

Infinitely many nonsolvable groups whose Cayley graphs are hamiltonian

Research Article

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Abstract: We show there are infinitely many finite groups G , such that every connected Cayley graph on G has a hamiltonian cycle, and G is not solvable. Specifically, we show that if A_5 is the alternating group on five letters, and p is any prime, such that $p \equiv 1 \pmod{30}$, then every connected Cayley graph on the direct product $A_5 \times \mathbb{Z}_p$ has a hamiltonian cycle.

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1. Introduction

It has been conjectured that every connected Cayley graph on every finite group has a hamiltonian cycle (unless the graph has less than three vertices). In support of this conjecture, the literature provides numerous infinite families of finite groups G , for which it is known that every connected Cayley graph on G has a hamiltonian cycle. (See [2] and its references for more information.) However, it seems that the union of these families contains only finitely many groups that are not solvable. This note puts an end to that unsatisfactory state of affairs:

Proposition 1.1. *There are infinitely many finite groups G , such that every connected Cayley graph on G has a hamiltonian cycle, and G is not solvable.*

Since the alternating group A_5 (of order 60) is a nonabelian simple group, and is therefore not solvable, the above is an immediate consequence of the following more specific result.

Proposition 1.2. *If p is a prime, such that $p \equiv 1 \pmod{30}$, then every connected Cayley graph on the direct product $A_5 \times \mathbb{Z}_p$ has a hamiltonian cycle.*

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The proof is based on a case-by-case analysis of Cayley graphs of the group A_5 . Most of the hamiltonian cycles were found by computer search (using a fairly naive backtracking algorithm).

Remark 1.3. *Rather than merely groups that are not solvable, it would be much more interesting to find infinitely many finite, simple groups G , such that every connected Cayley graph on G has a hamiltonian cycle. Regrettably, the known methods seem to be hopelessly inadequate for this problem.*

2. Preliminaries

Definition 2.1. *Let S be a subset of a finite group G . The Cayley graph of G with respect to the connection set S is the graph $\text{Cay}(G; S)$ whose vertices are the elements of G , and with edges $g - gs$ and $g - gs^{-1}$, for each $g \in G$ and $s \in S$.*

Notation 2.2. *Suppose S is a subset of a finite group G . For $s_1, \dots, s_m \in S \cup S^{-1}$, we use $(s_i)_{i=1}^m = (s_1, \dots, s_m)$ to denote the walk in $\text{Cay}(G; S)$ that visits (in order), the vertices*

$$e, s_1, s_1s_2, s_1s_2s_3, \dots, s_1s_2 \cdots s_m.$$

We use $(s_1, \dots, s_m)^k$ to denote the concatenation of k copies of the sequence $(s_i)_{i=1}^m$, and the following illustrates other notations that are often useful:

$$(a^2, b^{-3}, s_i)_{i=1}^3 = (a, a, b^{-1}, b^{-1}, b^{-1}, s_1, a, a, b^{-1}, b^{-1}, b^{-1}, s_2, a, a, b^{-1}, b^{-1}, b^{-1}, s_3).$$

Notation 2.3. *We use $\bar{\cdot} : A_5 \times \mathbb{Z}_p \rightarrow A_5$ to denote the natural projection (so $\overline{(x, y)} = x$).*

Our argument in Section 3 is based on the same outline as the proof in [3] of the following result.

Lemma 2.4 ([3]). *Every connected Cayley graph on A_5 has a hamiltonian cycle.*

The following result is the reason that the statement of Proposition 1.2 assumes $p \equiv 1 \pmod{30}$. A much weaker hypothesis would suffice in all other parts of the proof.

Corollary 2.5. *Let S be a minimal generating set of $A_5 \times \mathbb{Z}_p$, where p is prime, and $p \equiv 1 \pmod{30}$. If there exists $a \in S$, such that $\overline{S \setminus \{a\}}$ generates A_5 , then $\text{Cay}(A_5 \times \mathbb{Z}_p; S)$ has a hamiltonian cycle.*

Proof. Since $\gcd(|A_5|, p) = 1$, the minimality of S implies that $\langle S \setminus \{a\} \rangle = A_5$. (Namely, since $\gcd(|A_5|, p) = 1$, we have $\bar{g} \in \langle g \rangle$ for every $g \in A_5 \times \mathbb{Z}_p$. Therefore $A_5 = \langle \overline{S \setminus \{a\}} \rangle \subseteq \langle S \setminus \{a\} \rangle$. Since the minimality of S implies $a \notin \langle S \setminus \{a\} \rangle$, we conclude that $\langle S \setminus \{a\} \rangle = A_5$.) From Lemma 2.4, we know there is a hamiltonian cycle $(s_i)_{i=1}^{60}$ in $\text{Cay}(A_5; S \setminus \{a\})$. Since, by assumption, $p - 1$ is divisible by $30 = 2 \cdot 3 \cdot 5$ (and every element of A_5 has order 1, 2, 3, or 5), we know \bar{a}^{p-1} is trivial. This means $a^{p-1} \in \mathbb{Z}_p$ (so a^{p-1} centralizes A_5), so it is not difficult to verify that

$$(s_{2i-1}, a^{p-1}, s_{2i}, a^{-(p-1)})_{i=1}^{30}$$

is a hamiltonian cycle in $\text{Cay}(A_5 \times \mathbb{Z}_p; S)$.

For completeness, we sketch the verification that the given walk is a hamiltonian cycle. Since $\langle S \rangle = A_5 \times \mathbb{Z}_p$, and $S \setminus \{a\} \subseteq A_5$, we know that a projects nontrivially to \mathbb{Z}_p . Since $\gcd(|A_5|, p) = 1$, this implies $\mathbb{Z}_p \subseteq \langle a \rangle$, so every element g of $A_5 \times \mathbb{Z}_p$ can be written (uniquely) in the form $g = xa^r$ with $x \in A_5$ and $0 \leq r \leq p - 1$. Since $(s_i)_{i=1}^{60}$ is a hamiltonian cycle in a Cayley graph on A_5 , we have $x = s_1s_2 \cdots s_k$, for some k with $0 \leq k < 60$. If $k = 2i - 1$ is odd, then

$$g = xa^r = \left(\prod_{j=1}^{i-1} s_{2j-1}s_{2j} \right) s_{2i-1}a^r = \left(\prod_{j=1}^{i-1} (s_{2j-1}a^{p-1}s_{2j}a^{-(p-1)}) \right) s_{2i-1}a^r.$$

(because $a^{p-1} \in \mathbb{Z}_p$ is in the center of $A_5 \times \mathbb{Z}_p$). Also, if $k = 2i$ is even, then

$$g = xa^r = \left(\prod_{j=1}^{i-1} s_{2j-1} s_{2j} \right) s_{2i-1} s_{2i} a^r = \left(\prod_{j=1}^{i-1} s_{2j-1} a^{p-1} s_{2j} a^{-(p-1)} \right) s_{2i-1} a^{p-1} s_{2i} a^{-(p-1-r)}.$$

Thus, we see (in either case) that g is one of the vertices on the walk $(s_{2i-1}, a^{p-1}, s_{2i}, a^{-(p-1)})_{i=1}^{30}$. This means that the walk passes through all of the vertices in $\text{Cay}(A_5 \times \mathbb{Z}_p; S)$.

Also, note that the walk has the correct length $(60p)$ to be a hamiltonian cycle. Finally, by using once again the fact that a^{p-1} is in the center, we see that the terminal vertex of the walk is

$$\prod_{i=1}^{30} s_{2i-1} a^{p-1} s_{2i} a^{-(p-1)} = \prod_{i=1}^{30} s_{2i-1} s_{2i} = \prod_{i=1}^{60} s_i = e,$$

because $(s_i)_{i=1}^{60}$ is a (hamiltonian) cycle. □

Remark 2.6. For definiteness, we point out that we write our permutations on the left, so $gs(i) = g(s(i))$ for $g, s \in A_5$ and $i \in \{1, 2, 3, 4, 5\}$.

The remainder of this section records a few easy consequences of the following well-known, elementary observation.

Lemma 2.7 (“Factor Group Lemma” [4, §2.2]). *Suppose*

- N is a cyclic, normal subgroup of G ,
- $(s_i)_{i=1}^m$ is a hamiltonian cycle in $\text{Cay}(G/N; S)$, and
- the voltage $\Pi(s_i)_{i=1}^m$ generates N .

Then $(s_1, s_2, \dots, s_m)^{|N|}$ is a hamiltonian cycle in $\text{Cay}(G; S)$.

Corollary 2.8 ([2, Cor. 2.11]). *Suppose*

- N is a normal subgroup of G , such that $|N|$ is prime,
- the image of S in G/N is a minimal generating set of G/N ,
- there is a hamiltonian cycle in $\text{Cay}(G/N; S)$, and
- $s \equiv t \pmod{N}$ for some $s, t \in S \cup S^{-1}$ with $s \neq t$.

Then there is a hamiltonian cycle in $\text{Cay}(G; S)$.

Corollary 2.9. *Let S be a minimal generating set of $A_5 \times \mathbb{Z}_p$, such that \bar{S} is a minimal generating set of A_5 . If every element of \bar{S} has order 2, then $\text{Cay}(A_5 \times \mathbb{Z}_p; S)$ has a hamiltonian cycle.*

Notation 2.10. Let $C = (s_i)_{i=1}^m$ be a walk in a Cayley graph $\text{Cay}(G; S)$. For $s \in S$, we use $\text{wt}_C(s)$ to denote the difference between the number of occurrences of s and the number of occurrences of s^{-1} in C . (This is the net weight of the generator s in C .)

Lemma 2.11. *Let $S = \{a_1, \dots, a_k, b_1, \dots, b_\ell\}$ be a minimal generating set of $A_5 \times \mathbb{Z}_p$, such that \bar{S} is a minimal generating set of A_5 . Assume*

- $\ell \geq 1$, and $|\bar{a}_i| = 2$ for all i ,
- C_1, \dots, C_ℓ are hamiltonian cycles in $\text{Cay}(A_5; \bar{S})$, and

- $[\text{wt}_{C_i}(b_j)]$ is the $\ell \times \ell$ matrix whose (i, j) entry is $\text{wt}_{C_i}(b_j)$.

If $\det[\text{wt}_{C_i}(b_j)] \not\equiv 0 \pmod{p}$, then $\text{Cay}(A_5 \times \mathbb{Z}_p; S)$ has a hamiltonian cycle.

Proof. We may assume that $a_i \in A_5$ for $1 \leq i \leq k$, for otherwise Corollary 2.8 applies with $s = a_i$ and $t = a_i^{-1}$. Write $b_i = (\bar{b}_i, v_i)$ (with $v_i \in \mathbb{Z}_p$) for $1 \leq i \leq \ell$. Since S generates $A_5 \times \mathbb{Z}_p$, the vector $[v_1, \dots, v_\ell]$ must be nonzero in $(\mathbb{Z}_p)^\ell$. Then, since, by assumption, the matrix $[\text{wt}_{C_i}(b_j)]$ is invertible over \mathbb{Z}_p , this implies $[\text{wt}_{C_i}(b_j)][v_1, \dots, v_\ell]^T \neq \vec{0}$ in $(\mathbb{Z}_p)^\ell$, so there is some i , such that $\sum_{j=1}^\ell \text{wt}_{C_i}(b_j) v_j \neq 0$ in \mathbb{Z}_p .

We now show that this sum is precisely the voltage ΠC_i of the walk C_i , so Lemma 2.7 provides the desired hamiltonian cycle in $\text{Cay}(A_5 \times \mathbb{Z}_p; S)$. Write $C_i = (s_k)_{k=1}^n$, let $\pi = \prod_{k=1}^n s_k$ be the voltage of C_i , and let $n(s)$ and $n'(s)$, respectively, be the number of occurrences of s and s^{-1} in C_i . Since C_i is a (hamiltonian) cycle in $\text{Cay}(A_5; \bar{S})$, we know that $\bar{\pi}$ is trivial, so $\pi = \sum_{k=1}^n s_k^*$, where s_k^* is the projection of s_k to \mathbb{Z}_p . Noting that $(s^{-1})^* = -s^*$ for $s \in S$, we have

$$\pi = \sum_{k=1}^n s_k^* = \sum_{s \in S \cup S^{-1}} n(s) s^* = \sum_{s \in S} (n(s) - n'(s)) s^* = \sum_{s \in S} \text{wt}_{C_i}(s) s^* = \sum_{j=1}^\ell \text{wt}_{C_i}(b_j) v_j. \quad \square$$

3. Proof of Proposition 1.2

Assumptions 3.1. Let S be a minimal generating set of $A_5 \times \mathbb{Z}_p$ (and, in accordance with Notation 2.3, let \bar{S} be the image of S in A_5). We may assume \bar{S} is a minimal generating set of A_5 , for otherwise Corollary 2.5 applies. We may also assume, for every element s of S with $|\bar{s}| = 2$, that the projection of s to \mathbb{Z}_p is trivial, for otherwise Corollary 2.8 applies.

Case 1. Assume S has exactly two elements. Write $S = \{a, b\}$.

Subcase 1.1. Assume $|\bar{a}| = 2$ and $|\bar{b}| = 3$. To simplify matters, we show that, by applying an automorphism of A_5 , we may assume $\bar{a} = (1, 2)(3, 4)$ and $\bar{b} = (2, 4, 5)$. First of all, we may assume $\bar{a} = (1, 2)(3, 4)$, since every element of order 2 in A_5 is conjugate to this. Then, in order for $\langle \bar{a}, \bar{b} \rangle$ to be transitive, the support of \bar{b} must contain an element of each cycle of \bar{a} (including the 1-cycle (5)). So we may assume $\bar{b} = (2, 4, 5)$ (after conjugating by (1, 2) and/or (3, 4), if necessary).

Now, we have the following hamiltonian cycle in $\text{Cay}(A_5; \bar{S})$ (see note 4.1):

$$C_1 = ((\bar{a}, \bar{b}^2)^3, (\bar{a}, \bar{b}^{-2})^3, (\bar{a}, \bar{b}^2, \bar{a}, \bar{b}^{-2})^2)^2.$$

By using the fact that each left coset of $\langle \bar{b} \rangle$ appears as consecutive vertices in this cycle, we will show that

$$C_2 = ((a, b^{3p-1})^3, (a, b^{-(3p-1)})^3, (a, b^{3p-1}, a, b^{-(3p-1)})^2)^2$$

passes through all of the vertices in each left coset of $\langle b \rangle$, and is therefore a hamiltonian cycle in $\text{Cay}(A_5 \times \mathbb{Z}_p; S)$.

Note that $\overline{b^{3p-1}} = \bar{b}^2$ (since $|\bar{b}| = 3$). This implies that if we let x be the terminal vertex of the walk C_2 , then \bar{x} is the terminal vertex of the hamiltonian cycle C_1 , so \bar{x} is trivial. The projection of x to \mathbb{Z}_p is also trivial, because $\text{wt}_{C_2}(b) = 0$. Therefore, the walk C_2 is closed.

Now, since C_2 has the correct length to be a hamiltonian cycle, we need only show that it passes through every element of $A_5 \times \mathbb{Z}_p$. From the fact that $\overline{b^{3p-1}} = \bar{b}^2$, we see that the vertices of $\overline{C_2}$ are precisely the same elements of A_5 as the vertices of C_1 ; that is, the walk $\overline{C_2}$ passes through every element of A_5 . Thus, given any $v \in A_5 \times \mathbb{Z}_p$, the walk C_2 visits some vertex w with $\bar{w} = \bar{v}$; that is, v and w are in the same coset of \mathbb{Z}_p . Since $\mathbb{Z}_p \subseteq \langle b \rangle$, this implies that v and w are in the same left coset of $\langle b \rangle$. Also,

since there are never two consecutive appearances of a in C_2 , and every occurrence of b is contained in a string b^{3p-1} , we know that C_2 traverses every element of any left coset of $\langle b \rangle$ that it enters. In particular, C_2 traverses every element of the left coset of w , so it passes through v .

Subcase 1.2. Assume $|\bar{a}| = 2$ and $|\bar{b}| = 5$. We may assume $\bar{b} = (1, 2, 3, 4, 5)$, after conjugating by some permutation in S_5 . Since $|\bar{a}| = 2$, it has a fixed point, which we may assume is 5 (after conjugating by a power of \bar{b}). So $|\bar{a}|$ must be either $(1, 2)(3, 4)$, $(1, 3)(2, 4)$, or $(1, 4)(2, 3)$.

- For $\bar{a} = (1, 2)(3, 4)$, we have the following hamiltonian cycle (see note 4.2):

$$C = ((\bar{a}, \bar{b}, \bar{a}, \bar{b}^4)^2, \bar{a}, \bar{b}^2, \bar{a}, \bar{b}^{-1}, \bar{a}, \bar{b}^4, \bar{a}, \bar{b}, (\bar{a}, \bar{b}^2)^3, \bar{a}, \bar{b}^{-2}, \bar{a}, \bar{b}^4, \bar{a}, \bar{b}^{-2}, (\bar{a}, \bar{b})^2, \bar{a}, \bar{b}^{-1}, \bar{a}, \bar{b}^4, \bar{a}, \bar{b}^2).$$

Since $\text{wt}_C(\bar{b}) = 29 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

- For $\bar{a} = (1, 3)(2, 4)$, we have the following hamiltonian cycle (see note 4.3):

$$C = (\bar{a}, \bar{b}^4, \bar{a}, \bar{b}^{-1}, \bar{a}, \bar{b}, \bar{a}, \bar{b}^{-1}, \bar{a}, \bar{b}^{-4}, \bar{a}, \bar{b}^{-2}, (\bar{a}, \bar{b}^{-4}, \bar{a}, \bar{b}^2)^2, \bar{a}, \bar{b}^{-4}, \bar{a}, \bar{b}, \bar{a}, \bar{b}^{-1}, \bar{a}, \bar{b}^{-4}, \bar{a}, \bar{b}^{-2}, \bar{a}, \bar{b}^{-4}, \bar{a}, \bar{b}^2).$$

Since $\text{wt}_C(\bar{b}) = -19 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

- If $\bar{a} = (1, 4)(2, 3)$, then \bar{a} normalizes \bar{b} , so $\langle \bar{a}, \bar{b} \rangle \neq A_5$, which contradicts the fact that S is a generating set.

Subcase 1.3. Assume $|\bar{a}| = |\bar{b}| = 3$. The union of the supports of \bar{a} and \bar{b} must be all of $\{1, 2, 3, 4, 5\}$, since $\langle \bar{a}, \bar{b} \rangle$ is transitive. Since each support consists of three elements, the intersection must be a single element, which we may assume is 3. Then, by renumbering, we may assume the support of \bar{a} is $\{1, 2, 3\}$ and the support of \bar{b} is $\{3, 4, 5\}$. Therefore, either \bar{a} or \bar{a}^{-1} is $(1, 2, 3)$, and either \bar{b} or \bar{b}^{-1} is $(3, 4, 5)$. So we may assume $\bar{a} = (1, 2, 3)$ and $\bar{b} = (3, 4, 5)$. We have the following hamiltonian cycle (see note 4.4):

$$C_1 = (\bar{b}, \bar{a}, \bar{b}^2, \bar{a}^2, \bar{b}^{-2}, \bar{a}^{-2}, \bar{b}^2, \bar{a}^2, \bar{b}^2, \bar{a}^{-2}, \bar{b}^{-2}, \bar{a}^{-2}, \bar{b}^2, \bar{a}^2, \bar{b}^{-2}, \bar{a}, \bar{b}^{-2}, \bar{a}^{-2}, \bar{b}^2, \bar{a}^2, \bar{b}^{-1}, \bar{a}, \bar{b}, \bar{a}^2, \bar{b}^{-1}, \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}).$$

Note that $\text{wt}_{C_1}(\bar{a}) = 4$ and $\text{wt}_{C_1}(\bar{b}) = 0$. Conjugation by the permutation $(1, 4)(2, 5)$ interchanges \bar{a} and \bar{b} , and therefore yields a hamiltonian cycle C_2 with $\text{wt}_{C_2}(\bar{a}) = 0$ and $\text{wt}_{C_2}(\bar{b}) = 4$. Since $\det \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 16 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

Subcase 1.4. Assume $|\bar{a}| = 3$ and $|\bar{b}| = 5$. We may assume $\bar{b} = (1, 2, 3, 4, 5)$, by replacing it with a conjugate. Then the two fixed points of the 3-cycle \bar{a} are either consecutive or are separated by only one element (in circular order). Therefore, after conjugating by a power of \bar{b} , we may assume that one of the fixed points of \bar{a} is 5, and the other is either 3 or 4. Hence, \bar{a} is either $(1, 2, 4)$ or $(1, 2, 3)$ (or the inverse of one of these).

- If $\bar{a} = (1, 2, 4)$, then we have the following two hamiltonian cycles (see notes 4.5 and 4.6):

$$C_1 = (\bar{a}^2, \bar{b}^{-2}, \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}^{-1}, \bar{b}, \bar{a}^2, \bar{b}^{-1}, \bar{a}^{-2}, \bar{b}, \bar{a}^{-2}, \bar{b}^{-2}, \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}^{-1}, \bar{b}, \bar{a}^2, \bar{b}^{-1}, \bar{a}^2, \bar{b}, \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}^2, \bar{b}, \bar{a}^2, \bar{b}^{-1}, \bar{a}^{-2}, \bar{b}, \bar{a}^{-2}, \bar{b}^{-2}, \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}^{-1}, \bar{b}, \bar{a}^2, \bar{b}^{-1}, (\bar{a}^{-2}, \bar{b})^2)$$

and

$$C_2 = (\bar{a}^2, \bar{b}^{-2}, \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}^{-1}, \bar{b}, (\bar{a}^2, \bar{b}^{-1})^2, \bar{a}^2, \bar{b}^{-2}, \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}^{-1}, \bar{b}, \bar{a}^2, \bar{b}^{-1}, \bar{a}^2, \bar{b}, \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}^2, \bar{b}, \bar{a}^2, \bar{b}^{-1}, \bar{a}^{-2}, \bar{b}, \bar{a}^{-2}, \bar{b}^{-2}, \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}^{-1}, \bar{b}, \bar{a}^2, \bar{b}^{-1}, (\bar{a}^{-2}, \bar{b})^2)$$

Then $[\text{wt}_{C_1}(\bar{a}), \text{wt}_{C_1}(\bar{b})] = [-9, -5]$ and $[\text{wt}_{C_2}(\bar{a}), \text{wt}_{C_2}(\bar{b})] = [-1, -7]$. Since $\det \begin{bmatrix} -9 & -5 \\ -1 & -7 \end{bmatrix} = 58 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

- If $\bar{a} = (1, 2, 3)$, then we have the following two hamiltonian cycles (see notes 4.7 and 4.8):

$$C_1 = (\bar{a}^2, \bar{b}^{-2}, \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}^{-1}, \bar{b}^3, (\bar{a}^2, \bar{b})^2, \bar{a}, \bar{b}^{-1}, (\bar{a}^2, \bar{b})^2, \bar{a}^2, \bar{b}^{-2}, \bar{a}^2, \bar{b}, \bar{a}, \bar{b}^{-1}, \\ \bar{a}^{-2}, \bar{b}, \bar{a}^2, \bar{b}^{-1}, \bar{a}^{-2}, \bar{b}, \bar{a}^{-1}, \bar{b}^{-2}, \bar{a}^{-1}, \bar{b}^{-1}, \bar{a}^{-2}, \bar{b}, \bar{a}^2, (\bar{b}^{-1}, \bar{a}^{-2})^2, \bar{b})$$

and

$$C_2 = (\bar{a}^2, \bar{b}^{-1}, (\bar{a}^2, \bar{b})^2, \bar{a}^2, \bar{b}^{-1}, \bar{a}^{-2}, \bar{b}, \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}^2, \bar{b}, \bar{a}^2, \bar{b}^{-1}, \bar{a}^{-2}, \bar{b}^2, \\ (\bar{a}^{-2}, \bar{b}^{-1})^2, (\bar{a}^{-2}, \bar{b})^2, \bar{a}^2, \bar{b}, \bar{a}^2, \bar{b}^{-1}, \bar{a}, \bar{b}, \bar{a}^{-2}, \bar{b}, \bar{a}^2, (\bar{b}^{-1}, \bar{a}^{-2})^2, \bar{b})$$

Then $[\text{wt}_{C_1}(\bar{a}), \text{wt}_{C_1}(\bar{b})] = [5, -1]$ and $[\text{wt}_{C_2}(\bar{a}), \text{wt}_{C_2}(\bar{b})] = [-1, 3]$. Since $\det \begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix} = 14 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

Subcase 1.5. Assume $|\bar{a}| = |\bar{b}| = 5$. By applying an automorphism of A_5 , we may assume $\bar{a} = (1, 2, 3, 4, 5)$. From Sylow's Theorems, we know that A_5 has precisely six Sylow 5-subgroups. One of them is $\langle \bar{a} \rangle$, and $\langle \bar{a} \rangle$ acts transitively on the other 5 by conjugation. So we may assume, after conjugating by a power of \bar{a} , that $\langle \bar{b} \rangle = \langle (1, 2, 3, 5, 4) \rangle$. Then, by replacing b with its inverse if necessary, we may assume \bar{b} is either $(1, 2, 3, 5, 4)$ or $(1, 3, 4, 2, 5)$.

- If $\bar{b} = (1, 2, 3, 5, 4)$, then we have the following hamiltonian cycle (see note 4.9):

$$C_1 = (\bar{b}, \bar{a}^{-1}, \bar{b}, \bar{a}, \bar{b}^4, \bar{a}, \bar{b}^2, \bar{a}^{-1}, \bar{b}^2, \bar{a}^{-1}, \bar{b}^{-1}, \bar{a}^{-1}, \bar{b}, \bar{a}^{-1}, \bar{b}, \bar{a}, \bar{b}^2, \bar{a}^{-1}, \bar{b}^2, \bar{a}^{-1}, \bar{b}^{-1}, \\ \bar{a}, \bar{b}^{-2}, \bar{a}^{-1}, \bar{b}^{-4}, \bar{a}^{-1}, \bar{b}^{-1}, \bar{a}, \bar{b}^{-4}, \bar{a}, \bar{b}^{-2}, \bar{a}^{-1}, \bar{b}^{-1}, \bar{a}, \bar{b}^{-2}, \bar{a}^{-1}, \bar{b}^4, \bar{a}, \bar{b}^{-2}, \bar{a})$$

Then $[\text{wt}_{C_1}(\bar{a}), \text{wt}_{C_1}(\bar{b})] = [-2, 0]$. Conjugating by the permutation $(4, 5)$ interchanges \bar{a} and \bar{b} , and therefore yields a hamiltonian cycle C_2 with $[\text{wt}_{C_2}(\bar{a}), \text{wt}_{C_2}(\bar{b})] = [0, -2]$. Since $\det \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = 4 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

- If $\bar{b} = (1, 3, 4, 2, 5)$, then we have the following two hamiltonian cycles (see notes 4.10 and 4.11):

$$C_1 = (\bar{a}^4, \bar{b}^{-1}, \bar{a}^{-4}, \bar{b}, \bar{a}^4, \bar{b}^{-1}, \bar{a}^2, \bar{b}, \bar{a}^{-2}, \bar{b}, \bar{a}^{-1}, \bar{b}, \bar{a}^{-4}, \bar{b}^{-1}, \\ \bar{a}^{-1}, \bar{b}^{-1}, \bar{a}^4, \bar{b}, (\bar{a}^{-4}, \bar{b}^{-1})^2, (\bar{a}^2, \bar{b})^2, \bar{a}^4, (\bar{b}^{-1}, \bar{a})^2, \bar{b})$$

and

$$C_2 = (\bar{a}^4, \bar{b}^{-1}, \bar{a}^{-4}, \bar{b}, \bar{a}^4, \bar{b}^{-1}, \bar{a}^2, \bar{b}, \bar{a}^{-1}, \bar{b}^{-1}, \bar{a}^{-4}, \bar{b}^{-1}, \bar{a}, \bar{b}^{-1}, \\ \bar{a}^{-2}, \bar{b}^{-1}, \bar{a}^4, \bar{b}, (\bar{a}^{-4}, \bar{b}^{-1})^2, (\bar{a}^2, \bar{b})^2, \bar{a}^4, (\bar{b}^{-1}, \bar{a})^2, \bar{b})$$

Then $[\text{wt}_{C_1}(\bar{a}), \text{wt}_{C_1}(\bar{b})] = [4, 0]$ and $[\text{wt}_{C_2}(\bar{a}), \text{wt}_{C_2}(\bar{b})] = [6, -4]$. Since $\det \begin{bmatrix} 4 & 0 \\ 6 & -4 \end{bmatrix} = -16 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

Case 2. Assume S has at least three elements.. We claim that S has exactly three elements, so we may write $S = \{a, b, c\}$ with $|\bar{a}| \leq |\bar{b}| \leq |\bar{c}|$. To show this, write $\bar{S} = \{s_1, \dots, s_r\}$, and suppose $r \geq 4$. Let $H_i = \langle s_1, \dots, s_i \rangle$ for each i , and note that, since \bar{S} is a minimal generating set, we have $H_{i-1} \subsetneq H_i$. Since $|A_5| = 2^2 \cdot 3 \cdot 5$ is the product of only 4 primes, we must have $r = 4$ and $|H_i : H_{i-1}|$ is prime for $i = 1, \dots, 4$. Since A_4 is the only subgroup of prime index in A_5 (up to conjugacy), we may assume $H_3 = A_4$. Then H_2 must be the Sylow 2-subgroup of A_4 , since that is the only subgroup of prime index. So $H_3 = A_4$ is generated by s_1 and s_3 , contradicting the minimality of \bar{S} .

Subcase 2.1. Assume $|\bar{c}| = 5$. Since \bar{a} and \bar{b} cannot both normalize $\langle \bar{c} \rangle$ (but every proper subgroup of A_5 whose order is divisible by 5 has order 5 or 10), we see that either $\langle \bar{a}, \bar{c} \rangle = A_5$ or $\langle \bar{b}, \bar{c} \rangle = A_5$, which contradicts the minimality of S .

Subcase 2.2. Assume $|\bar{a}| = |\bar{b}| = |\bar{c}| = 3$. The minimality of \bar{S} implies that $\langle \bar{a}, \bar{b} \rangle \neq A_5$, so there must be at least two elements in the intersection of the supports of \bar{a} and \bar{b} . The supports cannot

be equal, since $\bar{a} \neq \bar{b}^{\pm 1}$. Therefore, the intersection of the supports consists of two points, which we may assume are 1 and 5. Then we may assume $\bar{a} = (1, 2, 5)$ and $\bar{b} = (1, 3, 5)$. Now, for the same reason, the support of \bar{c} must contain exactly two points from the support of \bar{a} and exactly two points from the support of \bar{b} , and it must also contain 4 (since 4 is fixed by both \bar{a} and \bar{b} , but not by A_5). This implies that the support of \bar{c} is $\{1, 4, 5\}$, so $\bar{c} = (1, 4, 5)^{\pm 1}$. Therefore (perhaps after replacing c by its inverse), we have

$$\bar{S} = \{(1, 2, 5), (1, 3, 5), (1, 4, 5)\} = \{(1, j, n) \mid 1 < j < n\} \quad \text{for } n = 5.$$

So [1, App. D] provides the following hamiltonian cycle in $\text{Cay}(A_5; \bar{S})$ (see note 4.12):

$$R_1 = \left(((\bar{a}^2, \bar{b})^2, \bar{a}^2, \bar{c})^2, \bar{a}, \bar{b}, (\bar{b}, \bar{a}^2)^2, \bar{c}^2, \bar{a}^2, \bar{c}, \bar{b}, \bar{a}, \bar{b}, \bar{c}, \bar{a}, (\bar{c}, \bar{b}^2)^2, (\bar{a}^2, \bar{b})^2, \bar{a}, \bar{c}^2, \bar{a}, \bar{b}^2, (\bar{a}, \bar{c}^2)^2 \right).$$

We have $[\text{wt}_{R_1}(\bar{a}), \text{wt}_{R_1}(\bar{b}), \text{wt}_{R_1}(\bar{c})] = [29, 17, 14]$. Conjugating by $(2, 3, 4)$ and $(2, 3, 4)^2$ cyclically permutes $\{\bar{a}, \bar{b}, \bar{c}\}$, and therefore yields hamiltonian cycles R_2 and R_3 , such that

$$[\text{wt}_{R_2}(\bar{a}), \text{wt}_{R_2}(\bar{b}), \text{wt}_{R_2}(\bar{c})] = [17, 14, 29] \quad \text{and} \quad [\text{wt}_{R_3}(\bar{a}), \text{wt}_{R_3}(\bar{b}), \text{wt}_{R_3}(\bar{c})] = [14, 29, 17].$$

Since

$$\det \begin{bmatrix} 29 & 17 & 14 \\ 17 & 14 & 29 \\ 14 & 29 & 17 \end{bmatrix} = 11,340 = 2^2 \cdot 3^4 \cdot 5 \cdot 7 \not\equiv 0 \pmod{p},$$

Theorem 2.11 applies.

Subcase 2.3. Assume $|\bar{a}| = 2$ and $|\bar{b}| = |\bar{c}| = 3$. We claim that we may assume $\bar{S} = \{(12)(45), (1, 2, 3), (1, 2, 4)\}$. Arguing as in the first paragraph of Subcase 2.2 (and renumbering), we may assume $\bar{b} = (1, 2, 3)$ and $\bar{c} = (1, 2, 4)$. Since $\langle \bar{a}, \bar{b}, \bar{c} \rangle = A_5$ is transitive on $\{1, 2, 3, 4, 5\}$, we know that 5 is in the support of \bar{a} . Also, since $\langle \bar{a}, \bar{b} \rangle \neq A_5$, we know that the support of \bar{b} does not contain precisely one element of each of the cycles of \bar{a} (and similarly for the support of \bar{c}).

If one of the cycles of \bar{a} is disjoint from the support of \bar{b} , then the cycle must be $(4, 5)$. The support of \bar{c} contains precisely one element of this cycle, and cannot be disjoint from the other cycle, so it must contain the entire cycle. The only 2-element subset of $\{1, 2, 3\}$ contained in the support of \bar{c} is $\{1, 2\}$, so $\bar{a} = (1, 2)(4, 5)$, as desired.

We may now assume that no cycle of \bar{a} is disjoint from the support of \bar{b} or the support of \bar{c} . (This will lead to a contradiction.) This assumption implies that the cycle $(x, 5)$ in \bar{a} must be either $(1, 5)$ or $(2, 5)$. We may assume it is $(1, 5)$ (after conjugating by $(1, 2)$ and replacing \bar{b} and \bar{c} by their inverses, if necessary). The other cycle either is $(2, 3)$ (in which case, $\langle \bar{a}, \bar{c} \rangle = A_5$), or is either $(2, 4)$ or $(3, 4)$ (in which case, $\langle \bar{a}, \bar{b} \rangle = A_5$), which contradicts the minimality of \bar{S} . This completes the proof of the claim.

We have the following two hamiltonian cycles (see notes 4.13 and 4.14):

$$C_1 = (\bar{a}, \bar{c}^{-1}, \bar{a}, \bar{b}, \bar{a}, \bar{c}, \bar{a}, \bar{b}^2, \bar{a}, \bar{b}, \bar{c}, \bar{b}^{-1}, \bar{a}, \bar{b}^{-2}, \bar{a}, \bar{c}^{-1}, \bar{a}, \bar{c}^{-1}, \bar{b}^2, \bar{c}, \bar{b}^{-2}, \bar{a}, \bar{b}^{-2}, \bar{c}, \bar{a}, \bar{b}, \bar{c}^{-1}, \bar{b}^{-1}, \bar{a}, \bar{c}^{-1}, \bar{a}, \bar{b}^{-2}, \bar{a}, \bar{b}^{-1}, \bar{c}, \bar{a}, \bar{b}^2, \bar{a}, \bar{b}, \bar{c}, \bar{b}^{-1}, \bar{a}, \bar{c}^{-1}, \bar{b}^2, \bar{c}, \bar{a}, \bar{b}, \bar{c}^{-1}, \bar{a}, \bar{b}^{-1}, \bar{a}, \bar{b})$$

and

$$C_2 = (\bar{a}, \bar{c}^{-1}, \bar{a}, \bar{b}, \bar{a}, \bar{c}, \bar{a}, \bar{b}^2, \bar{a}, \bar{b}, \bar{c}, \bar{b}^{-1}, \bar{a}, \bar{c}^{-1}, \bar{b}^2, \bar{c}, \bar{a}, \bar{b}, \bar{c}^{-1}, \bar{b}^{-1}, \bar{a}, \bar{b}^{-2}, \bar{a}, \bar{c}^{-1}, \bar{b}, \bar{a}, \bar{b}^2, \bar{a}, \bar{c}, \bar{a}, \bar{b}, \bar{c}^{-1}, \bar{b}^{-1}, \bar{a}, \bar{c}^{-1}, \bar{a}, \bar{c}^{-1}, \bar{b}^2, \bar{c}, \bar{b}^{-2}, \bar{a}, \bar{b}^{-2}, \bar{c}, \bar{a}, \bar{b}^2, \bar{a}, \bar{b}, \bar{c}^{-1}, \bar{a}, \bar{b}^{-1}, \bar{a}, \bar{b})$$

Then $[\text{wt}_{C_1}(\bar{b}), \text{wt}_{C_1}(\bar{c})] = [1, 0]$ and $[\text{wt}_{C_2}(\bar{b}), \text{wt}_{C_2}(\bar{c})] = [7, -2]$. Since $\det \begin{bmatrix} 1 & 0 \\ 7 & -2 \end{bmatrix} = -2 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

Subcase 2.4. Assume $|\bar{a}| = |b| = 2$ and $|\bar{c}| = 3$. We may assume $\bar{c} = (1, 2, 3)$.

- Suppose \bar{a} interchanges 4 and 5. This means that one of the 2-cycles in \bar{a} is (4, 5). The other 2-cycle must be contained in $\{1, 2, 3\}$, so, after conjugating by a power of \bar{c} , we may assume $\bar{a} = (1, 2)(4, 5)$. Since $\langle \bar{a}, \bar{b}, \bar{c} \rangle$ is transitive on $\{1, 2, 3, 4, 5\}$, the permutation \bar{b} cannot have (4, 5) as one of its 2-cycles. So no 2-cycle in \bar{b} is disjoint from the support of \bar{c} . Since $\langle \bar{b}, \bar{c} \rangle \neq A_5$ (by the minimality of \bar{S}), this implies that one of the 2-cycles must be contained in the support of \bar{c} , which is $\{1, 2, 3\}$. So \bar{b} fixes either 4 or 5. We may assume it is 5 that is fixed (after conjugating by (4, 5) if necessary). Then \bar{b} is either (1, 2)(3, 4) or (1, 3)(2, 4) or (2, 3)(1, 4). Since the last two are conjugate by (1, 2) (which centralizes \bar{a} and inverts \bar{b}), they do not both need to be considered.

– If $\bar{b} = (1, 2)(3, 4)$, we have the following hamiltonian cycle (see note 4.15):

$$C = \left(\bar{a}, \bar{c}^{-1}, (\bar{a}, \bar{b})^2, \bar{a}, \bar{c}^{-1}, \bar{a}, \bar{b}, \bar{c}^{-1}, \bar{b}, \bar{c}, (\bar{b}, \bar{a})^2, \bar{c}, (\bar{a}, \bar{b})^2, \bar{a}, \bar{c}, (\bar{a}, \bar{b})^2, \right. \\ \left. \bar{a}, \bar{c}^2, \bar{b}, \bar{c}^{-1}, \bar{b}, (\bar{a}, \bar{b})^2, \bar{c}^{-2}, ((\bar{a}, \bar{b})^2, \bar{a}, \bar{c}^{-1})^2, \bar{c}^{-1}, \bar{b}, \bar{a}, \bar{c}^{-1}, (\bar{a}, \bar{b})^2 \right).$$

Since $\text{wt}_C(\bar{c}) = -5 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

– If $\bar{b} = (1, 3)(2, 4)$, we have the following hamiltonian cycle (see note 4.16):

$$C = \left((\bar{a}, \bar{b})^4, \bar{c}, (\bar{a}, \bar{b})^4, \bar{a}, \bar{c}^{-1}, \bar{b}, \bar{a}, \bar{b}, \bar{c}^{-1}, (\bar{b}, \bar{a})^3, \bar{c}^{-1}, \right. \\ \left. \bar{b}, (\bar{a}, \bar{b})^2, \bar{c}, \bar{b}, \bar{a}, \bar{c}^{-1}, (\bar{b}, \bar{a})^4, \bar{b}, \bar{c}, (\bar{a}, \bar{b})^4, \bar{a}, \bar{c}^{-1}, \bar{b} \right).$$

Since $\text{wt}_C(\bar{c}) = -2 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

- We may now assume that neither \bar{a} nor \bar{b} interchanges 4 and 5. Since $\langle \bar{a}, \bar{c} \rangle \neq A_5$, and \bar{a} does not interchange 4 and 5, we see that \bar{a} must fix either 4 or 5. Similarly for \bar{b} . Furthermore, \bar{a} and \bar{b} cannot both fix 4 (or 5), since $\langle \bar{a}, \bar{b}, \bar{c} \rangle$ is transitive. So one of them fixes 4, and the other fixes 5. We may assume it is \bar{a} that fixes 5. Then we may assume $\bar{a} = (1, 2)(3, 4)$, after conjugating by a power of \bar{c} . Then \bar{b} must be either (1, 2)(3, 5) or (1, 3)(2, 5) or (1, 5)(2, 3). However, the last two are conjugate by (1, 2) (which centralizes \bar{a} and inverts \bar{c}), so they do not both need to be considered.

– If $\bar{b} = (1, 2)(3, 5)$, then we have the hamiltonian cycle (see note 4.17):

$$C = \left(\bar{a}, \bar{c}^{-1}, \bar{a}, \bar{c}, \bar{b}, (\bar{a}, \bar{b})^2, \bar{c}, (\bar{a}, \bar{b})^2, (\bar{a}, \bar{c})^2, \bar{a}, \bar{b}, \bar{a}, \bar{c}^{-2}, \bar{a}, \bar{c}^{-1}, \bar{b}, \bar{a}, \right. \\ \left. \bar{c}, \bar{b}, \bar{a}, \bar{b}, \bar{c}^2, ((\bar{a}, \bar{b})^2, \bar{a}, \bar{c})^2, (\bar{a}, \bar{c}^{-1})^2, (\bar{a}, \bar{b})^2, \bar{a}, \bar{c}^{-2}, (\bar{a}, \bar{b})^2 \right).$$

Since $\text{wt}_C(\bar{c}) = 1 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

– If $\bar{b} = (1, 3)(2, 5)$, then we have the following hamiltonian cycle (see note 4.18):

$$C = \left(\bar{a}, \bar{b}, \bar{c}^{-1}, (\bar{a}, \bar{b})^2, \bar{a}, \bar{c}, \bar{b}, (\bar{a}, \bar{b})^4, \bar{c}, (\bar{a}, \bar{b}, \bar{a}, \bar{c}^{-1})^2, \bar{b}, \bar{a}, \bar{b}, \bar{c}^2, \right. \\ \left. (\bar{a}, \bar{b})^2, \bar{a}, \bar{c}, ((\bar{b}, \bar{a})^2, \bar{c}^{-1})^2, (\bar{b}, \bar{a})^2, \bar{b}, \bar{c}^{-1}, (\bar{b}, \bar{a})^2, \bar{c}^{-1}, \bar{b} \right).$$

Since $\text{wt}_C(\bar{c}) = -2 \not\equiv 0 \pmod{p}$, Theorem 2.11 applies.

Subcase 2.5. Assume $|\bar{a}| = |\bar{b}| = |\bar{c}| = 2$. Since all generators are of order 2, Theorem 2.9 applies. □

4. Details of hamiltonian cycles in A_5

To aid the reader in validating the many hamiltonian cycles in A_5 that appeared in the proof of Theorem 1.2, this final section provides a list of the vertices in the order that they are visited by each cycle.

4.1. A hamiltonian cycle in $\text{Cay}(A_5; \bar{a}, \bar{b})$ with $\bar{a} = (1, 2)(3, 4)$ and $\bar{b} = (2, 4, 5)$.

	e	$\bar{a} \rightarrow$	$(1, 2)(3, 4)$	$\bar{b} \rightarrow$	$(1, 2, 3, 4, 5)$	$\bar{b} \rightarrow$	$(1, 2, 5, 3, 4)$	$\bar{a} \rightarrow$	$(1, 5, 3)$
$\bar{b} \rightarrow$	$(1, 5, 2, 4, 3)$	$\bar{b} \rightarrow$	$(1, 5, 4, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 2, 5, 4)$	$\bar{b} \rightarrow$	$(1, 3, 2)$	$\bar{b} \rightarrow$	$(1, 3, 2, 4, 5)$
$\bar{a} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 4)(3, 5)$	$\bar{a} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 3)$
$\bar{b}^{-1} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{a} \rightarrow$	$(1, 5, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 5)(2, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 2)$	$\bar{a} \rightarrow$	$(2, 5)(3, 4)$
$\bar{b} \rightarrow$	$(2, 3, 4)$	$\bar{b} \rightarrow$	$(3, 4, 5)$	$\bar{a} \rightarrow$	$(1, 2)(3, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 3, 5, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 4, 3, 5)$
$\bar{a} \rightarrow$	$(1, 4, 5)$	$\bar{b} \rightarrow$	$(1, 4)(2, 5)$	$\bar{b} \rightarrow$	$(1, 4, 2)$	$\bar{a} \rightarrow$	$(2, 4, 3)$	$\bar{b}^{-1} \rightarrow$	$(2, 5, 3)$
$\bar{b}^{-1} \rightarrow$	$(2, 3)(4, 5)$	$\bar{a} \rightarrow$	$(1, 3, 5, 4, 2)$	$\bar{b} \rightarrow$	$(1, 3, 5)$	$\bar{b} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 5, 2, 3)$
$\bar{b} \rightarrow$	$(1, 4, 2, 5, 3)$	$\bar{b} \rightarrow$	$(1, 4, 3)$	$\bar{a} \rightarrow$	$(1, 2, 4)$	$\bar{b} \rightarrow$	$(1, 2)(4, 5)$	$\bar{b} \rightarrow$	$(1, 2, 5)$
$\bar{a} \rightarrow$	$(1, 5)(3, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 2, 3, 4)$	$\bar{a} \rightarrow$	$(1, 3)(2, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 3)(4, 5)$
$\bar{b}^{-1} \rightarrow$	$(1, 3)(2, 4)$	$\bar{a} \rightarrow$	$(1, 4)(2, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 5, 3, 2)$	$\bar{a} \rightarrow$	$(2, 4)(3, 5)$
$\bar{b} \rightarrow$	$(3, 5, 4)$	$\bar{b} \rightarrow$	$(2, 3, 5)$	$\bar{a} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 3, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 3, 4, 2, 5)$
$\bar{a} \rightarrow$	$(1, 5)(2, 3)$	$\bar{b} \rightarrow$	$(1, 5, 3, 2, 4)$	$\bar{b} \rightarrow$	$(1, 5, 4, 3, 2)$	$\bar{a} \rightarrow$	$(2, 5, 4)$	$\bar{b}^{-1} \rightarrow$	$(2, 4, 5)$
$\bar{b}^{-1} \rightarrow$	e								

4.2. A hamiltonian cycle in $\text{Cay}(A_5; \bar{a}, \bar{b})$ with $\bar{a} = (1, 2)(3, 4)$ and $\bar{b} = (1, 2, 3, 4, 5)$.

	e	$\bar{a} \rightarrow$	$(1, 2)(3, 4)$	$\bar{b} \rightarrow$	$(2, 4, 5)$	$\bar{a} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{b} \rightarrow$	$(2, 5, 4)$
$\bar{b} \rightarrow$	$(1, 5)(2, 3)$	$\bar{b} \rightarrow$	$(1, 3, 4)$	$\bar{b} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{a} \rightarrow$	$(1, 4)(3, 5)$	$\bar{b} \rightarrow$	$(1, 2, 5, 4, 3)$
$\bar{a} \rightarrow$	$(1, 5, 4)$	$\bar{b} \rightarrow$	$(1, 2, 3)$	$\bar{b} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{b} \rightarrow$	$(2, 4)(3, 5)$	$\bar{b} \rightarrow$	$(1, 4, 3, 2, 5)$
$\bar{a} \rightarrow$	$(1, 5)(2, 4)$	$\bar{b} \rightarrow$	$(1, 4)(2, 3)$	$\bar{b} \rightarrow$	$(1, 3)(4, 5)$	$\bar{a} \rightarrow$	$(1, 2, 3, 5, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 5)$
$\bar{a} \rightarrow$	$(1, 2, 4, 3, 5)$	$\bar{b} \rightarrow$	$(1, 4)(2, 5)$	$\bar{b} \rightarrow$	$(1, 5, 4, 2, 3)$	$\bar{b} \rightarrow$	$(1, 3, 2)$	$\bar{b} \rightarrow$	$(3, 4, 5)$
$\bar{a} \rightarrow$	$(1, 2)(3, 5)$	$\bar{b} \rightarrow$	$(2, 5)(3, 4)$	$\bar{a} \rightarrow$	$(1, 5, 2)$	$\bar{b} \rightarrow$	$(2, 3, 4)$	$\bar{b} \rightarrow$	$(1, 3, 2, 4, 5)$
$\bar{a} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{b} \rightarrow$	$(1, 3, 2, 5, 4)$	$\bar{b} \rightarrow$	$(1, 5, 3)$	$\bar{a} \rightarrow$	$(1, 2, 5, 3, 4)$	$\bar{b} \rightarrow$	$(1, 5, 2, 4, 3)$
$\bar{b} \rightarrow$	$(1, 4, 2)$	$\bar{a} \rightarrow$	$(2, 4, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 3)(2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 2, 3, 4)$
$\bar{b} \rightarrow$	$(1, 3)(2, 4)$	$\bar{b} \rightarrow$	$(1, 4, 5, 3, 2)$	$\bar{b} \rightarrow$	$(3, 5, 4)$	$\bar{b} \rightarrow$	$(1, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5)(3, 4)$
$\bar{b}^{-1} \rightarrow$	$(2, 5, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 3, 5, 4, 2)$	$\bar{a} \rightarrow$	$(2, 3)(4, 5)$	$\bar{b} \rightarrow$	$(1, 3, 5)$	$\bar{a} \rightarrow$	$(1, 2, 3, 4, 5)$
$\bar{b} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 3)$	$\bar{b} \rightarrow$	$(1, 2)(4, 5)$
$\bar{b} \rightarrow$	$(2, 3, 5)$	$\bar{b} \rightarrow$	$(1, 3, 4, 2, 5)$	$\bar{b} \rightarrow$	$(1, 5, 3, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 2, 5, 3)$	$\bar{b} \rightarrow$	$(1, 5, 4, 3, 2)$
$\bar{b} \rightarrow$	e								

4.3. A hamiltonian cycle in $\text{Cay}(A_5; \bar{a}, \bar{b})$ with $\bar{a} = (1, 3)(2, 4)$ and $\bar{b} = (1, 2, 3, 4, 5)$.

	e	\bar{a}	$(1, 3)(2, 4)$	\bar{b}	$(1, 4, 5, 3, 2)$	\bar{b}	$(3, 5, 4)$	\bar{b}	$(1, 2, 5)$
\bar{b}	$(1, 5, 2, 3, 4)$	\bar{a}	$(1, 4, 3, 5, 2)$	\bar{b}^{-1}	$(1, 2, 4, 5, 3)$	\bar{a}	$(2, 5, 3)$	\bar{b}	$(1, 5)(3, 4)$
\bar{a}	$(1, 4, 2, 3, 5)$	\bar{b}^{-1}	$(2, 4, 5)$	\bar{a}	$(1, 3)(2, 5)$	\bar{b}^{-1}	$(1, 2, 3, 5, 4)$	\bar{b}^{-1}	$(1, 4, 5)$
\bar{b}^{-1}	$(2, 4, 3)$	\bar{b}^{-1}	$(1, 5, 3, 4, 2)$	\bar{a}	$(1, 4)(3, 5)$	\bar{b}^{-1}	$(1, 3, 2, 4, 5)$	\bar{b}^{-1}	$(2, 3, 4)$
\bar{a}	$(1, 4, 3)$	\bar{b}^{-1}	$(1, 5, 3, 2, 4)$	\bar{b}^{-1}	$(1, 3, 4, 2, 5)$	\bar{b}^{-1}	$(2, 3, 5)$	\bar{b}^{-1}	$(1, 2)(4, 5)$
\bar{a}	$(1, 3, 2, 5, 4)$	\bar{b}	$(1, 5, 3)$	\bar{b}	$(1, 2)(3, 4)$	\bar{a}	$(1, 4)(2, 3)$	\bar{b}^{-1}	$(1, 5)(2, 4)$
\bar{b}^{-1}	$(2, 5)(3, 4)$	\bar{b}^{-1}	$(1, 2)(3, 5)$	\bar{b}^{-1}	$(1, 3)(4, 5)$	\bar{a}	$(2, 5, 4)$	\bar{b}	$(1, 5)(2, 3)$
\bar{b}	$(1, 3, 4)$	\bar{a}	$(1, 4, 2)$	\bar{b}^{-1}	$(1, 5, 2, 4, 3)$	\bar{b}^{-1}	$(1, 2, 5, 3, 4)$	\bar{b}^{-1}	$(1, 3, 5)$
\bar{b}^{-1}	$(2, 3)(4, 5)$	\bar{a}	$(1, 2, 5, 4, 3)$	\bar{b}	$(1, 5, 2)$	\bar{a}	$(1, 3, 5, 2, 4)$	\bar{b}^{-1}	$(1, 2, 3, 4, 5)$
\bar{a}	$(1, 4, 3, 2, 5)$	\bar{b}^{-1}	$(2, 4)(3, 5)$	\bar{b}^{-1}	$(1, 3, 4, 5, 2)$	\bar{b}^{-1}	$(1, 2, 3)$	\bar{b}^{-1}	$(1, 5, 4)$
\bar{a}	$(1, 3, 5, 4, 2)$	\bar{b}^{-1}	$(1, 4, 5, 2, 3)$	\bar{b}^{-1}	$(1, 2, 4)$	\bar{a}	$(1, 3, 2)$	\bar{b}^{-1}	$(1, 5, 4, 2, 3)$
\bar{b}^{-1}	$(1, 4)(2, 5)$	\bar{b}^{-1}	$(1, 2, 4, 3, 5)$	\bar{b}^{-1}	$(3, 4, 5)$	\bar{a}	$(1, 4, 2, 5, 3)$	\bar{b}	$(1, 5, 4, 3, 2)$
\bar{b}	e								

4.4. A hamiltonian cycle C_1 in $\text{Cay}(A_5; \bar{a}, \bar{b})$ with $\bar{a} = (1, 2, 3)$ and $\bar{b} = (3, 4, 5)$.

	e	\bar{b}	$(3, 4, 5)$	\bar{a}	$(1, 2, 4, 5, 3)$	\bar{b}	$(1, 2, 4, 3, 5)$	\bar{b}	$(1, 2, 4)$
\bar{a}	$(1, 4)(2, 3)$	\bar{a}	$(1, 3, 4)$	\bar{b}^{-1}	$(1, 3, 5)$	\bar{b}^{-1}	$(1, 3)(4, 5)$	\bar{a}^{-1}	$(2, 3)(4, 5)$
\bar{a}^{-1}	$(1, 2)(4, 5)$	\bar{b}	$(1, 2)(3, 5)$	\bar{b}	$(1, 2)(3, 4)$	\bar{a}	$(2, 4, 3)$	\bar{a}	$(1, 4, 3)$
\bar{b}	$(1, 4, 5)$	\bar{b}	$(1, 4)(3, 5)$	\bar{a}^{-1}	$(1, 5, 3, 2, 4)$	\bar{a}^{-1}	$(1, 2, 5, 3, 4)$	\bar{b}^{-1}	$(1, 2, 5)$
\bar{b}^{-1}	$(1, 2, 5, 4, 3)$	\bar{a}^{-1}	$(3, 5, 4)$	\bar{a}^{-1}	$(1, 5, 4, 3, 2)$	\bar{b}	$(1, 5, 2)$	\bar{b}	$(1, 5, 3, 4, 2)$
\bar{a}	$(2, 4)(3, 5)$	\bar{a}	$(1, 4, 2, 5, 3)$	\bar{b}^{-1}	$(1, 4)(2, 5)$	\bar{b}^{-1}	$(1, 4, 3, 2, 5)$	\bar{a}	$(1, 5)(3, 4)$
\bar{b}^{-1}	$(1, 5, 3)$	\bar{b}^{-1}	$(1, 5, 4)$	\bar{a}^{-1}	$(1, 3, 2, 5, 4)$	\bar{a}^{-1}	$(1, 2, 3, 5, 4)$	\bar{b}	$(1, 2, 3)$
\bar{b}	$(1, 2, 3, 4, 5)$	\bar{a}	$(1, 3, 2, 4, 5)$	\bar{b}	$(1, 3, 5, 2, 4)$	\bar{a}^{-1}	$(1, 5, 2, 3, 4)$	\bar{b}^{-1}	$(1, 5)(2, 3)$
\bar{b}^{-1}	$(1, 5, 4, 2, 3)$	\bar{a}	$(1, 3, 5, 4, 2)$	\bar{a}	$(2, 5, 4)$	\bar{b}	$(2, 5, 3)$	\bar{b}	$(2, 5)(3, 4)$
\bar{a}	$(1, 5, 2, 4, 3)$	\bar{a}	$(1, 4, 3, 5, 2)$	\bar{b}^{-1}	$(1, 4, 5, 3, 2)$	\bar{b}^{-1}	$(1, 4, 2)$	\bar{a}^{-1}	$(1, 3)(2, 4)$
\bar{a}^{-1}	$(2, 3, 4)$	\bar{b}^{-1}	$(2, 3, 5)$	\bar{a}	$(1, 3)(2, 5)$	\bar{b}	$(1, 3, 4, 2, 5)$	\bar{a}	$(1, 5)(2, 4)$
\bar{a}	$(1, 4, 2, 3, 5)$	\bar{b}^{-1}	$(1, 4, 5, 2, 3)$	\bar{a}^{-1}	$(2, 4, 5)$	\bar{a}^{-1}	$(1, 3, 4, 5, 2)$	\bar{b}^{-1}	$(1, 3, 2)$
\bar{a}	e								

4.5. A hamiltonian cycle C_1 in $\text{Cay}(A_5; \bar{a}, \bar{b})$ with $\bar{a} = (1, 2, 4)$ and $\bar{b} = (1, 2, 3, 4, 5)$.

	e	$\bar{a} \rightarrow$	$(1, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 2, 4, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 5, 3, 4)$
$\bar{a}^{-1} \rightarrow$	$(3, 4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 3)(2, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{b} \rightarrow$	$(1, 3, 5, 4, 2)$
$\bar{a} \rightarrow$	$(3, 5, 4)$	$\bar{a} \rightarrow$	$(1, 2, 3, 5, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 5)(2, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 2, 5)$
$\bar{b} \rightarrow$	$(1, 5, 2, 3, 4)$	$\bar{a}^{-1} \rightarrow$	$(2, 5)(3, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 4)$
$\bar{a}^{-1} \rightarrow$	$(2, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 2)(4, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 3)(2, 4)$	$\bar{b} \rightarrow$	$(1, 4, 5, 3, 2)$
$\bar{a} \rightarrow$	$(2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5, 3, 2, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 3, 4, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5)(3, 4)$	$\bar{a} \rightarrow$	$(1, 2, 3, 4, 5)$
$\bar{b} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{a}^{-1} \rightarrow$	$(2, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{a} \rightarrow$	$(1, 4, 2, 5, 3)$
$\bar{a} \rightarrow$	$(1, 5, 3)$	$\bar{b} \rightarrow$	$(1, 2)(3, 4)$	$\bar{a} \rightarrow$	$(2, 3, 4)$	$\bar{a} \rightarrow$	$(1, 3, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 5)(2, 3)$
$\bar{a}^{-1} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 2, 4, 5)$	$\bar{b} \rightarrow$	$(1, 4)(3, 5)$	$\bar{a}^{-1} \rightarrow$	$(2, 4)(3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 2)(3, 5)$
$\bar{b}^{-1} \rightarrow$	$(1, 3)(4, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 4)(2, 3)$	$\bar{a}^{-1} \rightarrow$	$(2, 4, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 4, 2, 3)$
$\bar{a}^{-1} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{b} \rightarrow$	$(1, 5, 2)$	$\bar{a} \rightarrow$	$(2, 4, 5)$	$\bar{a} \rightarrow$	$(1, 4)(2, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 4, 3, 5)$
$\bar{a}^{-1} \rightarrow$	$(1, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{b} \rightarrow$	$(1, 3, 2, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(2, 3)(4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 4, 3, 2)$
$\bar{b} \rightarrow$	e								

4.6. A second hamiltonian cycle C_2 in $\text{Cay}(A_5; \bar{a}, \bar{b})$ with $\bar{a} = (1, 2, 4)$ and $\bar{b} = (1, 2, 3, 4, 5)$.

	e	$\bar{a} \rightarrow$	$(1, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 2, 4, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 5, 3, 4)$
$\bar{a}^{-1} \rightarrow$	$(3, 4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 3)(2, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{b} \rightarrow$	$(1, 3, 5, 4, 2)$
$\bar{a} \rightarrow$	$(3, 5, 4)$	$\bar{a} \rightarrow$	$(1, 2, 3, 5, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 5)$	$\bar{a} \rightarrow$	$(1, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5)(2, 4)$
$\bar{b}^{-1} \rightarrow$	$(2, 5)(3, 4)$	$\bar{a} \rightarrow$	$(1, 5, 2, 3, 4)$	$\bar{a} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 4)$
$\bar{a}^{-1} \rightarrow$	$(2, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 2)(4, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 3)(2, 4)$	$\bar{b} \rightarrow$	$(1, 4, 5, 3, 2)$
$\bar{a} \rightarrow$	$(2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5, 3, 2, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 3, 4, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5)(3, 4)$	$\bar{a} \rightarrow$	$(1, 2, 3, 4, 5)$
$\bar{b} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{a}^{-1} \rightarrow$	$(2, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{a} \rightarrow$	$(1, 4, 2, 5, 3)$
$\bar{a} \rightarrow$	$(1, 5, 3)$	$\bar{b} \rightarrow$	$(1, 2)(3, 4)$	$\bar{a} \rightarrow$	$(2, 3, 4)$	$\bar{a} \rightarrow$	$(1, 3, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 5)(2, 3)$
$\bar{a}^{-1} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 2, 4, 5)$	$\bar{b} \rightarrow$	$(1, 4)(3, 5)$	$\bar{a}^{-1} \rightarrow$	$(2, 4)(3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 2)(3, 5)$
$\bar{b}^{-1} \rightarrow$	$(1, 3)(4, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 4)(2, 3)$	$\bar{a}^{-1} \rightarrow$	$(2, 4, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 4, 2, 3)$
$\bar{a}^{-1} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{b} \rightarrow$	$(1, 5, 2)$	$\bar{a} \rightarrow$	$(2, 4, 5)$	$\bar{a} \rightarrow$	$(1, 4)(2, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 4, 3, 5)$
$\bar{a}^{-1} \rightarrow$	$(1, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{b} \rightarrow$	$(1, 3, 2, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(2, 3)(4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 4, 3, 2)$
$\bar{b} \rightarrow$	e								

4.7. A hamiltonian cycle C_1 in $\text{Cay}(A_5; \bar{a}, \bar{b})$ with $\bar{a} = (1, 2, 3)$ and $\bar{b} = (1, 2, 3, 4, 5)$.

	e	$\bar{a} \rightarrow$	$(1, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 4, 2, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 4)(2, 5)$
$\bar{a}^{-1} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 2, 3, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 5)$	$\bar{b} \rightarrow$	$(1, 2, 5, 3, 4)$
$\bar{b} \rightarrow$	$(1, 5, 2, 4, 3)$	$\bar{b} \rightarrow$	$(1, 4, 2)$	$\bar{a} \rightarrow$	$(2, 3, 4)$	$\bar{a} \rightarrow$	$(1, 3)(2, 4)$	$\bar{b} \rightarrow$	$(1, 4, 5, 3, 2)$
$\bar{a} \rightarrow$	$(3, 4, 5)$	$\bar{a} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{b} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{a} \rightarrow$	$(2, 5)(3, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 2)(3, 5)$
$\bar{a} \rightarrow$	$(2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5, 3)$	$\bar{b} \rightarrow$	$(1, 2)(3, 4)$	$\bar{a} \rightarrow$	$(2, 4, 3)$	$\bar{a} \rightarrow$	$(1, 4, 3)$
$\bar{b} \rightarrow$	$(1, 2)(4, 5)$	$\bar{a} \rightarrow$	$(2, 3)(4, 5)$	$\bar{a} \rightarrow$	$(1, 3)(4, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 4)(2, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 5)(2, 4)$
$\bar{a} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{a} \rightarrow$	$(1, 3, 4, 2, 5)$	$\bar{b} \rightarrow$	$(1, 5, 3, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4)(3, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 3, 2, 4, 5)$
$\bar{a}^{-1} \rightarrow$	$(1, 2, 3, 4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 5)$	$\bar{b} \rightarrow$	$(1, 2, 3, 5, 4)$	$\bar{a} \rightarrow$	$(1, 3, 2, 5, 4)$	$\bar{a} \rightarrow$	$(1, 5, 4)$
$\bar{b}^{-1} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 2, 4, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 5)(3, 4)$	$\bar{b} \rightarrow$	$(1, 2, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 4)$
$\bar{b}^{-1} \rightarrow$	$(1, 5)(2, 3)$	$\bar{b}^{-1} \rightarrow$	$(2, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 5, 4, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{a}^{-1} \rightarrow$	$(2, 4, 5)$
$\bar{a}^{-1} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{b} \rightarrow$	$(2, 4)(3, 5)$	$\bar{a} \rightarrow$	$(1, 4, 2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 3)(2, 5)$
$\bar{a}^{-1} \rightarrow$	$(2, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{a}^{-1} \rightarrow$	$(3, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 4, 3, 2)$
$\bar{b} \rightarrow$	e								

4.8. A second hamiltonian cycle C_2 in $\text{Cay}(A_5; \bar{a}, \bar{b})$ with $\bar{a} = (1, 2, 3)$ and $\bar{b} = (1, 2, 3, 4, 5)$.

	e	$\bar{a} \rightarrow$	$(1, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 4, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 5, 4, 2)$
$\bar{a} \rightarrow$	$(2, 5, 4)$	$\bar{b} \rightarrow$	$(1, 5)(2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 5)$	$\bar{a} \rightarrow$	$(1, 2, 5)$	$\bar{b} \rightarrow$	$(1, 5, 2, 3, 4)$
$\bar{a} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4)(2, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 4, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 5)(3, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 3, 2, 5)$
$\bar{b} \rightarrow$	$(1, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 2, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 2, 3, 5, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 5)$	$\bar{a} \rightarrow$	$(1, 2, 3, 4, 5)$
$\bar{a} \rightarrow$	$(1, 3, 2, 4, 5)$	$\bar{b} \rightarrow$	$(1, 4)(3, 5)$	$\bar{a} \rightarrow$	$(1, 2, 5, 3, 4)$	$\bar{a} \rightarrow$	$(1, 5, 3, 2, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 3, 4, 2, 5)$
$\bar{a}^{-1} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 5)(2, 4)$	$\bar{b} \rightarrow$	$(1, 4)(2, 3)$	$\bar{b} \rightarrow$	$(1, 3)(4, 5)$	$\bar{a}^{-1} \rightarrow$	$(2, 3)(4, 5)$
$\bar{a}^{-1} \rightarrow$	$(1, 2)(4, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 3)$	$\bar{a}^{-1} \rightarrow$	$(2, 4, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 2)(3, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 3)$
$\bar{a}^{-1} \rightarrow$	$(2, 5, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 2)(3, 5)$	$\bar{b} \rightarrow$	$(2, 5)(3, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 2, 4, 3)$
$\bar{b} \rightarrow$	$(1, 4, 2)$	$\bar{a} \rightarrow$	$(2, 3, 4)$	$\bar{a} \rightarrow$	$(1, 3)(2, 4)$	$\bar{b} \rightarrow$	$(1, 4, 5, 3, 2)$	$\bar{a} \rightarrow$	$(3, 4, 5)$
$\bar{a} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 3, 4)$	$\bar{a} \rightarrow$	$(1, 2, 4)$	$\bar{b} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{a}^{-1} \rightarrow$	$(2, 4, 5)$
$\bar{a}^{-1} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{b} \rightarrow$	$(2, 4)(3, 5)$	$\bar{a} \rightarrow$	$(1, 4, 2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 3)(2, 5)$
$\bar{a}^{-1} \rightarrow$	$(2, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{a}^{-1} \rightarrow$	$(3, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 4, 3, 2)$
$\bar{b} \rightarrow$	e								

4.9. A hamiltonian cycle C_1 in $\text{Cay}(A_5; \bar{a}, \bar{b})$ with $\bar{a} = (1, 2, 3, 4, 5)$ and $\bar{b} = (1, 2, 3, 5, 4)$.

	e	$\bar{b} \rightarrow$	$(1, 2, 3, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 5)$	$\bar{b} \rightarrow$	$(1, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 4, 5, 2)$
$\bar{b} \rightarrow$	$(2, 4, 3)$	$\bar{b} \rightarrow$	$(1, 4)(3, 5)$	$\bar{b} \rightarrow$	$(1, 2, 5)$	$\bar{b} \rightarrow$	$(1, 5, 4, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 2)$
$\bar{b} \rightarrow$	$(3, 5, 4)$	$\bar{b} \rightarrow$	$(1, 2, 5, 3, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 5)$	$\bar{b} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{b} \rightarrow$	$(1, 5, 3, 4, 2)$
$\bar{a}^{-1} \rightarrow$	$(1, 3)(2, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(2, 4, 5)$	$\bar{b} \rightarrow$	$(1, 4)(2, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 5)(2, 4)$
$\bar{b} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 5, 4, 2)$	$\bar{b} \rightarrow$	$(2, 5)(3, 4)$	$\bar{b} \rightarrow$	$(1, 5, 3, 2, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 4, 2, 5)$
$\bar{b} \rightarrow$	$(1, 5, 2, 4, 3)$	$\bar{b} \rightarrow$	$(1, 4, 5, 3, 2)$	$\bar{a}^{-1} \rightarrow$	$(1, 3)(2, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 3, 4, 5)$	$\bar{a} \rightarrow$	$(1, 3, 5, 2, 4)$
$\bar{b}^{-1} \rightarrow$	$(2, 3, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 2)(4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 3, 2, 4, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 2, 3, 4)$
$\bar{b}^{-1} \rightarrow$	$(2, 5, 4)$	$\bar{b}^{-1} \rightarrow$	$(1, 2)(3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 3)(4, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 5)(2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 4)$
$\bar{b}^{-1} \rightarrow$	$(2, 3)(4, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 2, 5, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 4, 3, 5)$	$\bar{a} \rightarrow$	$(1, 4)(2, 5)$
$\bar{b}^{-1} \rightarrow$	$(2, 4)(3, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 2)(3, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 4)$
$\bar{b}^{-1} \rightarrow$	$(2, 5, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{a}^{-1} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{b} \rightarrow$	$(1, 4, 2)$	$\bar{b} \rightarrow$	$(2, 3, 5)$
$\bar{b} \rightarrow$	$(1, 3, 2, 5, 4)$	$\bar{b} \rightarrow$	$(1, 5)(3, 4)$	$\bar{a} \rightarrow$	$(1, 2, 4)$	$\bar{b}^{-1} \rightarrow$	$(3, 4, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 5, 4, 3, 2)$
$\bar{a} \rightarrow$	e								

4.10. A hamiltonian cycle C_1 in $\text{Cay}(A_5; \bar{a}, \bar{b})$ with $\bar{a} = (1, 2, 3, 4, 5)$ and $\bar{b} = (1, 3, 4, 2, 5)$.

	e	$\bar{a} \rightarrow$	$(1, 2, 3, 4, 5)$	$\bar{a} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5, 4, 3, 2)$
$\bar{b}^{-1} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(2, 4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 2)(3, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 2, 5, 4)$
$\bar{b} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{a} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{a} \rightarrow$	$(2, 5, 4)$	$\bar{a} \rightarrow$	$(1, 5)(2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 4)$
$\bar{b}^{-1} \rightarrow$	$(1, 5, 2)$	$\bar{a} \rightarrow$	$(2, 3, 4)$	$\bar{a} \rightarrow$	$(1, 3, 2, 4, 5)$	$\bar{b} \rightarrow$	$(1, 2)(3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 3)(4, 5)$
$\bar{a}^{-1} \rightarrow$	$(1, 4)(2, 3)$	$\bar{b} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 4)(3, 5)$	$\bar{b} \rightarrow$	$(1, 5, 4, 2, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 4)(2, 5)$
$\bar{a}^{-1} \rightarrow$	$(1, 2, 4, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(3, 4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 2)$	$\bar{b}^{-1} \rightarrow$	$(1, 5)(2, 4)$	$\bar{a}^{-1} \rightarrow$	$(2, 5)(3, 4)$
$\bar{b}^{-1} \rightarrow$	$(1, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{a} \rightarrow$	$(2, 4)(3, 5)$	$\bar{a} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 4)$
$\bar{b} \rightarrow$	$(1, 3)(2, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 2, 3, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 2, 5)$	$\bar{a}^{-1} \rightarrow$	$(3, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 5, 3, 2)$
$\bar{b}^{-1} \rightarrow$	$(1, 3, 4, 2, 5)$	$\bar{a}^{-1} \rightarrow$	$(2, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 2)(4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 3, 2, 4)$
$\bar{b}^{-1} \rightarrow$	$(1, 3, 5, 4, 2)$	$\bar{a} \rightarrow$	$(2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5)(3, 4)$	$\bar{b} \rightarrow$	$(1, 4, 2)$	$\bar{a} \rightarrow$	$(2, 3)(4, 5)$
$\bar{a} \rightarrow$	$(1, 3, 5)$	$\bar{b} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{a} \rightarrow$	$(2, 4, 3)$	$\bar{a} \rightarrow$	$(1, 4, 5)$	$\bar{a} \rightarrow$	$(1, 2, 3, 5, 4)$
$\bar{a} \rightarrow$	$(1, 3)(2, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 5, 3, 4)$	$\bar{a} \rightarrow$	$(1, 5, 2, 4, 3)$
$\bar{b} \rightarrow$	e								

4.11. A second hamiltonian cycle C_2 in $\text{Cay}(A_5; \bar{a}, \bar{b})$ with $\bar{a} = (1, 2, 3, 4, 5)$ and $\bar{b} = (1, 3, 4, 2, 5)$.

	e	$\bar{a} \rightarrow$	$(1, 2, 3, 4, 5)$	$\bar{a} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5, 4, 3, 2)$
$\bar{b}^{-1} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(2, 4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 2)(3, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 2, 5, 4)$
$\bar{b} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{a} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{a} \rightarrow$	$(2, 5, 4)$	$\bar{a} \rightarrow$	$(1, 5)(2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 4)$
$\bar{b}^{-1} \rightarrow$	$(1, 5, 2)$	$\bar{a} \rightarrow$	$(2, 3, 4)$	$\bar{a} \rightarrow$	$(1, 3, 2, 4, 5)$	$\bar{b} \rightarrow$	$(1, 2)(3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 3)(4, 5)$
$\bar{b}^{-1} \rightarrow$	$(1, 4)(2, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 2, 4, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(3, 4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 3, 2)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 4, 2, 3)$
$\bar{b}^{-1} \rightarrow$	$(1, 4)(3, 5)$	$\bar{a} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 4)(2, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 5)(2, 4)$	$\bar{a}^{-1} \rightarrow$	$(2, 5)(3, 4)$
$\bar{b}^{-1} \rightarrow$	$(1, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{a} \rightarrow$	$(2, 4)(3, 5)$	$\bar{a} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 4)$
$\bar{b} \rightarrow$	$(1, 3)(2, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 2, 3, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 2, 5)$	$\bar{a}^{-1} \rightarrow$	$(3, 5, 4)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 5, 3, 2)$
$\bar{b}^{-1} \rightarrow$	$(1, 3, 4, 2, 5)$	$\bar{a}^{-1} \rightarrow$	$(2, 3, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 2)(4, 5)$	$\bar{a}^{-1} \rightarrow$	$(1, 4, 3)$	$\bar{a}^{-1} \rightarrow$	$(1, 5, 3, 2, 4)$
$\bar{b}^{-1} \rightarrow$	$(1, 3, 5, 4, 2)$	$\bar{a} \rightarrow$	$(2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5)(3, 4)$	$\bar{b} \rightarrow$	$(1, 4, 2)$	$\bar{a} \rightarrow$	$(2, 3)(4, 5)$
$\bar{a} \rightarrow$	$(1, 3, 5)$	$\bar{b} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{a} \rightarrow$	$(2, 4, 3)$	$\bar{a} \rightarrow$	$(1, 4, 5)$	$\bar{a} \rightarrow$	$(1, 2, 3, 5, 4)$
$\bar{a} \rightarrow$	$(1, 3)(2, 5)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{b}^{-1} \rightarrow$	$(1, 2, 5, 3, 4)$	$\bar{a} \rightarrow$	$(1, 5, 2, 4, 3)$
$\bar{b} \rightarrow$	e								

4.12. A hamiltonian cycle R_1 in $\text{Cay}(A_5; \bar{a}, \bar{b}, \bar{c})$ with $\bar{a} = (1, 2, 5)$, $\bar{b} = (1, 3, 5)$, and $\bar{c} = (1, 4, 5)$.

	e	$\bar{a} \rightarrow$	$(1, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 2)$	$\bar{b} \rightarrow$	$(1, 3, 2)$	$\bar{a} \rightarrow$	$(2, 5, 3)$
$\bar{a} \rightarrow$	$(1, 5)(2, 3)$	$\bar{b} \rightarrow$	$(1, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3)(2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 3)$	$\bar{c} \rightarrow$	$(1, 4, 3)$
$\bar{a} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{a} \rightarrow$	$(1, 5, 2, 4, 3)$	$\bar{b} \rightarrow$	$(2, 4, 3)$	$\bar{a} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 4, 3, 2)$
$\bar{b} \rightarrow$	$(1, 2)(3, 4)$	$\bar{a} \rightarrow$	$(2, 5)(3, 4)$	$\bar{a} \rightarrow$	$(1, 5)(3, 4)$	$\bar{c} \rightarrow$	$(1, 3, 4)$	$\bar{a} \rightarrow$	$(1, 2, 5, 3, 4)$
$\bar{b} \rightarrow$	$(1, 4)(2, 5)$	$\bar{b} \rightarrow$	$(1, 3, 2, 5, 4)$	$\bar{a} \rightarrow$	$(1, 5, 3, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4)(2, 3)$	$\bar{b} \rightarrow$	$(1, 2, 3, 5, 4)$
$\bar{a} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4)(3, 5)$	$\bar{c} \rightarrow$	$(3, 5, 4)$	$\bar{c} \rightarrow$	$(1, 3, 5)$	$\bar{a} \rightarrow$	$(1, 2)(3, 5)$
$\bar{a} \rightarrow$	$(2, 3, 5)$	$\bar{c} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{b} \rightarrow$	$(1, 5, 4, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3)(2, 4)$	$\bar{b} \rightarrow$	$(2, 4)(3, 5)$
$\bar{c} \rightarrow$	$(1, 2, 4, 3, 5)$	$\bar{a} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{c} \rightarrow$	$(1, 3, 5, 4, 2)$	$\bar{b} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{b} \rightarrow$	$(1, 4, 2)$
$\bar{c} \rightarrow$	$(1, 2)(4, 5)$	$\bar{b} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{b} \rightarrow$	$(1, 4, 5, 3, 2)$	$\bar{a} \rightarrow$	$(2, 3)(4, 5)$	$\bar{a} \rightarrow$	$(1, 3, 2, 4, 5)$
$\bar{b} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{a} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3)(4, 5)$	$\bar{b} \rightarrow$	$(3, 4, 5)$	$\bar{a} \rightarrow$	$(1, 2, 3, 4, 5)$
$\bar{c} \rightarrow$	$(1, 5, 2, 3, 4)$	$\bar{c} \rightarrow$	$(2, 3, 4)$	$\bar{a} \rightarrow$	$(1, 3, 4, 2, 5)$	$\bar{b} \rightarrow$	$(1, 4, 2, 5, 3)$	$\bar{b} \rightarrow$	$(2, 5, 4)$
$\bar{a} \rightarrow$	$(1, 5)(2, 4)$	$\bar{c} \rightarrow$	$(1, 2, 4)$	$\bar{c} \rightarrow$	$(2, 4, 5)$	$\bar{a} \rightarrow$	$(1, 4, 5)$	$\bar{c} \rightarrow$	$(1, 5, 4)$
$\bar{c} \rightarrow$	e								

4.13. A hamiltonian cycle C_1 in $\text{Cay}(A_5; \bar{a}, \bar{b}, \bar{c})$ with $\bar{a} = (1, 2)(4, 5)$, $\bar{b} = (1, 2, 3)$, and $\bar{c} = (1, 2, 4)$.

	e	$\xrightarrow{\bar{a}}$	$(1, 2)(4, 5)$	$\xrightarrow{\bar{c}^{-1}}$	$(1, 5, 4)$	$\xrightarrow{\bar{a}}$	$(1, 2, 5)$	$\xrightarrow{\bar{b}}$	$(1, 5)(2, 3)$
$\xrightarrow{\bar{a}}$	$(1, 3, 2, 5, 4)$	$\xrightarrow{\bar{c}}$	$(1, 5, 4, 3, 2)$	$\xrightarrow{\bar{a}}$	$(2, 5, 3)$	$\xrightarrow{\bar{b}}$	$(1, 5, 3)$	$\xrightarrow{\bar{b}}$	$(1, 2)(3, 5)$
$\xrightarrow{\bar{a}}$	$(3, 5, 4)$	$\xrightarrow{\bar{b}}$	$(1, 2, 5, 4, 3)$	$\xrightarrow{\bar{c}}$	$(1, 5, 4, 2, 3)$	$\xrightarrow{\bar{b}^{-1}}$	$(2, 5, 4)$	$\xrightarrow{\bar{a}}$	$(1, 5, 2)$
$\xrightarrow{\bar{b}^{-1}}$	$(1, 3)(2, 5)$	$\xrightarrow{\bar{b}^{-1}}$	$(2, 3, 5)$	$\xrightarrow{\bar{a}}$	$(1, 3, 5, 4, 2)$	$\xrightarrow{\bar{c}^{-1}}$	$(1, 2, 3, 5, 4)$	$\xrightarrow{\bar{a}}$	$(1, 3, 5)$
$\xrightarrow{\bar{c}^{-1}}$	$(1, 4, 2, 3, 5)$	$\xrightarrow{\bar{b}}$	$(1, 3, 4, 2, 5)$	$\xrightarrow{\bar{b}}$	$(1, 5)(2, 4)$	$\xrightarrow{\bar{c}}$	$(1, 4, 5)$	$\xrightarrow{\bar{b}^{-1}}$	$(1, 3, 2, 4, 5)$
$\xrightarrow{\bar{b}^{-1}}$	$(1, 2, 3, 4, 5)$	$\xrightarrow{\bar{a}}$	$(1, 3, 4)$	$\xrightarrow{\bar{b}^{-1}}$	$(1, 4)(2, 3)$	$\xrightarrow{\bar{b}^{-1}}$	$(1, 2, 4)$	$\xrightarrow{\bar{c}}$	$(1, 4, 2)$
$\xrightarrow{\bar{a}}$	$(2, 4, 5)$	$\xrightarrow{\bar{b}}$	$(1, 4, 5, 2, 3)$	$\xrightarrow{\bar{c}^{-1}}$	$(1, 5, 2, 4, 3)$	$\xrightarrow{\bar{b}^{-1}}$	$(2, 5)(3, 4)$	$\xrightarrow{\bar{a}}$	$(1, 5, 3, 4, 2)$
$\xrightarrow{\bar{c}^{-1}}$	$(1, 2, 5, 3, 4)$	$\xrightarrow{\bar{a}}$	$(1, 5)(3, 4)$	$\xrightarrow{\bar{b}^{-1}}$	$(1, 4, 3, 2, 5)$	$\xrightarrow{\bar{b}^{-1}}$	$(1, 2, 4, 3, 5)$	$\xrightarrow{\bar{a}}$	$(1, 4)(3, 5)$
$\xrightarrow{\bar{b}^{-1}}$	$(1, 5, 3, 2, 4)$	$\xrightarrow{\bar{c}}$	$(1, 4, 5, 3, 2)$	$\xrightarrow{\bar{a}}$	$(2, 4, 3)$	$\xrightarrow{\bar{b}}$	$(1, 4, 3)$	$\xrightarrow{\bar{b}}$	$(1, 2)(3, 4)$
$\xrightarrow{\bar{a}}$	$(3, 4, 5)$	$\xrightarrow{\bar{b}}$	$(1, 2, 4, 5, 3)$	$\xrightarrow{\bar{c}}$	$(1, 4, 2, 5, 3)$	$\xrightarrow{\bar{b}^{-1}}$	$(2, 4)(3, 5)$	$\xrightarrow{\bar{a}}$	$(1, 4, 3, 5, 2)$
$\xrightarrow{\bar{c}^{-1}}$	$(1, 3, 5, 2, 4)$	$\xrightarrow{\bar{b}}$	$(1, 4)(2, 5)$	$\xrightarrow{\bar{b}}$	$(1, 5, 2, 3, 4)$	$\xrightarrow{\bar{c}}$	$(1, 3, 4, 5, 2)$	$\xrightarrow{\bar{a}}$	$(2, 3, 4)$
$\xrightarrow{\bar{b}}$	$(1, 3)(2, 4)$	$\xrightarrow{\bar{c}^{-1}}$	$(1, 2, 3)$	$\xrightarrow{\bar{a}}$	$(1, 3)(4, 5)$	$\xrightarrow{\bar{b}^{-1}}$	$(2, 3)(4, 5)$	$\xrightarrow{\bar{a}}$	$(1, 3, 2)$
$\xrightarrow{\bar{b}}$	e								

4.14. A second cycle C_2 in $\text{Cay}(A_5; \bar{a}, \bar{b}, \bar{c})$ with $\bar{a} = (1, 2)(4, 5)$, $\bar{b} = (1, 2, 3)$, and $\bar{c} = (1, 2, 4)$.

	e	$\xrightarrow{\bar{a}}$	$(1, 2)(4, 5)$	$\xrightarrow{\bar{c}^{-1}}$	$(1, 5, 4)$	$\xrightarrow{\bar{a}}$	$(1, 2, 5)$	$\xrightarrow{\bar{b}}$	$(1, 5)(2, 3)$
$\xrightarrow{\bar{a}}$	$(1, 3, 2, 5, 4)$	$\xrightarrow{\bar{c}}$	$(1, 5, 4, 3, 2)$	$\xrightarrow{\bar{a}}$	$(2, 5, 3)$	$\xrightarrow{\bar{b}}$	$(1, 5, 3)$	$\xrightarrow{\bar{b}}$	$(1, 2)(3, 5)$
$\xrightarrow{\bar{a}}$	$(3, 5, 4)$	$\xrightarrow{\bar{b}}$	$(1, 2, 5, 4, 3)$	$\xrightarrow{\bar{c}}$	$(1, 5, 4, 2, 3)$	$\xrightarrow{\bar{b}^{-1}}$	$(2, 5, 4)$	$\xrightarrow{\bar{a}}$	$(1, 5, 2)$
$\xrightarrow{\bar{c}^{-1}}$	$(1, 4)(2, 5)$	$\xrightarrow{\bar{b}}$	$(1, 5, 2, 3, 4)$	$\xrightarrow{\bar{b}}$	$(1, 3, 5, 2, 4)$	$\xrightarrow{\bar{c}}$	$(1, 4, 3, 5, 2)$	$\xrightarrow{\bar{a}}$	$(2, 4)(3, 5)$
$\xrightarrow{\bar{b}}$	$(1, 4, 2, 5, 3)$	$\xrightarrow{\bar{c}^{-1}}$	$(1, 2, 4, 5, 3)$	$\xrightarrow{\bar{b}^{-1}}$	$(3, 4, 5)$	$\xrightarrow{\bar{a}}$	$(1, 2)(3, 4)$	$\xrightarrow{\bar{b}^{-1}}$	$(1, 4, 3)$
$\xrightarrow{\bar{b}^{-1}}$	$(2, 4, 3)$	$\xrightarrow{\bar{a}}$	$(1, 4, 5, 3, 2)$	$\xrightarrow{\bar{c}^{-1}}$	$(1, 5, 3, 2, 4)$	$\xrightarrow{\bar{b}}$	$(1, 4)(3, 5)$	$\xrightarrow{\bar{a}}$	$(1, 2, 4, 3, 5)$
$\xrightarrow{\bar{b}}$	$(1, 4, 3, 2, 5)$	$\xrightarrow{\bar{b}}$	$(1, 5)(3, 4)$	$\xrightarrow{\bar{a}}$	$(1, 2, 5, 3, 4)$	$\xrightarrow{\bar{c}}$	$(1, 5, 3, 4, 2)$	$\xrightarrow{\bar{a}}$	$(2, 5)(3, 4)$
$\xrightarrow{\bar{b}}$	$(1, 5, 2, 4, 3)$	$\xrightarrow{\bar{c}^{-1}}$	$(1, 3)(2, 5)$	$\xrightarrow{\bar{b}^{-1}}$	$(2, 3, 5)$	$\xrightarrow{\bar{a}}$	$(1, 3, 5, 4, 2)$	$\xrightarrow{\bar{c}^{-1}}$	$(1, 2, 3, 5, 4)$
$\xrightarrow{\bar{a}}$	$(1, 3, 5)$	$\xrightarrow{\bar{c}^{-1}}$	$(1, 4, 2, 3, 5)$	$\xrightarrow{\bar{b}}$	$(1, 3, 4, 2, 5)$	$\xrightarrow{\bar{b}}$	$(1, 5)(2, 4)$	$\xrightarrow{\bar{c}}$	$(1, 4, 5)$
$\xrightarrow{\bar{b}^{-1}}$	$(1, 3, 2, 4, 5)$	$\xrightarrow{\bar{b}^{-1}}$	$(1, 2, 3, 4, 5)$	$\xrightarrow{\bar{a}}$	$(1, 3, 4)$	$\xrightarrow{\bar{b}^{-1}}$	$(1, 4)(2, 3)$	$\xrightarrow{\bar{b}^{-1}}$	$(1, 2, 4)$
$\xrightarrow{\bar{c}}$	$(1, 4, 2)$	$\xrightarrow{\bar{a}}$	$(2, 4, 5)$	$\xrightarrow{\bar{b}}$	$(1, 4, 5, 2, 3)$	$\xrightarrow{\bar{b}}$	$(1, 3, 4, 5, 2)$	$\xrightarrow{\bar{a}}$	$(2, 3, 4)$
$\xrightarrow{\bar{b}}$	$(1, 3)(2, 4)$	$\xrightarrow{\bar{c}^{-1}}$	$(1, 2, 3)$	$\xrightarrow{\bar{a}}$	$(1, 3)(4, 5)$	$\xrightarrow{\bar{b}^{-1}}$	$(2, 3)(4, 5)$	$\xrightarrow{\bar{a}}$	$(1, 3, 2)$
$\xrightarrow{\bar{b}}$	e								

4.15. A hamiltonian cycle in $\text{Cay}(A_5; \bar{a}, \bar{b}, \bar{c})$ with $\bar{a} = (1, 2)(4, 5)$, $\bar{b} = (1, 2)(3, 4)$, and $\bar{c} = (1, 2, 3)$.

	e	$\bar{a} \rightarrow$	$(1, 2)(4, 5)$	$\bar{c}^{-1} \rightarrow$	$(1, 3)(4, 5)$	$\bar{a} \rightarrow$	$(1, 2, 3)$	$\bar{b} \rightarrow$	$(1, 3, 4)$
$\bar{a} \rightarrow$	$(1, 2, 3, 4, 5)$	$\bar{b} \rightarrow$	$(1, 3, 5)$	$\bar{a} \rightarrow$	$(1, 2, 3, 5, 4)$	$\bar{c}^{-1} \rightarrow$	$(1, 5, 4)$	$\bar{a} \rightarrow$	$(1, 2, 5)$
$\bar{b} \rightarrow$	$(1, 5)(3, 4)$	$\bar{c}^{-1} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{b} \rightarrow$	$(1, 5)(2, 4)$	$\bar{c} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{b} \rightarrow$	$(1, 3, 2, 4, 5)$
$\bar{a} \rightarrow$	$(1, 4)(2, 3)$	$\bar{b} \rightarrow$	$(1, 3)(2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{c} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{a} \rightarrow$	$(2, 3, 4)$
$\bar{b} \rightarrow$	$(1, 3, 2)$	$\bar{a} \rightarrow$	$(2, 3)(4, 5)$	$\bar{b} \rightarrow$	$(1, 3, 5, 4, 2)$	$\bar{a} \rightarrow$	$(2, 3, 5)$	$\bar{c} \rightarrow$	$(1, 3)(2, 5)$
$\bar{a} \rightarrow$	$(1, 5, 4, 2, 3)$	$\bar{b} \rightarrow$	$(1, 3, 2, 5, 4)$	$\bar{a} \rightarrow$	$(1, 5)(2, 3)$	$\bar{b} \rightarrow$	$(1, 3, 4, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 2, 3, 4)$
$\bar{c} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{c} \rightarrow$	$(1, 4)(2, 5)$	$\bar{b} \rightarrow$	$(1, 5, 2, 4, 3)$	$\bar{c}^{-1} \rightarrow$	$(2, 5)(3, 4)$	$\bar{b} \rightarrow$	$(1, 5, 2)$
$\bar{a} \rightarrow$	$(2, 5, 4)$	$\bar{b} \rightarrow$	$(1, 5, 4, 3, 2)$	$\bar{a} \rightarrow$	$(2, 5, 3)$	$\bar{b} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{c}^{-1} \rightarrow$	$(1, 4, 2, 5, 3)$
$\bar{c}^{-1} \rightarrow$	$(2, 4)(3, 5)$	$\bar{a} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{b} \rightarrow$	$(2, 4, 5)$	$\bar{a} \rightarrow$	$(1, 4, 2)$	$\bar{b} \rightarrow$	$(2, 4, 3)$
$\bar{a} \rightarrow$	$(1, 4, 5, 3, 2)$	$\bar{c}^{-1} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{a} \rightarrow$	$(1, 4, 3)$	$\bar{b} \rightarrow$	$(1, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 5)$
$\bar{b} \rightarrow$	$(1, 2, 4, 3, 5)$	$\bar{a} \rightarrow$	$(1, 4)(3, 5)$	$\bar{c}^{-1} \rightarrow$	$(1, 5, 3, 2, 4)$	$\bar{c}^{-1} \rightarrow$	$(1, 2, 5, 3, 4)$	$\bar{b} \rightarrow$	$(1, 5, 3)$
$\bar{a} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{c}^{-1} \rightarrow$	$(3, 5, 4)$	$\bar{a} \rightarrow$	$(1, 2)(3, 5)$	$\bar{b} \rightarrow$	$(3, 4, 5)$	$\bar{a} \rightarrow$	$(1, 2)(3, 4)$
$\bar{b} \rightarrow$	e								

4.16. A hamiltonian cycle in $\text{Cay}(A_5; \bar{a}, \bar{b}, \bar{c})$ with $\bar{a} = (1, 2)(4, 5)$, $\bar{b} = (1, 3)(2, 4)$, and $\bar{c} = (1, 2, 3)$.

	e	$\bar{a} \rightarrow$	$(1, 2)(4, 5)$	$\bar{b} \rightarrow$	$(1, 3, 2, 5, 4)$	$\bar{a} \rightarrow$	$(1, 5)(2, 3)$	$\bar{b} \rightarrow$	$(1, 2, 4, 3, 5)$
$\bar{a} \rightarrow$	$(1, 4)(3, 5)$	$\bar{b} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{a} \rightarrow$	$(2, 5)(3, 4)$	$\bar{b} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{c} \rightarrow$	$(1, 3, 4, 5, 2)$
$\bar{a} \rightarrow$	$(2, 3, 4)$	$\bar{b} \rightarrow$	$(1, 4, 3)$	$\bar{a} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{b} \rightarrow$	$(2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5, 4, 3, 2)$
$\bar{b} \rightarrow$	$(1, 2, 3, 5, 4)$	$\bar{a} \rightarrow$	$(1, 3, 5)$	$\bar{b} \rightarrow$	$(1, 5)(2, 4)$	$\bar{a} \rightarrow$	$(1, 4)(2, 5)$	$\bar{c}^{-1} \rightarrow$	$(1, 3, 5, 2, 4)$
$\bar{b} \rightarrow$	$(1, 5, 2)$	$\bar{a} \rightarrow$	$(2, 5, 4)$	$\bar{b} \rightarrow$	$(1, 3)(4, 5)$	$\bar{c}^{-1} \rightarrow$	$(2, 3)(4, 5)$	$\bar{b} \rightarrow$	$(1, 2, 5, 4, 3)$
$\bar{a} \rightarrow$	$(1, 5, 3)$	$\bar{b} \rightarrow$	$(2, 4)(3, 5)$	$\bar{a} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{b} \rightarrow$	$(1, 5, 2, 3, 4)$	$\bar{a} \rightarrow$	$(1, 3, 4, 2, 5)$
$\bar{c}^{-1} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{b} \rightarrow$	$(1, 5)(3, 4)$	$\bar{a} \rightarrow$	$(1, 2, 5, 3, 4)$	$\bar{b} \rightarrow$	$(1, 4, 5, 3, 2)$	$\bar{a} \rightarrow$	$(2, 4, 3)$
$\bar{b} \rightarrow$	$(1, 2, 3)$	$\bar{c} \rightarrow$	$(1, 3, 2)$	$\bar{b} \rightarrow$	$(1, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 5)$	$\bar{c}^{-1} \rightarrow$	$(1, 3, 2, 4, 5)$
$\bar{b} \rightarrow$	$(1, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 4)$	$\bar{b} \rightarrow$	$(1, 3, 5, 4, 2)$	$\bar{a} \rightarrow$	$(2, 3, 5)$	$\bar{b} \rightarrow$	$(1, 5, 2, 4, 3)$
$\bar{a} \rightarrow$	$(1, 4, 2, 5, 3)$	$\bar{b} \rightarrow$	$(3, 4, 5)$	$\bar{a} \rightarrow$	$(1, 2)(3, 4)$	$\bar{b} \rightarrow$	$(1, 4)(2, 3)$	$\bar{c} \rightarrow$	$(1, 3, 4)$
$\bar{a} \rightarrow$	$(1, 2, 3, 4, 5)$	$\bar{b} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 3, 2, 4)$	$\bar{b} \rightarrow$	$(1, 2)(3, 5)$	$\bar{a} \rightarrow$	$(3, 5, 4)$
$\bar{b} \rightarrow$	$(1, 5, 4, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3)(2, 5)$	$\bar{b} \rightarrow$	$(2, 4, 5)$	$\bar{a} \rightarrow$	$(1, 4, 2)$	$\bar{c}^{-1} \rightarrow$	$(1, 3)(2, 4)$
$\bar{b} \rightarrow$	e								

4.17. A hamiltonian cycle in $\text{Cay}(A_5; \bar{a}, \bar{b}, \bar{c})$ with $\bar{a} = (1, 2)(3, 4)$, $\bar{b} = (1, 2)(3, 5)$, and $\bar{c} = (1, 2, 3)$.

	e	$\bar{a} \rightarrow$	$(1, 2)(3, 4)$	$\bar{c}^{-1} \rightarrow$	$(1, 4, 3)$	$\bar{a} \rightarrow$	$(1, 2, 4)$	$\bar{c} \rightarrow$	$(1, 4)(2, 3)$
$\bar{b} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{b} \rightarrow$	$(1, 3, 2, 4, 5)$	$\bar{a} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{b} \rightarrow$	$(1, 3)(2, 4)$
$\bar{c} \rightarrow$	$(1, 4, 2)$	$\bar{a} \rightarrow$	$(2, 4, 3)$	$\bar{b} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{a} \rightarrow$	$(2, 4, 5)$	$\bar{b} \rightarrow$	$(1, 4, 5, 3, 2)$
$\bar{a} \rightarrow$	$(2, 4)(3, 5)$	$\bar{c} \rightarrow$	$(1, 4, 2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5, 3, 2, 4)$	$\bar{c} \rightarrow$	$(1, 4)(3, 5)$	$\bar{a} \rightarrow$	$(1, 2, 4, 5, 3)$
$\bar{b} \rightarrow$	$(1, 4, 5)$	$\bar{a} \rightarrow$	$(1, 2, 4, 3, 5)$	$\bar{c}^{-1} \rightarrow$	$(1, 5)(3, 4)$	$\bar{c}^{-1} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{a} \rightarrow$	$(1, 5)(2, 4)$
$\bar{c}^{-1} \rightarrow$	$(1, 3, 4, 2, 5)$	$\bar{b} \rightarrow$	$(1, 5, 4, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 2, 5, 4)$	$\bar{c} \rightarrow$	$(1, 5, 4)$	$\bar{b} \rightarrow$	$(1, 2, 5, 3, 4)$
$\bar{a} \rightarrow$	$(1, 5, 3)$	$\bar{b} \rightarrow$	$(1, 2, 5)$	$\bar{c} \rightarrow$	$(1, 5)(2, 3)$	$\bar{c} \rightarrow$	$(1, 3, 5)$	$\bar{a} \rightarrow$	$(1, 2, 3, 4, 5)$
$\bar{b} \rightarrow$	$(1, 3)(4, 5)$	$\bar{a} \rightarrow$	$(1, 2, 3, 5, 4)$	$\bar{b} \rightarrow$	$(1, 3, 4)$	$\bar{a} \rightarrow$	$(1, 2, 3)$	$\bar{c} \rightarrow$	$(1, 3, 2)$
$\bar{a} \rightarrow$	$(2, 3, 4)$	$\bar{b} \rightarrow$	$(1, 3, 5, 4, 2)$	$\bar{a} \rightarrow$	$(2, 3)(4, 5)$	$\bar{b} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{a} \rightarrow$	$(2, 3, 5)$
$\bar{c} \rightarrow$	$(1, 3)(2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 2, 3, 4)$	$\bar{c}^{-1} \rightarrow$	$(1, 4)(2, 5)$	$\bar{a} \rightarrow$	$(1, 5, 2, 4, 3)$	$\bar{c}^{-1} \rightarrow$	$(2, 5)(3, 4)$
$\bar{a} \rightarrow$	$(1, 5, 2)$	$\bar{b} \rightarrow$	$(2, 5, 3)$	$\bar{a} \rightarrow$	$(1, 5, 3, 4, 2)$	$\bar{b} \rightarrow$	$(2, 5, 4)$	$\bar{a} \rightarrow$	$(1, 5, 4, 3, 2)$
$\bar{c}^{-1} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{c}^{-1} \rightarrow$	$(3, 5, 4)$	$\bar{a} \rightarrow$	$(1, 2)(4, 5)$	$\bar{b} \rightarrow$	$(3, 4, 5)$	$\bar{a} \rightarrow$	$(1, 2)(3, 5)$
$\bar{b} \rightarrow$	e								

4.18. A hamiltonian cycle in $\text{Cay}(A_5; \bar{a}, \bar{b}, \bar{c})$ with $\bar{a} = (1, 2)(3, 4)$, $\bar{b} = (1, 3)(2, 5)$, and $\bar{c} = (1, 2, 3)$.

	e	$\bar{a} \rightarrow$	$(1, 2)(3, 4)$	$\bar{b} \rightarrow$	$(1, 4, 3, 2, 5)$	$\bar{c}^{-1} \rightarrow$	$(1, 2, 4, 3, 5)$	$\bar{a} \rightarrow$	$(1, 4, 5)$
$\bar{b} \rightarrow$	$(1, 3, 4, 5, 2)$	$\bar{a} \rightarrow$	$(2, 3, 5)$	$\bar{b} \rightarrow$	$(1, 5, 3)$	$\bar{a} \rightarrow$	$(1, 2, 5, 3, 4)$	$\bar{c} \rightarrow$	$(1, 5, 3, 2, 4)$
$\bar{b} \rightarrow$	$(1, 2, 3, 5, 4)$	$\bar{a} \rightarrow$	$(1, 3)(4, 5)$	$\bar{b} \rightarrow$	$(2, 4, 5)$	$\bar{a} \rightarrow$	$(1, 4, 3, 5, 2)$	$\bar{b} \rightarrow$	$(1, 5)(3, 4)$
$\bar{a} \rightarrow$	$(1, 2, 5)$	$\bar{b} \rightarrow$	$(1, 3, 2)$	$\bar{a} \rightarrow$	$(2, 3, 4)$	$\bar{b} \rightarrow$	$(1, 4, 2, 5, 3)$	$\bar{c} \rightarrow$	$(1, 5, 3, 4, 2)$
$\bar{a} \rightarrow$	$(2, 5, 3)$	$\bar{b} \rightarrow$	$(1, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 4)$	$\bar{c}^{-1} \rightarrow$	$(1, 4)(2, 3)$	$\bar{a} \rightarrow$	$(1, 3)(2, 4)$
$\bar{b} \rightarrow$	$(2, 5, 4)$	$\bar{a} \rightarrow$	$(1, 5, 4, 3, 2)$	$\bar{c}^{-1} \rightarrow$	$(1, 2, 5, 4, 3)$	$\bar{b} \rightarrow$	$(2, 4, 3)$	$\bar{a} \rightarrow$	$(1, 4, 2)$
$\bar{b} \rightarrow$	$(1, 3, 4, 2, 5)$	$\bar{c} \rightarrow$	$(1, 5)(2, 4)$	$\bar{c} \rightarrow$	$(1, 4, 2, 3, 5)$	$\bar{a} \rightarrow$	$(1, 3, 2, 4, 5)$	$\bar{b} \rightarrow$	$(1, 2)(4, 5)$
$\bar{a} \rightarrow$	$(3, 5, 4)$	$\bar{b} \rightarrow$	$(1, 5, 2, 4, 3)$	$\bar{a} \rightarrow$	$(1, 4)(2, 5)$	$\bar{c} \rightarrow$	$(1, 5, 2, 3, 4)$	$\bar{b} \rightarrow$	$(1, 4)(3, 5)$
$\bar{a} \rightarrow$	$(1, 2, 4, 5, 3)$	$\bar{b} \rightarrow$	$(2, 3)(4, 5)$	$\bar{a} \rightarrow$	$(1, 3, 5, 4, 2)$	$\bar{c}^{-1} \rightarrow$	$(1, 5, 4, 2, 3)$	$\bar{b} \rightarrow$	$(2, 4)(3, 5)$
$\bar{a} \rightarrow$	$(1, 4, 5, 3, 2)$	$\bar{b} \rightarrow$	$(1, 2, 3, 4, 5)$	$\bar{a} \rightarrow$	$(1, 3, 5)$	$\bar{c}^{-1} \rightarrow$	$(1, 5)(2, 3)$	$\bar{b} \rightarrow$	$(1, 2)(3, 5)$
$\bar{a} \rightarrow$	$(3, 4, 5)$	$\bar{b} \rightarrow$	$(1, 4, 5, 2, 3)$	$\bar{a} \rightarrow$	$(1, 3, 5, 2, 4)$	$\bar{b} \rightarrow$	$(1, 5, 4)$	$\bar{c}^{-1} \rightarrow$	$(1, 3, 2, 5, 4)$
$\bar{b} \rightarrow$	$(1, 2, 4)$	$\bar{a} \rightarrow$	$(1, 4, 3)$	$\bar{b} \rightarrow$	$(2, 5)(3, 4)$	$\bar{a} \rightarrow$	$(1, 5, 2)$	$\bar{c}^{-1} \rightarrow$	$(1, 3)(2, 5)$
$\bar{b} \rightarrow$	e								

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