Using simulation in the assessment of voting procedures: An epistemic instrumental approach

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Abstract
In this paper, we argue that computer simulations can provide valuable insights into the performance of voting methods on different collective decision problems. This could improve institutional design, even when there is no general theoretical result to support the optimality of a voting method.

To support our claim, we first describe a decision problem that has not received much theoretical attention in the literature. We outline different voting methods to address that collective decision problem. Under certain criteria of assessment akin to extensions of the Condorcet Jury Theorem, we run simulations for the methods using MATLAB, in order to compare their performance under various conditions. We consider and respond to concerns about the use of simulations in the assessment of voting procedures for policymaking.

Keywords
Voting methods, equation-based simulation, instrumental qualities, social policymaking, epistemic democracy

1 Introduction

Not all collective decision problems are born equal. They vary according to the number of participants that must make the decision (ranging from a pair to a full nation); they also differ regarding what must be chosen (for instance, it may be an only option, a ranking, or even a grade over several options); there is also variation on the views that count to make a choice (only those of some or the views of all the participants) and on how much they count (for instance, some views may be weighted or all might be counted equally). Nowadays, there is no shortage of decision problems that involve the participation and acquiescence of very large groups, expecting to meet certain political ideals. Different procedures can be used to make a collective decision by means of a voting method. In order to fit the ideals of our democratic societies, voting methods should incorporate the views of every member of the group to yield collective outcomes, by means of a specific aggregation function. Aiming to achieve certain goals, some of these methods can provide clear advantages over others, at least under certain conditions. Those goals may include procedural qualities such as inclusiveness, equality, and fairness; but they may also include instrumental qualities, such as reliability or welfare.

In this paper we explore one crucial aspect in which the use of computer simulations can provide insights for social policymaking concerning voting methods. For some collective decision problems, under certain assessment criteria, there is an optimal method;[1] but it is not necessarily the same for all choice situations. Hence, it would be in our best interest to have the means to select the best collective decision-making procedure, if there is one, for each situation. Astounding general results are known for some kinds of decision problems;[2] theoretical results, however, are hard to come by. We argue that when no theoretical result is available computer simulations can provide useful guidance of the procedure's performance on different choice situations.

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According to the enterprise that Rohit Parikh labeled 'social software',[3] this could be a remarkable advantage for institutional design.

In order to support our claim, we first describe a general kind of choice situation that has not received much attention in the existing research literature. Next, we outline different procedures to aggregate individual opinions in order to produce a collective outcome. We then set criteria for the assessment of epistemic instrumental value, that is: ways to determine the ability of a voting method to track the correct outcome, assuming there is one. Using MATLAB, we run some insightful simulations to compare the performance of our voting methods under various conditions. We conclude by identifying some crucial concerns about the use of computer simulations in order to assess voting procedures for policymaking.

2 Collective decision problems

Problems concerning the assignment of economic or human resources, to be decided by a set of agents, are addressed in the literature on social choice.[4] Therein, assignment mechanisms are designed to maximize, in a certain sense, a function of social choice; these mechanisms are defined over the set of preferences among the decision-makers. Preferences are private information of the agents that they manifest by means of a ballot. The ballot then allows to make a social choice through an aggregation function. The agents' strategic behavior plays a fundamental role in problems concerning the assignment of goods. Social choice assignment mechanisms try to avoid incentives for individuals to falsify their true preferences. Thus, non-manipulable mechanisms are the most valued and looked for in committees of decision-makers.

In this paper, we focus on the process of candidate choice selection by a set of agents (voters). In describing some methods that are common in the literature, we heavily rely on the hypothesis that voters are responding to the ballot sincerely, according to their true preferences, and not voting strategically. Even if many of the procedures we chose exhibit the property of non-manipulability, we always perform simulations based on the true preference of voters. The study of manipulability of voting systems has been the target of other theoretical[5,6,7] and empirical[8,9] avenues of research, some of them using simulations.[10] Recent research has shown how, from a strategic approach, stochastic simulation can be used as a tool to learn about optimal behavior and Nash equilibria in a sequential voting model.[11] In this paper we assume instead that voters' preferences are not manipulable.

When the number of candidates is two, simple plurality rule does not present any inconsistency and is the most used method. However, when it is extended to more than two candidates several controversies arise. Arrow's impossibility theorem[12] highlights the problem and roughly states that virtually all voting schemes on three or more choices must be manipulable.[13] For instance, consider a case represented in Table 1, in which 55 voters must choose among five candidates: {*a, b, c, d, e*}.

	Preferences				
Number of voters	1	2	3	4	5
18	a	d	e	c	b
12	b	e	d	c	a
10	c	b	e	d	a
9	d	c	e	b	a
4	e	b	d	c	a
2	e	c	d	b	a

Table 1. A case in which 55 voters must choose among five candidates {*a, b, c, d, e*} according to their preferences.

Using plurality rule *a* is the winner since she was voted first 18 times, while *b*, *c*, *d*, and *e* were voted 12, 10, 9, and 6 times, respectively. However, *a* was voted last 37 times. Borda count, which was introduced in 1770 by Jean-Charles de Borda and will be described below, would declare *d* winner. The reader is invited to verify that claim.

Another social choice procedure in the literature is that of choosing the Condorcet winner. This procedure is currently used in parliament elections in countries such as Nauru and Slovakia. We can describe it briefly as follows: Each voter avowals her preference order and *x* is declared Condorcet winner if, comparing it to another *y* –in the sense of who appears first in the order–, *x* is better than *y*. In our previous example, *e* is the Condorcet winner, as can be inferred from Table 2.

Comparisons	Results
e vs a	37 - 18
e vs b	33 - 22
e vs c	36 - 19
e vs d	28 - 27

Table 2. Pairwise comparisons to choose the Condorcet winner on the case in Table 1.

However, there might not be a Condorcet winner. Consider the setting in Table 3, where 21 voters must choose among candidates {*a*, *b*, *c*}.

Number of voters	Preferences		
	1	2	3
8	a	b	c
7	b	c	a
6	c	a	b

Table 3. A case in which 21 voters must choose among candidates {a, b, c} and there is no Condorcet winner.

In this case *a* beats *b* by 14-7, *b* beats *c* by 15-6, *c* beats *a* by 13-8.

We commonly find ourselves in a situation in which there is no Condorcet winner. This is called a Condorcet Paradox.[2,4] In the simulations we describe in Section 4 it can be appreciated that as the number *n* of voters increases, the probability of there being no Condorcet winner tends to a particular value. Combinatorial and probabilistic methods have been previously used to address Condorcet paradoxes;[14] the frequencies of undesirable events in electoral processes, particularly preference cycles have also been studied from a computational perspective.[15]

Our aim is to examine the performance of the methods themselves to address a specific choice problem; to achieve this aim we will use the criteria outlined in Section 3. Setting aside procedural assessments of axiomatization, such as the property of fairness, we claim that computational simulations can provide empirical data on several questions concerning how likely is it that a method yields a successful outcome, that is, how likely is it that a method chooses the best candidate. There has been previous work on simulation to compare plurality, Borda, and Condorcet methods, using frequential data to compare their performance.[16,17] We contribute to this discussion by analyzing additional voting methods and providing a rationale for their general assessment, which is illustrated with a collective decision-making problem that has not received much theoretical attention.

2.1 A collective decision problem and notation

Consider the problem of assigning *k* positions to *k* candidates. Each position must be occupied by one and only one candidate. Besides, assume that the assignment is determined by means of a voting procedure with *n* voters. We are interested in displaying electoral mechanisms –counting and voting processes– that solve the assignment problem. For practical reasons, we will restrict our attention to $k = 3$, though we hope that the proposal we outline can be naturally extended in future research. Various selection mechanisms can be found in the literature on social choice; but when $k \geq 3$, Arrow's impossibility theorem states that no voting system can produce group decisions from individual preferences while also satisfying some additional criteria.[18(pp87-90)] Some paradoxes with $k = 3$ are discussed by Brams.[19]

We will consider two variants of the problem: 1) one with priority of positions for assignment, and 2) another without priority of positions for assignment. As an example of the former, we can picture the election of president, vice president, and secretary when one position is more important than the other(s) or must take office beforehand. As for the latter, we can think about the assignment of positions to players in a team sport, such as being in the far left, in the middle, in the far right, where it makes no sense to compare the relative importance of each position.

In the literature on operational research we can find the assignment problem as one in which there is a performance data table among candidates and positions, and only one person, the decision-maker, looks for a solution that maximizes performance.[20(pp435-445)] However, in this paper we analyze preferences (probability distributions) among the n voters that will make the decision. The approach we develop in Section 4 uses computational simulation to display conditions for the voters' preference distribution that warrant, for each method, that the probability of choosing the best assignment (on the assumption that there is one) approaches 1.

When investigating the frequency with which each method chooses the ideal assignment, we assume that assignment to be the one that is objectively correct or more accurate than the alternative options to be voted on. Even though the nature and extent of the collective decision problems in which there is such an assignment are controversial, there is broad agreement in acknowledging that some public problems are of this kind.[21(pp38-47)] In developing our approach to examine several voters' preference distributions, we assume that *we know* what the ideal assignment is –actually, we fix it– and then we compare the performance of the methods. However, the ability of the methods to choose the ideal assignment does not require that someone knows which one it is, but only that such an assignment exists and that the voters' preference distribution (even if their preferences are subjective) surpasses a certain competence threshold in identifying it.

We will first define the notation that we use to describe the voting methods:

- $T = \{A, B, C\}$, the set of "candidates".
- A "feasible option" will be any vector with one or three components, each component will be a member of T and each member of T will appear at most once. Thus, each voter will approve or reject a feasible option, with the following interpretation:
  - (α): Indicates that candidate α (with $\alpha \in T$) is being chosen for the first position.
  - (α, β, γ): Indicates that candidates α, β, γ (members of T) are being chosen for positions 1, 2, and 3, respectively.
- A "conditioned feasible option" will be any vector with three components, one of which will be a member of $\hat{T} = \{\hat{A}, \hat{B}, \hat{C}\}$, the two remaining components will be different members of T and each member of T will appear at most once. Thus, each voter will approve or reject a

conditioned feasible option, with the following interpretation:

- $(\hat{\alpha}, \beta, \gamma)$: Indicates that candidate α (with $\alpha \in T\backslash\{\beta,\gamma\}$) has been fixed in position 1, and β and γ are being chosen for positions 2 and 3, respectively.
- $(\beta, \hat{\alpha}, \gamma)$: Indicates that candidate α (with $\alpha \in T\backslash\{\beta,\gamma\}$) has been fixed in position 2, and β and γ are being chosen for positions 1 and 3, respectively.
- $(\beta, \gamma, \hat{\alpha})$: Indicates that candidate α (with $\alpha \in T\backslash\{\beta,\gamma\}$) has been fixed in position 3, and β and γ are being chosen for positions 1 and 2, respectively.

- For each feasible option and each conditioned feasible option (*"one or three components"*) we will denote the total numbers of approving voters with #(*"one or three components"*).
- The following conditions must be verified by feasible options and conditioned feasible options:
  - $\#(A,B,C) + \#(A,C,B) + \#(B,C,A) + \#(B,A,C) + \#(C,A,B) + \#(C,B,A) = n$; the total votes are distributed among the 6 possible ways of choosing candidates for the positions.
  - $\#(A) + \#(B) + \#(C) = n$; the total of votes for the first position must be equal to the total of voters.
  - $\#(\hat{A},B,C) + \#(\hat{A},C,B) = \#(B,\hat{A},C) + \#(C,\hat{A},B) = \#(B,C,\hat{A}) + \#(C,B,\hat{A}) = \#(\hat{B},A,C) + \#(\hat{B},C,A) = \#(A,\hat{B},C) + \#(C,\hat{B},A) = \#(A,C,\hat{B}) + \#(C,A,\hat{B}) = \#(\hat{C},A,B) + \#(\hat{C},B,A) = \#(A,\hat{C},B) + \#(B,\hat{C},A) = \#(A,B,\hat{C}) + \#(B,A,\hat{C}) = n$; once a candidate is fixed in a position, the votes are distributed in the only two possible options to occupy the remaining positions.
- A winning assignment will be represented in the form $[\alpha, \beta, \gamma]$ and it will indicate that α, β, γ were assigned to positions 1, 2, and 3, respectively.

We can now describe the selection mechanisms (voting methods) on which the computational simulations are based.

2.2 Voting methods for a problem involving priority on positions

(I) *Plurality rule on a single candidate with priority.* $[\alpha, \beta, \gamma]$ is the winning assignment if and only if $\#(\alpha) > \#(\beta) > \#(\gamma)$. In this case, the positions are being assigned based on which candidate has a greater number of votes.

(II) *Plurality rule with runoff with priority.* Let $\alpha \in T$ be such that $\#(\alpha) > \#(\beta)$ and $\#(\alpha) > \#(\gamma)$. Let β be such that $\#(\hat{\alpha}, \beta, \gamma) > \#(\hat{\alpha}, \gamma, \beta)$. Then $[\alpha, \beta, \gamma]$ is the winning assignment. In the first part of this process, the voter is asked to choose a candidate for the first position. The candidate with the greater number of votes takes the first position. In the second stage (runoff), the occupant of the second position is chosen among the two remaining candidates that were not winners on the first stage.

(III) *Plurality rule on a full ticket.* In this process, each voter is asked to choose one of the six possible full tickets (as it turns out, for this method it is not significant whether there is a priority or not on the positions): If $(\alpha,\beta,\gamma) \in S_3 = \{(A,B,C),(A,C,B),(B,A,C),(B,C,A),(C,A,B),(C,B,A)\}$ and $\#(\alpha,\beta,\gamma) > \#(x,y,z)$ for every $(x,y,z) \in S_3 \backslash \{(\alpha,\beta,\gamma)\}$, then $[\alpha,\beta,\gamma]$ is the winning assignment. This method picks up the most voted ticket.

(IV) *Borda count with priority.* The six possible full tickets are identified, and the voter is asked to choose among them. For each candidate, a Borda count $(\#Borda)$ is performed, as follows:

$$\#Borda(A) = 2\big(\#(A,B,C) + \#(A,C,B)\big) + \#(B,A,C) + \#(C,A,B)$$

$$\#Borda(B) = 2\big(\#(B,A,C) + \#(B,C,A)\big) + \#(A,B,C) + \#(C,B,A)$$

$$\#Borda(C) = 2\big(\#(C,A,B) + \#(C,B,A)\big) + \#(A,C,B) + \#(B,C,A)$$

If $\#Borda(\alpha) > \#Borda(\beta) > \#Borda(\gamma)$, then $[\alpha,\beta,\gamma]$ is the winning assignment.

2.3 Voting methods for a problem involving no priority on positions

(V) *Plurality rule with runoff without priority.* This procedure is identical to the one described as (II) in Section 2.2. However, in order to account for changes in preferences after the first stage of election, considering there was no priority, the preference distribution was changed to "preserve initial positions choice" instead of "preserve ordering".

(VI) *Plurality rule on a full ticket.* This procedure is identical to the one described as (III) in Section 2.2. No changes in the preference distribution were required.

(VII) *Count for conditioned feasible options.* Each $(\alpha,\beta,\gamma) \in S_3$ is assigned an integer $G(\alpha,\beta,\gamma)$ which is calculated as follows:

$$P_\alpha(\alpha,\beta,\gamma) = \#(\alpha,\beta,\gamma) + \#(\alpha,\gamma,\beta)$$

$$M_\beta(\alpha,\beta,\gamma) = \#(\alpha,\beta,\gamma) + \#(\gamma,\beta,\alpha)$$

$$T_\gamma(\alpha,\beta,\gamma) = \#(\alpha,\beta,\gamma) + \#(\beta,\alpha,\gamma)$$

$$G(\alpha,\beta,\gamma) = P_\alpha(\alpha,\beta,\gamma) + M_\beta(\alpha,\beta,\gamma) + T_\gamma(\alpha,\beta,\gamma)$$

$G(\alpha,\beta,\gamma)$ counts the total votes for the ticket with α in first position, β in the second, and γ in the third.

If $(\alpha,\beta,\gamma) \in S_3 = \{(A,B,C),(A,C,B),(B,A,C),(B,C,A),(C,A,B),(C,B,A)\}$ and $G(\alpha,\beta,\gamma) > G(x,y,z)$ for all $(x,y,z) \in S_3 \backslash \{(\alpha,\beta,\gamma)\}$, then $[\alpha,\beta,\gamma]$ is the winning assignment. This method chooses the ticket that gets more votes on the feasible options along with their respective feasible conditioned options.

Do any of these methods solve our assignment problem? Which one performs *better*?

3 Criteria for assessment of voting procedures

It is often underappreciated just how difficult it is to assess a voting method for a specific decision problem. One reason for this is that, in many groups, legislations are previously placed to determine which method to use. Some of these methods, such as plurality rule, are widely used. However, as can be seen from the previous section, there are several different procedures worthy of the name 'plurality rule' when facing a specific decision problem. Another reason for the suspicion that there are simple criteria to assess voting methods is the pervasive belief that there are just a few of such methods. In common parlance, it is standard practice to qualify a group decision as dictatorial, elitist, or democratic. However, these labels usually conceal the fact that many genuinely different voting methods are gathered under those categories. Besides, there are many other methods that it would be difficult to classify among these categories. Actually, the number of possible decision-making procedures is overwhelming.

To put things in perspective, we can extend a technique developed by Christian List[22] to distinguish aggregative decision-making procedures (voting methods). First, there is an "agenda" that contains the issues to be voted on. Then, there are "inputs" of the procedure: voters' attitudes towards issues on the agenda (for instance, their preferences, opinions, and so forth). Finally, there are "outputs" of the procedure: collective outcomes towards issues on the agenda. We can distinguish voting methods by identifying them with "aggregation functions". An aggregation function takes values on all possible input conditions and assigns to each one a specific output. Two voting methods differ if and only if there is a set of data such that they assign a different output to the same combination of values for (at least) one possible input. According to this technique of counting, "if there are x admissible combinations of individual inputs and y admissible collective outputs, there are y^x possible decision procedures". [22(p272)]

Consider the decision problem detailed in the previous section. The agenda consists in assigning only one of the three candidates $\{A, B, C\}$ to each of the three positions (1, 2, 3), leaving no vacancies. Hence, there are six possible options on the agenda: the running tickets (A, B, C), (A, C, B), (B, A, C), (B, C, A), (C, A, B), and (C, B, A). Let us assume, for starters, that there is only one voter. If she cannot abstain, then she has only six possible choices (votes). As a result of each vote, the group (from which she is the only member) can pick up any of the 6 tickets. If she chooses ticket (A, B, C), a procedure could yield her choice as a group decision; but there are also different procedures that can pick up any of the other five tickets instead, choosing (A, C, B), (B, A, C), (B, C, A), (C, A, B), or (C, B, A) when she votes for (A, B, C). The same would apply to any other vote introduced as an input. A complete specification of a procedure should indicate all possible outcomes for each possible input. Thus, for this (tiny) group there are 6^6 (*i.e.*, 46,656) voting methods. Only one of them always identifies the individual inputs (her vote) with the group outputs (the collective decision) – thus being, at the same time, dictatorial, elitist, and democratic–. Many other methods could qualify as rigged elections, vetoes on specific candidates, or more bizarre electoral systems. The number of voting methods increases exponentially with an increase in the number of voters. For three voters, there will be 216 possible voting inputs (combinations of all the six possible choices for each voter). And from each one of them, the group can pick up any of the six tickets. Thus, this group has 6^{216} voting methods. This vastly exceeds the humanly conceivable procedures, but it allows us to catch a glimpse of some of them. For example, many of these voting methods will be anti-dictatorial: that is, they will yield a collective outcome that is always different from the choice of one voter, who never gets her way.

This should make clear that the question "how many different collective decision procedures are there?" is not an easy one. Using List's technique for counting them, the answer depends both on the admissible combinations of individual inputs and on the possible outcomes and, hence, on the kind of decision problem at hand. Even if this technique for counting methods seems too farfetched, we can expect the quantity of possible voting methods to be undoubtedly immense, even for very small groups. If we want to make a sensible choice of method to address a specific decision problem, we must narrow down the space of possibilities within this gigantic class. In fact, too many of the potential methods would be non-starters for most of us. For instance, what would be the point of having an institutionally rigged voting system that always selects the first (or -nth) option? Would it even make sense to go out on election day? Thus, some procedural constraints are often listed to limit the voting methods worthy of attention.[2] The most interesting ones seem to concern ways in which the outcome may reflect the overall group opinion. Additionally, our democratic ideals seem to require the use of procedures that comply with certain necessary qualities to ensure that all voters' inputs are taken fairly into account, at least to some extent. Along these lines, a "procedural assessment" of voting methods can be offered.

But there is another way to frame the problem of selecting a voting method for a specific decision problem. We can ask: which methods are most likely to produce successful outcomes?[2,4] More specifically, we can explore which methods are prone to produce outcomes that are correct, under several conditions. If those conditions are usually obtained, then the methods that produce correct outcomes –that "track the truth"– offer another kind of advantage. By basing our choice of methods on the answer to these questions, we are performing an "epistemic instrumental assessment" on them. This is the assessment of voting methods for which we wish to identify criteria. Our approach is thus premised on the idea that there is indeed a "correct" ticket for the assignment problem of Section 2: there is an ideal ordering for the group to choose. Following one of the major treads of epistemic democracy, we assume that "the existence of such an ordering provides a productive way to think about group choice problems".[1(p60)] Epistemic democracy assumes that people are reliable, that is: they are mostly right, most of the time. This assumption has been explored and supported on empirical grounds.[23] In assessing voting procedures, we frame the issue as a problem concerning how to best capitalize on people's reliability. Here, we draw on an intuition from extensions of the Condorcet Jury Theorem.[21] In that sense, this paper aligns to the growing literature that explores the potential payoffs of democratic decision-making procedures.[24,25,26,27,28] Actually, our approach via simulations was initially inspired by the "playful" and "exploratory" spirit of Kai Spiekermann's and Robert Goodin's recent book, which aims to "'see what happens' when we vary the many interrelated conditions that might affect the overall epistemic performance of modern democratic government";[21(pv)] although their main focus is on a different kind of decision problem.[21(pp33-36)]

We can now state our quest for assessment criteria more clearly. Which traits of a voting method exhibited in a simulation would provide (non-conclusive) evidence for the claim that one method "performs well" for a decision problem? Which features would support the claim that a voting method "performs better" than another for a decision problem? Having a sound answer to these questions would provide useful tools for institutional design. Especially, it would allow us to run some tests on voting mechanisms over kinds of decision problems prior to their implementation. Our proposal consists in assessing voting methods on a decision problem focusing on the following criteria:

(1) *Triviality validation*. If all voters prefer the ideal outcome, then the voting method always yields the ideal outcome as a result. This ensures that the method can perform at its best under ideal conditions on the preference distribution: it will never yield a suboptimal outcome when provided with ideal inputs.

(2) *Convergence to optimum*. The probability of the group choosing the ideal outcome converges to 1 as the size of the population increases. This is one of the promising ideas sparking from the Condorcet Jury Theorem. Some voting methods can produce better outcomes than the average voter, provided a competence threshold is surpassed; and their accuracy increases as the size of the voting population grows.

(3) *Fast-rate convergence*. The method's ability to arrive at the ideal outcome has a fast rate convergence to 1 (it does not require a very large population). This criterion provides a further refinement on Convergence to optimum. It offers a comparative basis to acknowledge when a voting method is better at capitalizing the initial reliability of the preference distribution than others.

(4) *Initial low competence*. The initial competence of the voting population to ensure that their collective output matches the ideal outcome converging to 1 is less demanding than that of alternative voting methods. This criterion is also a refinement on Convergence to optimum. It allows to compare the initial threshold of competence required for the group to improve on the average voter.

(5) *Constant behavior*. The probabilistic results of applying the same method to the same initial values are consistent on many trials. This is to ensure that the qualities praised by the aforementioned criteria are not merely random results.

How do the voting methods we described in Section 2 perform according to these criteria?

4 Comparing voting methods through simulations

In order to illustrate these criteria of assessment, we ran simulations in MATLAB of the voting methods for the decision problem described in Section 2. Here is an outline of how our simulations worked.

4.1 General settings and procedure

Different sizes for a population can be specified. For each simulation, a population ranging from 1 to n individuals is selected. A random number corresponding to one of the six possible ticket choices is assigned to every individual, according to the intended proportions. Assuming voter competence, the ideal ticket is always assigned as a default to a greater number of individuals. If the outcome of the voting procedure matches the ideal ticket, the simulation is deemed to be successful. In order to assess the procedures according to their instrumental epistemic value, we assume that there is an ideal ticket. Without loss of generality, we always assume that (A, B, C) is the ideal assignment –but only a percentage of the population knows it; alternatively, the general population is reliable tracking that ideal assignment up to a certain degree. Then, we run computer simulations of the voting methods. Each voting procedure is repeated in k

iterations for each population of n individuals, preserving the intended proportions. In our experiment, we always used 100 iterations of each procedure. The probability of choosing the ideal ticket for each population of n individuals is determined as the ratio of successful outcomes divided by the total number of iterations. A selection performance curve is then drawn with the ordered pairs (n, P_w), where n stands for the total population and P_w is the probability of choosing the ideal ticket.

The programs start by selecting a population size and assigning an ideal probability distribution by means of preference intervals on each candidate. Aiming to assess (2) *Convergence to optimum* and (3) *Fast-rate convergence*, different population sizes were provided as input. Variation in preference intervals allows to probe conditions for the assessment of (4) *Initial low competence*. In order to simulate an individual, a random number between 0 and 1 is created by means of the "rand" function in MATLAB (which follows a uniform distribution). At this point, different codes are used to specify each voting method. The next section provides an outline of each program specification. Once the voting procedure delivers a ticket with a complete preference order, it is determined whether the preference order corresponds to the ideal ticket. The simulation is then repeated several times, in order to calculate the probability of selecting the ideal ticket.

4.2 Specific procedure for voting methods

(a)Voting methods for a problem involving priority on positions

(I) *Plurality rule on a single candidate with priority.* A preference for a ticket is assigned to every individual, thus determining on which interval of the preference distribution lies its value. These preferences are ordered from greater to smaller on a candidate by candidate comparison according to the number of individuals assigned to each preference.

(II) *Plurality rule with runoff.* A preference for a ticket is assigned to every individual, thus determining on which interval of the preference distribution lies its value. The candidate with the greater number of individual votes is determined and assigned to the first position. After the first position is assigned, the second position is determined by identifying the candidate every individual ranked highest.

(III) *Plurality rule on a full ticket.* A preference for a ticket is assigned to every individual, thus determining on which interval of the preference distribution lies its value. These preferences are ordered from greater to smaller on a full ticket comparison according to the number of individuals assigned to each preference.

(IV) *Borda count with priority.* A preference for the first position of the ticket is assigned to every individual, thus determining on which interval of the preference distribution lies its value. A preference for the second position of the ticket is assigned to every individual, thus determining on which interval of the preference distribution lies its value (in order to avoid making the same choice as in the previous step, a proportion is followed). A Borda count is performed according to the positions chosen by every individual.

(b)Voting methods for a problem involving no priority on positions

(V) *Plurality rule with runoff without priority.* A preference for a ticket is assigned to every individual, thus determining on which interval of the preference distribution lies its value. The candidate with the higher number of votes is assigned to a (non-ranked) first position. After the first position is assigned, the second position is assigned in a way that best preserves the desired positions of the originally preferred ticket for each voter.

(VI) *Plurality rule on a full ticket.* This procedure is the same as the one involving priority.

(VII) *Count for conditioned feasible options.* A preference for a ticket is assigned to every individual, thus determining on which interval of the preference distribution lies its value. A count of all the conditioned feasible options is performed according to the formula specified for this procedure, thus yielding a total count for each ticket.

For significant comparisons, we assign the same initial preference values to all methods on each simulation scenario. For expository purposes, we omit the graphs showing that these methods comply with Triviality validation (which assigns probability 1 to the ideal ticket) and Constant behavior (which consists of performing multiple simulations with the same values and comparing their graphs). The reader can perform experiments of her own using our codes.

4.3 Assessment of methods using simulation's results

Roughly, the results of comparing our voting methods under the criteria of assessment outlined in Section 3 are as follows. For this choice problem, in general, we found that plurality rule on a full ticket performs as well as the other voting methods and, specifically, it sometimes performs significantly better. It has a faster convergence to optimum rate (see Figures 2 and 10), that holds under different conditions that spoil the epistemic efficacy of other voting methods (see Figures 6, 7 and 8, as well as Figures 14, and 15). To detail our assessment, we begin with the decision problem involving positions with priority. If the proportion of voters preferring the ideal ticket (or voter competence) has only a slight advantage over the preference for other tickets, then there seems to be no significant difference in the performance of our four methods. The selection performance curve does not show a clear convergence to 1 for any method, even in a considerably large population (Figure 1).

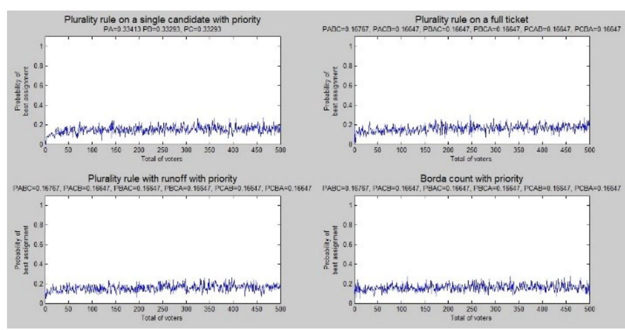

Figure 1. All possible tickets have a similar initial preference among the members of the population or, alternatively, the population is nearly as competent as a random device (*e.g.*, a fair dice) to choose the ideal ticket.

So, it seems that a homogeneous distribution of 1/6 probability and near values is below the threshold of optimal performance. However, if we increase the advantage of the ideal ticket over the alternatives, then the selection performance curve starts to show a significant increase even in small populations (Figures 2, 3 and 4).

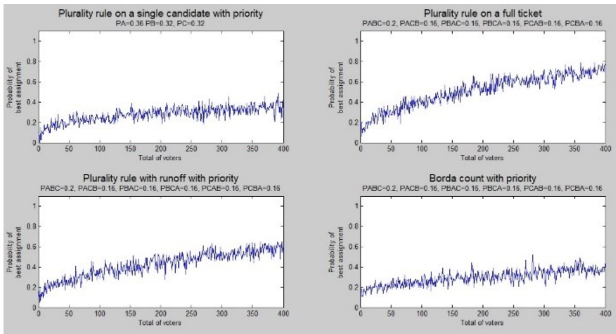

Figure 2. The population has some preference for the ideal ticket over any other ticket. This also reflects some preference for a single candidate (A) to take the first position, while both B and C are tied.

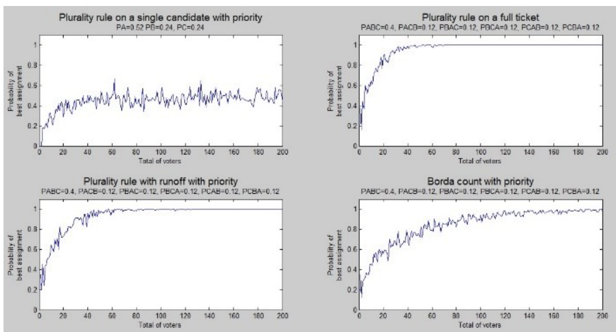

Figure 3. The population has a clearly higher preference for the ideal ticket over any other ticket. This also reflects a clear preference for a single candidate (A) to take the first position, while both B and C are tied.

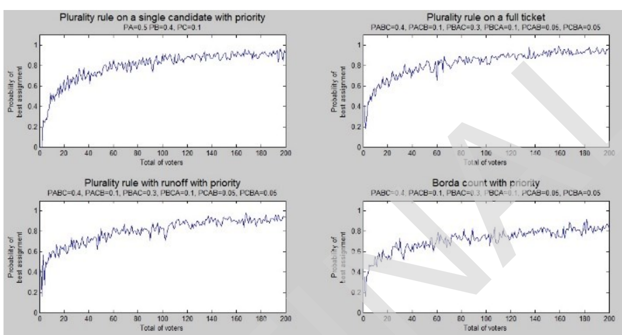

Figure 4. The preference ranking for each candidate mirrors the order of priority on positions of the ideal ticket. The ideal ticket is also preferred to any other ticket.

In these cases, plurality rule with runoff, plurality rule on a full ticket and Borda count always exhibit convergence to 1. In contrast, plurality rule on a single candidate stagnates when the preference for the ideal ticket does not favor a candidate for the second position (Figures 2 and 3). Putting those cases aside, the rate of convergence to 1 seems to visibly increase in proportion to the initial competence of the voters. When the ideal ticket has a relatively high probability (0.3) of being chosen but another ticket is almost equally high (0.29), all methods seem to stagnate or exhibit a very low converge to 1 rate, even in large populations (Figure 5).

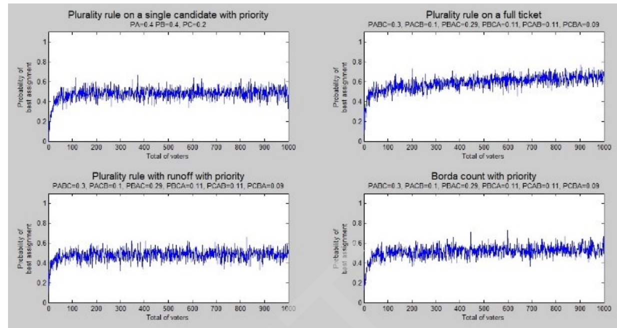

Figure 5. Although there is a slightly higher preference for the ideal ticket than for any other ticket on the population (ticket BCA is a close second), this reflects no preference for a single candidate to take the first position: the population is equally divided between A and B.

If there is a tie in preference between the ideal candidate for the first position (highest rank) and the ideal candidate for the last position (lowest rank), even if there is a preference for the ideal ticket, all voting methods behave poorly with the exception of plurality rule on a full ticket (Figure 6). As a matter of fact, when any candidate for the first position is preferred over the ideal one, the only voting method that shows a selection performance curve converging to 1 is plurality rule on a full ticket, even if there is a clear preference for the ideal ticket (Figures 6, 7 and 8).

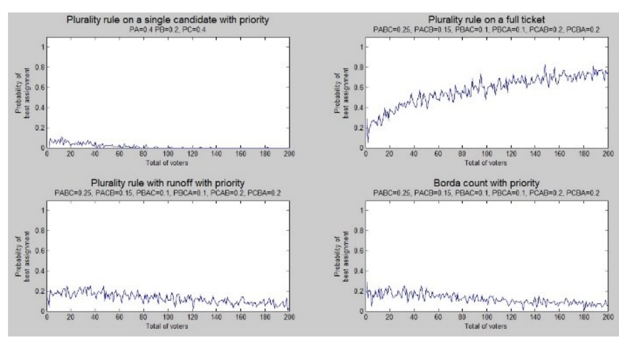

Figure 6. There is a higher preference for the ideal ticket than for any other ticket on the population (tickets CAB and CBA are tied in a not-too-close second). However, the preference for a single candidate to take the first position is higher for C than for A.

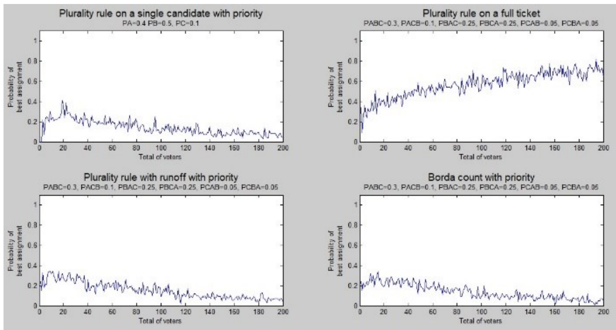

Figure 7. There is a higher preference for the ideal ticket than for any other ticket on the population (tickets BAC and BCA are tied in a not-too-close second). However, the preference for a single candidate to take the first position is higher for B than for A.

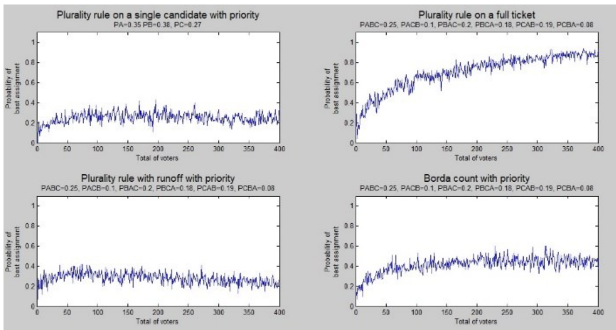

Figure 8. There is a higher preference for the ideal ticket than for any other ticket on the population (tickets BAC and CAB are not-too-close second and third). However, the preference for a single candidate to take the first position is higher for B than for A.

Thus, as it turns out, voting on a single candidate is not a very reliable procedure for choosing a ticket when there is priority on the positions to assign. Actually, it almost always yields worse results than the other three methods we examined. (This should give us pause to think about record charts based on the popularity of a single band or an artist). Plurality rule on a full ticket seems to score higher on all our criteria.

For the decision problem involving positions without priority we compared two of the previous voting methods (Plurality rule with runoff and Plurality rule on a full ticket) to a new procedure (Count for conditioned feasible options). Even if it does not stand out as clearly as in the previous decision problem, Plurality rule on a full ticket performed better than other methods (Figures 10, 14, and 15). As in our previous problem, there seems to be no significant difference in the performance of these methods when there is a homogeneous distribution of preferences among all tickets: there is no convergence to 1 for any method, under these conditions (Figure 9).

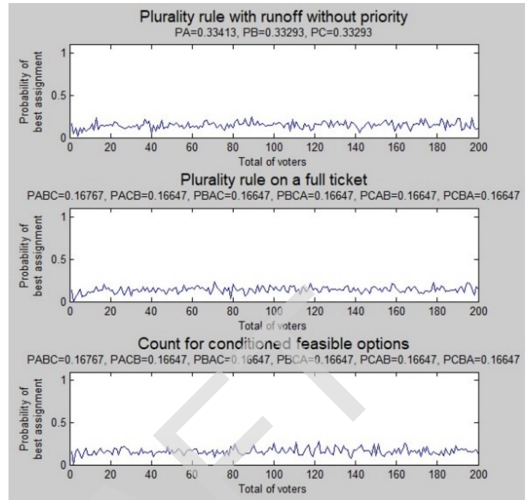

Figure 9. As in Figure 1, all possible tickets have a similar initial preference among the members of the population or, alternatively, the population is nearly as competent as a random device (*e.g.*, a fair dice) to choose the ideal ticket.

However, if we increase the advantage of the ideal ticket over all the other alternatives, then the selection performance curve of all methods starts to show a significant increase even in small populations (Figures 10 and 11).

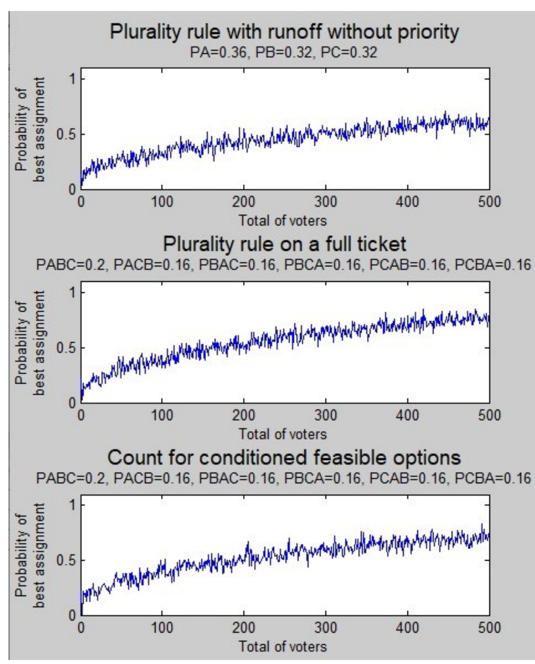

Figure 10. When the preference for the ideal ticket over any other ticket is barely significant, the reliability of the outcomes as the population grows starts to be noticeable.

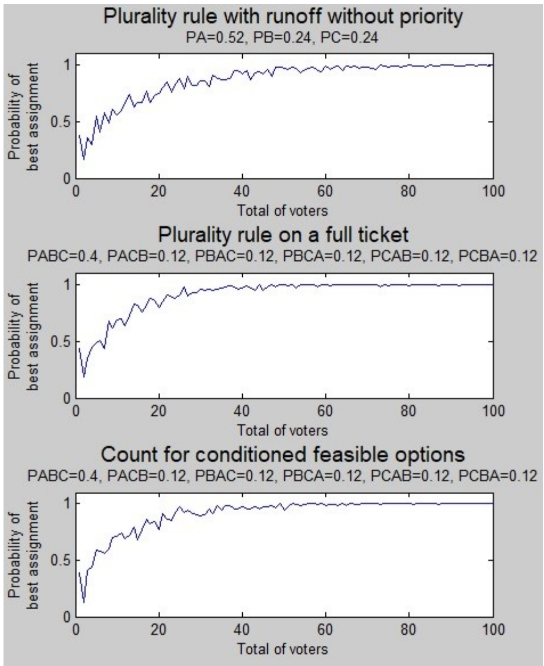

Figure 11. With a higher preference for the ideal ticket over any other ticket, all voting methods display Convergence to optimum even in a very small population.

There are differences in their performance when the ideal ticket, even having a higher probability, has one, two, or three main competitors (Figures 12, 13, 14 and 15). With only one other competing ticket on second place with a similar preference, all methods show a decrease in their convergence rate; and some seem to stagnate on nearly ½ (Plurality rule with runoff) or even underperform (Count on conditioned feasible options). Surprisingly, with more than one competitor, Plurality rule on a full ticket seems to thrive. On those conditions, the remaining methods seem to stagnate or even show a Convergence to 0 on their accuracy as the population grows.

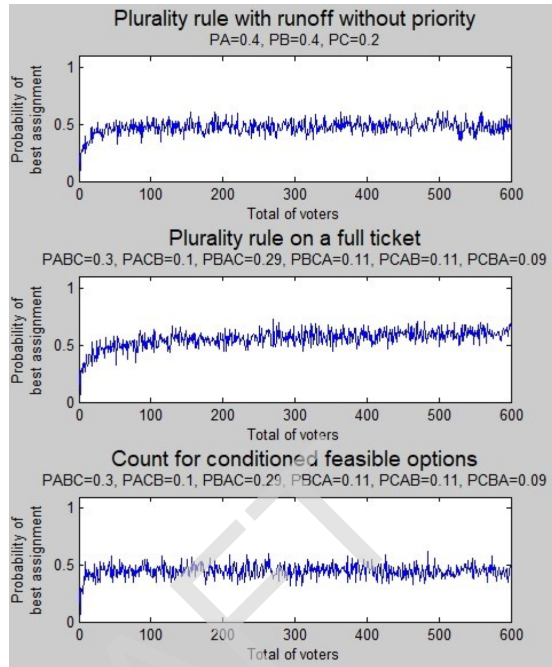

Figure 12. If the ideal ticket has a high probability, but there is another ticket with a preference nearly as high, the methods seem to stagnate.

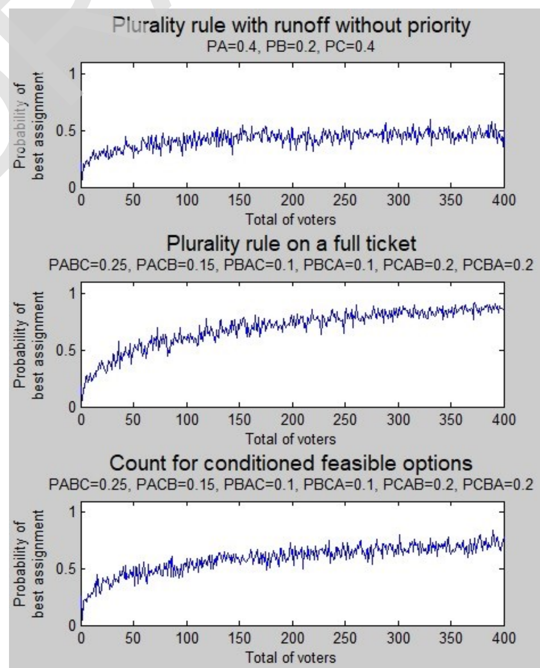

Figure 13. If the ideal ticket has a not-too-high probability, but there are two tickets tied in a not-too-close second place preference, the first method seems to stagnate, while the convergence rate of the other two is still significant.

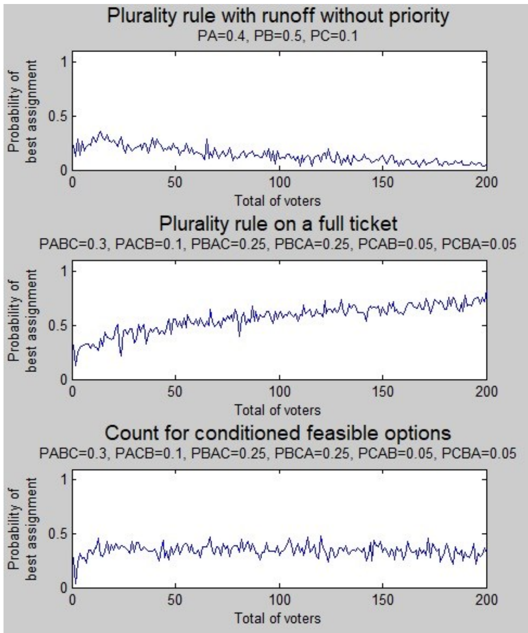

Figure 14. If the ideal ticket has a high probability, but there are two tickets tied in a not-too-close second place preference significantly above all other tickets, the first method starts converging to 0, while the convergence rate of the second is still significant and the third seems to stagnate.

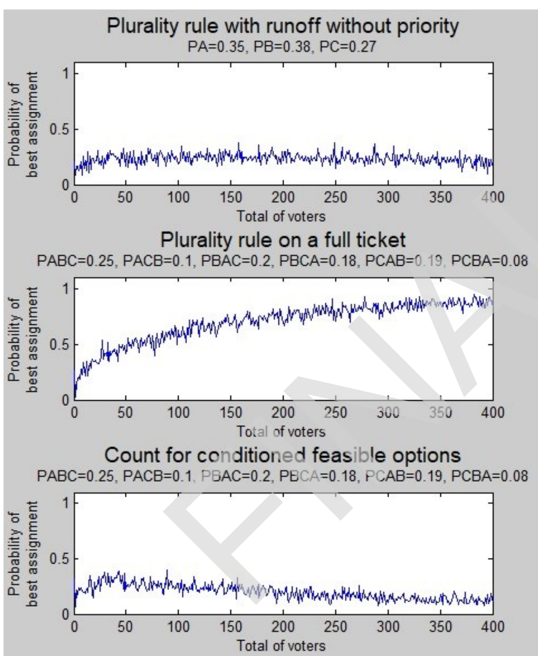

Figure 15. If the ideal ticket has a not-too-high probability, but there are three competing tickets almost tied, the first method seems to stagnate, the third shows convergence to 0, and the second exhibits Convergence to optimum as well as a significant Fast convergence rate.

Thus, when there is no priority on the assigned position, Plurality rule on a full ticket seems to be a good method when a choice must be made on all the tickets; choosing the candidates for each position on different stages according to Plurality rule with runoff seems to be a bad idea. As for the method of Count for conditioned feasible options there is a divided verdict. Its performance seems to slightly supersede that of Plurality rule with runoff in certain conditions (Figures 10, 13, and 14), while it underperforms in others (Figure 15).

5 Concerns over societal applications

If our discussion so far is on the right track, the use of simulations could improve the design of new institutions or reform of the existing ones. In order to appreciate this, it is enlightening to recall that there were heated disputes over the voting methods' mathematical details in the origins of modern democracies, at the end of the 18th century. Competing ideas were championed by thinkers as Jean-Charles Borda and Nicolas de Condorcet, sparking off the emergence of the field of social choice theory.[4] Many of the problems that those thinkers wanted to address required savvy and technical knowledge, that only a few people had at the time; for other problems, there is still no agreed solution, but their approaches have inspired educated guesses (sometimes conflicting). However, many of their impressive accomplishments are now part of our electoral systems. In a sense, the use of simulations could extend the reach of this project, making it both more available to non-expert users and more fruitful in its (tentative) results. It can also help to revise or even rule out choice procedures that we intuitively would be predisposed to accept. It is natural to place computer simulation near the core of what Rohit Parikh called "social software": the development of theories on "how to construct a social procedure which brings about a desired result in an efficient and reliable way, at least when such a thing is possible at all".[3(p90)]

Being little less than a full-blown revolution, such an extension of the original project is bound to raise concerns. Although we are cautiously optimistic about the prospects of using computer simulations for institutional design, in this section we will address some sources of those concerns.

(a) It is unclear that there are real life decision problems and voting processes that satisfy the conditions specified for the simulations. In specifying the problem in a way that is amenable to computational treatment, one must rely on assumptions that portrait "…a highly construed, artificial decision problem that is unlikely to occur in real-life settings".[25(p97),28] Some of the assumptions embodied in the description are clearly unrealistic (*e.g.*, random preferences on each new vote) and for some parameters (*e.g.*, voter competence), there is no clear way to determine which range of values might provide an accurate description, even if there is a general rationale.[23] As has been diagnosed for other results deemed important for social applications, from this it could be argued that "the computational experiment (…) offers no support for the social applications proposed by the authors".[29(p1025)] Although there is a kernel of truth in this concern, our approach might withstand such setback by recognizing such concern as the inevitable result of using idealization to formally represent complex phenomena. This commonly occurs in mathematical practice, and it also has widely acknowledged remedies: we can explore several extensions and alternatives to the original setting. Nonetheless, when the aim is to apply the results outside of the Math Lab, some cautionary warning and a reminder of this is always advisable. Although we address an aspect of this in (e), below, it is important to keep in mind that our assessment criteria are not primarily meant to be used as a descriptive tool for predicting voting behavior, but rather as providing insights into the optimality of the voting methods themselves.

(b) The procedure for assessing voting methods here outlined is not exhaustive. As the remarks on Section 3 concerning the number of voting methods make plain, there remains a vast uncharted territory of non-explored voting methods. In fact, when we designed our example, there were several brainstorming meetings to propose alternative voting methods to compete. Our proposal provides some insight on how well a voting method performs, conditional on there being "well-behaved" methods in our initial pool for assessment. Using this procedure does not warrant that the best method will be selected. But this should not overshadow some remarkable qualities of our criteria for assessment. First, the methodology we outlined does identify non-comparative traits of some "good" voting methods, provided they are considered for assessment. That task is performed by our criteria of Triviality validation, Convergence to optimum and Constant behavior. In addition, there are other criteria that allow for comparative assessment. While this comparison must be made considering a small number of methods, it could prove useful. Finally, although the procedure of assessment is contingent on conceivable voting methods, there is a pragmatic requirement that would probably discard "unimaginable" voting methods. In order to be incorporated in a democratic society, a voting method should be "transparent". In this sense, its details should be easily explainable to the voting population; otherwise, the method could be construed as a black box that does not accurately reflect the views of the overall group.

(c) Some simulations require additional information than that of (initial) competence and aggregation function. In order to compare the performance of voting methods on a scale, sometimes additional conditions must be imposed on the simulations. And there might be a natural way to decide which of the alternative ways of supplying those data are worthy of exploring. Actually, when considering other methods (see Appendix) that required deciding how to assign preference on pairwise comparisons, some of the exchanges among the authors were less than amicable (at the end of the day, we all joined the playful spirit of "let's run it and see what happens"). This is a technical concern and it is important to take note of it. However, insofar as the assumptions are made explicit –as is always required in the design of simulations–, one can confidently endorse the claim that a voting method shows an outstanding performance, "conditional on those assumptions".

(d) Non-democratic methods may fare better than democracy according to instrumental criteria for assessment. In describing our proposal there have been many praises on democracy. However, there is no guarantee that democratic voting methods will outperform non-democratic decision procedures for certain problems under certain conditions. In fact, democracy might be instrumentally worse than some non-democratic decision-making procedures. An important disclosure is required at this point. Although our examples did not consider explicitly non-democratic alternatives, our criteria for assessment are not politically oriented and tailor-made to support democracy. As does other related work,[30,31] we focused our discussion on ascertaining which (presumably) democratic decision-making procedures would perform better, under certain conditions. But those methods could be additionally compared to non-democratic alternatives; and democracy might not shine as bright under that light. However, further exploration of these issues falls outside the scope of our current interests. Besides, recalling that the logical space of non-democratic procedures is an unimaginable immense territory, we remain skeptical that there is always an obvious winner in that comparison.

(e) Simulations may obscure some psychologically important aspects of choice situations and voting processes. Different voting methods use different kinds of information to aggregate individual inputs into the collective outcome. That information is usually called a "ballot". In our examples, it might be a preference for a candidate to occupy a certain position, a preference for a full ticket, or a grade on a pair of candidates. Some voting procedures involve several stages. In order to compare voting methods, our criteria required that we use the same information on all methods. So, we assumed that each voter had a preference over a full ticket from the start. If that information was not fully required on the first ballot, we made the simulation "ration it", under certain specific patterns. This is a gross idealization. Real voters might not have an explicit preference for a full ticket unless there is a request for one. And even if they do have such a preference, it is not clear how that imposes additional constraints on their voting behavior. Besides, the quirks of human psychology –such as cognitive biases, strategic behavior, and group conformity– might introduce additional forces into the preference distribution once a voting method is established. These are all empirical concerns, targeted by descriptive research,[32,33] that our methodology does not address by itself. But, if we keep an open mind,

instead of obstacles, those concerns might open further avenues for new enquiry. For instance, the provisional result of simulations –when conjoined with psychological experiments or massive amounts of data– could be useful to support empirical hypotheses about cognitive biases on several settings.

6 Concluding remarks

As we have shown, computer simulations can be useful to assess the performance of different voting methods when they are applied to collective decision problems. Although we were not arguing for the optimality of a specific voting method, we outlined a methodology to assess these and other methods under epistemically instrumental optimality criteria. We argued that this could greatly improve the resources and capabilities for innovation and reform in matters of institutional design, even when there is no general theoretical result to support the optimality of a voting method. We are not implying that further inquiry into voting procedures from a theoretical perspective should be discouraged. On the contrary, theoretical results, when they are available, provide a better understanding and stronger foundations to prefer a voting procedure over another. However, solving social decision problems cannot be delayed until the mathematical tribunal reaches a verdict. Besides, not only is the use of simulations of voting methods compatible with theoretical research of a more mathematical vein, but simulations can provide useful insights for exploring new theoretical avenues. In order to recognize this, it is not necessary to assume that simulations provide direct evidence for the truth or falsity of a result. Instead, we can appreciate that simulated results can offer heuristic guidance on which aspects of decision problems or voting methods may hold more promise.

Appendix

In discussing possible ways to determine the remaining preferences on conditioned feasible options we came up with the following method, which is inspired by the Hungarian solution to the assignment problem. An alternative for performing conditioned voting could decide among feasible options in a different way than those defined in Section 2. In order to describe it, we need a different kind of ballot in which voters must rate each candidate on a matrix. Thus, the entry on row i and column j of the matrix specifies the voter's rating assigned to candidate's j occupying position i. Consider, for instance, the matrix in Table 4.

i\j	A	B	C
1	2	8	10
2	1	9	6
3	8	7	9

Table 4. An example of ratings on candidates (*i*) occupying positions (*j*) to solve the assignment problem.

Here, a voter assigns 10 to candidate C occupying position 1. This resembles the classic assignment problem. If we solve the assignment problem, we would have the result of $[C, B, A]$ as the voter's preference. But now let us suppose that B is assigned "by decree" to occupy position 1. Then, preferences on a conditioned voting are expressed by the submatrix that results from erasing row 1 and column 2 on the previous matrix. Thus, we have the submatrix in Table 5.

i\j	A	C
2	1	6
3	8	9

Table 5. The same ratings as in Table 4, conditioned on the assumption that B occupies position 1.

Applying the solution (to the 2×2 assignment problem) we have C occupying position 3 and, therefore, $[\hat{B}, C, A]$ would be the final assignment.

Thus, we could describe this method as follows: *Assignment method with conditioned voting*. Each $(\alpha, \beta, \gamma) \in S_3$ is assigned an integer $H(\alpha, \beta, \gamma)$ which is calculated as follows:

$$H(\alpha, \beta, \gamma) = \#[\hat{\alpha}, \beta, \gamma] + \#[\alpha, \hat{\beta}, \gamma] + \#[\alpha, \beta, \hat{\gamma}]$$

$H(\alpha, \beta, \gamma)$ is counting the total votes for ticket (α, β, γ) plus the votes on the conditioned feasible options that hold the order prescribed by that ticket.

If $(\alpha, \beta, \gamma) \in S_3 = \{(A, B, C), (A, C, B), (B, A, C) (B, C, A), (C, A, B), (C, B, A)\}$ and $H(\alpha, \beta, \gamma) > H(x, y, z)$ for all $(x, y, z) \in S_3 \backslash \{(\alpha, \beta, \gamma)\}$, then $[\alpha, \beta, \gamma]$ is the winning assignment. This method chooses the ticket that gets more votes on the feasible options along with their respective feasible conditioned options.

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Conflict of interest

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