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Transfer in problem solving.

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ABSTRACT

TRANSFER IN PROBLEM SOLVING

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An experiment was conducted in order to show the transfer of strategies in a problem solving situation. Subjects were preexposed to problems solutions to which involved one of the two strategy components essential in solving the third problem. Preexposure required that subjects actually solve the problems and then try to describe strategy or strategies they used in solving them. No effects of transfer were found. The results are interpreted in terms of other factors which might influence transfer in problem solving such as subjects' overall mathematical skill and the instructional methods.

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Introduction

Although it seems that problem solving is an important part of our lives and transfer in problem solving is an important aspect of the process, there is no large body of literature relevant to this topic. In spite of a large amount of research on problem solving in general, little progress has been made to provide an articulate body of empirical relationships (Schulz, 1960). It seems that there really are no studies which test what gets transferred and when, how, and under what conditions it gets transferred. And it seems that in order to fully understand problem solving behavior we must have some understanding of the functional relationships between antecedent variables and problem solving performance.

In general terms, transfer can be described as an effect of previous performance on Task A on the performance on Task B which is significantly different from the performance on Task B without prior experience with Task A. The degree of transfer may be influenced by a number of factors such as the degree of similarity between two tasks, the degree of familiarity with the Task A, or the time interval between the occurrence of two tasks. A good example of negative transfer is functional fixedness which refers to the fact that sometimes people get so "fixed" on their perception of an object that they cannot see other uses for it (Duncker, 1945; Maier, 1931).

Related Research

Positive and negative transfer have both been shown to occur in anagram solving. Dominowski and Ekstrand (1967) conducted a study where they tested the effects of previous exposure to solutions (direct priming), of previous exposure to words related to solutions (associative priming), and of previous exposure to words unrelated to solutions (inappropriate priming) on anagram solving. Thus, one group of subjects was preexposed to a list of words which contained the solutions to the anagrams. The second group was preexposed to a list of words which were related to solutions to anagrams. The third group was preexposed to a list of unrelated words but was instructed that they were related to solutions of anagrams. Greatest positive transfer occurred with direct priming, some positive transfer occurred with associative priming, and negative transfer occurred with inappropriate priming. However in Dominowski and Ekstrand's study subjects had been informed of the relation between the problems and the lists. The results might have been different if this information was withheld.

The importance of the instructions has been shown in a study by Reed, Ernst, and Banerji (1974) where the transfer did not occur when subjects were not told the relationship between the two problems. The problems used in the study involved moving two sets of three objects accross a boundary with certain restrictions as to the way it could be done (the so-called Missionaries and Cannibals and the Jealous Husbands problems). The problems had similar problem states and were related to each other in such a way that the latter was a special case of the former (i.e., it had the same goal but additional restrictions on what were permissible moves). In the first experiment, half of the subjects were given the more difficult problem first (Jealous Husbands problem) and half were given the easier problem first (Missionaires and Cannibals problem). There were no instructions given as to the realtionship between the problems. Since no significant transfer was found, two additional experiments were carried out. In the second experiment the subjects were asked to solve the same problem twice, and the third was a replication of the first experiment except that the subjects were explicitly told about the relationship between the problems. Three measures of transfer were taken in each experiment: the total solution time, the total number of moves, and the total number of illegal moves. Significant transfer was found when subjects were asked to solve the same problem twice. However in the third experiment, positive transfer was found when subjects solved the more difficult problem first, but no transfer was found when they solved the easier problem first.

The asymetrical aspects of transfer were also found by Cook (1936). His study used the disc transfer problem (known also as the Tower of Hanoi problem). The task is that given three pegs (A, B, and C) and a stack of discs in decreasing order of size from bottom to top on Peg A, the subject is supposed to move the stack to Peg B (using C as a temporary target) by moving one disc at a time and never placing a larger disc on top of a smaller one. In the first experiment, three groups of subjects solved either a two, or a three, or a four disc problem. The total number of errors, number of moves, and solution time were mesured. The errors were defined as the number of moves used to solve the problem in excess of the minimum number of moves necessary to solve it. In the second experiment different groups of subjects were given all three problems in increasing or decreasing order of difficulty. The number of errors, total solution time and the total number of moves were compared for the three disc problem when it was solved after a two disc problem and after a four disc problem. The same comparison was made for a four disc problem when it was solved as a first and as a third problem. Transfer was found when subjects were given the more difficult problem first, but no significant transfer was found when they were given the easier problem first.

It seems that there are several prerequisites for positive transfer to occur. First of all the subject must recognize the analogy between the problems. Secondly, he must be able to recall some information about the solution of the analogous problem. Thirdly he must translate this information to fit with the givens of the present problem. Fourth, this translation must reduce the number of operations that would have to be considered without the availability of the analogous problem (Reed, Ernst, Banerji, 1974). However their data as well as subjects' reports seem to suggest that transfer occurs on a higher level than individual muves. This is supported by the fact that subjects showed little improvement even though there were many moves in common between Missonaries and Cannibals and Jealous Husbands problems.

A more detailed analysis of the transfer between different problem states has been presented by Thomas (1974). His study involved the Missonaries and Cannibals problem described previously. In the first experiment one group (control) simply solved the problem from the beginning to end. The experimental group, first solved the problem from the middle state to the end and then was given the whole problem. The results of the experiment showed that the experimental group solved the first half of the problem in fewer number of moves than the control group did, therefore showing a positive transfer from their experience in solving the second half of the problem (the mean number of moves made by the two groups were compared). However the experimental group showed no improvement in their performance on the second half of the problem when they were solving it as a part of the whole problem. Also surprising is the fact that the experimental group did better on the second part of the problem on their first trial than the control group. Since the control group had previous experience solving the first half of the problem one would expect them to do better. However, the two halves of the problem are symmetrical with resepect to one another. This means that if at state n in the first half of the problem three Missionaries, one Cannibal and the boat are on the starting side of the river and two Cannibals are on the opposite side of the river, then at the corresponding state in the second half of the problem the three Missionaries, two Cannibals and the boat should be on the opposite side of the river for the solution to be reached. This may cause a negative transfer to occur from the first to the second half of the problem.

The two parts of the problem used in the above experiment are then related in such a way that similar problem states occur at different stages of the solution process (different number of objects on both sides of the river, different number of objects still left to move--see table 9). If we assume that the problem solving process involves a selection of moves from a constant set (Restle & Davis, 1962) then at each stage of the Missionaries and Cannibals problem the problem solver had to make a choice. The hypothesis that Thomas (1974) tested was that the probability of choosing a move depended only on the state of the problem regardless of the number of times that state has occured. However the results showed that the probability of occurrence of correct response increased across the solution of the problem (Thomas, 1974). Similar results were obtained by Greeno (1974) who also suggested that humans organize their solution to problems at least a few moves ahead.

Discussion of Transfer in Problem Solving

Although it seems intuitevly obvious that transfer is involved in problem solving, so far it has been very difficult to demonstrate the effects of transfer in an experimental situation. The problem may lie in the difficulty of constructing a specific environment or conditions necessary for transfer to occur. Research up to date has concentrated on creating situations where individual moves could be transferred from one problem to the other. However the negative results of these studies suggest that when (and if) transfer can be demonstrated, it is not likely to be individual moves that are transferred. Therefore this study was concerned with a more general "unit" of problem solving, that is with problem solving strategies and their transfer. The problem utilized here was the Tower of Hanoi problem mentioned previously. The reason for choosing this particular problem was that it seemed to have some strategy components that underly two general problem solving strategies: the method of working backwards and the subgoal method.

The subgoal method involves breaking the problem down into parts, solving each part separately and reassembling them in order to solve the problem. For example in case of the Tower of Hanoi problem one approach to the solution of the problem is to break it into subgoals. Let us call the Peg where the stack of discs has been originally placed and where it is supposed to be moved from, the Source Peg, the Peg where the stack of discs is supposed to be moved to, the Goal Peg, and the remaining third Peg, the Temporary Target. If we assume that Peg A is the Source Peg, Peg B the Goal Peg, and Peg C the Temporary Target, solving a five disc problem from A to B can be broken down to solving a four disc problem from A to C, moving the largest disc to B and solving a four disc problem from C to B. Then the next subgoal would be to solve a four disc problem from C to B by solving a three disc problem from C to A, moving second to the largest disc to B and solving a three disc problem from A to B. This involves solving a two disc problem from A to C, moving third to the largest disc to B and solving a two disc problem from C to B. And so on. This kind of procedure, involving a repetitive application of an operator, is sometimes referred to as recursion. The subgoal method is realted to another problem solving heuristic called simplifying which involves setting the original problem aside and solving a simpler problem first. In case of the Tower of Hanoi problem. however, those two strategies really mean the same thing since solving a problem with a smaller number of discs (simplifying) is a subgoal of a problem with a larger number of discs.

When using the method of working backward, the person concentrates on the goal rather than the givens and considers it as a starting point in solving a problem (Wickelgren, 1974). However the goal is considered not in order to draw inferences from it but rather to determine a preceeding statement. The solution to the Tower of Hanoi problem makes use of the working backward strategy in a very fundamental way. In order to end up with a stack of discs on the correct peg, the largest disc which is on the bottom of the stack has to be placed there first. Therefore all other discs have predetermined goal pegs where they are temporarily placed. So instead of starting to solve the problem from the givens (disc on top of the stack and three pegs) the problem solver concentrates on the goal (stack of discs on the goal peg). This will make him realize that in order to achieve the goal state the largest disc has to be placed first on the goal peg. After reaching this conclusion he can determine the conditions under which this subgoal can be achieved, which is having all discs except for the largest one on the temporary peg (not the source peg and not the goal peg). This will predetermine the temporary goal peg for the second to the largest disc. And this routine applied to the whole stack (without actually moving the discs) will determine the temporary goal pegs for all discs including the one on top of the stack which is the only one that can be legally moved. This working backward procedure has to be applied over and over again whenever a subgoal has been reached.

The purpose of the present experiment was to investigate conditions under which the transfer of strategies in problem solving would occur. Therefore the subjects were preexposed to certain problems, solutions to which involved one of the general strategies used in solving the Tower of Hanoi problem. (either working backward or subgoal method). For some subjects the problems involved the working backwards strategy, for others the problems involved only the subgoal strategy, and still another group was preexposed to one problem from each category. The other question which this study tried to get at was: if the transfer of strategies does occur, at what level of problem solving process does it occur? Since the Tower of Hanoi problem involves a recursive technique we may be able to divide the problem solving process into two stages: learning the recursive unit and applying it. If the subjects transferred the strategy or strategies from the first set of problems to the Tower of Hanoi problem this transfer should only have affected the first stage of the process. Since in the case of the Tower of Hanoi problem the recursive operator is applied to a smaller and smaller number of discs, learning stage would take place up until the subject moves the largest disc to the goal peg. The process of moving the rest of the discs to the goal peg involves simply a repeated application of the method used in the learning stage.

Method

Subjects. The subjects were 54 college students enrolled in psychology courses.

Materials. Six problems, solutions to two of which involves a sub-goal method; two, working backward method; two, unrelated problems.

1. Nine men and two boys want to cross a river, using an inflatable raft that will either carry one man or two boys. How many times must the boat cross the river in order to accomplish this goal?

2. Ingrid brings a quantity of hats to sell at the Saturday market. In the morning she sells her hats for $3 each, grossing $18. In the afternoon she reduces her price to $2 each and sells twice as many. What was Ingrid's gross income for the day from the sale of hats?

3. Three people play a game in which one person loses and two people win each game. The one who loses must double the amount of money that each of the other two players has at the time. The three players agree to play three games. At the end of three games each player has lost one game and each person has $8. What was the original stake of each player?

4. Given a jar that will hold exactly 7 quarts of water, a jar that will hold exactly 3 quarts of water, no other containers holding water, but an infinite supply of water describe a sequence of fillings and emptyings of water jars that will result in achieving 5 quarts of water.

5. The Nelsons have gone out for the evening, leaving their chil-

dren with a new babysitter, Nancy. Among the many instructions the Nelsons gave Nancy before they left was that three of their children were consistent liars and only one of them consistently told the truth. As she was preparing dinner for the children one of them broke a vase in the next room. Nancy rushed in and asked who broke the vase. These were the children's statements:

Betty: Steve broke the vase.

Steve: John broke it.

Laura: I didn't break it.

John: Steve lied when he said I broke the vase.

Knowing that only one of these statements is true Nancy quickly determined which child broke the vase. Who was it?

6. Without your pencil leaving the paper, draw four straight lines through the following three-by-three array of nine dots:

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• • •

• • •

Tower of Hanoi problem. You are given three pegs: A, B, and C. Four (five) discs are placed on Peg A in decreasing order of size (the largest disc on the bottom, the smallest one on top). You can only move one disc at a time, and you can never place a larger disc on top of a smaller one. Your task is to move all four (five) discs from Peg A to Peg B so that at the end of the problem Peg B will contain four (five) discs in the order they are now on Peg A.

<u>Procedure</u>. There were three experimental groups and two control

groups. Experimental group one was preexposed to two problems involving the subgoal method in their solutions. Experimental group two was preexposed to two problems which involved working backwards method in their solutions. Experimental group three was preexposed to one problem involving the subgoal method in its solution and one problem involving the working backwards method. Control group one was preexposed to two unrelated problems. Preexposure in all cases required that subjects actually solve the problems. All four groups were then asked to describe the strategy or strategies they used in solving the problems. Then they were told of the relationship between the problems they solved and the Tower of Hanoi problem. This means that experimental group one and two were told that the strategy they used in solving two problems would be helpful in solving the Tower of Hanoi problem; experimental group three was told that the strategies they used in solving each of the two problems would be helpful to them in solving the Tower of Hanoi problem; control group one was told that methods that they used in solving the two problems would not be helpful in solving the Tower of Hanoi. Then all groups were given a four and a five disc Tower of Hanoi problem to solve. Control group two was just given the Tower of Hanoi problem to solve without being preexposed to any other problems. Since the subjects solved the Tower of Hanoi at the computer terminal the description of the problem was given both in terms of pegs and in termas of the computer display which involved lines instead of pegs and digits instead of discs. The total number of moves, the total solution time, the time to make individual moves, and the time taken to solve subparts of the problem (Tower of Hanoi) were measured for all groups. After the three experimental groups and two control groups had participated in the experiment, an additional third control group was added to the experiment. This group solved only the five disc Tower of Hanoi problem. All other conditions and measures were the same as for the other groups.

Results

For the purpose of a more detailed analysis the Tower of Hanoi problem was divided into two halves. The first half of the problem includes the moves from the beginning of the problem up to the point of moving the largest disc to the goal peg. The second half of the problem consists of moving the rest of the discs on top of the largest one. So solving the four disc problem would consist of solving a three disc problem, moving the largest disc (first half) and solving a three disc problem again (second half). Solving the five disc problem would consist of solving a four disc problem, moving the largest disc (first half) and solving a four disc problem to the peg where the largest disc has been moved (second half).

Table 1 and Table 2 present means and medians of the times and the number of moves taken to complete the whole four disc problem as well as the subparts of the four disc problem by different experimental groups. The minimum number of moves necessary to solve the first half and the second half of the problem is 8 and 7 moves respectively. The minimum number of moves necessary to solve the whole problem is 15. The number of subjects that solved the problem in each group was nine. As it can be clearly seen from the tables there are no significant differences between the experimental and the control groups. However, all of the groups do show a marked decrease in both the number of moves and the time taken to solve the second half of the problem as compared with the first half. The mean number of moves used to solve the second half of the problem is slightly less for the experimental than the control groups.

Table 3 and Table 4 show the means and the medians respectively of the time and the number of moves taken to solve the whole five disc problem and the subparts of it. The minimum number of moves necessary to solve the five disc problem is 31, to solve the first half of it is 16, to solve the second half of it is 15. There were nine subjects tested in each group, however only eight subjects solved the five disc problem in Control 1 and Control 2, and only six subjects solved the problem in three experimental groups and third control group. Again there are no significant differences between the experimental and the control groups. The analysis also showed no significant differences between the second and third control group which suggests no overall transfer effects from the four to five disc problem (Table 5). However, again all of the groups show a decrease in the number of moves and the time taken to solve the second half of the problem.

However a significant difference was found between the second and third control group in both the time and the number of moves it took to move the top three discs off the source peg as a subpart of a five disc problem. For the difference in time a Mann-Whitney U=6 was found which is significant at .01 one tailed. For the difference in number of moves a Mann-Whitney U=9 was found which is significant at .03 one tailed (Table 4a). Since the only difference between the second and third control groups was that Control 2 was preexposed to a four disc problem it seems that what people did learn through this preexposure was the ability to solve a three disc problem.

A significant difference was also found between the time it took to solve a four disc problem by itself and to solve a four disc problem as a second half of the five disc problem. The values of the Wilcoxon T's for the corresponding groups are given in Table 6. The difference in the number of moves in this case was not significant for any of the groups. However the analysis of the two halves of the five disc problem in the third control group showed that transfer occurred between the time and the number of moves it took to solve the four disc problem as a part of the first half of the problem and the second half (Mann-Whitney U(time)=0, significant at .001; U(moves)=2, significant at .004).

Similar analysis done on the second control group which was pre-exposed to the four disc problem but not to the preliminary problems yielded also a significant difference (Mann-Whitney U(time)=4, significant at .001; U(moves)=13, significant at .025). This shows the same amount of transfer as in the third control group for the time involved but less transfer for the moves involved. However a comparison of the time and the number of moves taken to solve the four disc problem by itself and a four disc problem as a first part of the five disc problem showed no significant difference. This may be due to the fact that in order to solve the five disc problem subjects had to move the top four discs to Peg C and not to Peg B as was the case when solving a four disc problem.

The most critical move in solving the Tower of Hanoi problem is then, the very first move when the subject has a choice of moving the smallest disc to either the goal peg or the temporary target. The correct first move for the five disc problem was to move the smallest disc to Peg B (the goal peg). Table 7 shows the percentage of subjects in each group who made the first move to the correct peg. The second and the third experimental group which have the highest percentage of correct first moves were the only two groups preexposed to working backwards strategy which is essential in making this move. Subjects who made the correct first move were the ones who solved the problem in the least number of moves (as compared to the other subjects in the group) for all groups. This effect of preexposure to the working backwards strategy was not evident for the first move while solving a four disc problem (Table 8).

Another interesting observation is somewhat related to the question of "good" and "bad" problem solvers which I will address in more detail in the Discussion. By rating subjects from one to nine in each group on the basis of the time or the number of moves they took to solve a four disc problem all groups were divided into two subgroups: five "good" problem solvers and four "bad" problem solvers (it did not make the difference whether time or the number of moves was taken into consideration since for all groups subjects who took less moves to solve the problem also took less time). After using the same procedure with the five disc problem and comparing the results it was found that in Experimental 1, the five best problem solvers of the four disc problem were also the five best problem solvers of the five disc problem; in Experimental 2 and Control 2 of the five best solvers of the four disc problem, four were among the five best solvers of the five disc problem; and in Experimental 3 and Control 1, out of the five best solvers of the four disc problem three were among the five best solvers of the five disc problem.

Discussion

There seem to be at least two major problems with studying transfer in problem solving. One has to do with subjects' ability to solve problems in general. It seems intuitevly obvious that depending on the instructional method subjects learn different things about the concepts involved. Indeed it has been shown (Mayer and Greeno, 1972; Egan and Greeno, 1973) that different teaching methods result in structurally different learning outcomes. It seems that depending on the teaching method subjects encode the information in different ways. It has also been previously shown (Egan and Greeno, 1973) that different methods of teaching are more or less effective for people who are good or bad problem solvers. Their experiments dealt with teaching subjects basic concepts of probability using two different teaching methods. Subjects learning by the method of discovery proceeded by solving problems and generalizing with very little initial instructions. Subjects learning by rule were supposed to interpret initial instruction and apply it to problems. The results of this experiment showed that subjects who were found low in relevant mathematical abilities on the pretest performed better when instructed by the rule method then when instructed by the discovery method. This would suggest that "bad" problem solvers, or people lacking in skills necessary to solve problems may learn more efficiently when instructed by technique requiring interpretation and application of a rule. In the transfer situation this may imply that depending on person's ability to solve problems in general he or she may be transferring different elements of the problem solving process. Therefore one of the reasons for the insignificance of the overall results in this study might have been the lack of any pretest which would evaluate subjects' overall mathematical ability or preparation. This hypothesis seems to be supported by the fact that most subjects who did well in solving the four disc problem also did better than other subjects in solving the five disc problem.

A second problem that this study found had to do with the transfer procedure itself. In the three experimental groups subjects were preexposed to problems which involved a strategy or strategies also involved in the solution of the Tower of Hanoi problem. However whatever strategy they were forced to use, that short preexposure to it was probably overshadowed by their previous experience in problem solving. In spite of the fact that they were explicitly told that the strategies they used to solve preliminary problems were going to be helpful to them in solving the disc transfer problem, they may not have tried to use those strategies when actually solving the Tower of Hanoi problem. However this effect again might have been influenced by the fact that subjects' general problem solving ability was not taken into consideration. Preexposure to a strategy or strategies by using it in order to solve an otherwise unrelated problem might have been helpful to subjects skilled in problem solving in general but not to subjects whose problem solving ability was low.

Further analysis also showed no transfer effects from a four to a five disc problem. This may be due to the fact that in order to solve a four disc problem the subjects had to move the top disc (first move) to Peg C (temporary target), whereas in order to solve the five disc problem the top disc had to be moved to Peg B (the goal peg). However this result would suggest that subjects did not learn the deeper structure of the problem by solving the four disc problem. The transfer from solving the four disc problem by itself to solving the four disc problem as a second half of the five disc problem was in the speed of solution and not in the number of moves involved. This finding suggests that it is only some kind of overall skill in moving the discs around that gets transferred. Furthermore there are other data that suggest that this skill does not have to be acquired through solving a four disc problem first, but it can be learned during the process of solving a five disc problem by itself: the transfer effects in both the time and the number of moves involved from the first to the second half of the five disc problem solved by the third control group. What seems to be involved in the above transfer effect is an ability to solve a three disc problem which is demonstrated by the difference in time and the number of moves taken to move the top three discs between the second and third control group.

There are two basic processes involved in the Tower of Hanoi problem. One involves the process of moving the stack of discs from one peg to the other without specifying to which peg the stack is supposed to be moved to. When solving this type of problem (moving the stack of discs to either one of the two empty pegs) the first move is not crucial to achieving the solution. Therefore in this situation understanding of the deep structure of the problem which involves the method of working backwards is not essential to the problem solver.

The more difficult process is involved when the problem arises of moving a stack of discs to a particular peg. In this case the first move is essential and the correct first move can be easily determined by using the method of working backwards. The result of this study suggests that humans when faced with a relatively easy problem situation learn the surface structure before they can acquire the deep structure of the problem. What people learned in this situation was a subroutine to solve a three disc problem. Through solving a four disc problem they acquired then, a certain amount of facility in moving the discs from one peg to the other but they did not learn the deeper structure of the problem and therefore they did not apply the method of working backwards to the five disc problem.

However, almost all of the subjects who did solve the five disc problem and who made the first move to the incorrect peg realized their error early. The result of making the wrong first move could be that the stack of discs ends up being moved to the temporary target peg instead of to the goal peg. People usually realize that they made a mistake when they are faced with moving the second to the largest disc to the goal peg instead of to the temporary target where it should be moved to. At this point they have two choices: they can go on and move the second to the largest disc to the goal prg, and proceed with the solution so that they actually move the whole stack of discs to the wrong peg and then start over again, or they can correct the error by moving the stack of discs which is on the temporary target peg to the goal peg and then moving the second to the largest disc to the correct peg. Almost all of the successful solvers of the five disc problem took the second choice, that is corrected the error. Most of the subjects who were unable to finish the problem took the first choice. Thus it seems that most successful problem solvers in this situation even if they did not learn the deep structure of the problem through their experience with a four disc problem showed some effects of learning it during the course of solving the five disc problem.

In spite of the fact that the overall difference between the experimental groups and the control groups were almost nonexistant it seems that the preexposure to the method of working backwards had some effect. This is shown by the significantly higher percentage of subjects who made the first move to the right peg when solving the five disc problem in the second and third experimental group. These were the only two groups which were preexposed to the method of working backwards by solving one or two problems solutions to which involved using that method. However this preexposure might have effected only the "good" problem solvers and therefore the overall difference was insignificant.

The results of this study then seem to suggest that there are at least three interrelated factors which may be involved in transfer in problem solving: subjects' ability or skill in solving mathematical problems in general, the method of instruction, and the level of problem solving process (surface vs deep structure) which gets transferred.

The significance of the first of those factors can be tested by a quite simple procedure. The subjects could be given a pretest of their mathematical skills which could be composed in part of background questions and in part of the problems of the type of preliminary problems used in this experiment. On the basis of the results of this pretest subjects could be classified as bad or good problem solvers. Then all of the subjects would be given a four and then a five disc problem to solve. If there really is a difference of performance between good and bad problem solvers in this situation the results of this experiment should show a difference in time and the number of moves taken to solve the Tower of Hanoi problem by subjects which differ in their mathematical skill. It should also show some differences in transfer from a four to a five disc problem depending on subjects' problem solving ability.

If the results of the above described experiment would shoe that there exists an effect of the general problem solving ability on performance on the Tower of Hanoi problem, then another experiment could be designed which would test the effect of this difference on subjects' ability to acquire deep or surface structure of the problem. This experiment would again use a pretest to classify subjects as good or bad problem solvers. Then subjects in each category would be randomly assigned to two groups. One group would be asked to solve the four disc problem from Peg A to Peg B. The second group would be asked to solve the four disc problem from Peg A to either Peg B or Peg C, i.e. it would simply be told to move the stack of discs to either one of the empty pegs. Then all groups would be given a five disc problem to solve, to Peg B. This study should show different transfer effects depending on the training group (solving the four disc problem to either B or C, or B) and possibly depending on whether the subject is a good or a bad problem solver. Subjects who are asked to solve the four disc problem to a particular peg are actually being trained in the deeper structure of the problem. Subjects who are asked to solve the four disc problem to either of the empty pegs are trained in the surface structure of the problem since they do not have to worry about where the stack of discs ends up. On the second task (five disc problem) all subjects however are faced with a problem the solution to which involves the deeper structure of the problem. If good problem solvers pick up the deeper structure of the problem faster than the bad problem solvers they should do better on the five disc problem than the bad problem solvers regardless of their training on the four disc problem. It may happen however that both good and bad problem solvers trained on the surface structure of the problem will not learn its deeper structure but good problem solvers will pick it up when trained accordingly. The results could also be tested against a control group who would just solve the five disc problem.

In order to get at the effect of instruction or pretraining in more detail a third experiment could be designed. It actually could be designed in two ways. In both cases subjects would be again classified as good or bad problem solvers on the basis of the results of a pretest. In the first case one group of subjects would be first given a description of the two problem solving strategies used in the Tower of Hanoi problem (the subgoal method and the working backwards method). Then they would be asked to solve a four disc problem followed by a five disc problem. The second group would be first asked to solve the four disc problem. Then they would be given a written descriptions of the methods involved and then they would be asked to solve a five disc problem. The third group would be asked to solve the four disc problem. Then they would be asked to describe the problem solving strategies that they think are involved in solving the problem. Then they would be given the five disc problem to solve.

In the second version of the experiment the subjects would be divided into two groups. One group would be given the desciption of the subgoal and working backward methods first. Then they would be given two problems of the type used in this study: one involving a method of working backwards and the other the subgoal method in their solution. Then they would be given a four and a five disc problem to solve. The second group would be given the two problems first and would be asked to describe the strategy they used after solving each problem. Then they would be given a four and a five disc problem to solve.

In both versions of the experiment the four and five disc problem would be solved to a designated peg. One control group would be preexposed to the decsription of strategies and/or problems unrelated to the Tower of Hanoi problem. The second control group would not be preexposed to any strategies or problems. In all cases the subjects would be told whether or not the problems and strategies are related to the Tower of Hanoi problem.

The results of both versions of the above described experiment should show the effect of instructions on the transfer as well as on the subjects' ability to solve problems. However in the first case the transfer would involve very similar problem situations, whereas in the second case it would involve quite distinct problem situations. The second version is actually very similar to the present investigation only it takes into consideration both the instructions and the subjects' ability to solve problems which seem to have a crucial effect on transfer in problem solving.

Table 1

Group Means for the Four Disc Problem

	E-1	E-2	E-3	C-1	C-2
total moves	24.25	20.50	24.00	23.55	21.55
total time	17.11	13.27	15.24	16.16	13.30
1st half moves	15.50	12.00	15.22	14.44	12.55
1st half time	14.24	9.47	12.16	12.09	9.52
2nd half moves	8.75	8.50	8.77	9.10	9.00
2nd half time	3.47	3.39	3.09	4.06	3.38

Table 2

Group Medians for the Four Disc Problem

	E-1	E-2	E-3	C-1	C-2
total moves	28	22	24	24	18
total time	15.03	10.30	14.16	17.09	13.46
1st half moves	21	9.5	17	12	13
1st half time	11.20	9.00	10.39	11.03	10.23
2nd half moves	9	8.5	8	8	9
2nd half time	2.38	3.20	3.10	2.59	3.12

Table 3

Group Means for the Five Disc Problem

	E-1	E-2	E-3	C-1	C-2	C-3
total moves	49.83	49.66	44.20	50.00	51.37	50.17
total time	19.33	21.21	20.43	20.46	23.05	27.05
1st half moves	30.66	29.83	24.40	28.75	31.50	30.70
1st half time	13.39	13.28	11.58	13.27	16.16	19.34
2nd half moves	19.16	19.83	19.80	21.25	19.87	19.16
2nd half time	5.01	7.52	8.45	7.20	6.49	6.41

Table 4

Group Medians for the Five Disc Problem

	E-1	E-2	E-3	C-1	C-2	C-3
total moves	48	36.5	40	48	45	47.5
total time	19.30	16.30	22.09	20.00	21.10	24.11
1st half moves	30	20	18	29.5	27	27
1st half time	14.00	9.40	12.56	13.30	14.00	17.09
2nd half moves	18	16.5	21	21.5	19.5	19.5
2nd half time	5.10	8.07	7.37	6.30	6.10	7.09

Table 4a

Means and Medians for the moves and the time taken by C-2 and C-3 to move the first three discs when solving the five disc problem.

	C-2		C-3	
	Mean	Median	Mean	Median
moves	8.75	8	10.66	10
time	4.28	4.27	7.42	8.15

Table 5

Mann Whitney U's for the difference between C-2 and C-3 in time and number of moves involved in solving the five disc problem (all insignificant).

	U
total time	20
total moves	23
1st half time	22
1st half moves	21
2nd half time	25
2nd half moves	21

Table 6

Wilcoxon Matched Pairs T's for the transfer in time it took to solve the four disc problem by itself and as a second half of the five disc problem for corresponding groups.

Group	T	α
E-1	3	.02
E-2	6	.05
E-3	5	.05
C-1	2	.01
C-2	3	.02

Table 7

Percentage of subjects who made the first move to the correct peg when solving a five disc problem for corresponding groups (n=9).

Group	%
E-1	11
E-2	44
E-3	44
C-1	22
C-2	33
C-3	22

Table 8

Percentage of subjects who made the first move to the correct peg when solving the four disc problem for corresponding groups (n=9).

Group	%
E-1	55
E-2	67
E-3	67
C-1	44
C-2	67

Table 9

A step by step solution to the Missionaries and Cannibals problem (M---Missionary, C---Cannibal, b---boat).

M M M C C C

1. M M C C
   M C b

2. M M M C C b
   C

3. M M M
   C C C b

4. M M M C b
   C C

5. M C
   M M C C b

6. M M C C b
   M C

7. C C
   M M M C b

8. C C C b
   M M M

9. C
   M M M C C b

10. C C b
    M M M C

11.
    M M M C C C b

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