[C(n\Delta) - E\{C(n\Delta)]^T\}$$

$$\Gamma(n\Delta+\Delta) = \Phi\Gamma(n\Delta)\Phi^T + Q(n\Delta) \quad . . . . (28)$$

An algorithm was developed for the generation of the means and variances for the 50 year study period. A complete listing of this program and sample outputs are provided in Appendix A.

The only special consideration for the application of this method involved the determination of the appropriate time step for solution. Subsequent sensitivity analysis indicated that the model was insensitive to this parameter. Comparison of runs with 10, 73, 200, 365, 730 time-steps per year were made. The means of the sediment and lake phosphorus concentrations were identical to three significant figures for all runs. The standard deviations of these runs varied between the runs of 10 and 73, but not above. The 5-day time step (73 time steps/yr) was deemed adequate for the comparisons studied in this application.

Application of Monte Carlo Method

The discrete stochastic model (Equation 20) was used to generate samples for the Monte Carlo experiment. Each sample consisted of a 50 year simulation of phosphorus levels in Lake Washington; 3,840 samples were generated to permit estimation of the means and variances of the system. A complete program listing and sample output are provided in Appendix A.

The white noise disturbances were represented by selection of a random normal deviate for each time step. An algorithm generated pseudo-random normal deviates for a standard normal curve. The resulting deviates were modified to represent the desired variance, $Q(n\Delta)/\Delta$, prior to their use in Equation 20.

The number of samples required for the Monte Carlo experiment was initially estimated by application of Equation 29 (Hahn and Shapiro, 1967):

$$N = \left[\frac{Z\sigma'}{E}\right]^2 \quad . . . . (29)$$

in which

- E = maximum desired error on selected state variables
- Z = normal deviate corresponding to desired confidence level in projections of selected state variables
- σ' = estimate of the process standard deviation on the selected state variable

The maximum allowable error, E, was assumed to equal $\pm$ 0.1 μg/l and $\pm$ 1.0 mg/l for the water column and sediment phosphorus, respectively. A confidence level of 95 percent was selected. Variances produced by the method of Generation of Moment Equations were used to define the process standard deviation, σ'. If these values had not been available, it would have been necessary to estimate them by a preliminary Monte Carlo experiment. The application of Equation 29 to the Lake Washington system defined minimum sample generation to be 6,552 for water column phosphorus concentrations and 176 for the sediment concentrations. The simulation requirements of the water column concentrations are limiting and, therefore, define the required number of simulations to be 6,552.

Iterative sample sets of 60, 60, 120, 240, 480, 960, 1920 simulations were generated to produce an accumulated sample size of 3,840. Comparison of the mean and variances with the 1962 and 1980 values produced by the other two methods indicated that additional refinement of predicted values would not warrant the cost of generating additional samples. Figure 2 illustrates the convergence of the Monte Carlo generated means (dash lines) to the theoretically exact means. The actual number of samples generated for the Monte Carlo experiment was kept at 3,840. The full 6,552 sample simulations would be required to achieve the desired confidence level if the theoretically exact means were not available.

Application of First Order Technique

The analytical solution of Equation 15 may be expressed as:

$$C(t) = e^{Ft}[C_0 + F^{-1}(I-e^{-Ft})M + \int_0^t e^{-Fs}\tilde{W}(s)ds] \quad . . . (30)$$

in which

- I = identity matrix

The expected values, $E\{C(t)\}$, are given by:

$$E\{\tilde{C}(t)\} = e^{Ft}\left[C_0 + F^{-1}(I-e^{-Ft})M\right] \quad (31)$$

The covariance matrix, $\Gamma_{C(t)}$, is defined by Equations 32 and 33.

$$\Gamma_{C(t)} = e^{Ft}(\Gamma_{C_0} + \Gamma_{z(t)})e^{F^T t} \quad (32)$$

in which

$$z(t) = \int_0^t e^{-Fs}\,\tilde{W}(s)\,ds \quad (33)$$

The covariance matrix, $\Gamma_{z(t)}$, of the white noise expression is further defined as

$$\Gamma_{z(t)} = \int_0^t \int_0^t e^{-Fs}\,Q\delta(s-u)e^{-F^T u}\,ds\,du \quad (34)$$

in which
$\delta(s-u)$ = the dirac delta function

Equation 30 treats the phosphorus loading rate, M, as a constant. Equations 31 and 32 were therefore initialized yearly to reflect the yearly variations in M. The computer routine developed to evaluate Equations 31 and 32 is given in Appendix A.

Comparison of Techniques

Figures 3 and 4 illustrate the projected water column and exchangeable sediment phosphorus concentrations for Lake Washington and show confidence bands around expected values. The assumed phosphorus loading uncertainty, $\sigma_t = .3M(t)$, lead to significant uncertainty in both phosphorus projections. The uncertainty associated with the phosphorus in the water column, C_c, tended to be a coefficient of variation of 16 percent. The exchangeable sediment phosphorus, C_s, displayed a coefficient of variation which grew slowly to about 2.5 percent where it appeared to stablize.

These results were anticipated because it is logical to expect that the phosphorus levels in the lake are sensitive to uncertainty in loadings. Because it is difficult to specify the uncertainty associated with the loading assumptions made by Lorenzen, Smith, and Kimmel (1976), a series of runs were produced reflecting a wide range of uncertainty. Figure 5 presents the levels of uncertainty expected in the water column and sediment phosphorus projections in 1980 for a given level of uncertainty in loading. These results should be interpreted carefully because the slopes of the lines vary from

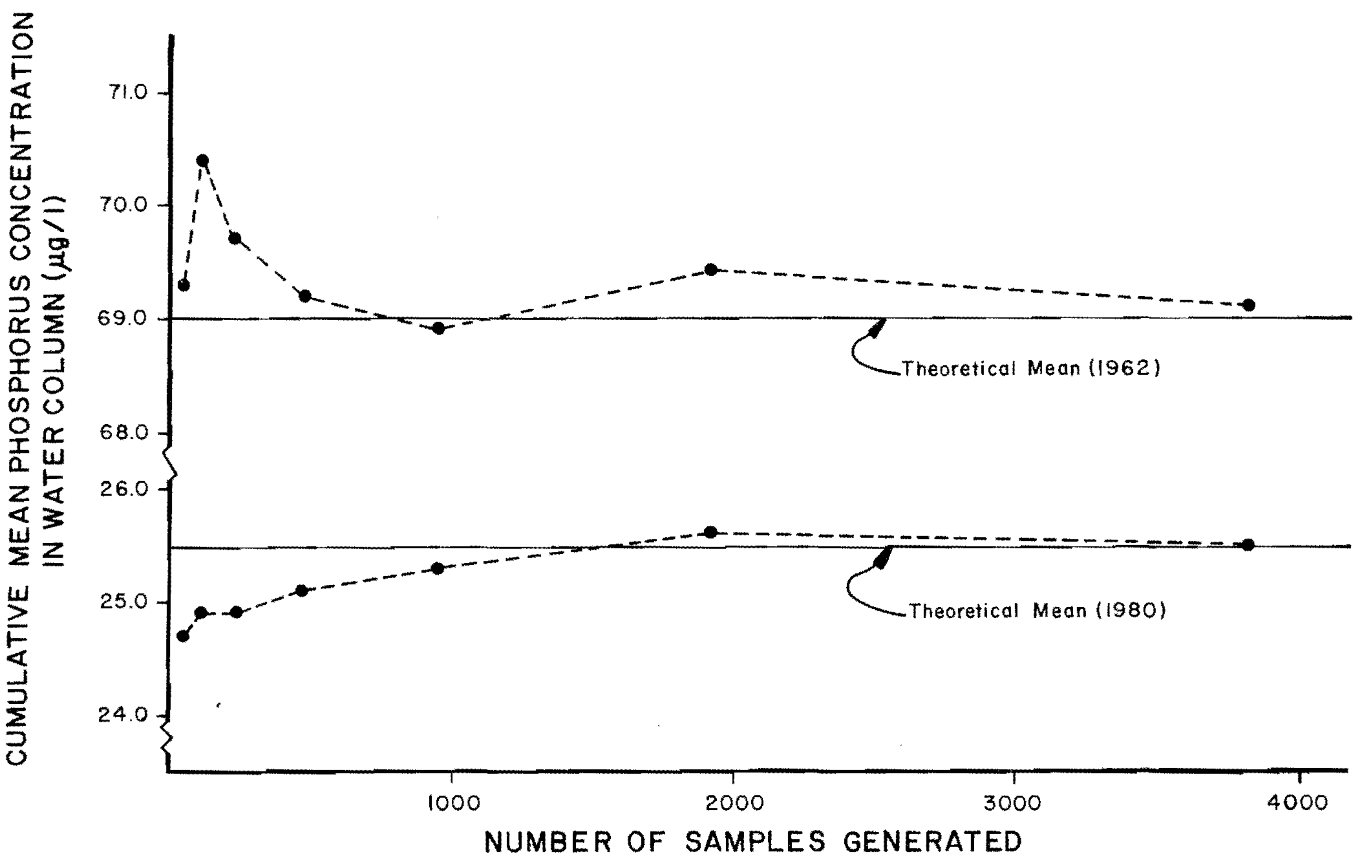

Figure 2. Convergence of Monte Carlo experiment.

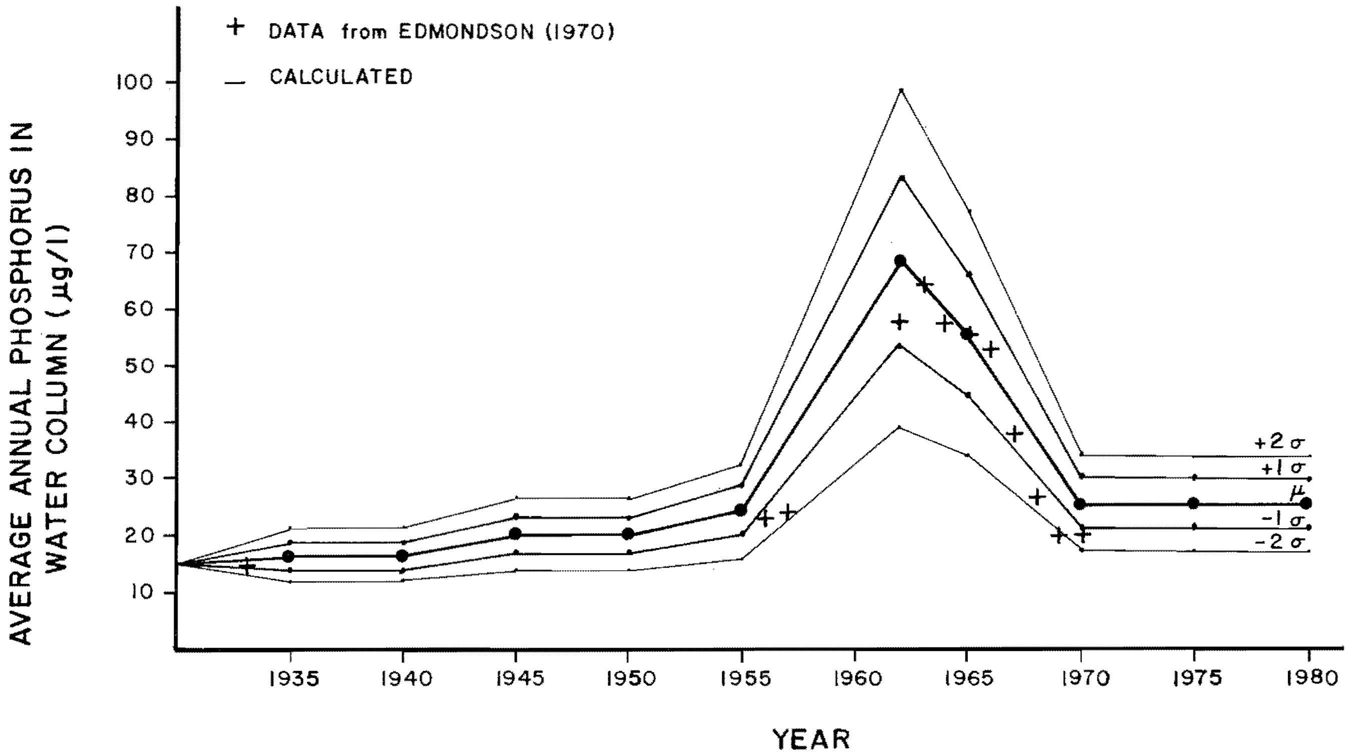

Figure 3. Projected phosphorus concentrations in water column of Lake Washington with loading uncertainty, PF = 0.3, produced by Monte Carlo, First Order, and Generation of Moment Equations techniques.

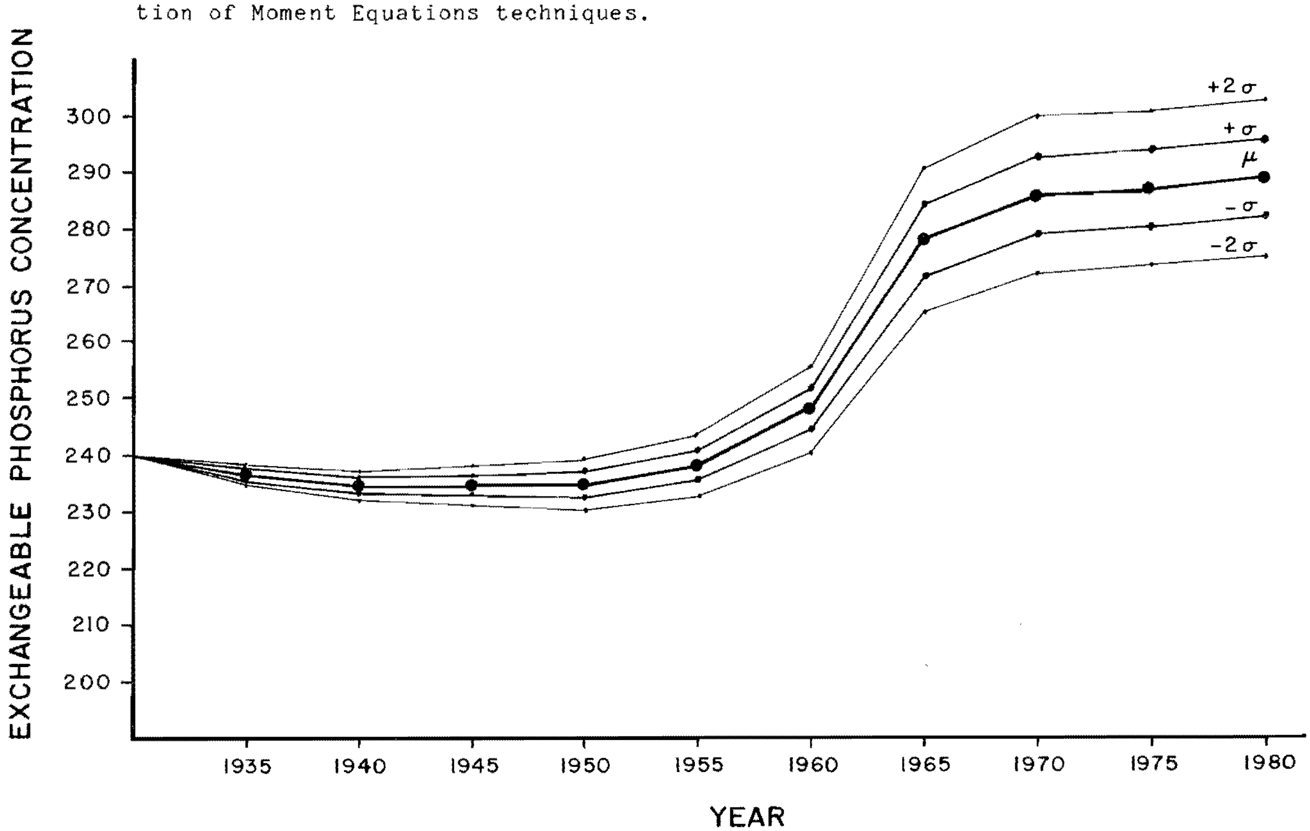

Figure 4. Simulation exchangeable sediment phosphorus concentration in Lake Washington with loading uncertainty, PF = 0.3, produced by Monte Carlo, First Order, and Generation of Moment Equations techniques.

year to year. Figure 5 is valid for the year 1980 only. These linear relationships result from the assumption of negligible uncertainty in the initial conditions (i.e. all elements of $\psi = 0$). In this example, however, it is unlikely that initial uncertainty would have any effect on the 1980 projections as the phosphorus model rapidly dampens historic uncertainty.

A comparison of the simulated curves and observed data in Figure 3 suggests that the assumed level of loading uncertainty (PF=.3) was conservative. Even though the mean values appear on the high side and rise and fall more slowly than the data, the simulated confidence bands are compatible with the observed data. All but one observation falls within two standard deviations of the mean and 60 percent within one standard deviation. Uncertainty associated with the modeling assumptions, flow variations, estimated detention times, etc., has not been considered here and would be expected to increase the projected uncertainty. It was therefore concluded that the level of uncertainty associated with the phosphorus loading rate (PF=.3) was probably too high.

Table 1 allows comparison of values predicted by each of the three stochastic methods. In all cases the First Order and Generation of Moment Equations methods produced identical (three significant figures) mean values. The Monte Carlo method differed slightly in its prediction of water column phosphorus concentrations (C_c) during the peak loading period of the 1960s. Theoretically, these discrepancies would have disappeared if a larger sample size had been used. The values of sediment phosphorus concentrations (C_s) projected by the Monte Carlo method corresponded exactly with those of the other techniques. This was expected as application of Equation 29 indicated that this variable required far fewer samples to stabilize in the mean than C_w. It is apparent that although minor variations in the standard deviations (Table 2) projected by the three methods exist, the methods are in close agreement.

The close agreement between these methods in terms of both means and variances was expected. For the linear phosphorus model both the First Order and the Generation of Moment Equation methods produce theoretically exact results. The Monte Carlo

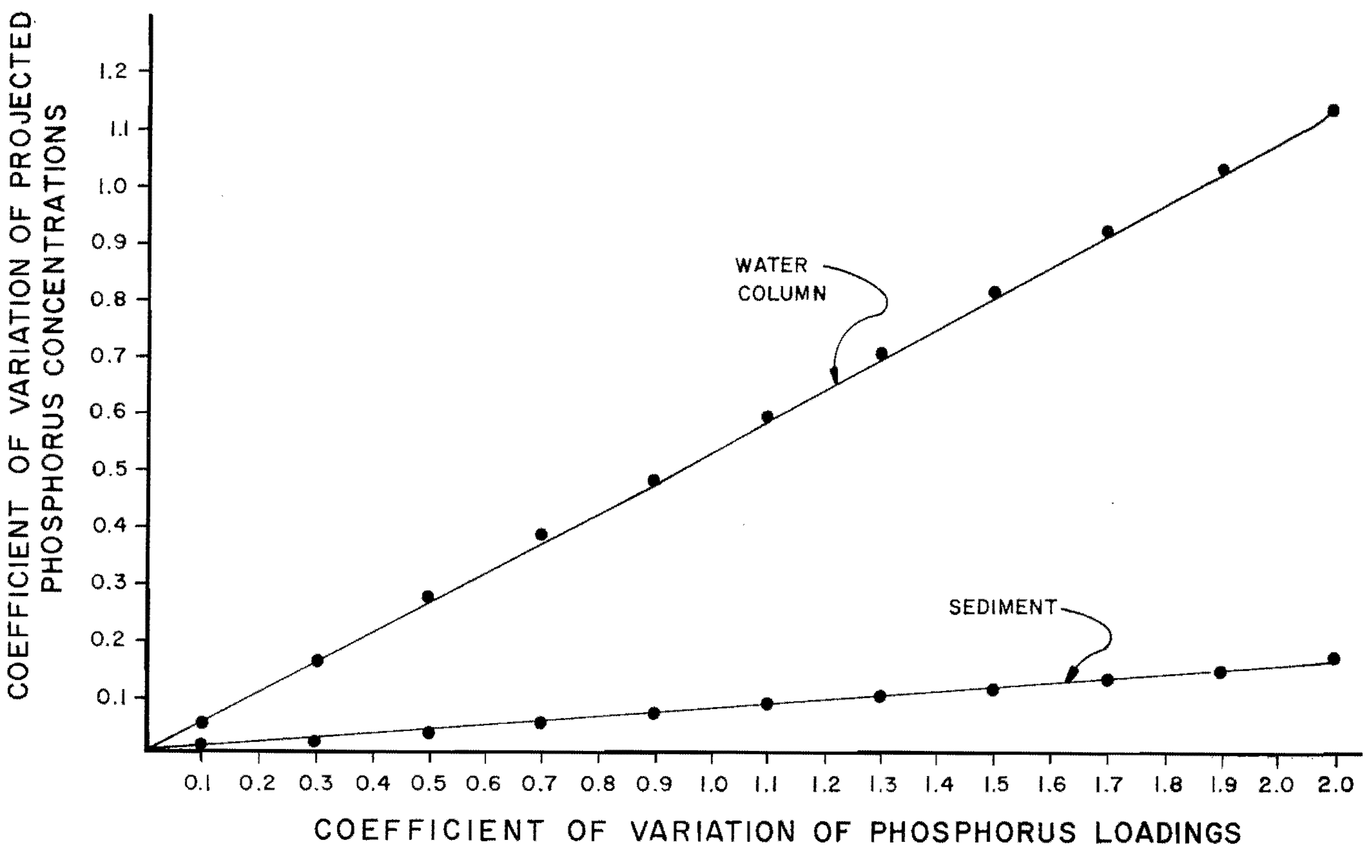

Figure 5. Uncertainty associated with the 1980 phosphorus projections with variations in loading uncertainty.

Table 1. Comparison of predicted mean values of phosphorus concentrations in Lake Washington.

Year	Phosphorus Concentration					
	Water Column (μg/l)			Sediment (mg/l)		
	First Order	Moment Equations	Monte Carlo[b]	First Order	Moment Equations	Monte Carlo[b]
1930[a]	15.0	15.0	15.0	240.	240.	240.
1935	16.3	16.3	16.4	237.	237.	237.
1940	16.3	16.3	16.3	235.	235.	235.
1945	19.8	19.8	19.8	235.	235.	235.
1950	19.8	19.8	19.8	235.	235.	235.
1955	24.3	24.3	24.3	238.	238.	238.
1960	55.1	55.1	54.8	248.	248.	248.
1965	55.1	55.1	55.4	278.	278.	278.
1970	25.6	25.6	25.6	286.	286.	286.
1975	25.4	25.4	25.4	287.	287.	288.
1980	25.5	25.5	25.5	289.	289.	289.

[a]Defined as initial condition.
[b]Based on 3840 samples.

Table 2. Comparison of simulated standard deviations associated with water column and sediment phosphorus concentrations.

Year	Standard Deviation					
	Water Column (μg/l)			Sediment (mg/l)		
	First Order	Moment Equations	Monte Carlo[b]	First Order	Moment Equations	Monte Carlo[b]
1930[a]	0.0	0.0	0.0	0.0	0.0	0.0
1935	2.31	2.32	2.33	0.817	0.818	0.819
1940	2.31	2.32	2.34	1.22	1.22	1.23
1945	3.13	3.15	3.18	1.68	1.68	1.70
1950	3.13	3.15	3.20	2.06	2.06	2.06
1955	4.16	4.18	4.20	2.54	2.54	2.53
1960	12.1	12.1	12.4	3.57	3.57	3.55
1965	10.4	10.5	10.4	6.37	6.37	6.38
1970	4.12	4.13	4.12	6.89	6.89	6.88
1975	4.11	4.13	4.15	6.83	6.83	6.84
1980	4.11	4.13	4.20	6.76	6.76	6.78

[a]Defined as initial conditions.
[b]Based on 3840 samples.

technique converges upon these same values with increasing sampling size. The difference in these methods lies in characteristics other than the results they produce.

Items which reflect the effort and expense associated with each technique (Table 3) of analysis along with more qualitative considerations (Table 4) can be used to compare or select a stochastic technique analysis for analysis. The number of lines in the programs and the compilation times indicate the relative programming effort required for application of each technique. These items show that the Monte Carlo required a more complex solution routine. The differences between the First Order and Generation of Moment Equation methods are probably more coincidental than representative. This particular comparison depends largely upon the numerical solution routine

Table 3. Indices of programming effort and cost associated with application of stochastic techniques to the phosphorus model.

Item	First Order	Moment Equation	Monte Carlo
Lines of programming	93	64	155
Core storage (words)	830	551	1830
Typical compilation time (sec)	3.00	1.80	3.09
Typical run time (sec)	0.77	1.76	8,973
Typical run cost	$0.15	$0.20	$219.92

required for solution of the discrete differential equation for the latter technique. The statement that the Monte Carlo programs tend to be more complex must be moderated somewhat by the fact that the other techniques require derivations external to the program. The exact techniques may also require more highly trained personnel since they are more theoretically complex. This must be considered when total cost is estimated for an application of these techniques.

The relative magnitude of the run time and run cost are representative of what would be expected of these methods. In this comparison a major drawback of the Monte Carlo method becomes apparent. The run time and cost of the First Order and Generation of Moment Equation techniques are negligible when compared to a run time of nearly 9,000 secs and a cost of $220 for the Monte Carlo technique. Of the former, the First Order technique is less expensive due to the efficiency of exact solutions when compared to numerical solution techniques. Although a minor cost of the study, computer costs can become a major factor when a more complex set of equations are used.

<u>Summary and Conclusions of Preliminary Comparison</u>

It is apparent that the more sophisticated methods, i.e., First Order and Generation of Moment Equations have a number of advantages. The most important of these considerations is that they produce theoretically exact solutions. The reliability of the Monte Carlo simulation is largely dependent upon the number of samples generated and, to a certain extent, chance. One must therefore always consider the possibility that a given Monte Carlo experiment is not accurate. This sampling uncertainty is not a factor with the exact methods.

The exact methods also reflect significant savings in computer costs. Programming effort and storage requirements are reduced when compared to the Monte Carlo techniques. Run costs are minimal, more comparable to deterministic analysis than the high Monte Carlo costs. In this comparison, the run costs for the exact methods were less than 0.1 percent of the run costs associated with the Monte Carlo approach.

The advantages of the Monte Carlo technique lie in its flexibility and lack of theoretical complexity. It may be applied by personnel with little training in stochastic techniques. The Monte Carlo technique can be applied to either exact or numerical solutions. First Order and Generation of Moment Equations in contrast are strictly limited to exact and numerical solutions, respectively. A Monte Carlo routine can be readily superimposed upon an existing deterministic model without complex external derivations. Although a given MonteCarlo experiment is always subjected to sampling

Table 4. Qualitative comparison of stochastic techniques.

Characteristics	First Order	Moment Equations	Monte Carlo
External Derivations	Moderate	Moderate	Minimal
Programming Effort	Minimal	Minimal	Moderate
Storage Requirement	Minimal	Minimal	Moderate
Run Cost	Minimal	Minimal	High
Special Consideration	Exact Solution Required	Differential Equation Required	Timestep, Sample Requirements, Result Uncertainty
Theoretical Complexity	Moderate	Moderate	Minimal
Solution	Exact For Linear Systems	Exact For Linear Systems	Statical Approximation

uncertainty, results for nonlinear systems are likely to be more accurate than those of the First Order or Moment Equation methods. The latter methods are only approximate for nonlinear systems.

The selection of a technique for application to a given system must be based upon the characteristics of that system. It is apparent from the preceding comparison that the exact methods offer significant advantages in linear applications. The high run cost and sampling uncertainty associated with the Monte Carlo technique make it virtually obsolete in such applications. The disadvantages may be partially or wholly offset for analysis dealing with nonlinear systems and higher-order moments.

Selection between First Order and Generation of Moment Equations methods depends primarily upon the format of the analysis. First Order is applicable to exact solutions, while Generation of Moments Equations technique is applicable to stochastic differential equations. The latter technique is generally more applicable to problems with non-constant coefficients as exact solutions for such problems are difficult to obtain. For large systems, the lower run costs of the First Order technique may become a selection criterion.

COLORADO RIVER SYSTEM

Introduction

This section presents an application of the Moment Generation Technique to analysis of uncertainties associated with salinity modeling in the Colorado River system. The objective of this study was to demonstrate the capability of this technique to estimate uncertainties actually associated with such water quality modeling efforts. This application is based upon the salinity analysis developed for the Colorado River Regional Assessment Study (or CRRAS) (UWRL, 1975) which analyzed the impact of PL 92-500 upon water quality in the Colorado River system. Results developed here do not define confidence bands for the CRRAS salinity study. Limits on the sources of uncertainty considered, procedural changes, and expansion of the data base were undertaken to achieve the objectives of this report.

System Description

CRRAS presented salinity (total dissolved solids) as the overriding water quality problem in the Colorado River. The salinity problems are due in large part to the nature of the system. The Colorado River is over 1,400 miles long. Its drainage area includes over 242,000 square miles with the lowest production of water per unit area (1.15 in/yr; Jensen, 1976) of any major river basin in the country. Annual precipitation varies from over 50 inches in the mountainous headwaters of Colorado, Wyoming, and Utah to less than 6 inches in the desert areas of Arizona, New Mexico, Nevada, and California. Inversely, river salinity varies from less than 50 mg/l in the high elevation headwaters to more than 850 mg/l at the Imperial Dam near the Mexican border (Andersen and Hanks, 1976).

The combined effects of increased consumptive water use and salt-loading along the water course have raised salinity to the point where it threatens to make the water unusable for important downstream uses. Certain dissolved solids interfere with specific uses. Magnesium and calcium, for instance, contribute to the "hardness" of the waters which has adverse effects upon municipal and industrial uses. High concentrations of sodium ions have undesirable effects on plant growth by altering the soil structure when used for irrigation (UWRL, 1975). The importance of preventing further increase in the salinity levels in the Colorado River becomes apparent when one considers 15 million people utilize its waters for domestic water supply, irrigation, industrial, and recreational purposes. The urgency is further increased by the need for the United States to keep its treaty commitments to Mexico by providing usable water from the Colorado River.

The build-up of salinity in the Colorado River is in part the result of sequential use of the waters. These uses contribute to the high salinity level by physically adding salts or by concentrating them through consumptive use of water. Evapotranspiration, for instance, reduces the volume of water carrying the residual salts thus leading to increased salinity. The low rainfall in much of the Colorado River system assures repeated reuse of waters. The result is a rapid increase in salinity along the watercourse.

O'Brien (1976) identifies the sources of salinity in decreasing contribution as: 1) natural sources, 2) irrigation sources, 3) reservoir evaporation, 4) out-of-basin export, and 5) municipal and industrial sources. One half (Andersen and Hanks, 1976) to two thirds (UWRL, 1975) of the salinity concentration is attributed to the natural sources. This is largely the result of the arid nature of the great areas of range and forest lands.

Following natural sources, agriculture is the largest contributor to salinity, although only 1.5 percent of the basin is presently irrigated, nearly 27 percent could be if the basin's irrigable land were fully developed. Refined irrigation practices have been identified as one of the major areas where better management can reduce salinity. As to the other sources, studies have been conducted to reduce reservoir evaporation and out-of-basin export losses (Jensen, 1976). Salinity reduction through regulation of present municipal and industrial uses is expected to have only a minor effect. This category may become more important as energy industries develop within the basin.

The Colorado River Regional Assessment Study

CRRAS estimated the present and projected salinity levels in the Colorado River. Management options were compared to a 1972 baseline by application of a steady state salt balance program, SALT. This section briefly describes this previous modeling effort.

As a steady state model, the program SALT was intended to represent long term salinity levels that would result from various management options. Salinity (or total dissolved solids) was considered a conservative constituent. The model was, therefore, basically an accounting routine based on a mass balance for flow and total dissolved solids of designated reaches of the Colorado River system.

A reach refers to a segment of river. Flow and salts are contributed to a reach by sources along the reach and by other tributary segments (upstream reaches). Nodes represent points on the river where contributions to the associated reach are summed and tabulated. Agricultural loadings were calculated by subdivisions of the area along a reach which are referred to as hydrologic subbasins.

Figure 6, illustrates the schematic of the Colorado River system used with the program SALT. Eighteen nodes were selected for accumulation of salt loadings and flows from upstream reaches. Loads or withdrawals from reaches were permitted by any of two flow options and five salinity options. Loads were essentially identified as 1) point sources or diversions, 2) agricultural loadings calculated for hydrologic subbasins, 3) natural or unknown diffuse loadings, or 4) input from upstream reaches. The total salt loading, S_n, or flow, Q_n, at any node, n, within the system was calculated as the sum of loadings from upstream nodes, ΣS_{n-1}, and inputs from point or diffuse sources along the reach. The node summation equation used for salinity calculations was

$$S_n = \Sigma S_{n-1} + S_{nat} + \Sigma S_{agri} + \Sigma S_L \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

in which

ΣS_{n-1} = salt loading from upstream reaches contributing directly to reach n (thousands tons/yr)

S_{nat} = salt loading resulting from natural and unknown diffuse sources in reach n (thousand tons/yr)

ΣS_{agri} = salt loading resulting from agricultural practices in reach n (thousand tons/yr)

ΣS_L = salt loading resulting from identified diversions, return flows, municipal, and industrial uses in reach n (thousand tons/yr)

Flows were similarly summed as:

$$Q_n = \Sigma Q_{n-1} + Q_{nat} + \Sigma Q_{agri} + \Sigma Q_L \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

in which

ΣQ_{n-1} = flow contributed by upstream reaches contributing directly to reach n (thousands of acre feet/yr)

Q_{nat} = flows resulting from natural and unknown diffuse sources in reach n (thousands of acre feet/yr)

ΣQ_{agri} = flows resulting from agricultural practices in reach n (thousands of acre feet/yr)

ΣQ_L = flows resulting from identified diversions return flows, reservoir evaporation losses, municipal and industrial uses in reach n (thousands of acre feet/yr)

The summation signs used in Equations 35 and 36 require summation of all sources (flow or salts) contributing to or withdrawing from the reach associated with node n. Several upstream nodes (S_{n-1} or Q_{n-1}) may contribute to reach n. Agricultural loads (S_{agri} and Q_{agri}) were computed by CRRAS for hydrologic subbasins contributing to each river segment. Individual point loadings were estimated (S_L and Q_L). These inputs were summed to achieve total loadings from each source classification. The natural loading terms (S_{nat} and Q_{nat}) were estimated for each reach in the calibration process. These natural loading terms are not associated with a reach summation sign since they were treated as reach constants.

Application of the salt loading summation equation was hindered by the diffuse character of the loadings. It was estimated by CRRAS that 84 percent of the total salt loading is derived from natural or man-manipulated diffuse sources. For instance, better water management of agricultural sources was considered a likely future option. Only 8 percent of the irrigated area was associated with identifiable artificial drains. The remaining 92 percent of the flows and salts are contributed from agriculture by diffuse loadings. This made separation of the natural and agricultural contributions in Equations 35 and 36 difficult, and yet such separation is essential to permit assessment of the impact of changes of irrigation efficiencies upon salinity in the Colorado River.

Calibration of the program SALT upon 1972 conditions was undertaken by CRRAS to provide estimates of diffuse loads. The total steady state salt load at a given node, S_n, was estimated from historic data. Point sources, S_L, were identified. Separation of diffuse natural and agricultural sources was accomplished by estimation of agricultural loadings. The natural and unknown diffuse source loadings, S_{nat}, of reach n were therefore defined by rearrangement of Equation 35 to

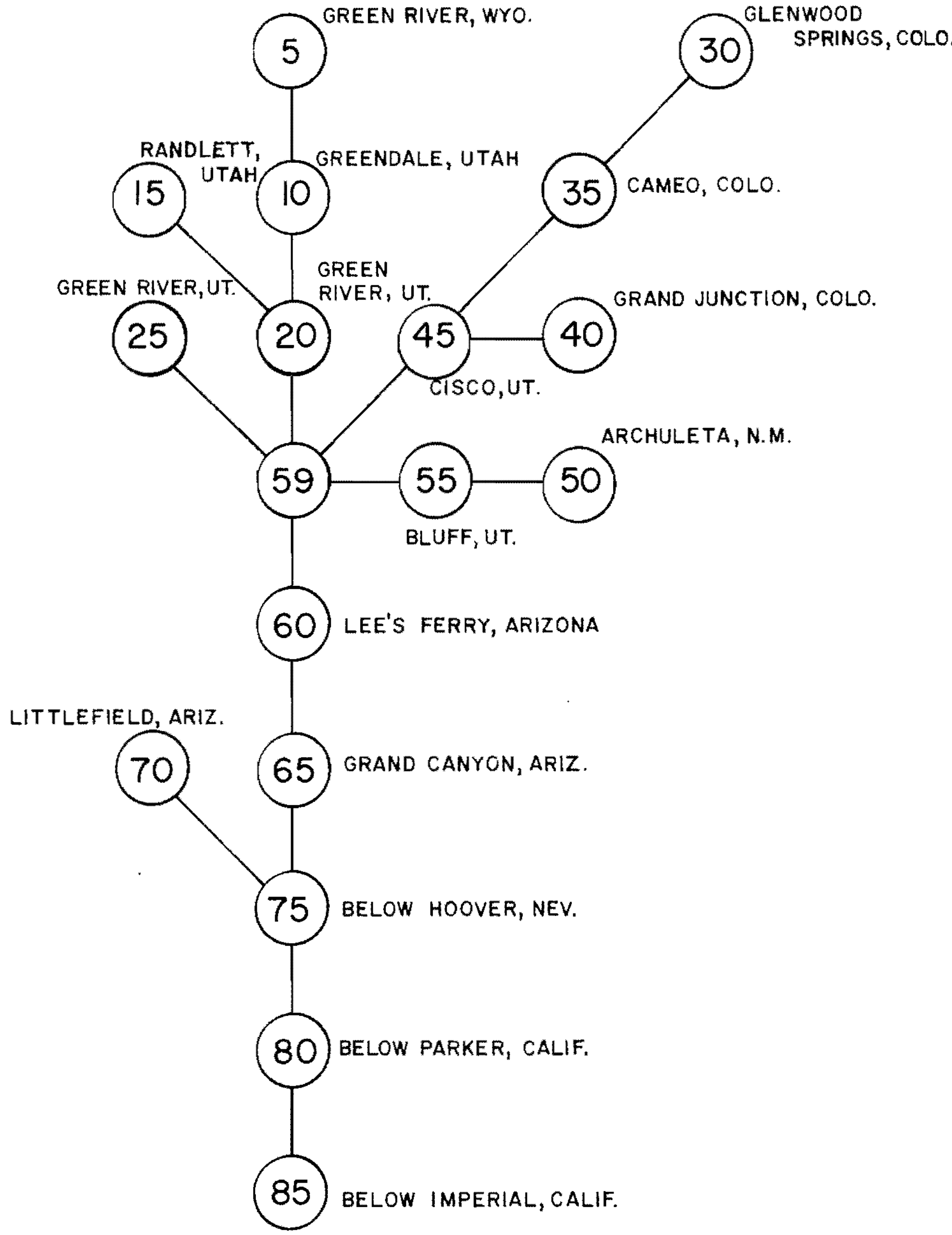

Figure 6. Schematic of Colorado River system for programs SALT and SALTEZ.

$$S_{nat} = S_n - \Sigma S_{n-1} - \Sigma S_{agri} - \Sigma S_L \quad . \ . \ . \ . \ . \ . \quad (37)$$

Natural loading terms calculated by Equation 37 include natural loadings and loadings from unidentified or unclassified sources. Any salt or flow input not falling into the category of agricultural or identified point source in the 1972 calibration run was assigned to the natural loading terms, S_{nat} or Q_{nat}. Further, any error made in the estimation of agricultural or point source loadings would have been compensated for by this procedure. This computational procedure defines the term S_{nat} (i.e. natural and unknown diffuse source loadings) and has significant impact upon the uncertainty analysis.

Separation of natural and agricultural salinity loads faced one additional difficulty. A relationship between agricultural salt loading and irrigation efficiencies and flows was required for analysis of farm water management options. The one used by CRRAS was

$$S_{agri} = \Phi_B \left[\frac{1-E}{E}\right] Q \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (38)$$

in which

Φ_B = the base leaching factor (tons/ac/ft)

E = irrigation efficiency as a decimal fraction (dimensionless)

Q = irrigation diversion flow (thousands of acre feet/yr)

This equation represents a compromise between two theoretical approaches to salt release by irrigated lands. The first assumes the contribution results solely from the concentrating effects of evapotranspiration. Under this approach, salt loading varies only as a function of irrigation efficiency. Salt released by basic weathering processes or leached residual salts are not considered. The second approach assumes that salt pickup is related to the volume of percolating water. Equation 38 represents aspects of both these theoretical approaches.

Sources of Uncertainty in Program SALT Applications

It was the purpose of this study to estimate the uncertainties associated with simulated management options like those developed in the Colorado Regional Assessment Study (UWRL, 1975). The first source of uncertainty is our ability to estimate steady state conditions from historic data. The term S_n (Equation 35) in the 1972 calibration runs theoretically represents the total salt loading at a node, n, that would result from the steady state existence of 1972 conditions. Actually, the value for S_n must be estimated from limited historic data subject to numerous uncertainties in the forms of measurement errors and natural variability. In addition, the data base is often in a dynamic state of change rather than constant as the steady state assumption implied. As a result, any estimation of a steady state value of S_n is uncertain. Although this uncertainty was considered an uncertainty of estimation, it is the direct result of measurement errors, natural variability, and changes in the data base.

The second source of uncertainty to be considered is the use of Equation 38 to represent agricultural salinity loadings. The selection of this relationship as a source of uncertainty is best explained by a quote from the Colorado River Regional Assessment Study (UWRL, 1975, p. 136):

> Unfortunately, the processes involved in salt loading in both the agricultural and natural system are not well understood, and in the absence of this understanding and adequate data, the linear derived leaching factor was employed.

The apparent consternation of the previous authors and the magnitude of diffuse sources (84 percent of total salinity loading) identified the linear base leaching factor, Φ_B, as a likely source of significant uncertainty.

The uncertainty associated with Φ_B was considered a modeling uncertainty because it would result from incorrectness of the theoretical model (Equation 38). Estimation error resulting from inadequate data base, measurement uncertainty, and natural variability, whenever these factors contributed to an incorrect assignment of a Φ_B value, was also included. In practice the modeling and estimation errors were impossible to separate.

Stochastic Model Development

The definition of the specific uncertainties to be analyzed permitted the development of equations to represent the stochastic model of the SALT runs. These equations define the relationship between of the constants and random variables in the program SALT salinity summation. The general form of the stochastic model is:

$$\Delta\tilde{S}_n = \tilde{S}_{nat} + \Sigma\tilde{S}_{agri} + \Sigma S_L \quad \text{. . . . . (39)}$$

in which

$\Delta\tilde{S}_n$ = the incremental salt load at node n, i.e., $\Delta\tilde{S}_n = \tilde{S}_n - \Sigma\tilde{S}_{n-1}$

The terms $\Delta\tilde{S}_n$, $\tilde{S}_{nat}$, and $\tilde{S}_{agri}$ are random variables reflecting uncertainties in the estimation of total, natural, and agricultural loadings. The point loading terms, S_L, are assumed to be known constants. As would be expected from Equation 35, the random variables, $\tilde{S}_{nat}$, is computed directly from the random variables $\Delta\tilde{S}_n$ and $\tilde{S}_{agri}$ used in the SALT model calibration. Utilizing a superscript to represent variables of the 1972 calibration run, the natural loading term, $\tilde{S}_{nat}$, was defined by:

$$\tilde{S}_{nat} = \Delta\tilde{S}_n^{72} - \Sigma\tilde{S}_{agri}^{72} - \Sigma S_L^{72} \quad \text{. . . . . . . (40)}$$

No superscript is associated with the $\tilde{S}_{nat}$ term as it was assumed to remain unchanged through time in the SALT runs. Equation 39 may be expressed for a simulation of year, Y, as:

$$\Delta\tilde{S}_n^Y = \Delta\tilde{S}_n^{72} + \Sigma(\tilde{S}_{agri}^Y - \tilde{S}_{agri}^{72}) + \Sigma(S_L^Y - S_L^{72}) \quad \text{. . (41)}$$

This equation may expanded by assigning the uncertainties of the term, $\tilde{S}_{agri}$, to the linear base leaching factor, Φ_B. The stochastic model of this analysis which resulted from the incorporation of Equation 38 into Equation 41 is

$$\tilde{S}_n^Y = \Sigma\tilde{S}_{n-1}^Y + \Delta\tilde{S}_n^{72} + \Sigma\tilde{\Phi}_B\left\{\left[\frac{1-E^Y}{E^Y}\right]Q^Y - \left[\frac{1-E^{72}}{E^{72}}\right]Q^{72}\right\}$$

$$+ \Sigma(S_L^Y - S_L^{72}) \quad \text{. . . . . (42)}$$

The expected value for the total node salt loading for a year, Y, is:

$$E(\tilde{S}_n^Y) = S_n^Y = \Sigma S_{n-1}^Y + \Delta S_n^{72} + \Sigma \Phi_B \left\{ \left[\frac{1-E^Y}{E^Y} \right] Q^Y - \left[\frac{1-E^{72}}{E^{72}} \right] Q^{72} \right\} + \Sigma (S_L^Y - S_L^{72}) \quad . . . . . (43)$$

Assuming independence of $\tilde{S}_n^{72}$ and $\tilde{\Phi}_B$ the equation for the variance, $\sigma_{S_n} Y^2$ is:

$$\sigma^2_{S_n^Y} = \Sigma \sigma^2_{S_{n-1}^Y} + \sigma^2_{\Delta S_n^{72}} + \Sigma \sigma^2_{\Phi_B} \left\{ \left[\frac{1-E^Y}{E^Y} \right] Q^Y - \left[\frac{1-E^{72}}{E^{72}} \right] Q^{72} \right\}^2 \quad . . . . (44)$$

The uncertainties associated with SALT projections were assumed to depend only upon uncertainties in estimation of the total salt loadings to a node and in the modeling parameter, Φ_B. As Equation 44 illustrates, modeling uncertainty in Φ_B contributes to uncertainty in S_n only when agricultural loading is assumed to vary as a result of changes in efficiencies, E, or return flows, Q. If the agricultural loading remains unchanged Equation 44 reduces to:

$$\sigma^2_{S_n^Y} = \Sigma \sigma^2_{S_{n-1}^Y} + \sigma^2_{\Delta S_n^{72}} \quad . . . . (45)$$

The separation of the agricultural loading term, S_{agri}, from the natural and unidentified diffuse salt loading term, S_{nat}, was important only when modification of agricultural practice was part of the management option. Otherwise, the reliability of the simulation was dependent upon the reliability of the original 1972 steady state estimate of total salinity loadings.

General Description of Steady State Stochastic Salinity Model

The program SALTEZ is a steady state stochastic model designed for modeling levels of salinity in river systems. The program SALTEZ was a modification of the program SALT which was previously applied (UWRL, 1975) to the Colorado River system. The program SALTEZ has the capability to model means, variances, and skewness resulting from independent stochastic inputs of salinity. Skewness calculations were not performed for the Colorado River Analysis.

The program conducts a mass balance on conservative salts and flow. Beginning at the headwaters of the system, loads and depletion are accumulated to define the state of the system at given points (nodes) along the river. Figure 7 illustrates the schematic for the SALTEZ test run. The test run schematic defines five reaches. Each reach contributes flow and salts to an associated node through loading options. There are two flow and five salinity loading options. Flow loadings may be defined as: 1) An input or withdrawal (thousands of acre feet per year) or 2) the product of an area (thousands of acres) and a consumptive use factor (feet/year).

The salinity inputs may be defined (see Figure 8) as: 1) A load or depletion, S_L (thousands of tons per year); 2) the product of flow, W_L (thousands of acre feet per year) and a specified concentration, C_L, (tons per acre feet); 3) the product of area, A (thousands of acres) and a salt load factor, S_A, (tons per acre per year); 4) a diversion taken as the product of a flow withdrawal (thousands of acre feet per year) and the salinity concentration at the node calculated by the model, S_n'; 5) an agricultural loading taken as a function of flow, Q_d (thousands of acre feet per year), efficiency, E, and a leaching factor, Φ_B (tons per acre per year).

Up to 500 loads may be defined by various combinations of the flow and salinity input options. A sign convention of positive (+) for salts or flows into the river, and negative (-) for withdrawals from the river was designated.

The program SALTEZ permits inputs and in-stream salinity to be represented as

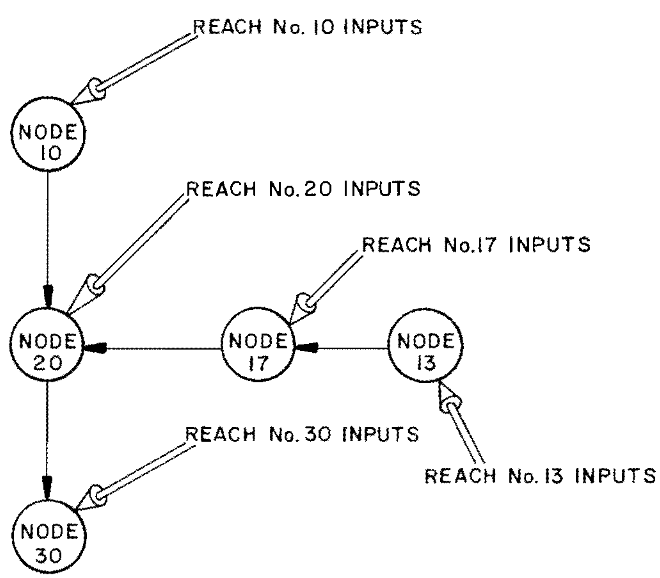

Figure 7. Schematic of SALTEZ test run.

random variables rather than constants. Uncertainties or variations in selected input terms may be defined by their second and third central moments. Figure 8 illustrates the five salinity loading options and their optional stochastic terms. The terms that may be defined as random variables are (by option): 1) The salt loading term, $\tilde{S}_L$; 2) the specified concentration, $\tilde{C}_L$; 3) the irrigated area, $\tilde{A}$, and/or the salt load factor, $\tilde{S}_A$; 4) none, model computes intermediate salinity, $\tilde{S}_n$; and 5) the factor [(1-E)/E] and/or the base leaching factor $\tilde{\Phi}_B$.

The stochastic model of the salinity level at node n is expressed by

$$\tilde{S}_n = \Sigma\tilde{S}_{n-1} + \Sigma\tilde{S}_L + \Sigma\{W_L * \tilde{C}_L\} + \Sigma\{\tilde{A} * \tilde{S}_A\} - \Sigma\{(Q_d/Q_{n'})\tilde{S}_{n'}\} + \Sigma\left\{Q_d\left[\frac{1-E}{E}\right]\tilde{\Phi}_B\right\} \quad \ldots\ldots\ldots \quad (46)$$

Equation 46 is approximate as diversions from the mainstream were assumed to occur after accounting for all other salinity loads contributing to a node. The intermediate mainstream flow, $\tilde{Q}_{n'}$, and the intermediate mainstream salinity, $\tilde{S}_{n'}$, are computed

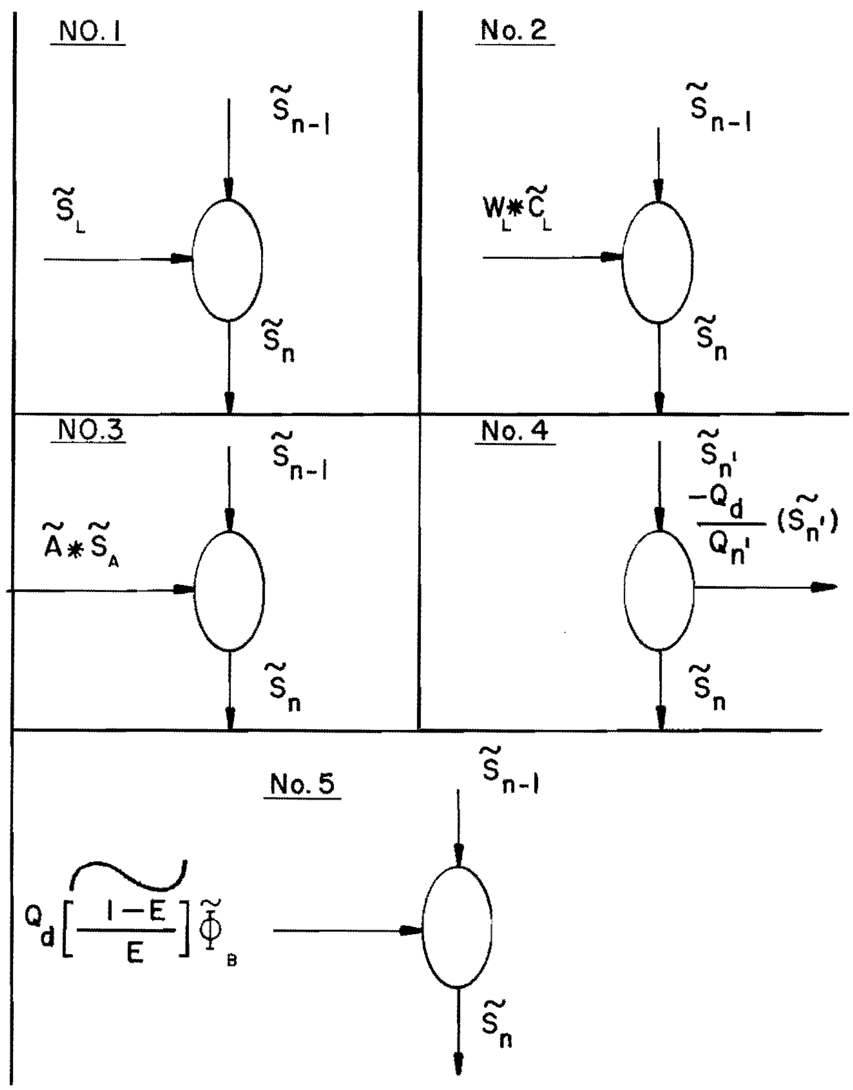

Figure 8. Stochastic salinity options for the program SALTEZ.

prior to computation of the node salinity, $\tilde{S}_n$. These intermediate summations reflect upstream contributions and all salinity and flow inputs for the reach except those defined by the diversion option (#4). Intermediate values are used to compute the salt removal resulting from the diversion option, thus permitting solution of Equation 46. This procedure is undertaken for computation of the mean, variance, and skewness equations.

If the stochastic terms within Equation 46 are assumed to be independent, the mean salinity, S_n, at node n is defined by

$$S_{n'} = \Sigma S_{n-1} + \Sigma S_L + \Sigma\{W_L * C_L\} + \Sigma\{A * S_A\}$$

$$+ \Sigma\left\{Q_d\left[\frac{1-E}{E}\right]\Phi_B\right\} \quad . . . . (47)$$

$$S_n = [1 - \Sigma(Q_d/Q_n)]S_{n'} \quad . . . . (48)$$

The variance of salinity at node n, $\sigma_{S_n}^2$ may be derived by the Generation of Moment Equations techniques as follows:

$$\sigma^2_{S_{n'}} = \Sigma\sigma^2_{S_{n-1}} + \Sigma\sigma^2_{S_L} + \Sigma\{W_L^2\ \sigma^2_{C_L}\}$$

$$+ \Sigma\{\sigma_A^2\ \sigma^2_{S_A} + A^2\ \sigma^2_{S_A} + S_A^2 + \sigma_A^2\}$$

$$+\Sigma\left\{Q_d^2\left[\sigma^2_{\left[\frac{1-E}{E}\right]}\ \sigma_{\Phi_B}^2 + \left[\frac{1-E}{E}\right]^2\ \sigma^2_{\Phi_B} + \Phi_B^2\ \sigma^2_{\left[\frac{1-E}{E}\right]}\right]\right\}$$

$$. . (49)$$

$$\sigma^2_{S_n} = \left[1 - \Sigma\left(\frac{Q_d}{Q_{n'}}\right)\right]^2 \sigma^2_{S_{n'}} \quad . . . . (50)$$

APPLICATION OF THE PROGRAM SALTEZ TO THE COLORADO RIVER

Calibration Requirements

Application of the program SALTEZ required three distinct tasks. The first was estimation of the uncertainties associated with the assumed 1972 steady state values of incremental salt loadings, $\Delta \tilde{S}_n$. Uncertainties associated with $\Delta \tilde{S}_n$ were defined for 17 of the nodes within the Colorado River system. The second task involved estimation of the uncertainties associated with the agricultural base leaching factor, Φ_B. Data from the 49 subbasins within the systems were utilized to estimate the variance associated with the use of this term. The third task was modification of the program SALTEZ to resolve differences in format between the stochastic model of the CRRAS SALT application (Equation 44) and the generalized format of the program SALTEZ (Equation 49).

Estimation of Uncertainties Associated with $\Delta \tilde{S}_n$

The programs SALT and SALTEZ are steady state models. As such they require steady state values for inputs. When the 1972 calibration of the program SALT was undertaken by CRRAS the steady state value of the term $\Delta \tilde{S}_n$ was estimated from a limited amount of historic salinity data. This estimation of the steady value was uncertain because of the dynamic nature of the historic data. The uncertainties in the salt loadings are the combined result of natural variability in climate, flow and salinity phenomena, measurement and calculation errors, and man related perturbations of the Colorado River system. The uncertainties defined in this section specifically relate to the uncertainties associated with estimating a steady state value of the total salt load at a node from this nonsteady state data base.

The parameter for defining confidence bands around predicted means is the standard error. This term essentially defines the standard deviation of sample means. In the 1972 program SALT calibration, the steady state values for incremental salt loadings were defined from the mean of the historic data collected for the 34 year period 1940-1974. The uncertainty associated with the steady state S_n was equivalent to the standard error associated with the mean of the historic incremental salt loadings.

Table 5 lists the statistical characteristics for the 17 nodes of the Colorado River system as previously defined (Figure 6). This data summarizes incremental salt loadings for the 34 year period, 1940-1974. The mean incremental salt load, standard deviation, and standard error of each unmodified data set are presented. Salinity data used in the development of Table 5 were taken from a 34 year (1940-1974) summary of salinity data (USBR, 1977). CRRAS derived its estimates of salt loadings from Hyatt et al. (1970) and USBR (1975). Some minor differences exist between these data bases.

If the historic data were utilized in an unmodified form, the square of the standard error would define the variance associated with ΔS_n. However, since it was recognized that segments of the Colorado River have displayed a trend of decreasing salt loadings with time due to increased consumptive uses, use of such long term salinity loadings to estimate the 1972 steady state levels would be improper. An attempt was made to extract temporal trends from the historic data (Table 6). This analysis showed that only the mainstream Colorado River nodes (30, 45, 60, 65, 80, 85) displayed trends in their incremental salt loadings.

The extraction of these trends did not in any case decrease the standard error associated with the 1972 steady state estimations of ΔS_n. This can be seen by comparison of the 1972 standard error of the trend analysis (Table 6) with the standard errors of the unmodified data (Table 5). This occurs because the uncertainties of estimating a mean value at a point along a regression line differ from those of estimating the mean of a population sample. The 1972 standard error, for instance, was one of the largest annual standard errors because it lies along the fringe of the temporal data. This made estimates of the 1972 mean value highly sensitive to small errors in the regression slope coefficient, B. No such uncertainty is associated with estimates of means from populations displaying no trends since all points can be assumed to represent a single point in time for steady state estimates.

The value of the trend analysis was that it identified those historic data sets (nodes 30, 45, 60, 65, 80, 85) which could not be

Table 5. Summary of statistical characteritics of historic incremental salinity loads prior to trend analysis.

Node	Node Location	Mean Salt Load[a] (TT/Y)	Standard Deviation (TT/Y)	Standard Error (TT/Y)
5	Green River above Green River, WY	552.	132.	22.6
10	Green River near Greendale, UT	361.	222.	38.1
15	Duchesne River above Randlett, UT	404.	123.	21.1
20	Green River above Green River, UT	1284.	348.	59.7
25	San Rafael River Basin, Green River, UT	210.	80.9	13.9
30	Colorado River above Glenwood Springs	595.	79.5	13.6
35	Colorado River near Cameo, Colo.	936.	127.	21.7
40	Gunnison River above Grand Junction	1454.	282.	48.4
45	Colorado River above Cisco, UT	1119.	318.	54.5
50	San Juan River above Archuleta, NM	198.	88.0	15.1
55	San Juan River above Bluff, UT	785.	323.	55.4
60	Colorado River above Lee's Ferry, AZ	2558.	1451.	249.
65	Colorado River above Grand Canyon, AZ	1088.	420.	72.
70	Virgin River above Littlefield, AZ	349.	76.8	13.2
75	Colorado River below Hoover Dam, Ariz.-Nev.	362.	1941.	333.
80	Colorado River below Parker Dam, Ariz.-Calif.	910.	523.	89.6
85	Colorado River at Imperial Dam, Ariz.-Calif.	146.	367.	62.9

[a]1940-1974 (U.S. Bureau of Reclamation, 1977).

Table 6. Results of trend analysis of historic incremental salinity loads.

Node	Node Location	Linear Regression[a] Coefficients: Intercept A (TT/Y)	Slope B (TT/Y^2)	r^2	1972 Standard Deviation (TT/Y)	1972 Standard Error (TT/Y)
5	Green River above Green River, WY	542.	0.60	0.002	141.	41.1
10	Green River near Greendale, UT	290.	4.3	0.038	231.	67.7
15	Duchesne River above Randlett, UT	448.	-2.7	0.046	127.	37.2
20	Green River above Green River, UT	1339.	-3.3	0.009	368.	108.
25	San Rafael River Basin, Green River, UT	241.	-1.8	0.051	83.6	24.5
30	Colorado River above Glenwood Springs	602.	-0.40	0.25	84.3	24.6
35	Colorado River near Cameo, Colo.	991.	-3.3	0.068	130.	38.1
40	Gunnison River above Grand Junction	1597.	-8.6	0.092	285.	83.5
45	Colorado River above Cisco, UT	1422.	-18.4	0.33	276.	80.7
50	San Juan River above Archuleta, NM	222.	-1.4	0.026	92.2	27.0
55	San Juan River above Bluff, UT	851.	-4.0	0.015	341.	99.7
60	Colorado River above Lee's Ferry, AZ	3510.	-57.6	0.16	1415.	414.
65	Colorado River above Grand Canyon, AZ	1499.	-24.9	0.35	359.	105.
70	Virgin River above Littlefield, AZ	384.	-2.1	0.07	78.5	23.0
75	Colorado River below Hoover Dam, Ariz.-Nev.	1253.	54.0	0.077	1981.	580.
80	Colorado River below Parker Dam, Ariz.-Calif.	-277.	-38.3	0.53	380.	111.
85	Colorado River at Imperial Dam, Ariz.-Calif.	-46.2	-11.6	0.10	369.	108.

[a]1940-1974 (U.S. Bureau of Reclamation, 1977).

used directly (Table 5) to estimate the steady state mean. For those sets which display trends, the appropriate standard error for the 1972 steady state estimation of ΔS_n was defined from Table 6. The standard error from the unmodified data (Table 5) was used to define the variance for those data sets not displaying significant salinity trends ($r^2 < 0.1$). These cutoff of $r^2 < 0.1$ gave a 95 percent assurance that existing trends were not neglected (Steel and Torrie, 1960). Table 7, presents the values of variance selected for $\Delta \tilde{S}_n$ of each node. It also identifies the source of the standard error used in calculating the variance.

Estimation of Uncertainties Associated with Φ_B

The estimation of the uncertainty associated with Φ_B included two types of error. The first, modeling uncertainty ,resulted from the coarseness of the functional relationships between Φ_B, E, and Q in representation of actual phenomenon. The second reflects error resulting from the estimation of the values of Φ_B used by CRRAS in the program SALT modeling effort. The methodology used to define the uncertainties associated with Φ_B included both types of error and is described below.

In the Colorado River Regional Assessment Study, Equation 38 was used to define the agricultural salinity loading for management runs through calculation of Φ_B for each subbasin. Values for E and Q were derived from historic data. The agricultural loading S_{agri} was calculated by

$$S_{agri} = \frac{\Delta S}{\left(\frac{\Delta Q + ET_{agri}}{W_d - ET_{agri}}\right)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (51)$$

Table 7. Variance of ΔS_n used in program SALTEZ simulation.

Node	Variance of $\Delta \tilde{S}_n$ $(TT/Y)^2$	Source of Standard Error
5	511	Table 5
10	1,452	Table 5
15	445	Table 5
20	3,564	Table 5
25	193	Table 5
30	605	Table 6, Trend analysis
35	471	Table 5
40	2,343	Table 5
45	6,512	Table 6, Trend analysis
50	228	Table 5
55	3,069	Table 5
60	171,369	Table 6, Trend analysis
65	11,025	Table 6, Trend analysis
70	174	Table 5
75	110,889	Table 5
80	12,321	Table 6, Trend analysis
85	11,664	Table 6, Trend analysis

in which

ΔS = change in salt load resulting from natural and agricultural flows within a subbasin (thousands of tons/yr)

ΔQ = change in flow resulting from natural and agricultural flows within a subbasin (thousands of acre feet/yr)

W_d = flow diverted for agricultural purposes (thousands of acre feet/yr)

ET_{agri} = evapotranspiration losses of water diverted for agri culture (thousands of acre feet/yr)

The values of Φ_B calculated by this procedure were then used by CRRAS in Equation 38 to calculate salinity loadings from agriculture in management runs. The procedure that was developed to define uncertainties associated with Φ_B accounts for deviation from reality of the functional relationship (Equation 38) and estimation errors accumulated from approximation of E, Q, and S_{agri}.

The underlying assumption of the term Φ_B is that salinity pickup is proportional to the flow through the soil column. This assumption presumes a chemical equilibrium between the soil and percolating waters. If this theoretical relationship were entirely correct each soil type would display a characteristic base leaching factor. One could theoretically define the base leaching factor for a subbasin by calculation of the weighted mean of soil type contributions:

$$\Phi_B = \Phi_1 F_1 + \Phi_2 F_2 + \Phi_3 F_3 + \ldots \Phi_n F_n \quad . \; . \; . \quad (52)$$

in which

Φ_n = characteristic base leaching factor of soil type n (tons per acre foot)

F_n = fraction of subbasin of soil type n

Table 8 lists by subbasin the distribution of geologic types and base leaching factors developed in the Colorado River Regional Assessment Study (UWRL, 1975). Each subbasin results in an agricultural input of salinity to the system. Equation 52 was applied to these data. Deviation from a best fit regression analysis of the soil types and subbasin base leaching factors represented accumulated uncertainty from modeling and estimation errors. Underlying this conclusion are two basic assumptions: 1) all soil types are equally likely to be used for agriculture; 2) the geologic types accurately represent the soil type distributions in the subbasin. It is very unlikely that either of these assumptions was entirely correct. This estimation of uncertainty, therefore, must include some error from these assumptions.

The regression analysis revealed a weak correlation ($r^2 = 0.33$) between the eight

Table 8. Geologic characteristics and base leaching factors for Colorado River system subbasins (after UWRL, 1975).

Subbasin No.	Subbasin Name	Geologic Type[a] (% of Total Basin)								ϕ_B (Tons/ac./ft.)
		1	2	3	4	5	6	7	8	
UG1	New York River Basin	40	20						40	0.10
UG2	Green River above LaBarge, WY	10	70	5	5	5	5			0.23
UG3	Green River above Fontenelle Reservoir	15	50	10	10	5	10			0.62
UG4	Big Sandy Creek Basin	10	85						5	0.76
UG6	Green River above Green River, WY	15	60	25						8.15
UG7	Black Fork River Basin	15	50	15			5		15	0.49
UG8	Green River above Flamming Gorge Dam	5	55	2	3		5		30	0.175
UG9	Little Snake River Basin	10	60	20				2	8	0.22
UG10	Yampa River Basin	5	15	53			2	5	8	0.21
UG11	Green River above Jensen, UT	5	40	5	5		20		25	1.07
UG12	Ashley Creek Basin	5	10	40	10	5	10		20	0.58
UG13	Duchesne River above Duchesne, UT	5	60	5	10	5	5		10	0.47
UG14	Duchesne River above Randlett, UT	5	55			5	10		25	0.34
UG15	White River Basin	5	50	20	3	2	10	10		0.49
UG16	Price River Basin	10	5	60	25					1.76
UG17	Green River above Green River, UT	5	80	10	3			2		1.20
UG18	San Rafael River Basin, Green River, UT	5	15	25	25	20	10			0.91
UM1	Colorado River above Hot Sulfur Springs	1	30					15	60	0.074
UM2	Eagle River Basin			20			45	10	25	0.335
UM3	Colorado River above Glenwood Springs		15	10	10	5	25	15	10	0.26
UM4	Roaring Fork River Basin	5		15			30	25	25	0.30
UM5	Colorado River above Plateau Creek	5	45	20	5	3	15	5	2	1.42
UM6	Plateau Creek Basin		80	10				10		0.23
UM7	Tomichi Creek Basin	5		10			15	30	40	0.129
UM8	Gunnison River above North Fork Gunnison		10	20			5	50	15	0.077
UM9	Uncomphagre River Basin	10		30	30			25	5	0.456
UM10	Gunnison River above Grand Junction	2	2	40	40			8		1.10
UM11	Colorado River above Colorado-Utah Line	2	15	55	28					1.19
UM12	Colores River Basin	5		35	40		5	10	5	0.29
UM13	Colorado River above Cisco, UT		5	30	45		10		10	1.28
UM14	Colorado River above Lee's Ferry, AZ			25	45	5	15	10		0.34
US1	San Juan River above Arbola	5	2	60	3			30		0.141
US2	San Juan River above Archulets, NM	5	40	30	5			8	12	0.158
US3	Animas River Basin	5	25	15	3	2	20	10	20	0.30
US4	San Juan River above Farmington	5	90	5						1.30
US5	La Platta River Basin		5	90	5					0.18
US6	San Juan River above Shiprock	2	10	85	3					1.17
US7	San Juan River above Bluff, UT	3	6	10	40	25	15	1		0.59
LM1	Colorado River above Grand Canyon, AZ	5		20	30	30		10	5	0.20
LM2	Virgin River above Littlefield, AZ	5			15	60			30	1.13
LM3	Muddy River Basin below Hoover Dam	45	10					45		0.16
LM4	Colorado River above Hoover Dam	10	15			45	15		15	0
LM5	Bill Williams River above Alamo	15	5				10	30	40	0.40
LM6	Colorado River Hoover to Parker below Parker Dam		50					25	25	0.24
LM7	Colorado River Parker to Imperial below Imperial Dam	70	10					20		0.24
LM8	United States - Mexico Border	80	5					15		ϕ
LL1	Little Colorado River above Hunt	20	30			20			30	0.056
LL2	Little Colorado River above Holbrook	15	15			40	5	15	10	0.34
LL3	Little Colorado River above Cameron	10	10	15	10	30	10	5	10	0.50

[a]From Utah Water Research Laboratory (1975).

Description of geological classifications used:

1) Unconsolidated continental deposits: Fluvial and glacial fluvial deposits beneath and bordering streams terraces. Include pediment gravels and sand dunes.
2) Continental rocks: Lacustrine deposits of shale, siltstone, fire-grained sandstone. Includes the Wasatch, Green River, Uintah and Bridges formations.
3) Continental and marine rocks: Shale and sandstone. Includes the Mancos, Mesa Verde, and related formations.
4) Predominantly continental rock: Massive quartzose sandstone, interbedded sandstone and mudstone, and conglomerate. Includes Glen Canyon, San Rafael groups, Morrison and Dakota Formations.
5) Continental and marine rocks: Mudstone, siltstone and shale, conglomerate. Includes Moenkopi and chinle formations.
6) Marine rocks: Limestone, quartzite, shale, and evaporites with quartzose sandstone. Includes the Leadville, Hermosa, Cutler, Weber and related formations.
7) Igneous rocks: Volcanic and intrusive basalt, andesite, diorite, and others. Includes lava flows and flows related ejectamenta and intrusive laccoliths.
8) Igneous and metamorphic rocks: Schist, granite greiss, granite, and granite permatite. Forms the basement complex upon which Units 7 to 1 rests.

Table 9. Comparison of actual and predicted estimates for B.

Subbasin No.	Φ_B (T/A/F)	Predicted Φ_B (T/A/F)	Deviation (T/A/F)
UG1	0.100	0.267	-0.167
UG2	0.230	0.386	-0.156
UG3	0.620	0.752	-0.132
UG4	0.760	0.680	-0.080
UG6	8.150	0.628	7.522
UG7	0.490	0.587	-0.098
UG8	0.175	0.482	-0.307
UG9	0.220	0.638	-0.418
UG10	0.210	0.721	-0.511
UG11	1.070	0.552	0.518
UG12	0.580	0.620	-0.040
UG13	0.470	0.684	-0.214
UG14	0.340	0.525	-0.185
UG15	0.490	0.638	-0.148
UG16	1.760	0.825	0.935
UG17	1.200	0.728	0.472
UG18	0.910	0.830	0.080
UM1	0.074	0.007	0.067
UM2	0.335	0.459	-0.124
UM3	0.260	0.433	-0.173
UM4	0.300	0.222	0.078
UM5	1.420	0.698	0.722
UM6	0.230	0.629	-0.399
UM7	0.129	-0.002	0.131
UM8	0.077	-0.032	0.110
UM9	0.456	0.438	0.018
UM10	1.100	0.767	0.333
UM11	1.190	0.854	0.336
UM12	0.290	0.689	-0.399
UM13	1.280	0.809	0.471
UM14	0.340	0.769	-0.428
US1	0.141	0.383	-0.242
US2	0.158	0.555	-0.397
US3	0.300	0.464	-0.164
US4	1.300	0.744	0.556
US5	0.180	0.824	-0.644
US6	1.170	0.809	0.361
US7	0.590	0.857	-0.267
LM1	0.200	0.648	-0.448
LM2	1.130	0.505	0.625
LM3	0.160	0.012	0.148
LM4	ϕ	ϕ	ϕ
LM5	0.400	-0.046	0.446
LM6	0.240	0.194	0.046
LM7	0.240	0.268	-0.028
LM8	ϕ	ϕ	ϕ
LL1	0.056	0.420	-0.364
LL2	0.340	0.422	-0.082
LL3	0.500	0.612	-0.112

geologic types and the base leaching factor. Table 9 presents the calculated Φ_B, the predicted Φ_B, and the deviation associated with each subbasin. Individual estimates of the standard error associated with a subbasin were not developed because only one estimate of Φ_B was available for each subbasin. Estimates of variance made upon such limited data would have been highly unreliable. The alternate approach of developing a mean standard error for all subbasins was employed. The mean standard deviation was found to be 0.399 (T/A/F). A variance of 0.159 $(T/A/F)^2$ was, therefore, associated with the use of Φ_B for all subbasins.

These uncertainties apply only to the procedures followed by the Colorado River Regional Assessment Study to define Φ_B. Other methods of estimating Φ_B result in a different set of uncertainties. The base leaching factor was calculated from an estimation of historic agricultural salt loadings byy CRRAS. Such a procedure involves a different set of uncertainties than would, for instance, be associated with the estimation of Φ_B from field measurements.

Modification of the Program SALTEZ

Minor modification of the generalized program SALTEZ was required for this application to the Colorado River. The modification was required because there was a significant deviation between the generalized assumptions of the program SALTEZ and the procedures used by CRRAS to estimate calibration data for the program SALT.

The generalized program SALTEZ is based upon an assumption of independence between individual salt loads. The general form of the stochastic model (Equation 39) is compatible with this assumption. However, the procedure (Equation 51) followed by CRRAS for estimation of the natural loading term, $\tilde{S}_{nat}$, violated this assumption. The natural loading terms were estimated by the difference between the total salt loading and the estimated agricultural and point loads (Equation 40). Clearly, the term S_{nat} was not independent of $\tilde{S}_n$ and $\tilde{S}_{agri}$. Rearrangement was required to permit development of variance equations. The resulting equation (Equation 44) had a format that was not compatible with Equation 49 of the generalized program SALTEZ. Equation 49 for the variance of the intermediate salinity, S_n', was therefore modified to:

$$\sigma^2_{S_n'^{Y}} = \sigma^2_{\Delta S_n^{72}} + \Sigma\sigma^2_{S_{n-1}^{Y}} + \Sigma\sigma^2_{\Phi_B}\left(\left[\frac{1-E^Y}{E^Y}\right]Q^Y - \left[\frac{1-E^{72}}{E^{72}}\right]Q^{72}\right)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (53)$$

The program SALTEZ was further modified to permit simultaneous input of two efficiencies, E^Y and E^{72} or consumed flows, Q^Y and Q^{72} used in Equation 53.

The fifth salinity option (Figure 8) was also temporarily modified in the program SALTEZ (see Appendix B). This option permitted calculation of the uncertainties specific to Φ_B in Equation 53. The variance associated with the $\Delta\tilde{S}_n^{72}$ parameter was accumulated by node, and input as the variance of the nodal slack term. Table 7 listed these variance terms. Reference to the modified program SALTEZ in the balance of this report refers to the version of SALTEZ that includes the above modifications.

RESULTS OF STEADY STATE STOCHASTIC SALINITY SIMULATIONS

Six stochastic simulations of salinity in the Colorado River system were undertaken with the program SALTEZ. Three of these simulations were of the baseline conditions for 1977, 1983, and 1995. The remaining three reflected of the three irrigation efficiencies (E_1, E_2, E_3) for the year 1983.

The baseline simulations reflect the baseline runs made with the program SALT in the Colorado River Assessment Study. These baseline runs assumed a 14 million acre foot per year virgin flow at Lee's Ferry, Arizona, and the most likely level of development of agriculture, energy, and water export as estimated by CRRAS. They were compared with simulations of management options. The 1977, 1983, and 1995 SALTEZ simulations correspond with the runs #1, #2, and #3 of the Colorado River Assessment Study.

Table 10 summarizes the results of the baseline simulations. The projected flows and mean salt loads display a decreasing trend with time. This reflects increased consumptive use of water. The coefficients of variation range from 5.7 (1983) to 5.8 percent (1977 and 1995) of the projected steady state salt load at Imperial Dam, California. Figure 9, illustrates the 95 percent confidence bands (normality assumed) associated with the projected salinity load for the 1977 baseline simulation.

The management simulations reflect application of different irrigation efficiencies to the 1983 baseline run. The three superimposed irrigation efficiencies reflect different irrigation management alternatives. The level 1 efficiency (E_1) reflected estimates of efficiencies resulting from on-farm management requiring no capital investment. The level 2 efficiency reflects upgrading of conveyance systems to an assumed 95 percent efficiency. The level 3 efficiency reflects upgrading of both conveyance and on-farm management techniques to technological limits. Table 11 compares the irrigation efficiencies used with the 1983 baseline and management simulations. The level 3 efficiency was assumed equal to 76 percent for all subbasins. The 1983-E_1, 1983-E_2, and 1983-E_3 simulations correspond with runs #7, #8, #9 of the Colorado River Assessment Study.

The results of the 1983 agricultural management simulation are presented in Table 12. The flows for these and the 1983 baseline projection were identical. This presumes complete utilization of water rights regardless of efficiency of application. The salt load decreases with increasing efficiency for each subbasin. Since the mean efficiency increases with efficiency level (see Table 11) the salt load at Imperial Dam decreases from E_1 through E_3. Individual subbasins, for example UG1-6, may show decreases in efficiency from level 1 to level 2. This resulted in an associated salt load increase from E_1 to E_2 for some nodes (nodes 5 and 10). The standard deviations at Imperial Dam varies from 6.8 (1983-E_1) to 10.3 percent (1983-E_3) of the steady state salt load. Figure 10 illustrates the 95 percent confidence bands associated with the projected salinity load for the 1983-E_3 simulation.

Discussion of Results

In interpretation of the results presented here one should consider a number of factors. Foremost are the limitations of the study in terms of the types of uncertainty considered. This analysis was limited to the propagation of uncertainty from the two sources $\Delta \tilde{S}_n$ and $\tilde{\Phi}_B$. All other parameters were assumed to be deterministic. The confidence bands projected here reflect only the two sources of uncertainty under the deterministic baseline conditions assumed by CRRAS. In particular, the flow regime and point sources were considered deterministically. The confidence bands in Figures 9 and 10 were positioned by assuming normality of the uncertainty distributions. Although this assumption appears reasonable it has not been demonstrated. Estimation of uncertainties associated with SALT simulations other than those presented here would be best undertaken with reapplication of the program SALTEZ since the variance estimates may vary widely between scenarios.

The mean values produced by the program SALTEZ simulations are identical to the deterministic values produced by the program SALT in CRRAS. The compatibility of the Generation of Moment Equations technique with the program SALT format permitted addition of the variance equations without significant

Table 10. Summary of results for Colorado River System baseline runs.

Node No.	1977-Baseline Flow (TAF/Y)	1977-Baseline Salt Load (TT/Y)	1977-Baseline σ (TT/Y)	1983-Baseline Flow (TAF/Y)	1983-Baseline Salt Load (TT/Y)	1983-Baseline σ (TT/Y)	1995-Baseline Flow (TAF/Y)	1995-Baseline Salt Load (TT/Y)	1995-Baseline σ (TT/Y)
5	2217.	885.	22.6	2207.	880.	22.6	2171.	870.	25.1
10	2734.	1711.	44.3	2711.	1703.	44.4	2668.	1689.	45.7
15	385.	397.	22.4	318.	403.	25.3	233.	390.	25.3
20	5134.	3223.	77.7	5001.	3264.	78.9	4796.	3296.	81.5
25	25.4	130.	13.9	25.4	130.	13.9	25.4	130.	13.9
30	1753.	690.	24.6	1655.	680.	24.6	1623.	677.	24.6
35	2947.	1759.	32.8	2834.	1748.	32.8	2752.	1719.	32.8
40	1377.	1194.	65.3	1358.	1191.	65.3	1286.	1231.	74.5
45	4083.	3766.	109.	3945.	3753.	109.	3753.	3771.	115.
50	702.	220.	17.0	663.	213.	17.0	643.	210.	17.0
55	1450.	1156.	58.0	1209.	1366.	79.1	931.	1629.	127.
59	10693.	8274.	146.	10180.	8512.	157.	9505.	8826.	191.
60	10471.	8022.	439.	9924.	8244.	443.	9177.	8523.	456.
65	10977.	9033.	451.	10430.	9255.	455.	9682.	9535.	468.
70	149.	391.	13.2	149.	391.	13.2	149.	391.	13.2
75	10183.	9787.	559.	9576.	9943.	558.	8766.	10137.	564.
80	9118.	8990.	522.	8319.	8905.	508.	6455.	7756.	438.
85	8154.	9169.	534.	7248.	9095.	522.	5287.	7827.	457.

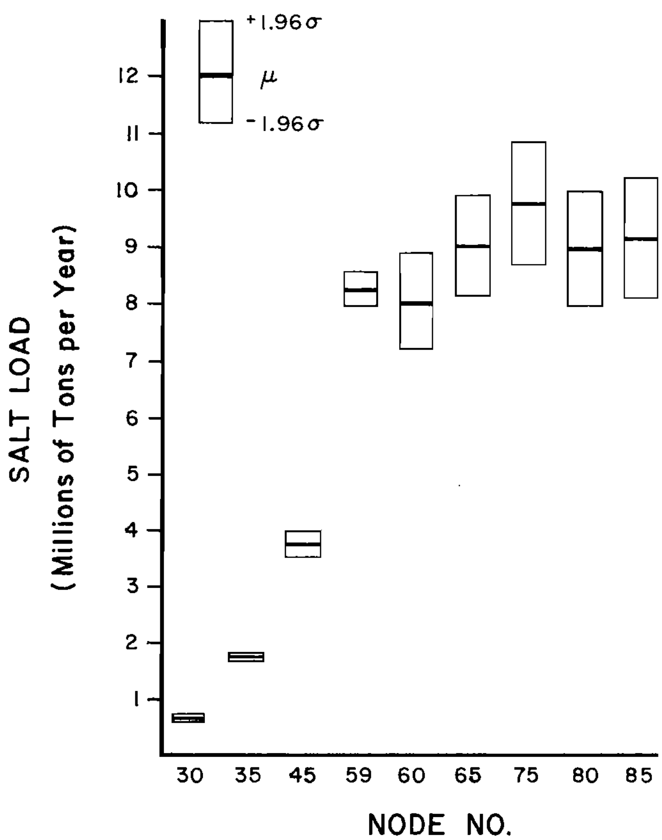

Figure 9. Mean values and 95 percent confidence bands (normality assumed) for the 1977 base run.

alteration of the existing program SALT capabilities. The same equations that produced the deterministic simulations of SALT are used in SALTEZ to produce mean values. Further, information developed through manipulation of data in this report was only used to define variances. No attempt was made to redefine the means produced by CRRAS in this would have beyond both the scope and objectives of the study.

Although the procedures followed in the Colorado River Assessment Study were used as a basis for developing the rationale for uncertainty definition, differences in the data base existed. Salinity data used in the development of $\Delta \tilde{S}_n$ in this report was taken from a 34 year (1940-1974) summary of salinity data (USBR, 1977). In the previous study estimates of salt loadings were derived primarily from Hyatt et al. (1970) and USBR (1975). Some minor differences, therefore, exist between these data. Further, trend analysis was incorporated into this estimation of $\Delta \tilde{S}_n$. This procedure was not included in the steady-state estimation of $\Delta \tilde{S}_n$ in the Colorado River Assessment Study. The effect of this difference in procedure is not known.

Finally, the uncertainty propagation presented here includes an undetermined contribution from the assumptions included in the development of variance estimates from $\tilde{\Phi}_B$. The use of geological types to represent agricultural soil types percentages within subbasins may have contributed variance not related to the uncertainty of $\tilde{\Phi}_B$.

The principal value of the SALTEZ simulations presented here lies in the

Table 11. Comparison of irrigation efficiencies for SALTEZ simulations.

Subbasin No.	Irrigation Efficiencies (Percent)			
	Base Runs	E_1 Runs	E_2 Runs	E_3 Runs
UG1	22	44	24	76
UG2	34	54	36	76
UG3	47	71	49	76
UG4	46	58	50	76
UG6	22	44	24	76
UG7	68	68	72	76
UG8	51	57	55	76
UG9	36	54	38	76
UG10	39	51	47	76
UG11	42	45	58	76
UG12	61	61	76	76
UG13	39	46	52	76
UG14	49	49	66	76
UG15	35	47	43	76
UG16	63	63	76	76
UG17	53	63	71	76
UG18	50	50	66	76
UM1	32	46	43	76
UM2	27	43	36	76
UM3	36	43	48	76
UM4	42	43	56	76
UM5	60	60	76	76
UM6	72	72	76	76
UM7	20	40	26	76
UM8	42	47	55	76
UM9	25	43	33	76
UM10	34	47	45	76
UM11	31	42	46	76
UM12	60	60	65	76
UM13	54	54	76	76
UM14	51	51	74	76
US1	30	45	41	76
US2	43	43	59	76
US3	42	45	58	76
US4	55	55	76	76
US5	55	55	76	76
US6	55	55	76	76
US7	57	57	71	76
LM1	60	60	60	76
LM2	67	67	69	76
LM3	75	75	75	76
LM4	60	60	60	76
LM5	65	65	72	76
LM6	53	53	59	76
LM7	53	61	53	76
LL1	58	58	76	76
LL2	58	58	76	76
LL3	58	58	76	76
MEAN	47.6	53.7	58.1	76

definition of confidence bands associated with the salinity projections (Figures 9 and 10). These confidence bands effectively define our ability to model steady state salinity in the Colorado River system under the procedures followed in this report. They do not reflect the actual variability that can be expected in future salinity measurements. Only uncertainty associated with selected relationships of the salinity model have been considered. Natural variability was only considered when it had a direct impact on estimation of steady state parameters. Given a future management scenario the confidence bands define the model's capability of predicting a steady state salinity

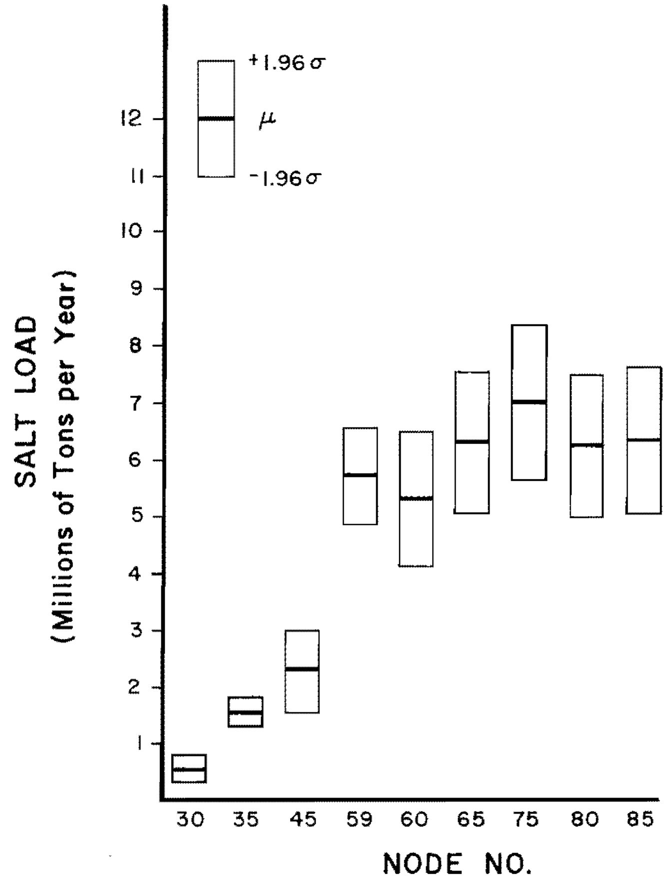

Figure 10. Mean values and 95 percent confidence bands (normality assumed) for the 1983 E_3 run.

level. Or in other words, they define the resolution of the modeling effort. This provides the management agency with information useful in deciding whether the modeling results warrant implementation of various management alternatives. For instance, since the coefficient of variation of the runs presented here varied from 5.7 to 10.3 percent,the management agency would in all likelihood consider the predicted means reliable. This knowledge would permit them to make management decisions with confidence.

The results of this stochastic analysis are presented in terms of salinity loadings rather than concentration. The coefficients of variation of the simulated salt loadings can be applied directly to simulated concentrations. This procedure has been avoided because concentrations represent a combination of flow and mass and only the latter was considered stochastic in this study. The deterministic treatment of the flow regime was dictated by the limits of this study and by the lack of an accepted methodology for handling the interdependence of flow and salinity. Until such a methodology is developed it is considered appropriate to

Table 12. Summary of results for 1983 agricultural management simulations of the Colorado River system.

Node No.	1983-E_1			1983-E_2			1983-E_3		
	Flow (TAF/Y)	Salt Load (TT/Y)	σ (TT/Y)	Flow (TAF/Y)	Salt Load (TT/Y)	σ (TT/Y)	Flow (TAF/Y)	Salt Load (TT/Y)	σ (TT/Y)
5	2207.	48.	83.4	2207.	855.	26.0	2207.	682.	120.
10	2711.	1568.	91.8	2711.	1669.	46.3	2711.	1484.	127.
15	318.	387.	25.6	318.	259.	61.6	318.	189.	95.
20	5001.	3049.	121.	5001.	2911.	102.	5001.	2552.	190.
25	25.4	130.	13.9	25.4	80.6	20.6	25.4	60.4	25.6
30	1655.	639.	44.3	1655.	636.	49.6	1655.	582.	97.3
35	2834.	1706.	49.4	2834.	1634.	57.6	2834.	1570.	104.
40	1358.	844.	205.	1358.	960.	134.	1358.	593.	309.
45	3945.	3101.	236.	3945.	3070.	186.	3945.	2365.	368.
50	663.	209.	19.6	663.	197.	35.3	663.	185.	52.4
55	1209.	1310.	73.3	1209.	1006.	75.8	1209.	760.	154.
59	10180.	7590.	275.	10180.	7068.	226.	10180.	5737.	443.
60	9924.	7321.	497.	9924.	6672.	472.	9924.	5334.	607.
65	10430	8333.	508.	10430.	7667.	484.	10430.	6328.	616.
70	149.	391.	13.2	149.	383.	13.3	149.	360.	15.0
75	9576.	9030.	601.	9576.	8364.	581.	9576.	7011.	693.
80	8319.	8090.	546.	8319.	7489.	528.	8319.	6271.	626.
85	7248.	8225.	556.	7248.	7680.	542.	7248.	6336.	652.

present results in terms of loading rather than concentrations.

The SALTEZ simulations also provide information for comparing various sources of the total uncertainty at Imperial Dam. In these runs, uncertainty stems from three sources (Table 13). The major source is the estimation of the 1972 calibration value for $\Delta\tilde{S}_n$. The variance contributed from $\Delta\tilde{S}_n$ remains constant in all runs based upon 1972 calibration. The second source of variance is the process of estimation of the base leaching factors, Φ_B. The contribution from this source varies widely. In the 1977-baseline run it contributes only 0.8 percent of the total variance. In the 1983-E_3 simulation it is responsible for 37 percent. The program SALT simulations were clearly sensitive to changes in agricultural parameters. The third source of uncertainty considered stems from projected diversion flows in the simulations. Although the diversion flows were presumed completely known, the associated instream salinities, $\tilde{S}_n$, were uncertain. The variance associated with diversions varied as a function of their size and the accumulated upstream variance. The diversions reduced the total variance in all of the simulations presented here. In the 1995-base run this source of variance reduced the total variance by 42 percent.

Analysis of the sources of variance can contribute significantly to our understanding of the modeling process. It identifies those relationships or components of the model that warrant further refinement. It is possible,

Table 13. Sources of variances at Imperial Dam, California.

Run Identification	Total Salt Load (TT/Y)	Variance $\Delta\tilde{S}_n$ $(TT/Y)^2$	Variance[a] Diversions $(TT/Y)^2$	Variance Φ_B $(TT/Y)^2$	Total Variance $(TT/Y)^2$
1977-Baseline	9169.	336,895	- 54,876	2,799	284,814
1983-Baseline	9095.	336,895	- 73,133	9,219	272,985
1995-Baseline	7827.	336,895	-152,275	23,830	208,447
1983 E_1	8225.	336,895	- 84,465	57,125	309,559
1983 E_2	7680.	336,895	- 79,159	36,372	294,110
1983 E_3	6336.	336,895	-111,417	200,185	425,665

[a]Negative signs indicate variance was subtracted.

for instance, that another method of estimation of $\tilde{\Phi}_B$ could significantly decrease the variance of the agricultural management simulations. Comparison of the cost of such a refinement with the value of increased resolution would determine whether such efforts were warranted.

Comparison of Programs SALT and SALTEZ

Table 14 compares the cost of the stochastic program SALTEZ with that of the deterministic program SALT. As would be

Table 14. Comparison of programming effort and cost of the stochastic program SALTEZ and the deterministic program SALT.

Item	Program SALT	Program SALTEZ
Lines of programming	348	569
Core storage (words)	2,243	2,999
Typical compilation time (secs)	5.5	9.5
1977-base Run time (secs)	3.5	7.1
1977-base Run cost	$0.72	$1.17

expected, the stochastic analysis requires more programming effort, core storage, and simulation time. The method of Generation of Moments Equations showed itself to be a particularly effective method for developing the uncertainty estimations. The items in Table 14 indicate the program SALTEZ requires 1.6 the modeling costs associated with the program SALT. The costs associated with both programs were so small as to make this a negligible increase.

The major price paid for the additional stochastic information would stem from the estimation of the uncertainty associated with selected parameters. The stochastic model inherently requires more external data manipulation than the deterministic approach. The magnitude of this additional effort depends largely upon the specific type of uncertainty generated. Estimation of the uncertainty associated with $\Delta\tilde{S}_n$, for instance, would cause a negligible cost increase over the deterministic process. Estimation of uncertainties associated with individual point loads could be much more costly because of the numerous different types of point loads and the need to gather data and analysis data on each.

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The two case studies presented in this report demonstrate the feasibility of applying stochastic techniques to the area of water quality. The three techniques considered in the preliminary comparison, i.e., Monte Carlo, First Order, and Generation of Moment Equations all proved to be applicable to a dynamic water quality model. In all cases, assumptions or approximations required for application of the stochastic techniques to the phosphorus model were compatible with those made with the deterministic model. Finally the estimates of variance were in close agreement between the three techniques. This indicated all three approaches would be reliable in applications of this type.

This preliminary comparison revealed differences between the techniques. Although the Monte Carlo technique was clearly the most expensive in terms of computer time, its flexibility and ease of application was apparent. The main drawback of this technique was the large number of simulations required to achieve statistical significance of results. The First Order and Generation of Moment Equation techniques required only a single simulation to produce estimates of mean and variance. Unfortunately both these techniques become approximate for nonlinear systems. Further, both require personnel with stochastic training to derive the stochastic equations used for solution.

The stochastic simulations of Lake Washington also revealed the water column phosphorus was very responsive to uncertainty in annual phosphorus loadings. If a coefficient of variation of 30 percent was placed upon the estimated phosphorus loading rate, the water column phosphorus was estimated with a relative standard deviation of about 16 percent. This is in contrast to the insensitive sediment phosphorus levels (2.5 percent).

The second case study was the application of the Generation of Moment Equations technique to the Colorado River system. The technique was readily applied to an existing linear salinity model developed by UWRL (1975). Computation time and run costs for the stochastic simulations were approximately 1.6 times the costs for similar deterministic runs. Two aspects of this application were particularly important. First, it was demonstrated that estimates of uncertainty could be made from data bases similar to those used for deterministic calibrations. Secondly, it indicated that the steady state estimations of salinity loadings for selected management alternatives were reasonable. Coefficient of variations of projected salinity projections varied from 5.7 to 10.3 percent.

The major source of uncertainty in the steady state salinity projections was the estimation of the incremental salinity loadings, ΔS_n, used for the model calibration. Uncertainty from estimation of the agricultural base leaching factor, Φ_B, became important when major changes in farm water management were considered. The total uncertainty defined the reliability of the steady state salinity modeling effort. this information could allow water management agencies to place the appropriate weight upon results of simulations. Identification of the sources of uncertainty also defines those areas limiting the reliability of the modeling effort.

Recommendations for Further Study

If the full value of stochastic analysis of water quality problems is to be realized certain areas should be more fully developed. Possibly the most important is identification of those sources of uncertainty that should be considered. This implies expansion of the type of analysis presented in this report until all major sources of uncertainty have been identified. Only then can the variance on the output variables realistically represent the actual model reliability.

The second area is the development of accepted techniques for estimation of variances. Careful study should be made of alternate methods of estimating the variances associated with selected sources of uncertainty. This should permit the development of a number of standard approaches to estimating uncertainties, thus minimizing error and aiding interpretation of results.

Techniques should be developed or demonstrated to cope with characteristic problems of water quality. For instance, accepted methodologies must be developed for dealing with the interdependence of flow and constituent concentrations. This problem was avoided in this study by assuming a deterministic flow regime. The feasibility of applying the stochastic techniques to uncertain decay coefficients should be demonstrated. Finally, the reasonableness of assuming normality for distributions of water quality parameters should be investigated.

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APPENDIX A

PROGRAM LISTINGS AND SAMPLE OUTPUTS FOR PRELIMINARY

COMPARISON OF STOCHASTIC TECHNIQUES

FIRST ORDER ANALYSIS: PROGRAM LISTING AND

OUTPUT FOR PHOSPHORUS MODEL

```
       B6700/B7700   F O R T R A N   C O M P I L A T I O N   M A R K   2.8.060 WEDNESDAY, 04/26/78    09:35 AM
C      FIRST ORDER ANALYSIS OF PHOSPHORUS MODEL                                        C  000:0000:5
                                                                                        START OF SEGMENT 002
       REAL M(70),K1,K2,K3                                                             C  002:0000:0
        DIMENSION ANS(6,51)                                                            C  002:0000:0
       DATA M/10*45.,10*61.,6*81.,89.,140.,190.,240.,280.,288.,257.,                   C  002:0000:0
     1 257.,196.,196.,34*80./                                                          C  002:0000:0
       PF=.3                                                                           C  002:0000:0
       V=3.8*10.**9.                                                                   C  002:0002:3
       K1=36.                                                                          C  002:0006:2
       K2=.0012                                                                        C  002:0007:1
       K3=.6                                                                           C  002:0009:3
       VS=10.**7.                                                                      C  002:000B:3
       Q=9.*10**8.                                                                     C  002:000D:4
       A=10**8.                                                                        C  002:0010:0
       ANS(1,1)=.015                                                                   C  002:0011:5
       ANS(2,1)=240.                                                                   C  002:0014:1
       ANS(4,1)=0.                                                                     C  002:0015:2
       ANS(3,1)=0.                                                                     C  002:0016:3
       DO 10 K=1,70                                                                    C  002:0017:4
   1)  M(K)=M(K)*10**6./V                                                              C  002:0019:0
       X2=(K1*A+Q)/V                                                                   C  002:001E:5
       X3=K2*A/V                                                                       C  002:0021:1
       X4=(1-K3)*K1*A/VS                                                               C  002:0023:0
       X5=K2*A/VS                                                                      C  002:0025:4
       B=(X2+X5)                                                                       C  002:0027:3
       C=X2*X5-X3*X4                                                                   C  002:0028:5
       S=C                                                                             C  002:002B:1
       TK=-S                                                                           C  002:002B:4
       AA=X5/TK                                                                        C  002:002C:2
       AB=X3/TK                                                                        C  002:002D:4
       AC=X4/TK                                                                        C  002:002F:0
       AD=X2/TK                                                                        C  002:0030:2
       R=-1./(2.*B)                                                                    C  002:0031:4
       AL=(-B-SQRT(B**2.-4.*C))/2.                                                     C  002:0033:3
       BE=C/AL                                                                         C  002:0037:3
       AT=EXP(-AL)                                                                     C  002:0038:5
       BT=EXP(-BE)                                                                     C  002:003A:3
       DA=1./(AL-BE)*AT                                                                C  002:003C:1
       DB=1./(BE-AL)*BT                                                                C  002:003E:2
       DO11=(AL+X5)*DA+(BE+X5)*DB                                                      C  002:0040:3
       DO12=+X3*(DA+DB)                                                                C  002:0043:5
       DO21=+X4*(DA+DB)                                                                C  002:0045:4
       DO22=(AL+X2)*DA+(BE+X2)*DB                                                      C  002:0047:3
       AT=EXP( AL)                                                                     C  002:004A:5
```

```
      BT=EXP( BE)
      DA=1./(AL-BE)*AT
      DB=1./(BE-AL)*BT
      CO11=(AL+X5)*DA+(BE+X5)*DB
      CO12=+X3*(DA+DB)
      CO21=+X4*(DA+DB)
      CO22=(AL+X2)*DA+(BE+X2)*DB
      WRITE(6,901) PF
901   FORMAT (1H1,T27,'EFFECTS OF VARIATIONS IN LOADING UPON PHOSPHORUS
    1 MODEL,FIRST ORDER ANALYSIS'/T53,'PROBABILITY FACTOR=',F4.2)
      WRITE (6,915)
915   FORMAT (1H ,T30,'YEAR',T43,'LAKE',T56,'STANDARD',T71,'SEDIMENT',
    1  T86,'STANDARD',/T43,'CONC',T55,'DEVIATION',T73,'CONC',T85,
    2  'DEVIATION',/T43,'MG/L',T58,'MG/L',T73,'MG/L',T88,'MG/L')
      NYEAR=1930
      DOG=SQRT(ANS(3,1))
      CAT=SQRT(ANS(4,1))
      WRITE (6,916)NYEAR,ANS(1,1),DOG,ANS(2,1),CAT
916   FORMAT (1H ,T30,I4,4(5X,E10.3))
      DO 100 I=1,50
      VW=ANS(3,I)
      VS=ANS(4,I)
      VM=(PF*M(I))**2.
      B1=R*VM*(1.+AA*S*AA)
      B2=R*S*AA*VM*AC
      B3=R*S*AC*VM*AA
      B4=R*S*AC*VM*AC
      Z1=B1-(DO11*B1+DO12*B3)*DO11-(DO11*B2+DO12*B4)*DO12
      Z2=B2-(DO11*B1+DO12*B3)*DO21-(DO11*B2+DO12*B4)*DO22+ANS(5,I)
      Z3=B3-(DO21*B1+DO22*B3)*DO11-(DO21*B2+DO22*B4)*DO12+ANS(6,I)
      Z4=B4-(DO21*B1+DO22*B3)*DO21-(DO21*B2+DO22*B4)*DO22
      ANS(1,I+1)=CO11*(ANS(1,I)+AA*(1.-DO11)*M(I)-AB*DO21*M(I))+
    $ CO12*(ANS(2,I)+AC*(1.-DO11)*M(I)-AD*DO21*M(I))
      ANS(2,I+1)=CO21*(ANS(1,I)+AA*(1.-DO11)*M(I)-AB*DO21*M(I))+
    $ CO22*(ANS(2,I)+AC*(1.-DO11)*M(I)-AD*DO21*M(I))
      ANS(3,I+1)=(CO11*(VW+Z1)+CO12*Z3)*CO11+(CO11*Z2+CO12*(VS+
    $ Z4))*CO12
       ANS(4,I+1)=(CO21*(VW+Z1)+CO22*Z3)*CO21+(CO21*Z2+CO22*(VS+Z4))
    $ *CO22
      ANS(5,I+1)=(CO11*(VW+Z1)+CO12*Z3)*CO21+(CO11*Z2+CO12*(VS+
    $ Z4))*CO22
       ANS(6,I+1)=(CO21*(VW+Z1)+CO22*Z3)*CO11+(CO21*Z2+CO22*(VS+Z4))
    $ *CO12
      NYEAR=1930+I
      DOG=SQRT(ANS(3,I+1))
      CAT=SQRT(ANS(4,I+1))
      WRITE (6,916)NYEAR,ANS(1,I+1),DOG,ANS(2,I+1),CAT
100   CONTINUE
      CALL EXIT
      END
```

```
  C   002:004C:2
  C   002:004D:5
  C   002:0050:0
  C   002:0052:1
  C   002:0055:3
  C   002:0057:2
  C   002:0059:1
  C   002:005C:3
        FIB IS 0006 LONG
  C   002:0063:2
  C   002:0063:2
  C   002:0063:2
  C   002:0067:2
  C   002:0067:2
  C   002:0067:2
  C   002:0067:2
  C   002:0068:2
  C   002:006A:1
  C   002:006C:0
  C   002:0079:2
  C   002:0079:2
  C   002:007A:0
  C   002:007C:1
  C   002:007E:2
  C   002:0080:4
  C   002:0083:5
  C   002:0086:4
  C   002:0089:3
  C   002:008C:2
  C   002:0092:1
  C   002:0099:5
  C   002:00A1:2
  C   002:00A7:1
  C   002:00AF:2
  C   002:00B6:3
  C   002:00BF:0
  C   002:00C6:1
  C   002:00CC:2
  C   002:00CE:0
  C   002:00D5:0
  C   002:00D5:5
  C   002:00DC:0
  C   002:00DD:4
  C   002:00E4:4
  C   002:00E5:3
  C   002:00E7:1
  C   002:00EA:0
  C   002:00EC:5
  C   002:00FB:2
  C   002:00FD:3
  C   002:00FE:2
SEGMENT 002 IS 010F LONG
```

EFFECTS OF VARIATIONS IN LOADING UPON PHOSPHORUS MODEL,FIRST ORDER ANALYSIS
PROBABILITY FACTOR=0.30

YEAR	LAKE CONC MG/L	STANDARD DEVIATION MG/L	SEDIMENT CONC MG/L	STANDARD DEVIATION MG/L
1930	.150E-01	0.	.240E+03	0.
1931	.160E-01	.220E-02	.239E+03	.197E+00
1932	.162E-01	.230E-02	.239E+03	.403E+00
1933	.163E-01	.231E-02	.238E+03	.569E+00
1934	.163E-01	.231E-02	.238E+03	.704E+00
1935	.163E-01	.231E-02	.237E+03	.817E+00
1936	.163E-01	.231E-02	.237E+03	.916E+00
1937	.163E-01	.231E-02	.236E+03	.100E+01
1938	.163E-01	.231E-02	.236E+03	.108E+01
1939	.163E-01	.231E-02	.235E+03	.116E+01
1940	.163E-01	.231E-02	.235E+03	.122E+01
1941	.187E-01	.307E-02	.235E+03	.130E+01
1942	.195E-01	.313E-02	.235E+03	.140E+01
1943	.197E-01	.313E-02	.235E+03	.150E+01
1944	.198E-01	.313E-02	.235E+03	.159E+01
1945	.198E-01	.313E-02	.235E+03	.168E+01
1946	.198E-01	.313E-02	.235E+03	.177E+01
1947	.198E-01	.313E-02	.235E+03	.184E+01
1948	.198E-01	.313E-02	.235E+03	.192E+01
1949	.198E-01	.313E-02	.235E+03	.199E+01
1950	.198E-01	.313E-02	.235E+03	.206E+01
1951	.229E-01	.408E-02	.235E+03	.213E+01
1952	.239E-01	.415E-02	.236E+03	.223E+01
1953	.242E-01	.416E-02	.236E+03	.234E+01
1954	.243E-01	.416E-02	.237E+03	.244E+01
1955	.243E-01	.416E-02	.238E+03	.254E+01
1956	.243E-01	.416E-02	.238E+03	.264E+01
1957	.256E-01	.454E-02	.239E+03	.273E+01
1958	.339E-01	.698E-02	.241E+03	.287E+01
1959	.441E-01	.953E-02	.243E+03	.314E+01
1960	.551E-01	.121E-01	.248E+03	.357E+01
1961	.647E-01	.142E-01	.253E+03	.416E+01
1962	.690E-01	.147E-01	.260E+03	.483E+01
1963	.656E-01	.134E-01	.267E+03	.545E+01
1964	.647E-01	.132E-01	.273E+03	.596E+01
1965	.551E-01	.104E-01	.278E+03	.637E+01
1966	.523E-01	.101E-01	.282E+03	.665E+01
1967	.336E-01	.501E-02	.285E+03	.683E+01
1968	.279E-01	.421E-02	.286E+03	.689E+01
1969	.262E-01	.413E-02	.286E+03	.690E+01
1970	.256E-01	.412E-02	.286E+03	.689E+01
1971	.255E-01	.412E-02	.287E+03	.688E+01
1972	.254E-01	.411E-02	.287E+03	.687E+01
1973	.254E-01	.411E-02	.287E+03	.685E+01
1974	.254E-01	.411E-02	.287E+03	.684E+01
1975	.254E-01	.411E-02	.287E+03	.683E+01
1976	.254E-01	.411E-02	.288E+03	.681E+01
1977	.255E-01	.411E-02	.288E+03	.680E+01
1978	.255E-01	.411E-02	.288E+03	.679E+01
1979	.255E-01	.411E-02	.288E+03	.678E+01
1980	.255E-01	.411E-02	.289E+03	.676E+01

MONTE CARLO METHOD: PROGRAM LISTING AND OUTPUT

FOR PHOSPHORUS MODEL

```
        B6700/B7700    F O R T R A N    C O M P I L A T I O N    M A R K    2.8.060 WEDNESDAY, 04/26/78    09:38 AM

FILE  12(KIND=DISK, TITLE="STOREMONTE",MYUSE=IO,MAXRECSIZE=14,BLOCKSIZE=        C  00000001
     1 420,AREASIZE=30,AREAS=998)                                               C  00000002
                                                                                 START OF SEGMENT 002
      REAL K1,K2,K3,M(70)                                                       C  002:0000:0
C     APPLICATION OF THE MONTE CARLO APPROACH TO A PHOSPHORUS MODEL             C  002:0000:0
C     RON MALONE NOV 7,1977                                                     C  002:0000:0
      REAL MEAN(50,2)                                                           C  002:0000:0
      DIMENSION CONC(50,2),STDEV(50,2)                                          C  002:0000:0
      DATA M/10*45.,10*61.,6*81.,89.,140.,190.,240.,280.,288.,257.,257.         C  002:0000:0
     1,196.,196.,34*80./                                                        C  002:0000:0
      READ (5,926) IT,PF,PFC1,PFCS1,MONTE,NUM,NTYR                              C  002:0000:0
                                                                                    FIB IS 0006 LONG
  926 FORMAT (I5,3F10.0,3I5)                                                    C  002:0013:2
C     IF "IT" IS NEGATIVE OR ZERO "STOREMONTE" WILL BE CLEARED                  C  002:0013:2
C     IF PFC1 IS EQUAL TO ZERO INTIAL UNCERTAINITY WILL BE BYPASSED             C  002:0013:2
      IF(IT.GT.0) GO TO 13                                                      C  002:0013:2
      CNT=0.                                                                    C  002:0014:3
      WRITE (12,972) MEAN,STDEV,CNT                                             C  002:0015:1
  972 FORMAT (4E18.11)                                                          C  002:001F:2
   13 CONTINUE                                                                  C  002:001F:2
      REWIND 12                                                                 C  002:001F:2
      READ (12,972)MEAN,STDEV,CNT                                               C  002:0020:5
      WRITE (6,927) MEAN,STDEV,CNT                                              C  002:002B:2
                                                                                    FIB IS 0006 LONG
  927 FORMAT (1H ,10(E10.3,2X))                                                 C  002:0035:2
C     SEED NUMBER SELECTION                                                     C  002:0035:2
      IF(IR.GT.0) GO TO 603                                                     C  002:0035:2
      IR=TIME(1)                                                                C  002:0036:3
  353 IF (IR.LT.2097152)GO TO 403                                               C  002:0038:0
      IR=IR/2                                                                   C  002:003A:4
      GO TO 353                                                                 C  002:003C:1
  403 IF (IR.GT.524288)GO TO 503                                                C  002:003C:4
      IR=IR*2                                                                   C  002:003F:4
      GO TO 403                                                                 C  002:0041:1
  503 FNN=FLOAT(IR)/2.                                                          C  002:0041:4
      NN1=IFIX(FNN)                                                             C  002:0043:0
      NN2=2*NN1                                                                 C  002:0043:4
      IF (IR-NN2)603,553,603                                                    C  002:0045:1
  553 IR=IR-1                                                                   C  002:0046:3
  603 CONTINUE                                                                  C  002:0047:5
      TOC=IR                                                                    C  002:0047:5
      C1=.015                                                                   C  002:0048:4
      CS1=240.                                                                  C  002:004A:3
      STDC1=PFC1*C1                                                             C  002:004B:2
```

```
      STDCS1=PFCS1*CS1                                                  C   002:004C:4
      IF(PFC1.LE.0.)STDC1=0.                                            C   002:004E:0
      IF(PFC1.LE.0.)STDCS1=0.                                           C   002:004F:5
      TYR=NTYR                                                          C   002:0051:4
      DELT=1.                                                           C   002:0052:3
      DO 10 K=1,70                                                      C   002:0053:1
   10 M(K)=M(K)/ TYR*10.**6                                             C   002:0054:0
      K1=36./TYR                                                        C   002:0059:5
      Q=9.*10.**8/TYR                                                   C   002:005B:1
      K2=.0012/TYR                                                      C   002:005E:0
      K3=.6                                                             C   002:0061:0
      A=10**8                                                           C   002:0063:3
      V=3.8*10.**9                                                      C   002:0065:2
      VS=10.**7                                                         C   002:0069:2
      CO11=1.-DELT*(K1*A+Q)/V                                           C   002:006B:3
      CO12=DELT*K2*A/V                                                  C   002:006E:4
      CO21=DELT*(K1-K1*K3)*A/VS                                         C   002:0071:0
      CO22=1.-DELT*K2*A/VS                                              C   002:0074:1
      JTO=2                                                             C   002:0076:5
      INUM=NUM                                                          C   002:0077:4
      DO 200 N=1,MONTE                                                  C   002:0078:3
      CS1=240.                                                          C   002:007A:0
      C1=.015                                                           C   002:007A:5
      IF (PFC1.LE.0.) GO TO 14                                          C   002:007C:3
      CS1=CS1+RNOR(IR)*STDCS1                                           C   002:007D:4
      C1=C1+RNOR(IR)*STDC1                                              C   002:0080:2
   14 CONTINUE                                                          C   002:0083:0
      DO 100 I=1,NUM                                                    C   002:0083:0
      STD  =PF*M(I)*SQRT(TYR)                                           C   002:0084:0
      DO 50 J=1,NTYR                                                    C   002:0087:1
      X=(M(I)+RNOR(IR)*STD)/V                                           C   002:0088:0
      C2=CO11*C1+CO12*CS1+X                                             C   002:008B:5
      CS2=CO21*C1+CO22*CS1                                              C   002:008E:4
      C1=C2                                                             C   002:0091:0
      CS1=CS2                                                           C   002:0091:5
   50 CONTINUE                                                          C   002:0092:4
      CONC(I,1)=C2                                                      C   002:0094:5
      CONC(I,2)=CS2                                                     C   002:0096:3
  100 CONTINUE                                                          C   002:0098:2
      IF (N.EQ.MONTE)INUM=-NUM                                          C   002:009A:3
      CALL DEV (CONC,JTO,INUM,MEAN,STDEV,CNT)                           C   002:009D:0
  200 CONTINUE                                                          C   002:00A1:1
      WRITE (6,913)                                                     C   002:00A3:2
  913 FORMAT(1H1,T26,'EFFECTS OF VARIATIONS IN LOADING UPON PHOSPHORUS  C   002:00A7:2
     1MODEL,MONTE CARLO METHOD')                                        C   002:00A7:2
      WRITE (6,914) INUM,PF,TOC                                         C   002:00A7:2
  914 FORMAT (1H ,T26,'NUMBER OF RUNS=',I4,5X,'PROBABILITY FACTOR=',    C   002:00B1:2
     1 F4.2,5X,'SEED=',I8)                                              C   002:00B1:2
      WRITE (6,915)                                                     C   002:00B1:2
  915 FORMAT (1H ,T30,'YEAR',T43,'LAKE',T56,'STANDARD',T71,'SEDIMENT',  C   002:00B5:2
     1  T86,'STANDARD',/T43,'CONC',T55,'DEVIATION',T73,'CONC',T85,      C   002:00B5:2
     2  'DEVIATION',/T43,'MG/L',T58,'MG/L',T73,'MG/L',T88,'MG/L')       C   002:00B5:2
C       OUTPUT INTIAL CONDITIONS                                        C   002:00B5:2
```

```
      NYEAR=1930                                                         C  002:00B5:2
      C1=.015                                                            C  002:00B6:2
      CS1=240.                                                           C  002:00B8:3
      WRITE (6,916) NYEAR,C1,STDC1,CS1,STDCS1                            C  002:00B9:2
      DO 500 L=1,NUM                                                     C  002:00C6:2
      NYEAR=1930+L                                                       C  002:00C7:0
      WRITE (6,916) NYEAR,(MEAN(L,KT),STDEV(L,KT),KT=1,2)                C  002:00C8:4
  500 CONTINUE                                                           C  002:00DA:2
  916 FORMAT (1H ,T30,I4,4(5X,E10.3))                                    C  002:00DC:3
      LOCK 12                                                            C  002:00DC:3
      CALL EXIT                                                          C  002:00DE:0
      END                                                                C  002:00DE:5
                                                                       SEGMENT 002 IS 00F7 LONG
```

```
                                                                        START OF SEGMENT 006
      SUBROUTINE DEV(X,J,NUM,XSUM,XSQ,CNT)                               C  006:0000:0
C        SUBROUTINE DEV CALCULATES THE MEAN AND STANDARD DEVIATION FOR EACH  C  006:0000:0
C     ELEMENT OF THE "X" ARRAY,THE INTEGER "NUM" DEFINES THE NUMBER OF   C  006:0000:0
C     ELEMENTS IN X,IF NUM IS POSITIVE ONLY INTERMEDIATE CALCULATIONS ARE  C  006:0000:0
C     MADE,IF NUM IS NEGATIVE THE MEAN AND STANDARD DEVIATION ARE        C  006:0000:0
C     CALCULATED FOR EACH ELEMENT OF X USING THE PREVIOUSLY DEFINED      C  006:0000:0
C     INTERMEDIATE VALUES        RON MALONE NOV. 3,1977                  C  006:0000:0
      DIMENSION X(50,2),XSUM(50,2),XSQ(50,2)                             C  006:0000:0
      CNT=CNT+1                                                          C  006:0000:0
      INUM=IABS(NUM)                                                     C  006:0001:1
      DO 100 KT=1,J                                                      C  006:0002:2
      DO 100 I=1,INUM                                                    C  006:0003:0
      XSUM(I,KT)=XSUM(I,KT)+X(I,KT)                                      C  006:0004:0
      XSQ(I,KT)=XSQ(I,KT)+X(I,KT)**2.                                    C  006:000A:1
  100 CONTINUE                                                           C  006:0010:4
      IF (NUM.GT.0)GO TO 20                                              C  006:0015:0
      REWIND 12                                                          C  006:0016:1
      WRITE (12,972) XSUM,XSQ,CNT                                        C  006:0017:4
  972 FORMAT (4E18.11)                                                   C  006:0023:2
      DO 200 KT=1,J                                                      C  006:0023:2
      DO 200 I=1,INUM                                                    C  006:0024:0
  901 FORMAT (1H ,3(E10.3,5X))                                           C  006:0025:0
      XSUM(I,KT)=XSUM(I,KT)/CNT                                          C  006:0025:0
      XSQ(I,KT)=((XSQ(I,KT)-CNT*XSUM(I,KT)**2.)/(CNT-1.))**.5            C  006:0028:5
  200 CONTINUE                                                           C  006:0033:1
      NUM=CNT                                                            C  006:0037:3
   20 CONTINUE                                                           C  006:0038:3
      RETURN                                                             C  006:0038:3
      END                                                                C  006:0039:0
                                                                       SEGMENT 006 IS 0044 LONG
```

```
      FUNCTION RNOR(IR)
      DATA I/0/
      IF(I.GT.0)GO TO 30
10    X=2.0*RANDOM(IR)-1.0
      Y=2.0*RANDOM(IR)-1.0
      S=X*X+Y*Y
      IF(S.GE.(1.0))GO TO 10
      S=SQRT(-2.0*ALOG(S)/S)
      RNOR=X*S
      GO2=Y*S
      I=1
      GO TO 40
30    RNOR=GO2
      I=0
40    RETURN
      END
```

```
    START OF SEGMENT 008
  C   008:0000:0
  C   008:0000:0
  C   008:0000:0
  C   008:0001:1
  C   008:0003:4
  C   008:0006:1
  C   008:0008:1
  C   008:0009:0
  C   008:000C:2
  C   008:000D:4
  C   008:000F:0
  C   008:000F:4
  C   008:0010:1
  C   008:0011:0
  C   008:0011:4
  C   008:0012:1
SEGMENT 008 IS 0019 LONG
```

EFFECTS OF VARIATIONS IN LOADING UPON PHOSPHORUS MODEL,MONTE CARLO METHOD
NUMBER OF RUNS=3842 PROBABILITY FACTOR=0.30 SEED= 2094209

YEAR	LAKE CONC MG/L	STANDARD DEVIATION MG/L	SEDIMENT CONC MG/L	STANDARD DEVIATION MG/L
1930	.150E-01	0.	.240E+03	0.
1931	.160E-01	.223E-02	.239E+03	.198E+00
1932	.162E-01	.231E-02	.239E+03	.405E+00
1933	.163E-01	.232E-02	.238E+03	.573E+00
1934	.164E-01	.230E-02	.238E+03	.707E+00
1935	.164E-01	.233E-02	.237E+03	.819E+00
1936	.163E-01	.231E-02	.237E+03	.913E+00
1937	.163E-01	.231E-02	.236E+03	.100E+01
1938	.163E-01	.236E-02	.236E+03	.108E+01
1939	.164E-01	.236E-02	.235E+03	.116E+01
1940	.163E-01	.234E-02	.235E+03	.122E+01
1941	.187E-01	.308E-02	.235E+03	.130E+01
1942	.194E-01	.316E-02	.235E+03	.140E+01
1943	.196E-01	.319E-02	.235E+03	.150E+01
1944	.197E-01	.316E-02	.235E+03	.160E+01
1945	.198E-01	.318E-02	.235E+03	.170E+01
1946	.198E-01	.312E-02	.235E+03	.179E+01
1947	.198E-01	.313E-02	.235E+03	.187E+01
1948	.198E-01	.312E-02	.235E+03	.193E+01
1949	.198E-01	.312E-02	.235E+03	.200E+01
1950	.198E-01	.320E-02	.235E+03	.206E+01
1951	.229E-01	.412E-02	.235E+03	.213E+01
1952	.238E-01	.416E-02	.236E+03	.224E+01
1953	.241E-01	.415E-02	.236E+03	.234E+01
1954	.242E-01	.422E-02	.237E+03	.244E+01
1955	.243E-01	.420E-02	.238E+03	.253E+01
1956	.243E-01	.415E-02	.238E+03	.263E+01
1957	.257E-01	.455E-02	.239E+03	.272E+01
1958	.338E-01	.703E-02	.240E+03	.286E+01
1959	.440E-01	.961E-02	.243E+03	.312E+01
1960	.548E-01	.124E-01	.248E+03	.355E+01
1961	.647E-01	.143E-01	.253E+03	.417E+01
1962	.691E-01	.146E-01	.260E+03	.483E+01
1963	.657E-01	.133E-01	.266E+03	.546E+01
1964	.649E-01	.134E-01	.273E+03	.598E+01
1965	.554E-01	.104E-01	.278E+03	.638E+01
1966	.523E-01	.100E-01	.282E+03	.667E+01
1967	.336E-01	.496E-02	.285E+03	.684E+01
1968	.279E-01	.418E-02	.286E+03	.689E+01
1969	.261E-01	.423E-02	.286E+03	.689E+01
1970	.256E-01	.412E-02	.286E+03	.688E+01
1971	.255E-01	.407E-02	.287E+03	.687E+01
1972	.255E-01	.414E-02	.287E+03	.686E+01
1973	.255E-01	.416E-02	.287E+03	.685E+01
1974	.255E-01	.420E-02	.287E+03	.684E+01
1975	.254E-01	.415E-02	.288E+03	.684E+01
1976	.254E-01	.420E-02	.288E+03	.682E+01
1977	.254E-01	.408E-02	.288E+03	.681E+01
1978	.255E-01	.425E-02	.288E+03	.680E+01
1979	.254E-01	.421E-02	.288E+03	.680E+01
1980	.255E-01	.420E-02	.289E+03	.678E+01

GENERATION OF MOMENT EQUATIONS TECHNIQUE: PROGRAM

LISTING AND OUTPUT FOR PHOSPHORUS MODEL

```
     B6700/B7700   F O R T R A N   C O M P I L A T I O N   M A R K   2.8.060 WEDNESDAY, 04/26/78    09:39 AM

                                                                                START OF SEGMENT 002
      REAL K1,K2,K3,P(70),M(70)                                               C  002:0000:0
C BAYESIAN APPROACH APPLIED TO PHOSPHORUS MODEL  RON MALONE 1/11/78           C  002:0000:0
      DATA P/10*45.,10*61.,6*81.,89.,140.,190.,240.,280.,288.,257.,257.       C  002:0000:0
     1,196.,196.,34*80./                                                       C  002:0000:0
  102 READ (5,903,END=101) PF,Y221,Y121,Y111,NUM,NTYR                        C  002:0000:0
                                                                                  FIB IS 0006 LONG
  903 FORMAT (4F10.0,2I5)                                                      C  002:0012:0
      TYR=NTYR                                                                 C  002:0012:0
      DELT=1.                                                                  C  002:0012:5
      K1=36./TYR                                                               C  002:0013:3
      K2=.0012/TYR                                                             C  002:0014:5
      K3=.6                                                                    C  002:0017:0
      CS1=240.                                                                 C  002:0019:3
      C1=.015                                                                  C  002:001A:2
      Q=9.*10.**8/TYR                                                          C  002:001C:3
      A=10**8                                                                  C  002:001F:2
      V=3.8*10.**9                                                             C  002:0021:1
      VS=10.**7                                                                C  002:0025:2
      DO 10 K=1,70                                                             C  002:0027:3
   10 M(K)=P(K)*10.**6.                                                        C  002:0029:0
      C011=1.-DELT*(K1*A+Q)/V                                                  C  002:002F:0
       C012=DELT*K2*A/V                                                        C  002:0032:1
       C021=DELT*(K1-K1*K3)*A/VS                                               C  002:0034:3
      C022=1.-DELT*K2*A/VS                                                     C  002:0037:4
      Y211=Y121                                                                C  002:003A:2
      NYEAR=1930                                                               C  002:003B:1
      Y1RT=SQRT(Y111)                                                          C  002:003C:1
      Y2RT=SQRT(Y221)                                                          C  002:003D:4
      WRITE (6,913)                                                            C  002:003F:1
                                                                                  FIB IS 0006 LONG
  913 FORMAT (1H1,T26,'EFFECTS OF VARIATIONS IN LOADING UPON PHOSPHORUS      C  002:0043:2
     1 MODEL ,BAYESIAN APPROACH')                                              C  002:0043:2
      WRITE (6,914) PF,NTYR                                                    C  002:0043:2
  914 FORMAT (1H ,T42,'PROBALITY FACTOR=',F4.2,5X,'TIME STEPS/YEAR=',         C  002:004B:2
     1 I4)                                                                     C  002:004B:2
      WRITE (6,915)                                                            C  002:004B:2
  915 FORMAT (1H ,T39,'LAKE',T52,'STANDARD',T66,'SEDIMENT',T80,              C  002:004F:2
     1 'STANDARD'/T29,'YEAR',T40,'CONC',T52,'DEVIATION',T68,'CONC',T80,     C  002:004F:2
     3 'DEVIATION',T93,'COVARIANCE'/T40,4('MG/L',10X),T93,'(MG/L)**2')       C  002:004F:2
      WRITE (6,918) NYEAR,C1,Y1RT,CS1,Y2RT,Y121                               C  002:004F:2
  918 FORMAT (1H ,T29,I4,5(4X,E10.3))                                          C  002:005D:2
      DO 100 I=1,NUM                                                           C  002:005D:2
```

```
      ZD=DELT *M(I)/V/TYR                                                       C  002:005E:0
       QM=(PF*M(I)*DELT)**2./V**2.                                              C  002:0061:0
      DO 50 J=1,NTYR                                                            C  002:0064:4
      C2=CO11*C1+CO12*CS1+ZD                                                    C  002:0066:0
       CS2=CO21*C1+CO22*CS1                                                     C  002:0068:5
      Y112=CO11*(CO11*Y111+CO12*Y211)+CO12*(CO11*Y121+CO12*Y221)+QM/TYR         C  002:006B:1
      Y122=CO21*(CO11*Y111+CO12*Y211)+CO22*(CO11*Y121+CO12*Y221)                C  002:0071:2
      Y222=CO21*(CO21*Y111+CO22*Y211)+CO22*(CO21*Y121+CO22*Y221)                C  002:0076:4
      C1=C2                                                                     C  002:0078:5
      CS1=CS2                                                                   C  002:007C:4
      Y111=Y112                                                                 C  002:007D:3
      Y121=Y122                                                                 C  002:007E:2
      Y211=Y122                                                                 C  002:007F:1
      Y221=Y222                                                                 C  002:0080:0
   50 CONTINUE                                                                  C  002:0080:5
      NYEAR=1930+I                                                              C  002:0083:0


      Y1RT=SQRT(Y111)                                                           C  002:0084:4
      Y2RT=SQRT(Y221)                                                           C  002:0086:1
      WRITE (6,918) NYEAR,C1,Y1RT,CS1,Y2RT,Y121                                 C  002:0087:4
  100 CONTINUE                                                                  C  002:0096:2
      GO TO 102                                                                 C  002:0098:3
  101 CONTINUE                                                                  C  002:0099:0
      CALL EXIT                                                                 C  002:0099:0
      END                                                                       C  002:0099:5
    002:009D:1  IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 002:0000
                                                                                 SEGMENT 002 IS 00AE LONG
```

EFFECTS OF VARIATIONS IN LOADING UPON PHOSPHORUS MODEL ,BAYESIAN APPROACH

PROBALITY FACTOR=0.30   TIME STEPS/YEAR= 73

YEAR	LAKE CONC MG/L	STANDARD DEVIATION MG/L	SEDIMENT CONC MG/L	STANDARD DEVIATION MG/L	COVARIANCE (MG/L)**2
1930	.150E-01	0.	.240E+03	0.	0.
1931	.160E-01	.221E-02	.239E+03	.197E+00	.309E-03
1932	.163E-01	.231E-02	.239E+03	.404E+00	.529E-03
1933	.163E-01	.232E-02	.238E+03	.569E+00	.609E-03
1934	.163E-01	.232E-02	.238E+03	.704E+00	.637E-03
1935	.163E-01	.232E-02	.237E+03	.818E+00	.649E-03

1936	.163E-01	.232E-02	.237E+03	.917E+00	.656E-03
1937	.163E-01	.232E-02	.236E+03	.100E+01	.661E-03
1938	.163E-01	.232E-02	.236E+03	.108E+01	.666E-03
1939	.163E-01	.232E-02	.235E+03	.116E+01	.670E-03
1940	.163E-01	.232E-02	.235E+03	.122E+01	.675E-03
1941	.187E-01	.308E-02	.235E+03	.130E+01	.938E-03
1942	.195E-01	.314E-02	.235E+03	.140E+01	.113E-02
1943	.197E-01	.315E-02	.235E+03	.150E+01	.120E-02
1944	.198E-01	.315E-02	.235E+03	.159E+01	.122E-02
1945	.198E-01	.315E-02	.235E+03	.168E+01	.124E-02
1946	.198E-01	.315E-02	.235E+03	.177E+01	.125E-02
1947	.198E-01	.315E-02	.235E+03	.184E+01	.126E-02
1948	.198E-01	.315E-02	.235E+03	.192E+01	.126E-02
1949	.198E-01	.315E-02	.235E+03	.199E+01	.127E-02
1950	.198E-01	.315E-02	.235E+03	.206E+01	.128E-02
1951	.229E-01	.409E-02	.235E+03	.213E+01	.172E-02
1952	.239E-01	.417E-02	.236E+03	.223E+01	.203E-02
1953	.242E-01	.418E-02	.236E+03	.234E+01	.215E-02
1954	.243E-01	.418E-02	.237E+03	.244E+01	.220E-02
1955	.243E-01	.418E-02	.238E+03	.254E+01	.222E-02
1956	.243E-01	.418E-02	.238E+03	.264E+01	.224E-02
1957	.256E-01	.455E-02	.239E+03	.273E+01	.246E-02
1958	.339E-01	.702E-02	.241E+03	.287E+01	.441E-02
1959	.442E-01	.957E-02	.243E+03	.314E+01	.826E-02
1960	.551E-01	.121E-01	.248E+03	.357E+01	.138E-01
1961	.647E-01	.142E-01	.253E+03	.417E+01	.202E-01
1962	.690E-01	.148E-01	.260E+03	.484E+01	.243E-01
1963	.656E-01	.134E-01	.267E+03	.546E+01	.235E-01
1964	.647E-01	.133E-01	.273E+03	.597E+01	.223E-01
1965	.551E-01	.105E-01	.278E+03	.637E+01	.178E-01
1966	.523E-01	.101E-01	.282E+03	.665E+01	.148E-01
1967	.335E-01	.501E-02	.285E+03	.683E+01	.890E-02
1968	.278E-01	.423E-02	.286E+03	.689E+01	.516E-02
1969	.261E-01	.414E-02	.286E+03	.690E+01	.386E-02
1970	.256E-01	.413E-02	.286E+03	.689E+01	.346E-02
1971	.255E-01	.413E-02	.287E+03	.688E+01	.333E-02
1972	.254E-01	.413E-02	.287E+03	.687E+01	.329E-02
1973	.254E-01	.413E-02	.287E+03	.685E+01	.327E-02
1974	.254E-01	.413E-02	.287E+03	.684E+01	.326E-02
1975	.254E-01	.413E-02	.287E+03	.683E+01	.326E-02
1976	.254E-01	.413E-02	.288E+03	.681E+01	.325E-02
1977	.255E-01	.413E-02	.288E+03	.680E+01	.325E-02
1978	.255E-01	.413E-02	.288E+03	.679E+01	.324E-02
1979	.255E-01	.413E-02	.288E+03	.678E+01	.324E-02
1980	.255E-01	.413E-02	.289E+03	.676E+01	.323E-02

APPENDIX B

PROGRAM SALTEZ-SUPPLEMENTARY DOCUMENTATION

Input Requirements

The input requirements for the program SALTEZ are presented in Table B-1. Abbreviated units are summarized in Table B-2. Only nine types of cards are required to supply the data necessary for a SALTEZ run. Cards 1 through 4 are required for each simulation. Card 3 contains a provision for including or bypassing calculation of skewness. Cards 4 and 5 define default values for proportion variance and skewness terms associated with the stochastic salinity options. Card 5 should be omitted from all runs in which the skewness calculations are bypassed.

Proportional variance and skewness terms were utilized to reduce external calculations. This approach has the advantage of permitting definition of system factors that may be used for any stochastic input term lacking specific information. It also permits the loads to be redefined without external redefinition of individual variances and skews. Equation B-1 defines the relationship between the proportional variance factor (relative standard deviation or RSD) and the variance, σ_C^2, of a random variable, $\tilde{C}$ with mean C.

$$\sigma_C^2 = (RSD * C) ** 3 \quad . . . . \quad (B\text{-}1)$$

In a similar fashion the proportional skewness factor (PSF) is defined by Equation B-2 where τ_C is the skewness of $\tilde{C}$.

$$\tau_C = (PSF * C) ** 3 \quad . . . . \quad (B\text{-}2)$$

The RSD and PSF of a specific input term is defined by the system default values (Cards 4 and 5), unless specific value(s) are provided by inclusion of Cards 7 and 8 following the associated input information Card 6. An option is also available for the direct input of variance and skewness values on Cards 7 and 8. The program automatically scans the card following each node or input information Card 6 to determine its type, Figure B-1, the SALTEZ flow chart, illustrates the scanning procedure followed by the program. If the card following a Card 6 is not another Card 6 or system termination Card 9 it is assumed to be a RSD Card 7. The information on the RSD Card 7 will supersede the system RSD values for the input defined by the immediate prior Card 6 only. Unless skewness calculations have been bypassed, a PSF Card 8 must follow each RSD Card 7. This information similarly supersedes system PSF values. This procedure permits the user to add or delete RSD and PSF cards for specific inputs without otherwise altering the data deck.

One Card 6 must be included for each node or input in the system. For inputs, each Card 6 defines the associated node, title, flow option, salinity option, and means for that input. Information related to uncertainty and skewness must be defined by a following RSD Card 7 and PSF Card 8 unless system default values are to be used. The structure of the system is defined by the node information cards. The node cards are automatically distinguished from input cards by specification of the downstream node. Only the node card number, the downstream code number, and title should be specified on a node information Card 6. Additional information, RSD Card 7 and PSF Card 8, may result in error.

Node information cards must be structured upstream to downstream. All headwater nodes must precede their junction node with other headwaters. Input information cards must precede their associated node information card. There are no limits on the number of branches or headwaters as long as the total number of nodes is less than 99. The program SALTEZ automatically checks for improperly sequenced nodes (see Figure B-1). Improperly placed input information are not automatically deleted. Misplacement of an input information Card 6 will cause error in the node accumulation.

Program Output

Output from the program SALTEZ consists of five tables. This appendix contains sample SALTEZ outputs. The first table echoes the input data. All information contained on input and node information Cards 6, RSD Cards 4 and 7, and PSF Cards 5 and 8 are listed when this optional table is requested (IOPE = 0).

Table B-1. Input requirements for program SALTEZ.

Card	Columns	Format	Name	Description
CARD 1:	Mandatory all runs			
1	1-80	20A4	TITL(I)	Project title
CARD 2:	Mandatory all runs			
2	1-80	20A4	STITL(I)	Project subtitle
CARD 3:	Mandatory all runs			
3_1	1-2	I2	IOPW	Write option, ≥ 1 output information for all inputs
3_2	3-4	I2	IOPE	Option for echoing input data = 0, echo input data = 1, do not echo input data
3_3	7-8	I2	ISKW	Option for computing skewness = 0, skewness not calculated = 1, skewness computed
CARD 4:	Mandatory all runs			
4_1	41-48	F8.0	ASLD	System default value for the relative standard deviation associated with salt load when IOPS(IP) = 1. (dimensionless)
4_2	49-56	F8.0	ASCN	System default value for the relative standard deviation associated with the salt concentration when IOPS(IP) = 2. (dimensionless)
4_3	57-64	F8.0	ABA	System default value for the relative standard deviation associated with the area for IOPS(IP) = 3. (dimensionless)
4_4	70-74	F5.0	ASLF	System default value for the relative standard deviation of salt load factor for IOPS(IP) = 3 or efficiency for IOPS(IP) = 5. (dimensionless)
4_5	75-80	F6.0	APHI	System default value for the relative standard deviation of base leaching factor for IOPS(IP) = 5. (dimensionless)
CARD 5:	Optional; include when ISKW = 1			
5_1	41-48	F8.0	TSLD	System default value for the proportional skewness factor associated with the salt load when IOPS(IP) = 1. (dimensionless)
5_2	49-56	F8.0	TSCN	System default value for the proportional skewness factor associated with the salt concentration when IOPS(IP) = 2. (dimensionless)
5_3	57-64	F8.0	TBA	System default value for the proportional skewness factor associated with the area for IOPS(IP) = 3. (dimensionless)
5_4	70-74	F5.0	TSLF	System default value for the proportional skewness factor associated with the salt load factor for IOPS(IP) = 3 or efficiency for IOPS(IP) = 5. (dimensionless)
5_5	75-80	F6.0	TPHI	System default value for the proportional skewness factor associated with the base leaching factor for IOPS(IP) = 5. (dimensionless)
CARD 6:	Mandatory for each input or node			
6_1	2-3	I2	INIP(IP)	Node code number associated with input IP
6_2	4-5	I2	IDNDS(IN)	Blank except for node card where code number of next downstream node is required
6_3	6-29	6A4	DESC(IP,6)	24 character alpha numeric title for input IP
6_4	30	I1	STA(IP)	State code number associated with input IP
6_5	31	I1	IOPF(IP)	Flow option at input IP ≤ 2: Flow read in ≥ 3: Flow computed as product of area and consumptive use factor
6_6	32	I1	IOPS(IP)	Salt option at input IP 1: Salt load read in, 2: Salt load computed as product of concentration and inflow 3: Salt load computed as product of area and salt load factor 4: Salt depletion computed as product of instream concentration and diversion flow 5: Salt load computed as a function of flow, irrigation efficiency and base leaching factor
6_7	33-40	F8.0	Q(IP)	Flow associated with input IP(TAF/Y). Use when IOPF(IP) ≤ 2.
6_8	41-48	F8.0	SLD(IP)	Salt load associated with input IP(TT/Y). Use when IOPS(IP) = 1.

Table B-1. Continued.

Card	Columns	Format	Name	Description
6_9	49-56	F8.0	SCN(IP)	Salt concentration for input IP (T/AF). Use when IOPS(IP) = 2.
6_{10}	57-64	F8.0	A(IP)	Area for input IP (TA). Use when IOPS(IP) = 3 or IOPF(IP) = 3.
6_{11}	65-69	F5.0	CUF(IP)	Consumptive use factor for input IP (AF/A/Y). Use when IOPF = 3.
6_{12}	70-74	F5.0	SLF(IP)	Salt load factor (T/A/Y) for input IP when IOPS(IP) = 3 or efficiency (dimensionless) when IOPS(IP) = 5.
6_{13}	75-80	F6.0	PHI(IP)	Base leaching factor for input IP (T/A/F)
CARD 7:	Optional for each input or node. Use to override system default values for relative standard deviations (card 4)			
7_1	2-3	I2	IAN(IP)	Input options for card 7 and card 8 associated with input IP. IAN(IP) = 0: Override factor will be relative standard deviations for variance modification and proportion skewness factors for skew calculations IAN(IP) = 1: Override factor will be variance and skew
7_2	41-48	F8.0	SLDEZ(IP)	Override factor for variance associated with salt load when IOPS(IP) = 1.
7_3	49-56	F8.0	SCNEZ(IP)	Override factor for variance associated with salt concentration when IOPS(IP) = 2.
7_4	57-64	F8.0	AEZ(IP)	Override factor for variance associated with area when IOPS(IP) = 3.
7_5	70-74	F5.0	SLFEZ(IP)	Override factor for variance of salt load factor when IOPS(IP) = 3 or efficiency when IOPS(IP) = 5.
7_6	75-80	F6.0	PHIEZ(IP)	Override factor for base leaching factor when IOPS(IP) = 5.
CARD 8:	Optional for each input or node; include with card 7 unless ISKW = 0.			
8_1	41-48	F8.0	SLDTZ(IP)	Override factor for skew of salt load when IOPS(IP) = 1.
8_2	49-56	F8.0	SCNTZ(IP)	Override factor for skew associated with salt concentration when IOPS(IP) = 2.
8_3	57-64	F8.0	ATZ(IP)	Override factor for skew associated with area when IOPS(IP) = 3.
8_4	70-74	F5.0	SLFTZ(IP)	Override factor for skew of salt load when IOPS(IP) = 3 or efficiency when IOPS(IP) = 5.
8_5	75-80	F6.0	PHITZ(IP)	Override factor for skew of the base leaching factor when IOPS(IP) = 5.
CARD 9:	Mandatory for all runs			
9_1	2-3	I2	INIP(IP)	Flag signaling end of system

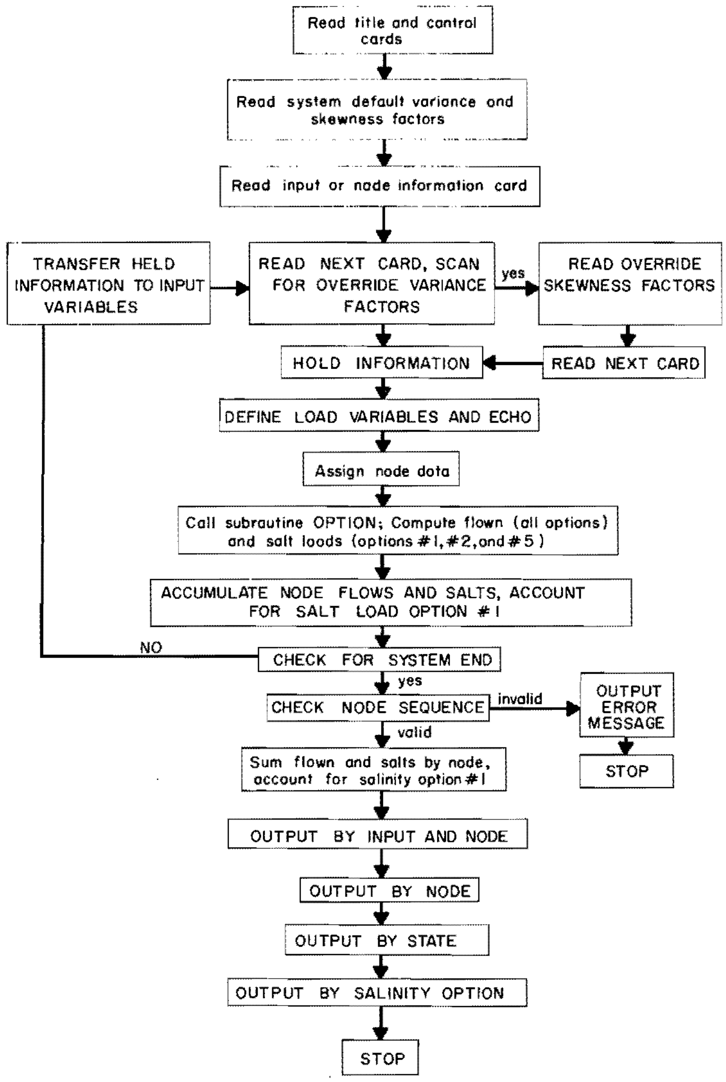

Figure B-1. Flow chart for SALTEZ.

Table B-2. Abbreviated units of SALTEZ parameters.

Abbreviated Unit	Description
T/A/F	Tons per acre per foot
TAF/Y	Thousands of acre-feet per year
TT/Y	Thousands of tons per year
T/AF	Tons per acre foot
TA	Thousands of acres
AF/A/Y	Acre feet per acre per year
T/A/Y	Tons per acre per year

The remaining three tables summarize the results of a SALTEZ simulation by node, state, and salinity options. The first table lists accumulated flows and salt loads for each node of the system. This table also lists the standard deviation and skews associated with the salinity load. The next table lists flows and salt loads contributed to the system by each state. Similarly, the last table presents the flows, salt loads, variance, and net skew contributed to the entire system by salinity option.

Variable Listing

The variable listing for the program SALTEZ is presented in Table B-3. A complete program listing is provided in this appendix.

Table B-3. Definition of program SALTEZ variables.

Name	Description
TITL (20)	80 character alpha numeric run title
STITL (20)	80 character alpha numeric run subtitle
IP	Input sequence number $\leq$ 500
ISTA(IP)	State identification number of load IP, ISTA(IP) $\leq$ 20
DESC(IP,6)	24 character alpha numeric input title
IOPF(IP)	Flow option for input IP IOPF(IP) $\leq$ 2: Q(IP) read in IOPF(IP) = 3: Q(IP) defined as product of consumptive use factor, CUF(IP) and area, A(IP)
IOPS(IP)	Salinity option for input IP IOPS(IP) = 1: SLD(IP) read in IOPS(IP) = 2: SLD(IP) defined as product of flow, Q(IP) and salt concentration, SCN(IP) IOPS(IP) = 3: SLD(IP) defined as product of area, A(IP) and salt load factor, SLF(IP) IOPS(IP) = 4: SLD(IP) defined as product of flow, Q(IP) and calculated intermediate node concentration IOPS(IP) = 5: SLD(IP) calculated as a function of efficiency factor, SLF(IP), flow, Q(IP), and base leaching factor, PHI(IP)
IN	Node sequence number $\leq$ 99
ICIN(IN)	Identification number of node IN
INIC(IC)	Node sequence number, IN, associated with the node identification number IC
IDLIM(IN,I)	I = 1: Lowest input sequence number, IP, associated with node, IN I = 2: Highest input sequence number, IP, associated with node, IN, represents the null set input of node IN
IDNDS(IN)	Identification number of next downstream node receiving flow from node IN
QN(IN)	Accumulated flow from inputs of reach, IN, to node IN(TAF/Y)
SN(IN)	Accumulated salt load from all inputs of reach, IN, to node IN (TT/Y)
Q(IP)	Flow associated with input, IP (TAF/Y)
SCN(IP)	Salt concentration for input, IP. Used with IOPS(IP) = 2. (T/AF)
SLD(IP)	Salt load associated with input, IP(TT/Y)
A(IP)	Area associated with project, IP. Used for IOPF(IP) = 3 and/or IOPS(IP) = 3. (TA)
CUF(IP)	Consumptive use factor associated with input, IP. Used with IOPF(IP) = 3
SLF(IP)	Salt load factor associated with input, IP, for IOPS(IP) = 3 (T/A/Y) or efficiency for IOPS(IP) = 5 (dimensionless)
QNS(IN)	Accumulated flow at node IN from all sources (TAF/Y)
SNS(IN)	Accumulated salt load at node IN from all sources (TT/Y)

Table B-3. Continued.

Name	Description
STAS(IS)	Accumulated salt loads for inputs in state IS. (TT/Y)
IC	Code identification number of a node
PHI(IP)	Base leaching factor associated with input IP (T/AF)
SLDEZ(IP)	Relative standard deviation associated with SLD(IP) when IOPS(IP) = 1 (dimensionless)
SCNEZ(IP)	Relative standard deviation associated with SCN(IP) (dimensionless)
SLFEZ(IP)	Relative standard deviation associated with SLF(I) (dimensionless)
PHIEZ(IP)	Relative standard deviation associated with PHI(IP) (dimensionless)
SVAR(IP)	Variance of input IP (TT^2/Y^2)
SUMVAR(IN)	Accumulated variance from reach IN (TT^2/Y^2)
AEZ(IP)	Relative standard deviation associated with A(IP) for IOPS(IP) = 3 (dimensionless)
TOTVAR(IN)	Accumulated variance at node IN from all sources (TT^2/Y^2)
BESC(6)	Dummy array for transfer of DESC(IP,6)
ASLD	System default value for SLDEZ(IP)
ASCN	System default value for SCNEZ(IP)
ABA	System default value for AEZ(IP)
ASLF	System default value for SLFEZ(IP)
APHI	System default value for PHIEZ(IP)
SLDTZ(IP)	Proportional skewness factor associated with SLD(IP) when IOPS(IP) = 1 (dimensionless)
SCNTZ(IP)	Proportional skewness factor associated with SCN(IP) (dimensionless)
SLFTZ(IP)	Proportional skewness factor associated with SLF(IP) (dimensionless)
PHITZ(IP)	Proportional skewness factor associated with PHI(IP)
SKEW(IP)	Skewness of input IP (TT^3/Y^3)
SUMSKW(IN)	Accumulated variance from reach IN (TT^3/Y^3)
ATZ(IP)	Proportional skewness factor associated with A(IP) (dimensionless)
TOTOSKW(IN)	Accumulated skewness at node IN from all sources (TT^3/Y^3)
TSLD	System default value for SLDTZ(IP)
TSCN	System default value for SCNTZ(IP)
TBA	System default value for ATZ(IP)
TSLF	System default value for SLFTZ(IP)
TPHI	System default value for PHITZ(IP)
IAN(IP)	Override option for variance and skew IAN(IP) = 0: Proportional variance and skew factors are input IAN(IP) = 1: Variance and skew factors are input directly

PROGRAM SALTEZ LISTING

```
B6700   F O R T R A N   C O M P I L A T I O N   M A R K   2.9.190    FRIDAY, 07/28/78    11:51 AM

                                        S A L T E Z
                                        = = = = = =

                                                                        START OF SEGMENT 002
                                                                    FORMAT SEGMENT IS 0007 LONG
                                                                    FORMAT SEGMENT IS 0003 LONG
      COMMON  TITL(20),RIV(50,10),IRN(99),ISTA(500),DESC(500,6)       00000000  C  002:0000:0
     *       ,IDPF(500),IDPS(500),ICIN(99),INIC(99),IDLIM(99,2)       00000010  C  002:0000:0
     *       ,IDNDS(99),QN(99),SN(99),Q(500),SCN(500),SLD(500),A(500) 00000020  C  002:0000:0
     *       ,CUF(500),SLF(500),QNS(99),SNS(99),STITL(20),INIP(500)   00000030  C  002:0000:0
     * ,PHI(500),SLDEZ(500),SCNEZ(500),SLFEZ(500),PHIEZ(500),SVAR(500)          C  002:0000:0
     * ,SUMVAR(99),AEZ(500),TOTVAR(99),BESC(6),SLDTZ(500),SCNTZ(500),          C  002:0000:0
     * SLFTZ(500),PHITZ(500),SKEW(500),SUMSKW(99),ATZ(500),TOTSKW(99),          C  002:0000:0
     * ISKW,IAN(500)                                                            C  002:0000:0
      DIMENSION STAQ(20),STAS(20)                                     00000090  C  002:0000:0
       DIMENSION DSALT(5),DFLOW(5),DVAR(5),DSKEW(5)                             C  002:0000:0
      READ(5,100) (TITL(I),I=1,20)                                    00000100  C  002:0000:0
                                                                            FIB IS 0006 LONG
      READ(5,100) (STITL(I),I=1,20)                                   00000110  C  002:000C:2
  100 FORMAT(20A4)                                                    00000120  C  002:0019:2
  102 FORMAT(1H // 1H ,20A4)                                          00000150  C  002:0019:2
      READ(5,104) IOPW,IOPE,NRIV,ISKW                                           C  002:0019:2
  104 FORMAT(4I2)                                                               C  002:0027:2
       READ (5,901) ASLD,ASCN,ABA,ASLF,APHI                                     C  002:0027:2
       IF (ISKW.NE.1)GO TO 771                                                  C  002:0036:2
       READ (5,901) TSLD,TSCN,TBA,TSLF,TPHI                                     C  002:0038:0
  771  CONTINUE                                                                 C  002:0047:2
      GOTO 200                                                        00000690  C  002:0047:2
      DO 2 I=1,NRIV                                                   00000700  C  002:0047:5
      READ(5,108) IRIV,(RIV(IRIV,J),J=1,10)                           00000710  C  002:0049:0
  108 FORMAT(I2,10A4)                                                 00000720  C  002:0059:2
      WRITE(6,110) IRIV,(RIV(IRIV,J),J=1,10)                          00000730  C  002:0059:2
                                                                            FIB IS 0006 LONG
  110 FORMAT(1H ,I2,1X,10A4)                                          00000740  C  002:0068:2
    2 CONTINUE                                                        00000750  C  002:0068:2
  200 CONTINUE                                                        00000760  C  002:006A:3
       IFISH=0                                                                  C  002:006A:3
      DO 5 I=1,99                                                     00000770  C  002:006B:1
      QN(I)=0.0                                                       00000780  C  002:006C:0
       SUMSKW(I)=0.                                                             C  002:006D:5
       SUMVAR(I)=0.0                                                            C  002:006F:4
      SN(I)=0.0                                                       00000790  C  002:0071:3
      QNS(I)=0.0                                                      00000800  C  002:0073:2
      SNS(I)=0.0                                                      00000810  C  002:0075:1
       TOTSKW(I)=0.                                                             C  002:0077:0
       TOTVAR(I)=0.                                                             C  002:0078:5
    5 CONTINUE                                                        00000820  C  002:007A:4
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      IP=1                                                                 C   002:007C:5
      IN=1                                                                 C   002:007D:3
      IOLIM(1,1)=1                                                         C   002:007E:1
      IF(IOPE.LT.1) CALL HEAD1(TITL,STITL)                                 C   002:007F:3
       WRITE (6,906) ASLD,ASCN,ABA,ASLF,APHI                               C   002:0083:0
  906  FORMAT(1H ,T10,'DEFAULT RELATIVE DEVIATIONS',T64,F5.2,T72,F5.2,     C   002:008F:2
     * T80,F5.2,T97,F5.2,T106,F5.2)                                        C   002:008F:2
       IF (ISKW.NE.1)GO TO 772                                             C   002:008F:2
       WRITE (6,941) TSLD,TSCN,TBA,TSLF,TPHI                               C   002:0091:0
  941  FORMAT (1H ,T2,'DEFAULT PROPORTIONAL SKEWNESS FACTORS',T64,F5.2,    C   002:009D:2
     * T72,F5.2,T80,F5.2,T97,F5.2,T106,F5.2)                               C   002:009D:2
  772  CONTINUE                                                            C   002:009D:2

      READ(5,112) IZ1,IZ2,IZ3,(DESC(1,I),I=1,6),ISTA(1),IOPF(1)            C   002:009D:2
     *           ,IOPS(1),Q(1),SLD(1),SCN(1),A(1),CUF(1),SLF(1),PHI(1)     C   002:00B2:3
 7222  READ (5,112) IB1,IB2,IB3,(BESC(I),I=1,6),BISTA,IBOPF,IBOPS,BQ,      C   002:00C0:2
     * BSLD,BSCN,BBA,BCUF,BSLF,BPHI                                        C   002:00E6:0
       IF (IB2.NE.0) GO TO 7223                                            C   002:00F4:2
       IAN(IP)=IB1                                                         C   002:00F5:3
      IFISH=1                                                              C   002:00F7:3
       SLDEZ(IP)=BSLD                                                      C   002:00F8:1
       SCNEZ(IP)=BSCN                                                      C   002:00FA:1
       AEZ(IP)=BBA                                                         C   002:00FC:1
       SLFEZ(IP)=BSLF                                                      C   002:00FE:1
       PHIEZ(IP)=BPHI                                                      C   002:0100:1
C      *******  START PART ONE                                             C   002:0102:1
C      *******  TEMPORARY MODIFICATION TO PROGRAM SALTEZ FOR APPLICATION   C   002:0102:1
C      *******  TO COLORADO RIVER ASSESSMENT STUDY PROCEDURE,TO DELETE     C   002:0102:1
C      *******  REMOVE CARDS HERE AND IN SUBROUNTINE OPTION                C   002:0102:1
       IF (IOPS(IP).NE.5)GO TO 170                                         C   002:0102:1
       PHIEZ(IP)=BPHI*((1.-SLF(IP))/SLF(IP)*CUF(IP)*A(IP)-                 C   002:0104:3
     $ (1.-BSLF)/BSLF*BCUF*BBA)**2.                                        C   002:010C:3
       CCUF=BCUF                                                           C   002:010F:3
       CPHI=BPHI                                                           C   002:0110:2
  170 CONTINUE                                                             C   002:0111:1
C      *******  END OF PART ONE                                            C   002:0111:1
       IF (ISKW.NE.1)GO TO 773                                             C   002:0111:1
       READ (5,901) SLDTZ(IP),SCNTZ(IP),ATZ(IP),SLFTZ(IP),PHITZ(IP)        C   002:0112:5
  773  CONTINUE                                                            C   002:0128:2
       READ (5,112) IB1,IB2,IB3,(BESC(I),I=1,6),BISTA,IBOPF,IBOPS,BQ,      C   002:0128:2
     * BSLD,BSCN,BBA,BCUF,BSLF,BPHI                                        C   002:0141:0
       GO TO 7224                                                          C   002:014F:2
 7223  CONTINUE                                                            C   002:014F:5
       SLDEZ(IP)=ASLD                                                      C   002:014F:5
       SCNEZ(IP)=ASCN                                                      C   002:0151:5
       AEZ(IP)=ABA                                                         C   002:0153:5
       SLFEZ(IP)=ASLF                                                      C   002:0155:5
       PHIEZ(IP)=APHI                                                      C   002:0157:5
       IF (ISKW.NE.1)GO TO 774                                             C   002:0159:5
       SLDTZ(IP)=TSLD                                                      C   002:015B:3
       SCNTZ(IP)=TSCN                                                      C   002:015D:3
       ATZ(IP)=TBA                                                         C   002:015F:3
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      SLFTZ(IP)=TSLF                                                        C  002:0161:3
      PHITZ(IP)=TPHI                                                        C  002:0163:3
  774 CONTINUE                                                              C  002:0165:3
 7224 CONTINUE                                                              C  002:0165:3
 901  FORMAT (40X,3F8.0,5X,F5.0,F6.0)                                       C  002:0165:3
  112 FORMAT(I1,2I2,6A4,3I1,4F8.0,2F5.0,F6.0)                      00000880 C  002:0165:3
      INIP(IP)=IZ2                                                 00000890 C  002:0165:3
    1  IF(IOPE.GE.1) GO TO 7227                                             C  002:0167:3
      WRITE(6,114) IP, IZ1,IZ2,IZ3,(DESC(IP,I),I=1,6),ISTA(IP),IOPF(IP)     C  002:0168:4
     *          ,IOPS(IP),Q(IP),SLD(IP),SCN(IP),A(IP),CUF(IP),SLF(IP) 00000930 C  002:017E:0
     *          ,PHI(IP)                                           00000940 C  002:018F:3
      IF (IFISH.NE.1) GO TO 7227                                            C  002:0195:2
      IF(IAN(IP).NE.0)GO TO 555                                             C  002:0196:3
      WRITE (6,902) SLDEZ(IP),SCNEZ(IP),AEZ(IP),SLFEZ(IP),PHIEZ(IP)         C  002:0198:4
  902 FORMAT (1H ,T10,'OVERRIDE RELATIVE DEVIATIONS',T64,F5.2,T72,F5.2,     C  002:01AA:2
     * T80,F5.2,T97,F5.2,T106,F5.2)                                         C  002:01AA:2
      GO TO 556                                                             C  002:01AA:2
  555 CONTINUE                                                              C  002:01AA:5
C     ******** START PART THREE                                             C  002:01AA:5
      WRITE (6,1956) SLDEZ(IP),SCNEZ(IP),AEZ(IP),CCUF,SLFEZ(IP),            C  002:01AA:5
     * CPHI                                                                 C  002:0139:0
 1956 FORMAT (1H ,T20,'SPECIAL VARIANCE VALUES',T61,F8.1,T69,F8.3,T77,      C  002:01BD:2
     * F8.4,T86,F8.3,T94,F8.3,T103,F8.4)                                    C  002:01BD:2
      CCUF=0.                                                               C  002:01BD:2
      CPHI=0.                                                               C  002:01BE:0
      GO TO 556                                                             C  002:01BE:4
C     ********* END PART THREE                                              C  002:01BF:1
      WRITE (6,956)SLDEZ(IP),SCNEZ(IP),AEZ(IP),SLFEZ(IP),PHIEZ(IP)          C  002:01BF:1
  957 FORMAT (1H ,T25,'OVERRIDE SKEW',T61,F8.4,T69,F8.4,T77,F8.4,           C  002:01D1:2
     * T94,F8.4,T103,F8.4)                                                  C  002:01D1:2
  956 FORMAT (1H ,T20,'OVERRIDE VARIANCE',T61,F8.1,T69,F8.4,T77,F8.4,       C  002:01D1:2
     * T94,F8.4,T103,F8.4)                                                  C  002:01D1:2
  556 CONTINUE                                                              C  002:01D1:2
      IF (ISKW.NE.1)GO TO 775                                               C  002:01D1:2
  942 FORMAT (1H ,T2,'OVERRIDE PROPORTIONAL SKEWNESS FACTOR',T64,           C  002:01D3:0
     * F5.2,T72,F5.2,T80,F5.2,T97,F5.2,T106,F5.2)                           C  002:01D3:0
      IF (IAN(IP).NE.0) GO TO 557                                           C  002:01D3:0
      WRITE (6,942) SLDTZ(IP),SCNTZ(IP),ATZ(IP),SLFTZ(IP),PHITZ(IP)         C  002:01D5:1
      GO TO 558                                                             C  002:01E7:2
  557 CONTINUE                                                              C  002:01E7:5
      WRITE (6,957) SLDTZ(IP),SCNTZ(IP),ATZ(IP),SLFTZ(IP),PHITZ(IP)         C  002:01E7:5
  558 CONTINUE                                                              C  002:01F9:2
  775 CONTINUE                                                              C  002:01F9:2
      IFISH=0                                                               C  002:01F9:2
 7227 CONTINUE                                                              C  002:01FA:0
  114 FORMAT(1H0,I3,2X,I1,3X,I2,2X,I2,2X,6A4,2X,I1,1X,I1,1X,I1,F10.1 00000950 C  002:01FA:0
     *         ,F9.1,F8.3,F8.1,1X,2F8.3,F10.4)                     00000960 C  002:01FA:0
      IF(IZ3.LT.1) GOTO 4                                          00000970 C  002:01FA:0
C                                                                  00000980 C  002:01FB:1
C                    ASSIGN NODE DATA                              00000990 C  002:01FB:1
C                                                                  00001000 C  002:01FB:1
      ICIN(IN)=IZ2                                                 00001010 C  002:01FB:1
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      INIC(IZ2)=IN                                          C  002:01FD:1
      IDLIM(IN,2)=IP                                        C  002:01FF:1
      IDNDS(IN)=IZ3                                         C  002:0201:1
      IDLIM(IN+1,1)=IP+1                                    C  002:0203:1
      IRN(IN)=IZ1                                           C  002:0205:4
      IN=IN+1                                               C  002:0207:4
      GOTO 50                                               C  002:0209:0
C                                                           C  002:0209:3
C                ACCUMULATE NODE FLOWS AND SALT             C  002:0209:3
C                                                           C  002:0209:3
    4 CALL OPTION(IP)                                       C  002:0209:3
      IF(IOPS(IP).EQ.4) GOTO 50                             C  002:020A:4
      QN(IN)=QN(IN)+Q(IP)                                   C  002:020D:0
      SN(IN)=SN(IN)+SLD(IP)                                 C  002:0210:3
  50   CONTINUE                                             C  002:0214:0
       IF (ISKW.NE.1)GO TO 776                              C  002:0214:0
       IF (IOPS(IP).EQ.1)SKEW(IP)=(SLDTZ(IP)*SLD(IP))**3.   C  002:0215:4
       IF (IOPS(IP).EQ.1.AND.IAN(IP).NE.0)SKEW(IP)=SLDTZ(IP) C  002:021D:0
       SUMSKW(IN)=SUMSKW(IN)+SKEW(IP)                       C  002:0224:0
  776  CONTINUE                                             C  002:0227:3
       IF(IOPS(IP).EQ.1)SVAR(IP)=(SLDEZ(IP)*SLD(IP))**2.    C  002:0227:3
       IF (IOPS(IP).EQ.1.AND.IAN(IP).NE.0)SVAR(IP)=SLDEZ(IP) C  002:022E:3
       SUMVAR(IN)=SUMVAR(IN)+SVAR(IP)                       C  002:0235:3
       IP=IP+1                                              C  002:0239:0
       INIP(IP)=IB2                                         C  002:023A:2
       IF (IB2.LT.1)GO TO 7225                              C  002:023C:0
       IZ1=IB1                                              C  002:023D:1
       IZ2=IB2                                              C  002:023E:0
       IZ3=IB3                                              C  002:023E:5
       DO 7226 JB=1,6                                       C  002:023F:4
 7226  BFSC(IP,JB)=BFSC(JB)                                 C  002:0241:0
       ISTA(IP)=BISTA                                       C  002:0247:2
       IOPF(IP)=IBOPF                                       C  002:0249:3
       IOPS(IP)=IBOPS                                       C  002:024B:3
       Q(IP)=BQ                                             C  002:024D:3
       SLD(IP)=BSLD                                         C  002:024F:3
       SCN(IP)=BSCN                                         C  002:0251:3
       A(IP)=BBA                                            C  002:0253:3
       CUF(IP)=BCUF                                         C  002:0255:3
       SLF(IP)=BSLF                                         C  002:0257:3
       PHI(IP)=BPHI                                         C  002:0259:3
       GO TO 7222                                           C  002:025B:3
 7225  CONTINUE                                             C  002:025C:0
C                                                           C  002:025C:0
C                CHECK NODE SEQUENCE                        C  002:025C:0
C                                                           C  002:025C:0
      NPOINT=IP-1                                           C  002:025C:0
      NNODE=IN-1                                            C  002:025D:2
      IF(ICIN(1).GE.IDNDS(1).AND.NNODE.NE.1) GOTO 67        C  002:025E:4
      IF(NNODE.EQ.1) GOTO 77                                C  002:0261:5
      DO 60 IN=2,NNODE                                      C  002:0263:0
      JEND=IN-1                                             C  002:0264:0
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      DO 70 J=1,JEND                                                        C  002:0265:2
      IF(ICIN(IN).LE.ICIN(J)) GOTO 67                                       C  002:0266:0
   70 CONTINUE                                                              C  002:0269:2
      IF(ICIN(IN).GE.IDNOS(IN)) GOTO 65                                     C  002:0268:3
   60 CONTINUE                                                              C  002:026E:5
      GOTO 77                                                               C  002:0271:0
   65 IF(IN.EQ.NNODE) GO TO 77                                              C  002:0271:3
   67 WRITE(6,78) ICIN(IN)                                                  C  002:0272:5
   78 FORMAT(1H0,'NODE ',I2,'  IS OUT OF SEQUENCE OR FEEDS AN UPSTREAM'     C  002:027A:2
     *       ,' NODE')                                                      C  002:027A:2
      STOP                                                                  C  002:027A:2
   77 CONTINUE                                                              C  002:027B:1
      IF(IOPW.LE.2)                                                         C  002:027B:1
     *CALL HEAD(TITL,STITL)                                                 C  002:027C:0
C                                                                           C  002:027E:5
C                   SUM FLOW AND SALT AT NODE IN (ACCOUNT FOR IOPS=4)       C  002:027E:5
C                                                                           C  002:027E:5
      DO 10 IN=1,NNODE                                                      C  002:027E:5
      QNS(IN)=QN(IN)+QNS(IN)                                                C  002:0280:0
      SNS(IN)=SN(IN)+SNS(IN)                                                C  002:0283:3
       TOTSKW(IN)=TOTSKW(IN)+SUMSKW(IN)                                     C  002:0287:0
       TOTVAR(IN)=TOTVAR(IN)+SUMVAR(IN)                                     C  002:028A:3
      N1=IDLIM(IN,1)                                                        C  002:028E:0
      N2=IDLIM(IN,2) -1                                                     C  002:028F:5
      DO 53 I=N1,N2                                                         C  002:0292:1
      IF(IOPS(I).NE.4) GOTO 53                                              C  002:0293:0
      IF(Q(I).LE.0.0000001) GOTO 54                                         C  002:0295:2
      WRITE(6,116) I                                                        C  002:0298:4
  116 FORMAT(1H /// 1H ,'POSITIVE FLOW USED WITH IOPS = 4 AT IP =',I3)      C  002:029F:2
      STOP                                                                  C  002:029F:2
   54 IF(QNS(IN).GT.0.000001) GOTO 502                                      C  002:02A0:1
      WRITE(6,504) I                                                        C  002:02A3:4
  504 FORMAT(1H0,'CONCENTRATION CALCULATION ATTEMPTED WITH FLOW EQUAL'      C  002:02AA:2
     *      ,' TO OR LESS THAN ZERO , IP=',I3)                              C  002:02AA:2
      SLD(I)=0.0                                                            C  002:02AA:2
      GOTO 506                                                              C  002:02AC:1
  502 X=SNS(IN)/QNS(IN)                                                     C  002:02AC:4
      SLD(I)=X*Q(I)                                                         C  002:02B0:0
  506 QN(IN)=QN(IN)+Q(I)                                                    C  002:02B3:3
       DOG=(Q(I)/QNS(IN))**2.*TOTVAR(IN)                                    C  002:02B7:0
       TOTVAR(IN)=TOTVAR(IN)+DOG                                            C  002:02BC:1
       SUMVAR(IN)=SUMVAR(IN)+DOG                                            C  002:02BE:4
       SVAR(I)=DOG                                                          C  002:02C1:1
       IF (ISKW.NE.1)GO TO 777                                              C  002:02C3:1
       CAT=(Q(I)/QNS(IN))**3.*TOTSKW(IN)                                    C  002:02C4:5
       TOTSKW(IN)=TOTSKW(IN)+CAT                                            C  002:02CA:2
       SKEW(I)=CAT                                                          C  002:02CC:5
       SUMSKW(IN)=SUMSKW(IN)+CAT                                            C  002:02CE:5
  777  CONTINUE                                                             C  002:02D1:2
      SN(IN)=SN(IN)+SLD(I)                                                  C  002:02D1:2
      QNS(IN)=QNS(IN)+Q(I)                                                  C  002:02D4:5
      SNS(IN)=SNS(IN)+SLD(I)                                                C  002:02D8:2
```

```
 53 CONTINUE                                                                C  002:020B:5
    IF(IN.EQ.NNODE) GOTO 520                                                C  002:020E:0
    IC=IDNOS(IN)                                                            C  002:020F:2
    K=INIC(IC)                                                              C  002:02E1:1
    QNS(K)=QNS(K)+QNS(IN)                                                   C  002:02E2:4
    SNS(K)=SNS(K)+SNS(IN)                                                   C  002:02E5:5
     TOTSKW(K)=TOTSKW(K)+TOTS"W(IN)                                         C  002:02E9:2
     TOTVAR(K)=TOTVAR(K)+TOTV R(IN)                                         C  002:02EC:5
520 IF(IOPW.GT.2) GOTO 10                                                   C  002:02F0:2
C                                                                           C  002:02F1:4
C                 OUTPUT FOR FROJECTS AND NODES                             C  002:02F1:4
C                                                                           C  002:02F1:4
    N1=IOLIM(IN,1)                                                          C  002:02F1:4
    N2=IOLIM(IN,2)-1                                                        C  002:02F3:3
    DO 22 IP=N1,N2                                                          C  002:02F5:5
    CALL WRITEP(IN,IP)                                                      C  002:02F7:0
 22 CONTINUE                                                                C  002:02F8:4
     CALL WRITEN(IN)                                                        C  002:02FA:5
 10 CONTINUE                                                                C  002:02FC:0
C                                                                           C  002:02FE:1
C                 OUTPUT FOR NODES                                          C  002:02FE:1
C                                                                           C  002:02FE:1
    WRITE(6,220) (TITL(I),I=1,20)                                           C  002:02FE:1
    WRITE(6,222)(STITL(I),I=1,20)                                           C  002:030A:2
220 FORMAT(1H1,20A4)                                                        C  002:0316:2
222 FORMAT(1H0,20A4)                                                        C  002:0316:2
    WRITE(6,224)                                                            C  002:0316:2
    WRITE(6,225)                                                            C  002:031A:2
    WRITE(6,227)                                                            C  002:031E:2
224 FORMAT(1H0,43X,'   FLOW       SALT      SALT        SALT',             C  002:0322:2
   *  T85,'STANDARD')                                                       C  002:0322:2
225 FORMAT(1H ,43X,'              LOAD      CONC        CONC',             C  002:0322:2
   *  T85,'DEVIATION',T100,'SKEWNESS')                                      C  002:0322:2
227 FORMAT(1H ,43X,'  (TAF/Y)   (TT/Y)    (T/AF)     (MG/L)',              C  002:0322:2
   *  T87,'TT/Y',T100,'((TT/Y)**3)')                                        C  002:0322:2
    DO 235 IN=1,NNODE                                                       C  002:0322:2
    IF(ABS(QNS(IN)).LT.0.00001) QNS(IN)=0.00001                             C  002:0323:0
    X=SNS(IN)/QNS(IN)                                                       C  002:032A:1
    X1=X*735.8                                                              C  002:032D:3
    IP=IOLIM(IN,2)                                                          C  002:032F:4
     DOG=(TOTVAR(IN))**.5                                                   C  002:0331:3
     IF (ISKW.NE.1)GO TO 778                                                C  002:0335:3
    WRITE(6,230) ICIN(IN),IDNOS(IN),(DESC(IP,I),I=1,6),QNS(IN),SNS(IN)      C  002:0337:1
   *,X,X1,DOG,TOTSKW(IN)                                                    C  002:0348:3
     GO TO 883                                                              C  002:0355:2
778  CONTINUE                                                               C  002:0355:5
    WRITE(6,230) ICIN(IN),IDNOS(IN),(DESC(IP,I),I=1,6),QNS(IN),SNS(IN)      C  002:0355:5
   *,X,X1,DOG                                                               C  002:036A:1
883  CONTINUE                                                               C  002:0372:2
230 FORMAT(1H0,4X,'NODE ',I2,' (',I2,') ',6A4,1X,F10.1,F10.1,F8.3           C  002:0372:2
   *        ,F12.0,T84,F10.2,T99,E10.3)                                     C  002:0372:2
235 CONTINUE                                                                C  002:0372:2
```

```
C                                                          C  002:0374:3
C                 WRITE NODES BY RIVER                     C  002:0374:3
C                                                          C  002:0374:3
      GOTO 202                                             C  002:0374:3
      NR=0                                                 C  002:0375:0
      IR=1                                                 C  002:0375:4
   32 ISKIP=0                                              C  002:0376:2
      DO 30 IN=1,NNODE                                     C  002:0377:0
      IF(IRN(IN).NE.IR) GOTO 30                            C  002:0378:0
      IF(ISKIP.GT.0) GOTO 34                               C  002:037A:2
      ISKIP=1                                              C  002:037B:3
      K=IRN(IN)                                            C  002:037C:1
      WRITE(6,120) IRN(IN),(RIV(K,L),L=1,10)               C  002:037E:0
  120 FORMAT(1H /// 1H ,I2,2X,10A4)                        C  002:038D:2
   34 X=SNS(IN)/QNS(IN)                                    C  002:038D:2
      WRITE(6,122) ICIN(IN),QNS(IN),SNS(IN),X              C  002:0390:4
  122 FORMAT(I3,3F15.3)                                    C  002:039F:2
   30 CONTINUE                                             C  002:039F:2
      IR=IR+1                                              C  002:03A1:3
      IF(IR.LE.NRIV) GOTO32                                C  002:03A2:5
  202 CONTINUE                                             C  002:03A3:5
C                                                          C  002:03A3:5
C                 OUTPUT CHANGES BY STATE                  C  002:03A3:5
C                                                          C  002:03A3:5
      DO 302 I=1,20                                        C  002:03A3:5
      STAQ(I)=0.0                                          C  002:03A5:0
      STAS(I)=0.0                                          C  002:03A6:3
  302 CONTINUE                                             C  002:03A8:0
      J=0                                                  C  002:03AA:1
      DO 300 IP=1,NPOINT                                   C  002:03AA:5
      DO 301 IN=1,NNODE                                    C  002:03AC:0
      IF(IDLIM(IN,2).EQ.IP) GOTO 300                       C  002:03AD:0
  301 CONTINUE                                             C  002:03AF:2
      IF(ISTA(IP).LT.1) GOTO 300                           C  002:03B1:3
      K=ISTA(IP)                                           C  002:03B3:4
      IF(K.GT.J) J=K                                       C  002:03B5:3
      STAQ(K)=STAQ(K)+Q(IP)                                C  002:03B7:2
      STAS(K)=STAS(K)+SLD(IP)                              C  002:03BA:3
  300 CONTINUE                                             C  002:03BD:4
      WRITE(6,220) (TITL(I),I=1,20)                        C  002:03BF:5
      WRITE(6,222)(STITL(I),I=1,20)                        C  002:03CC:2
      WRITE(6,304)                                         C  002:03D8:2
  304 FORMAT(1H0,'STATE',8X,'FLOW',3X,'SALT LOAD')         C  002:03DC:2
      DO 308 K=1,J                                         C  002:03DC:2
      WRITE(6,306) K,STAQ(K),STAS(K)                       C  002:03DD:0
  306 FORMAT(1H0,2X,I2,3X,F10.1,2X,F9.0)                   C  002:03E8:2
  308 CONTINUE                                             C  002:03E8:2
C                                                          C  002:03EA:3
C        OUTPUT BY SALINITY OPTION                         C  002:03EA:3
C                                                          C  002:03EA:3
       DO 310 K=1,5                                        C  002:03EA:3
       DSALT(K)=0.                                         C  002:03EC:0
```

```
      DFLOW(K)=0.                                                         C  002:03ED:3
      DVAR(K)=0.                                                          C  002:03EF:0
      DSKEW(K)=0.                                                         C  002:03F0:3
  310 CONTINUE                                                            C  002:03F2:0
      DO 311 IP=1,NPOINT                                                  C  002:03F4:1
      IF (IOPS(IP).LE.0) GO TO 311                                        C  002:03F5:0
      JD=IOPS(IP)                                                         C  002:03F7:1
      DSALT(JD)=DSALT(JD)+SLD(IP)                                         C  002:03F9:0
      DFLOW(JD)=DFLOW(JD)+Q(IP)                                           C  002:03FB:5
      DVAR(JD)=DVAR(JD)+SVAR(IP)                                          C  002:03FF:0
      DSKEW(JD)=DSKEW(JD)+SKEW(IP)                                        C  002:0402:1
  311 CONTINUE                                                            C  002:0405:2
      WRITE (6,220) (TITL(I),I=1,20)                                      C  002:0407:3
      WRITE (6,222) (STITL(I),I=1,20)                                     C  002:0413:2
      WRITE (6,946)                                                       C  002:041F:2
  946 FORMAT (1H0,T29,'CONTRIBUTIONS BY SALINITY OPTIONS'/T41,            C  002:0423:2
     * '  OPTION    OPTION    OPTION    OPTION    OPTION'/T45,            C  002:0423:2
     * '#1',T55,'#2',T65,'#3',T75,'#4',T85,'#5')                          C  002:0423:2
      WRITE (6,947) DFLOW                                                 C  002:0423:2
  947 FORMAT (1H ,'TOTAL FLOW CONTRIBUTED (TAF/Y)',T41,5(F8.2,2X))        C  002:042A:2
      WRITE (6,943) DSALT                                                 C  002:042A:2
  943 FORMAT (1H ,'TOTAL SALTS CONTRIBUTED (TT/Y)',T41,5(F8.2,2X))        C  002:0431:2
      WRITE (6,944) DVAR                                                  C  002:0431:2
  944 FORMAT (1H ,'TOTAL VARIANCE CONTRIBUTED (TT/Y)**2.',T41,            C  002:0438:2
     *  5(F8.0,2X))                                                       C  002:0438:2
      WRITE (6,945) DSKEW                                                 C  002:0438:2
  945 FORMAT (1H ,'NET SKEW CONTRIBUTED (TT/Y)**3.',T41,5(E10.3))         C  002:043F:2
     STOP                                                                 C  002:043F:2
     END                                                                  C  002:0440:1
                                                                SEGMENT 002 IS 0490 LONG

                                                                    START OF SEGMENT 007
     SUBROUTINE OPTION(IP)                                                C  007:0000:0
     COMMON  TITL(20),RIV(50,10),IRN(99),ISTA(500),DESC(500,6)            C  007:0000:0
    *        ,IOPF(500),IOPS(500),ICIN(99),INIC(99),IOLIM(99,2)           C  007:0000:0
    *        ,IONDS(99),QN(99),SN(99),Q(500),SCN(500),SLD(500),A(500)     C  007:0000:0
    *        ,CUF(500),SLF(500),QNS(99),SNS(99),STITL(20),INIP(500)       C  007:0000:0
    * ,PHI(500),SLDEZ(500),SCNEZ(500),SLFEZ(500),PHIEZ(500),SVAR(500)     C  007:0000:0
    * ,SUMVAR(99),AEZ(500),TOTVAR(99),BESC(6),SLDTZ(500),SCNTZ(500),      C  007:0000:0
    * SLFTZ(500),PHITZ(500),SKEW(500),SUMSKW(99),ATZ(500),TOTSKW(99),     C  007:0000:0
    * ISKW,IAN(500)                                                       C  007:0000:0
C                                                                         C  007:0000:0
C                 FLOW OPTIONS                                            C  007:0000:0
C                                                                         C  007:0000:0
     IF(IOPF(IP).LT.3) GOTO 10                                            C  007:0000:0
     Q(IP)=A(IP)*CUF(IP)                                                  C  007:0002:2
  10 IF(IOPS(IP).LT.1.OR.IOPS(IP).EQ.4) GO TO 20                          C  007:0006:5
C                                                                         C  007:000B:0
C                 SALT OPTIONS                                            C  007:000B:0
C                                                                         C  007:000B:0
     K=IOPS(IP)                                                           C  007:000B:0
     GOTO(30,1,3,20,5),K                                                  C  007:000C:5
   1 SLD(IP)=SCN(IP)*Q(IP)                                                C  007:0013:0
```

```
      SVAR(IP)=(SCNEZ(IP)*SCN(IP))**2.*Q(IP)**2.                         C   007:0017:3
      IF(IAN(IP).NE.0)SVAR(IP)=SCNEZ(IP)*Q(IP)**2.                       C   007:001E:1
      SKEW(IP)=(SCNTZ(IP)*SCN(IP))**3.*Q(IP)**3.                         C   007:0025:1
      IF(IAN(IP).NE.0)SKEW(IP)=SCNTZ(IP)*Q(IP)**2.                       C   007:002C:3
     GO TO 30                                                            C   007:0033:3
   3 SLD(IP)=A(IP)*SLF(IP)                                               C   007:0034:0
      DOG=(AEZ(IP)*A(IP))**2.                                            C   007:0038:3
      IF(IAN(IP).NE.0)DOG=AEZ(IP)                                        C   007:003C:1
      CAT=(SLFEZ(IP)*SLF(IP))**2.                                        C   007:0040:1
      IF(IAN(IP).NE.0)CAT=SLFEZ(IP)                                      C   007:0043:5
      SVAR(IP)=DOG*CAT+A(IP)**2.*CAT+SLF(IP)**2.*DOG                     C   007:0047:5
      IF (ISKW.NE.1)GO TO 779                                            C   007:004F:0
      TDOG=(ATZ(IP)*A(IP))**3.                                           C   007:0050:4
      IF(IAN(IP).NE.0)TDOG=ATZ(IP)                                       C   007:0054:4
      TCAT=(SLFTZ(IP)*SLF(IP))**3.                                       C   007:0058:4
      IF(IAN(IP).NE.0)TCAT=SLFTZ(IP)                                     C   007:005C:4
      SKEW(IP)=TDOG*TCAT+3.*A(IP)*DOG*TCAT+3.*SLF(IP)*TDOG*CAT+          C   007:0060:4
    * 6.*A(IP)*SLF(IP)*DOG*CAT+A(IP)**3.*TCAT+SLF(IP)**3.*TDOG           C   007:0069:0
 779  CONTINUE                                                           C   007:0073:0
     GOTO 30                                                             C   007:0073:0
   5 IF(SLF(IP).EQ.0.0) SLF(IP)=0.0000001                                C   007:0073:3
C     *******  START OF PART TWO                                         C   007:0079:1
      SVAR(IP)=PHIEZ(IP)                                                 C   007:0079:1
     SLD(IP)=-Q(IP)*((1.0-SLF(IP))/SLF(IP))*PHI(IP)                      C   007:007C:1
      GO TO 30                                                           C   007:0084:1
C     *******  END OF PART TWO                                           C   007:0084:4
      SLD(IP)=-Q(IP)*((1.0-SLF(IP))/SLF(IP))*PHI(IP)                     C   007:0084:4
      UBIRD=((1.-SLF(IP))/SLF(IP))                                       C   007:008C:4
      BIRD=UBIRD**2.                                                     C   007:0090:2
      DOG= SLFEZ(IP)**2.*BIRD                                            C   007:0091:1
      IF(IAN(IP).NE.0)DOG=SLFEZ(IP)                                      C   007:0093:5
      CAT=(PHIEZ(IP)*PHI(IP))**2.                                        C   007:0097:5
      IF(IAN(IP).NE.0)CAT=PHIEZ(IP)                                      C   007:009B:3
      SVAR(IP)=Q(IP)**2.*(DOG*CAT+BIRD*CAT+PHI(IP)**2.*DOG)              C   007:009F:3
      IF (ISKW.NE.1)GO TO 780                                            C   007:00A7:1
      TBIRD=UBIRD**3.                                                    C   007:00A8:5
      TDOG=SLFTZ(IP)**3.*TBIRD                                           C   007:00AA:2
      IF(IAN(IP).NE.0)TDOG=SLFTZ(IP)                                     C   007:00AD:2
      TCAT=(PHITZ(IP)*PHI(IP))**3.                                       C   007:00B1:2
      IF(IAN(IP).NE.0)TCAT=PHITZ(IP)                                     C   007:00B5:2
      SKEW(IP)=-Q(IP)**3.*(TDOG*TCAT+3.*UBIRD*DOG*TCAT+3.*PHI(IP)*       C   007:00B9:2
    * TDOG*CAT+6.*UBIRD*PHI(IP)*DOG*CAT+TBIRD*TCAT+PHI(IP)**3.*TDOG)     C   007:00C1:3
 780  CONTINUE                                                           C   007:00CA:2
     GOTO 30                                                             C   007:00CA:2
  20 SLD(IP)=0.0                                                         C   007:00CA:5
      SVAR(IP)=0.0                                                       C   007:00CC:4
      SKEW(IP)=0.0                                                       C   007:00CE:3
  30 RETURN                                                              C   007:00D0:2
     END                                                                 C   007:00D0:5
                                                   SEGMENT 007 IS 00D8 LONG
```

```
                                                                        START OF SEGMENT 008
      SUBROUTINE WRITEN(IN)                                             C  008:0000:0
      COMMON  TITL(20),RIV(50,10),IRN(99),ISTA(500),DESC(500,6)         C  008:0000:0
     *       ,IOPF(500),IOPS(500),ICIN(99),INIC(99),IDLIM(99,2)         C  008:0000:0
     *       ,IONDS(99),QN(99),SN(99),Q(500),SCN(500),SLD(500),A(500)   C  008:0000:0
     *       ,CUF(500),SLF(500),QNS(99),SNS(99),STITL(20),INIP(500)     C  008:0000:0
     * ,PHI(500),SLDEZ(500),SCNEZ(500),SLFEZ(500),PHIEZ(500),SVAR(500)  C  008:0000:0
     * ,SUMVAR(99),AEZ(500),TOTVAR(99),BESC(6),SLDTZ(500),SCNTZ(500),   C  008:0000:0
     * SLFTZ(500),PHITZ(500),SKEW(500),SUMSKW(99),ATZ(500),TOTSKW(99),  C  008:0000:0
     * ISKW,IAN(500)                                                    C  008:0000:0
      IP=IDLIM(IN,2)                                                    C  008:0000:0
      IF(ABS(QNS(IN)).LT.0.00001) QNS(IN)=0.00001                       C  008:0001:5
      X=SNS(IN)/QNS(IN)                                                 C  008:0009:1
      WRITE(6,1) ICIN(IN),IONDS(IN),(DESC(IP,I),I=1,6),QN(IN),SN(IN)    C  008:000C:3
     *          ,QNS(IN),SNS(IN),X                                      C  008:0020:5
    1 FORMAT(1H0,4X,'NODE ',I2,' (',I2,') ',6A4,6X,F10.1,F9.1          C  008:002A:2
     *     ,38X,F10.1,F9.1,F8.3 // 1H )                                 C  008:002A:2
      RETURN                                                            C  008:002A:2
      END                                                               C  008:002A:5
                                                           SEGMENT 008 IS 0032 LONG


                                                                        START OF SEGMENT 009
      SUBROUTINE HEAD1(TITL,STITL)                                      C  009:0000:0
      DIMENSION TITL(20),STITL(20)                                      C  009:0000:0
      WRITE(6,1) (TITL(I),I=1,20)                                       C  009:0000:0
      WRITE(6,2) (STITL(I),I=1,20)                                      C  009:000C:2
      WRITE(6,3)                                                        C  009:0019:2
      WRITE(6,15)                                                       C  009:001D:2
      WRITE(6,11)                                                       C  009:0021:2
      WRITE(6,12)                                                       C  009:0025:2
      WRITE(6,13)                                                       C  009:0029:2
      WRITE(6,14)                                                       C  009:002D:2
    1 FORMAT(1H1,20A4)                                                  C  009:0031:2
    2 FORMAT(1H0,20A4)                                                  C  009:0031:2
    3 FORMAT(1H0,'ECHO INPUT DATA')                                     C  009:0031:2
   15 FORMAT(1H0,5X,'1   2-3 4-5',8X,'6-29',15X                         C  009:0031:2
     *              ,'30-32     33-40     41-48     49-56    57-64    65-69'  C  009:0031:2
     *              ,'    70-74      75-80')                            C  009:0031:2
   11 FORMAT(1H ,43X,'INPUT      INPUT     INPUT     INPUT    INPUT    CONSUM'  C  009:0031:2
     *              ,'    SALT       PHI')                              C  009:0031:2
   12 FORMAT(1H ,43X,'CODES      FLOW      SALT      SALT     AREA      USE  '  C  009:0031:2
     *              ,'    LOAD ')                                       C  009:0031:2
   13 FORMAT(1H ,43X,'                     LOAD      CONC              FACTOR'  C  009:0031:2
     *              ,'  FACTOR')                                        C  009:0031:2
   14 FORMAT(1H ,43X,'                    (TT/Y)    (T/AF)   (T-ACR)   AF/A/Y'  C  009:0031:2
     *              ,'    T/A/Y      T/A/F')                            C  009:0031:2
      RETURN                                                            C  009:0031:2
      END                                                               C  009:0031:5
                                                           SEGMENT 009 IS 003A LONG
```

```
      SUBROUTINE HEAD(TITL,STITL)
      DIMENSION TITL(20),STITL(20)
      WRITE(6,8) (TITL(I),I=1,20)
      WRITE(6,9) (STITL(I),I=1,20)
    8 FORMAT(1H1,20A4)
    9 FORMAT(1H0,20A4)
      WRITE(6,15)
   15 FORMAT(1H0,5X,'    2-3 4-5',8X,'6-29',15X
     *           ,'30-32      33-40     41-48    49-56    57-64  65-69'
     *           ,'  70-74  75-80')
      WRITE(6,1)
      WRITE(6,2)
      WRITE(6,3)
      WRITE(6,4)
    1 FORMAT(1H ,43X,'INPUT      INPUT     INPUT    INPUT    INPUT  CONSUM'
     *           ,'  SALT    PHI       FLOW      SALT      SALT')
    2 FORMAT(1H ,43X,'CODES      FLOW      SALT     SALT     AREA     USE  '
     *           ,'  LOAD                         LOAD      CONC')
    3 FORMAT(1H ,43X,'                     LOAD     CONC              FACTOR'
     *           ,' FACTOR')
    4 FORMAT(1H ,43X,'                    (TT/Y)   (T/AF)   (T-ACR) AF/A/Y'
     *           ,'  T/A/Y (T/A/F)  (TAF/Y)   (TT/Y)    (T/AF)')
      RETURN
      END
```

```
                START OF SEGMENT 00A
            FORMAT SEGMENT IS 000B LONG
00003480  C  00A:0000:0
00003490  C  00A:0000:0
00003500  C  00A:0000:0
00003510  C  00A:000C:2
00003520  C  00A:0019:2
00003530  C  00A:0019:2
00003540  C  00A:0019:2
00003550  C  00A:001D:2
00003560  C  00A:001D:2
00003570  C  00A:001D:2
00003580  C  00A:001D:2
00003590  C  00A:0021:2
00003600  C  00A:0025:2
00003610  C  00A:0029:2
00003620  C  00A:002D:2
00003630  C  00A:002D:2
00003640  C  00A:002D:2
00003650  C  00A:002D:2
00003660  C  00A:002D:2
00003670  C  00A:002D:2
00003680  C  00A:002D:2
00003690  C  00A:002D:2
00003700  C  00A:002D:2
00003710  C  00A:002D:5
            SEGMENT 00A IS 0036 LONG
```

```
      SUBROUTINE WRITEP(IN,IP)
      COMMON  TITL(20),RIV(50,10),IRN(99),ISTA(500),DESC(500,6)
     *        ,IOPF(500),IOPS(500),ICIN(99),INIC(99),IDLIM(99,2)
     *        ,IDNDS(99),QN(99),SN(99),Q(500),SCN(500),SLD(500),A(500)
     *        ,CUF(500),SLF(500),QNS(99),SNS(99),STITL(20),INIP(500)
     * ,PHI(500),SLDEZ(500),SCNEZ(500),SLFEZ(500),PHIEZ(500),SVAR(500)
     * ,SUMVAR(99),AEZ(500),TOTVAR(99),BESC(6),SLDTZ(500),SCNTZ(500),
     * SLFTZ(500),PHITZ(500),SKEW(500),SUMSKW(99),ATZ(500),TOTSKW(99),
     * ISKW,IAN(500)
      IF(IOPF(IP).NE.0) GOTO 10
      IF(IOPS(IP).NE.0) GOTO 1
      GOTO 4
   10 K=IOPF(IP)
      GOTO(1,1,3),K
    1 IF(IOPS(IP).EQ.0) GOTO 4
      K=IOPS(IP)
      GOTO(4,5,6,4,6),K
    4 WRITE(6,101) IP, INIP(IP),(DESC(IP,I),I=1,6),ISTA(IP),IOPF(IP)
     *,IOPS(IP)    ,Q(IP),SLD(IP)
      RETURN
    5 WRITE(6,102) IP, INIP(IP),(DESC(IP,I),I=1,6),ISTA(IP),IOPF(IP)
     *,IOPS(IP)    ,Q(IP),SLD(IP),SCN(IP)
      RETURN
```

```
                START OF SEGMENT 00B
00003720  C  00B:0000:0
00003730  C  00B:0000:0
00003740  C  00B:0000:0
00003750  C  00B:0000:0
00003760  C  00B:0000:0
          C  00B:0000:0
          C  00B:0000:0
          C  00B:0000:0
          C  00B:0000:0
00003820  C  00B:0000:0
00003830  C  00B:0002:1
00003840  C  00B:0004:2
00003850  C  00B:0004:5
00003860  C  00B:0006:4
00003870  C  00B:000C:0
00003880  C  00B:000E:1
00003890  C  00B:0010:0
00003900  C  00B:0016:0
00003910  C  00B:0029:2
00003920  C  00B:0034:2
00003930  C  00B:0034:5
00003940  C  00B:0048:1
00003950  C  00B:0055:2
```

```
    6 WRITE(6,103) IP, INIP(IP),(DESC(IP,I),I=1,6),ISTA(IP),IOPF(IP)    C  00B:0055:5
     *,IOPS(IP)    ,Q(IP),SLD(IP),A(IP),SLF(IP)                          C  00B:0069:1
     * ,PHI(IP)                                                          C  00B:0075:4
      RETURN                                                             C  00B:0078:2
    3 IF(IOPS(IP).EQ.0) GOTO 12                                          C  00B:007B:5
      K=IOPS(IP)                                                         C  00B:007E:0
      GOTO(12,14,16,12,16),K                                             C  00B:007F:5
   12 WRITE(6,104) IP, INIP(IP),(DESC(IP,I),I=1,6),ISTA(IP),IOPF(IP)    C  00B:0086:0
     *,IOPS(IP)    ,Q(IP),SLD(IP),A(IP),CUF(IP)                          C  00B:0099:2
      RETURN                                                             C  00B:00A9:2
   14 WRITE(6,105) IP, INIP(IP),(DESC(IP,I),I=1,6),ISTA(IP),IOPF(IP)    C  00B:00A9:5
     *,IOPS(IP)    ,Q(IP),SLD(IP),SCN(IP),A(IP),CUF(IP)                  C  00B:00BD:1
      RETURN                                                             C  00B:00CF:2
   16 WRITE(6,106) IP, INIP(IP),(DESC(IP,I),I=1,6),ISTA(IP),IOPF(IP)    C  00B:00CF:5
     *,IOPS(IP)     ,Q(IP),SLD(IP),A(IP),CUF(IP),SLF(IP)                 C  00B:00E3:1
     * ,PHI(IP)                                                          C  00B:00F2:1
      RETURN                                                             C  00B:00F8:2
  101 FORMAT(1H0,I3,6X,I2,6X,6A4,I3,2I2,F9.1,F9.1)                       C  00B:00F8:5
  102 FORMAT(1H0,I3,6X,I2,6X,6A4,I3,2I2,F9.1,F9.1                        C  00B:00F8:5
     *        ,F8.3)                                                     C  00B:00F8:5
  103 FORMAT(1H0,I3,6X,I2,6X,6A4,I3,2I2,F9.1,F9.1                        C  00B:00F8:5
     *        ,8X,F8.1,8X,F7.2,F10.4)                                    C  00B:00F8:5
  104 FORMAT(1H0,I3,6X,I2,6X,6A4,I3,2I2,F9.1,F9.1                        C  00B:00F8:5
     *        ,8X,F8.1,F8.2)                                             C  00B:00F8:5
  105 FORMAT(1H0,I3,6X,I2,6X,6A4,I3,2I2,F9.1,F9.1                        C  00B:00F8:5
     *        ,F8.3,F8.1,F8.2)                                           C  00B:00F8:5
  106 FORMAT(1H0,I3,6X,I2,6X,6A4,I3,2I2,F9.1,F9.1                        C  00B:00F8:5
     *        ,8X,F8.1,F8.2,F7.2,F10.4)                                  C  00B:00F8:5
      END                                                                C  00B:00F8:5
                                                    SEGMENT 00B IS 010A LONG
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PROGRAM SALTEZ OUTPUT FOR 1977-BASE RUN

COLORADO RIVER FLOW SALINITY STUDY

RUN FOR 1977-VARIANCE BASED UPON DELTA S

ECHO INPUT DATA

	1	2-3	4-5	6-29	30-32 INPUT CODES	33-40 INPUT FLOW	41-48 INPUT SALT LOAD (TT/Y)	49-56 INPUT SALT CONC (T/AF)	57-64 INPUT AREA (T-ACR)	65-69 CONSUM USE FACTOR AF/A/Y	70-74 SALT LOAD FACTOR T/A/Y	75-80 PHI T/A/F
				DEFAULT RELATIVE DEVIATIONS			0.00	0.00	0.00		0.00	0.00
1	1	5	0	UG 1 1977 WY G A	7 3 5	0.0	0.0	0.000	52.4	-1.100	0.220	0.1090
				SPECIAL VARIANCE VALUES			0.0	0.000	52.4000	-1.100	0.220	0.2480
2	1	5	0	UG 2 B 1977 WY G A	7 3 5	0.0	0.0	0.000	116.0	-1.200	0.340	0.2600
				SPECIAL VARIANCE VALUES			0.0	0.000	116.0000	-1.200	0.340	0.2480
3	1	5	0	UG 3 B 1977 WY G A	7 3 5	0.0	0.0	0.000	4.3	-1.600	0.470	1.1600
				SPECIAL VARIANCE VALUES			0.0	0.000	4.3000	-1.600	0.470	0.2480
4	1	5	0	UG 4 BA1977 WY G A	7 3 5	0.0	0.0	0.000	21.4	-1.700	0.460	1.0800
				SPECIAL VARIANCE VALUES			0.0	0.000	21.4000	-1.700	0.460	0.2480
5	1	5	0	UG 6 1977 WY G A	7 3 5	0.0	0.0	0.000	1.8	-1.300	0.220	10.4000
				SPECIAL VARIANCE VALUES			0.0	0.000	1.8000	-1.300	0.220	0.2480
6	1	5	0	UPRR CHAMPLIN WY 1977	7 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
7	1	5	0	SEEDSKADEE SHALE 1977	7 2 2	0.0	0.0	0.510	0.0	0.000	0.000	0.0000
8	1	5	0	SEEDSKADEE POWER 1977	7 2 2	-30.0	0.0	0.470	0.0	0.000	0.000	0.0000
9	0	5	0	SLACK 14000	0 1 1	2489.0	665.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES			511.0	0.000	0.0000	0.000	0.000	0.0000
10	0	5	10	GREEN RIVER WYO (2170)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
11	1	10	0	UG 7 1977 WY G A	7 3 5	0.0	0.0	0.000	82.4	-1.500	0.680	0.6800
				SPECIAL VARIANCE VALUES			0.0	0.000	77.4000	-1.500	0.680	0.2480
12	1	10	0	UG 8 1977 UT G A	6 3 5	0.0	0.0	0.000	28.0	-2.000	0.510	0.2000
				SPECIAL VARIANCE VALUES			0.0	0.000	28.0000	-2.000	0.510	0.2480
13	1	10	0	WESTAVCO PR MI 1977	7 2 2	-8.0	0.0	0.420	0.0	0.000	0.000	0.0000
14	0	10	0	EVAP FLAM GR	0 1 1	-74.0	0.0	0.000	0.0	0.000	0.000	0.0000
15	0	10	0	SLACK 14000	0 1 1	779.0	779.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES			1452.0	0.000	0.0000	0.000	0.000	0.0000

16	0	10	20	GREENDALE, UT (2345)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
17	2	15	0	UG13 1977 UT D A	6 3 5	0.0	0.0	0.000	52.0	-2.000	0.390	0.3500
				SPECIAL VARIANCE VALUES			0.0	0.000	52.0000	-2.000	0.390	0.2480
18	2	15	0	UG14 1977 UT D A	6 3 5	0.0	0.0	0.000	150.0	-2.000	0.490	0.7300
				SPECIAL VARIANCE VALUES			0.0	0.000	141.0000	-2.000	0.490	0.2480
19	2	15	0	CUP BONNEVILLE 1977	6 2 2	-25.0	0.0	0.160	0.0	0.000	0.000	0.0000
20	2	15	0	CUP UINTAH DIVER 1977	6 2 2	0.0	0.0	0.160	0.0	0.000	0.000	0.0000
21	0	15	0	SLACK 14000	0 1 1	814.0	116.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES			445.0	0.000	0.0000	0.000	0.000	0.0000
22	0	15	20	RANDLETT, UT (3020)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
23	1	20	0	UG10 1977 CO Y A	3 3 5	0.0	0.0	0.000	79.4	-1.700	0.390	0.2300
				SPECIAL VARIANCE VALUES			0.0	0.000	79.4000	-1.700	0.390	0.2480
24	1	20	0	UG 9 1977 CO Y A	3 3 5	0.0	0.0	0.000	25.7	-1.900	0.360	0.2400
				SPECIAL VARIANCE VALUES			0.0	0.000	25.7000	-1.900	0.360	0.2480
25	1	20	0	UG11 1977 UT G A	6 3 5	0.0	0.0	0.000	5.4	-2.200	0.420	1.8500
				SPECIAL VARIANCE VALUES			0.0	0.000	5.4000	-2.200	0.420	0.2480
26	1	20	0	UG12 1977 UT G A	6 3 5	0.0	0.0	0.000	27.4	-2.000	0.600	0.9800
				SPECIAL VARIANCE VALUES			0.0	0.000	27.4000	-2.000	0.600	0.2480
27	1	20	0	UG15 1977 UT W A	6 3 5	0.0	0.0	0.000	34.8	-2.100	0.350	0.5440
				SPECIAL VARIANCE VALUES			0.0	0.000	34.8000	-2.100	0.350	0.2480
28	1	20	0	UG16 1977 UT P A	6 3 5	0.0	0.0	0.000	19.6	-2.200	0.630	2.5500
				SPECIAL VARIANCE VALUES			0.0	0.000	19.6000	-2.200	0.630	0.2480
29	1	20	0	UG17 1977 UT G A	6 3 5	0.0	0.0	0.000	20.0	-2.000	0.530	2.5300
				SPECIAL VARIANCE VALUES			0.0	0.000	20.2000	-2.000	0.530	0.2480
30	1	20	0	UT COLD SHALE 1977	6 2 2	0.0	0.0	0.530	0.0	0.000	0.000	0.0000
31	1	20	0	WHITE POWER 1977	3 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
32	1	20	0	YAMPA POWER 1977	3 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
33	1	20	0	HAYDEN POWER 1977	3 2 2	-4.0	0.0	0.470	0.0	0.000	0.000	0.0000
34	1	20	0	NW EXTENSION POW 1977	3 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
35	1	20	0	CRAIG SLATER POW 1977	3 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
36	0	20	0	CARBON 1 2 PWR 1977	6 2 2	-3.0	0.0	0.470	0.0	0.000	0.000	0.0000

37	1	20	0	CYS LAR DIV	1977	7 2 2	-1.0	0.0	0.190	0.0	0.000	0.000	0.0000
38	1	20	0	FOUR COUNTY DIV	1977	3 2 2	0.0	0.0	0.190	0.0	0.000	0.000	0.0000
39	0	20	0	SLACK 14000		0 1 1	2430.0	755.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES				3564.0	0.000	0.0000	0.000	0.000	0.0000
40	0	20	59	GREEN RIVER, UT	3285)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
41	3	25	0	UG18 1977	UT A A	6 3 5	0.0	0.0	0.000	39.3	-2.000	0.500	1.2900
				SPECIAL VARIANCE VALUES				0.0	0.000	39.3000	-2.000	0.500	0.2480
42	3	25	0	HUNTINGTON CANY	1977	6 2 2	-12.0	0.0	0.470	0.0	0.000	0.000	0.0000
43	0	25	0	SLACK 14000		0 1 1	116.0	34.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES				193.0	0.000	0.0000	0.000	0.000	0.0000
44	0	25	59	GREEN RIVER, UT	(3285)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES				0.0	0.000	23.9000	-1.300	0.320	0.2480
46	4	30	0	UM 2 1977	CO E A	3 3 5	0.0	0.0	0.000	22.0	-2.000	0.270	0.3300
				SPECIAL VARIANCE VALUES				0.0	0.000	22.0000	-2.000	0.270	0.2480
47	4	30	0	UM 3 1977	CO C A	3 3 5	0.0	0.0	0.000	73.5	-1.900	0.360	0.2900
				SPECIAL VARIANCE VALUES				0.0	0.000	73.5000	-1.900	0.360	0.2480
48	4	30	0	DEN ENGLEWOOD	1977	3 2 2	0.0	0.0	0.100	0.0	0.000	0.000	0.0000
49	4	30	0	HOMESTAKE	1977	3 2 2	-20.0	0.0	0.100	0.0	0.000	0.000	0.0000
50	0	30	0	SLACK 14000		0 1 1	1988.0	575.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES				605.0	0.000	0.0000	0.000	0.000	0.0000
51	0	30	35	GLENWOOD SPRINGS CO(725)		0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
52	4	35	0	FRYING PAN	1977	3 2 2	-55.0	0.0	0.060	0.0	0.000	0.000	0.0000
53	4	35	0	UM 4 1977	CO C A	3 3 5	0.0	0.0	0.000	29.5	-2.100	0.420	0.3200
				SPECIAL VARIANCE VALUES				0.0	0.000	29.5000	-2.100	0.420	0.2480
54	4	35	0	UM 5 1977	CO C A	3 3 5	0.0	0.0	0.000	54.7	-1.700	0.600	1.8000
				SPECIAL VARIANCE VALUES				0.0	0.000	54.7000	-1.700	0.600	0.2480
55	4	35	0	COLO SHALE	1977	3 2 2	0.0	0.0	0.510	0.0	0.000	0.000	0.0000
56	0	35	0	SLACK 14000		0 1 1	1404.0	933.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES				471.0	0.000	0.0000	0.000	0.000	0.0000
57	0	35	45	CAMEO COLD	(955)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
58	5	40	0	UM 7 1977	CO U A	3 3 5	0.0	0.0	0.000	54.9	-1.300	0.200	0.1370
				SPECIAL VARIANCE VALUES				0.0	0.000	54.9000	-1.800	0.200	0.2480

59	5	40	0	UM 8 1977 CO U A	3 3 5	0.0	0.0	0.000	62.8	-1.400	0.420	0.0970
				SPECIAL VARIANCE VALUES			0.0	0.000	62.8000	-1.400	0.420	0.2480
60	5	40	0	UM 9 1977 CO U A	3 3 5	0.0	0.0	0.000	104.2	-2.300	0.250	0.5600
				SPECIAL VARIANCE VALUES			0.0	0.000	104.2000	-2.300	0.250	0.2480
61	5	40	0	UM10 1977 CO U A	3 3 5	0.0	0.0	0.000	39.6	-2.400	0.340	1.2400
				SPECIAL VARIANCE VALUES			0.0	0.000	39.6000	-2.400	0.340	0.2480
62	4	40	0	DALLAS CREEK POW 1977	3 2 2	-12.0	0.0	0.470	0.0	0.000	0.000	0.0000
63	4	40	0	ASP DALLAS FISH 1977	3 2 2	0.0	0.0	0.370	0.0	0.000	0.000	0.0000
64	0	40	0	SLACK 14000	0 1 1	1883.0	517.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES			2343.0	0.000	0.0000	0.000	0.000	0.0000
65	0	40	45	GRAND JUNCTION CO (1525)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
66	4	45	0	UM 6 1977 CO C A	3 3 5	0.0	0.0	0.000	21.9	-2.300	0.720	0.3100
				SPECIAL VARIANCE VALUES			0.0	0.000	21.9000	-2.300	0.720	0.2480
67	4	45	0	UM11 1977 CO C A	3 3 5	0.0	0.0	0.000	86.0	-2.300	0.310	1.5700
				SPECIAL VARIANCE VALUES			0.0	0.000	86.0000	-2.300	0.310	0.2480
68	4	45	0	UM12 1977 UT D A	6 3 5	0.0	0.0	0.000	43.2	-2.200	0.600	0.3700
				SPECIAL VARIANCE VALUES			0.0	0.000	43.2000	-2.200	0.600	0.2480
69	4	45	0	UM13 1977 UT C A	6 3 5	0.0	0.0	0.000	2.7	-2.400	0.540	1.2900
				SPECIAL VARIANCE VALUES			0.0	0.000	2.7000	-2.400	0.540	0.2480
70	4	45	0	SAN MIGUEL POWER 1977	3 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
71	4	45	0	REC MI USE OTHER 1977	3 2 2	0.0	0.0	0.370	0.0	0.000	0.000	0.0000
72	0	45	0	SLACK 14000	0 1 1	109.0	96.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES			6512.0	0.000	0.0000	0.000	0.000	0.0000
73	0	45	59	CISCO, UT (1905)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
74	6	50	0	US 1 1977 CO S A	3 3 5	0.0	0.0	0.000	11.8	-1.800	0.300	0.1470
				SPECIAL VARIANCE VALUES			0.0	0.000	11.8000	-1.800	0.300	0.2480
75	6	50	0	US 2 1977 NM S A	4 3 5	0.0	0.0	0.000	64.5	-1.700	0.420	0.1850
				SPECIAL VARIANCE VALUES			0.0	0.000	64.5000	-1.700	0.420	0.2480
76	6	50	0	SAN JUAN CHAMA 1977	4 2 2	-11.0	0.0	0.160	0.0	0.000	0.000	0.0000
77	0	50	0	SLACK 14000	0 1 1	844.0	186.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES			288.0	0.000	0.0000	0.000	0.000	0.0000
78	0	50	55	ARCHULETA NM (3555)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000

79	6	55	0	US 3 1977	NM S A	4 3 5	0.0	0.0	0.000	33.3	-2.400	0.420	0.3300
				SPECIAL VARIANCE VALUES				0.0	0.000	33.3000	-2.400	0.420	0.2480
80	6	55	0	US 4 1977	NM S A	4 3 5	0.0	0.0	0.000	12.3	-2.600	0.550	2.5000
				SPECIAL VARIANCE VALUES				0.0	0.000	12.3000	-2.600	0.550	0.2480
81	6	55	0	US 5 1977	NM S A	4 3 5	0.0	0.0	0.000	30.7	-2.600	0.550	0.3500
				SPECIAL VARIANCE VALUES				0.0	0.000	30.7000	-2.600	0.550	0.2480
82	6	55	0	US 6 1977	NM S A	4 3 5	0.0	0.0	0.000	16.5	-2.700	0.510	1.8700
				SPECIAL VARIANCE VALUES				0.0	0.000	14.5000	-2.700	0.510	0.2480
83	6	55	0	US 7 1977	UT S A	6 3 5	0.0	0.0	0.000	79.7	-2.300	0.570	0.8700
				SPECIAL VARIANCE VALUES				0.0	0.000	79.7000	-2.300	0.570	0.2480
84	6	55	0	ANIMAS LA PLATA	1977	4 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
85	6	55	0	FOUR CORNERS POW	1977	4 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
86	6	55	0	SAN JUAN POWER	1977	4 2 2	-6.0	0.0	0.470	0.0	0.000	0.000	0.0000
87	6	55	0	GALLUP DIVERSION	1977	4 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
88	6	55	0	NM OTHER POWER	1977	4 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
89	6	55	0	USBR PROJ ENERGY	1977	4 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
90	6	55	0	USBR PROJ ENERGY	1977	3 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
91	6	55	0	FOUR CORNERS POW	1977	3 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
92	6	55	0	ANIMAS LA PLATA	1977	3 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
93	6	55	0	EL PASO COAL GAS	1977	4 2 2	0.0	0.0	0.350	0.0	0.000	0.000	0.0000
94	6	55	0	WESCO COAL GAS	1977	4 2 2	0.0	0.0	0.350	0.0	0.000	0.000	0.0000
95	6	55	0	REC MI USE OTHER	1977	4 2 2	0.0	0.0	0.400	0.0	0.000	0.000	0.0000
96	0	55	0	SLACK 14000		0 1 1	1173.0	614.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES				3069.0	0.000	0.0000	0.000	0.000	0.0000
97	0	55	59	BLUFF UT	(3795)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
98	0	59	60	LOADING TO POWELL		0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
99	4	60	0	UM14 1977	AZ C A	1 3 5	0.0	0.0	0.000	44.3	-2.000	0.510	2.3600
				SPECIAL VARIANCE VALUES				0.0	0.000	44.3000	-2.000	0.510	0.2480
100	4	60	0	NAVAJO POWER	1977	1 2 2	-33.0	0.0	0.470	0.0	0.000	0.000	0.0000
101	7	60	0	ST GEORGE POWER	1977	6 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000

102	4	60	0	EMERY CO POWER	1977	6 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
103	4	60	0	FREMONT POWER	1977	6 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
104	4	60	0	KAIPAROWITZ POW	1977	6 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
105	4	60	0	ESCALANTE POWER	1977	6 2 2	0.0	0.0	0.470	0.0	0.000	0.000	0.0000
106	0	60	0	LAKE POWELL EVAP		0 1 1	-600.0	0.0	0.000	0.0	0.000	0.000	0.0000
107	0	60	0	SLACK 14000		0 1 1	500.0	-438.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES				171369.0	0.000	0.0000	0.000	0.000	0.0000
108	0	60	65	LEE,S FERRY AZ	(3800)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
109	4	65	0	LL 1 1977	AZ L A	1 3 5	0.0	0.0	0.000	12.0	-4.000	0.580	0.1200
				SPECIAL VARIANCE VALUES				0.0	0.000	12.0000	-4.000	0.580	0.2480
110	4	65	0	LL 2 1977	AZ L A	1 3 5	0.0	0.0	0.000	10.0	-4.000	0.580	0.4600
				SPECIAL VARIANCE VALUES				0.0	0.000	10.0000	-4.000	0.580	0.2480
111	4	65	0	LL 3 1977	AZ L A	1 3 5	0.0	0.0	0.000	6.0	-4.000	0.580	0.6400
				SPECIAL VARIANCE VALUES				0.0	0.000	6.0000	-4.000	0.580	0.2480
112	4	65	0	LM 1 1977	AZ C A	1 3 5	0.0	0.0	0.000	5.0	-4.500	0.600	0.2000
				SPECIAL VARIANCE VALUES				0.0	0.000	5.0000	-4.500	0.600	0.2480
113	0	65	0	SLACK 14000		0 1 1	640.0	980.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES				11025.0	0.000	0.0000	0.000	0.000	0.0000
114	0	65	75	GRAND CANYON AZ	(4025)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
115	7	70	0	LM 2 1977	AZ V A	1 3 5	0.0	0.0	0.000	34.0	-3.000	0.670	1.6900
				SPECIAL VARIANCE VALUES				0.0	0.000	34.0000	-3.000	0.670	0.2480
116	0	70	0	SLACK 14000		0 1 1	251.0	306.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES				174.0	0.000	0.0000	0.000	0.000	0.0000
117	0	70	75	LITTLEFIELD AZ	(4150)	0 0 0	0.0	0.0	0.000	0.0	0.000	0.000	0.0000
118	4	75	0	LM 4 1977	AZ C A	1 3 5	0.0	0.0	0.000	10.0	-4.500	0.600	0.0000
				SPECIAL VARIANCE VALUES				0.0	0.000	10.0000	-4.500	0.600	0.2480
119	4	75	0	LM 3 1977	AZ C A	1 3 5	0.0	0.0	0.000	115.0	-4.500	0.750	0.5600
				SPECIAL VARIANCE VALUES				0.0	0.000	115.0000	-4.500	0.750	0.2480
120	4	75	0	SOUTH NEV PROJ 1977		5 1 4	-42.0	0.0	0.000	0.0	0.000	0.000	0.0000
121	0	75	0	LAKE MEAD EVAP		0 1 1	-880.0	0.0	0.000	0.0	0.000	0.000	0.0000
122	0	75	0	SLACK 14000		0 1 1	542.0	306.0	0.000	0.0	0.000	0.000	0.0000
				SPECIAL VARIANCE VALUES				110889.0	0.000	0.0000	0.000	0.000	0.0000