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ABSTRACT

A chance-constrained separable programming model of water allocations between agriculture and energy production was developed in order to examine the effect of the variability of water supplies in Utah. Using an incomplete gamma function, based on method of moments estimation of parameters, the water flows at 85, 90, and 95 percent probabilities of occurrence were generated. These flows were then used as constraints in the allocation model. Results indicate that water quality could be a more significant constraint on irrigated agriculture than water quantity in the face of large scale energy development, and that variability of water availability alone is likely not to be a significant factor in economic growth in Utah.

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INTRODUCTION AND OBJECTIVES

Introduction

The interest demonstrated in developing Utah's energy resources (Snyder et al. 1981; Keith 1981) has brought to light certain issues which are of general importance to the state and of specific importance to public policy planners. It is critical to examine the effects energy development might have on air and water quality, and on water availability for agricultural production. It is of particular interest to examine what effects a highly variable water supply has on water use. Much of the planned water development in Utah is targeted at increasing water availability in low flow periods in order to assure production.

In this study, a methodology was developed to study the variations in water availability and to relate these variations to changes in agricultural and energy production and environmental externalities. The results provide a basis for the formulation of public policies that would optimize the state's development of its energy, water, and agricultural resources.

To accommodate both the public and private goals, the water management system should embody a strategy for efficiently and equitably apportioning available water under conditions of uncertainty. All hydrologic phenomena are subject to variations in quantity with some probability for periodic water shortage. These shortages may prevent the satisfaction of the entitlements of all water rights holders.

The firm, if it is to embark on a long run production in an activity that uses water extensively as an input, is interested in determining the probability of obtaining needed water and the acquisition cost at different probabilities. Depending upon the importance of water cost relative to the operation cost for the firm, it will decide whether to obtain its water through buying senior water rights, by filing for unappropriated water, or by some combination of both.

The use of water in Utah is supervised by the state. The state's responsibilities for the use of water are:

> The state must superimpose controls upon the initiation of uses, the exercise of water rights, the division of water among users, and the reallocation of water rights to new users as needs change. A modern water law system must not only promote the welfare of water users, it must accomplish the state's social and economic objectives, coordinate private activities with state projects, protect the interests of the public in common uses and environmental values, and integrate the activities of individual and corporate users into comprehensive state water plans for water development and management. (Trelease 1977, p. 388)

In Utah, the primary responsibilities in this area are detailed in the Utah Code Annotated, and the Division of Water Rights is assigned to carry out the above objectives. An allocation model

developed by Snyder et al. (1981) can be modified by incorporating probability constraints, which will provide some quantitative results with respect to the optimal water allocations under conditions of uncertainty. This model currently includes consideration of various energy developments and air and water quality, as well as two-season water availability.

Objectives

The purpose of this study is to examine the potential effect of variability of water supplies on water use in Utah. More specifically, the objectives are to:

1. Obtain the necessary data to determine the seasonal surface water availability in each of Utah's major drainages (called Hydrologic Sub Units or HSUs).

2. Develop a model for fitting the data to a probabilistic distribution.

3. Develop and verify the computer programs to obtain the probabilistic levels of surface water availability for each HSU based on a comparison between the actual data and the calculated probabilistic levels in each HSU.

4. Incorporate the probabilistic water data into the allocation model.

5. Analyze the results obtained from the allocation models with respect to their water policy implications.

MODEL DEVELOPMENT

The approach to developing the chance-constrained allocation model consisted of identifying an existing allocation model of Utah and altering its constraints to reflect water variability.

The Programming Model

The separable programming allocation model used in this study has been described in Snyder et al. (1981) in detail, and a mathematical description and data used may be found in the Appendix. Figure 1 is a schematic diagram of the model's structure. The model maximizes net profits to the agriculture and electrical energy sectors, subject to several resource availabilities, including water, and other constraints for Utah, the Green River Basin in Wyoming, and the Yampa and White River Basins in Colorado. Implicitly, the model assumes freely transferable water rights, even across state boundaries. Selected synthetic fuels industries are included in the model as are preservation of current wetlands, existing municipal and industrial water use and increases in municipal water demands associated with energy development. The State of Utah and Department of Energy projections of synfuels industries for 1990 include 177,000 barrel per day oil shale production in the White River drainage in Utah, 200,000 barrel per day oil shale production in the Piceance Basin in Colorado,, and 250 million cubic feet per day coal gasification plant in Emery County, Utah (Utah Consortium for Energy Research and Education 1981).

The water constraints are based on two seasons: January through June (the "high" runoff period) and July through December ("low" runoff period). Clearly, the aggregation of 6 months of flow contains an implicit assumption of stream regulation within each season. However, since much of the early runoff occurs during the growing season (April through June), the early period regulated availability is not a crucial problem. The late-season flow, however, may be more severe in any one month, week, or day than the modeling approach using 6 month averages suggests. Since the model is complex and large, including more than two flow periods increases research and computing costs. The two-season approach was felt to be a reasonable compromise by the research team, particularly since some on-site storage is anticipated for the synfuels and generation industries.

The Chance-Constrained Approach

The allocation model developed by Snyder et al. (1981) is a relatively large separable programming model, but the water constraints in the model can be relatively simply altered to a chance-constrained format in order to assess the effects of water variability. Chance-constrained programming as developed by Charnes and Cooper (1959, 1963) and described by Wagner (1975) and Hillier and Lieberman (1967) can be applied in a simplified way:

$$\text{Maximize} \quad \sum_{j=1}^{n} c_j x_j \qquad \ldots\ldots\ldots (1)$$

subject to

$$\sum_{j=1}^{k} a_{ij} x_j = b_i \qquad \ldots\ldots\ldots (2)$$

(first stage)

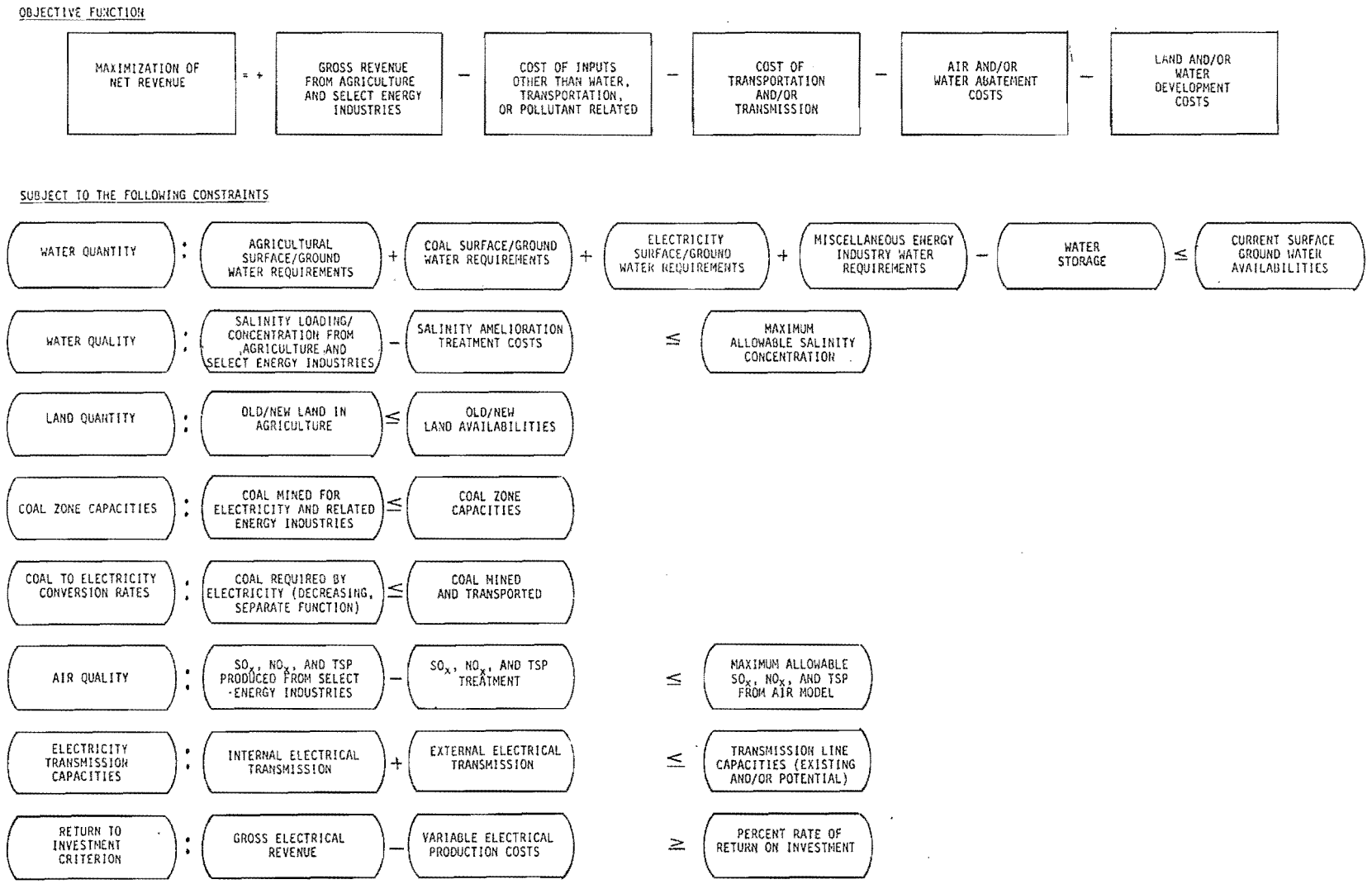

Figure 1. General economic feasibility model.

for $i = 1, \ldots, n$

and

$$P\left[\sum_{j=1}^{k} a_{ij} x_j \leq b_i\right] \geq p_i \quad \ldots\ldots (3)$$

(chance constraints)

for $i = n+1, \ldots, m$

and

$$\text{all } x_i \geq 0$$

where c_j are the objective function coefficients, x_j are the decision variables, a_{ij} are the constraint coefficients, b_i are the right hand side values, and p_i is the probabilities that the i^{th} constraint will be satisfied. There are j variables and i constraints. The chance constraints can be substituted by the deterministic equivalents:

$$\sum_{j=1}^{k} a_{ij} x_j \leq B_i \quad \ldots\ldots (4)$$

for $i = n + 1, \ldots, m$

where B_i is the largest number satisfying

$$P\left[b_i \leq B_i\right] \geq p_i \quad \ldots\ldots (5)$$

Thus, the water availability associated with a given probability level can be utilized as a right-hand-side. This gives a model that can be solved through the usual programming techniques. One of the problems with this technique is that excesses in the availability of B_i are not examined. However, for water allocations policies in Utah, excess water is not a crucial problem. The approach has been used successfully for a nonlinear, seasonal-stochastic model for water by Bishop and Narayanan (1977).

Hydrologic data fitting is a necessary first step to determine the right-hand-side associated with specific probabilities. Haan (1979) and Salas (1980) extensively examined various approaches and probability density functions for applicability. In choosing a probability density function, some thought has to be given to the availability of practical techniques for estimating its parameters.

In order to develop the chance constrained surface water availabilities, a theoretical model for their probability distribution had to be constructed. In many HSUs in Utah stream flow is regulated; furthermore, return flows play a significant role in downstream availability. For these reasons, it is not clear that stream flows recorded by many gaging stations accurately represent the variability of surface water production. Consequently, the theoretical models were developed so that the variability of headwater flows, defined by gaging stations above either storage facilities or significant consumptive use associated with human activities, was applied to total surface water availability. Thus, seasonal availabilities were "normalized" using headwater data.

In addition, offstream inflows in the basin are hard to measure since all records of precipitation are averaged over broad areas (climatological study units or CSUs) which do not have boundary resemblance to the HSUs (in fact, one CSU encompasses several HSUs) (Jeppson et al. 1968). Therefore, the extension of the headwater variability over the rest of the basin will yield an approximation that may be superior to any figure calculated from the integration of climatological data over the area of the HSU below the headstream measuring stations.

Normalization

For the i^{th} HSU, the total measured headwater flow is

$$TH_{ik} = \sum_{j=1}^{n} h_{ijk} \quad \ldots\ldots\ldots (6)$$

where h_{ij} is the j^{th} stream flowing into the HSU in season k in HSU i. This TH_{ik} is related to the measured water budget (WB_i) for the HSU through the expected value of TH_i and a parameter γ_i that will account for unmeasured headwaters and other runoffs into the HSU:

$$E(WB_i) = (1 + \gamma_i)E(TH_i)$$

$$= E[(1 + \gamma_i)TH_i] \quad \ldots\ldots (7)$$

In the best of cases, γ_i will be low, and, in general, we would expect that:

$$0 < \gamma_i < 1$$

In none of the HSUs in Utah do we get $\gamma_i < 0$ which would imply greater headwater flow than downstream flow. Given Equation 7, we also can obtain the variance of the water availability

$$V(WB_i) = E[(1+\gamma_i)TH_i - E[(1+\gamma_i)TH_i]]^2$$

$$= (1+\gamma_i)^2 E[TH_i - E(TH_i)]^2 \quad \ldots (8)$$

Thus, there are two descriptors of the water availability (mean and variance) and the surface water availabilities normalized for the sample years in each HSU:

$$x_{ik} = (1 + \gamma_i)TH_{ik} \quad \ldots\ldots (9)$$

Data Fitting

To fit the observed data for surface water availability to a probability density function, certain characteristics of the sample have to be determined. Among these are the range of the data, skewness, mean, and variance. Continuous distribution functions such as the normal, log normal, gamma, Weibull, and Gumbel are used in practice (Salas 1980).

The normal distribution is widely used when certain conditions hold, such as zero skew, symmetry, and tails that asymptotically approach zero as x approaches large and small values (Haan 1977). Given that hydrologic data are bounded at the low end ($x_i \geq 0$), the normal distribution is not a suitable distribution, particularly when the variance is large. The normal distribution can be used on skewed data if the data are transformed. Transformation is often done by using a log normal distribution with

$$y_i = \log(x_i) \quad \ldots\ldots\ldots (10)$$

where y_i is normally distributed with mean μ_y and variance σ_y^2. If the biases in the sample mean and variance are small, this is a good approach; but if the bias is large, it is not (Salas 1980). In the latter case, it is preferable to model the skewed series with the appropriate distribution. A three parameter lognormal function can be used in cases of skew ($Y_i = \log[X_i - c_i]$) according to Lettenmaier and Burges (1980). Given the possibility of large skews from the relatively sparce data sets for some headwaters in Utah, alternative functions were examined.

For extreme value distributions on bounded series ($x_i \geq 0$), such as occur with flooding peaks, the Gumbel and Weibull distributions are used. The Weibull is used for minimum values. The minimum values from a log normal follow this distribution closely. The Gumbel is used for maximum or minimum stream flow values (Haan 1979). These distributions are generally fit with extreme values in the sample and are not usually suited for overall modeling of a time series of flows.

A particular form of the Weibull that is used for hydrologic flows (Haan 1979) is the gamma distribution. This is a two-parameter distribution.

If necessary, a nonzero lower bound can be used, making it a three-parameter distribution. The gamma distribution has several advantages: assumption of a nonzero lower bound ($x_i > 0$), asymmetric distribution around the mode (positively skewed), a wide variety of shapes depending on the two parameters (α and β), and acceptance for use in annual or semiannual hydrological data (Haan 1979). Although the lognormal distribution has also been widely used, the gamma was selected for these reasons. There also is a transformation of gamma distribution data into a symmetrical distribution given by:

$$y = \sqrt{x} \quad \ldots\ldots\ldots\ldots (11)$$

but it is not an exactly normal distribution (Salas 1980).

By using the gamma distribution, we assume that the surface water availability (x) in each HSU has the density function:

$$f(x; \alpha, \beta) = \begin{cases} \dfrac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} & \text{for } x > 0 \\ & \alpha, \beta > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Then, for a desired probability level for the surface water availability, x*,

$$\int_0^{x^*} f(x; \alpha, \beta)dx = p \quad 0 < p \leq 1 \quad \ldots (12)$$

where p is the desired area under the tail of the distribution. This equation is also expressed as:

$$F(x^*; \alpha, \beta) = p \quad \ldots\ldots\ldots (13)$$

or by using the inverse function,

$$x^* = F^{-1}(\alpha, \beta, p) \quad \ldots\ldots\ldots (14)$$

With this expression, x* can be calculated when α and β are known. Since α and β are unknown, the alternative approach uses estimates of α and β from which a point estimate for x* is obtained.

Parameter Estimation

There are various methods to estimate α and β. Two methods that are widely used are the maximum likelihood and the method of moments (Haan 1979).

The maximum likelihood estimators are not unbiased; however, as the number of observations increases (tends to ∞), they become asymptotically unbiased. In addition, maximum likelihood estimators are sufficient and consistent. If an efficient estimator exists, the maximum likelihood estimator, after correction for bias, will be efficient. The method of moments will equate the first m moments of the distribution to the first m sample moments. Then the resultant m equations can be solved for m unknown parameters. Since only two parameters (α and β) are to be estimated, the first two moments have to be calculated. The method of moments will, in general, not always produce the same estimates as maximum likelihood. However, it is not always possible to obtain the maximum likelihood of estimators except through iterative numerical solutions. The accuracy of the method of moments can suffer if the moments are large. If a sample from the population is used, the estimates are not the most efficient (Kendall 1979).

By assuming we have n random observations, $x_1, \ldots, x_n$, then their joint probability function is $\phi_X(x, \alpha, \beta)$, and the likelihood function is:

$$L(\alpha, \beta) = \prod_{i=1}^{n} \phi_x(x_i; \alpha, \beta) \quad \ldots . (15)$$

Given that x is gamma distributed with parameters α and β, the joint

density function, $\phi(\hat{\alpha}, \hat{\beta})$, would be asymptotically normally distributed so that

$$\phi(\hat{\alpha}, \hat{\beta}) \sim N\left[\begin{bmatrix}\alpha \\ \beta\end{bmatrix}, \begin{bmatrix}\sigma^2_\alpha & \sigma^2_{\alpha\beta} \\ \sigma^2_{\alpha\beta} & \sigma^2_\beta\end{bmatrix}^{-1}\right] \quad . . .(16)$$

where

$$\sigma^2_\alpha = -E\left(\frac{\partial^2 \text{Log } L}{\partial\, \alpha^2}\right) \quad . . . . . .(17)$$

$$\sigma^2_{\alpha\beta} = -E\left(\frac{\partial^2 \text{Log } L}{\partial\, \alpha\, \partial\, \beta}\right) \quad . . . . . .(18)$$

$$\sigma^2_\beta = -E\left(\frac{\partial^2 \text{Log } L}{\alpha\beta^2}\right) \quad . . . . . .(19)$$

and L is the likelihood function. In this case,

$$L = \left|\frac{1}{\Gamma(\alpha)\beta^\alpha}\right|^n \prod_{i=1}^{n} (x_i^{\alpha-1} e^{-x_i/\beta}) \quad . . . .(20)$$

By obtaining the first order conditions with respect to α and β, we obtain the parameter estimates $\hat{\alpha}$ and $\hat{\beta}$. In practice, the expression used is:

$$\frac{\partial \ln L}{\partial \alpha} = \frac{L'_\alpha}{L} \quad . . . . . .(21)$$

and

$$\frac{\partial \ln L}{\partial\, \beta} = \frac{L'_\beta}{L} \quad . . . . . .(22)$$

where $L > 0$.

Now we obtain the maximum likelihood estimate of x* by the invariance property:

$$\hat{x}^* = F^{-1}(\hat{\alpha}, \hat{\beta}, p) \quad . . . . . .(23)$$

since $\hat{x}^*$ is a maximum likelihood estimate. Therefore, under general conditions, $\hat{x}^*$ is a consistent estimator of x*. Thus,

$$E(\hat{x}^*) = F^{-1}(\alpha, \beta, p) \quad . . . . . .(24)$$

As the number of observations tends to infinity, the variance of $\hat{x}^*$ becomes asymptotically zero.

This method is not used in the empirical model because of the difficulties in estimating $\hat{\alpha}$ and $\hat{\beta}$ and analytically differentiating the gamma function where α is unknown. Although the maximum likelihood estimation is preferred (Haan 1979), this study seems to be a case in which it is more practical to use the method of moments (Kendall 1977).

For the method of moments approach, a moment-generating function is defined. Then the first two moments are evaluated for t = 0 and equated to the sample moments.

The moment generating function (MGF) is given by:

$$M_x(t) = E(e^{tx}) = \frac{1}{\Gamma(\alpha)\beta^\alpha} \int_0^\infty e^{tx} x^{\alpha-1} e^{-x/\beta} dx$$

$$= \frac{1}{\Gamma(\alpha)\beta^\alpha} \int_0^\infty e^{-x(-t+1/\beta)} x^{\alpha-1} dx \quad . . .(25)$$

By manipulating this equation, the first and second ordinary moments can be evaluated at t = 0. The first ordinary moment is:

$$\left.\frac{d\, M_x(t)}{dt}\right|_{t=0} = M_1 = \alpha\beta(1-\beta t)^{-\alpha-1} = \alpha\beta \quad . . . . .(26)$$

and the second ordinary moment is:

$$\left.\frac{d^2M_x(t)}{dt^2}\right|_{t=0} = M_2 = \alpha\beta^2(\alpha+1)(1-\beta t)^{-x-2}$$

$$= \alpha\beta^2(\alpha+1) \quad \ldots\ldots(27)$$

Setting M_1 and M_2 equal to the sample moments, then

$$M_1 = \hat{\mu} = \hat{\alpha}\hat{\beta} \quad \ldots\ldots\ldots\ldots(28)$$

and

$$\hat{\sigma}^2 = M_2 - M_1^{2\,1} \quad \ldots\ldots\ldots(29)$$

The derivation of the variance equation leads to

$$\hat{\sigma}^2 = \hat{\alpha}^2\hat{\beta}^2 + \hat{\alpha}\hat{\beta}^2 - \hat{\alpha}^2\hat{\beta}^2 = \hat{\alpha}\hat{\beta}^2 \quad \ldots\ldots(30)$$

By simultaneously solving Equations 28 and 30, the estimates of $\hat{\alpha}$ and $\hat{\beta}$ for use in the gamma distribution are:

$$\hat{\alpha} = \frac{\hat{\mu}}{\hat{\beta}} \quad \ldots\ldots\ldots\ldots(31)$$

Since,

$$\beta^2 = \frac{\hat{\sigma}}{\hat{\alpha}} = \frac{\hat{\sigma}^2}{\frac{\hat{\mu}}{\hat{\beta}}} = \frac{\hat{\sigma}^2\hat{\beta}}{\hat{\mu}} \quad \ldots\ldots\ldots(32)$$

then

$$\hat{\beta} = \frac{\hat{\sigma}^2}{\hat{\mu}} \quad \ldots\ldots\ldots\ldots(33)$$

and, by substituting into Equation 31,

$$\hat{\alpha} = \frac{\hat{\mu}^2}{\hat{\sigma}^2} \quad \ldots\ldots\ldots\ldots(34)$$

Given a vector of desired probabilities, we can use Equation 13 to determine

$$F(x^*;\ \hat{\alpha},\ \hat{\beta}) = p_m \quad \ldots\ldots\ldots(35)$$

for m = 1, . . . , M

By expansion

$$\frac{1}{\Gamma(\hat{\alpha})\hat{\beta}^{\hat{\alpha}2}}\int_0^{x^*} x^{\hat{\alpha}-1}e^{-x/\hat{\beta}}dx = p_m \quad \ldots\ldots(36)$$

for m = 1, . . . , M

where the left hand side is the incomplete gamma function.

The incomplete gamma distribution can be transformed into a three-parameter distribution by the addition of a lower bound component. There are three possiblities for c: it can be zero (the two-parameter case); it can be calculated; and it can be the sample low flow (x_{min}). The latter alternatives might produce a better fit whenever the sample data are not close to zero.

Detailed explanations are given in Kendall (1979) on these estimation techniques. Maximum likelihood and method of moments estimations were used in this study because they are relatively simple and robust.

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[1]The sample's first and second moments are:

$$M_1 = \sum_i x_i/n\ ;\ M_2 = \sum_i x_i^2/n\ ;$$

and $\hat{\sigma}^2 = \Sigma x_i^2/n - (\Sigma\ x_i/n)^2$.

EMPIRICAL APPLICATION

In the original model, Utah was divided into various Hydrological Study Units (HSUs), listed in Table 1 and depicted in Figure 2. They form part of two major drainages: the Colorado River Basin and the Great Basin.

The Wyoming and Colorado drainages are treated as inflows to the Utah model, although each basin includes agricultural and energy-related production.

Data Collection

There are various sources of data for surface water availability but the primary source is the United States Geological Service (USGS) streamflow data, collected at stream gaging stations in each drainage. These data are

Table 1. Hydrological study units in Utah.

HSU No.	Basin Name	Drainage
1	Western Desert	Great Basin
2	Bear River	Great Basin
3	Ogden River	Great Basin
4	Jordan River	Great Basin
5	Sevier River	Great Basin
6	Cedar Beaver	Great Basin
7.1	Green River	Colorado River
7.2	Uintah River	Colorado River
7.3	Lake Fork	Colorado River
7.4	Rock Creek	Colorado River
7.5	Headwaters of Strawberry and Duchesne Rivers	Colorado River
7.W	White River in Utah	Colorado River
8.1	Price River	Colorado River
8.2	West of Colorado and East of Wasatch including the Colorado River inflows to Utah[a]	Colorado River
9	South and East of Colorado River	Colorado River
10	Virgin River	Colorado River
WY	Wyoming Inflow	Colorado River
CY	Colorado Yampa	Colorado River
CW	Colorado White	Colorado River

[a]Modeling the upper main stem would have required extensive data collection and was beyond the scope of this study.

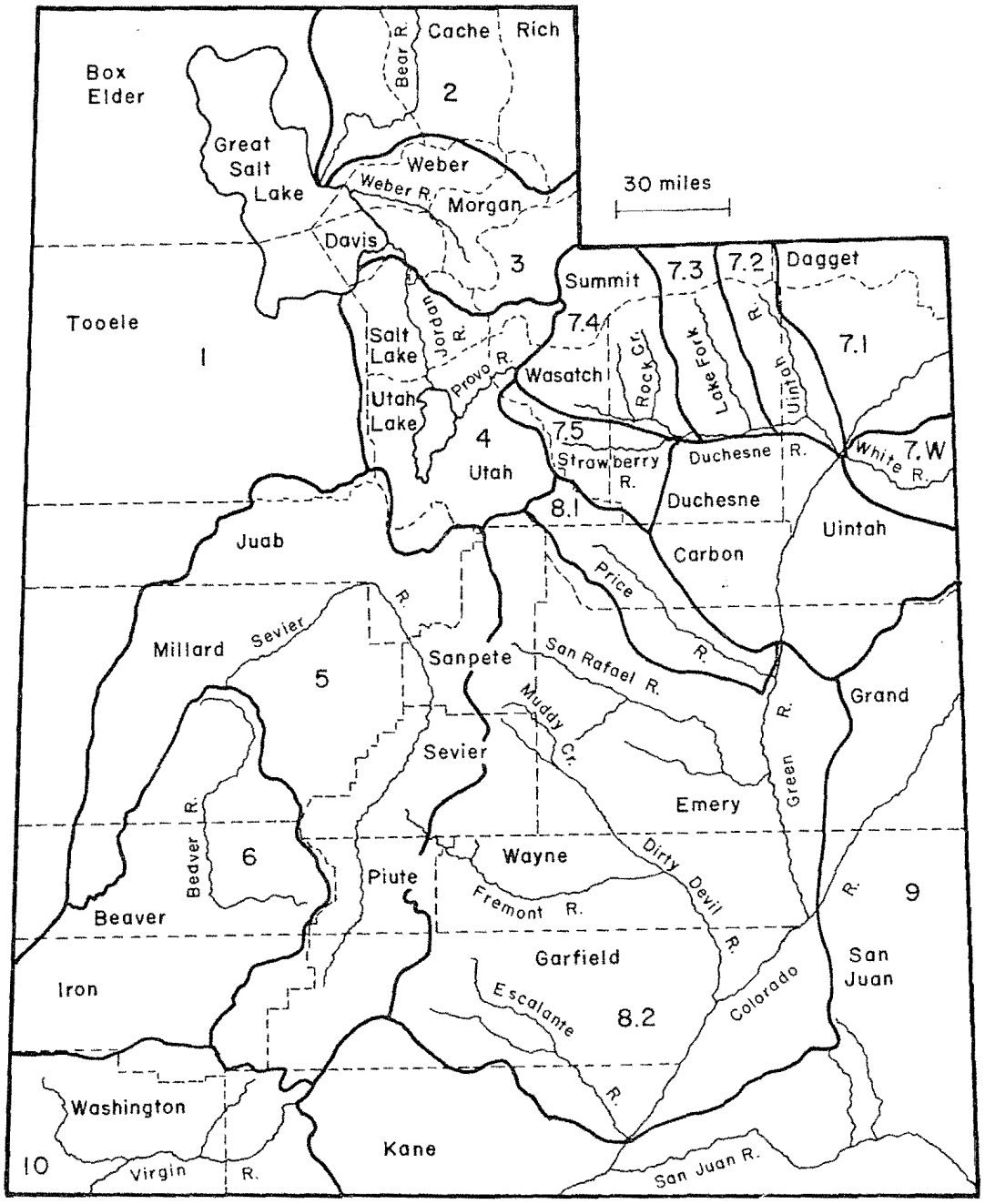

Figure 2. County boundaries, major drainage systems, and hydrologic study units of Utah.

readily available for most streams for a varying number of years at each station. The daily measurements reflect the precipitation less consumptive use upstream of the station. In addition to these data are the original sources of the surface water availability budgets for the HSUs as defined by King (1972). He added consumptive use to the existing flows and then compensated for recharge of groundwater to obtain estimates of average water availabilities. Given the needs of this study, the primary data source was the USGS streamflow data tape (WATSTORE) for the State of Utah, which covers both the Colorado River drainage and the Great Basin drainage. Table 2 indicates the average seasonal availabilities of surface water.

Empirical Model Development

The estimation of water availability from historical streamflow data was done in various steps. The first step was to extract the headstream flow data for each HSU from the USGS data tape. This was done in order to create a data file for each HSU. The second step was to accumulate the data for each season and normalize it against the average surface water availabilities found in King et al. (1972). Note that these "availabilities" are the production of surface water within each HSU. The programming model calculates outflows from each HSU to the immediate downstream HSU and adds those outflows to total water availability for the downstream HSU. Thus, reduced water flow in one HSU effects the total flow in every downstream HSU. The descriptive statistics were then calculated. The final step was to use alternative probability levels for each HSU by season and compare the distribution function against these levels to obtain the estimated availabilities. The last step was repeated under the various assumptions with respect to the intercept for the distribution. A flowchart of the system is shown in Figure 3. To preserve the integrity of the calculations in this last step, the subroutine MDGAM from the IMSL library was used to calculate the incomplete gamma function. The observed probabilities indicate any gross abnormality in fit.

Table 2. Average seasonal surface water availabilities by HSU (ac ft x 10^3).

HSU	Season 1 January-June	Season 2 July-December
1	424.85	188.15
2	519.37	413.63
3	445.78	320.06
4	273.00	265.69
5	196.60	213.40
6	41.30	37.70
7.1	2,216.60	1,148.80
7.2	166.74	92.91
7.3	685.39	360.09
7.4	314.08	168.81
7.5	296.85	286.64
7.W	21.00	9.00
8.1	122.45	79.45
8.2	4,829.70	1,820.20
9	1,427.70	714.25
10	173.49	70.12
WY	1,114.23	682.97
CY	967.00	483.50
CW	354.20	177.15

Probabilistic Water Availabilities

For all the HSU (except 1 and 4) the best overall fit was obtained by using a lower bound defined by the lowest observed flow. The availabilities for the two seasons were obtained for probabilities of 85 percent, 90 percent, and 95 percent, and are shown in Table 3. For HSU 1 (Western Desert) there were not enough measured data to account for the average water budget. Given the nature of the basins (arid, extensive, and subject to wide variations in rainfall over the basin), the average was assumed to be the best measure available. In HSU 4 (Jordan River) the surface water availability is

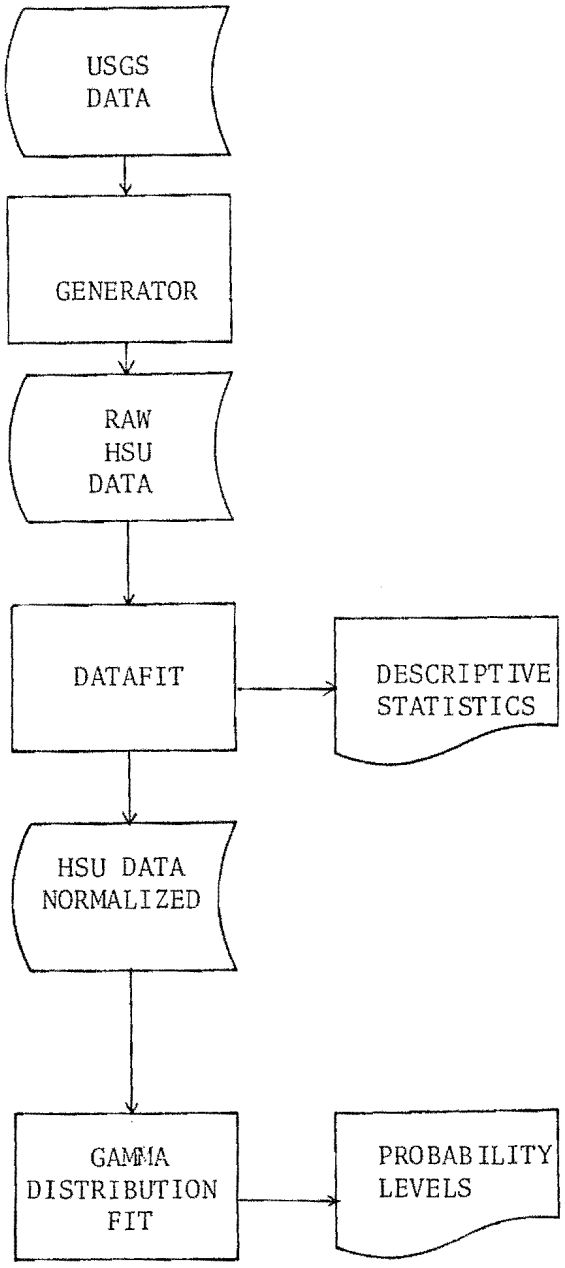

Figure 3. System flow chart.

so highly regulated that the measured low flow season water budget would be available under circumstances equivalent to 95 percent certainty of availabilities. It should also be noted that in HSUs 3, 5, and 6, and for the 90 percent and 95 percent probabilities in HSU 7.1, the "low" flow season has more water available than the "high" flow season. These are not the expected results. The anomolies are due to limited data sets for gaging stations above impoundments, although the average availability in HSU 5 follows the same pattern. It was determined that the estimated values should be used rather than using variances about the averages from King et al. (1972), partially because the study treated new storage, and existing storage in these regions is likely to cause similar allocations, and partially because there is little justification for applying estimated deviations to the average availabilities. The availabilities were then used as the right hand sides (upper limits) of water availability for each HSU in the programming model.

Table 3. Probabilistic seasonal surface water availabilities by HSU in Utah (acre/feet).

HSU	SEASON 1 (Jan-Jun)			SEASON 2 (Jul-Dec)		
	85%	90%	95%	85%	90%	95%
2	337210.	305143.	261619.	280956.	256891.	223907.
3	216960.	183642.	141265.	238393.	222640.	200633.
5	103440.	89154.	70651.	154378.	143215.	127709.
6	16898.	13731.	9869.	18660.	15858.	12278.
7.1	13663.	10337.	6579.	12410.	10815.	8726.
7.2	117229.	108039.	95351.	67859.	63092.	56456.
7.3	542198.	513427.	427734.	230242.	207615.	177011.
7.4	194441.	174025.	146612.	68496.	55550.	39791.
7.5	199298.	181741.	157739.	214183.	200920.	181466.
7.W	11835.	10366.	8433.	5489.	4896.	4103.
8.1	59580.	51440.	40880.	21234.	16086.	10263.
8.2	1231670.	893040.	52280.	1160030.	1045240.	890060.
9	658370.	550310.	414540.	342368.	288769.	220949.
10	51922.	39144.	34769.	43679.	39149.	33060.
CW	101305.	75536.	46891.	58409.	54077.	29686.
CY	357608.	283685.	194773.	169435.	132694.	89594.
WY	640798.	563848.	462177.	403930.	357742.	296340.

ALLOCATION EFFECTS OF WATER VARIABILITY

Initially, the programming model was run with average seasonal availabilities in order to have a basis for comparison with chance-constrained solutions. Current nondegradation policies dictated that the maximum average annual salinity levels for the base model were those which existed in 1972. These levels are consistent with the Federal Water Pollution Control Act Amendments (PL 92-500), the Colorado River Basin Salinity Control Act (PL 92-320) and the Colorado River Salinity Forum recommendations. Alternative water practices for irrigated agriculture were mitigating possibilities. The treatments considered were sprinkler irrigation and canal lining, both of which reduce salt loading from irrigated return flows. Franklin (1982) indicated that some publicly financed salinity controls could be justifiably implemented to reduce the impact of salinity restrictions on agriculture. However, with privately financed treatments, agricultural production was constrained in HSUs 1, 5, 7.4, 9 and 10 by the salinity levels.

For the lower surface water availability in the chance-constrained model at the 85 percent probability level with the salinity constraints in place, the solution was infeasible, because the salt loading could not be reduced sufficiently to meet the 1972 standard by any combination of privately financed treatments or retirement of land. The natural loading was not reduced proportionately to the decrease in water availabilities (Jeppson 1968), so that the salt concentration rose more than the elimination of agricultural loading could compensate. Clearly, the lower the availabilities, the more constraining the salinity standards were.

In PL 92-500, only long-term average annual salinity levels are expected to be maintained. The relaxation of these limits when water availability is, reduced is expected. Thus, a base case solution with no salinity constraints (Base NSC) was needed to separate the effects of salinity constraint relaxation from those of water reduction in the chance-constrained models.

There were some important differences between the Base NSC solution and the previous solution (Base). The agricultural land presently under irrigation (Class I, II, III, and IIIP) was increased in most cases to the current maximums. In addition, the amount of irrigation (partial irrigation to full) was also increased (see Table 4). These differences indicate that, as energy resources are developed, some irrigation will be reduced to compensate for increased salinity. Accompanying the increases in irrigation as the salinity constraint is relaxed, was the drop in the shadow price for water to zero in all HSUs except 5, 6, and 8.1. Agricultural production is the marginal use of water, that is, the value of marginal product for water is lower than for energy producers. Thus the two solutions indicated that electrical production would not change, since water was not constraining on electricity production. Some treatment was indicated in HSU 6, but no wide scale treatment was justifiable for private investment.

With reductions in surface water availabilities (to 85 percent, 90 percent, and 95 percent probabilities) there was no decrease in irrigated acres with the exception of HSUs 6, 7.4, 10,

Table 4. Changes in presently irrigated agriculture by HSU in Utah (acres).

HSU		Base	Base NSC	85% NSC	90% NSC	95% NSC
1	Western Desert	13803.	40000.			
2	Bear River	212000.	237548.			
3	Ogden River	144366.	144366.			
4	Jordan River	179478.	179478.			
5	Sevier River	272200.	282701.		[6800.][a]	[26700.]
6	Cedar Beaver	71500.	75866.	[3120.]	[3120.] (500.)[b]	[3120.] (1300.)
7.1	Green River	4600.	4600.			
7.W	White River, Utah	0.	0.			
7.2	Lake Fork	21000.	21000.			
7.3	Rock Creek	36000.	36000.			
7.4	Headwaters of Strawberry and Duchesne Rivers	27911.	27911.			(6220.)
7.5	Uintah River	20000.	20000.			
8.1	Price River	17944.	18000.		[700.]	[1100.]
8.2	West of Colorado and East of Wasatch	51510.	62500.			
9	South and East of Colorado River	9585.	11442.			
10	Virgin River	20300.	20300.	[3400.]	[3400.]	[3400.] (659.)
WY	Wyoming	184116.	251185.			
CY	Colorado Yampa	36374.	36374.			
CW	Colorado White	5753.	22371.	(5099.)	[200.] (5503.)	[1500.] (8664.)

[a] [acres] = acres reduced from full to partial irrigation.

[b] (acres) = acres eliminated from production.

and CW. A closer examination of the solutions showed that instead of reducing the acreage under irrigation, the model reduced application in some HSUs (full to either partial irrigation for two seasons or irrigation for one season only). As a result, the foregone profits from decrements to water supply increase as availability decreases, as seen in the increasing shadow price. Table 4 shows the base case solution, the base case with no salinity constraint (Base NSC) solution and the differences between this last solution and the chance-constrained solutions. As water availability was reduced, the shadow price remained at zero with the exception of HSUs 5, 6, 7W, CW, 7.4, 8.1, and 10 (Table 5). This is to be expected because all profitable agricultural land, even the marginally profitable, will be under some form of irrigation the water is available or salinity standards are relaxed.

As surface water availabilities were reduced, excess early runoff could be transferred to the second season through a storage activity. With one exception, storage was not indicated. Agricultural profits at the margin were not large enough to pay for the construction of storage facilities and electrical producers were able to "purchase" the existing water rights from agriculture by paying higher than the agricultural shadow prices. In HSU 8.1 (Price River) 620 and 6443 acre/feet of storage were indicated with the 90 percent and 95 percent probability model solutions, respectively. The second season shadow prices for water in HSU 8.1 were correspondingly quite high, compared to the other HSUs (Table 5).

Table 5. Shadow price of water ($ per ac ft).

		Season 1 (January-June)				Season 2 (July-December)			
HSU		Base Case	85%	90%	95%	Base Case	85%	90%	95%
5	Sevier River[a]	4.41	5.27	5.27	5.27				
6	Cedar Beaver	6.13	6.13	6.13	6.13				
7W	White River					0.00	0.00	6.34	19.87
CW	Colorado White					0.00	0.00	6.34	19.87
7.4	Headwaters of Strawberry and Duchesne Rivers					0.00	0.74	7.78	9.14
8.1	Price River	1.40	2.26	2.26	2.26	1.40	26.28	34.08	34.09
10	Virgin River	0.00	0.00	0.00	4.77				

[a]Note that since water availability is least in the early, or "high" flow, season in HSUs 5 and 6, the shadow price is positive in that season.

This was due to the water requirement for coal gasification in the HSU. Interestingly enough, it is in this drainage that some storage has been developed by Utah Power and Light.

The State of Utah plans to develop storage capacity on the White River to provide water for a 177,000 barrel per day oil shale industry, but no storage facility entered the model solution because of an implicit assumption of the free transfer of water rights from Colorado irrigators to Utah shale producers. For this reason, additional constraints were imposed to generate alternative solutions consistent with Utah Division of Water Resources planning. The current levels of irrigated agriculture in the White River Basin of Colorado were used as minimum constraints. Under this case, sufficient outflow was still available for shale, even at the 95 percent level of flows. Next, it was assumed that the Ute Indian water right in the White River (approximately 30,000 acre-feet) could not be utilized by the shale industry. A small amount of storage (less than 100 acre-feet) was indicated by the model. Finally, the lowest observed flows in the White River for any one month within each season for the past 60 years was used as the monthly flow, so that seasonal flows were equal to the lowest flow ever experienced in that season multiplied by 6. Under these constraints, approximately 10,000 acre-feet of storage was indicated in the model solution. These low flows were consistent with a 100 percent probability level, since this flow was used as a lower bound in the estimation of moments.

Electrical production does not change from the base case when the salinity constraints are relaxed (Base NSC) but with the water availability reductions, there is some shifting of production (see Table 6). For the 85 percent probability level for surface

Table 6. Changes in electrical production (MWH).

	Plant	Base	Base NSC	85% NSC	90% NSC
7	East Juab	10735200.	10735200.	46800.	46800.
8	East Basin	665780.	665780.		
9	Sanpete Sevier	2690040.	2690040.		
10	Warner Valley	2817149.	2817149.	(309223.)	(72006.)
11	Western Box Elder	1752000.	1752000.	(1687016.)	(1687016.)
12	Northwest Box Elder	3832398.	3832398.	243532.	6305.
15	Northeast Millard	5693816.	5693816.		
16	Milford-Black Rock	2944668.	2944668.		
17	Iron County	864578.	864573.		
18	Southwest Emery	750887.	750887.		
19	West Carbon	2295393.	2295393.		
20	East Carbon	1721545.	1721545.		
21	S. W. Emery	1147696.	1147696.		
22	East Grand	210220.	210220.		
N1	Harry Allen	723440.	723440.		
NM1	Star Lake	34063.	34063.	(124.)	(124.)
C1	Barstow	419629.	419629.	979134.	979134.
C2	Cadiz	6590086.	6590086.	707564.	707564.
W1	Kemmerer	3190997.	3190997.	19228.	19228.

water, a shift out of Western Box Elder to California plants at Barstow and Cadiz was indicated, as were some relatively smaller shifts within Utah. The reduction in total profit to irrigated agriculture in HSU 1, which is a function of tradeoffs among HSUs 1, 2, 3, and 4 in providing minimum inflows to the Great Salt Lake, is sufficiently high to make the combination of Barstow-Cadiz plants more profitable using New Mexico coal than the Box Elder plants using Utah coal. These shifts are the result of a very small difference in electrical generation profitability among the four plants which is offset by a small loss in agricultural profits. Whether such a shift would occur in reality is questionable. However, the similarity of electrical generation profitability among the plants is itself of interest.

The 90 percent probability level had only a minor adjustment between Warner Valley and Northwest Box Elder and the 95 percent model had no shifts in production sites. The profitability changes which were induced by such shifts are apparently exhausted at the 90 percent probability level.

CONCLUSIONS AND RECOMMENDATIONS

The results from the chance-constrained model indicate that, in general, water quantity is not a significant constraint in regional economic growth in Utah, particularly if water rights are relatively freely transferable. Even under the most severe case examined, that of a flow which is assured with 95 percent probability, only marginally significant changes in irrigated agriculture were evidenced, while major increases in the energy sectors from the current level were indicated. HSU 5, the Sevier River Basin, is the only basin with reductions in irrigation on more than a few thousand acres.

Water quality constraints, however, have a significant impact on agriculture, particularly given increasing energy development, since privately financed treatments appear to be economically infeasible for most farmers. Only a publically financed system, such as some proposed by the Salinity Forum, could be expected to reduce the concentrating effects of increased water withdrawals for energy use.

The development of storage also appears unwarranted in most cases, because the marginal user of water, agriculture, cannot afford to pay for development. However, storage facilities could be developed publically if it is deemed a desirable way to eliminate risk of water shortages for agriculture, even though the value of production is low. Under relatively stringent assumptions, some storage does enter the solution for the White River.

Results from the study appear to indicate that, at least in the near term, other impediments to large scale development may be much more important than water availability. It would be probable that irrigators could sell their water rights to energy producers at much higher prices than they could pay, as was the case of the Intermountain Power Plant. Alternative water applications, such as partial irrigation, might be expected to mitigate a part of the irrigation effects. A closer examination of the effects of nondegradation stream standards on both energy and agriculture appears to be the most crucial current water-related problem.

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APPENDIX

EMPIRICAL MODEL

(Taken from Snyder et al. 1981)

The Programming Model

The theoretical model can be applied by using mathematical programming. However, the optimality conditions determined for the theoretical model will change because activities become the "output" rather than products in the programming model. Thus, the Hicksian condition that the marginal rate of product transformation must equal the price ratio of those products holds only if each activity is directly related to a specific product. Given a functional relationship between activities and products, the optimality conditions determined previously will hold in general (Naylor 1966).

For large scale, complex problems such as this study examines, nonlinear classical programming is infeasible. For this reason a linear programming model is utilized in this study. A linear programming approach requires the acceptance of some rather stringent assumptions: 1) marginal and average costs are assumed constant and equal, and 2) average and marginal revenue are likewise constant and equal. With no resource constraints, production would either not occur or would be nonunique and unlimited. The use of demand and cost functions would be desirable, but the data required to estimate such functions is overwhelming. In the absence of such data, it will be assumed that the size of the existing and projected electrical power facilities proposed by the power companies are made in response to the actual and anticipated demand. The projected capacities will function as proxies for demand and will serve to constrain production accordingly.

The profit maximizing objective function will include agriculture and electrical power generation only. The basic model structure is adapted from Glover et al. (1979). The notation to be used is:

$$\text{Maximize } Z = \sum_{i=1}^{L}\sum_{j=1}^{M}\sum_{r=1}^{N} b_{ij}^{r} X_{ij}^{r} - \sum_{q=1}^{S}\sum_{r=1}^{N} \theta_{q}^{r} WA_{q}^{r}$$

$$- \sum_{q=1}^{S}\sum_{\substack{k=1\\k\neq r}}^{N}\sum_{r=1}^{N} d_{qk}^{r} MA_{qk}^{r} + \sum_{t=1}^{H}\sum_{r=1}^{N} W_{t}^{r} \Big|_{t}^{r}$$

$$- \sum_{h=1}^{G}\sum_{t=1}^{H}\sum_{k=1}^{T}\sum_{r=1}^{N} (\phi_{h}^{r} + \Pi_{ht}^{kr})\, Y_{ht}^{kr}$$

$$- \sum_{q=1}^{S}\sum_{r=1}^{N} \sigma_{q}^{r} WE_{q}^{r} - \sum_{q=1}^{S}\sum_{\substack{k=1\\k\neq 1}}^{N}\sum_{r=1}^{N} \mu_{qk}^{r} ME_{qk}^{r}$$

$$- \sum_{w=1}^{C}\sum_{r=1}^{N} J_{w} TR_{rw} - \sum_{x=1}^{D}\sum_{r=1}^{N} J_{x} TR_{rx} \qquad \ldots\ldots\ldots (21)$$

subject to:

Water Constraints

groundwater availability

$$\sum_{q=1}^{S} p_{qk}^{r}\,(k=r)\,(WA_{q}^{r} + WE_{q}^{r}) + \sum_{q=1}^{S} \vartheta_{q}^{r} WTLG_{q}^{r} - (RFA^{r}) \leq GW^{r}$$

$$r = 1,\ldots,N \qquad \ldots\ldots\ldots (22)$$

surface water availability

$$\sum_{q=1}^{S} WN_{q}^{r} (WA_{q}^{r} + WE_{q}^{r}) - \sum_{q=1}^{S}\sum_{\substack{k=1\\k\neq r}}^{N} V_{qk}^{r} (MA_{qk}^{r} + ME_{qk}^{r})$$

$$+ \sum_{q=1}^{S} p_{qk}^{r}\,(k=r)\, WTLS_{q}^{r} + S_{k}^{r} OF_{k}^{r}$$

$$+ \sum_{q=1}^{S}\sum_{\substack{k=1\\k\neq r}}^{N}\sum_{r=1}^{N} O_{qk}^{r} (EXA_{qk}^{r} + EXE_{qk}^{r})$$

$$- (1-\lambda)^{r} (RFA^{r} + RFE^{r}) \leq SW^{r} \qquad r = 1,\ldots,N$$

$$\ldots\ldots\ldots (23)$$

return flow from agriculture

$$\sum_{q=1}^{S} (1-\eta)_{q}^{r} WA_{q}^{r} + \sum_{q=1}^{S}\sum_{\substack{k=1\\k\neq r}}^{N} (1-\eta)_{qk}^{r} MA_{qk}^{r} - RFA^{r} = 0$$

$$r = 1,\ldots, N$$

$$\ldots\ldots\ldots (24)$$

wetland requirements

$$\sum_{q=1}^{S} J_{q}^{r} WTLS_{q}^{r} + \sum_{q=S+1}^{B} \nabla_{q}^{r} WTLG_{q}^{r} = WLREQ^{r}$$

$$r = 1,\ldots, N$$

$$\ldots\ldots\ldots (25)$$

Table 1. Variable notation.

Z	regional profit function
i	land class
j	type of crop grown
r,k	study region
α	seasons
q	source of water
b_{ij}	net revenue per acre of j^{th} crop grown on i^{th} land class exclusive of water cost
X_{ij}	j^{th} crop grown on land class i
θ_q	unit cost of water delivery from q^{th} source to agricultural use
WA_q	amount of water used by agriculture from q^{th} source
d_{qk}	unit cost of transferring water from q^{th} source in region k to agriculture
MA_{qk}	amount of water imported from q^{th} source in region k
h	raw energy product
t	converted energy product
W_t	price of the t^{th} final energy product
F_t	amount of the final energy product
ϕ_h	unit cost of extraction and conversion of h^{th} raw energy product
Π_{ht}	unit cost of transporting h^{th} raw product to t^{th} conversion process
Y^{rk}_{ht}	amount of h^{th} raw product transported to the t^{th} conversion process from the r^{th} region to the k^{th} region
σ_q	unit cost of delivering water from source q to energy use
WE_q	amount of water used from source q to energy
μ_{qk}	unit cost of transferring water from q^{th} source in region k to energy
ME_{qk}	amount of water imported from source q in region k to energy
W	agricultural effluents
∂_w	treatment cost for effluents from agriculture
TR_w	amount of agricultural effluent treated
X	energy effluents
O_x	treatment cost for effluents from energy
TR_x	amount of energy effluent treated
a_{ij}	percent of j^{th} crop grown on i^{th} land class
PIL_i	total acres of presently irrigable land of class i
$PCDL_i$	total acres of presently cultivated dry land of class i
$POIL_i$	total acres of potentially irrigable land of class i
$POCDL_i$	total acres of potentially cultivated dry land of class i
E_{ij}	rotational coefficient of j^{th} crop on i^{th} land class
$WTLS_q$	wetland requirements met from surface water of q^{th} source
$WTLG_q$	wetland requirements met from ground water of q^{th} source
$WTLREQ$	total wetland water requirements
F^r_h	amount of h^{th} raw energy product in the r^{th} region
I^r_h	total amount of h^{th} output in the r^{th} region
B_{ht}	efficiency of conversion process for h^{th} raw product
F_t	amount of t^{th} final energy product
EMA_h	amount of h^{th} raw energy material available
S_{ij}	consumptive use water requirements in acre feet per acre of the j^{th} crop on the i^{th} land class
η_q, η_{qk}	efficiency parameter of water used by agriculture
$(1-\eta)_q$, $(1-\eta)_{qk}$	return flow coefficients of water in agriculture
RFA	return flows from agriculture
GW	total amount of ground water available
OF_k	stream outflow of local surface water from region k
EXA_{qk}, EXE_{qk}	amount of water exported from source q in region k to agriculture and energy, respectively
λ	ground water recharge coefficient
$(1-\lambda)$	portion of return flow which augments surface water availabilities
SW	total amount of surface water available
EMA_w	agricultural w^{th} effluent emissions
EME_x	energy x^{th} effluent emissions
F_{th}	tons of effluent that occur from the use of the h^{th} raw product in the t^{th} conversion process
$NTEMA_w$	net effluent of w^{th} pollutant from agriculture
$NTEME_x$	net effluent of x^{th} pollutant from energy
$MAXEA_w$	allowable maximum for w^{th} effluent from agriculture
$MAXED_x$	allowable maximum for x^{th} effluent from energy

J_q, q, e_{ht}, P_h, F_h, M_h, n_h, U_h, P_q, ∂_q, P_{qk}, W_q, V_{qk}, F_q, S_k, O_{qk}, E_n, and E_m are the coefficients associated with given variables.

land availability

$$\sum_{j=1}^{M} a_{ij}^{r} X_{ij}^{r} \leq PIL_{i}^{r} \qquad \begin{matrix} i = 1,\ldots, L \\ r = 1,\ldots, N \end{matrix} \quad . \quad . \quad (26)$$

$$\sum_{j=1}^{M} a_{ij}^{r} X_{ij}^{r} \leq PCDL_{i}^{r} \qquad \begin{matrix} i = 1,\ldots, L \\ r = 1,\ldots, N \end{matrix} \quad . \quad . \quad (27)$$

$$\sum_{j=R+1}^{M} a_{ij}^{r} X_{ij}^{r} \leq POIL_{i}^{r} \qquad \begin{matrix} i = 1,\ldots, L \\ r = 1,\ldots, N \end{matrix} \quad . \quad . \quad (28)$$

$$\sum_{j=1}^{M} a_{ij} X_{ij}^{r} \leq POCDL_{i}^{r} \qquad \begin{matrix} i = 1,\ldots, L \\ r = 1,\ldots, N \end{matrix} \quad . \quad . \quad (29)$$

Agricultural Production

crop rotation

$$\sum_{j=1}^{M} E_{ij}^{r} X_{ij}^{r} \gtreqless 0 \qquad \begin{matrix} i = 1,\ldots, L \\ r = 1,\ldots, N \end{matrix} \quad . \quad . \quad (30)$$

agriculture water requirements

$$\sum_{i=1}^{L} \sum_{j=1}^{M} \delta_{ij}^{r} X_{ij}^{r} - \sum_{q=1}^{S} \eta_{q}^{r} WA_{q}^{r} - \sum_{\substack{k=1 \\ k\neq r}}^{N} \sum_{q=1}^{S} \eta_{qk}^{r} MA_{qk}^{r} = 0$$

$$r = 1,\ldots, N \quad . \quad . \quad (31)$$

Energy Production

intermediate energy flow and final outputs

$$\sum_{t=1}^{H} \sum_{k=1}^{T} \alpha_{dt}^{kr} Y_{ht}^{kr} - f_{ht}^{r} \Big|_{t}^{r} = 0$$

$$\begin{matrix} h = 1,\ldots, G \\ r = 1,\ldots, N \end{matrix} \quad . \quad . \quad (32)$$

efficiency of conversion process

$$\sum_{t=1}^{H} \sum_{k=1}^{T} \beta_{ht}^{kr} Y_{ht}^{kr} - M_{t}^{r} \Big|_{t}^{r} = 0$$

$$\begin{matrix} h = 1,\ldots, G \\ r = 1,\ldots, N \end{matrix} \quad . \quad . \quad (33)$$

capacity of the plants, resource availability, and transmission facilities

$$\eta_{t}^{r} \Big|_{t}^{r} \leq MEMA_{t}^{r} \qquad \begin{matrix} h = 1,\ldots, G \\ r = 1,\ldots, N \end{matrix} \quad . \quad . \quad (34)$$

energy water requirement

$$\sum_{t=1}^{H} U_{t}^{r} \Big|_{t}^{r} - \sum_{q=1}^{S} P_{qk}^{r} WE_{q}^{r} - \sum_{q=1}^{S} \sum_{\substack{k=1 \\ k\neq r}}^{T} P_{qk}^{r} ME_{qk}^{r} = 0$$

$$r = 1,\ldots, N \quad . \quad . \quad (35)$$

gross emissions

agriculture

$$\sum_{w=1}^{C} (EMA_{rw} - \Sigma E_{w}^{r} RFA^{r}) = 0$$

$$r = 1,\ldots, N \quad . \quad . \quad (36)$$

energy

$$\sum_{x=1}^{D} (EME_{rx} - \sum_{t=1}^{H} E_{xt}^{r} \Big|_{t}^{r})$$

$$r = 1,\ldots, N \quad . \quad . \quad (37)$$

emission treatment level

agriculture

$$\sum_{w=1}^{C} EMA_{rw} - \sum_{w=1}^{C} TR_{rw} - \sum_{w=1}^{C} NTEMA_{rw} = 0$$

$$r = 1,\ldots, N \quad . \quad . \quad (38)$$

energy

$$\sum_{x=1}^{D} EME_{rx} - \sum_{x=1}^{D} TR_{rx} - \sum_{x=1}^{D} NTEME_{rx} = 0$$

$$r = 1,\ldots, N \quad . \quad . \quad (39)$$

environmental constraints

$$\sum_{w=1}^{C} NTEMA_{rw} \leq \sum_{w=1}^{C} MAXE_{A_{rw}}$$

$$r = 1,\ldots, N \quad . \quad . \quad (40)$$

$$\sum_{x=1}^{D} NTEME_{rx} \leq \sum_{x=1}^{D} MAXE_{D_{rx}}$$

$$r = 1,\ldots, N \quad . \quad . \quad (41)$$

Definition of variables and terms:

i = class of land (I, II, III, IV, ...)

j = type of crop grown

r,k = study regions

q = source of water (present surface and groundwater and new development surface and groundwater, etc.)

b_{ij}^{r} = net revenue associated with one acre of the j^{th} crop grown in the i^{th} class of land in region r, excluding water costs

θ_{q}^{r} = unit cost of delivering water from q^{th} source in region r to agriculture use

WA_q^r = amount of water used by agriculture from q^{th} source in region r to agriculture use

d_{qk}^r = unit cost of transferring water from region k to region r of q^{th} type (present and new transfer for agriculture use)

MA_{qk}^r = amount of imported water from region k to region r of q^{th} type for agriculture use

a_{ij}^r = the coefficient associated with X_{ij}

PIL_i^r = available total acres of presently irrigable dry land i in region r

$PCDL_i^r$ = available total acres of presently cultivated dry land i in region r

$POIL_i^r$ = available total acres of potentially irrigable land i in region r

$POCDL_i^r$ = available total acres of potentially cultivated dry land i in region r

E_{ij}^r = the rotational coefficient of the j^{th} crop with i^{th} land class in region r

$WTLS_q^r$ = wetland water requirements met from surface water of q^{th} type in region r

$WTLC_q^r$ = wetland water requirements met from groundwater of q^{th} type in region r

$WLREQ^r$ = wetland water requirements in region r

X_{ij}^r = j^{th} crop acreage grown in i^{th} land class in region r

h = the raw energy product (coal, crude oil, tar sands, oil shale, natural gas, etc.)

t = the converted energy product (gasified coal, liquified coal, coal slurry, electricity, refined oil, etc.)

W_t^r = price of the t^{th} final product in region r

F_t^r = amount of the t^{th} final product in region r

ϕ_h^r = unit cost of extraction and conversion of the h^{th} energy product in region r

Π_{ht}^{kr} = unit cost of transporting the h^{th} product to the t^{th} conversion process plant from region r to region k

Y_{ht}^{kr} = amount of the h^{th} product transported to the t^{th} conversion process plant from region r to region k

σ_q^r = unit cost of delivering water from source q to energy use in region r

WE_q^r = amount of water used from source q to energy use in region r

μ_{qk}^r = unit cost of transferring water from source q in region k to energy use in region r

ME_{qk}^r = amount of imported water from source q in region k to energy use in region r

f_{ht}^r = input requirements for the h^{th} input per unit of the t^{th} output

$|_t$ = total amount of t^{th} final output in region r (note that for some regions, a final output may be a raw energy product)

β_{ht}^{kr} = the efficiency of the t^{th} conversion process for the h^{th} raw product in region r

$MEMA_t^r$ = amount of the t^{th} energy material available in region r

ℓ_{ht}^{kr} = the coefficient associated with Y_{ht}^{kr}

α_i = the augmented M & I water requirements

δ_{ij}^r = consumptive use water requirements per acre in acre feet of j^{th} crop in the i^{th} land class in region r

η_q^r, η_{qk}^r = efficiency parameter of water use by agriculture in region r

g_t^r = consumptive use water requirement in acre feet to produce one unit of the t^{th} energy product

RFA^r, RFE^r = return flows from agriculture and energy in region r, respectively

GW^r = total amount of groundwater available in region r

SW^r = total amount of local surface water available in region r

OF_k^r = stream outflow of local surface water from region r to k

$WTLG_q^r$ = wetland requirement taken from groundwater availability in region r

$WTLS_q^r$ = wetland requirement taken from local surface water availability in region r

EXA^r_{qk}, EXE^r_{qk} = amount of water exported from source q in region r to agriculture and energy production in region k, respectively

λ^r = the recharge coefficient of groundwater from return flow in region r

$(1-\lambda)^r$ = the recharge coefficient of local surface water from return flow in region r

$(1-\eta)^r$ = the return flow coefficients of agriculture and energy in region r, respectively

$J^r_q, Y^r_q, S^r_k, M^r_t, N^r_h, U^r_t, P^r_{qk}, \partial^r_q, WN, V^r_{qk}, \Gamma^r_q, O^r_{qk}$ = the efficiency of use coefficients associated with the given activity in region r

E^r_w = emission rate for the w^{th} pollutant from agricultural return flow in region r

E^r_{xt} = emission rate for the x^{th} pollutant from the t^{th} energy product in the r^{th} region

x = energy effluents

w = agricultural effluents

EMA_{rw} = agricultural w^{th} emissions for the r^{th} region

$J_{w(or\ x)}$ = cost of treatment per unit emissions of the w^{th} or x^{th} pollution

EMC_{rx} = energy emissions of x^{th} pollution for r^{th} region

$TR_{rw(or\ x)}$ = treatments of w^{th} or x^{th} pollutant for region r

$NTEMA_{rw}$ = net effluent of w^{th} pollutant for agriculture in region r

$NTEME_{rx}$ = net effluent of x^{th} pollutant for energy for region r

$MAXE_{A_{rw}}$ = allowable maximum for the w^{th} effluent for agriculture in region r

$MAXE_{E_{rx}}$ = allowable maximum for the x^{th} effluent for energy in region r

Note that the maximum for each sub or superscript can vary as the scope of the model is expanded or narrowed.

The Electricity Sector

The following equations further detail the electrical generation sector and associated coal activity.

Objective function

$$\sum_k \sum_r \sum_d EPROFIT^k_{dr} + \sum_k \sum_r \sum_{d=1} \sum_{m=1}^{R} \left(W^r_c \, CM^{km}_{dr} - \sum_{m=1}^{R} \phi^r_c \, CLCST^{km}_{dr} \right) \quad . \quad . \quad . \quad . \quad . \quad (42)$$

electricity profit

$$\sum_k \sum_r \left(W^k_e \, ELEC^k_{dr} - \phi^k_e \, ELEC^k_{dr} - W^k_c \, CM^{km}_{dr} - \sum_z \Pi^{rk}_{ce} \, CT^{km}_{drz} - \sum_{z=1}^{Z} \Pi^{rk}_{ee} \, TRM^k_{dr} - \theta^{dk}_q \, ENWREQ - J_{ex} \, TR_{exr} \right) = \sum_k \sum_r EPROFIT^k_{dr} \quad . \quad . \quad (43)$$

coal requirement

$$ELEC^k_{dr} - \sum_{m=1}^{R} \sum_{z=1}^{Z} \beta^{rk}_{ce} \, CM^{km}_{dr} = 0 \quad . \quad . \quad . \quad . \quad (44)$$

(Note that the coal requirement, or conversion ratio, may vary for each coal source, but it is constant for a given coal source.)

water requirement

$$g^r_e \, ELEC^k_{dr} = ENWREQ^k_{dr} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

(Includes M and I augmented water requirement.)

electricity transmission (each MWH produced must be transmitted)

$$ELEC^k_{dr} = TRM^k_{dr} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

coal transport

$$\beta^{rk}_{ce} \, ELEC^k_{dr} = \sum_{z=1}^{Z} CT^{km}_{drz} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

demand constraints

$$\sum_{r=1}^{N} \sum_{d=1}^{Q} ELEC^k_{dr} \leq DMND_k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

coal mining constraints

$$\sum_{z=1} \sum_{d=1}^{Q} \sum_{k=1}^{N} CT^{km}_{drz} = CM^m_r \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

$$CM^m_r \leq CMMAX^m_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

coal transportation constraints

$$CT^k_{drz} \leq CTMAX^{km}_{drz} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

transmission constraints

$$\sum_{d=1}^{Q} \sum_{r=1}^{N} TRM^k_{dr} \leq TRMMAX^k_{drz} \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

gross pollution

$$\sum_{k=1}^{N} E_{xe} \, ELEC_{dr}^{k} + W_k CM_{dr}^{km} = EME_{xer} \quad . \quad . \quad . \quad (53)$$

(Note the adjustment of pollutant produced based on the coal used as compared to a standard coal.)

treatment of pollution

$$EME_{xer} - TR_{exr} = NTEME_{exr} \quad . \quad . \quad . \quad . \quad . \quad (54)$$

(Treatment levels are incremented.)

maximum allowable pollution

$$NTEME_{exr} \leq MAXE_{E_{exr}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

fixed investment levels (by plant)

$$\sum_{n=1}^{N} N_{(trm)} \, TRM_{dr}^{k} + N_{(trt)} \, TR_{dr} + \sum_{k=1}^{N} N_e \, ELEC_{dr}^{k} = INV_{edr} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

(Fixed investment is determined for output (per MWH), transmission (per MWH), and treatment (per ton removed).)

minimum profitability constraint

$$\sum_{k=1}^{N} EPROFIT_{dr}^{k} - PCT_e \, INV_{edr} \quad . \quad . \quad . \quad . \quad (57)$$

(A plant must meet or exceed an exogenously specified return to fixed investment.)

Super and subscripts are the same as those listed above with the following exceptions:

e,c = electricity and coal production, respectively (would be subsumed under subscript t or h)

d = electricity plant identification number ($d = 1, \ldots, Q$)

m = mine identification number ($m = 1, \ldots, R$)

N_e = investment cost per MWH produced

$N_{(trm)}$ = investment cost of transmission per MWH

$N_{(trt)}$ = investment cost per ton of pollutant treated

PCT_e = established rate of return on investment for electrical generation

z = coal transportation route and/or type ($z = 1, \ldots, Z$)

w_k = coefficient of pollution adjustment for each coal source

$EPROFIT_{dr}^{k}$ = profit to the d^{th} plant from the r^{th} region from sales to the k^{th} region

CM_{dr}^{km} = coal mined in the m^{th} mine in the r^{th} region sent to the d^{th} electrical plant in the k^{th} region

$CLCST_{dr}^{km}$ = cost of coal from the m^{th} mine in the r^{th} region to the d^{th} plant in the k^{th} region

$ELEC_{dr}^{k}$ = electrical production in the d^{th} plant in the r^{th} region sold in the k^{th} region

CT_{drz}^{km} = coal transportation from the m^{th} mine in the r^{th} region to the d^{th} plant in the k^{th} region by the z^{th} route

$ENWREQ_{dr}$ = water required for the d^{th} plant in the r^{th} region

TRM_{drz}^{k} = transmission of electricity from the d^{th} plant in the r^{th} region to the k^{th} region by the z^{th} route

$DMND_k$ = maximum demand for electricity in region k

$CMMAX_r^m$ = maximum coal available annually from the m^{th} mine in the r^{th} region

$CTMAX_{drz}^{km}$ = maximum transportation capacity of the z^{th} route to the d^{th} plant in the k^{th} region from the m^{th} mine in the r^{th} region (note that the capacity may involve sums of transport in some cases)

$TRMMAX_{drz}^{k}$ = maximum electrical transmission capacity of the z^{th} line from the r^{th} region to the k^{th} region

INV_{edr} = investment cost of the d^{th} electrical plant in the r^{th} region

θ_q^{dk} = cost of water from source q by plant k

W_e^k = gate price of electricity at plant k

ϕ_e^k = variable cost of production excluding coal, water from and pollution treatment at generating plant k

Π_{ce}^{rk} = transport cost for coal from mine C in region r to plant k

Π_{ee}^{rk} = transmission cost for energy from plant k to region k

The coal sector is composed primarily of the revenues from mine mouth sales less the

production costs in the objective function (Equation 42) and the water requirements associated with coal mining (almost entirely M and I demand increases). For each coal source, there exists a conversion rate to electricity based on a 10,000 Btu heat rate adjusted for coal quality (Equation 44). This configuration implicitly assumes a constant conversion rate for each coal irrespective of plant size at any one site. Since coal conversion rates are the major component of production cost savings to larger plants (i.e., decreasing production costs and plant size increases) the model assumes constant cost production relationship for a given coal source.

Input-Output Model

Final demand (export)

$$FD_{ar} + \alpha_{ar} W_t^r |_t^r + W_t^r X_{ij}^r = TFD_{ar} \quad . \quad . \quad . \quad (58)$$

Input-output constraints

$$\sum_{a=1}^{A} \sum_{b=1}^{B} \psi_{abr} TFD_{ar} = RGO_{ar} \quad . \quad . \quad . \quad . \quad (59)$$

Regional gross output

$$\sum_{a=1}^{A} RGO_{ar} = TRGO_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

in which

a,b = economic sector

FD_{ar} = existing final demand in the a^{th} sector in the r^{th} region

TFD_{ar} = total (augmented) final demand in the a^{th} sector of the r^{th} region

RGO_{ar} = regional gross output (sales) in the a^{th} sector of the r^{th} region

$TRGO_r$ = total regional gross output in the r^{th} region

ψ_{abr} = proportion of each dollar of output sold to the b^{th} sector by the a^{th} sector

Objective Function Coefficients

Water Costs

Water costs (Table 2) specific to each HSU were obtained from King (1972) and Glover et al. (1980) and updated to 1977 prices using irrigation and water cost indices found in the Engineering News Record (1978). The cost per acre foot of delivering water to agricultural production for both existing and new water are included for surface water sources as well as groundwater supplies. The cost per acre foot of water imports, both present and new, are also shown for each HSU. Similar information is included for the energy sector.

Agricultural Costs and Revenues

Net revenue coefficients from agriculture for the entire 1977 season are shown in Table 3. Crop productivities by county had been previously determined by Christensen et al. (1973b) and updated by Davis et al. (1975). Productivity rates by HSU were then multiplied by appropriate crop prices[1] to determine gross revenue per acre. Variable costs (Glover et al. 1979; U.S. Department of Agriculture 1978a), excluding water transfer and application costs, were then subtracted from gross revenue figures to determine net revenue on a per acre basis.

In order to determine the impact of seasonal water availabilities, seasonal net revenue was also computed (Table 4) by assuming productivity to be directly proportional to the quantity of water consumptively used (Office of the State Engineer 1962). For example, if alfalfa consumed 31 percent of its annual water requirement within the first 6 months, productivity was assumed to be 31 percent of the annual production rate also. Production costs were divided between seasons proportional to the growing periods for all crops except barley and nurse crops where costs were assumed proportional to production.

The new land development costs on an annual basis shown in Table 5 were obtained from Keith et al. (1978) and modified utilizing information from the U.S. Department of Agriculture (1969a, 1969b, 1978b) and the Engineering News Record Construction Index (1978). While these costs include charges for land clearing and leveling, no attempt has been made to include the expenditures necessary to raise the actual productivity of the new land to a level consistent with land currently under production. The net revenue associated with new agricultural land was therefore, at a level somewhat higher than would actually prevail in the market. Another possible complication is that land ownership, whether state, federal, or private, is not taken into account so that all land suited for crop production is made available for production. These development costs were then subtracted from both the full-season and partial-season net revenue figures to determine the net revenue for new land development.

[1]An 8-year price average was determined for each crop to eliminate the annual variability which often is found in agricultural prices.

Table 2. Cost components for supplying water to agriculture and energy in Utah for 1977 (annual cost in $/ac-ft).

	AGRICULTURE								ENERGY							
	Local Surface Water		Groundwater		Surface Water Transfers				Present Water		New Water		Surface Water Transfers			
HSU	Present	New	Present	New	To HSU	Present	To HSU	New	Surface	Ground	Surface	Ground	To HSU	Present	To HSU	New
1	2.27	15.77	3.74	4.55	4	5.58			51.09	72.13	135.39	126.98	4	65.53		
2	2.27	14.30	5.28	6.02			1	60.87	51.09	72.13	135.39	126.98			3	210.263
							3	77.55								
3	2.27	16.50	6.02	6.75	2	5.58	4	66.90	60.11	90.16	177.32	148.43	4	63.66	4	246.298
					4	3.76										
4	2.27	15.77	8.22	9.02			5	55.17	60.11	90.16	177.32	148.43			5	246.298
5	2.27	14.30	5.28	6.02			6	67.63	54.10	72.13	156.28	126.98			6	200.637
6	2.27	13.57	6.75	7.48					54.10	72.13	156.28	126.98				
7.W	2.27	15.77	3.08	3.74	4	3.76			54.10	72.13	156.28	126.98	4	63.66		
CW	2.27	15.77	3.08	3.74	4	3.76			54.10	72.13	156.28	126.98	4	63.66		
CY	2.27	15.77	3.08	3.74	4	3.76			54.10	72.13	156.28	126.98	4	63.66		
WY	2.27	15.77	3.08	3.74	4	3.76			54.10	72.13	156.28	126.98	4	63.66		
7.1	2.27	15.77	3.08	3.74	4	3.76			54.10	72.13	156.28	126.98	4	63.66		
7.2	2.27	15.77	3.08	3.74	4	3.76			54.10	72.13	156.28	126.98	4	63.66		
7.3	2.27	15.77	3.08	3.74	4	3.76	3	95.58	54.10	72.13	156.28	126.98	4	63.66	3	264.930
							4	84.91					7.4	2.27	5	248.609
							5	79.21								
							7.4	2.27								
7.4	2.27	15.77	3.08	3.74	4	3.76			54.10	72.13	156.28	126.98	4	63.66		
7.5	2.27	15.77	3.08	3.74	4	3.76	4	75.91	54.10	72.13	156.28	126.98	4	63.66	5-Ute	254.682
															5-Bonn	245.359
8.1	2.27	15.77			5	5.58			66.12		201.37		5	63.66		
8.2	2.27	15.77			5	5.58	4	78.92	66.12		201.37		5	63.66	4	248.466
							5	52.16							5	248.466
9	2.27	15.77							60.11		177.32					
10	2.27	15.77	3.74	4.55	6	3.76	6	67.63	60.11	72.13	177.32	126.98	6	57.447	6	218.344

Table 3. Net revenue for full season agricultural production by land class and HSU in Utah, 1977, ($/acre).

Crop	Land Class I	II	III	IV	Land Class I	II	III	IV	Land Class I	II	III	IV
	HSU #1				HSU #2				HSU #3			
Alfalfa (Full)	85.40	67.23	57.19		98.39	78.20	60.82		103.46	92.57	73.39	
Alfalfa (Partial)	60.51	44.65	41.93		75.03	62.05	58.38		80.11	69.96	68.06	
Barley	97.72	76.98	64.44		99.30	83.70	69.86		100.30	92.08	72.60	
Nurse Crop	47.92	31.39	23.51		58.00	42.58	32.69		45.34	50.03	38.30	
Corn Grain	111.02	76.35	40.52		104.92	73.68	37.06		104.92	73.68	78.39	
Corn Silage	192.02	180.18	149.52		183.88	178.73	155.47		185.01	184.98	168.08	
Apple (N)[a]	576.96	518.03	435.54		569.32	544.50	432.90		569.32	512.38	432.90	
Apple (M)[b]	2912.76	2380.18	2228.99		2906.20	2374.68	2223.25		2906.20	2374.68	2223.25	
Peach (N)	596.12	471.21	399.20		592.90	469.84	393.58		571.30	469.84	393.58	
Peach (M)	2642.94	2228.51	2089.35		2633.31	2224.39	2085.92		2633.31	2224.39	2085.92	
St. Cherry (N)	(143.07)[d]	(132.41)	(133.94)		(147.64)	(136.93)	(136.22)		(147.64)	(136.93)	(136.22)	
St. Cherry (M)	857.52	634.00	432.78		845.70	628.84	427.74		845.70	628.84	427.74	
Sr. Cherry (N)	205.80	99.34	85.70		141.90	98.48	83.08		141.90	98.48	83.08	
Sr. Cherry (M)	1261.03	1030.16	965.43		1256.20	1027.15	959.70		1256.20	1027.15	959.70	
Dry Wheat				11.70				11.12				15.20
Dry Beans												
Alfalfa (Full)-(P)[c]			37.94				32.50				44.08	
Alfalfa (Partial)-(P)			27.27				19.88				28.00	
Barley-(P)			28.72				36.65				46.85	
Nurse Crop-(P)			9.60				14.19				17.10	
	HSU #4				HSU #5				HSU #6			
Alfalfa (Full)	105.16	88.66	71.81		77.41	64.96			92.54	69.91	70.60	
Alfalfa (Partial)	81.80	68.83	56.39		53.88	45.67			67.58	63.06	60.62	
Barley	100.30	89.93	73.41		75.93	61.10			90.78	76.62	62.88	
Nurse Crop	55.34	49.86	36.19		30.75	19.45			32.85	31.45	21.55	
Corn Grain	122.15	70.99	44.55		69.48	33.71			111.44	65.89	28.59	
Corn Silage	185.01	183.49	157.57		188.27	163.44			208.18	188.64	172.75	
Apple (N)	569.32	512.38	432.90									
Apple (M)	2906.20	2374.68	2223.25									
Peach (N)	571.30	469.84	393.58									
Peach (M)	2633.31	2224.39	2085.92									
St. Cherry (N)	(147.64)	(136.93)	(136.22)									
St. Cherry (M)	845.70	628.84	427.74									
Sr. Cherry (N)	141.90	98.48	83.08									
Sr. Cherry (M)	1256.20	1027.15	959.70									
Dry Wheat				8.52				8.24				8.68
Dry Beans												
Alfalfa (Full)-(P)			40.31				40.63				40.16	
Alfalfa (Partial)-(P)			28.58				22.48				26.16	
Barley-(P)			37.06				34.86				32.22	
Nurse Crop-(P)			15.34				10.28				12.46	

Table 3. Continued.

Crop	Land Class I	II	III	IV	Land Class I	II	III	IV	Land Class I	II	III	IV
	HSU #7.$_i$ (i=1,...,5,W,CW,CY, and WY)				HSU #8.$_j$ (j=1,2)				HSU #9			
Alfalfa (Full)		72.28	65.61		97.54	87.93	80.41		110.23	86.24	73.65	
Alfalfa (Partial)		59.31	43.37		73.59	69.89	72.10		84.59	55.08	39.53	
Barley		78.20	72.55		94.19	84.30	70.08		102.64	79.46	62.70	
Nurse Crop		30.56	18.86		44.63	36.65	26.33		51.16	31.81	18.90	
Corn Grain		75.06	30.26		97.02	62.26	35.46		76.14	51.82	30.26	
Corn Silage		185.47	156.98		190.41	184.48	161.80		188.29	188.87	163.08	
Apple (N)[a]												
Apple (M)[b]												
Peach (N)												
Peach (M)												
St. Cherry (N)												
St. Cherry (M)												
Sr. Cherry (N)												
Sr. Cherry (M)												
Dry Wheat				7.48				7.56				5.22
Dry Beans												33.14
Alfalfa (Full)-(P)			46.80				52.87				48.31	
Alfalfa (Partial)-(P)			21.70				43.05				23.81	
Barley-(P)			35.05				32.84				23.62	
Nurse Crop-(P)			11.16				14.37				11.98	

	HSU #10		
Alfalfa (Full)	143.59	103.40	70.69
Alfalfa (Partial)	69.45	47.75	31.39
Barley	85.54	73.24	57.80
Nurse Crop	32.59	24.12	12.68
Corn Grain	145.74	98.26	65.06
Corn Silage	241.50	244.79	195.84
Apple (N)	520.72	472.32	402.14
Apple (M)	2867.14	2295.90	2162.77
Peach (N)	558.76	441.69	371.98
Peach (M)	2574.10	2175.21	2048.17
St. Cherry (N)			
St. Cherry (M)			
Sr. Cherry (N)			
Sr. Cherry (M)			
Dry Wheat			9.98
Dry Beans			
Alfalfa (Full)-(P)			23.00
Alfalfa (Partial)-(P)			3.03
Barley-(P)			17.71
Nurse Crop-(P)			(0.86)

[a]N = Nurse crop. [b]M = Mature crop. [c]P = Pasture. [d]Negative values are enclosed in parentheses.

Table 4. Net revenue in first-half season for selected crops by land class and HSU in Utah, 1977 ($/acre).

Crop	Land Class I	II	III	IV	Land Class I	II	III	IV	Land Class I	II	III	IV
	HSU #1				HSU #2				HSU #3			
Alfalfa (Full)	4.41	1.25	1.94		18.31	12.87	9.68		20.42	18.78	16.81	
Alfalfa (Partial)	0.64	(1.79)[a]	1.08		13.82	10.93	12.48		15.94	14.46	17.74	
Barley	53.76	42.30	35.44		69.56	58.59	48.91		70.21	64.45	50.82	
Nurse Crop	26.36	17.27	12.93		38.08	29.81	22.89		38.73	35.67	25.44	
Corn Grain	(14.09)	(20.27)	(26.15)		(14.36)	(19.95)	(26.27)		(6.00)	(12.85)	(20.68)	
Corn Silage	18.02	21.46	18.75		12.25	21.09	21.51		22.60	31.94	33.42	
Dry Wheat				11.70				11.12				15.20
Dry Beans												
Alfalfa (Full)-(P)[b]			1.28				5.17				10.10	
Alfalfa (Partial)-(P)			0.70				4.25				7.30	
Barley-(P)			17.03				31.72				39.47	
Nurse Crop-(P)			6.63				7.54				17.10	

	HSU #4				HSU #5			HSU #6			
Alfalfa (Full)	20.85	17.04	14.54		4.86	4.98		6.92	5.63	6.32	
Alfalfa (Partial)	16.36	13.47	12.33		1.34	2.74		3.33	4.24	7.04	
Barley	70.21	62.96	50.39		53.16	42.76		50.84	42.91	35.23	
Nurse Crop	38.73	36.66	25.34		21.53	13.61		15.62	17.61	12.08	
Corn Grain	(1.02)	(13.49)	(18.52)		(12.67)	(20.01)		(13.88)	(22.63)	(28.60)	
Corn Silage	22.60	31.34	(29.58)		35.36	32.44		22.56	24.56	25.12	
Dry Wheat				8.52			8.24				8.68
Dry Beans											
Alfalfa (Full)-(P)			8.16			3.11				3.59	
Alfalfa (Partial)-(P)			6.25			1.35				3.04	
Barley-(P)			31.65			30.42				19.70	
Nurse Crop			15.34			10.28				9.24	

Table 4. Continued.

Crop	Land Class I	Land Class II	Land Class III	Land Class IV	Land Class I	Land Class II	Land Class III	Land Class IV	Land Class I	Land Class II	Land Class III	Land Class IV
	HSU #7.$_i$ (i=1,...,5,W,WY,CW, and CY)				HSU #8.$_j$ (j=1,2)				HSU #9			
Alfalfa (Full)		5.68	7.21		12.81	12.02	14.82		14.92	9.91	9.75	
Alfalfa (Partial)		5.54	3.26		8.87	10.19	14.26		10.99	4.27	2.84	
Barley		44.57	41.35		52.75	47.21	39.23		57.48	44.50	35.10	
Nurse Crop		17.41	10.75		25.85	20.52	14.70		28.65	17.81	10.58	
Corn Grain		(17.57)	(25.47)		(9.65)	(16.62)	(20.83)		(19.93)	(23.39)	(25.47)	
Corn Silage		25.52	23.26		26.22	30.63	29.07		19.70	26.37	24.82	
Dry Wheat				7.48				7.56				5.22
Dry Beans												33.14
Alfalfa (Full)-(P)			2.33				9.74				6.40	
Alfalfa (Partial)-(P)			1.63				8.51				1.71	
Barley-(P)			18.45				19.67				23.62	
Nurse Crop-(P)			11.16				9.95				11.98	

Crop	HSU #10			
Alfalfa (Full)	53.18	36.47	23.92	
Alfalfa (Partial)	22.45	13.93	7.87	
Barley	74.42	63.71	50.29	
Nurse Crop	28.38	20.99	11.02	
Corn Grain	63.82	41.82	26.61	
Corn Silage	107.99	110.91	89.06	
Dry Wheat				9.98
Dry Beans				
Alfalfa (Full)-(P)			7.78	
Alfalfa (Partial)-(P)			3.03	
Barley-(P)			17.71	
Nurse Crop-(P)			(0.86)	

[a]Negative values are enclosed in parentheses.

[b]P = Pasture.

Table 5. Annualized costs of preparing potentially irrigable land for production by land class in Utah for 1977 ($/acre).

HSU	Land Class			
	I	II	III	IV
1	22.17	25.06	27.25	30.39
2	24.59	27.48	29.67	32.80
3	27.00	29.90	32.08	37.64
4	29.42	32.31	34.50	37.64
5	-	27.48	29.67	32.80
6	27.00	29.90	32.08	37.67
7.1	-	25.06	27.25	30.39
7.W	-	25.06	27.25	30.39
WY	-	25.06	27.25	30.39
CW	-	25.06	27.25	30.39
CY	-	25.06	27.25	30.39
7.2	-	25.06	27.25	30.39
7.3	-	25.06	27.25	30.39
7.4	-	25.06	27.25	30.39
7.5	-	25.06	27.25	30.39
8.1	22.17	25.06	27.25	30.39
8.2	22.17	25.06	27.25	30.39
9	22.17	25.06	27.25	30.25
10	22.17	25.06	27.25	30.25

The energy sector consists mainly of electrical generation and the associated coal use.

Energy Resource Costs and Revenues

Coal mining costs and coal revenues were determined for 21 mines or mine areas in Utah, Wyoming, and Colorado. Mine specific extraction costs were not available. Costs vary among mines with overburden depth, seam thickness, in-mine flooding, actual coal conditions, and coal mine capacities. The estimates of coal mining costs as shown in Table 6 were determined from information found in Anderson (1977, 1979), Stradley (1977), the U.S. Department of Energy (1978a, 1979), and the U.S. Department of the Interior (1975, 1976b, 1976c). Where necessary, these costs have been updated using mining cost indices prepared by the U.S. Department of Commerce (1978). The f.o.b. mine selling price estimates specifically allow for mine types, current or projected output rates, mining methods, sulfur content, and Btu heat value (U.S. Department of Energy 1978a).

Transportation methods include belt, truck, rail, and slurry. Coal transportation costs from vaious mines to power plants by alternative methods were estimated in earlier research (Glover et al. 1979). Some adjustments were made for distances and tonnages by using specific observations from the sources cited above as well as current negotiated rates furnished by Union Pacific Railroad (1979). These costs are displayed in Table 7. It should be noted that these costs are only estimates. Truck transportation rates are sensitive to mileages and tonnages. Railroad rates are dependent upon the load turn-around time, car ownership, and new construction requirements, as well as distances and tonnages. Given the large potential increases in transportation, existing shipments are seldom comparable to the proposed shipments. Slurry transportation rates are very sensitive to volumes shipped and substantially less sensitive to distances (Anderson 1979). Belt shipments are restricted to mine mouth generating facilities only. New facilities construction was included where applicable.

Electricity Costs and Revenues

Electricity costs, prices, and net revenues by HSU (Table 8) were obtained from the Utah Division of Public Utilities (1979) and the U.S. Department of Energy (1978b). Variable costs excluding fuel were determined on a plant by plant basis. Revenue is available only as a company-wide average price of approximately $21.41 per MWH. Proposed power plants are assumed to experience costs and profitability similar to the latest Huntington unit. The Huntington unit was chosen because it is the only major Utah power plant which contains a sizable amount of pollution control equipment.

The Huntington plant is currently experiencing a variable cost of $2.56 per MWH, which generates an average net revenue of $18.85 per MWH.

Salinity Abatement Costs

Agricultural pollution abatement costs per ton of salt removed (Table 9) were obtained from Glover et al. (1979) and updated using appropriate cost indices for canal lining and sprinkler systems (Engineering News Record 1978). The term "salinity" is used as a proxy for total dissolved solids (TDS). Agriculture contributes to salinity through irrigation. First, there is some direct loading from fertilizers being applied to the soil. Second, the natural salts found within the soil are added to the stream flows through the leaching process and return flows. Salinity concentrations are also increased by consumptive water use. The principal methods of salinity control in agriculture are the installation of sprinklers (Treatment 1) and canal lining (Treatment 2). It is assumed that producers of coal-fired electrical power will follow a total containment policy (i.e., pond evaporation), even though there is some evidence that they might not continue to do so. For instance, the water from the Huntington units is currently being used to irrigate a variety of crops under a project initiated by Utah Power and Light Company (Hanks et al. 1977; Hanks et al. 1978).

Air Pollution Abatement Costs

Estimated pollution treatment costs for electrical power plants expressed in dollars per ton of effluent removed are shown for the removal of SO_2 (Table 10). Costs ranged from $1067/ton removed to $209/ton removed (Martin 1976). The emission rates

Table 6. Estimated mining costs and at mine selling prices for coal mines in Utah, Wyoming, and Colorado ($/ton).

COAL SOURCE	MINE OR COAL FIELD INFORMATION						MINING COST AND COAL PREMIUM			SELLING PRICE FOR MINE
	Mine Type	Seam Depth	Btu Content/ lb.	Planned Mine Capacity in mmtpy	% Sulfur Content	Mine Method	Mining Cost/Ton	Premium for Low Sulfur	Premium for Btu Content	
Utah										
1. Alton	S	12	10,772	9.5	1.3	strip	6.39	-	1.00	7.39
2. Bookcliffs	U	7-10	12,762	5.0	0.5	C/L[a]	10.94	1.62	3.00	15.56
3. Braztah	U	10*	12,300	6.5	0.5	C/L	10.94	1.62	3.00	15.56
4. Carbon Fuel	U	10*	12,850	4.0	0.2	C/L	10.94	3.24	3.00	17.18
5. Castlegate	U	10*	12,870	4.0*	0.2	C/L	10.94	3.24	3.00	17.18
6. Deer Creek	U	13	12,800	2.2	0.5	C/L	10.94	1.62	3.00	15.56
7. Deseret	U	10-13	12,830	4.0*	0.6	C/L	10.94	0.81	3.00	14.75
8. Henry Mtns. (Emery)	U	6	12,480	4.0*	0.96	C[b]	16.59	-	3.00	19.59
9. Henry Mtns. (General)	U	10	12,833	10.0	2.03	C/L	10.94	-	3.00	13.94
10. Hiawatha Quads	U	10-20	12,744	2.0	0.59	C/L	12.40	1.62	3.00	17.02
11. Huntington Canyon	U	5-14	13,300	2.0	0.6	C/L	12.40	0.81	4.00	17.21
12. Kaiparowits	U	12	11,999	6.0	0.87	C/L	10.94	-	2.00	12.94
13. Kolob	S	11	11,700	6.0	2.51	strip	6.39	-	2.00	8.39
14. Salina Canyon	U	10*	11,360	2.0	0.45	C/L	10.94	2.43	2.00	15.37
15. Swisher	U	6-10	12,700	4.0	0.6	C/L	10.94	0.81	3.00	14.75
16. Wasatch Plateau	U	10	12,589	4.0	0.6	C	10.94	0.81	3.00	14.75
17. Wilberg	U	13	12,280	2.2	0.5	C/L	10.94	1.62	3.00	15.56
Wyoming										
1. Evanston	U	9-11	10,450	2.0	0.4	C/L	12.40	2.43	1.00	15.83
2. Kemmerer	S	25	9,683	5.0	0.5	strip	6.39	1.62	-	8.01
3. Powder River	S	20	8,360	5.0	0.5	strip	6.39	1.62	-	8.01
4. Rock Springs	a.U	9-11	9,210	1.5	0.6	C/L	13.86	0.81	-	14.67
	b.S	40	9,210	8.0	0.6	strip	6.39	0.81	-	7.20
Colorado										
1. Yampa	S	25-50	10,598	3.0	0.47	strip	6.51	2.43	1.00	9.94

* = estimates due to poor data

[a] C/L = combination of continuous and longwall techniques

[b] C = continuous mining technique only

Table 7. Range of transport costs for coal per ton mile, 1977 ($/t mi).

	Max. Cost	Min. Cost
Truck	$0.090	$0.065
Rail	0.030	0.018
Slurry	0.035	0.030
Belt	0.07	0.07

Table 8. Estimated electricity costs, price, and net revenue of existing or proposed power plants by HSU for 1977 ($/MWH).

HSU	Plant	Price Average	Average Variable Costs	Average Net Revenue
1	Lucin	21.41	2.56	18.85
	Kelton	21.41	2.56	18.85
4	Gadsby (#1,#2)	21.41	2.77	18.64
	Hale	21.41	3.27	18.14
	Nephi	21.41	2.56	18.85
5	Axtell-Gunnison	21.41	2.56	18.85
	IPP	21.41	2.56	18.85
6	Milford-Black Rock	21.41	2.56	18.85
	Beryl-Lund	21.41	2.56	18.85
7	Moon Lake	21.41	2.56	18.85
8.1	Carbon (#1,#2)	21.41	2.56	18.85
	Helper	21.41	2.56	18.85
8.2	Huntington	21.41	2.56	18.85
	Emery	21.41	3.02	18.39
	Garfield	21.41	2.56	18.85
10	Warner Valley	21.41	2.56	18.85

Table 9. Agricultural pollution abatement methods and costs by HSU for 1977 ($/ton of salt removed).

HSU	Sprinkler Cost	Canal Lining Cost
1	41.91	29.09
2	41.91	29.09
3	41.91	29.09
4	41.91	29.09
5	15.23	21.85
6	15.23	29.09
7.1	33.38	29.09
7.W	33.38	29.09
WY	33.38	29.09
CW	33.38	29.09
CY	33.38	29.09
7.2	61.64	29.09
7.3	8.29	30.75
7.4	21.19	30.75
7.5	15.23	30.75
8.1	72.60	21.85
8.2	10.48	23.04
9	27.41	26.95
10	21.09	30.75

Table 10. SO_2, NO_x, and particulate emissions and control costs per ton removed.

	(Tons/Hour)		$/Ton Removed	
	Max.	Min.	"Acceptable Control"	Maximum Removal
SO_2	0.015	0.0019	$757.00	$770.00
NO_x	0.0056	0.0031		$133.00
Particulates	0.046	0.019		$151.00

were calculated under the assumption that the power plants would operate at only 80 percent of their nameplate capacity to allow for boiler shut-downs. Treatment costs for SO_2 removal depend on site conditions, quantity of sulfur to be removed, plant capacity and whether the system is new or in production. Since the majority of Utah, Wyoming, and Colorado coal has a low sulfur content, the costs per ton removed are fairly high. The few cases of high sulfur content coal found in Utah are adjusted accordingly (Martin 1976; Battelle 1978). NO_x and particulate emissions are also listed in Table 10. Cost per ton of NO_x removal at 20 percent control was estimated to be $133/ton while removal of particulates at 99 percent control was estimated at $151/ton (Glover et al. 1979; Martin 1976).

Matrix A Coefficients

Agriculture

The rotational constraints listed in Keith et al. (1978) were modified to include only those crops currently grown. For instance, sugar beet processing has declined significantly since the closure of the Garland beet processing facility (Decker 1979). The modified rotational constraints are listed in Table 11.

The consumptive use of water by selected crops on a seasonal basis is shown in Table 12. Crop productivity was adjusted seasonally according to these consumption rates. Irrigation efficiency coefficients and agricultural water return flow coefficients (Table 13) for each HSU were obtained from Keith et al. (1978). The irrigation coefficients indicate the percent of water applied to the crops that is consumed on the average by the crops or other vegetation. The return flow coefficients represent the distribution of water that is not consumed by the crop.

As discussed previously, the return flow of water from agriculture generally carries an increased concentration of salinity which increases the salt carried by a region's surface and groundwater. Coefficients used to measure the impact of irrigated agriculture on these water flows (Table 13) were obtained from Glover et al. (1979).

Table 11. Rotational constraints for selected crops in Utah.

1.	Alfalfa full + Alfalfa partial $\geq$ Barley
2.	Barley $\geq$ Nurse crop
3.	Alfalfa full + Alfalfa partial $\leq$ 5 (Nurse crop)
4.	Alfalfa full + Alfalfa partial + Barley + Nurse crop $\geq$ 7 (Corn grain + Corn silage)
5.	Mature applies $\geq$ 2.3 (Nurse apples)
6.	Mature peaches $\geq$ 2.0 (Nurse peaches)
7.	Mature sweet cherries $\geq$ 2.0 (Nurse sweet cherries)
8.	Mature sour cherries $\geq$ 2.6 (Nurse sour cherries)
9.	Alfalfa full + Alfalfa partial + Barley + Nurse crop + Corn grain + Corn silage $\geq$ 30 (Mature apples)
10.	Alfalfa full + Alfalfa partial + Barley + Nurse crop + Corn grain + Corn silage $\geq$ 15 (Mature peaches)
11.	Alfalfa full + Alfalfa partial + Barley + Nurse crop + Corn grain + Corn silage $\geq$ 27 (Mature sweet cherries)
12.	Alfalfa full + Alfalfa partial + Barley + Nurse crop + Corn grain + Corn silage $\geq$ 25 (Mature sour cherries)

Energy

Levels of output and plant efficiencies determine the quantity of energy material that is required. The amount of coal required for a specific coal-fired electrical power plant depends on the heat rate (Btu required per megawatt hour) of the plant and the Btu content of the coal. Existing or proposed power plants have heat rates which varied from 9400 to 12,000 Btu per MWH. Given that each coal has a different Btu content, each power plant was matched with one or more possible coal sources with the appropriate average Btu content by coal source (Anderson 1977, 1979; U.S. Department of the Interior 1975, 1977). The coal feed rate for each plant was determined under the assumption that the plant is operating at 100 percent of nameplate capacity (Perkins 1974; Painter 1974). Any other operating capacity can be found by multiplying these feed rates by the percentage of operation time.

Water requirements for the production of electricity used were 0.1258×10^{-2} acre feet per MWH (Keith et al. 1978).

Emission factors measured in tons per hour per megawatt for each coal source were calculated using a method similar to that employed by Painter (1974), Perkins (1974), and the Federal Energy Administration (1976) (Appendix B). Emission calculations depend on plant heat rates, the Btu content of the coal, the actual chemical composition of the coal, and the plant operating time.

Right-hand Side Values

The right-hand side (RHS) values are those values which serve as limits on the resources within a region.

Water Resources

The total surface water available within a region (net of municipal and industrial requirements) was obtained from King et al. (1972) and Keith et al. (1978). These availabilities were then modified to reflect the seasonal flows which occur throughout the year, as recorded by Utah State University (1968). Regional water flows were further adjusted for existing storage facilities (United States Department of Agriculture 1978b, 1978c). Surface water availabilities for Season 1 (January - June) and Season 2 (July - December) as shown in Table 14 exclude water used in the current production of petroleum (Keith et al. 1978).

Groundwater availability (Keith et al. 1978) was modeled such that any or all pumping could occur in either of the two seasons (Table 14). Finally, wetland requirements and present or new imports (King 1972) were divided equally between the two seasons.

Agricultural Land

The land available in each of four land classes (Table 15) by HSU was obtained from Keith et al. (1978) with an allowance made for potentially irrigable land as well as presently irrigated land. Land class IV included all presently and potentially cultivable land net of present or potentially irrigable land within the optimal solution set in order to allow dry land crops to be grown on any land if unprofitable in other uses. Fruit crops were restricted to present acreages of 630 acres in HSU 1, 1,633 acres in HSU 2, 1,422 acres in HSU 3, 8,021 acres in HSU 4, and 383 acres in HSU 10 (Keith et al. 1978; Utah Department of Agriculture 1978).

Coal Resources

Coal production projections for Utah, Wyoming, and Colorado were obtained from the U.S. Department of the Interior (1975, 1977) and were reduced to account for coal currently committed to other uses such as coking and household use (Table 16). The two levels of coal are related to an accelerated and a more likely mining rate scenario. The levels were used to examine the effects of coal availability. Approximate coal source locations are shown in Figure 1.

Clean Water Resources

Agriculture can have an adverse impact on the quality of the water used in its production processes (Utah State University 1975). It was assumed that a nondegradation restriction on salinity would be imposed.

Table 12. Seasonal consumptive use of water by selected crops in Utah (ac-ft).

HSU	Alfalfa Full		Alfalfa Partial		Barley		Nurse Crop		Corn Grain		Corn Silage		Nurse Apples		Mature Apples		Nurse Peaches		Mature Peaches	
	Season		Season		Season		Season		Season		Season		Season		Season		Season		Season	
	1[a]	2[b]	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
1	0.62	1.38	0.465	1.035	0.66	0.54	0.88	0.72	0.35	1.05	0.325	0.975	0.8	1.7	1.152	2.448	0.896	1.904	1.248	2.652
2	0.576	1.024	0.36	0.64	0.49	0.21	0.432	0.768	0.286	0.814	0.26	0.74	0.912	1.488	1.33	2.17	1.206	1.674	1.444	2.356
3	0.576	1.024	0.36	0.64	0.49	0.21	0.77	0.33	0.348	0.852	0.319	0.781	0.912	1.488	1.33	2.17	1.206	1.674	1.444	2.356
4	0.72	1.28	0.468	0.832	0.63	0.27	1.05	0.45	0.435	1.065	0.406	0.994	1.026	1.674	1.444	2.356	1.14	1.86	1.596	2.604
5	0.682	1.518	0.341	0.759	0.84	0.36	1.12	0.48	0.435	1.065	0.406	0.994	–	–	–	–	–	–	–	–
6	0.651	1.449	0.465	1.035	0.56	0.44	0.896	0.704	0.375	1.125	0.35	1.05	–	–	–	–	–	–	–	–
7.1	0.651	1.449	0.341	0.759	0.741	0.559	0.912	0.688	0.40	1.2	0.315	1.125	–	–	–	–	–	–	–	–
7.2	0.651	1.449	0.341	0.759	0.741	0.559	0.912	0.688	0.40	1.2	0.315	1.125	–	–	–	–	–	–	–	–
7.3	0.651	1.449	0.341	0.759	0.741	0.559	0.912	0.688	0.40	1.2	0.315	1.125	–	–	–	–	–	–	–	–
7.4	0.651	1.449	0.341	0.759	0.741	0.559	0.912	0.688	0.40	1.2	0.315	1.125	–	–	–	–	–	–	–	–
7.5	0.651	1.449	0.341	0.759	0.741	0.559	0.912	0.688	0.40	1.2	0.315	1.125	–	–	–	–	–	–	–	–
8.1	0.672	1.428	0.352	0.748	0.728	0.572	0.896	0.704	0.432	1.168	0.405	1.095	–	–	–	–	–	–	–	–
8.2	0.672	1.428	0.352	0.748	0.728	0.572	0.896	0.704	0.432	1.168	0.405	1.095	–	–	–	–	–	–	–	–
9	0.713	1.587	0.403	0.897	0.784	0.616	1.008	0.792	0.5	1.5	0.475	1.425	–	–	–	–	–	–	–	–
10	1.628	2.072	1.32	1.68	1.305	0.195	1.74	0.26	1.128	1.272	1.081	1.219	1.32	1.68	1.76	2.24	1.496	1.904	1.936	2.464

HSU	N. Sweet Cherries		M. Sweet Cherries		N. Sour Cherries		M. Sour Cherries		Alfalfa F. Pasture		Alfalfa P. Pasture		Barley Pasture		N. Crop Pasture	
	Season		Season		Season		Season		Season		Season		Season		Season	
	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
1	0.864	1.836	1.216	2.584	0.864	1.836	1.216	2.584	0.62	1.38	0.465	1.035	0.66	0.54	0.88	0.72
2	0.988	1.612	1.406	2.294	0.988	1.612	1.406	2.294	0.576	1.024	0.36	0.64	0.49	0.21	0.432	0.768
3	0.988	1.612	1.406	2.294	0.988	1.612	1.406	2.294	0.576	1.024	0.36	0.64	0.49	0.21	0.77	0.33
4	1.102	1.798	1.558	2.542	1.102	1.798	1.558	2.542	0.72	1.28	0.468	0.832	0.63	0.27	1.05	0.45
5	–	–	–	–	–	–	–	–	0.682	1.518	0.341	0.759	0.84	0.36	1.12	0.48
6	–	–	–	–	–	–	–	–	0.651	1.449	0.465	1.035	0.56	0.44	0.896	0.704
7.1	–	–	–	–	–	–	–	–	0.651	1.449	0.341	0.759	0.741	0.559	0.912	0.688
7.2	–	–	–	–	–	–	–	–	0.651	1.449	0.341	0.759	0.741	0.559	0.912	0.688
7.3	–	–	–	–	–	–	–	–	0.651	1.449	0.341	0.759	0.741	0.559	0.912	0.688
7.4	–	–	–	–	–	–	–	–	0.651	1.449	0.341	0.759	0.741	0.559	0.912	0.688
7.5	–	–	–	–	–	–	–	–	0.651	1.449	0.341	0.759	0.741	0.559	0.912	0.688
8.1	–	–	–	–	–	–	–	–	0.64	1.36	0.352	0.748	0.672	0.528	0.896	0.704
8.2	–	–	–	–	–	–	–	–	0.64	1.36	0.352	0.748	0.672	0.528	0.896	0.704
9	–	–	–	–	–	–	–	–	0.713	1.587	0.403	0.897	0.784	0.616	1.008	0.792
10	–	–	–	–	–	–	–	–	1.628	2.072	1.32	1.68	1.305	0.195	1.74	0.26

Note: N = Nurse, M = Mature, P = Partial, F = Full.

[a]Season 1: January-June; [b]Season 2: July-December.

Table 13. Irrigation efficiency and agricultural return flow coefficients and salt loading attributable to agriculture by HSU in Utah.

HSU	Irrigation Efficiency Coefficients (%)	To Surface (%)	To Ground (%)	Salt Loading (t/ac ft Return Flow)
1	0.4758	0.4742	0.0500	0.34
2	0.3423	0.6077	0.0500	0.34
3	0.3667	0.5833	0.0500	0.34
4	0.3891	0.5609	0.0500	0.34
5	0.3250	0.6250	0.0500	0.89
6	0.4553	0.4947	0.0500	0.89
7.1	0.3712	0.6288	0.0000	0.78
7.W	0.3712	0.6288	0.0500	0.592
WY	0.3712	0.6288	0.0500	0.232
CW	0.3712	0.6288	0.0500	0.5917
CY	0.3712	0.6288	0.0500	0.40
7.2	0.3712	0.6288	0.0000	0.58
7.3	0.3712	0.6288	0.0000	0.34
7.4	0.3712	0.6288	0.0000	0.34
7.5	0.3712	0.6288	0.0000	0.47
8.1	0.3750	0.6250	0.0000	1.49
8.2	0.3750	0.6250	0.0000	1.09
9	0.2000	0.8000	0.0000	0.58
10	0.5000	0.4500	0.0500	1.26

Table 14. Average groundwater availabilities by HSU in Utah.

HSU	Groundwater ac-ft x 10^3
1	184.00
2	94.00
3	62.00
4	127.00
5	335.00
6	127.00
7.1	1.49
7.2	6.98
7.3	11.65
7.4	13.59
7.5	6.47
7.W	0.50
8.1	-
8.2	-
9	-
10	10.00
WY	4.60
CW	6.00
CY	12.00

Table 15. Presently (1977) cultivated and potentially cultivable land acreage available by HSU in Utah (acres).

HSU	PILND I	PILND II	PILND III	PILND3P	PCLND IV	POILND I	POILND II	POILND III	POILND3P	POCLND IV
1	3,100	15,300	21,600	2,882	47,600	98,900	487,300	611,000	479,200	1,676,400
2	13,600	75,000	78,400	70,547	246,000	14,900	78,000	68,400	127,700	289,000
3	29,400	51,900	56,200	6,866	169,700	700	8,000	21,800	26,200	56,700
4	17,500	58,900	88,400	14,678	224,600	24,500	92,400	100,600	79,200	296,700
5	-	186,300	85,900	10,500	298,000	-	221,900	308,100	446,000	976,000
6	300	49,300	21,900	4,366	80,000	200	233,500	274,100	344,700	852,500
7.1	-	1,653	2,447	500	4,600	-	16,126	23,873	10,590	38,752
7.W	-	-	-	-	-	-	5,240	7,760	2,500	15,000
WY	-	33,240	134,105	-	352,900	-	70,100	124,790	158,010	281,185
CW	-	590	4,825	-	25,960	-	9,260	39,195	39,285	87,740
CY	-	1,560	12,810	-	68,910	-	37,040	156,780	157,140	350,960
7.3	-	18,545	27,454	17,000	36,000	-	13,150	19,467	13,957	51,070
7.4	-	10,815	16,010	1,085	42,000	-	16,274	24,091	17,272	63,200
7.5	-	34,470	51,029	14,500	20,000	-	13,184	19,517	13,992	51,200
8.1	-	7,719	9,141	6,400	18,000	-	20,400	22,400	15,800	58,600
8.2	933	19,689	30,887	25,750	62,500	7,000	92,400	96,400	50,000	245,800
9	976	2,050	1,500	4,160	1,900	5,400	132,000	290,000	106,000	533,000
10	3,200	11,900	5,200	620	21,000	7,800	37,600	103,400	95,300	244,100

PILND I = presently irrigated class I land.
PILND II = presently irrigated class II land.
PILND III = presently irrigated class III land.
PILND3P = presently irrigated pasture, class III.
PCLND IV = presently cultivated class IV land.

POILND I = potentially irrigable class I land.
POILND II = potentially irrigable class II land.
POILND III = potentially irrigable class III land.
POILND3P = potentially irrigable pasture, class III.
POCLND IV = potentially cultivable class IV land.

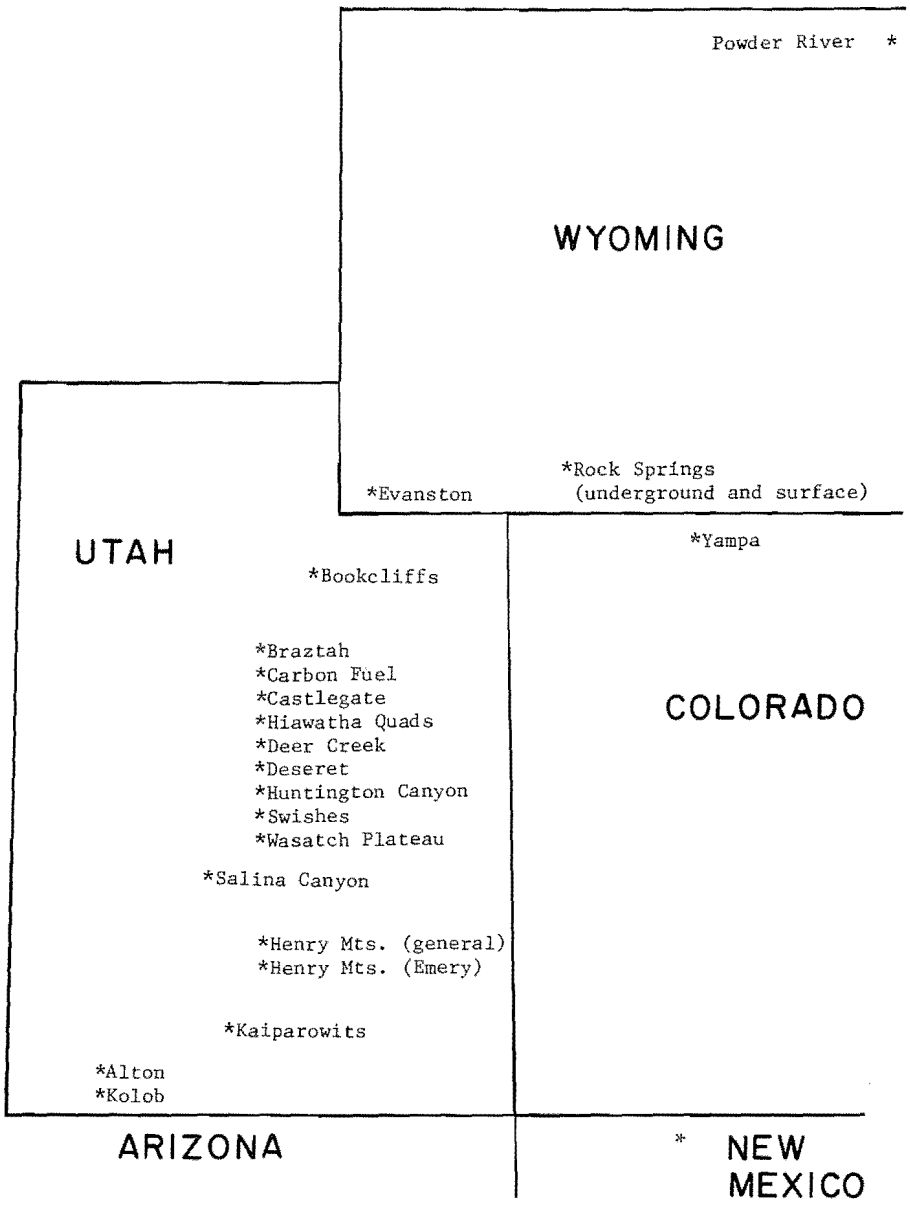

Figure 1. Coal source locations.

That is, salinity concentrations would be allowed to stay at current levels but could not exceed those levels if new irrigated agriculture were developed. These nondegradation limits (expressed in tons/year emitted) were obtained from Glover et al. (1979) as shown in Table 17. The model allowed an increase (decrease) in these limits associated with an increase (decrease) in surface flows.

Table 16. Projected coal source capacities.

Coal Source	Projected Coal Source Yearly Capacity (MM tons)	Estimated Coal Sources Capacity for Production of Electrical Power	
		Level 1	Level 2
Alton	9.50	4.00	2.00
Bookcliffs	5.00	2.00	1.00
Braztah	6.50	2.50	1.25
Carbon Fuel	4.00	4.00	2.00
Castlegate	4.00[a]	4.00	2.00
Deer Creek	2.20[a]	2.00	1.00
Deseret	2.00[a]	2.00	1.00
Henry Mts. (E)	4.00[a]	4.00	2.00
Henry Mts. (G)	10.00	10.00	5.00
Hiawatha Quads	2.00	2.00	1.00
Huntington Canyon	2.00	2.00	1.00
Kaiparowits	6.00	6.00	3.00
Kolob	6.00	6.00	3.00
Salina Canyon	2.00	2.00	1.00
Swisher	4.00	4.00	2.00
Wasatch Plateau	4.00	4.00	2.00
Wilberg	2.20	2.20	1.10
Evanston	2.00	2.00	1.00
Kemmerer	5.00	5.00	2.50
Powder River	5.00	5.00	2.50
Rock Springs (A) underground	1.50	1.50	0.75
Rock Springs (B) surface	8.00	4.00	2.00
Yampa	4.00	4.00	2.00
New Mexico	5.9	5.9	5.9

[a]Estimated annual output levels.

Table 17. Salinity loading limits for a nondegradation criterion at outflow of HSU.

HSU	Maximum Salinity Level (tons/year)
1	57,851
2	226,972
3	139,538
4	165,236
5	600,415
6	267,000
7.1	281,950
7.2	172,600
7.3	152,000
7.4	27,676
7.5	98,124
7.W	-
8.1	44,900
8.2	761,170
9	872,470
10	42,840
WY	222,670
CW	66,000
CY	64,000

Table 18. Maximum allowable increase in effluents measured in micrograms per cubic meter by air quality classifications for 1977.

	Particulates		Sulfur Dioxide		
	Annual Geometric Mean	24-Hour Maximum	Annual Geometric Mean	24-Hour Maximum	3-Hour Maximum
Class I	5	10	2	5	25
Class II	19	37	20	91	512
Class III	37	75	40	182	700

This study assumed given power plant locations so that a plume model utilizing the concept of "source to high terrain" would be most effective in determining limitations on power production (Figure 2). Furthermore, the 3-hour SO_2 effluent limit, the most limiting case, has been utilized (Wooldridge 1979b). The discharge of SO_2 was believed to be more restrictive than the discharge of particulates or NO_x throughout most of the state. Particulate control is effective at 95 percent control or better and is not considered a major problem. NO_x control on a commercial level has been limited to only 15 to 20 percent (Martin 1976) and is not expected to be a problem except for the Wasatch Front area which has already been designated as a nonattainment region. A nonattainment area is one in which no additional effluents can be emitted. The extent to which current emissions must be controlled is uncertain at this time.

The model used for the calculation of SO_2 effluent limitations (Appendix C) assumed the plume to be normally distributed with complete reflection at the earth's surface and at the top of a "mixed" layer. This represents a conservative approach (Wooldridge 1979b). Power production restrictions within each air shed were calculated for specific plants and coal sources. The annual maximum tonnage of SO_2 allowed was calculated, as well as SO_2 required to be removed; particulates were assumed to be controlled at the 99 percent level and nitrogen oxides are assumed controlled at the 20 percent level.

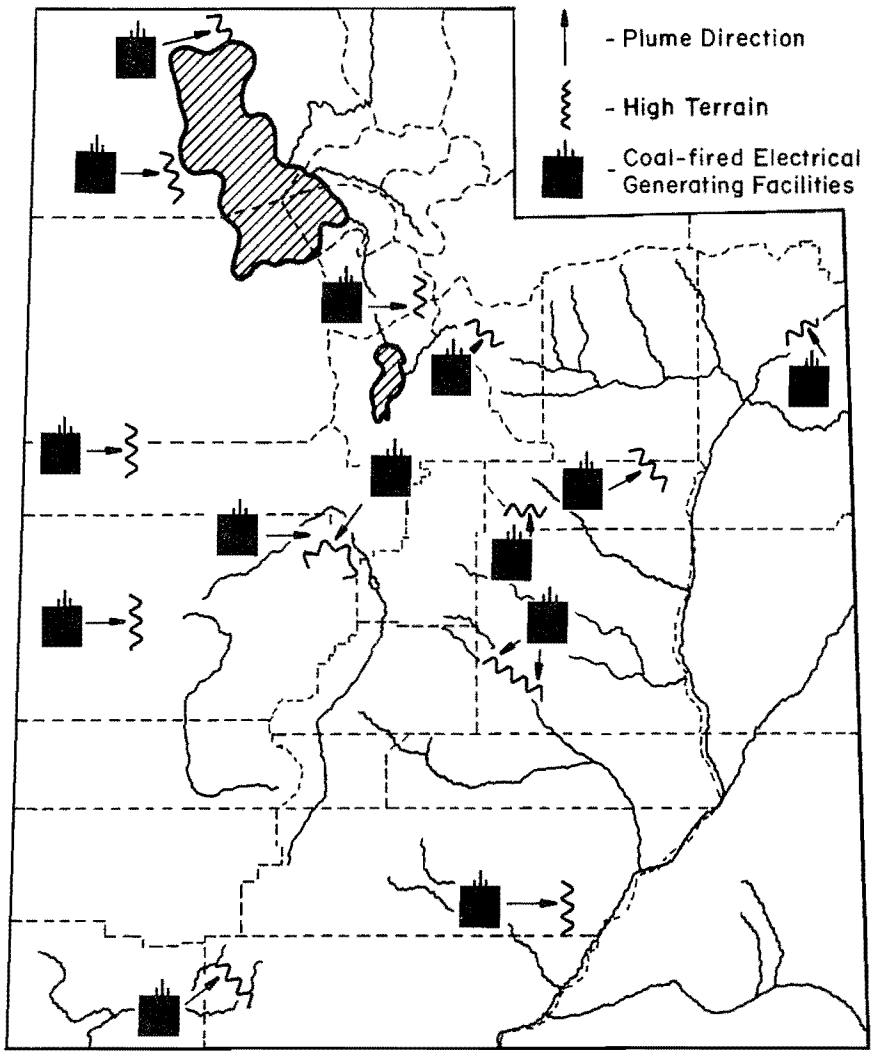

Figure 2. Effluent vectors and impingement points utilized in plume model.