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Reversible elements in rings*

Research Article

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Abstract: We study some properties related to zero divisors and reversibility in noncommutative rings.

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1. Introduction

All our rings will have a unity. A ring R is *reversible* if, for any $a, b \in R$, $ab = 0$ if and only if $ba = 0$. These rings are natural generalizations of commutative rings. Reversible rings were studied, in particular, by P.M. Cohn [1], Gutan and Kisielewicz [3], Kim and Lee [4] and many others. Our aim, in this short note, is to introduce elementwise definitions that are directly connected to reversibility but can be applied in a more flexible way. For $a \in R$, we will write $r(a) = \{x \in R \mid ax = 0\}$ and $l(a) = \{x \in R \mid xa = 0\}$. An element $a \in R$ is right reversible if $r(a) \subseteq l(a)$. The set of right reversible elements will be denoted by $rRev(R)$. A ring R is semi-commutative if for any $a, b \in R$, $ab = 0$ implies that $aRb = 0$. In other words a ring R is semi-commutative when the annihilator of an element is a two sided-ideal. The subsets $N(R)$ and $U(R)$ will respectively stand for the set of nilpotent elements and invertible elements of the ring R. A ring R is 2-primal if the set $N(R)$ coincides with the prime radical. Other notions will be defined when and where needed. The second section is devoted to the definition, characterizations and properties of the reversible set of a ring and its behavior relative to some ring constructions. Reversible nilpotent and idempotent elements are characterized and connections with 2-primal rings are established. In the third section we study some connections with other, more classical, notions such as strongly regular rings and McCoy rings. The paper ends with considerations related to elements having the property that $r(a) \neq 0$ implies $l(a) \neq 0$.

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2. Reversible set of a ring

Definition 2.1. *Let R be a ring with $1 \in R$. For an element a in R we write $r_R(a)$ or just $r(a)$ (resp. $l_R(a)$ or $l(a)$) the right (resp. left) annihilator of a. The element a is right (resp. left) reversible if $r(a) \subseteq l(a)$ (resp. $l(a) \subseteq r(a)$). An element which is both left and right reversible is a reversible element. The set of right (resp. left) reversible elements of R will be denoted by $rRev(R)$ (resp. $lRev(R)$). The set of reversible elements is denoted by $Rev(R)$. Observe that for any ring R, $Rev(R) = rRev(R) \cap lRev(R)$. A ring is reversible (cf. [1]) if $rRev(R) = R$ (equivalently $lRev(R) = R$).*

Example 2.2.

1. *Observe that $0 \in rRev(R)$. Also if $r(a) = 0$ than $a \in rRev(R)$. In particular, left invertible elements, right regular elements in a ring are right reversible.*
2. *If R is commutative or if R is domain then $rRev(R) = lRev(R) = R$.*
3. *Of course, if R is reduced then $rRev(R) = R = lRev(R)$. This is easily checked as follows: for any $a \in R$, if $ab = 0$ then $(ba)^2 = 0$ and hence $ba = 0$, showing that $rRev(R) = R$.*
4. *A ring R is semi-commutative if, for any $a \in R$, $r(a)$ is a 2-sided ideal. In other words R is semi-commutative if for any elements $a, b \in R$ we have $ab = 0$ implies that $aRb = 0$. In general, if an element $a \in rRev(R)$ is such that $r(a) \subseteq rRev(R)$, then $r(a)$ is a 2-sided ideal of R. In particular, a reversible ring is always semi-commutative.*
5. *Let k be a field. The set of 2×2 lower triangular matrices over k will be denoted $L_2(k)$. We have $rRev(L_2(k)) = \{\begin{pmatrix} \alpha & 0 \\ \beta & \gamma \end{pmatrix} \mid \alpha\gamma \neq 0\} \cup \{\begin{pmatrix} 0 & 0 \\ \beta & \gamma \end{pmatrix} \mid \beta \in k, \gamma \in k \setminus \{0\}\}$. Indeed, if $\alpha\gamma \neq 0$ then the corresponding matrix is invertible and hence it belongs to $rRev(R)$. It is easy to check that, for any $\alpha, \beta, \gamma \in k \setminus \{0\}$ and any $\delta \in k$, we have*
$$\begin{pmatrix} 0 & 0 \\ \beta & 0 \end{pmatrix} \notin rRev(R), \quad \begin{pmatrix} 0 & 0 \\ \delta & \gamma \end{pmatrix} \in rRev(R), \quad \begin{pmatrix} \alpha & 0 \\ \delta & 0 \end{pmatrix} \notin rRev(R).$$
6. *Let R be any ring. Observe that for $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ in $M_2(R)$ we have*
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$
This shows that for any ring R, $rRev(M_2(R)) \neq M_2(R)$.
7. *It is easy to check that if two rings R, S are such that $R \subseteq S$ then $rRev(S) \cap R \subseteq rRev(R)$. The reverse inclusion generally does not hold. Indeed, let R be a domain and σ an endomorphism of R with nonzero kernel. Consider the Ore extension $S = R[t; \sigma]$ with polynomials of the form $\sum X^i a_i$ and commutation rule $aX = X\sigma(a)$. If $a \in \ker(\sigma)$ then $aX = X\sigma(a) = 0$ but $Xa \neq 0$ this shows that $a \in rRev(R)$ but $a \notin rRev(S) \cap R$.*

The following proposition provides a characterization of right reversible elements.

Proposition 2.3. *Let R be a ring. For an element $a \in R$, the following are equivalent:*

(i) $a \in rRev(R)$,

(ii) $r(a) \subseteq C(a)$, *where* $C(a) = \{x \in R \mid ax = xa\}$ *is the centralizer of* a *in* R,

(iii) The correspondence $\varphi : aR \to Ra$ *defined by* $\varphi(ar) = ra$ *is a well-defined additive map,*

(iv) $a \in r(r(a))$,

(v) *For every* $b \in R$ *we have that* $(ab)^2 = ab$ *implies that* $(ba)^2 = ba$.

Proof. (i) $\Leftrightarrow$ (ii): This is clear.

(i) $\Rightarrow$ (iii): Notice that if $ar = ar'$ then $a(r - r') = 0$ and hence $(r - r')a = 0$ so that $ra = r'a$. Now we have $\varphi(ar) = ra = r'a = \varphi(ar')$. This shows that φ is well defined. The fact that the map is additive is clear. The converse implication is obvious.

(iii) $\Rightarrow$ (iv): For any $x \in r(a)$ we have $ax = 0$ and $0 = \varphi(ax) = xa$. This shows that $a \in r(r(a))$.

(iv) $\Rightarrow$ (v) Suppose $b \in R$ is such that $(ab)^2 = ab$, this gives that $bab - b \in r(a)$ hence by (iv) we $(bab - b)a = 0$, i.e., $(ba)^2 = ba$.

(v) $\Rightarrow$ (i): Suppose $ab = 0$. Then $(ab)^2 = ab = 0$ and the hypothesis shows that $0 = (ba)^2 = ba$ and so $ba = 0$. □

Corollary 2.4. *If* $a \in rRev(R)$ *is such that* $(ab)^2 = ab$ *then* $(ab)R \cong (ba)R$ *and* $R(ba) \cong R(ab)$ *and the idempotents* ab *and* ba *are isomorphic.*

Proof. This is a direct consequence of Proposition 2.3 above and Proposition 21.20 in [5]. □

Let us now show how the reversible notion behaves.

Theorem 2.5. *Let* R *be any ring. The following hold:*

(a) *The set* $rRev(R)$ *(respectively* $lRev(R)$*) is closed under product (,i.e., if* $a, b \in rRev(R)$*, then* $ab \in rRev(R)$*).*

(b) *If* R *and* S *are two rings and* $\varphi : R \longrightarrow S$ *is an isomorphism of rings then* $\varphi(rRev(R)) = rRev(S)$*. In particular, if* $u \in U(R)$ *is a unit in* R*, then* $a \in rRev(R)$ *if and only if* $uau^{-1} \in rRev(R)$*.*

(c) *If* $a \in rRev(R)$ *then, for invertible elements* $u, v \in U(R)$*, we have* $uav \in rRev(R)$*.*

(d) *If* R *is a prime ring we have* $\{a \in R \mid r(a) = 0\} = rRev(R)$*.*

(e) *If* a *is right invertible then* a *is right reversible if and only if* a *is left invertible (and hence invertible).*

(f) *If* R *and* S *are two rings then* $rRev(R \times S) = rRev(R) \times rRev(S)$*.*

(g) *If* R *is a semisimple ring then* $rRev(R) = U(R)$*, the set of invertible elements of* R*.*

(h) *An idempotent* $e \in R$ *is right reversible if and only if* $(e - 1)Re = 0$*.*

(i) *Let* $\{x_1, \ldots, x_n\}$ *be a subset of* $rRev(R)$*. If* $b \in r(x_1 x_2 \cdots x_n)$*, then* $RbRx_1Rx_2R \ldots Rx_nR = 0$*.*

(j) *If* $a \in R$ *is right reversible and nilpotent, then the ideal* RaR *is nilpotent.*

(k) *If* $a \in R$ *is such that the descending chain of left ideals* Ra^i *stabilizes then* $l(a) = 0$ *if and only if* $a \in U(R)$*.*

Proof. (a) Let $a, b \in rRev(R)$ and let $c \in r(ab)$. Then we have $abc = 0$ and since $a \in rRev(R)$, this gives $bca = 0$. Since $b \in rRev(R)$, we get $cab = 0$, and hence $c \in l(ab)$.

(b) and (c) These are left to the reader.

(d) If R is prime and $0 \neq a \in rRev(R)$ we have, for any $b \in r(a)$ and any $r \in R$, we have $abr = 0$ and hence $bra = 0$. This gives $bRa = 0$ and the primeness of R leads to $b = 0$, as required.

(e) If a is right invertible then there exists $b \in R$ such that $ab = 1$ and hence $a(ba - 1) = 0$. Since a is also right reversible we get that $(ba - 1)a = 0$. This gives $ba^2 = a$ and, right multiplying by b, we get $ba = 1$. The converse is clear.

(f) This is obvious.

(g) In the light of Wedderburn-Artin Theorem and Part (f) of this theorem, we may assume that R is a matrix ring over a division ring. In particular, R is a prime ring and so the point (d) shows that the right reversible elements are nonzero divisors. Now, a nonzero divisor matrix with coefficients in a division ring must be invertible, this easily yields the statement.

(h) This is clear.

(i) We, then, have $x_1x_2\cdots x_nbR = 0$, and since $x_1 \in rRev(R)$, we get $x_2x_3\cdots x_nbRx_1 = 0$. So $x_2x_3\cdots x_nbRx_1R = 0$. But $x_2 \in rRev(R)$, hence $x_3x_4\cdots x_nbRx_1Rx_2 = 0$. Continuing this process we get the desired result.

(j) Let us suppose that $a \in rRev(R)$ is such that $a^n = 0$, for some $n \in \mathbb{N}$. We then have $a \in r(a^{n-1})$ and the above statement (i) yields the result.

(k) There exists $n \in \mathbb{N}$ and $x \in R$ such that $a^n = xa^{n+1}$. Since $l(a) = 0$, this leads to $1 = xa$ and hence to $a = axa$ and also $1 = ax$, showing that $a \in U(R)$. □

Let us recall that $N(R) = \{x \in R \mid \exists n \in \mathbb{N} : x^n = 0\}$.

Corollary 2.6. *(1) For any ring R, $rRev(R) \cap N(R)$ is contained in the prime radical of R.*

(2) If all nilpotent elements of a ring are right reversible then the ring is 2-primal.

(3) In a semiprime ring a nilpotent element cannot be right or left reversible.

(4) If $a \in N(R) \cap rRev(R)$ and $b \in N(R)$ then $a + b \in N(R)$.

(5) If $a \in rRev(R)$, $Rr(a)$ is a proper (,i.e., different from R) two-sided ideal.

Proof. 1) This is clear from Theorem 2.5 (i).

2) This is an obvious consequence of Corollary 2.6 (1).

3) It is enough to use the fact that a semiprime ring does not have nonzero nilpotent two sided ideal

4) Let $l \in \mathbb{N}$, be such that $b^l = 0$. When we develop $(a + b)^l$ all monomials will be in the prime radical.

5) This is due to the fact that $Rr(a)$ is contained in $l(a)$. □

We observe that the ring R of upper triangular 2×2 matrices over a field is 2-primal but not reversible.

Example 2.7.

(a) Observe that $rRev(R)$ is in general not closed under addition. To give a concrete example let us consider the ring of 2×2 lower matrices over a field k. It is easy to check that the matrix $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}$ is not right reversible although the two matrices on the right hand side are, indeed, right reversible.

(b) In connection with Theorem 2.5 (j) we remark that we might have RaR nilpotent even if $a \notin rRev(R)$. This is the case of the element $a = e_{12}$ of the strictly upper 2×2 matrices over a field.

3. Connections with other notions

Proposition 3.1. *Let R be a semiprime ring and $a \in rRev(R)$. Then:*

(a) The right annihilator $r(a)$ is a two-sided ideal of R.

(b) for any $n \in \mathbb{N}$, we have $r(a) = r(a^n)$.

Proof. (a) If $b \in r(a)$ we have $bRa = 0$ and hence $(RaRb)^2 = 0$. Since R is semiprime this leads to $RaRb = 0$, which, in turn, implies that $aRb = 0$.

(b)For $n \in \mathbb{N}$, $n \geq 1$, and $a \in rRev(R), b \in r(a^n)$ we have $a^{n-1}(a^{n-1}b) = 0$. Since a^{n-1} is also in $rRev(R)$ we have that $a^{n-1}bRa^{n-1}b = 0$. The fact that R is semiprime leads to $a^{n-1}b = 0$,i.e., $b \in r(a^{n-1})$. The same method leads to $b \in r(a^{n-2})$ and the desired result follows by iterations. □

We remark that the above statement admits a partial converse: if a ring R is such that for any $a \in R$, there exists $l > 1$ such that $r(a) = r(a^l)$ then R is reduced and hence semiprime.

We also recall that a strongly regular ring is a ring R such that for every $a \in R$ there exists $x \in R$ such that $a = a^2x$. The following proposition is based on Exercise 12. 6A in Lam's book [6].

Proposition 3.2. *The following are equivalent:*

(i) The ring R is strongly regular,

(ii) The ring R is regular and reduced,

(iii) The ring R is regular and reversible.

Let us now give some applications to McCoy condition on polynomials. Let us first define $Rev(R) = rRev(R) \cap lRev(R)$ and say that a polynomal $f(x) \in R[x]$ is right McCoy if $r_{R[x]}(f(x)) \neq 0$ implies that there exists a nonzero $c \in R$ such that $f(x)c = 0$. We denote the set of right McCoy polynomials by $rMC(R[x])$.

Proposition 3.3. *For any ring R we have $Rev(R)[x] \subseteq rMC(R[x])$.*

Proof. Let $f(x) = \sum_{i=0}^{n} a_i x^i \in Rev(R)[x]$. If $r_{R[x]}(f(x)) = 0$ then clearly $f(x) \in rMC(R(x))$. So let us suppose that $0 \neq g(x) = \sum_{j=0}^{m} b_j x^j \in R[x]$ is of minimal degree such that $f(x)g(x) = 0$. We, then have $a_n b_m = 0$ and since $a_n \in Rev(R)$, we get $b_m a_n = 0$, this leads to $\deg(g(x)a_n) < \deg(g(x))$ and since $f(x)g(x)a_n = 0$, the minimality of $\deg(g(x))$ shows that we have $g(x)a_n = 0$ and hence also $a_n g(x) = 0$. We now have $a_{n-1}b_m = 0$ which leads to $b_m a_{n-1} = 0$ and hence $\deg(g(x)a_{n-1}) < \deg(g(x))$. Since we have $f(x)g(x)a_{n-1} = 0$ the minimality of $\deg(g(x))$ implies that $g(x)a_{n-1} = 0$. Since $a_{n-1} \in Rev(R)$ we thus conclude that $a_{n-1}g(x) = 0$. Continuing this process we will finally obtain that for all $i \in \{0, \ldots, n\}$, $a_i g(x) = 0$. In particular, we obtain $f(x)b_m = 0$, as desired. □

We also have the following properties also connected with the McCoy condition.

Proposition 3.4. *Let $f(x) = \sum_{i=0}^{n} a_i x^i, g(x) = \sum_{j=0}^{m} b_j x^j \in R[x]$ be such that $f(x)g(x) = 0$. Then:*

(a) If $a_0 \in rRev(R)$ then $g(x)a_0^{m+1} = 0$. In particular if $a_0^{m+1} \neq 0$, then $r(g(x)) \cap R \neq 0$.

(b) If $b_0 \in lRev(R)$, then $b_0^{n+1} f(x) = 0$. In particular if $b_0^{n+1} \neq 0$, then $l(f(x)) \cap R \neq 0$.

Proof. (a) With the notation as in the statement of the theorem, it is enough to prove that, for any $0 \leq i \leq m$, $b_i a_0^{i+1} = 0$. If $n = \deg(f(x)) < \deg(g(x)) = m$, we put $a_l = 0$ for any $n < l \leq m$. With this notation the equality $f(x)g(x) = 0$ gives, for any $0 \leq k \leq m$, $a_0 b_k + a_1 b_{k-1} + \cdots + a_k b_0 = 0$. In particular, $a_0 b_0 = 0$. Since $a_0 \in rRev(R)$ we also have $b_0 a_0 = 0$. This shows that the required equality mentioned above is valid for $i = 0$. Let $l < m$ and assume we have proved that $b_i a_0^{i+1} = 0$ for any $0 \leq i \leq l < m$. Multiplying the equation $a_0 b_{l+1} + a_1 b_l + \cdots + a_{l+1} b_0 = 0$ on the right by a_0^{l+1} we then get $a_0 b_{l+1} a_0^{l+1} = 0$ and hence, since $a_0 \in rRev(R)$, $b_{l+1} a_0^{l+2} = 0$. This yields the required equalities.

(b) The second part of the theorem is proved similarly. □

We now consider relations between reversible elements and other kind of classical elements.

Let us recall that an element $a \in R$ is strongly π-regular if there exists $n \in \mathbb{N}$ such that $a^n \in Ra^{n+1} \cap a^{n+1}R$. This is equivalent to asking that both chains $aR \supset a^2R \supset ... \supset a^nR \supset \dots$ and $Ra \supset Ra^2 \supset ... \supset Ra^n \supset \dots$ stabilize. The set of strongly π-regular elements is denoted by $sreg_\pi(R)$. R is a π regular ring if $sreg_\pi(R) = R$. Let us recall that Dischinger [2] showed that a ring R is strongly π-regular if and only if any descending chain condition $Ra \supset Ra^2 \supset ...$ stabilizes, i.e., only one of the above chain conditions is required for a ring to be strongly π-regular.

Let us mention that using the above, we can show that a right (resp. left) artinian ring is such that every element $a \in R$ with $r(a) = 0$ (resp. $l(a) = 0$) must be invertible. In particular any left or right artinian rings is strongly π-regular. This short discussion leads quickly to the following classical result.

Proposition 3.5. *If R is π-strongly regular then every left or right nonzero divisor is invertible.*

Prompted by this proposition, we introduce another elementwize condition. This concept is more general than the right reversible one. For this we define the following two sets:

$$S_r(R) = \{a \in R \mid r(a) \neq 0 \text{ , if } l(a) \neq 0\} \quad S_l(R) = \{a \in R \mid l(a) \neq 0 \text{ , if } r(a) \neq 0\}.$$

We say that the ring R satisfies the $\mathcal{R}$ (resp. $\mathcal{L}$) property if $S_r(R) = R$ (resp.$S_l(R) = R$).

Corollary 3.6. *Let R be any ring. Both $rRev(R)$ and $sreg_\pi(R)$ are contained in $S_r(R)$.*

Example 3.7. Consider the upper triangular matrix ring of the form

$$\begin{pmatrix} \mathbb{Z} & \mathbb{Z}/2\mathbb{Z} \\ 0 & \mathbb{Z} \end{pmatrix}$$

It is easy to check that the element $a = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$ is such that $r(a) \neq 0$ but $l(a) = 0$.

We have seen that it was not possible to pass the right reversible property from a ring to the matrix ring. In the next proposition we show that in some cases the property $S_r(R) = R$ goes up to the matrix ring $M_n(R)$.

Proposition 3.8. *(a) Let $a, u, v \in R$ such that u, v are invertible. Then $a \in S_r(R)$ if and only if $uav \in S_r(R)$. A similar result is true for $S_l(R)$.*

(b) Let R be such that $S_r(R) = R$ and suppose that every square matrix $A \in M_n(R)$ is diagonalizable. Then $S_r(M_n(R)) = M_n(R)$.

(c) Let R be a ring with a total left ring of quotient S. If $S_r(S) = S$ then $S_r(R) = R$.

(d) Let $R \subseteq S$ be rings such that ${}_RR$ is essential in ${}_RS$. If $S_r(S) = S$ then $S_r(R) = R$.

(e) Let $a \in R$ be a unit regular element (i.e., there exists an invertible element $u \in U(R)$ such that $a = aua$). Then $a \in S_r(R) \cap S_l(R)$.

Proof. (a) It easy to check that $r(uav) = v^{-1}r(a)$ and $l(uav) = u^{-1}l(a)$. So if we assume that $r(a) \neq 0$ implies $l(a) \neq 0$, then $r(uav) \neq 0$ implies $l(uav) \neq 0$.

(b) By Part (a) above, it is enough to show that a diagonal matrix A is such that $r(A) \neq 0$ also satisfies $l(A) \neq 0$. This is easy and left to the reader.

(c) This is easy as follows: let $a \in R$ be such that $r_R(a) \neq 0$. Hence $r_S(a) \neq 0$ and since $S_r(S) = S$, we have that $l_S(a) \neq 0$, so there exists elements $x, y \in R$ with $l_S(x) = 0$ and $s = x^{-1}y \in l_S(a)$. We then get that $0 \neq y \in l_R(a)$.

(d) Suppose $a \in R$ is such that $r_R(a) \neq 0$ then $r_S(a) \neq 0$. The fact that $a \in S_r(S)$ implies that $l_S(a) \neq 0$ and since R is essential in S, we obtain that $l_R(a) \neq 0$.

(e) Suppose that $a \in R$ is unit regular, i.e., there exists an invertible element u such that $a = aua$. Suppose that $l(a) = 0$, then since $(1 - au)a = 0$, we have that $1 = au$, and if $b \in r(a)$, we get that $u^{-1}b = auu^{-1}b = ab = 0$, this gives that $b = 0$. So that $a \in S_r(R)$. The fact that $a \in S_l(R)$ is obtained similarly. $\square$

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