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# Trajectory Optimization of a Reusable Launch Vehicle

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## Aim of this work

- Compare the optimization results between two direct optimal control methodologies applied to a complete model of an ascent vehicle.

- Compare the results with the original ones produced in [1] to understand how improvements in technology, i.e. numerical precision, can lead to better results thus making more appealing the development of certain technologies that were discarded in the past.

[1] D'Angelo S., Minisci E., Di Bona D., Guerra L. (2000) Optimization Methodology for Ascent Trajectories of Lifting-Body Reusable Launchers. Journal of Spacecraft and Rockets. Vol. 37, No. 6.

## Why the comparison?

- Both method selected are within the category of direct methods.
- For ascent trajectory problems, many difficulties can arise using indirect methods:
  - in order to have an accurate solution, indirect methods need the analytical development of necessary and sufficient conditions, which can be very challenging.
- The two methods selected differ in their very definition:
  - In pseudospectral collocation the states and controls are approximated by using polynomials evaluated at fixed collocation points [2] → faster
  - In multiple shooting the states are propagated according to a polynomial discretization of the controls in a certain interval → more precise

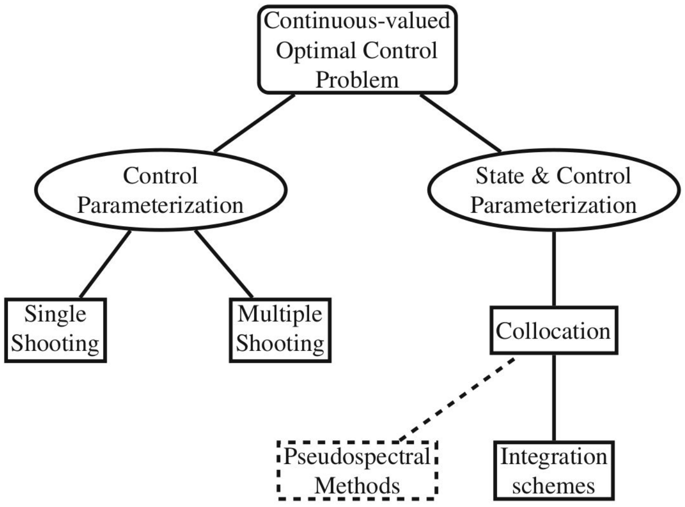


Figure 1: Direct Optimal Control methods classification [2]

[2] Böhme T.J., Frank B. (2017) Direct Methods for Optimal Control. In: Hybrid Systems, Optimal Control and Hybrid Vehicles. Advances in Industrial Control. Springer, Cham.

## Constrained Optimal Control: general definition

The problem at hand belong to the class of constrained optimal control problems.
Where:

- **J** is the objective function to be minimized
- **x** is the states vector
- **u** is the controls vector
- $\mathbf{x}_L$ , $\mathbf{u}_L$ are the lower bounds on states and controls
- $\mathbf{x}_U$ , $\mathbf{u}_U$ are the upper bounds on states and controls
- $\mathbf{x}_0$, $\mathbf{u}_0$ are the initial conditions on states and controls
- $\mathbf{x}_f$, $\mathbf{u}_f$ are the final conditions on states and controls

$$
\begin{aligned}
\underset{\mathbf{u}(t),t \in [t_0,t_f]}{\text{minimize}} \quad & J(\mathbf{x}(t),\mathbf{u}(t),t) \\
\text{subject to} \quad & \dot{\mathbf{x}}(t) = f(\mathbf{x}(t),\mathbf{u}(t),t), \\
& \mathbf{x}_L \leq \mathbf{x}(t) \leq \mathbf{x}_U, \\
& \mathbf{u}_L \leq \mathbf{u}(t) \leq \mathbf{u}_U, \\
& \mathbf{x}(t_0) = \mathbf{x}_0, \\
& \mathbf{u}(t_0) = \mathbf{u}_0, \\
& \mathbf{x}(t_f) = \mathbf{x}_f, \\
& \mathbf{u}(t_f) = \mathbf{u}_f
\end{aligned}
$$

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## Pseudo-Spectral Collocation with Legendre-Gauss-Lobatto nodes

The main characteristic of Pseudo-Spectral collocation is that the continuous trajectory problem is discretized by approximating the continuous states and controls with polynomials over a certain node distribution. In this work, Legendre polynomials with Legendre-Gauss-Lobatto node distribution was used.

Implications of this choice:

- some numerical problem could arise
- but the subsequent integration of the states with the collocation control laws leads to better results

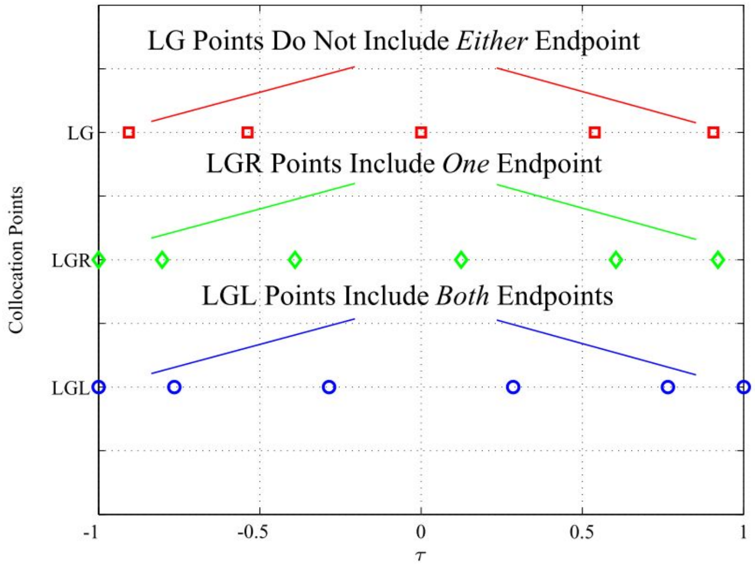



Figure 2: Comparison between LG, LGR and LGL nodes distribution [3]

[3] Garg D., Patterson M., Hager W., Rao A., Benson D., et al. (2017) An overview of three pseudospectral methods for the numerical solution of optimal control problems. <hal-01615132>.

## Multiple Shooting

The time interval $[T_0, T_f]$ is divided into N subintervals and then a single-shooting method, is performed over each subinterval $[t_i, t_{i+1}]$ on the states variables, starting from their value at the beginning of each subinterval. To enforce continuity of the states variables, the following equality condition is applied at the end of each interval:

$$\mathbf{x}_{i-1} - \mathbf{x}_i = 0\,, \quad i = 1, \dots, N+1$$

The controls in each subinterval can be kept constant or they can per parametrized with a polynomial interpolation.
In this work, a Piecewise Cubic Hermite Interpolating Polynomial (PCHIP) was used to interpolate the controls, to avoid overshooting that could have arise using a spline interpolation.

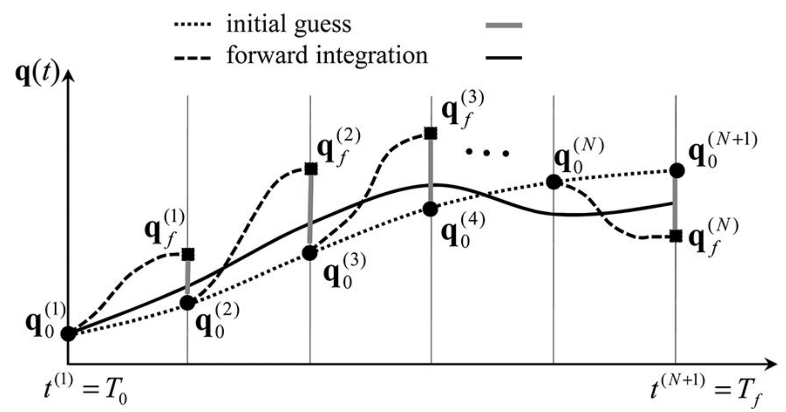



Figure 3: Multiple-Shooting method illustration [4]

[4] Brüls O., Bastos Jr. G., Seifried R. (2014) A stable inversion method for feedforward control of constrained flexible multibody systems. Journal of Computational and Nonlinear Dynamics 9.

## Problem: Ascent trajectory optimization of the FESTIP-FSS5 [1]

Vehicle characteristics:

- Lifting body
- Aerospike engine
- Vertical Lift-Off and Horizontal landing

Interest in revisit such concept since a single engine configuration for space and atmospheric flight can be more reliable than a configuration with two engines or an hybrid engine.

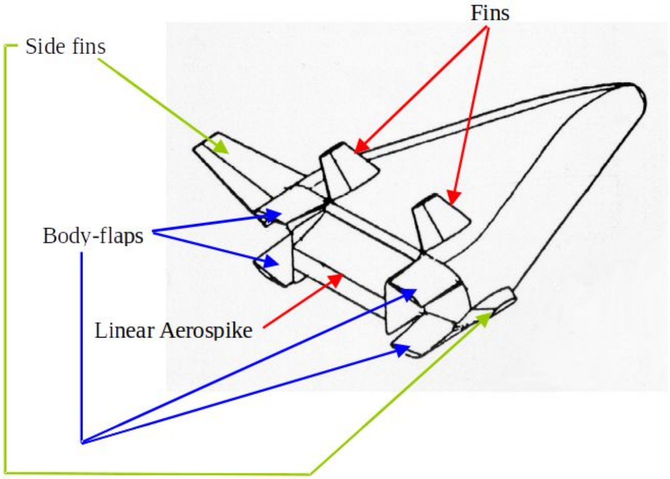



Figure 4: FESTIP-FSS5 concept vehicle [1]

[1] D'Angelo S., Minisci E., Di Bona D., Guerra L. (2000) Optimization Methodology for Ascent Trajectories of Lifting-Body Reusable Launchers. Journal of Spacecraft and Rockets. Vol. 37, No. 6.

## Mission Profile

Starting point → Kourou launch site:

- Longitude = 52.775 deg W
- Latitude = 5.2 deg N
- Initial flight path = 113 deg

Target orbit → circular orbit reached through Hohmann transfer:

- Height = 400 km
- Inclination = 51.6 deg

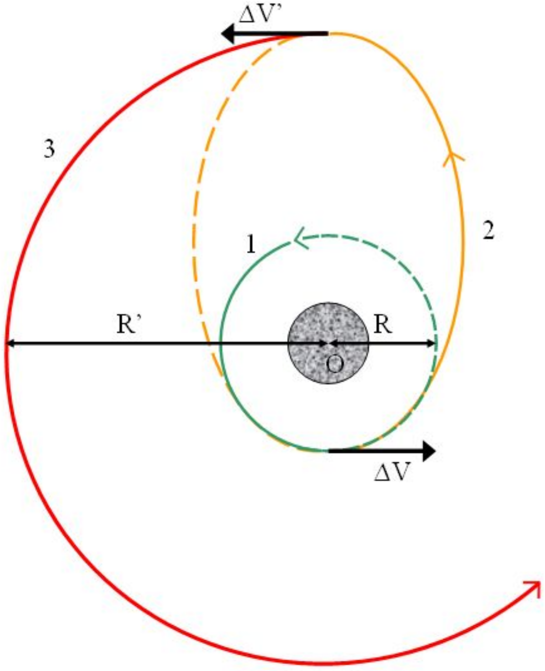



Figure 5: Hohmann transfer from one circular orbit to another [6]

[6] https://commons.wikimedia.org/wiki/File:Hohmann_transfer_orbit.png

## Continuous problem formulation

According to the motion equations previously discussed, the optimal control problem is defined by:

- 7 states:
  - v = velocity
  - χ = flight path angle
  - γ = path inclination angle
  - θ = longitude
  - λ = latitude
  - h = height
  - m = mass
- 5 controls:
  - α = angle of attack ( acts on the aerodynamic coefficients and pitch angle)
  - δ = linear throttle (acts on the Thrust)
  - $\delta_{flap}$ = body flap angle (acts on the aerodynamic coefficients)
  - τ = differential throttle (acts on the pitch angle and pitching moment)
  - μ = bank angle
- Objective function: minimization of fuel consumption

Paths bounds:

- $v > 0.0$ m/s
- 90 deg < χ < 270 deg
- -90.0 deg < γ < 90 deg
- -orbit incl. < λ < orbit incl.
- $h > 0.0$ m
- $m_{10}$ (10% of $m_0$)< m < $m_0$
- -2 deg < α < 40 deg
- 0.0 < δ < 1.0
- -20 deg < $\delta_{flap}$ < 30 deg
- -1.0 < τ < 1.0
- -90 deg < μ < 90 deg

Initial conditions:

- $v_0$ = 0.0 m/s
- $\chi_0$ = 113 deg
- $\gamma_0$ = 90 deg
- $\theta_0$ = -52.775 deg
- $\lambda_0$ = 5.2 deg
- $h_0$ = 0.0 m
- $m_0$ = 450400 kg
- $\alpha_0$ = 0.0 deg
- $\delta_0$ = 1.0
- $\delta_{flap0}$ = 0.0 deg
- $\tau_0$ = 0.0
- $\mu_0$ = 0.0 deg

Other constraints:

- $a_x \leq$ 30.0 m/s²
- $a_z \leq$ 15.0 m/s²
- $q \leq$ 40 kPa
- |Total Moment| $\leq$ 5 kNm

Final conditions:

- $v_f = v_{orbit}$
- $\chi_f = \chi_{orbit}$
- $\gamma_f$ = 0.0 deg

# Results

## Simpler problem comparison: Goddard rocket

$$\dot{R} = V_r$$
$$\dot{\theta} = \frac{V_t}{R}$$
$$\dot{V}_r = \frac{T_r}{m} - \frac{D_r}{m} - g + \frac{V_t^2}{R}$$
$$\dot{V}_t = \frac{T_t}{m} - \frac{D_t}{m} - g + \frac{V_t \cdot V_r}{R}$$
$$\dot{m} = -\frac{\sqrt{T_r^2 + T_t^2}}{g_0 \cdot I_{sp}}$$

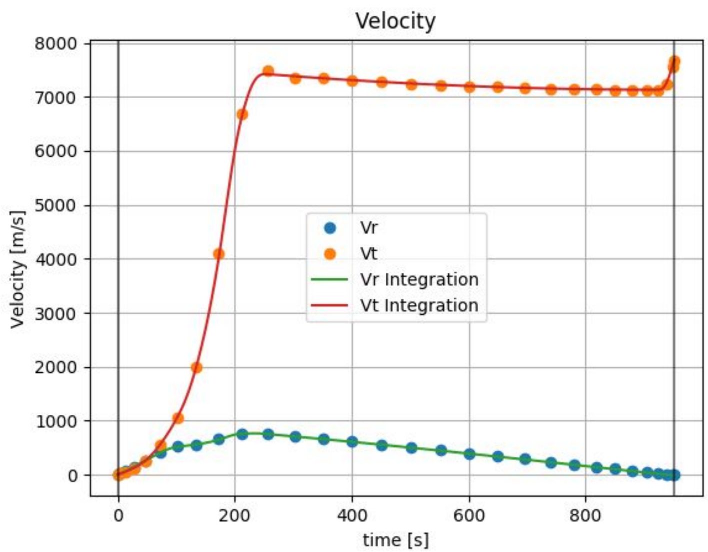
| | Collocation | Multiple Shooting continuous controls | Multiple Shooting |
|---|---|---|---|
| Objective Function | -0.04391696 | -0.04225785 | -0.04360999 |
| Final mass | Collocation: 4392 kg<br>Propagation value: **4366 kg** | 4226 kg | 4361 kg |
| Points/Leg and control points | 30 | 5 leg, 21 control points in total | 5 leg, 25 control points in total |
| Computational time (hh:mm:ss) | 0:01:50 | 3:06:43 | 8:22:32 |

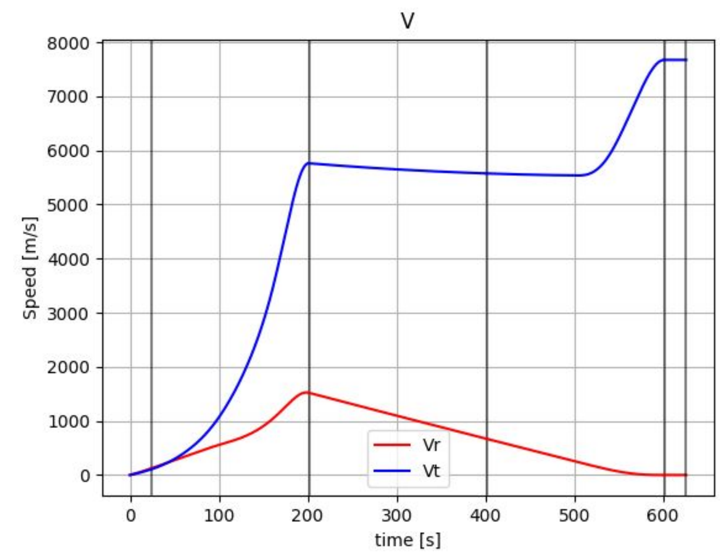
5 states:

- R
- θ
- $V_r$
- $V_t$
- m

2 Controls:

- $T_r$
- $T_t$

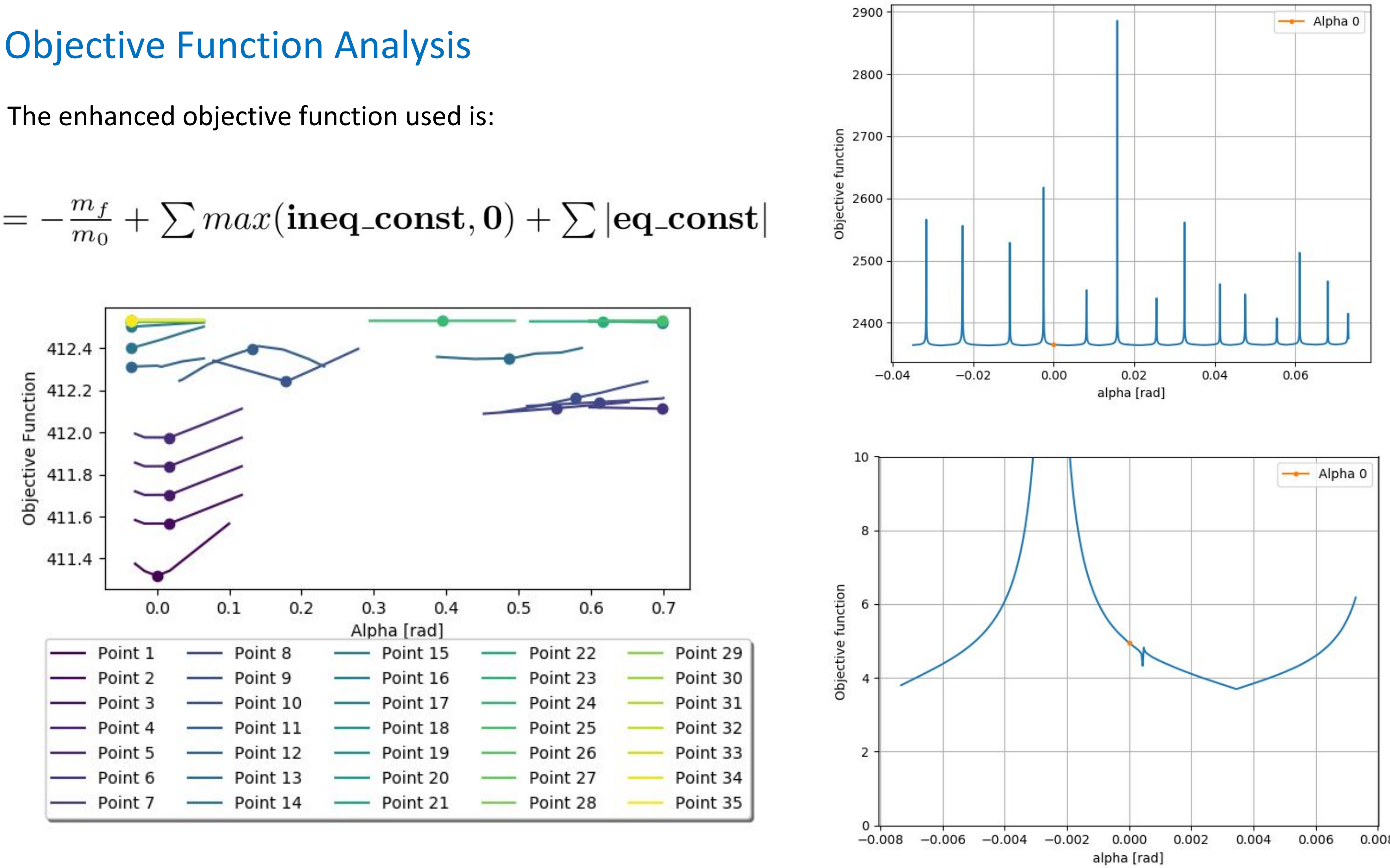


## Objective Function Analysis

The enhanced objective function used is:

$$J = -\frac{m_f}{m_0} + \sum max(\mathbf{ineq\_const}, 0) + \sum |\mathbf{eq\_const}|$$



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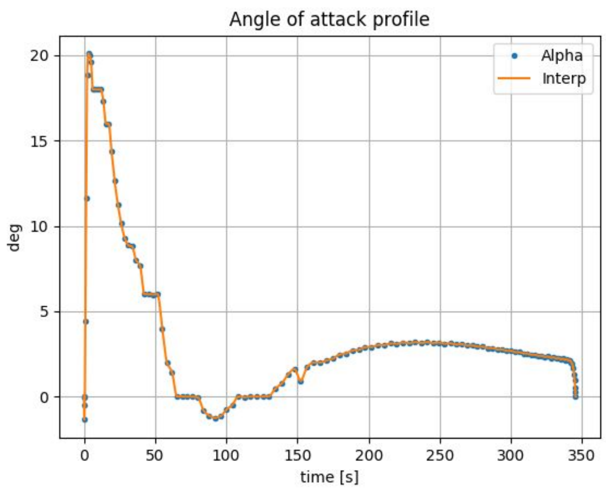
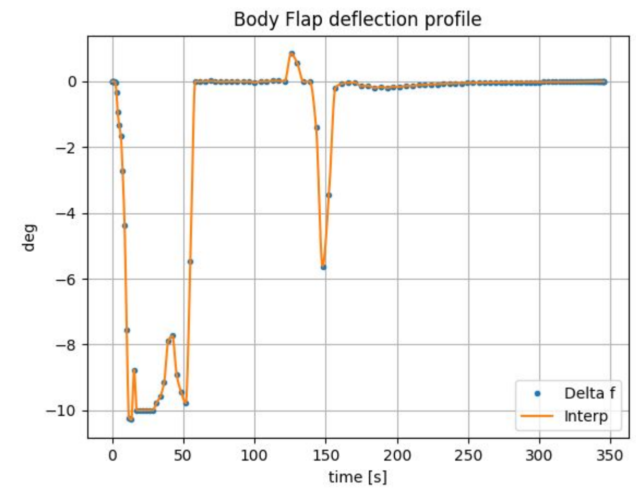
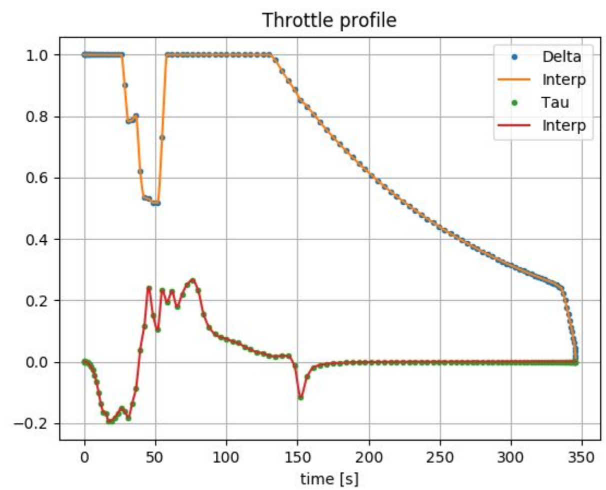
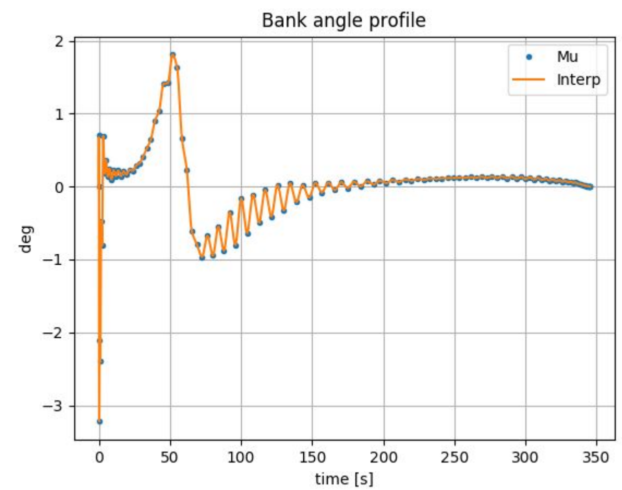
## Collocation: Controls

The collocation algorithm used to perform this simulation is part of the open source library OpenGoddard [5].




[5] OpenGoddard - trajectory optimization for python. https://github.com/istellartech/OpenGoddard. (2017)

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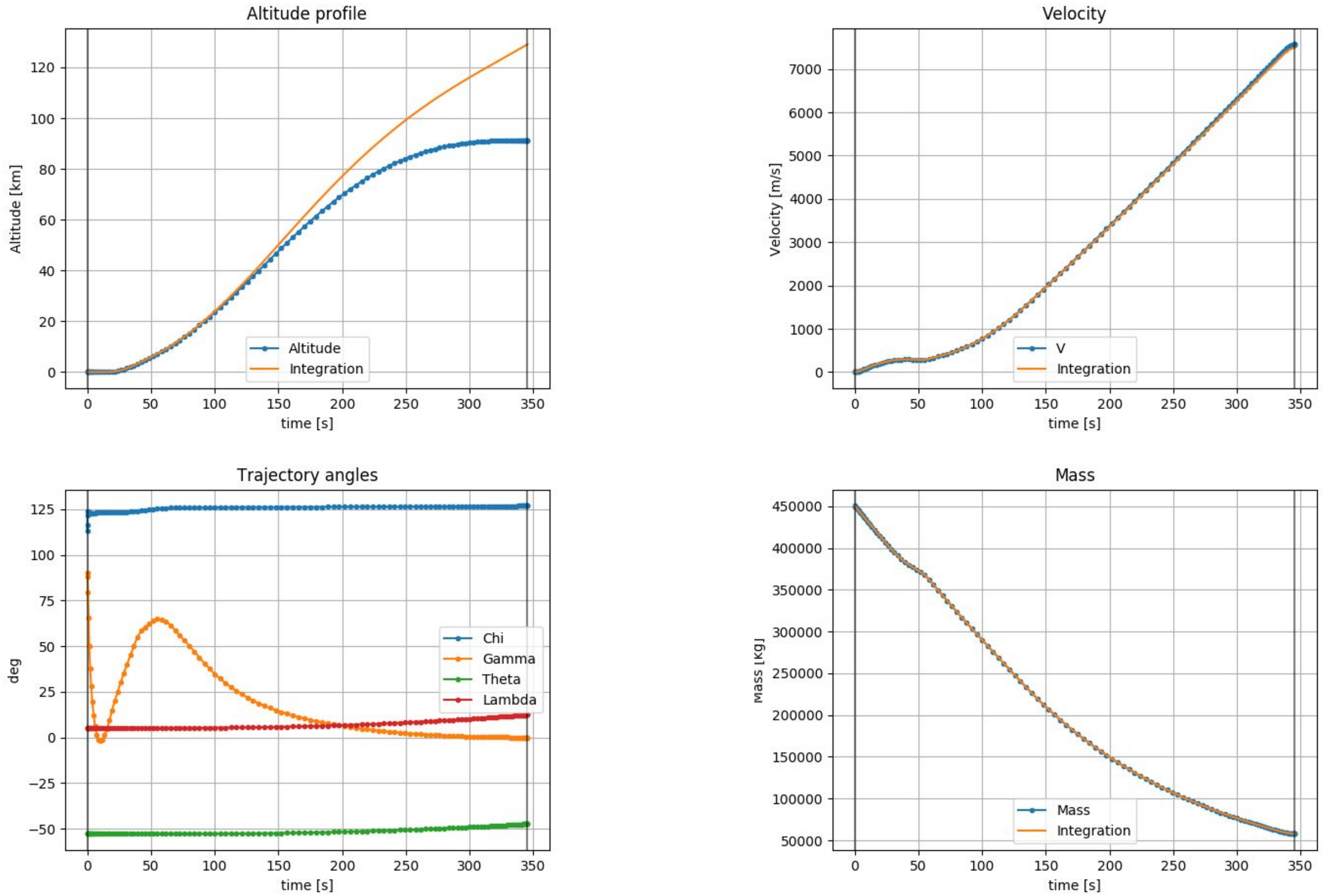
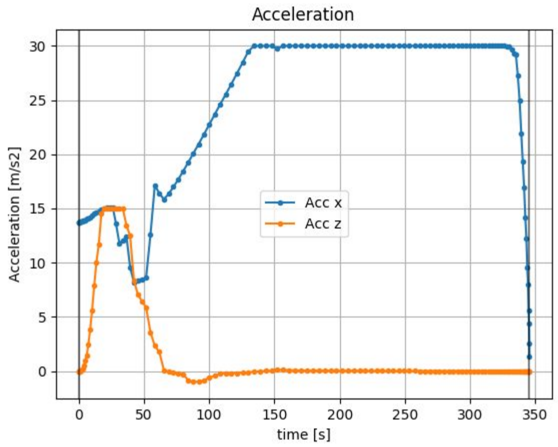
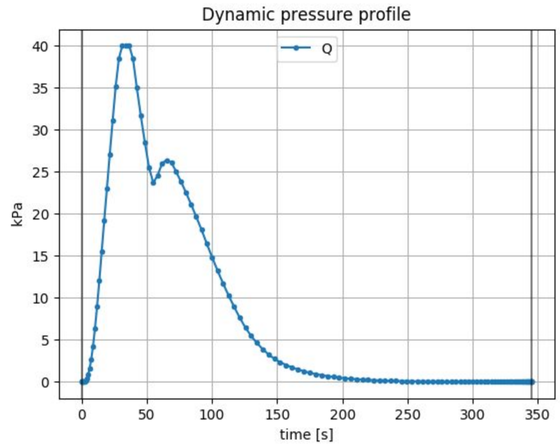
# Collocation: States

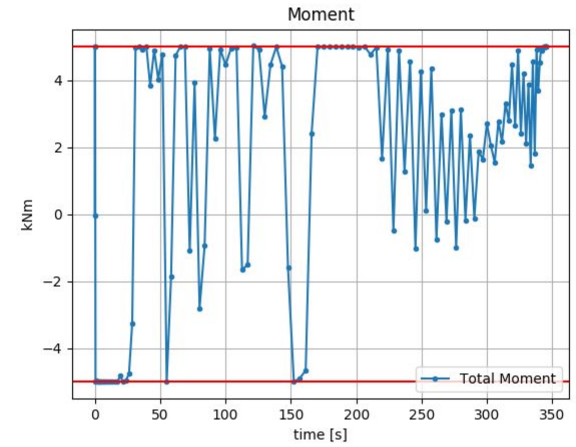


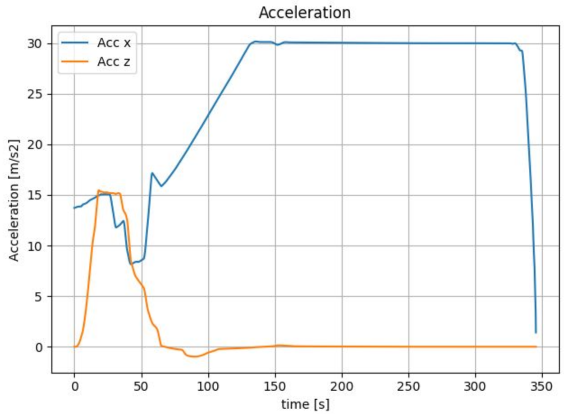
# Results

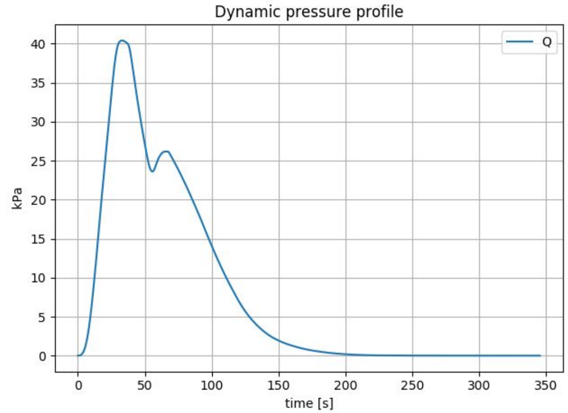
## Collocation: Constraints

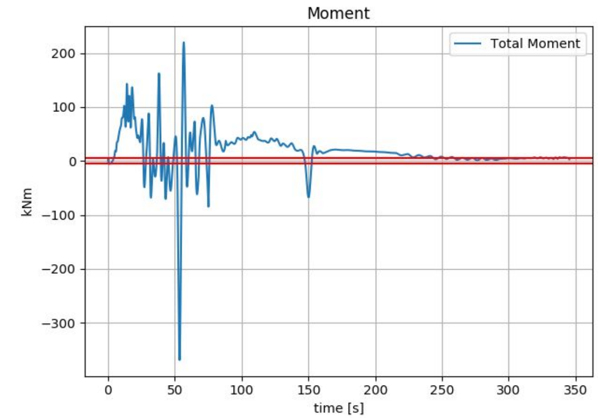


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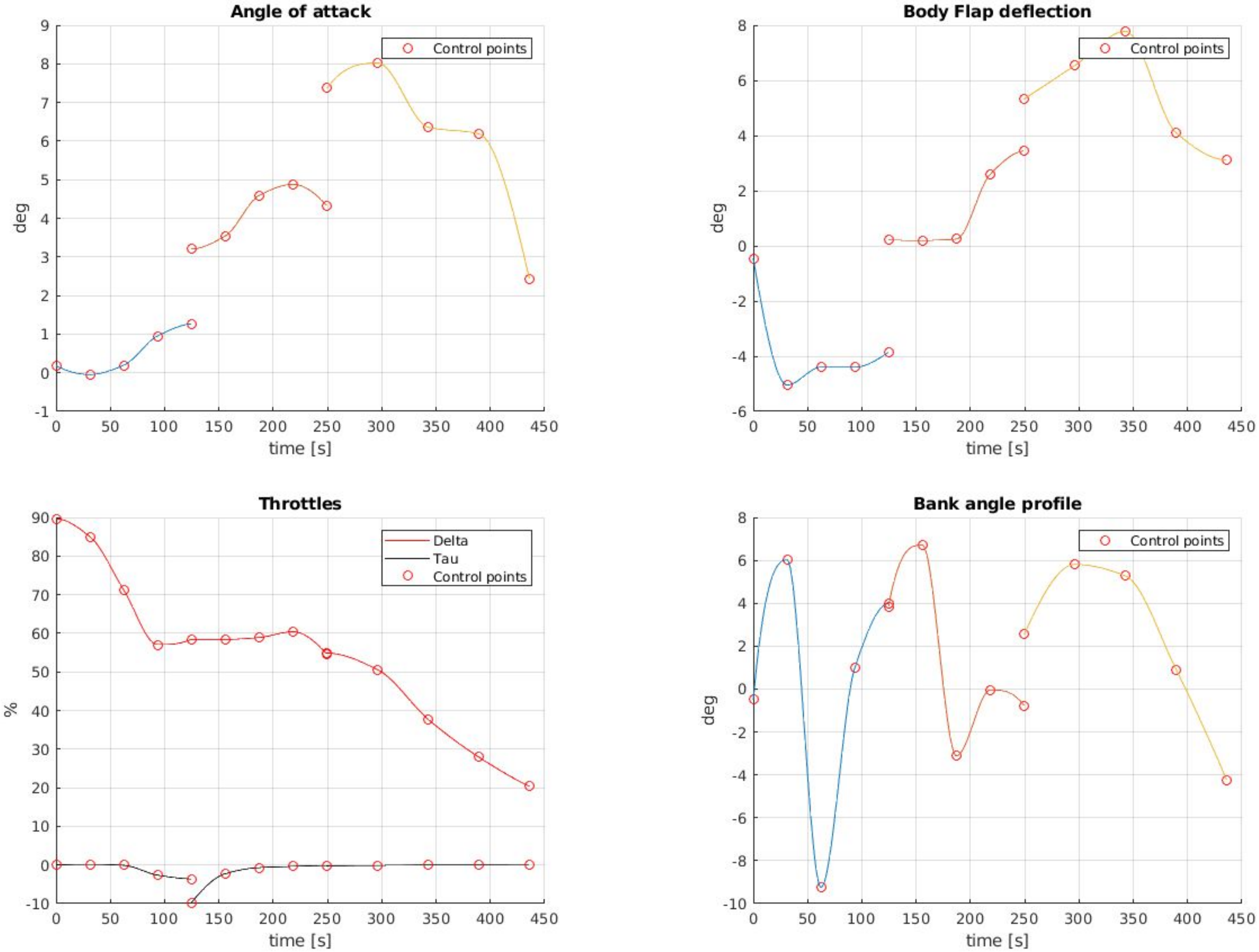
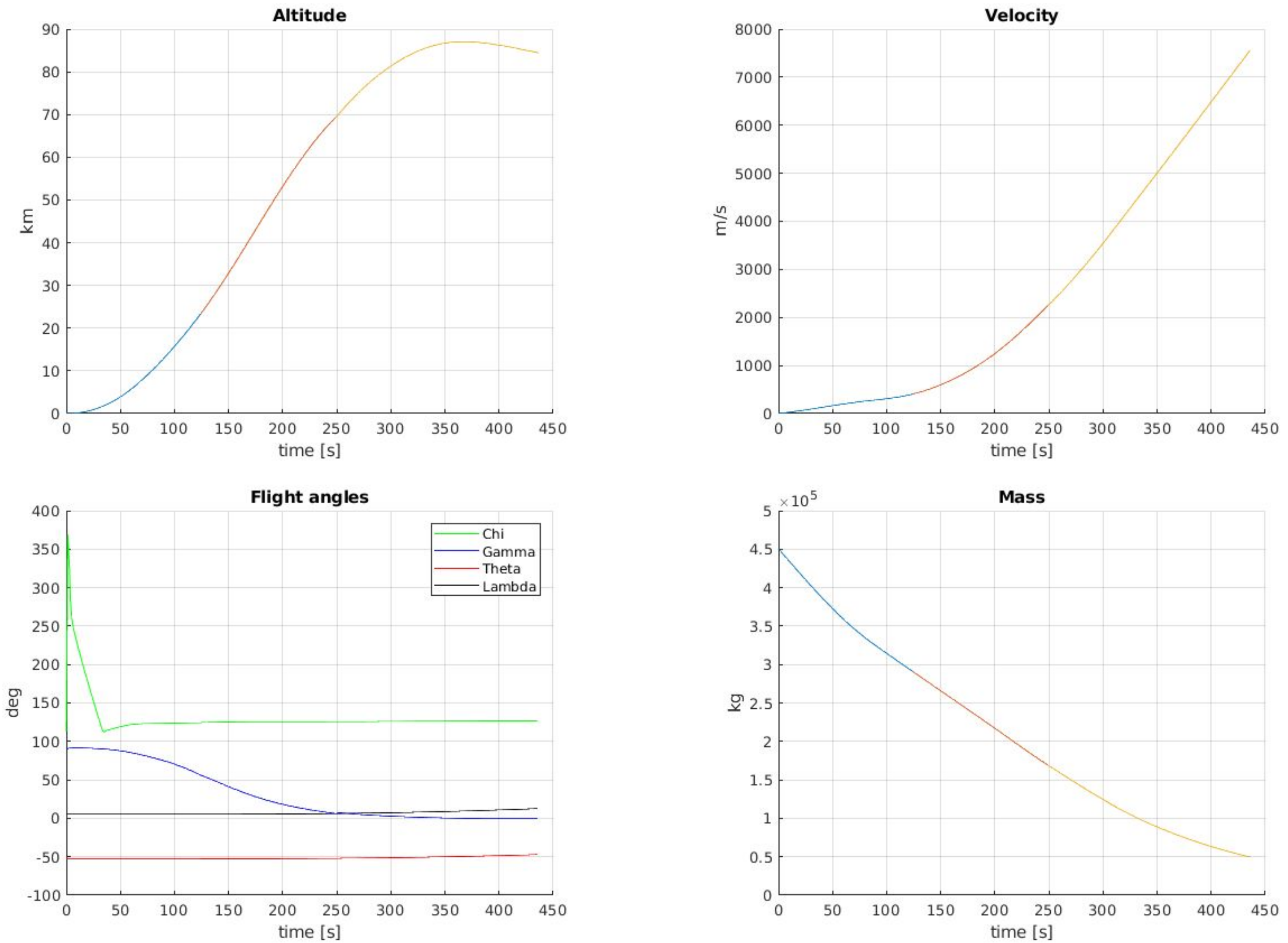
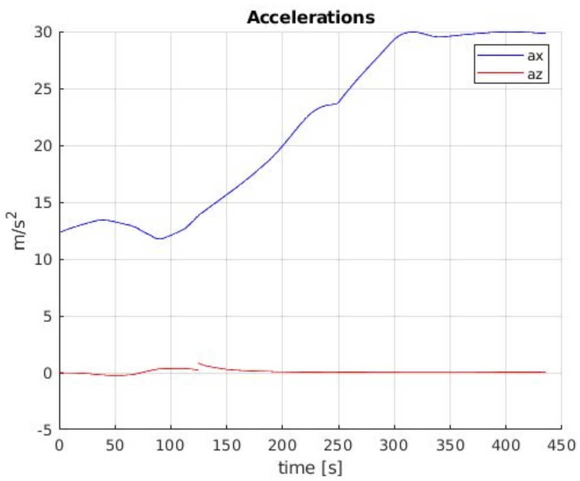
## Multiple Shooting: Controls

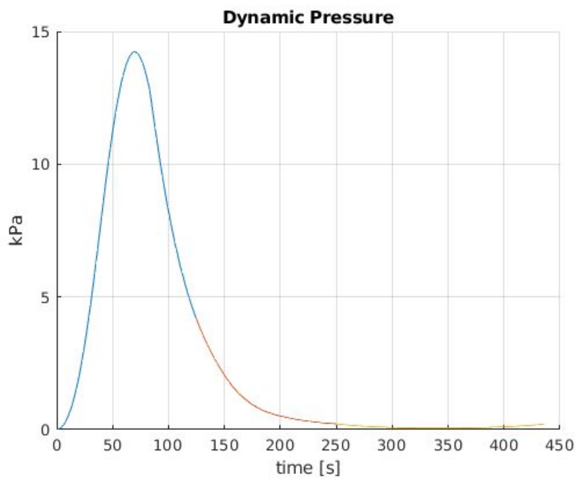


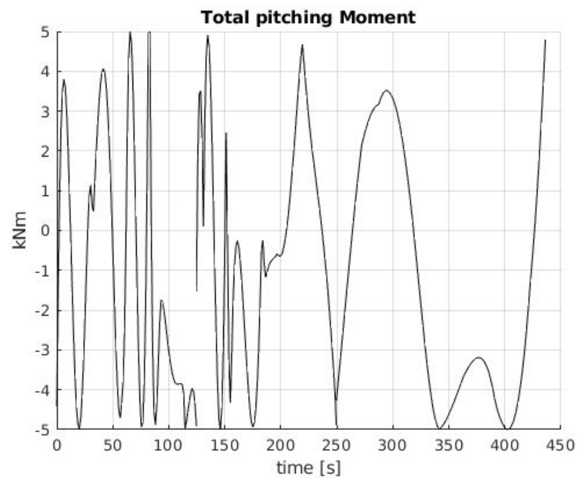
## Multiple Shooting: States



## Multiple Shooting: Constraints



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## Comparison

Both simulations were run on the same machine with 8 GB of RAM and an Intel®Core™ i7-6700 CPU @3.40 GHz x 8 processors.

| | Collocation | Multiple-Shooting |
|---|---|---|
| Objective Function Value | -0.1231 | -0.1101 |
| Number of collocation points / Number of legs and control points | 120 | 3 legs / 5 control points |
| Starting height of Hohmann transfer | 91.363 km | 84.476 km |
| Mass before Hohmann transfer | 57729 kg | 49604 kg |
| Final mass | 55434 kg | 49591 kg |
| Payload mass | 2.3 % = 10394.16 kg | 1.01% = 4549.04 kg |

# Conclusions

Multiple shooting method is strongly dependant on the definition of the initial guess, while collocation can be used also with a poor knowledge of that.

Nevertheless, the collocation approach must be guided to a physically meaningful solution by an appropriate definition of the bounds and constraints.

An hybrid approach using collocation to obtain a feasible first initial guess and then a multiple shooting to refine it could lead to better results.

# References

[1] D'Angelo S., Minisci E., Di Bona D., Guerra L. (2000) Optimization Methodology for Ascent Trajectories of Lifting-Body Reusable Launchers. Journal of Spacecraft and Rockets. Vol. 37, No. 6.

[2] Böhme T.J., Frank B. (2017) Direct Methods for Optimal Control. In: Hybrid Systems, Optimal Control and Hybrid Vehicles. Advances in Industrial Control. Springer, Cham.

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[6] https://commons.wikimedia.org/wiki/File:Hohmann_transfer_orbit.png

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