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# INTERVAL UNCERTAINTY ANALYSIS OF RELIABILITY OF SYSTEMS WITH COMPLEX INTERCONNECTIONS

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***Abstract***
*Industrial reliability and safety are important phenomena. The uncertainties play a key role in quantitative investigation of reliability of Systems with Complex Interconnections (SwCI). This paper proposes a modular approach matrix algebraic interval analysis method to investigate uncertainty of SwCIs reliability. Using results of uncertainty intervals of system reliability, maintenance cost and working expenditures of investigated manufacturing unit can be estimated, the maintenance management can receive specific supporting data to make correct decisions.*

## 1 Introduction

The reliability of engineering systems and their components, and risk analysis are highly important tools in analyzing and quantifying technical safety. Within the framework of the Synergy Demands of Automated Transport Systems project (EFOP-3.6.2-16-2017-00016) at Óbuda University, Institute of Mechatronics and Vehicle Engineering sensor networks and systems including their reliable and safe operation are examined [4].

Balogh and Hanka discussed applicability of Bayesian methods to probabilistic risk assessment and engineering design problems [1]. The attraction of Bayesian methods lies in their ability to integrate observed data and prior knowledge to form a posterior distribution estimate of a quantity of interest. Conceptually, Bayesian methods are desirable because they have the property of taking prior estimates and updating them with data over time. This proposed methodology might be useful to engineering managers for rare event risk analysis in other applications and other disciplines as well.

Theoretical background of reliability of engineering systems can be knowb by handbooks of Johanyák [2], [3] and book of Myers [6].

The reliability of Systems with Complex Interconnections (SwCIs) has become a crucial matter in several fields of engineering. The real complex systems, such as vehicle sensory networks, are not simply interconnected. The systems that have no so-called simple interconnections are the Systems with Complex Interconnection. The complex systems cannot be simplified by a combination of parallel and series blocks. The "traditional" reliability investigation methods – such as Reliability Block Diagram and Fault Tree Analysis – cannot be used to investigate reliabilities of the SwCIs. One approach to investigating the reliability parameters complex systems is Truth Table Method (TTM) that summarizes the probabilities of all the operating and non-operating states of the investigated system [2].

The main engineering task of mathematics is the mathematical modeling, model-based simulation and analysis or synthesis of technical systems. During mathematical model-based system investigation the modelers should meet different type, form and size of model uncertainties. Its reasons can be lack of knowledge of modelers' part or data inaccuracy [10].

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Model uncertainties play vital role in investigating the reliability of safety-critical systems with complex interconnections such as the sensory-network of the electric vehicles [8]. This uncertainty can be described as an interval [12]. This describing method is called interval uncertainty analysis.

The Bridge Structure System (BSS) such as sensors can be used as example of system reliability analysis.

The research which is reported in this paper is related basically to followings publications:

Oberkampf suggested a dialog between exploration experts of the reliability engineering, risk assessment, and information theory on the uncertainty representation, aggregation, and propagation [8] [9].

Möller and Beer explored an interval method of uncertainty modeling [5].

Pokorádi adapted the mathematical diagnostic methodology of aircraft gas turbine engines to determine system reliability sensitivity as Systems with Complex Interconnections (SwCI) [12] and as Bridge Structure Sensor, for example Wheatstone Like Bridge (WLB). These proposed methods are called TTM and Linear Sensitivity Model of System Reliability (LSMoSR). The paper showed a proposed method, and its applicability to investigate reliability of Bridge Structure Systems (BSS) by tree examples [11]; [12]; [13].

In paper [7] a real sensor and commutation network system of fully electric vehicle (Nissan Leaf Z0) was explored.

The present article applies the approach of the publications mentioned above. The main aim of this paper is to present a modular approach interval analysis method in order to determine the uncertainty of BSS's reliability.

The paper is organized as follows: Section 2 shows the determination method of SwCIs reliability, using TTM. Section 3 describes the model uncertainty. Section 4 presents the linear sensitivity model theoretically. Section 5 outlines the interval analysis of the most of BSS reliability. Section 6 summarizes the paper, outlines the prospective scientific work of the Author.

## 2 Determination of System Reliability

A BSS (see Figure 1) has five blocks, A; B; C; D; E. Their reliability can be characterized by reliability $r_i$ and probability of failure $p_i$.

The components have only two states – good (it is performing its required function – designated as the 1) and fault (it is not performing its required function – designated by 0). Sum of their probabilities should be one:

$$p_i + r_i = 1 \tag{1}$$

One of the approaches is to correctly calculate the reliability of BSS by sum of the probabilities of all good system states of the investigated system.

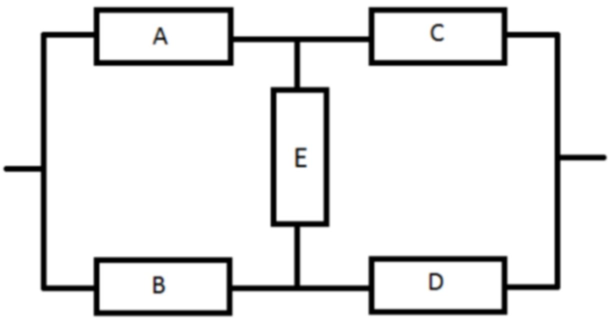


*Figure 1. Bridge Structure System*

A table listing the probabilities of each possible states for a system is referred as a Truth Table (TT).

The possible system states are summarized in the form of a Truth Table, shown in Table 1., with each component being assigned either a good or a fault state. The $Q_j$ column comprises the probabilities of each of the system states. Since the table covers all of the possible combinations, the sum of all of the state probabilities should be 1.

In general case the BSS is operating if a sign or matter can “go across” it. For example it can be an industrial plant, where there are two parallel production lines that should be connected by a buffer store to balance their fluctuation.

*Table 1. Truth Table of BSS*

| *i* | *A* | *B* | *C* | *D* | *E* | *System* | $Q_i$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | $p_A\ p_B\ p_C\ p_D\ p_E$ |
| 2 | 1 | 0 | 0 | 0 | 0 | 0 | $r_A\ p_B\ p_C\ p_D\ p_E$ |
| 3 | 0 | 1 | 0 | 0 | 0 | 0 | $p_A\ r_B\ p_C\ p_D\ p_E$ |
| 4 | 1 | 1 | 0 | 0 | 0 | 0 | $r_A\ r_B\ p_C\ p_D\ p_E$ |
| 5 | 0 | 0 | 1 | 0 | 0 | 0 | $p_A\ p_B\ r_C\ p_D\ p_E$ |
| 6 | 1 | 0 | 1 | 0 | 0 | 1 | $r_A\ p_B\ r_C\ p_D\ p_E$ |
| 7 | 0 | 1 | 1 | 0 | 0 | 0 | $p_A\ r_B\ r_C\ p_D\ p_E$ |
| 8 | 1 | 1 | 1 | 0 | 0 | 1 | $r_A\ r_B\ r_C\ p_D\ p_E$ |
| 9 | 0 | 0 | 0 | 1 | 0 | 0 | $p_A\ p_B\ p_C\ r_D\ p_E$ |
| 10 | 1 | 0 | 0 | 1 | 0 | 0 | $r_A\ p_B\ p_C\ r_D\ p_E$ |
| 11 | 0 | 1 | 0 | 1 | 0 | 1 | $p_A\ r_B\ p_C\ r_D\ p_E$ |
| 12 | 1 | 1 | 0 | 1 | 0 | 1 | $r_A\ r_B\ p_C\ r_D\ p_E$ |
| 13 | 0 | 0 | 1 | 1 | 0 | 0 | $p_A\ p_B\ r_C\ r_D\ p_E$ |
| 14 | 1 | 0 | 1 | 1 | 0 | 1 | $r_A\ p_B\ r_C\ r_D\ p_E$ |
| 15 | 0 | 1 | 1 | 1 | 0 | 1 | $p_A\ r_B\ r_C\ r_D\ p_E$ |
| 16 | 1 | 1 | 1 | 1 | 0 | 1 | $r_A\ r_B\ r_C\ r_D\ p_E$ |
| 17 | 0 | 0 | 0 | 0 | 1 | 0 | $p_A\ p_B\ p_C\ p_D\ r_E$ |
| 18 | 1 | 0 | 0 | 0 | 1 | 0 | $r_A\ p_B\ p_C\ p_D\ r_E$ |
| 19 | 0 | 1 | 0 | 0 | 1 | 0 | $p_A\ r_B\ p_C\ p_D\ r_E$ |
| 20 | 1 | 1 | 0 | 0 | 1 | 0 | $r_A\ r_B\ p_C\ p_D\ r_E$ |
| 21 | 0 | 0 | 1 | 0 | 1 | 0 | $p_A\ p_B\ r_C\ p_D\ r_E$ |
| 22 | 1 | 0 | 1 | 0 | 1 | 1 | $r_A\ p_B\ r_C\ p_D\ r_E$ |
| 23 | 0 | 1 | 1 | 0 | 1 | 1 | $p_A\ r_B\ r_C\ p_D\ r_E$ |
| 24 | 1 | 1 | 1 | 0 | 1 | 1 | $r_A\ r_B\ r_C\ p_D\ r_E$ |
| 25 | 0 | 0 | 0 | 1 | 1 | 0 | $p_A\ p_B\ p_C\ r_D\ r_E$ |
| 26 | 1 | 0 | 0 | 1 | 1 | 1 | $r_A\ p_B\ p_C\ r_D\ r_E$ |
| 27 | 0 | 1 | 0 | 1 | 1 | 1 | $p_A\ r_B\ p_C\ r_D\ r_E$ |
| 28 | 1 | 1 | 0 | 1 | 1 | 1 | $r_A\ r_B\ p_C\ r_D\ r_E$ |
| 29 | 0 | 0 | 1 | 1 | 1 | 0 | $p_A\ p_B\ r_C\ r_D\ r_E$ |
| 30 | 1 | 0 | 1 | 1 | 1 | 1 | $r_A\ p_B\ r_C\ r_D\ r_E$ |
| 31 | 0 | 1 | 1 | 1 | 1 | 1 | $p_A\ r_B\ r_C\ r_D\ r_E$ |
| 32 | 1 | 1 | 1 | 1 | 1 | 1 | $r_A\ r_B\ r_C\ r_D\ r_E$ |

The state probabilities resulting in an operating system are included in the rows 6; 8; 11; 12; 14; 15; 16; 22; 23; 24; 26; 27; 28; 30; 31 and 32. The

$$R_{sys} = Q_6 + Q_8 + Q_{11} + Q_{12} + Q_{14} + Q_{15} + Q_{16} + Q_{22} + \\ + Q_{23} + Q_{24} + Q_{26} + Q_{27} + Q_{28} + Q_{30} + Q_{31} + Q_{32} \quad (2)$$

sum of the operating system state probabilities included in this column is the reliability of the system.

In general case the BSS is operating if a sign or matter can "go across" it. The Figure 2 shows system reliabilities $R_{sys}$ in case of different reliabilities of component $r_i$. For example it can be an industrial plant, where there are two parallel production lines that should be connected by a buffer store to balance fluctuation of their productivities that are characteristics of applied technology.

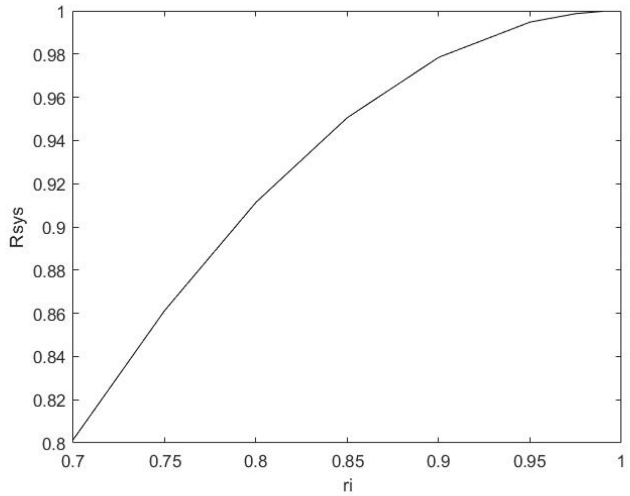


*Figure 2. System Reliabilities in Cases of Different Reliabilities of Components*

In so called "full" working state of a BSS the all components should operate. In this case the reliability $R_{full}$ can be calculated as the probability of the system state 32 of Truth Table of BSS (see Table 1.). The "full" system reliabilities are shown by Figure 3 in cases of different component.

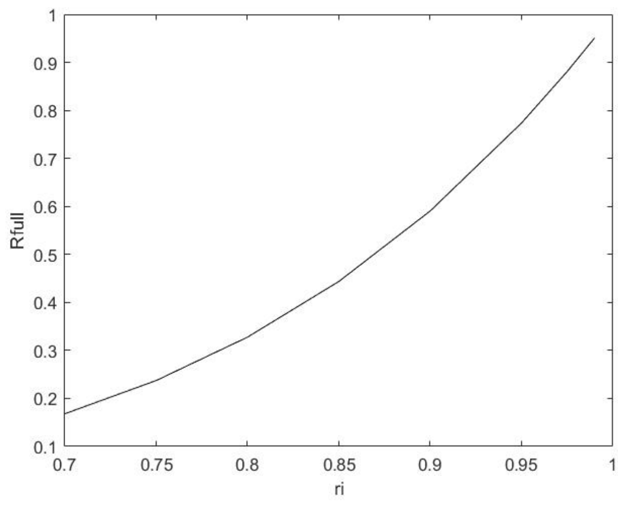


*Figure 3. "Full" System Reliabilities in Cases of Different Reliabilities of Components*

## 3 Model Uncertainties

The mathematical modeling is the description of the process occurring on the investigated system from the point of view of the given investigation by mathematical equation or system of equations. A real technical system is precise but complex. Additionally, a system of systems consists of large number of inter-connected systems and aggregates. But, the mathematical model should be simplified therefore can be imprecise.

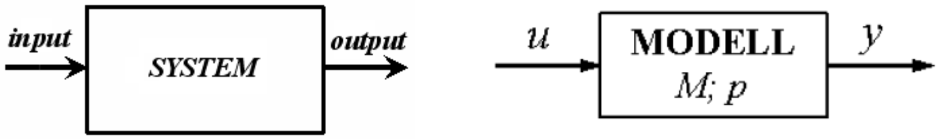


*Figure 4. System and Model*

Seeing Figure 4., a mathematical model has

$M$ – structure (e.g. stochastic, as reliability model of BSS in this study);
$p$ – inner system parameters (e.g. number of element of system in our case);
$u$ – input signs (e.g. reliabilities of elements of investigated BSS);

and responds by output parameters **y**.

For interpretation of types of uncertainty and their investigation methods, let

$$\mathbf{y} = f(\mathbf{x}) \tag{3}$$

general mathematical model, where **y** is the vector of dependent (output) variables, **x** is the vector of independent variables.

One of the most widely accepted types of uncertainties are aleatory and epistemic ones.

Model uncertainties are called as epistemic, if the modeler reduces the model improperly. This uncertainty may be comprised of substantial amounts of both objectivity and subjectivity.

The epistemic uncertainty means the incorrection of model structure $M$ (Figure 5.a).

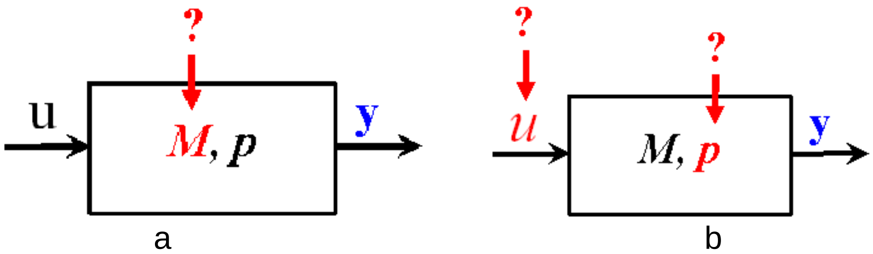


*Figure 5. Epistemic and Aleatory Uncertainty*

Aleatory uncertainties are inseparable variation associated with the modeled system or its environment and also are called parametric uncertainty. Their possible engineering sources are:

- inaccurate measuring;
- measuring noises;
- unconscionable digitalization;
- wrong statistical information.

The parametric uncertainty means anomalies of parameters (see Figure 5.b).

Referring to Equation (3), there are two fundamental approaches to investigate parametric uncertainties. The first method is the interval uncertainty analysis that characterizes a given uncertainty by

$$\mathbf{i}_y = f_i(\mathbf{i}_x) \tag{4}$$

general equation, where

$\mathbf{i}_x$ – vector of intervals of input variables;
$\mathbf{i}_y$ – vector of intervals of output variables;

Another fundamental investigation method is the probabilistic analysis that describes uncertainty by probability distributions. In this case the

$$\mathbf{d}_y = f_d(\mathbf{d}_x) \tag{5}$$

general equation is used, where
$\mathbf{d}_x$ – vector of distributions of independent variables;
$\mathbf{d}_y$ – vector of distributions of dependent variables;

During uncertainty analysis the functions $f_i$ and $f_d$ should be determined by applied mathematical model.

## 4 Linear Sensitivity Model

The theoretical method of setting up Linear Sensitivity Model can be read in references [12] and [13] in detail. The probabilities of possible system states (see Table 1.) can be described by

$$Q_j = \prod_{i=A}^{E} u_i(r_i) \tag{6}$$

general equation form.
If the state of component is good, the inner function $u_i = r_i$, then the sensitivity coefficient is:

$$K_{ji} = 1 \tag{7}$$

If the state of the component is faulty, the inner function $u_i = 1 - r_i$, then the sensitivity coefficient is:

$$K_{ji} = -\frac{r_i}{Q_j} \prod_{\substack{k=A \\ k \neq i}}^{E} u_i \tag{8}$$

In case of functions determining directly the probabilistic system parameters – see equations (2) – the sensitivity coefficients can be determined by

$$K_j = \frac{Q_j}{R_{sys}} \tag{9}$$

Following the general determinations mentioned above we can now set up the linear sensitivity models of system.
According to the references [10], [12] mentioned before the connection between relative changes of the independent and the dependent parameters can be described by

$$\mathbf{A}\delta\mathbf{y} = \mathbf{B}\delta\mathbf{x} \tag{10}$$

equation, where **A** and **B** coefficient matrices are independent and dependent parameters, and , are vectors relative changing of independent and dependent parameters. Using the

$$\mathbf{D} = \mathbf{A}^{-1}\mathbf{B} \tag{11}$$

relative sensitivity coefficient matrix of the investigated system, the equation

$$\delta\mathbf{y} = \mathbf{D}\delta\mathbf{x} \tag{12}$$

can be used for relative sensitivity investigations [12]. The independent parameter vector consists of component reliabilities:

$$\mathbf{x}^T = \begin{bmatrix} r_A & r_B & r_C & r_D & r_E \end{bmatrix} \tag{13}$$

The vector of dependent parameters consists of the probability of system reliability and probabilities of operating system states – see equation (2):

$$\mathbf{y}_{sys}^{T} = \left[ R_{sys} \quad Q_6 \quad Q_8 \quad Q_{11} \quad Q_{12} \quad Q_{14} \quad Q_{15} \quad Q_{16} \quad Q_{22} \quad Q_{23} \quad Q_{24} \quad Q_{26} \quad Q_{27} \quad Q_{28} \quad Q_{30} \quad Q_{31} \quad Q_{32} \right] \tag{14}$$

The coefficient matrix of the dependent parameters:

$$\mathbf{A}_{sys} = \begin{bmatrix}
1 & -K_6 & -K_8 & -K_{11} & -K_{12} & -K_{14} & -K_{15} & -K_{16} & -K_{22} & -K_{23} & -K_{24} & -K_{26} & -K_{27} & -K_{28} & -K_{30} & -K_{31} & -K_{32} \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \tag{15}$$

The coefficient matrix of independent parameters:

$$\mathbf{B}_{sys} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & K_{6B} & 1 & K_{6D} & K_{6E} \\
1 & 1 & 1 & K_{8D} & K_{8E} \\
K_{11A} & 1 & K_{11C} & 1 & K_{11E} \\
1 & 1 & K_{12C} & 1 & K_{12E} \\
1 & K_{14B} & 1 & 1 & K_{14E} \\
K_{15A} & 1 & 1 & 1 & K_{15E} \\
1 & 1 & 1 & 1 & K_{16E} \\
1 & K_{22B} & 1 & K_{22D} & 1 \\
K_{23A} & 1 & 1 & K_{23D} & 1 \\
1 & 1 & & K_{24D} & 1 \\
1 & K_{26B} & K_{26C} & 1 & 1 \\
K_{27A} & 1 & K_{27C} & 1 & 1 \\
1 & 1 & K_{28C} & 1 & 1 \\
1 & K_{30B} & 1 & 1 & 1 \\
K_{31A} & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1
\end{bmatrix} \tag{16}$$

## 5 Interval Uncertainty Analysis

In this study the $f_i$ function of BSS reliability will be determined based on the TTM mentioned above.

The first step is determining the vectors of relative maximum and minimum values of independent parameter values:

$$\begin{aligned}\delta\mathbf{x}_{\max} &= \mathbf{X}^{-1}\left(\mathbf{x}_{\max} - \mathbf{x}_{nom}\right)\\ \delta\mathbf{x}_{\min} &= \mathbf{X}^{-1}\left(\mathbf{x}_{\min} - \mathbf{x}_{nom}\right)\end{aligned} \tag{17}$$

where:

**X** – matrix of nominal values of independent variables

$$\mathbf{X} = \left\langle x_{1nom} \quad x_{2nom} \quad \dots \quad x_{Nnom} \right\rangle \; .$$

$\mathbf{x}_{nom}$ – vector of nominal values of independent variables;
$\mathbf{x}_{max}$ – vector of maximal values of independent variables;
$\mathbf{x}_{min}$ – vector of minimal values of independent variables

For interval uncertainty analysis the relative sensitivity model – see equation (12) – should be modified. The so-called “positive diagnostic matrix” and “negative diagnostic matrix”

$$\begin{aligned}\mathbf{D}_+ &= \left[ d_{ij+} = \begin{cases} d_{ij} & \text{if} \quad d_{ij} \geq 0 \\ 0 & \text{if} \quad d_{ij} < 0 \end{cases} \right]\\ \mathbf{D}_- &= \left[ d_{ij-} = \begin{cases} d_{ij} & \text{if} \quad d_{ij} < 0 \\ 0 & \text{if} \quad d_{ij} \geq 0 \end{cases} \right]\end{aligned} \tag{18}$$

should be introduced.

Knowing the matrices mentioned above, the vectors of relative minimum and maximum values of the dependent parameters:

$$\begin{bmatrix} \delta\mathbf{y}_{max} \\ \delta\mathbf{y}_{min} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_+ & \mathbf{D}_- \\ \mathbf{D}_- & \mathbf{D}_+ \end{bmatrix} \begin{bmatrix} \delta\mathbf{x}_{max} \\ \delta\mathbf{x}_{min} \end{bmatrix} \tag{19}$$

Knowing the elative minimum and maximum values, the measured minimum and maximum output parameter values should be determined

$$\begin{aligned}\mathbf{y}_{max} &= \mathbf{y}_{nom} + \mathbf{Y}\delta\mathbf{y}_{max}\\ \mathbf{y}_{min} &= \mathbf{y}_{nom} + \mathbf{Y}\delta\mathbf{y}_{min}\end{aligned} \tag{20}$$

where:

**Y** – matrix of nominal values of dependent variables

$$\mathbf{Y} = \left\langle y_{1nom} \quad y_{2nom} \quad \dots \quad y_{Mnom} \right\rangle \quad ;$$

$\mathbf{y}_{nom}$ – vector of nominal values of dependent variables.

### 5.1 Determinations of Uncertainty Intervals Depend on Reliabilities of Components (Theoretical Investigation)

Firstly the interval uncertainty analysis method was used to determine uncertainty intervals of system reliability. During simulation reliabilities of all components were same and different between maximum, minimum and nominal values were 0.02.

In general case the system reliability $R_{sys}$ was determined by equation (2) and the above mentioned method was used. The results are shown by Figure 6.

In so called “full” working state of a BSS the maximum and minimum values of uncertainty interval are the 17$^{th}$ elements of measured minimum and maximum output parameter vectors. The figure 7 shows the results of interval uncertainty analysis of “full” system reliability.

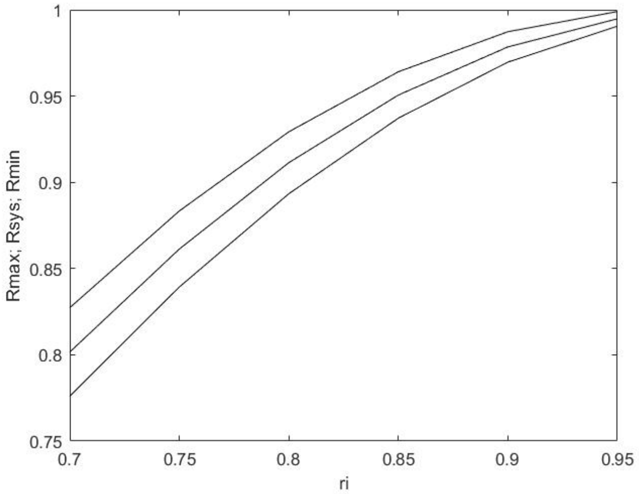


*Figure 6. Results of Interval Analysis of System Reliabilities in Cases of Different Reliabilities of Components ($\Delta r_i$=±0.02)*

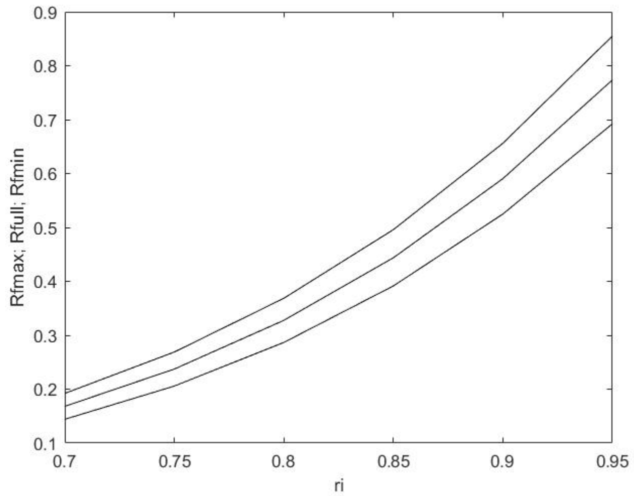


*Figure 7. Fig. Results of Interval Analysis of "Full" System Reliabilities in Cases of Different Reliabilities of Components ($\Delta r_i$=±0.02)*

### 5.2 Investigation of Production Line's Reliability (Practical Case Study)

The author aimed to investigate two parallel production lines that are connected by a buffer store to balance unsteadiness of their productivities. These two parallel production lines and buffer store system can be investigated as a BSS – from the point of view of its reliability.

When determining of the reliabilities of failures of sublines and storage statistically, remarkable uncertainties where observe. These statistical data are shown by Table 2 and Table 3 shows of the results of interval uncertainty analysis of the manufacturing line reliabilities.

*Table 2. Data of Reliability Intervals of Items*

| *i* | *A* | *B* | *C* | *D* | *E* |
|---|---|---|---|---|---|
| $r_{imax}$ | 0.885 | 0.874 | 0.911 | 0.877 | 0.958 |
| $r_{inom}$ | 0.871 | 0.872 | 0.901 | 0.881 | 0.955 |
| $r_{imin}$ | 0.861 | 0.870 | 0.899 | 0.884 | 0.951 |

*Table 3. Result of Interval Reliability Investigation*

| *j* | *Sys* | *„Full”* |
|---|---|---|
| $r_{jmax}$ | 0.97387 | 0.59192 |
| $r_{jnom}$ | 0.97092 | 0.57576 |
| $r_{jmin}$ | 0.96933 | 0.5661 |

These results can be used for the estimation of expected minimum and maximum values of maintenance cost and working expenditures of investigated manufacturing unit. Thus, the maintenance management can receive specific supporting data to make correct decisions. They can determine the maximum and minimum number of required spare parts to ensure continuous operation of investigated manufacturing line.

### 5.3 Discussions

The following conclusions can be drawn from the results of interval uncertainty analysis:

***A1:*** In general case, the system reliability is approaching 1 asymptotically when reliabilities of components increase (see Figure 2.).

***A2:*** In general case, the uncertainty interval of system reliability decreases if the reliabilities of the elements increase (see Figure 6.).

***B1:*** The “full” system reliabilities is approaching 1 when reliabilities of components increase exponentially (see Figure 3.).

***B2:*** The uncertainty interval of “full” system reliability increases if the reliabilities of the elements increase (see Figure 7.).

***C1:*** Using the proposed method the intervals of probabilities of the system can be determined. The results of interval uncertainty analysis of reliability give important information for the maintenance management to make correct decisions.

## 6 Closing Remarks

This paper presented a new interval uncertainty investigation method of SwCIs’ reliability. Its possibilities of use have been shown by way of theoretical investigation and a practical case study of the BSSs’ reliabilities.

The Author’s proposed prospective future research direction is the study of uncertainty analysis methodologies of systems with complex interconnections, such as vehicle sensory network, reliability based on probabilistic uncertainty analysis, probability-bounds analysis and Monte-Carlo Simulation.

## Acknowledgment

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