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Master Equation for a Localized Particle Driven by Poisson White Noise

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Fluctuations in nanosystems play an important role in forming their electric, magnetic, thermal and other properties. Usually, due to the central limit theorem of probability theory, these fluctuations obey Gaussian statistics. However, in some cases, e.g., when the system is subjected to Poisson white noise, that is a random sequence of δ-pulses, the system fluctuations are not Gaussian. Here, we derive the corresponding equation for the probability density function $P(x,t)$ of the system parameter $x(t)$ interpreted as a particle coordinate within an impenetrable box.

We assume that $x(t)$ satisfies the stochastic equation $dx(t)/dt = \xi(t)$, where $\xi(t)$ is Poisson white noise and $x(t) \in [-l, l]$ for all times $t \in [0, \infty)$. By introducing a discrete time $t = n\tau$ ($n = 0,1,2, \ldots$) with a time step τ, this equation can be rewritten in the differential form $\Delta x(t) = \Delta\eta(t)$, where $\Delta x(t) = x(t) - x(t-\tau)$ is the increment of the particle position and $\Delta\eta(t) = \int_{t-\tau}^{t} \xi(t')dt'$. Since Poisson noise $\xi(t)$ is white, the increments $\Delta\eta(n\tau)$ with different n are independent random variables distributed with the same probability density $p(\Delta\eta, \tau) = (1-\lambda\tau)\delta(\Delta\eta) + \lambda\tau q(\Delta\eta)$. Here, λ is the rate of the Poisson counting process, $\delta(x)$ is the Dirac δ function, and $q(x)$ is an arbitrary probability density function.

Next, using the definition $P(x,t) = \langle\delta(x - x(t))\rangle$, where the angular brackets denote averaging with respect to $\Delta\eta(t)$, and introducing the function $F(x) = -l, x$ and l if $x < -l$, $-l \leq x \leq l$ and $x > l$, respectively, we obtain $P(x,t) = \langle\delta[x - F(x(t-\tau) + \Delta\eta(\tau))]\rangle$. From this, calculating the ratio $[P(x,t) - P(x,t-\tau)]/\tau$ as $\tau \to 0$, one gets the master equation

$$\frac{1}{\lambda}\frac{\partial P(x,t)}{\partial t} = \int_{-l}^{l} P(x',t)[\delta(l+x)R(l+x') + \delta(l-x)R(l-x')]dx' + \int_{-l}^{l} P(x',t)q(x-x')dx' - P(x,t). \qquad (1)$$

In the steady state, when $\lim_{t\to\infty} P(x,t) = P_{st}(x)$, Eq. (1) is reduced to

$$\int_{-l}^{l} P_{st}(x')[\delta(l+x)R(l+x') + \delta(l-x)R(l-x')]dx' + \int_{-l}^{l} P_{st}(x')q(x-x')dx' - P_{st}(x) = 0. \qquad (2)$$