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A successful local fluid-to-fluid similarity theory for heat transfer to supercritical pressure fluids: merits and limitations

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ABSTRACT

A similarity theory, proposed with a limited success some years ago and subsequently refined in a more complex form in further efforts, has been applied in a recent published work to perform Direct Numerical Simulations (DNS) of heat transfer to turbulent flow, with four different fluids at supercritical pressure. The obtained results showed an exceptionally good behaviour of the theory in the addressed cases, suggesting that the initial proposal, though it had only limited success in the cases considered at that time, possibly caught some of the basic features to be preserved in scaling.

The theory, based on dimensionless definitions that provided a reasonable degree of universality in the analysis of flow stability, found immediate difficulties to be applied with a comparable success to heat transfer problems. These difficulties mainly stemmed from the fact that, while it is relatively easy to scale fluid density, having a major role in stability analyses, it is definitely much harder to scale at a comparable level of accuracy the fluid thermo-physical properties, relevant in heat transfer. The very good results obtained in the recent work by DNS stimulated new reflections that shed light on the merits and limitations on the old theory.

The present paper, starting from these recent results and discussing them in front of RANS calculations, is aimed to highlight the promising features of this theory, envisaging the missing steps that should be completed to make it more general, in order to give to its consequences a higher level of universality.

Keywords: fluid-to-fluid, scaling, heat transfer, supercritical pressure, supercritical water reactor (SCWR)

1. INTRODUCTION

Similarity theories are among the most powerful means for quantitatively predicting physical phenomena, e.g., by setting up suitable engineering correlations. Identifying relevant dimensionless parameters, often in the frame of geometrical similarity, is a way of translating conclusions obtained from a limited number of experiments to more general cases, thus improving both phenomena understanding and model predictive capabilities. In all the fields of physics and engineering, dimensionless numbers are introduced to translate the relations between competing phenomena, to identify prevailing effects, phenomenological thresholds and the relative relevance of driving forces, in the hope to get an essential picture of the observed data. As known, this methodology can be used on the basis of well assessed models or even in the lack of exact theories.

Fluids at supercritical pressure exhibit a considerable level of complexity in their behaviour, partly shared with the even more complex case of two-phase flows. Notwithstanding the lack of interfaces and of the related surface tension effects, the steep changes they show in thermodynamic and thermo-physical properties, e.g., as a function of temperature at a constant pressure, make it much harder to establish analogies between heat and mass transfer than in single-phase fluids. As a consequence, despite of numerous efforts in the past, reliable correlations for heat transfer and turbulence models for CFD predictions are still only applicable in simple cases, mostly for normal or enhanced heat transfer conditions, while deteriorated heat transfer represents still a challenge for most models, with only a few exceptions applicable in a limited range of operating conditions. This is of course a problem in the design of equipment making use of supercritical pressure fluids, including future nuclear Generation IV Supercritical Water Reactors (SCWRs). Classical references on this subject are [1]-[3] while recent accounts on ongoing research are reported in [4] and [5].

Part of this situation is due to a limited capability in predicting the complex phenomena occurring at the transition across the pseudocritical temperature (i.e., the temperature at which the specific heat at constant pressure has a high maximum) and in its neighbourhood. This is reflected by the fact that most similarity theories developed to date have shown significant limitations in this operating range. Recently, [6] and [7] coped with the problem of assessing existing similarity theories suitable for supercritical pressure fluid conditions, on the basis of comparisons with experimental data, reaching the rather sad conclusion that: *"The prospects of deriving relatively simple empirical means of accounting accurately for differences in the thermodynamic behaviour of different fluids at supercritical pressures do not appear to be very promising"* [7]. This conclusion, which can be only partly shared on the basis of the results to be presented herein and those of previously performed work ([8] and [9]), expresses in an effective way the sense of frustration for several decades of work on supercritical pressure fluids, which accumulated plenty of good experimental evidences, though yet the advancement of modelling tools for their prediction is somehow limited.

In this frame, an important aspect to be borne in mind is that supercritical pressure fluids can be considered "false friends" because, despite the fact that they could be conceived as single-phase fluids, their properties evolve continuously along any heated or cooled duct, so that their behaviour cannot be evaluated only basing on local conditions, being dependent on the previous history of the fluid starting from the entrance. This aspect is shared with two-phase flows, especially when considering deteriorated heat transfer (DHT) phenomena, being a milder counterpart of the thermal crisis occurring in subcritical pressure conditions. In fact, it is well known that in subcritical conditions at high enough heat flux-to-mass flux ratios, Departure from Nucleate Boiling (DNB) or Dry-Out (DO) may occur, depending on the local void fraction. These subcritical fluid phenomena are at different degrees conditioned by upstream fluid history, something accounted for by the classical critical length-critical quality relations or Tong factors (see e.g. [10] for a classical treatment of these aspects). So, it can be expected that also fluids at supercritical pressures, exhibiting so large changes in fluid density (leading almost to a two-phase mixture without real interfaces), may behave in analogy.

As it will be shown later on, the presently considered similarity theory mainly considers the capability of the fluid to expand or contract in bulk and at the wall owing to heating or cooling, depending on inlet conditions and a dimensionless power-to-flow ratio, i.e., on upstream fluid history as in the case of two-phase flow. Moreover, one of the parameters to be introduced, being the dimensionless specific enthalpy, has a close resemblance with the definition of quality in two-phase flows and represents one of the most important factors in this frame, owing to an interesting link to the values of a dimensionless fluid density, to be defined below.

As it will be recalled later on, starting with a first paper on the subject by Ambrosini [8], showing the first embryonal choices of the presently discussed similarity theory, but also highlighting the first problems in its application, a more complex form of the theory was developed in [9]. The latter work successfully compared the behaviours of four different fluids based on results of RANS calculations, in view of assessing a general similarity theory.

In this regard, it must be clarified that the recourse to computations for establishing a similarity rationale is presently necessary because it is really difficult (if not impossible) to find experiments made with different fluids suitable to assess the proposed dimensionless numbers. Just to mention one relevant aspect, data obtained with carbon dioxide have inlet conditions generally much closer to the pseudocritical point in dimensionless terms than those obtained with water, being the coolant of SCWRs; this makes any close similarity between CO_2 and H_2O very difficult or impossible to discover by empirical means, unless achieved in dedicated experiments that do not yet exist at the moment.

By the way, coming back to the very interesting work by Mouslim [6] and Mouslim and Tavoularis [7], it seems that the previous efforts by Ambrosini [8] and Pucciarelli and Ambrosini [9] were considered, though they were not given enough attention because they were based on inlet conditions and not judged of interest in

their purposes. However, it must be considered that recent experiments adopting supercritical CO_2 by Kline [11] showed the importance of entrance conditions on the occurrence of different heat transfer regimes, including “normal”, “enhanced” and “deteriorated” heat transfer. The list of examples based on experimental evidence supporting this conclusion is long. By the way, the prediction of Kline’s data by a turbulence model based on an algebraic evaluation of the turbulent heat transfer (see [12]and [13]) resulted a fair success for relatively low flow rates, being able to explain most of the experimentally observed features, including the termination of heat transfer deterioration at the transition to gas-like conditions.

Recently, He et al. [14] re-assessed the work by Ambrosini [8] by performing DNS analyses for the same four fluids and the same pressures considered in that paper and making use of the same similarity principles, with the noticeable addition of the preservation of the value of the inlet Reynolds number as a further constraint. While Ambrosini [8] mostly preserved the dimensionless inlet subcooling, the power-to-flow ratio and the Froude number, also making sensitivity analyses with a scaled geometry that implied a rough correspondence of the inlet Peclet number, He et al. [14] changed their geometry case by case in order to also impose the same inlet Reynolds number. As a result, while Ambrosini [8] had only a rough qualitative matching of the scaled behaviour among the different fluids, He et al. [14] actually reported rather improved similar trends of dimensionless variables at the wall as well as in the bulk fluid, in an exceptional display of good coherence. These results triggered interest for understanding the reasons of this better success, obtained as a modification of the similarity principles used in Ambrosini [8]; in particular, the discussion tried to clarify if imposing exactly the same inlet Reynolds number was the main reason for the obtained improvement. As it will be shown later on, there is another important detail which largely contributed to cause this success, with respect to the more modest qualitative indications obtained by Ambrosini [8], and the discussion is now bringing to interesting conclusions, to be described in the following.

2. BASES FOR THE CONSIDERED SIMILARITY THEORY

As mentioned in the Introduction, Mouslim [6] reports an overview of the different scaling theories proposed for supercritical pressure fluids to date. Here we briefly summarise the evolution of the many similarity theories that appeared in literature.

The theory due to Jackson and Hall [15] and other similar works were mainly making reference to the critical point as reference condition with respect to which it was suggested to scale the fluid behaviour, something coherent with the assumptions of the corresponding state theory. Actually, considering the behaviour of fluid properties at supercritical pressure, it later became evident that the pseudocritical temperature at each given pressure had to be considered as the most important bifurcation point, virtually discriminating between liquid-like and gas-like conditions. An example of the relevance given to this parameter is in the formulations proposed by Cheng et al. [16]; previously, in 2006 and 2008, Ambrosini and Sharabi [17] had recognised the overwhelming importance of the pseudocritical point in scaling the stability phenomena in heated channels with supercritical fluids. An interesting feature of the dimensionless relations proposed by [17] is the fact that the dimensionless density, defined as the ratio of the density to the one at the pseudocritical temperature at any given pressure, has a nearly unique trend as a function of a dimensionless enthalpy. This result was found to be largely independent of the fluid and of the considered supercritical pressure, with modest deviations in the liquid-like region (see e.g., [17]). The two definitions introduced in this regard are:

- dimensionless density: $\rho^* = \rho / \rho_{pc}$;
- dimensionless enthalpy: $h^* = \left(h - h_{pc} \right) \beta_{pc} / C_{p,pc}$.

The origin of these formulations, as repeatedly recognised in the early papers on the subject, was a sort of occasional finding discovered while searching for relations similar to those adopted for boiling channel stability (see e.g., [18]) suitable for providing a good degree of universality to the treatments adopting them. In this regard, additional parameters were introduced, again in analogy with boiling channel instabilities formalism (see e.g., [18]):

- the trans-pseudocritical number, as a sort of dimensionless power-to-flow ratio: $N_{TPC} = \left(\dot{Q} / W \right) \beta_{pc} / C_{p,pc}$, replacing the phase change number in two-phase flow;
- the sub-pseudocritical number, as a sort of dimensionless inlet subcooling with respect to the pseudo-critical threshold: $N_{SPC} = -h^*_{inlet} = -\left(h_{inlet} - h_{pc} \right) \beta_{pc} / C_{p,pc}$, replacing the subcooling number;
- the Froude number at the inlet, to be defined in later treatments either on the basis of the pipe length or of the diameter: i.e., $Fr = w^2_{inlet} / \left(gL \right)$ or $Fr = w^2_{inlet} / \left(gD \right)$.

The role of these parameters has been explained several times in previous papers (e.g., [17] together with earlier and later papers by Ambrosini and collaborators on the issue of stability of supercritical pressure

fluids). However, in the present context the following considerations can be drawn (see e.g., [9] for a similar discussion):

- given the good level of universality of the trend of dimensionless density as a function of dimensionless enthalpy, a firm link is established between heating and all the phenomena at the root of the relevant dynamic and heat transfer behaviours, e.g., the density difference between channel inlet and outlet (governing both instability and heat transfer phenomena) and across the channel radius (e.g., ruling the mixed convection phenomena which cause deteriorated heat transfer and its extinction approaching gas-like conditions, as shown by [13]);
- since along a heated channel the supercritical fluid continuously changes its properties, up to the dramatic changes occurring at the pseudo-critical point, the flow is never fully developed and a memory of the previous history is necessary to be kept in modelling heat transfer: this function is accomplished by the dimensionless power-to-flow ratio, N_{TPC}, and by the dimensionless inlet subcooling with respect to the pseudocritical point, expressed by N_{SPC}, that should be both preserved in the similarity;
- buoyancy effects, instead, are mainly governed by the Froude number, another number to be preserved at channel inlet.

For ease of the reader, an argument already discussed in [8] is repeated here with reference to Figure 1. As it can be noted, the expansion of the four considered fluids as a function of increasing values of h^* is nearly the same, with the above-mentioned slight deviation in the liquid-like region, and N_{TPC} represents the difference between the outlet and the inlet dimensionless enthalpies, whereas N_{SPC} is equal to the negative of the inlet dimensionless specific enthalpy (i.e., $N_{SPC} = -h^*_{inlet}$ in the figure). It can be therefore noted that N_{SPC} and N_{TPC} represent the operating region of the fluid in terms of capability to expand because of heating, something that is approximately the same for any fluid and any supercritical pressure. Indeed, this is the most important basis for the present fluid-to-fluid similarity theory.

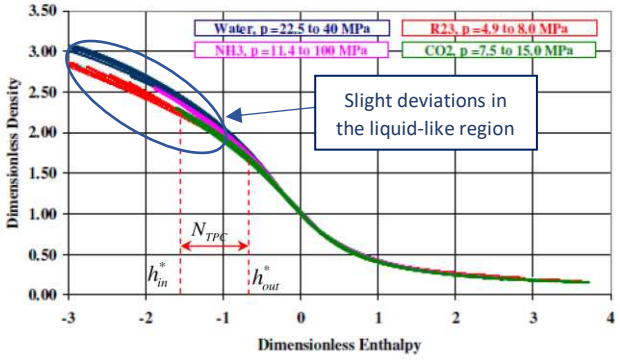


Figure 1. Relation between dimensionless density and specific enthalpy for four different fluids (adapted from [8]).

This said, local conditions also have a role. In particular, Pucciarelli and Ambrosini [9] highlighted the role of the Stanton number and of the Peclet number, whose values should not be too different for different fluids, and the difference can be minimised through the choice of the "similar" pressures for the different fluids (see the referenced papers for details).

Actually, a key point in the first attempts made by Ambrosini [8] which is found very important to explain the recent good results by He et al. [14], is that the supercritical pressures of the four fluids were selected in order to have the same peak value of the Prandtl number at the pseudocritical conditions (around 8 for water at 25 MPa). This selection of pressures was actually inspired by the interesting choice made by researchers at TU Delft in the design of the DeLight facility, where R23 at 5.7 MPa was used, just to get a trend of the Prandtl number similar to the one of water at 25 MPa (see e.g., [19]). This interesting principle was extended also to CO_2 and to the more exotic choice of NH_3, selected just for its greater similarity to water in terms of property trends. In fact, it must be noted that having the same Prandtl number for the reference and the model fluids can largely ease the achievement of similarity. Nevertheless, the trends reported in Figure 2 clearly show that even equalising the values of *Pr* at the pseudocritical points ($h^* = 0$) discrepancies continue to appear especially in the negative h^* region (i.e., in liquid-like conditions), while in the positive h^* region (gas-like conditions) the deviations are much more limited.

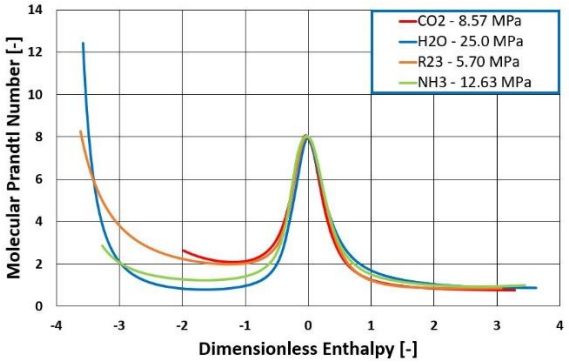


Figure 2. Prandtl number as a function of the dimensionless specific enthalpy for the selected fluids and operating pressures.

This argument helped to understand the results obtained by Ambrosini [8], achieved by RANS models available in the STAR-CCM+ CFD code [20] taking an experimental geometry from Pis'menny et al. [21] as reference case and making use of the above values of pressures for the different fluids (though the experiments by Pis'menny et al. [21] were run with water at 23.5 MPa). In fact, since the inlet dimensionless enthalpy was slightly higher than -1.5, the fluid at the entrance of the channel was subcooled with respect to the pseudocritical point in a region where the Prandtl numbers of the four fluids are quite different. This is shown in Figure 3, where it can be noted that a higher value of the Prandtl number in the liquid-like region of a fluid causes the deteriorated heat transfer to start earlier in that case.

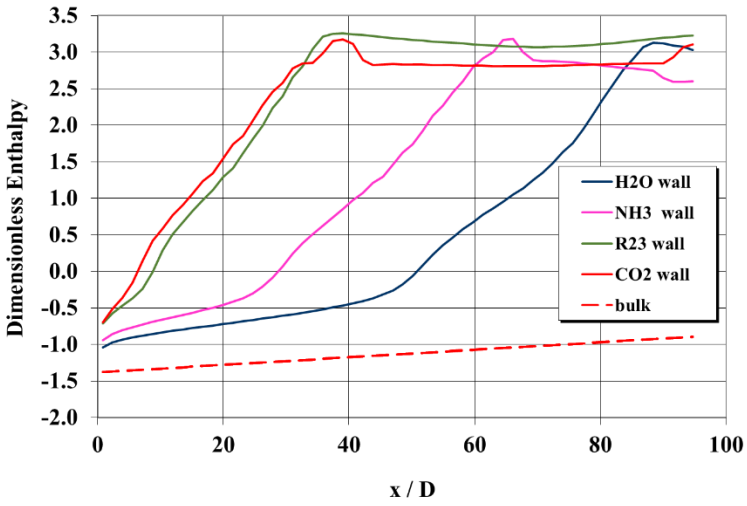


Figure 3. Wall and bulk dimensionless enthalpies for the cases considered in [8] for the four fluids and operating pressures (adapted from the data in the referenced paper)

Figure 3 is here useful also to state what is meant by "similarity" in the theory proposed by Ambrosini [8] as well as in the present one: owing to the link between dimensionless specific enthalpy and dimensionless density, it is considered that two fluids behave in a similar way when they have the same trends of the dimensionless specific enthalpy in bulk and at the wall. As it is noted in the figure, the bulk specific enthalpy has the same trend in the four cases, because the CFD calculations (see [8] for further details) were made by assigning the same dimensionless power-to-flow ratio (N_{TPC}), the same inlet subcooling ($N_{SPC} = -h^*_{inlet}$) and the same Froude number. The two first constraints assure the same observed linear increase of dimensionless specific enthalpy in bulk, being the red dashed line in the plot of Figure 3. However, owing to the different values of the Prandtl number in the key region where deterioration may first develop, the already described result of displaced deterioration occurrence was unfortunately obtained. Other attempts were made in order to get better results, which showed only slight improvements. However, a general qualitative similarity between the trends was observed, considering for instance the results obtained with higher inlet flow rate or larger pseudo-subcooling (see [8] for details).

The above results motivated the already mentioned step made by Pucciarelli [9] who developed in his PhD thesis a similarity rationale based on principles made more complex than in the previous attempt. However, the focus of this paper is not to discuss again that improvement, that is anyway worth per se, but the exceptionally accurate results obtained by He et al. [14] based on the proposal by Ambrosini [8], something which looked at first somehow strange in front of previous poorer results, though it can now be easily explained. The rest of the paper is devoted to summarise these results and to explain them in front of new computational information.

3. RECENT RESULTS BY DNS IN SUPPORT TO THE OLD THEORY

He et al. [14] ran DNS calculations for the system reported in Figure 4. Methodology and tools for performing the calculations are described in the referred paper and will not be repeated here. However, it is worth considering that the four fluids already mentioned in the previous section were adopted at the operating pressures selected in [8] for achieving a similar value of the peak Prandtl number. Moreover, the boundary conditions applied in dimensionless form (different from the one adopted by Ambrosini [8]) allowed for imposing the same N_{TPC}, N_{SPC} and the same inlet Reynolds and Froude numbers.

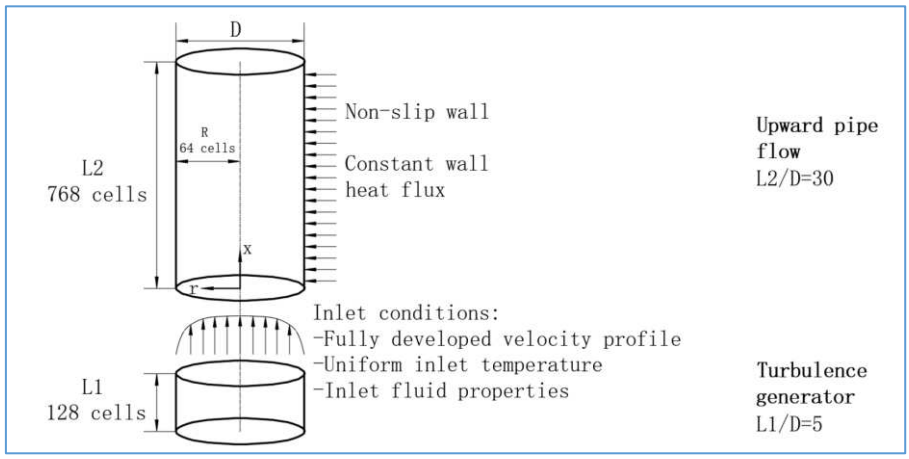


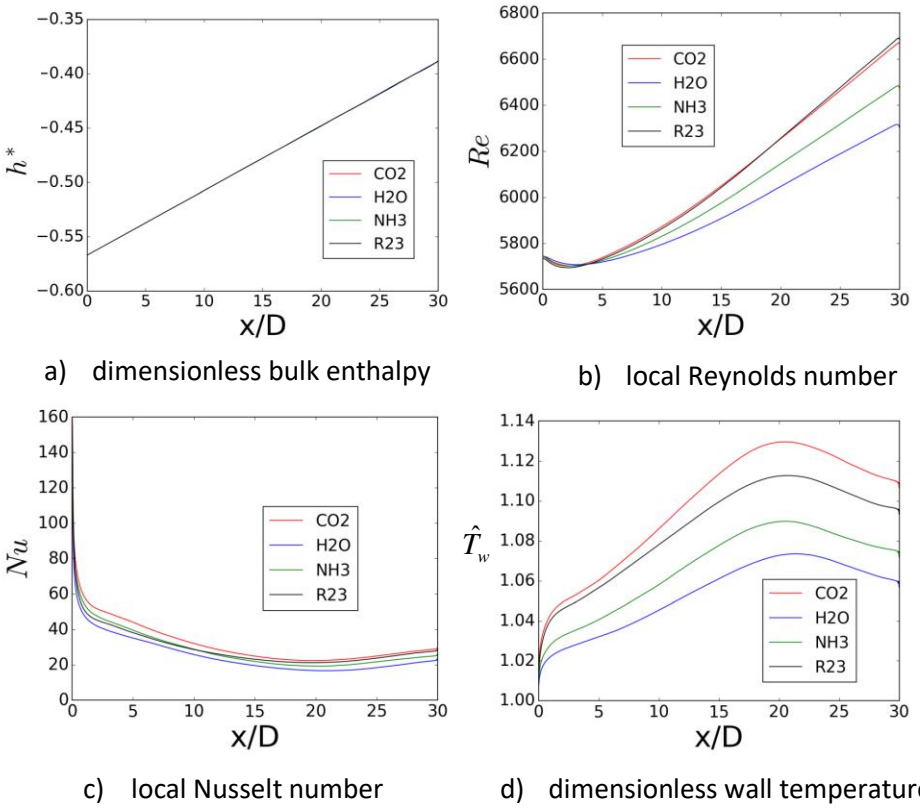
Figure 4. Computational domain adopted by He el al. [14] (adapted from the reference)

Table 1. Boundary conditions from the DNS calculation cases [14]

Case	q'' [W/m^2]	G_{in} [kg/m^{2}s]	h_{in} [J/kg]	D [m]	L [m]	N_{TPC}	N_{SPC}	Fr	Re_0	Pe_0
CO_2	30870	166.62	2.712x10^5	0.002	0.06	0.1783	0.5669	0.08249	5234.02	14954
H_2O	113739.32	128.68	1.815x10^6	0.0025	0.074	0.1783	0.5669	0.08249	5234.02	8806
NH_3	54050.83	94.01	1.064x10^6	0.0026	0.078	0.1783	0.5669	0.08249	5234.02	11390
R23	20884.68	179.76	2.439x10^5	0.0028	0.055	0.1783	0.5669	0.08249	5234.02	10954

These choices, summarised in Table 1, resulted particularly effective in achieving good results, in close similarity among the four fluids, as it is shown hereafter, though the simultaneous requirement to preserve the inlet Reynolds number and Froude resulted in the need to change the diameter and length of the pipe (preserving the L/D ratio) for each addressed fluid.

Figure 5 reports some of the most significant data obtained by He et al. [14] in relation to the similarity. As it can be noted from Figure 5a, the bulk dimensionless enthalpy (as defined in the present paper) is increasing linearly with the distance from the pipe inlet, as a consequence of the uniform value of the power-to-flow ratio. An important detail to be noted is that, with respect to the cases considered by Ambrosini [8], the dimensionless inlet enthalpy, still negative, is now closer to the pseudocritical condition ($h^* = 0$). Considering Figure 2 this assures that the bulk fluid is in the region of the liquid-like fluid where the values of the Prandtl number are very close for all fluids, being really a key point for the success of the similarity theory.



a) dimensionless bulk enthalpy b) local Reynolds number

c) local Nusselt number d) dimensionless wall temperature

Figure 5. Streamwise distribution of wall temperatures and integrated values of the four cases from He et al. [14] (adapted from the reference)

Figure 5 also shows that the Nusselt number assumes very similar values for the four fluids (Figure 5c) and that the axial evolution of the dimensionless temperature is also quite similar, exhibiting deterioration and recovery phenomena at the same values of x/D in the four cases (Figure 5d). Finally, it can be also noted that imposing the same values of the Reynolds number at the inlet, a "must" for turbulence similarity in single-phase fluids, is certainly playing a role on the good results achieved, though this number evolves in different ways along the pipe for the four fluids (Figure 5b). In this case, owing to the limited length of the domain and to the adopted parameters, the discrepancies are limited, but this cannot be always the case.

An even more striking similarity is revealed if the values of the dimensionless temperatures as defined by He et al.[14] ($\hat{T}_w$ in Figure 5d) are expressed in terms of the dimensionless specific enthalpy, h^*, as defined in this paper. As it can be noted in Figure 6, the values computed by the DNS analyses at the wall match very closely with each other, with exceptional accuracy considering that the matching of fluid properties is only approximate in dimensionless form. It can be also noted that the values of the dimensionless specific enthalpy at the wall belong to a range in which the values of the Prandtl number are very close to each other for the four fluids (see again Figure 2). This, together with the previous similar observation for the bulk dimensionless specific enthalpy, suggests that in the whole computational domain the four fluids have very close values of Pr, something that is now understood to be clearly at the root of the exceptionally good behaviour exhibited by the similarity theory in this case. In fact, comparing this behaviour with the much less

exciting ones shown by the plot in Figure 3, it is clearly seen that the most relevant difference is the more limited range of h^* considered in the DNS data, possibly together with the choice to preserve the same value of the Reynolds number at the inlet. Since it was shown that unavoidably the Reynolds number, despite being preserved in the channel, will then vary with fluid property changes, it is argued that such principle can be only approximately satisfied.

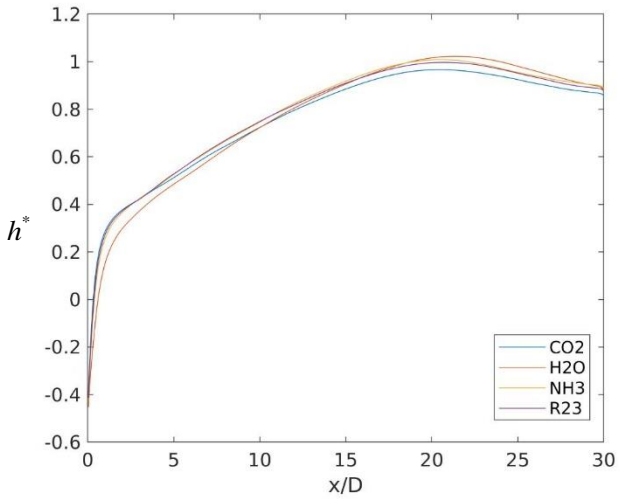


Figure 6. Values of the dimensionless specific enthalpy at the wall from the DNS calculations

As a consequence of the above, we can now further interpret also the success in representing the similarity in terms of Nusselt numbers for the different fluids. In this regard, it can be noted that the analysis performed by Pucciarelli and Ambrosini [9] expressed the Newton's law of convection in the following formalism, coherent with the dimensionless definitions adopted herein,

$$N_{TPC} = \overline{St}\left(h_w^* - h^*\right)\frac{4L}{D} \quad , \tag{1}$$

in which the averaged Stanton and Prandtl numbers are defined as:

$$\overline{St} = \frac{Nu}{Re\,\overline{Pr}} \qquad \overline{Pr} = \frac{\overline{C}_p \mu}{k} \tag{2}$$

In the above relationships h^* represents the bulk value of the dimensionless specific enthalpy, while the averaged specific heat, $\overline{C}_p$, entering the definitions of the averaged Prandtl and Stanton numbers is defined as

$$\overline{C}_p = \frac{h_w - h}{T_w - T} \tag{3}$$

with h and T being the "bulk" values of the fluid specific enthalpy and temperature. From the above definitions, it is then clear that if the bulk and the wall dimensionless specific enthalpy differences are preserved in the similarity and the Prandtl and Reynolds numbers are nearly the same, preserving the N_{TPC} results in preserving the Nusselt number as well.

We leave to the reader to consider in the paper by He et al. [14] the very close trends of axial fluid velocity, dimensionless density and temperature obtained by the DNS calculation for the different fluids, shifting now our interest to repeat similar analyses with RANS models, in order to get an independent check of the obtained results.

4. CONFIRMATION OF THE SIMILARITY THEORY BY RANS ANALYSES

In order to get the appropriate independent confirmation of the good results obtained by the DNS calculations and to accumulate further material for the present discussion, the same computational domain and boundary conditions considered by He et al. [14] were addressed by RANS turbulence models. However, making use of RANS models, it was possible to efficiently run the calculations in 2D axial-symmetric geometry, making use of mesh refinement close to the wall, with a number of meshes variable case by case, being anyway in the order of 200 axial nodes and 50 radial nodes with decreasing size along the radius from the centre to the wall. The analyses make use of both the Lien et al. model [22], implemented in the STAR-CCM+ code [20] and of a variant of it developed on the basis of the algebraic heat flux model (AHFM) being applied to an increasingly broad set of experimental data (see e.g. [23] and[12]). The latter has been found to greatly improve the predictions with respect to two-equations models as the Lien et al. [22] one in many conditions of interest. However, as it will be shown, regardless of the different levels of accuracy with respect to experimental data that was previously assessed for the adopted models, the presently proposed theory always provides similar behaviour for the four considered fluids, showing that its results are mainly consequence of the behaviour of the physical properties of the fluid and much less of the specific turbulence model adopted.

The RANS analyses performed to assess the similarity theory were made considering the same physical cases and boundary conditions adopted for the DNS cases (except for the mentioned 2D geometry). In order to get a fully developed velocity profile at the inlet of the pipe, velocity and turbulence distributions were computed for a relatively long pipe and fed as boundary conditions to the inlet cross section. We stress again the fact that the calculations were performed using the same boundary conditions as in the DNS cases, i.e., preserving N_{TPC}, N_{SPC} as well as the inlet values of the Reynolds and the Froude numbers. The axial and radial discretisation of the computational domain was performed following well assessed principles (see e.g., previous works as [23], [9], [13]), including the achievement of a y^+ lower than one close to the wall, as prescribed in the use of low-Reynolds number turbulence models.

The obtained results, discussed below, are expectedly slightly deviating from the DNS data, owing to the greater simplicity of the adopted RANS models, but they confirm the accuracy of the fluid-to-fluid similarity theory. Figure 7 reports the data of wall dimensionless specific enthalpy for the four fluids as a function of the dimensionless specific enthalpy of the bulk fluid. Remember that the latter variable linearly increases from the inlet to the outlet of the duct as a function of x/D, so that the comparison with the data in Figure 6 in terms of the latter variable is straightforward. As it can be noted, the trends are similar for the DNS and the RANS analyses, though expectedly not exactly the same. The RANS model, in fact, tends to predict an earlier dimensionless enthalpy peak with respect to DNS, though the maximum values are similar. However, it can be noted the presence of the same deterioration and recovery phases as predicted by DNS.

Indeed, both Figure 6 and Figure 7 show an excellent match of the trends of h^* at the wall for the four fluids, demonstrating that the similarity theory works almost perfectly, independently of the adopted model. This conclusion, to be later confirmed also by calculations with the model by Lien et al. [22] confirms as anticipated that the theory is mainly based on similarities in the fundamental behaviour of fluids, namely in their capability to expand because of heating, and that the chosen governing parameters, in the selected range of h^*, really represent excellent choices for establishing a similarity.

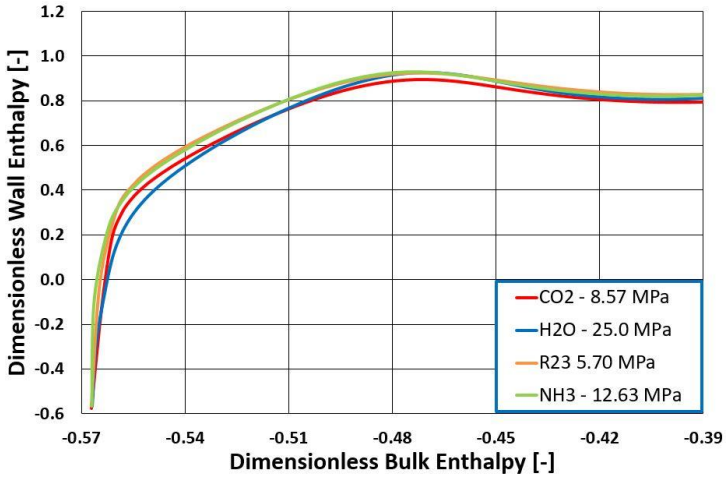
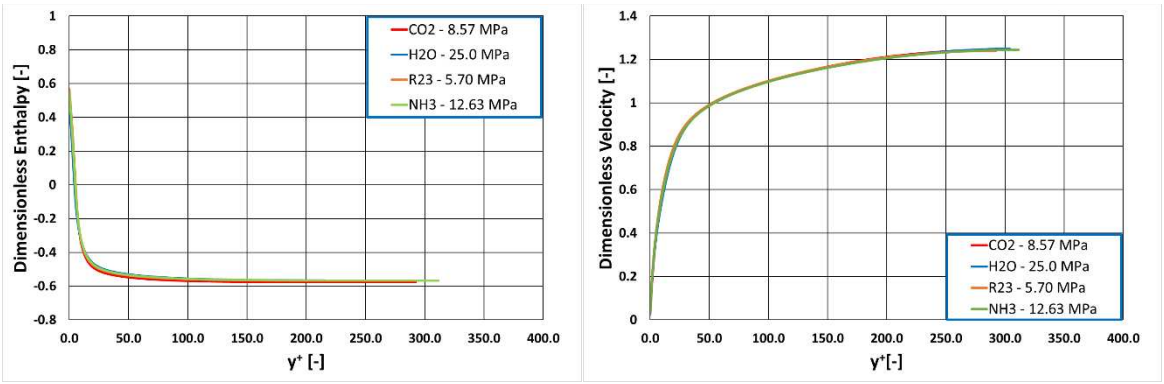


Figure 7. Values of the dimensionless specific enthalpy at the wall from the new RANS calculations with the AHFM model

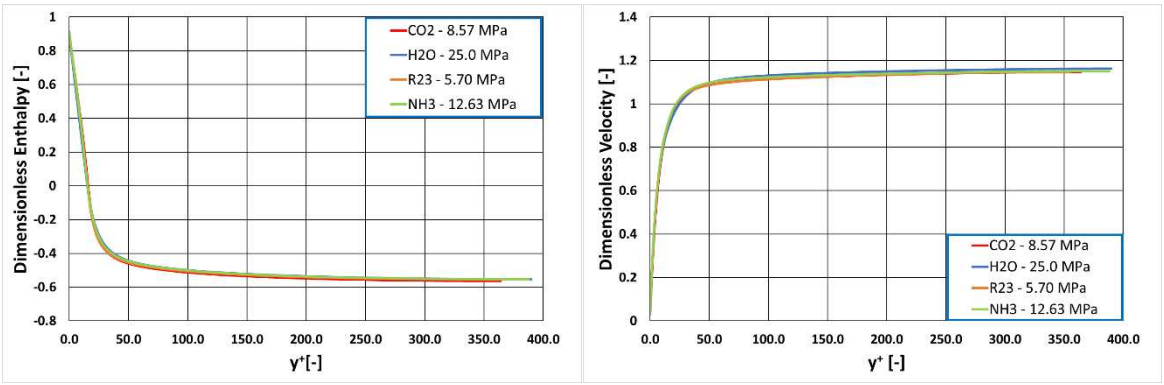
The plots in Figure 8 report the radial distributions of the dimensionless specific enthalpy and the axial component of fluid velocity normalised on the basis of the inlet velocity at three values of the bulk specific enthalpy along the pipe as a function of the classical dimensionless distance from the wall, $y^+ = w_\tau y/\nu$. The relation of the bulk h^* vs x/D is highlighted in Figure 9, showing that the three selected values of h^* at which radial profiles are illustrated are roughly corresponding to x/D = 5, 15 and 25. Moreover, for the sake of a further check, Figure 10 shows the same trends as a function of the distance from the wall normalised on the basis of the pipe radius; it is clear that, though the reported information in the two figures is the same, the two coordinates are not completely equivalent, since y^+ takes into account the different properties of the fluids and the shear stress at the wall. As it can be noted, both radial coordinates show an excellent matching of the radial distributions of these variables for the four fluids, showing that the similarity is nearly perfect, not only for the trends at the wall but also for those in the bulk fluid, highlighting a really successful set of assumptions.

In order to complete the picture of the radial distributions, Figure 11 presents the trends of the turbulent kinetic energy at the same three axial position as a function of the dimensionless distance from the wall. As it can be noted the match is also very good, except for the last axial location at which some discrepancy

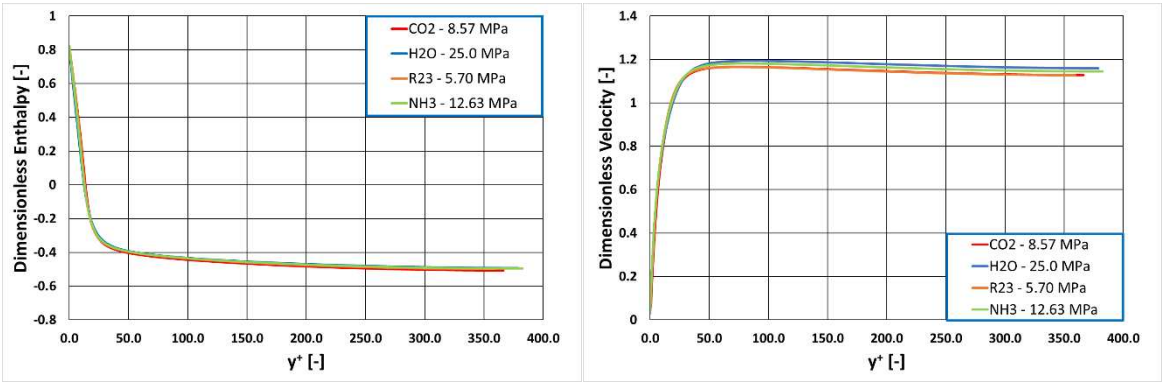
appears. Taking into account Figure 5b it can be argued that at that location the Reynolds number slightly differs for the three fluids owing to a change in properties; considering that this Reynolds number is quite low (for reasons related to the feasibility of the DNS calculations) discrepancies of that extent can be expected. In particular, in Figure 5b it can be noted that the trends of the Reynolds number for CO_2 and R23 are almost coincident, while those of water and NH_3 differ from each other and from the other trends; this matches exactly with what observed in Figure 11c, where a quite similar behaviour is noted for the corresponding curves.



a) Dimensionless bulk enthalpy = - 0.54 → x/D ≈ 5



b) Dimensionless bulk enthalpy = - 0.48→ x/D ≈ 15



c) Dimensionless bulk enthalpy = - 0.42 → x/D ≈ 25

Figure 8. Values of the dimensionless specific enthalpy and of the dimensionless velocity at different values of the dimensionless bulk enthalpy along the pipe as a function of y^+

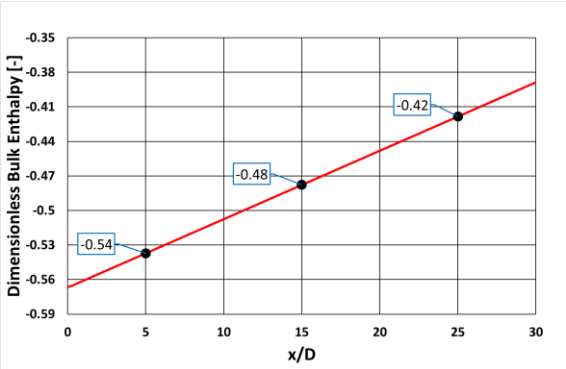
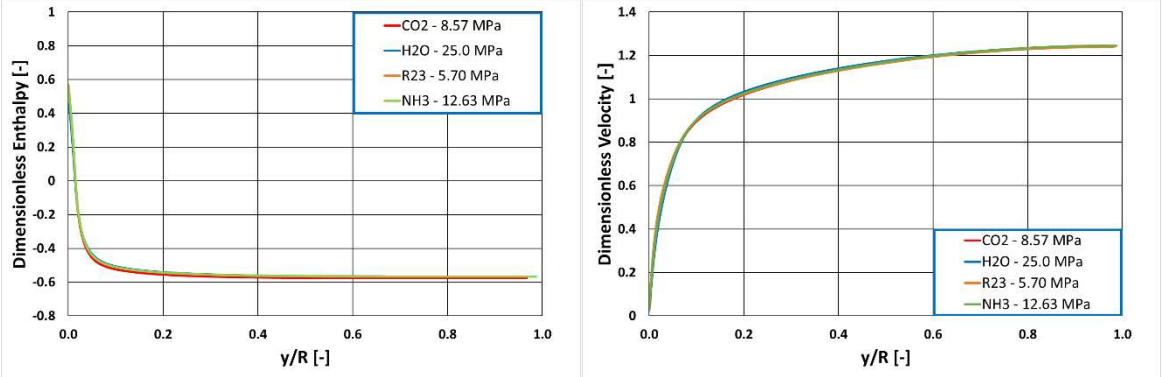
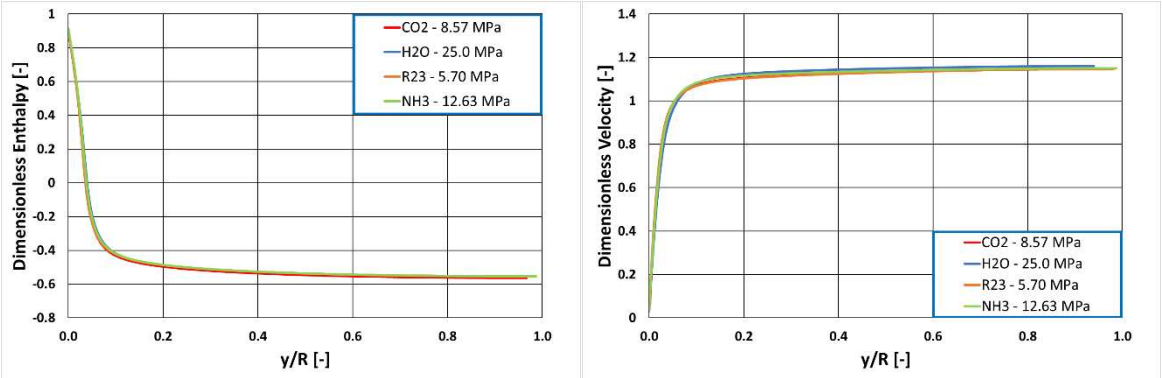


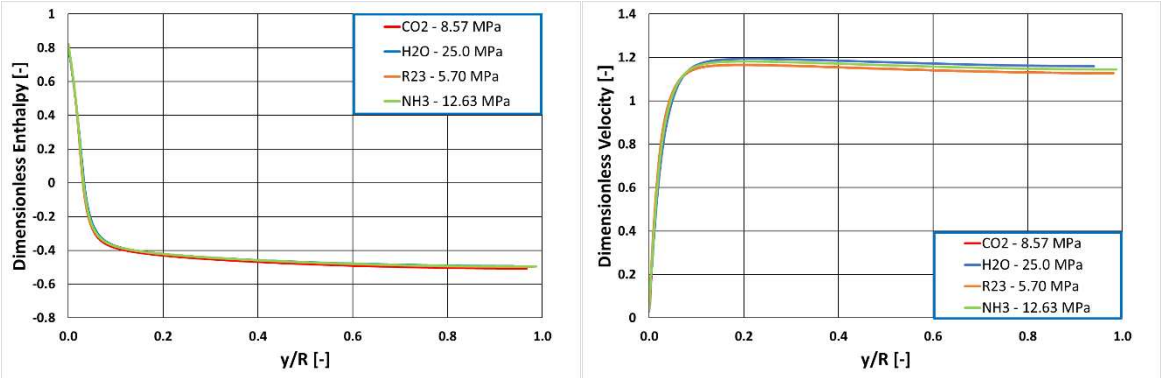
Figure 9. Relation between the dimensionless specific enthalpy and x/D



a) Dimensionless bulk enthalpy = - 0.54 → x/D ≈ 5

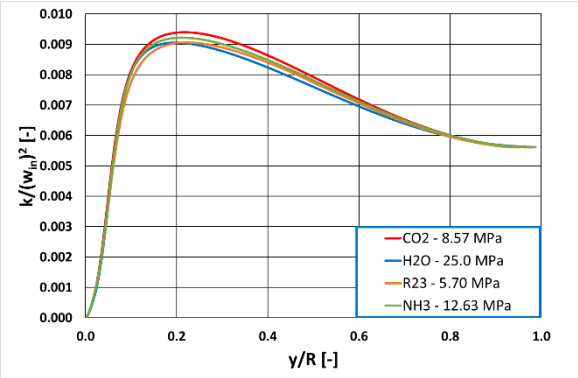


b) Dimensionless bulk enthalpy = - 0.48→ x/D ≈ 15

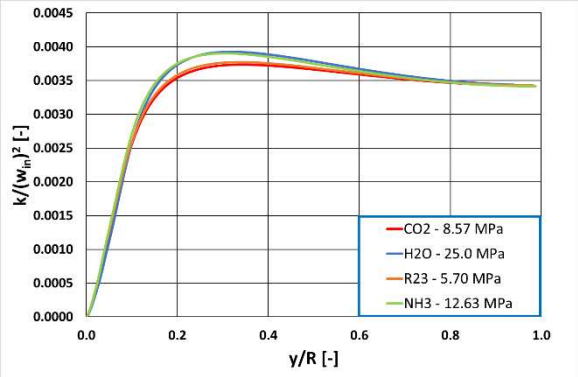


c) Dimensionless bulk enthalpy = - 0.42 → x/D ≈ 25

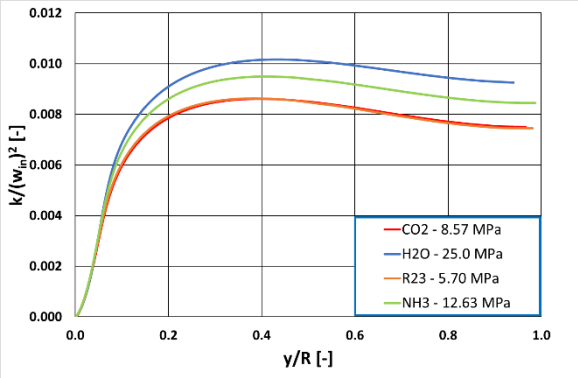
Figure 10. Values of the dimensionless specific enthalpy and of the dimensionless velocity at different values of the dimensionless bulk enthalpy along the pipe as a function of y/R



a) Dimensionless bulk enthalpy = - 0.54 → x/D ≈ 5



b) Dimensionless bulk enthalpy = - 0.48 → x/D ≈ 15



c) Dimensionless bulk enthalpy = - 0.42 → x/D ≈ 25

Figure 11. Values of the normalised dimensionless turbulence kinetic energy as a function of y/R

Finally, Figure 12 reports data for h^* at the wall obtained with the Lien et al. [22] model. It can be noted that this model is known to provide a good detection of the onset of deteriorated heat transfer, but often with a large overestimation of the wall temperature. The trends shown in Figure 7 and those in Figure 12 are somewhat different showing different laminarisation/recovery behaviours predicted by these models. Nevertheless, the results in Figure 12 again show that the trends in the four fluids are sufficiently close to each other to declare that similarity is still maintained.

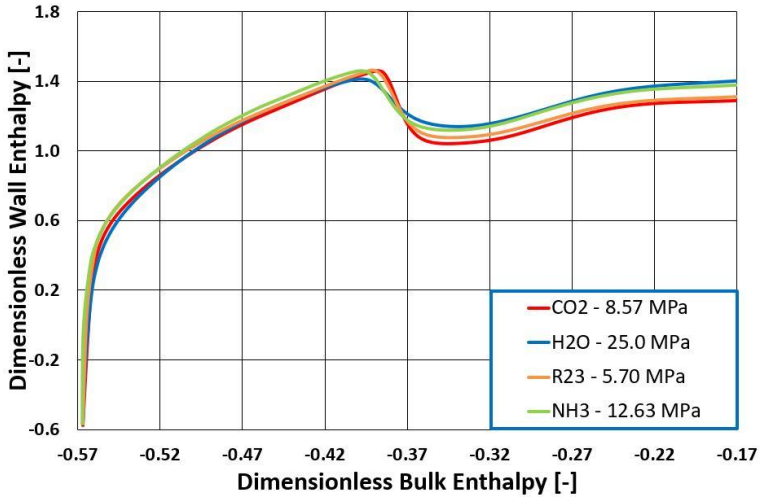


Figure 12. Values of the dimensionless specific enthalpy at the wall from the new RANS calculations with the Lien et al. [22] model

5. MERITS AND LIMITATIONS OF THE PROPOSED SIMILARITY THEORY

Basing on the previously described results, it can be noted that the similarity theory proposed in this paper and already used for the DNS data by He et al. [14], considering the choices made in [8], behaves exceptionally well in the range of conditions to which it was applied. However, considering the not very exciting results proposed by Ambrosini [8] and also by Ambrosini and De Rosa [24], some explanation of this improvement is in order.

With respect to the cases considered in the papers by Ambrosini [8] and Ambrosini and De Rosa [24], showing that deteriorated heat transfer started earlier in the simulations of a fluid with a greater molecular Prandtl number in the liquid-like range, two differences were introduced in the calculation cases proposed by He et al. [14] (and considered also in this paper):

- a much lower subcooling at the inlet with respect to the pseudocritical temperature was used;
- the inlet Reynolds number was preserved in all the cases.

Indeed the second of these two differences helps a lot in preserving the degree of turbulence, especially in the present low Re cases, as already mentioned. However, in front of previous much less successful results, the first item seems to be the most important one. In particular, the four calculation cases of Ambrosini and De Rosa [24], whose results are shown in Figure 13, show clearly that increasing values of the Prandtl number in the liquid-like region significantly moves forward heat transfer deterioration.

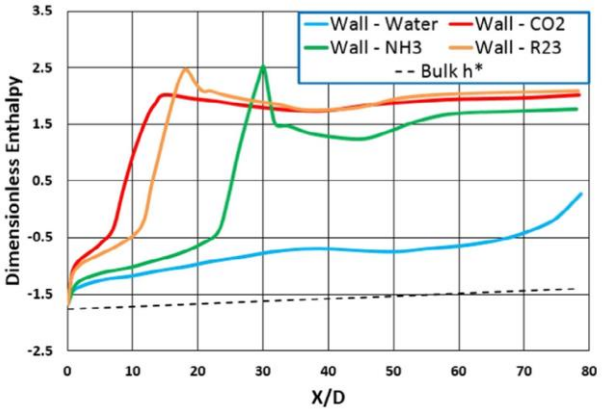


Figure 13. Trends from [24] obtained by the Lien et al. [22] model for the four fluids based on the boundary conditions of one of the Watts [25] water experimental data
(adapted from [9])

Indeed in these calculations [24] the inlet Reynolds number was not preserved, keeping the same diameter in all cases. This instils a final doubt to be clarified about whether the closeness of the values of the Pr or imposing the same Re at the inlet is the cause of the improved behaviour observed by He et al. [14].

In order to clarify this issue, two of the calculations performed by Ambrosini [8], whose results were displayed in Figure 3, were here repeated also imposing the same Reynolds number at the pipe inlet. The two calculations refer to water and CO_2 that, in front of the values of Pr in Figure 2, represent bounding cases,

since they refer to the highest and lowest values of the molecular Prandtl number. The boundary conditions adopted in the analyses are reported in Table 2.

Table 2. Boundary conditions for the additional calculations on the cases considered by Ambrosini [8]

	T_{inlet} [°C]	N_{SPC}	G [kg/(ms^2)]	Q'' [kW/m^2]	D [mm]	L [m]	N_{TPC}	Re_{inlet}	$Fr_{D,\,in}$
H_2O	300	1.38	509	390	6.28	0.6	0.515	34877	7.617
CO_2	-12.28	1.38	717.73	115.25	6.56	0.627	0.515	34877	7.617

The results of this analysis, made making use of the AHFM, are shown in Figure 14 finally clarifying that, in a range of dimensionless enthalpy where the molecular Prandtl numbers are considerably different, the similarity established by the present theory between the values of the dimensionless specific enthalpy at the wall is definitely lost.

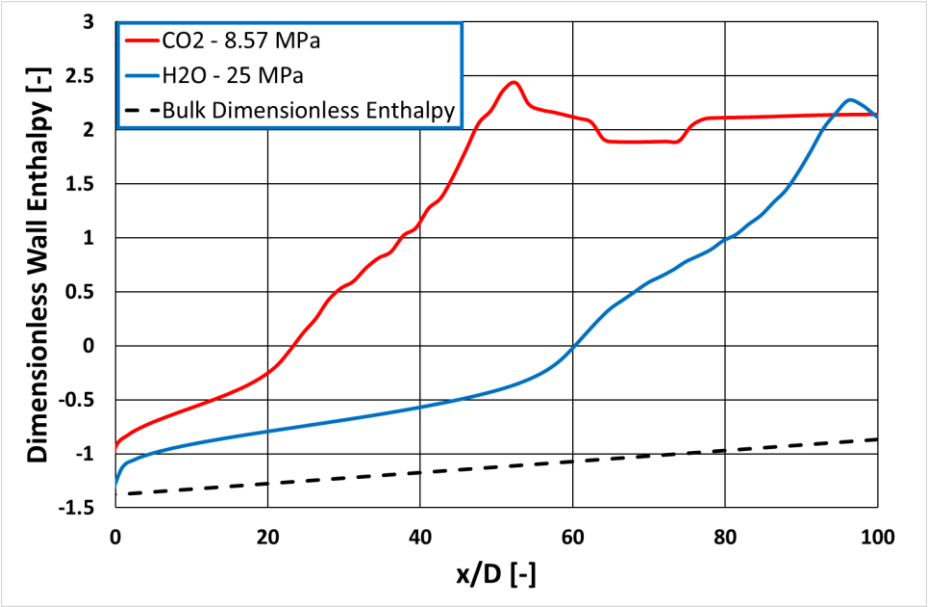


Figure 14. Results obtained for two of the cases addressed in [8] performed preserving the value of the Reynolds number at the inlet for water and CO_2

As an a posteriori consideration, it must be recognised that the value of the molecular Prandtl number is very much influencing heat transfer at the wall, as testified by its consideration in all the classical heat transfer theories and by its presence in most semi-empirical correlations. So, it can be concluded that the present similarity theory is very successful in the regions in which the molecular Prandtl number of the fluids is nearly the same, as in the cases considered by He et al. [14], while it is not as much accurate elsewhere. In view of Figure 2, it can be noted again that the most troublesome region is the liquid-like one, where it cannot be hoped to get matching values of the Prandtl number by a proper selection of the operating pressures. This represents a serious problem for all the trans-pseudocritical applications, as the ones concerning SCWR reactors, deserving a further reflection for its possible solution.

6. CONCLUSIONS

The results shown in this paper demonstrated that the ideas proposed in [8]and later developed by Pucciarelli and Ambrosini [9] and He et al. [14] are substantially correct in suggesting to preserve the values of dimensionless power-to-flow ratio, N_{TPC}, of the inlet subcooling, N_{SPC} and of the Froude number. In fact, preserving these numbers assures a similar behaviour of the dimensionless specific enthalpy in bulk and at the wall and, in part, of the buoyancy effects along the duct.

However, the mentioned works by Pucciarelli and Ambrosini [9] and He et al. [14] represent key improvements with respect to those embryonal ideas presented in [8], carried out on the basis of two different philosophies: the former, trying to generalise the theory for application over a broad range of Prandtl numbers, resulting in greater complexity, the latter suggested by the right belief that trying to preserve also the Reynolds number at least at the pipe inlet may give a better physical basis to the similarity theory in a turbulent flow field.

In this paper, following the rationale by He et al. [14], it was possible to show that the resulting theory is exceptionally accurate for relatively low values of $N_{SPC} = -h^*_{inlet}$, where the values of the Prandtl numbers for the four selected fluids and operating pressures show small differences, conditioned to an appropriate choice of the operating pressures. However, this cannot be said for applications involving larger degrees of pseudo-subcooling where it is hopeless to find similar values of the Prandtl number for water and other simulant fluids. Therefore, the theory needs to be extended, e.g., as done by Pucciarelli and Ambrosini [9] at the price of a greater complexity or by any other suitable rationale to be envisaged in the future. It must be explicitly mentioned that, though this work is mainly focused on deteriorated heat transfer conditions, it is believed that the similarity theory can be applied also to conditions of normal and enhanced heat transfer that are generally easier to predict; the assessment of such applicability is a further aspect to be considered in future work.

As a further conclusion, considering that nowadays available heat transfer correlations fail in predicting heat transfer deterioration, it can be argued that one of the reasons for this failure is the present neglect of parameters like N_{TPC}, N_{SPC} and of the Froude number, which indeed characterise the capability of the fluid to expand and generate those buoyancy forces that lead to laminarisation and eventually to deterioration.

This observation represents a powerful incentive to combine the reasoning at the basis of the presented similarity theory with the existing and only partially successful semi-empirical heat transfer relationships, aiming to achieve a better prediction of heat transfer phenomena for the interesting class of fluids being the ones operated at supercritical pressures.

NOMENCLATURE

Latin Letters

C_p	specific heat at constant pressure [J/(kgK)]
D	pipe diameter [m]
Fr	Froude number
g	gravity [m/s^2]
G	mass flux [kg/(m^{2}s)]
h	specific enthalpy [J/kg]
L	pipe length [m]
N_{SPC}	sub-pseudocritical number
N_{TPC}	trans-pseudocritical number
Nu	Nusselt number
Pe	Peclet number
Pr	Peclet number
q''	heat flux [W/m^2]
$\dot{Q}$	channel power [W]
Re	Reynolds number
St	Stanton number
T	temperature [K]
$\hat{T}_w$	dimensionless wall temperature in He et al. (2018) notation
w	velocity [m/s]
w_τ	shear velocity [m/s]
W	channel flow rate [kg/s]
x	pipe axial coordinate [m]
y	distance from the wall [m]

Greek Letters

β	isothermal expansion coefficient [1/K]
ρ	density [kg/m^3]
ν	kinematic viscosity [m^2/s]

Superscripts

*	starred variables imply dimensionless form
+	dimensionless value according to standard notation in turbulence theory

- overbar quantities are averaged between bulk and wall values

Subscripts

in, inlet	inlet value
pc	pseudocritical value
0	reference value (in He et al., 2018, paper)

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