From Time Asymmetry to Quantum Entanglement: The Humean Unification

Eddy Keming Chen*

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Abstract

Two of the most difficult problems in the philosophical foundations of physics are (1) what gives rise to the arrow of time and (2) what the ontology of quantum mechanics is. The first problem is puzzling since the fundamental dynamical laws of physics do not include an arrow of time. The second problem is puzzling since the quantum-mechanical wave function describes a non-separable reality that is remarkably different from the objects in our ordinary experiences.

In this paper, we propose a unified "Humean" solution to the two problems. Humeanism allows us to incorporate the Past Hypothesis and the Statistical Postulate into the best system, which we then use to simplify the quantum state of the universe. This allows us to confer the nomological status to the quantum state in a way that adds no significant complexity to the best system and solves the "supervenient-kind problem" facing the original version of the Past Hypothesis. We call this strategy the *Humean unification*. It brings together the origins of time asymmetry and quantum entanglement. On this theory, what gives rise to the arrow of time is also responsible for the non-separable phenomena in nature. The result is a more unified theory, with a separable mosaic, a best system that is simple and non-vague, less tension between quantum mechanics and special relativity, and more theoretical and dynamical unity. We then compare our proposals to those in the literature that focus on only one of the two problems. Our analysis further suggests that, in order to obtain a deeper understanding about the problems in philosophy of science, it can be tremendously illuminating to explore the full resources of Humeanism, even if one is not a Humean.

Keywords: arrows of time, quantum entanglement, wave function, density matrix, Humean supervenience, laws of nature, Best System Account, the Past Hypothesis, Statistical Postulate, the Mentaculus, objective probabilities, separability, narratability, vagueness, Lorentz invariance

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[*] Department of Philosophy, University of California, San Diego, 9500 Gilman Dr, La Jolla, CA 92093-0119. Website: www.eddykemingchen.net. Email: eddykemingchen@ucsd.edu

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1 Introduction

Two of the most puzzling phenomena in nature are time asymmetry and quantum entanglement. They have played important roles in the development of contemporary physics. The study of time asymmetry started a rigorous discipline of statistical mechanics with applications to many domains. The study of quantum entanglement produced profound insights about the foundations of quantum mechanics, as well as potential technological advances in quantum information and cryptography.

In philosophy of science, both problems are treated as useful data for evaluating leading theories about laws, chances, and ontology. They frequently come up in debates about Humeanism vs. anti-Humeanism in the metaphysics of science, serving as important case studies regarding questions such as whether the fundamental ontology is separable, whether laws supervene on the material ontology, and whether we should allow fundamental laws about initial conditions and "deterministic chances."

So far, however, they have largely been treated as distinct and unrelated problems in the foundations of physics and philosophy of science. Humeans have offered ingenious solutions to them by conferring nomological status to the Past Hypothesis, a promising explanation for the arrow of time in our patch of the universe, and (recently) to the quantum wave function, which is responsible for the phenomena of quantum entanglement. However, conferring nomological status is not always easy and could lead to tensions with other things Humeans may believe about laws of nature. For example, can the Past Hypothesis be a fundamental (Humean) law even if it is stated in a non-fundamental language, as an infinitely long disjunction, or with vague terms? Can the wave function be considered nomological if it is extremely complex and perhaps more complex than the mosaic it aims to summarize? There have been proposed answers but they seem to require further modifications of the Humean framework, which may not be fully satisfactory.

The purpose of this paper is to focus on some interconnections between the two problems and show that they are deeply related such as to permit a unified treatment in the Humean framework. The unification in the Humean framework shows that what is responsible for time's arrow can also be responsible for the non-separable phenomena in nature. We do this by adopting a new theory of quantum statistical mechanics and using the nomological status of the Past Hypothesis to select a natural initial quantum state of the universe and to argue for its nomological status. We call the general strategy the *Humean unification*. We show that it leads to not only novel solutions to both problems but also new insights about the relationship between Humeanism and foundations of physics. Humean unification suggests that, in order to obtain a deeper understanding about the problems in philosophy of science, it can be tremendously illuminating to explore the full resources of Humeanism, even if one is not a Humean.

We proceed as follows. In §2, we review the problems of time asymmetry and quantum entanglement in more details and discuss their relevance to the Humean framework. In §3, we review the Mentaculus theory, a promising and concrete theory of quantum statistical mechanics, and we construct a new theory called the *Wentaculus* that makes central use of density matrices and a new law called the *Initial Projection Hypothesis* that replaces the Past Hypothesis. In §4, we "Humeanize" the Wentaculus by arguing that the initial quantum state of the universe described by the Initial Projection Hypothesis can be interpreted nomologically rather than ontologically, which leads to a unified treatment of time asymmetry and quantum entanglement. In §5, we discuss the fruits of Humean unification. In §6, we compare and contrast Humean unification to other related proposals that focus on only one of the two problems.[1]

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[1] In this paper, we make use of ideas and methods from the metaphysics of science, philosophy of physics, and mathematical physics. We do not worry too much about whether the ideas are purely philosophical or scientific. A precise disciplinary boundary here may be difficult to draw. Indeed,

The issues we discuss here have ramifications that go beyond the plausibility of Humeanism. By choosing a *natural* initial quantum state, our theory provides novel insights about the foundations of quantum statistical mechanics. The new quantum theories admit a Humean interpretation, but I believe that they are also compatible with a non-Humean interpretation. I discuss this possibility in §7.[2]

2 Problems of Time Asymmetry and Quantum Entanglement

2.1 The Original Problems

In this section, we discuss the original problems of time asymmetry and quantum entanglement (A and B) as well as further problems they give birth to (A1-2 and B1-2).

The first problem can be stated as follows:

A. The Problem of Time Asymmetry: Why is there temporal asymmetry in the world when the fundamental dynamical laws are symmetric in time?

Time asymmetry is widespread in nature: ice cubes melt in a cup of hot water but do not spontaneously form in it; gas expands in a box but does not spontaneously contract; wine glasses break into pieces but the broken pieces do not spontaneously form wine glasses. In the language of thermodynamics, (isolated) physical systems (typically) evolve from states of lower entropy to states of higher entropy; but not the other way around. The phenomena are summarized by the Second Law of Thermodynamics: (isolated) physical systems (typically) do not decrease in entropy. However, the fundamental dynamical laws of physics, such as the Newtonian equation of motion, the Schrödinger equation, the Dirac equation, and Einstein field equations are (essentially) symmetric in time. They allow ice cubes to decrease in size and to increase in size, gas molecules to expand and to contract, and wine glasses to break into pieces and the pieces spontaneously form glasses. They allow (isolated) physical systems to increase in entropy as well as to decrease in entropy.

It has been argued that the origin of time asymmetry in our universe lies in a low-entropy boundary condition, now called the *Past Hypothesis*.[3] According to the Past Hypothesis, the universe "started" in a state of extremely low entropy. Starting from that state, *most likely* the universe will evolve according to the fundamental physical laws into higher entropy states, giving rise to the temporal asymmetry we observe. We add the probabilistic qualifier "most likely" because there exist

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we welcome the possibility that some ideas in philosophy may lead to new theoretical possibilities in foundations of physics and vice versa.

[2] This paper is the second part of a project called "Time's Arrow in a Quantum Universe." For other related papers in the project, see Chen (2018a,b, 2019a) and Chen (2020).

[3] Albert (2000) coins the term. See Feynman (2017), Goldstein (2001), Lebowitz (2008), Ehrenfest and Ehrenfest (2002), North (2011), and Penrose (1979) for more discussions about the low-entropy initial condition. See Earman (2006) for worries about the Past Hypothesis. See Goldstein et al. (2016) for a discussion about the possibility, and some recent examples, of explaining the arrow of time without the Past Hypothesis.

some initial conditions compatible with the Past Hypothesis that will go to lower-entropy states in the future. To provide a reason for neglecting those anti-entropic states, we add a uniform probability distribution to make them extremely unlikely. That distribution is specified by the *Statistical Postulate.* Loewer (2012) dubs the package of postulates—the dynamical laws, the Past Hypothesis, and the Statistical Postulate—the Mentaculus.

However, the Past Hypothesis and the Statistical Postulate give rise to difficult conceptual issues. Since they play a crucial role in explaining time asymmetry and the Second Law, and since they are incredibly simple, it has been argued that the Past Hypothesis is a fundamental law of nature[4] and the Statistical Postulate provides objective probabilities. But how can the Past Hypothesis be a law of nature if it is a (macroscopic) boundary condition? And how can the initial probability distribution be objective if the laws are deterministic?

A1. **The Status of the Past Hypothesis:** How can the Past Hypothesis be a fundamental law of nature if it is a (macroscopic) boundary condition?

A2. **The Status of the Statistical Postulate:** How can the initial probability distribution be objective if the laws are deterministic?

The second and seemingly unrelated problem is as follows:

B. The Problem of Quantum Entanglement: What is the nature of quantum entanglement?

Quantum mechanics is one of the most empirically successful theories. But it presents numerous conceptual puzzles. At the heart of them is the phenomenon of quantum entanglement. Quantum entanglement is a property of the quantum state, which is standardly represented by a wave function ψ. Two systems A and B are entangled when their joint state ψ_{AB} is not a product of their individual states ψ_A and ψ_B. We have good reasons to be *realist* about quantum mechanics and about the quantum state.[5] So we may have to postulate the quantum state in the world. If it is fundamental, then the fundamental ontology would be *non-separable*: the fundamental state of the world is not determined by the states of its parts. Quantum entanglement is a kind of *holism*.[6] However, this is not the only surprising consequence of quantum entanglement. David Albert (2015) has shown that if quantum entanglement is among the fundamental facts, i.e. in the mosaic, then Lorentz invariance of special relativity would conflict with a very natural principle called *narratability*: the full history of the world can be narrated in a single temporal sequence, and other ways of narrating it will be its geometrical transformations (e.g.

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[4] Suggestions that the Past Hypothesis is an additional law of nature can be found in Feynman (2017), Albert (2000), Goldstein (2001), Callender (2004), and Loewer (2012). The inference that the Past Hypothesis may be a *fundamental* law is based on the fact that it does not seem to be derived from anything else. In this paper, we set aside the interesting possibility raised by Carroll and Chen (2004).

[5] See Chen (2019b) for a survey of the realist proposals.

[6] See Miller (2016) for more discussions about the notion of holism here.

by Lorentz transformations). The conflict could be a problem for Everettian (and some GRW-type) theories that aspire to be (fully) Lorentz invariant.

B1. **The Problem of Non-Separability:** The state of the world is not determined by the states of its parts.

B2. **The Conflict between Lorentz Invariance and Narratability:** If quantum entanglement is in the mosaic, then Lorentz invariance conflicts with narratability.

2.2 Relevance to Humeanism

The problem of time asymmetry and the problem of quantum entanglement have been much discussed in foundations of physics and metaphysics of science. Both problems have come up when evaluating Humeanism: the first problem has been used to support Humeanism and the second one against Humeanism. Each of them has also inspired much interesting, original, and insightful work about the Humean framework in the metaphysics of science. These include: Loewer (1996), Cohen and Callender (2009), Callender and Cohen (2010), Miller (2013), Esfeld (2014), Bhogal and Perry (2015), Callender (2015), Albert (2015), Miller (2016), Esfeld and Deckert (2017).

So far, the two problems have been treated as distinct problems. We have seen impressive progress in developing interesting solutions to these two problems. Interestingly, the solutions both have something to do with laws of nature. However, the solutions not fully satisfactory. I will discuss some *prima facie* problems below.

The Humean framework in the metaphysics of science can be roughly characterized by the following theses:

- Humean Mosaic: the fundamental physical ontology is a separable mosaic. In the terminology of Lewis (1986), it consists in "local matter of particular fact."
- Best System Account of Lawhood: the fundamental laws are the axioms of the summary that best balances a host of theoretical virtues such as simplicity and strength.

Humean supervenience is the thesis that all there is is a Humean mosaic consisting of local matter of particular fact and all else supervenes on that. Loewer (2001, 2004) suggests that we should allow the best system to admit objective probabilities even when the laws are deterministic, so long as admitting them makes the system more informative without adding too much complexity. This is an important modification of the original Humean framework, but it is arguably continuous with the Mill-Ramsey-Lewis account. Given the success of statistical mechanics, it also represents an important advancement in our understanding of objective probabilities that play a central role in physics.[7] For the rest of this paper, I will adopt the modified Humean framework as the starting point to think about Humeanism.

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[7] For a different perspective, see Schaffer (2007).

On the one hand, in response to the problem of time asymmetry, it has been argued[8] that the (modified) Humean framework can solve the problem and the worries raised in A1 and A2. It is highly plausible that the Past Hypothesis and the Statistical Postulate belong to the Humean best summary, as they vastly increase the informativeness of the system without adding much complexity. This is true despite the fact that the Past Hypothesis describes a boundary condition and the fundamental dynamical laws may be deterministic. Hence, on the (modified) Humean account, the Past Hypothesis (PH) and the Statistical Postulate (SP) are as nomic as the dynamical equations of motion.

However, new problems arise as we consider the character of the two new Humean laws. For PH, there is a language problem.[9] Adapting the terminology of Cohen and Callender (2009), we can call it the "supervenient-kind problem": terms such as entropy is obviously not fundamental, and as such PH may not be fit to be a fundamental law *if the axioms of the best system require the vocabulary to be entirely in fundamental physical terms*.[10] They note that if we flesh out "low entropy" in terms of the microlanguage, it will be an infinitely long disjunction of microstates which does not seem to be simple at all. (The supervenient-kind problem is part of the motivation for "language-relativization" in the Better Best System Account, which is arguably a radical departure from the original Humean framework.)

In fact, the situation may be even worse: not only is an infinite disjunction probably too long to be an axiom of the best system, but also are the macroscopic terms such as entropy unlikely to correspond to exactly one set of disjuncts.[11] Given the inherent vagueness in the bridge between the macroscopic and the microscopic, it is plausible that there will be borderline cases of whether some microstates fall under the allowed range of states dictated by the Past Hypothesis. Macrostates have vague boundaries. Any precise boundary would seem artificial and arbitrary. To be sure, these problems do not refute the Humean understanding of the Past Hypothesis, but they seem to suggest that it may be premature to marry the Humean framework to something like the Mentaculus. Hence, Humeanism initially seemed friendly to treating the Past Hypothesis as a law, but upon closer inspection there are deep and difficult problems about language.

For SP, the Humean solution treats it as objective as with the postulates of quantum-mechanical probabilities. It would be desirable if we can unify the two or

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[8] See, for example, Callender (2004) and Loewer (2012).

[9] Some may respond that we can just replace the PH and SP by specifying a single probability distribution on the state space that does not suffer from the language problem described here. But it is plausible that, in the standard framework, the simplest way to specify SP is to first specify the initial macrostate (using something like PH) which will then serve as the "support" of the probability distribution. Nevertheless, new possibilities will be available in the Wentaculus framework. But we are getting ahead of ourselves.

[10] This is essentially Lewis's insistence that the terms of the best system must refer to "perfectly natural properties," or the fundamental properties picked out by fundamental physics. See Sider (2011) for a similar proposal for "structural" and "joint-carving" properties. What these amount to and whether this requirement is tenable is a controversial issue. But it is important to note that such properties play an important role in response to the "problem of the (x)Fx" and the new riddle of induction.

[11] I discuss this point more systematically in Chen (2020).

reduce one to the other.

On the other hand, the problem of quantum entanglement is *prima facie* threatening to Humeanism. First, Teller (1986) and Maudlin (2007) suggest that B1 is a problem for Humeanism, as the entanglement relations would make the mosaic non-separable. Second, if we would like to keep Lorentz invariance (e.g. for Everettian theories) in a non-separable mosaic, we would have to sacrifice narratability. This is undesirable. Since Lorentz transformations does not fully preserve the quantum-mechanical data as argued by Albert, in order to tell the story of the mosaic in a temporal sequence, we would have to specify the quantum state not just along one foliation but along all foliations of space-like hypersurfaces. Describing the Humean mosaic temporally will become infinitely more complex—a potentially undesirable result.

A promising response to B1 is also a "nomic" strategy: it recommends that Humeans allow the quantum state (represented by a wave function) into the best system. In quantum theories with additional ontologies beyond the quantum state (such as Bohmian mechanics, GRW spontaneous collapse theories with a matter density ontology or a flash ontology, and Everettian quantum theory with a matter density ontology), it may be tempting to think that the quantum state is part of the summary of the local ontology consisting in particles, matter density, or flashes in physical space. However, the quantum state may be too complex to be nomological. In fact, the typical quantum wave function is highly complex as a function on configuration space. In response to the complexity worry, one may follow Dürr et al. (1996), Goldstein and Teufel (2001), Goldstein and Zanghì (2013) to connect the nomological interpretation to the Wheeler-DeWitt equation in quantum gravity. As a solution to that equation, the universal wave function must be time-independent and thus *may* be simple. But for Humeanism, it seems premature to tie its tenability to a particular idea in quantum gravity, especially when it is not clear whether the Wheeler-DeWitt equation will survive future development in quantum gravity. (It has yet to play a central role in string theory.)

These concerns with the nomological interpretation of the wave function are by no means decisive refutations, but they suggest that we may need to think outside the box and look for other ways to solve the problems that avoid the above issues. Nevertheless, the nomological interpretation seems promising and on the right track, especially since a successful nomological interpretation can solve both problems—B1 and B2—at the same time. If the entanglement relations are not in the mosaic, then it can satisfy separability, narratability, as well as Lorentz invariance (for theories with such an aspiration). This is in contrast to the interesting proposal of the high-dimensional Humean interpretation of the wave function developed by Loewer (1996).

The treatments of the two problems are so far largely unrelated to each other. In the next three sections, we discuss some important connections between the two. The Humean unification will take advantage of their connections. We use the Past Hypothesis to simplify the quantum state (by choosing a natural, simple, unique, objective, but mixed quantum state), so that we can find solutions to both B1 and B2 without making the law system overly complex . We then use the chosen quantum state to connect the initial low-entropy macrostate to the microdynamics, providing a solution to the supervenient-kind problem and the vagueness problem.

3 Towards A New Theory

In this section, I first review the standard account of quantum mechanics in a time asymmetric universe. For concreteness, we focus on the quantum Mentaculus, a neo-Boltzmannian account of quantum statistical mechanics. Next, I propose an alternative account called the *Wentaculus.* It replaces the universal wave function with a (mixed-state) universal density matrix, the pure-state dynamics with mixed-state dynamics, and the Past Hypothesis with the Initial Projection Hypothesis. Here I focus on the conceptual ideas as much as possible, leaving most mathematical details to the footnotes.

3.1 The Mentaculus

Understanding the world we live in requires us to understand all the regularities in nature. As we discussed in §2, many regularities we are familiar with, such as ice melting, smoke dispersing, and face getting more wrinkled with time, are time-asymmetric. A large class of these time-asymmetric phenomena can be understood as entropic asymmetries in time: the past events have lower entropies than future events. To give a full account of entropic asymmetries of time in terms of scientific explanations, we can postulate some low-entropy boundary conditions and probability distributions beyond the fundamental dynamical equations. The field of statistical mechanics has devoted considerable energy in justifying the conjecture that something like a low-entropy initial condition (together with some probability distributions) will lead to a typical monotonic increase in entropy. For concreteness, we can focus on one particular proposal of Albert (2000), Loewer (2012), and Loewer (2016):

The Classical Mentaculus

1. **Fundamental Dynamical Laws (FDL):** the classical microstate of the universe is represented by a point in phase space[a] (encoding the positions and momenta of all particles in the universe) that obeys $F = ma$.

2. **The Past Hypothesis (PH)**: at a temporal boundary of the universe, the microstate of the universe lies inside M_0, a low-entropy macrostate that, given a choice of C-parameters,[b] corresponds to a small-volume set of points on phase space that are macroscopically similar.

3. **The Statistical Postulate (SP)**: given the macrostate M_0, we postulate a uniform probability distribution[c] over the microstates compatible with M_0.

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[a] The phase space is the $6N$-dimensional state space for a classical system with N point particles with precise locations and velocities in physical space.

[b] The C-parameters are certain conventional choices—the coarse-graining variables—that connect the macrostates to sets of microstates.

[c] The uniform probability distribution here is with respect to the canonical Lebesgue measure on phase space.

This is the classical-mechanical version of the Mentaculus theory. It is a version of the neo-Boltzmannian account of classical statistical mechanics. However, it is a pretty strong version as it specifies a particular low-entropy macrostate M_0 and a particular probability distribution (the uniform one). The detailed differences do not matter here. Most theorems and conjectures in statistical mechanics apply to it just as well as they apply to weaker versions of the PH and SP. We chose the Mentaculus not to commit ourselves to it but merely to write it down as a (concrete) representative of a standard way of thinking about time's arrow in a classical universe.

Next, we move to quantum statistical mechanics. Let us consider how to extend the classical Mentaculus to the quantum version. The key will be to replace the classical state space (phase space) with the quantum state space—the Hilbert space—and to reformulate Boltzmannian statistical mechanics in terms of resources in Hilbert space. Here we can follow the suggestions of Albert (2000)§7 and the mathematical framework of Goldstein et al. (2010a).

The Quantum Mentaculus

1. **Fundamental Dynamical Laws (FDL):** the quantum microstate of the universe is represented by a wave function Ψ that obeys the Schrödinger equation $i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$.
2. **The Past Hypothesis (PH)**: at a temporal boundary of the universe, the wave function Ψ_0 of the universe lies inside a low-entropy macrostate that, given a choice of C-parameters,[a] corresponds to $\mathscr{H}_{PH}$, a low-dimensional subspace of the total Hilbert space.
3. **The Statistical Postulate (SP)**: given the subspace $\mathscr{H}_{PH}$, we postulate a uniform probability distribution[b] over the wave functions compatible with $\mathscr{H}_{PH}$.

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[a] In addition to the ones mentioned in the classical Mentaculus, the C-parameters here also include conventional choices about the cut-off threshold of quantum state macrostate inclusion.

[b] The uniform probability distribution is with respect to the surface area measure on the unit sphere of $\mathscr{H}_{PH}$.

The quantum Mentaculus is a concrete version of a standard way of thinking about time's arrow in a quantum universe. Given this setup, the aim is to show that typical universal wave functions compatible with these postulates will evolve in such a way that most subsystems increase in entropy. There has been impressive results that are *highly suggestive* along this direction.[12]

Let us provide some explanations of the quantum Mentaculus. First, the quantum microstate of the universe is represented by a wave function Ψ. It corresponds to a unit-length vector in Hilbert space. The Hilbert space is an infinite dimensional state space for quantum theory. But a slightly more perspicuous picture of the wave function is to think of it as a function on the configuration space $\mathbb{R}^{3N}$. The configuration space is analogous to the phase space in classical mechanics except that it has only $3N$ dimensions instead of $6N$ dimensions (where N is the number of particles in the universe), and each point in the configuration space represents a possible configuration of particles in physical space in terms of their locations only. The wave function assigns values to every point in configuration space. How to interpret the wave function is a central question in the foundations of quantum mechanics.

But even before we engage in the philosophical questions about the interpretation of the wave function, it is important to realize there is a scientific question at the heart of quantum theory: what is the dynamics of quantum mechanics? The wave function changes over time and obeys the Schrödinger equation. Since the wave functions can superpose into other wave functions, and since the Schrödinger equation is linear, we encounter the notorious *quantum measurement problem*, about which we will return to shortly.

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[12] For example, see Goldstein et al. (2010b).

Second, to account for the temporal asymmetry of entropy, we introduce a low-entropy boundary condition—the Past Hypothesis (PH). It rules out the overwhelming majority of initial wave functions in the Hilbert space, leaving a small class of wave functions that will start in low entropy states. The actual initial wave function has to come from a particular subspace $\mathscr{H}_{PH}$ of the total Hilbert space. The Past Hypothesis subspace $\mathscr{H}_{PH}$ has very low entropy. In classical statistical mechanics, Boltzmann defines entropy of a phase point to be proportional to the logarithm of the volume of the macrostate that includes the phase point. Analogously, the Boltzmann entropy of a wave function is proportional to the logarithm of the dimension of the subspace it (almost entirely) belongs.[13] Hence, $\mathscr{H}_{PH}$ is a low-dimensional subspace. By comparison, PH for classical statistical mechanics postulates that the initial phase point lies inside a macrostate of very small volume.

Third, to make it overwhelmingly likely that the initial wave function is entropic, i.e. evolves to higher-entropy states, we introduce the quantum version of the Statistical Postulate (SP). It provides a uniform probability distribution over the initial wave functions in the subspace. Because of time-reversal invariance, it is plausible that there exist an infinity of "bad" wave functions that are anti-entropic (i.e. evolve to lower entropy). But the uniform probability distribution assigns much lower weight on them than on the entropic wave functions.[14] By comparison, SP in classical statistical mechanics is a uniform probability distribution on the classical phase points compatible with the Past Hypothesis.

These three postulates make up the quantum version of the Mentaculus. However, the Mentaculus cannot be the entire story of quantum mechanics in a time asymmetric universe. As we mentioned before, quantum mechanics itself faces the measurement problem. It seems that the Schrödinger evolution of the wave function is interrupted by sudden collapses. The wave function typically evolves into superpositions of macrostates, such as the cat being alive and the cat being dead. This can be represented by wave functions on the configuration space with disjoint macroscopic supports X and Y. During measurements, which are not precisely defined processes in the standard theory, the wave function undergoes random collapses. The probability that it collapses into any particular macrostate X is given by the Born rule.[15]

As such, quantum mechanics is not a candidate for a fundamental physical theory. It has two dynamical laws: the deterministic Schrödinger equation and the indeterministic collapse rule. What are the conditions for applying the former, and what are the conditions for applying the latter? Measurements and observations are extremely vague concepts. Take a concrete experimental apparatus for example. When should we treat it as part of the quantum system that evolves linearly and when should we treat it as an "observer," i.e. something that stands outside the quantum system and collapses the wave function? That is, in short, the quantum

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[13] See Goldstein et al. (2010a) for more rigorous definitions.

[14] Since the wave functions have to be normalized, they form a unit sphere in the subspace. So the distribution is only on the unit sphere $\mathscr{S}(\mathscr{H})$.

[15] That is, $P(X) = \int_X |\psi(x)|^2 dx$.

measurement problem.[16]

Various solutions have been proposed to solve the measurement problem. Bohmian mechanics (BM) solves it by preserving the Schrödinger dynamics, adding particles to the ontology, and an additional guidance equation for the particles' motion. Ghirardi-Rimini-Weber (GRW) theories postulate a spontaneous collapse mechanism, making wave function collapses independent of the observers. Everettian quantum mechanics (EQM) simply removes the collapse rules from standard quantum mechanics and suggest that there are many (emergent) worlds, corresponding to the branches of the wave function, which are all real. My aim here is not to adjudicate among these theories. Suffice it to say that they are all quantum theories that remove the centrality of observations and observers.

Both BM and GRW use probabilistic postulates to account for the Born rule. BM postulates the Quantum Equilibrium Distribution, which dictates that the initial particle configuration is distributed by the Born rule (see Dürr et al. (1992)). GRW postulates probabilistic modification of the Schrödinger equation by which the center of wave function collapses is distributed randomly according to (something close to) the Born rule. EQM, developed and defended by David Wallace (2012), is the only one that does not introduce any objective probabilities (but seeks to derive them from preference axioms). On BM and GRW, SP will postulate a fundamentally different kind of probabilities from the quantum mechanical probabilities. It would be desirable if they can be unified. On EQM, the aspiration is to come up with a theory that explains the probabilistic phenomena in nature, for which the objective statistical mechanical probabilities of SP will be an obstacle.[17]

Recent work in the foundations of quantum mechanics suggests that just as we can add particles in Bohmian mechanics (BM), we can add additional ontologies to GRW and Everettian theories: GRW with a flashy ontology (GRWf), GRW with a mass-density ontology (GRWm), and Everettian theory with a mass-density ontology (Sm).[18] Let us call them *quantum theories with additional ontologies*. Unlike Bohmian particles, these additional ontologies are not independent variables from the wave function.

The above quantum theories—BM, GRW, GRWm, GRWf, EQM, Sm—make plausible the view that I call *Wave Function Realism*: the universal quantum state is (1) ontic and (2) completely represented by the universal wave function. This is in contrast to the epistemic views about the wave function that maintain that the quantum state, represented by a wave function, corresponds to only our epistemic uncertainties over the actual state of the world.

In short, the quantum Mentaculus contains the quantum version of the Past Hypothesis and that of the Statistical Postulate that support the claim that typical initial microstates will be entropic. Such an understanding can be supplemented with further interpretations about the meaning of the wave function. But the mar-

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[16] See Bell (1990) and Myrvold (2017) for introductions to the quantum measurement problem.

[17] There have been some proposals of how to solve these problems, such as Albert (2000), Ch.7 and Wallace (2011). They rely on two plausible conjectures.

[18] Sm was introduced in Allori et al. (2010).

riage between the Mentaculus and Humeanism is not perfect; as we discuss in §2 and §6, it leads to issues that seem to cry out for a different approach:

- Non-separability and non-narratability problems if we keep the quantum state in the mosaic;
- Complexity problems if we move the quantum state from the mosaic to the best system;
- Supervenient-kind and vagueness problems of the Past Hypothesis;
- (Not a problem but worth mentioning: dualism of statistical mechanical probabilities and quantum mechanical probabilities.)

3.2 The Wentaculus

In this section, I construct an alternative framework—the Wentaculus. As we discuss in §4 and §5, the Wentaculus solves the problems above *and* provides additional theoretical virtues. I proceed in two steps: (1) Density Matrix Realism and (2) the Initial Projection Hypothesis. I will also explain how Humeanism provides motivations for this new framework.

3.2.1 Density Matrix Realism

In §3.1, we saw that the quantum Mentaculus assigns probabilities over wave functions. Now, there is a well-known method of encoding the probabilities in the quantum state itself. Instead of saying that the wave function lies inside some subspace $\mathscr{H}_v$ and that there is a uniform probability distribution over the wave functions in $\mathscr{H}_v$, quantum theory provides a compact way of putting these two pieces of information together into one mathematical gadget—a density matrix. The probability distribution over wave functions can be represented by a density matrix $\hat{W}_v$.[19]

We should not be misled by the language here. Even though we talk about "constructing a density matrix from a collection of wave functions," there is a more *intrinsic* way of understanding the density matrix that is independent of the wave functions. A density matrix is a well-defined object in its own right in Hilbert space.

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[19] More precisely, the density matrix is equal to an integral over wave functions inside the unit sphere of the subspace with respect to the uniform distribution given by the surface area measure: $\hat{W}_v = \int_{\mathscr{S}(\mathscr{H}_v)} \mu(d\psi) \left|\psi\right\rangle \left\langle\psi\right|$. Here is a more intuitive way of understanding the construction procedure. Start from the subspace $\mathscr{H}_v$. It is compatible with many vectors representing different initial wave functions. Take an arbitrary vector $\left|\psi\right\rangle$. We can construct a *projection operator* (projecting to $\left|\psi\right\rangle$) as $\left|\psi\right\rangle \left\langle\psi\right|$. If we apply $\left|\psi\right\rangle \left\langle\psi\right|$ to any other vector $\left|\phi\right\rangle$, it will first take the inner product $\left\langle\psi|\phi\right\rangle$ and output a scalar c, which measures "how much" of $\left|\phi\right\rangle$ overlaps with $\left|\psi\right\rangle$. Then it multiplies the scalar to $\left|\psi\right\rangle$, which yields $c\left|\psi\right\rangle$. Take all the vectors on $\mathscr{S}(\mathscr{H}_v)$, the unit sphere in the subspace, construct the corresponding projection operators, and then take the "weighted sum" over the projection operators. Since there is a continuous infinity of objects to sum over, instead of using an infinite sum, we use an integral over them with respect to the surface area measure on the unit sphere $\mu(d\psi)$. This construction produces a density matrix that represents the probability distribution over the initial wave function.

Instead of constructing it from wave functions, we can think of the above-mentioned density matrix as a simple object defined on subspace $\mathscr{H}_v$. The object contains no more and no less information than what is contained in the subspace itself. It is called the normalized projection, whose mathematical representation can be written as follows:

$$\hat{W}_v = \frac{\mathbb{I}_v}{dim\mathscr{H}_v}, \tag{1}$$

This is the *normalized projection* onto the subspace $\mathscr{H}_v$. The normalization is achieved by dividing the subspace identity operator $\mathbb{I}_v$ by the dimension of the subspace $dim\mathscr{H}_v$. The identity operator is restricted to the subspace: it does nothing to vectors contained inside the subspace and projects into the subspace everything that is not completely contained within.[20]

Given the intrinsic understanding of density matrices in Hilbert space, is there a sense we can provide an intrinsic understanding of it on configuration space that is independent from wave functions and equally objective? The answer is yes. We call this perspective Density Matrix Realism, in contrast to Wave Function Realism.

Just as we can think of the wave function as a function that assigns values to the configuration space $\mathbb{R}^{3N}$, we can think of the density matrix as a function that assigns values to the Cartesian product of the configuration space with itself. Moreover, we can also think of it as a function that assigns values to every ordered pair of points in configuration space.

Wave Function Realism is motivated by the idea that the wave function is central to the dynamics and the kinematics of quantum mechanics. In order to motivate Density Matrix Realism, we need to reformulate quantum mechanics directly in terms of a fundamental density matrix. Can we do that? The answer is yes.[21]

First, the density matrix has an evolution equation analogous to that of the wave function. While the wave function obeys the Schrödinger equation, the density matrix obeys its generalization to mixed states—the von Neumann equation:

$$i\hbar\frac{d\hat{W}(t)}{dt} = [\hat{H}, \hat{W}], \tag{2}$$

where the commutator bracket represents the linear evolution analogous to the linear evolution described by the Schrödinger equation.

Second, the Born rule distribution can be written in terms of the density matrix:

$$P(q)dq = W(q,q)dq \tag{3}$$

Third, we can reformulate BM, GRW, and EQM in terms of the density matrix.[22]

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[20] Since the diagonal entries of $\hat{W}_v$ add up to 1, it is a density matrix.

[21] Density Matrix Realism has already been suggested but not necessarily endorsed by some in the literature. For some recent examples, see Dürr et al. (2005), Maroney (2005), Wallace and Timpson (2010) and Wallace (2011, 2012). What is new in this paper is the combination of Density Matrix Realism with the Past Hypothesis in forming the Initial Projection Hypothesis (§2.2.2) and the argument for the Humean Unification based on that.

[22] For W-EQM, equation (2) is all there is to govern the fundamental quantum state W.

We can show that each reformulation of the realist quantum theory in terms of a universal density matrix W is empirically equivalent to its wave-function counterpart, if on the latter theories the uncertainty over the universal wave function is represented by a statistical density matrix W.[23] Therefore, these are also empirically adequate quantum theories without observers. We call these theories *W-Bohmian mechanics, W-GRW theory*, and *W-Everettian quantum mechanics*. Thus, we can think of W as the central dynamical object in quantum mechanics that produces quan-

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For W-EQM with a mass-density ontology, we can define the mass density function in terms of the density matrix:

$$m(x,t) = \text{tr}(M(x)W(t)), \tag{4}$$

where $M(x) = \sum_i m_i \delta(Q_i - x)$ is the mass-density operator, which is defined via the position operator $Q_i\psi(q_1, q_2, ...q_n) = q_i\psi(q_1, q_2, ...q_n)$. This allows us to determine the mass-density ontology at time t via $W(t)$.

For W-BM, we can postulate the guidance equation as follows:

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\text{Im}\frac{\nabla_{q_i}W(q,q',t)}{W(q,q',t)}(q = q' = Q), \tag{5}$$

Finally, we can impose a boundary condition similar to that of the Quantum Equilibrium Hypothesis:

$$P(Q(t_0) \in dq) = W(q,q,t_0)dq. \tag{6}$$

Since the system is also equivariant, if the probability distribution holds at t_0, it holds at all times. Equivariance holds because of the following continuity equation:

$$\frac{\partial W(q,q,t)}{\partial t} = -\text{div}(W(q,q,t)v),$$

where v denotes the velocity field generated via (5.) This theory is first described in Dürr et al. (2005) and Maroney (2005). Dürr et al. (2005) call this theory W-Bohmian mechanics.

For W-GRW (first suggested in Allori et al. (2013)), between collapses, the density matrix will evolve unitarily according to the von Neumann equation. It collapses randomly, where the random time for an N-particle system is distributed with rate $N\lambda$, where λ is of order 10^{-15} s^{-1}. At a random time when a collapse occur at "particle" k at time T^-, the post-collapse density matrix at time T^+ is the following:

$$W_{T^+} = \frac{\Lambda_k(X)^{1/2}W_{T^-}\Lambda_k(X)^{1/2}}{\text{tr}(W_{T^-}\Lambda_k(X))}, \tag{7}$$

with X distributed by the following probability density:

$$\rho(x) = \text{tr}(W_{T^-}\Lambda_k(x)), \tag{8}$$

where W_{T^+} is the post-collapse density matrix, W_{T^-} is the pre-collapse density matrix, X is the center of the actual collapse, and $\Lambda_k(x)$ is the collapse rate operator defined as follows:

$$\Lambda_k(x) = \frac{1}{(2\pi\sigma^2)^{3/2}}e^{-\frac{(Q_k-x)^2}{2\sigma^2}},$$

where Q_k is the position operator of "particle" k, and σ is a new constant of nature of order 10^{-7} m postulated in current GRW theories.

For W-GRWm, we can let the density matrix determine the mass density function on space-time by (4). For W-GRWf, we postulate flashes that are the space-time events at the centers (X) of the W-GRW collapses.

[23] This is because the predictions of quantum theory are probabilistic; it does not matter whether the density matrix we use to extract predictions is statistical or fundamental. See Dürr et al. (2005), Wallace (2016), and Chen (2019a) for more detailed arguments.

tum mechanical phenomena and determine the behaviors of "local beables." This makes possible an "ontic" interpretation of the density matrix: it is the complete description of the quantum state of the world; there is no more fundamental fact about which wave function is the actual one. Thus, the framework of Density Matrix Realism is a viable alternative to Wave Function Realism.

3.2.2 The Initial Projection Hypothesis

The quantum Mentaculus is most plausible in the framework of Wave Function Realism. On that view, the wave function represents the quantum state of the world, the Past Hypothesis is a constraint on the initial wave function, and the Statistical Postulate is a uniform probability distribution over possible wave functions.

In the framework of Density Matrix Realism, the wave function no longer represents the quantum state. Instead, a density matrix completely represents the initial state. Hence, we can consider reformulating the low-entropy boundary condition as the constraint on the density matrix. However, just as there are many wave functions compatible with $\mathscr{H}_{PH}$, there are many density matrices compatible with $\mathscr{H}_{PH}$, the Past Hypothesis subspace in the total Hilbert space. One could perhaps construct a probability distribution over the possible initial density matrices.

Interestingly, Density Matrix Realism provides a much simpler constraint that combines the Past Hypothesis and the Statistical Postulate that is unavailable in the wave function framework. We postulate that the initial density matrix is the simplest and most natural density matrix associated with $\mathscr{H}_{PH}$: its normalized projection. It can be expressed as follows:

$$\hat{W}_{IPH}(t_0) = \frac{\mathbb{I}_{PH}}{dim \mathscr{H}_{PH}}, \tag{9}$$

It is the identity operator on $\mathscr{H}_{PH}$ divided by the dimension of $\mathscr{H}_{PH}$. I label its Hilbert space representation as $\hat{W}_{IPH}(t_0)$. In the position representation, it is $W_{IPH}(q, q', t_0)$.

This constraint is motivated by Humeanism. The goal of a Humean theorist is to come up with the simplest and most informative summary of the history of the world. If we can avoid the postulation of a probability distribution by making the initial state unique, then the Humean theorist would be motivated to do so. As we shall see in the next section, the postulate leads to further benefits to Humeanism. In contrast, there is no obvious candidate for the simplest or most natural wave function compatible with the Past Hypothesis.

Therefore, I propose that we add the following postulate to any quantum theory in the framework of Density Matrix Realism:

Initial Projection Hypothesis: The initial quantum state of the universe is $\hat{W}_{IPH}(t_0)$.

The Initial Projection Hypothesis (IPH) plays a similar role as that of the Past Hypothesis (PH). They both rule out many available initial states on the state space to explain the time asymmetry in our universe. They carry the same information about initial entropy. PH selects the initial wave function to be one of the wave functions

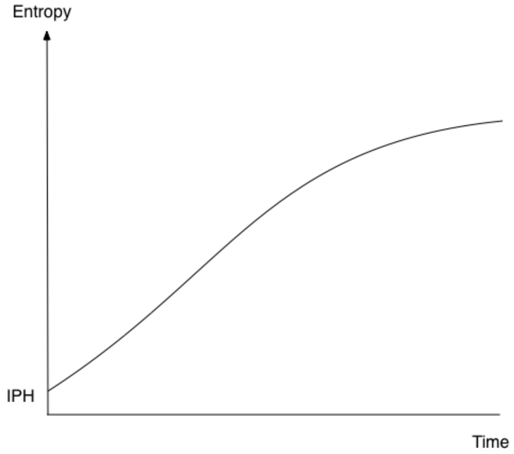

Figure 1: The expected growth of entropy under the Initial Projection Hypothesis (IPH) and the density-matrix dynamics.

inside $\mathscr{H}_{PH}$, and IPH selects the initial density matrix to be the (unique) normalized projection on $\mathscr{H}_{PH}$. Both have exactly the same amount of entropy—that of $\mathscr{H}_{PH}$.[24] However, there are some important differences between IPH and PH. First, IPH picks out a unique initial quantum state of the universe while PH does not. In so far as the Past Hypothesis subspace can be unambiguously specified given some coarse-graining variables, the normalized projection can be unambiguously specified, and IPH also unambiguously specifies the initial state as $\hat{W}_{IPH}(t_0)$.[25] In contrast, PH narrows down the initial wave function to be inside the subspace $\mathscr{H}_{PH}$, which is still compatible with an infinite number of different wave functions.

Second, IPH requires no further statistical mechanical probability distribution while PH needs to be supplemented with SP. Since IPH chooses a *unique* initial state, there is no need to add a probability weighting on the initial states compatible with IPH. However, PH is compatible with many wave functions, some of which will evolve to lower-entropy states. Hence, PH needs to be supplemented with a statistical mechanical probability distribution (SP) that assigns high weight to the "good" wave functions and low weight to the "bad" ones.

When we add IPH to Density Matrix Realism, we arrive at an alternative account of time's arrow in a quantum universe:

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[24] $S_B(\Psi_{PH}(t_0)) = S_B(W_{IPH}(t_0)) = k_B \log(\dim \mathscr{H}_{PH})$, where S_B is the Boltzmann entropy function, k_B is the Boltzmann constant, and "dim" counts the dimensionality of the subspace.

[25] There are additional subtleties about vagueness, which we explore in §5.2.

The Wentaculus

1. **Fundamental Dynamical Laws (FDL):** the quantum state of the universe is represented by a density matrix $\hat{W}(t)$ that obeys the von Neumann equation (2).[a]
2. **The Initial Projection Hypothesis (IPH)**: at a temporal boundary of the universe, the density matrix is the normalized projection onto $\mathscr{H}_{PH}$, a low-dimensional subspace of the total Hilbert space. (That is, the initial quantum state of the universe is $\hat{W}_{IPH}(t_0)$ as described in (9).)

[a] For GRW-type theories, the density matrix obeys the stochastic modification of the von Neumann equation described in footnote #22).

This is the W-version of the Mentaculus. Let us call it the *Wentaculus*. To solve the quantum measurement problem, we can construct Bohmian, Everettian, and GRW versions of the Wentaculus.[26] Let us call these theories W_{IPH}-quantum theories. In §4, we show that the Wentaculus naturally leads to a Humean unification of the origins of quantum entanglement and time asymmetry. Such a unification will bear many fruits (§5).

4 The Humean Unification

We have seen that the density matrix formalism opens up a new possibility for a time-asymmetric quantum-mechanical world: it can be described by the W_{IPH}-quantum theories of the Wentaculus. In this section, I show that Humeanism allows us to further simplify the theoretical structure, by unifying the sources of time asymmetry and quantum entanglement and removing the quantum state from the mosaic. First, I argue for the Nomological Thesis. Second, I show that Humeanism allows us to obtain a unified explanation of time asymmetry and quantum entanglement. Third, I discuss two new wrinkles brought up by the Humean unification.

4.1 The Nomological Thesis

As we discussed in §2, the classical Mentaculus consist in three postulates—the fundamental dynamical equations, the Past Hypothesis, and the Statistical Postulate—all of which can be admitted, by the best-system account, as Humean laws. The Past Hypothesis and the Statistical Postulate are not the usual dynamical laws. In particular, the Past Hypothesis is regarded as a Humean law even though it may look like just another contingent initial condition. Even before we get into Humeanism, there are pre-theoretical reasons (reasons that are conceptually prior to a systematic view about laws) that support its nomological status. For example, plausibly it plays a starring role in deriving the Second Law of Thermodynamics; and perhaps also in deriving the counterfactual asymmetries, the records asymmetry, the epistemic

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[26] The Wentaculus as it is will be sufficient for W_{IPH}-EQM. See §4.2 for the the Bohmian version.

asymmetry, and influence asymmetry in time. They support the idea that the Past Hypothesis is nomologically necessary and not merely contingent. Absent any further nomological explanation of the Past Hypothesis,[27] plausibly it is a law of nature and not a contingent initial condition.

The Humean best-system account provides another argument for the nomological status of the Past Hypothesis. Take for example the quantum Mentaculus. If we subtract the Past Hypothesis and the Statistical Postulate from the Mentaculus, the theory is much weaker. Let us call it the quantum Mentaculus$^-$:

The Quantum Mentaculus$^-$

Fundamental Dynamical Laws (FDL): the quantum microstate of the universe is represented by a wave function Ψ that obeys the Schrödinger equation.

This is the Mentaculus without PH and SP. Since it is time symmetric, it does not ground lawful generalizations such as the Second Law of Thermodynamics and many other temporal asymmetries. As it is, it is much less informative than the Mentaculus. The Mentaculus$^-$ would be much more informative if we add to it PH and SP.

Moreover, PH and SP are not very complex. The uniform surface area measure, specified by SP, is a simple probability measure on the subspace. PH is simple in the macro-language specified in terms of the macro-variables such as temperature, volume, densities, and entropy. In fact, PH could also be simple in the micro-language. A version of PH in the general relativistic cosmological context is the Weyl Curvature Hypothesis (WCH), which is a simple postulate about the initial geometry.[28] To have a complete quantum generalization of WCH would require a theory of quantum gravity, which is still work in progress. However, there are reasons to be hopeful. For example, the generalization of WCH in the Loop Quantum Cosmology program has yielded the Quantum Homogeneity and Isotropy Hypothesis (QHIH)

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[27] That is, we set aside in this paper the possibility explored by Carroll and Chen (2004).

[28] In the context of thinking about the origin of the Second Law of Thermodynamics in the early universe with high homogeneity and isotropy, and the relationship between space-time geometry and entropy, Penrose proposes a hypothesis:

> I propose, then, that there should be complete lack of chaos in the initial *geometry*. We need, in any case, some kind of low-entropy constraint on the initial state. But thermal equilibrium apparently held (at least very closely so) for the *matter* (including radiation) in the early stages. So the 'lowness' of the initial entropy was not a result of some special matter distribution, but, instead, of some very special initial spacetime geometry. The indications of [previous sections], in particular, are that this restriction on the early geometry should be something like: *the Weyl curvature* C_{abcd} *vanishes at any initial singularity*. (Penrose (1979), p.630, emphasis original)

The Weyl curvature tensor C_{abcd} is the traceless part of the Riemann curvature tensor R_{abcd}. It is not fixed completely by the stress-energy tensor and thus has independent degrees of freedom in Einstein's general theory of relativity. Since the entropy of matter distribution is quite high, the origin of thermodynamic asymmetry should be due to the low entropy in geometry, which corresponds very roughly to the vanishing of the Weyl curvature tensor. The Weyl Curvature Hypothesis is simple to state in the language of general relativity.

according to which the initial quantum state has to come from a small subset of possible states with low entropy. It retains the general features of WCH but also introduces some vagueness (about the proper duration of the Planck regime), which is to be expected for any hypothesis that relies on some kind of coarse-graining.[29]

We have good reasons to think that the quantum Mentaculus could be the best system. Thus, we have good reasons to think that PH and SP are parts of the best system. On the modified Humean theory of laws and objective probabilities, it follows that PH is a Humean law of nature and SP specifies objective probabilities in the world.

Similarly, the best system from the point of view of Density Matrix Realism is the Wentaculus. Given the crucial role IPH plays in the Wentaculus, it is plausible that IPH should be regarded as a Humean law if the Wentaculus is the best system. After all, IPH has the same informational content as PH. They both specify a low-entropy initial condition. Moreover, IPH is as simple as PH+SP. They pick out the same density matrix in the low-entropy subspace.

Hence, in so far as we have good reasons to take PH to be a Humean law if the quantum Mentaculus is true, we have equally good reasons to take IPH to be a Humean law if the Wentaculus is true. That is, if Wentaculus is the right theory of the actual world, then we have good reasons to confer (Humean) nomological status to IPH that are on a par with our reasons for conferring (Humean) nomological status to PH. But how do we know which is true: the Mentaculus or the Wentaculus? Here we encounter a case of underdetermination by evidence. The two theories are empirically equivalent: no amount of empirical evidence can settle the question which one is correct. However, we can use super-empirical virtues, some of which will be discussed in §5.

Both PH and IPH are Humean laws that are about the initial quantum state. As discussed before, it is controversial what the quantum states represent. But what is the nature of the initial quantum state? One promising answer suggests that it is nomological.

The Nomological Thesis: The initial quantum state of the world is nomological, i.e. it is on a par with laws of nature.

The Nomological Thesis, on the one hand, is in tension with the complexity issue in the quantum Mentaculus. Even though PH is simple, the wave function compatible with PH is unlikely to be simple enough to be nomological. The Humeans could follow Dürr et al. (1996) and claim that the Wheeler-DeWitt equation in quantum gravity would produce a time-independent wave function that may also be simple enough. Even though the Wheeler-DeWitt equation leads to fascinating scientific and interpretational questions, the Humeans who endorse this strategy faces several challenges. First, a technical challenge: given the time-independence of the wave function, the time asymmetry cannot be described in terms of the entropy of the universal wave function, and that would require significant changes to the Mentaculus program. Moreover, it does not follow that a time-independent wave function

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[29] For more details, see Ashtekar and Gupt (2016a) and Ashtekar and Gupt (2016b).

will definitely be simple. We need additional reasons to support that conjecture. Second, a strategy question: do Humeans want to tie the survival and success of Humeanism to a particular equation in quantum gravity, which has yet to play a central role in research programs such as string theory? It seems reasonable to seek alternative ways to defend the Nomological Thesis. If for nothing else, it would be a safer strategy to find multiple ways to reconcile Humeanism with fundamental physics.

On the other hand, the Wentaculus transforms the situation. Since IPH is a law, and since IPH completely specifies the initial quantum state, $W_{IPH}(t_0)$ is no more complex than IPH itself. So if IPH is simple enough to be nomological, then $W_{IPH}(t_0)$ is simple enough to be nomological. (In contrast, even though PH is simple enough, the initial wave function in the Mentaculus may not be simple.) Hence, in the Wentaculus (but not in the Mentaculus), we can easily remove the complexity obstacle by regarding the initial quantum state to be on a par with laws of nature.

We propose that the Humeans remove the initial quantum state from the mosaic and move it to the best system. Its values can be completely and in a simple way specified by the best system. Hence, moving $W_{IPH}(t_0)$ to the best system is not going to overburden the best system or make it more complex, since IPH already contains that information. This is to be contrasted with the situation in the Mentaculus: the PH does not contain all the information to pin down the initial microstate (wave function) while the IPH in the Wentaculus does contain all the information to pin down the initial microstate (density matrix). However, after we remove the quantum states from the mosaic, we need something to be still present in the mosaic. We can use the additional ontologies in quantum theories such as BM, Sm, GRWm, and GRWf. The fundamental ontology, in each of these theories, will be the particles, matter densities, or flashes, which are separable.

What about later quantum states $W_{IPH}(t_1)$, $W_{IPH}(t_2)$,..., and so on? Do we need to postulate them in the mosaic? That is not necessary. For unitary quantum theories such as BM and Sm, their information can be directly derived from the von Neumann equation, which is also in the best system. For stochastic theories, the initial quantum state (in the best system) can specify a complete probability distribution and conditional probabilities over possible mosaic histories.[30]

4.2 A Unified Explanation

If the initial quantum state is nomological, then the Humean best system contains all of its information: without adding any contingent fact from the mosaic, we can

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[30] The GRW route is different from the way we think about making predictions on GRW theories. That might mean that the GRW route of Humean unification is less natural than the route on unitary theories. Hence, Humean unification may be sensitive to empirical questions about whether GRW is correct, and whether quantum theory is exact. Even if one is bothered by this sensitivity, one need not give it too much weight all things considered. So far, all experimental tests to find violations of unitary dynamics and the deviations from exact Born rule have only but confirmed exact quantum theories such as BM and EQM; we have not found any confirmation of GRW over its rivals. For a review, see Feldmann and Tumulka (2012).

deduce its state at any time with just the best system alone. We do not need to, as we standardly do, independently specify the initial condition of the quantum state as a contingent fact in the mosaic. This opens up a new possibility for W$_{IPH}$-quantum theories with additional ontologies. For these theories, we can remove the quantum state from the mosaic without losing any information about entanglement and correlations, for such information is already contained in the best system. And the mosaic will not be empty—it will still contain the local beables such as particles, matter densities, and flashes, which make up pointers, tables, and chairs. From a Humean point of view, the best system (now the Wentaculus plus the values of the initial quantum state) supervenes on the mosaic.

Take W$_{IPH}$-BM for example. Let us write down the mosaic + best system package without the Humean unification:

The W$_{IPH}$-BM mosaic: particle trajectories $Q(t)$ on physical space-time and the quantum state $W_{IPH}(t)$.

The W$_{IPH}$-BM best system: four equations—the simplest and strongest axioms summarizing the mosaic:

(A) The von Neumann equation: $i\hbar \frac{\partial \hat{W}}{\partial t} = [\hat{H}, \hat{W}]$,

(B) The Initial Projection Hypothesis: $\hat{W}_{IPH}(t_0) = \frac{I_{PH}}{dim \mathscr{H}_{PH}}$

(C) The W-Quantum Equilibrium Hypothesis: $P(Q(t_0) \in dq) = W_{IPH}(q, q, t_0) dq$,

(D) The W-guidance equation: $\frac{dQ_i}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \frac{\nabla_{q_i} W_{IPH}(q, q', t)}{W_{IPH}(q, q', t)} (q = q' = Q)$.

What would it look like after Humean unification? It will have fewer things in the mosaic and fewer equations in the best system.

The W$_{IPH}$-BM mosaic: particle trajectories $Q(t)$ on physical space-time.

The W$_{IPH}$-BM best system: three equations—the simplest and strongest axioms summarizing the mosaic:

(A) The Initial Projection Hypothesis: $\hat{W}_{IPH}(t_0) = \frac{I_{PH}}{dim \mathscr{H}_{PH}}$

(B) The W-Quantum Equilibrium Hypothesis: $P(Q(t_0) \in dq) = W_{IPH}(q, q, t_0) dq$,

(C) The combined equation: $\frac{dQ_i}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \frac{\nabla_{q_i} \langle q | e^{-i\hat{H}t/\hbar} \hat{W}_{IPH}(t_0) e^{i\hat{H}t/\hbar} | q' \rangle}{\langle q | e^{-i\hat{H}t/\hbar} \hat{W}_{IPH}(t_0) e^{i\hat{H}t/\hbar} | q' \rangle} (q = q' = Q)$

In this theory, the mosaic no longer contains the quantum state. IPH still postulates the values of the initial quantum state $\hat{W}_{IPH}(t_0)$. But it is dispensable. The role it plays in the best system above is to specify the values of the initial probability distribution and the velocity field for particles. We can rewrite any occurrence of $\hat{W}_{IPH}(t_0)$ in terms of its explicit functional form. We can construct similar Humean interpretations of W$_{IPH}$-GRWm, W$_{IPH}$-GRWf, and W$_{IPH}$-Sm.

The nomological role of the quantum state here is similar to that of the Hamiltonian in classical mechanics. The Hamiltonian specifies the interactions or the "forces" among the component systems. The Hamiltonian is on par with the classical laws of motion as it is a simple part of the Hamiltonian equations. That is, if we expand the Hamiltonian function as a function of the variables for things in the mosaic (positions and velocities of particles), the equation is still simple.[31] Similarly, the quantum state is a simple part of the von Neumann equation. However, an important difference is that equation (C) is time-dependent, while the Hamiltonian equations of motion are time-independent.

I call such an interpretive strategy the *Humean Unification*. Humean Unification recommends that we remove the quantum state from the mosaic. How, then, does one explain the phenomena of quantum entanglement? How can systems at space-like separation be perfectly correlated, if there is no fact about quantum entanglement in the mosaic? The Humean Unification provides purely nomic explanations. There is a law that specifies the quantum entanglement of all the systems at t_0, from which we can use the von Neumann equation to derive quantum entanglement at a later time t. The equation for local beables (such as (4) and (5)) will then determine the behavior of objects in space-time: e.g. if Alice were to observe "Spin Up" then Bob would observe "Spin Down." (Both the observers and the observed systems will be made out of the local beables and not of the quantum state.) Moreover, the law that specifies the quantum initial condition—IPH—is the same law that specifies the low-entropy initial condition. Hence, IPH is the origin of both quantum entanglement and time asymmetry.

The Humean unification provides a unified view on the sources of quantum entanglement and time asymmetry. In the Mentaculus, they have distinct sources—one macroscopic Past Hypothesis and one microscopic wave function. But in the Wentaculus, it is one and the same density matrix.

The time-dependence of the equation of motion (such as the combined equation (C)) in the unified best system further suggests that there is intertwining between the two. In the Mentaculus picture, the theory as a whole is not time-translation-invariant, because the Past Hypothesis applies only at a particular time. However, we can still understand the sense in which the Mentaculus picture is still time-translation-invariant: we can separate the dynamics from the initial condition, even when both are Humean laws; the dynamics is invariant even when the lawful initial condition is not. But in the Wentaculus picture, after Humean unification, there

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[31] The Hamiltonian equations are:

$$\frac{\partial \boldsymbol{q_i}}{\partial t} = \frac{\partial H}{\partial \boldsymbol{p_i}}, \frac{\partial \boldsymbol{p_i}}{\partial t} = -\frac{\partial H}{\partial \boldsymbol{q_i}}. \tag{10}$$

The Hamiltonian function is specified as follows:

$$H(\boldsymbol{q}, \boldsymbol{p}) = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_{1 \leq k < l \leq N}\sum V_{k,l}(|q_k - q_l|), \tag{11}$$

where $V_{k,l}$ is a simple formula for the pair-wise interactions.

is no such clean separation. The two, as it were, are genuinely unified into one thing, so there is no sense in which the theory is fundamentally invariant under time translation.

The violation of time-translation invariance should be viewed not as a bug but as a feature of Humean unification. After all, is it reasonable to insist on saving the symmetry at the cost of everything else? Of course not. Does this particular violation make the theory less simple? No; in fact the theory becomes simpler because of it. Can we still make sense of the appearance of this symmetry? Yes. At the emergent level of effective dynamics and subsystem dynamics, we can still make sense of a time-translation-invariant non-fundamental dynamics. For many subsystems, they will have (non-fundamental) subsystem density matrices that still obey time-translation-invariant (effective) laws.[32]

4.3 New Wrinkles

The Humean unification is based on a new theoretical possibility opened up by the conjunction of Density Matrix Realism and the Initial Projection Hypothesis. However, it also leads to some new wrinkles that should be addressed. We focus on two here: the complexity worry and the classical maneuver.

(1) The complexity worry. The Initial Projection Hypothesis selects a density matrix that is mathematically equivalent to a "disjunction" of wave functions with a uniform measure over them. If any wave function in the disjunct is overly complex, shouldn't the whole disjunction, and therefore the initial density matrix, be overly complex? How is that compatible with the earlier claim that the Initial Projection Hypothesis as well as the initial density matrix are simple enough to be nomological? The intuition behind this worry is that the disjunction inherits whatever complexity that is in the disjuncts, and the initial density matrix $\hat{W}_{IPH}(t_0)$ will be highly complex, and perhaps even more so than typical wave functions. So our criticism about the standard wave function nomological view comes back to haunt us.

That is not quite right. We offer a counterexample to the intuition and a positive argument for the simplicity of the initial density matrix. Let us consider classical mechanics governed by $F = ma$. We can think of the content of $F = ma$ as given by the disjunction of all the solutions to that equation, namely the disjunction of all complete trajectories of any number of point particles that classical mechanics allows. Most of those trajectories will be highly complex. However, $F = ma$ is a simple law, even though it is informationally equivalent to the complete disjunction of its possible solutions. Positive argument: there are ways of understanding the density matrix that is independent of the collection of wave functions; the quantum state space (Hilbert space) permits a straightforward, intrinsic, and geometrical understanding of the initial density matrix selected by IPH. In fact, it is (modulo the normalization constant) equivalent to the subspace itself. While an individual vector

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[32] Even if one is worried about the loss of (fundamental) time-translation invariance, the theoretical cost should be viewed in the context of and in balance with the numerous theoretical benefits that we discuss in §5.

in the subspace (the wave function) may require many coordinate numbers to pick out, the subspace requires much less information to specify. The availability of the intrinsic understanding of the fundamental density matrix $\hat{W}_{IPH}(t_0)$ also responds to the supervenient-kind problem raised in §2.

(2) The classical maneuver. One might worry that the same "trick" can be played in the classical context, which somehow makes the Humean unification look too easy and perhaps trivial. On first glance, the suggested maneuver is to take the "probability distribution" (ρ) as "ontic." The same thing can presumably be taken in the classical context, where the probability distribution on phase space can be given a similar ontic interpretation, thus avoiding the problems in the classical domain as well. If that is possible, moreover, it seems to show that either we have proven too much, or that it does not depend on the details of quantum theory.

However, that is a mistake. First, it is much less natural to give an ontic interpretation of the probability distribution in classical statistical mechanics. If we use the same idea in the classical domain, we will likely get a many-worlds version of classical mechanics or lose determinism. The classical probability distribution ρ plays no dynamical role (unlike the density matrix in the W-quantum theories). Moreover, since ρ follows the Hamiltonian dynamics, it will in general be supported on many macroscopically distinct regions on phase space. If we reify ρ as ontic and do not modify the dynamics, then we arrive at a many-worlds theory. If we modify the dynamics to introduce objective "collapses" of ρ into one of the "branches," it will look much more artificial and complex than the original Hamiltonian theory. In contrast, on each of the three interpretations of QM, the artificial effects do not arise on the Wentaculus. The Bohmian version remains deterministic (and single-world), the GRW version remains stochastic (and single-world), and the Everettian / many-worlds version is still deterministic. On the other hand, even if a classical extension of our maneuver is possible, it is unclear how it makes the quantum case trivial, since presumably both require different choices of the ontology and the dynamics.

5 Fruits of Humean Unification

The Humean Unification has consequences for both the Humean mosaic and the best system. In this section, we list some fruits of this project.

5.1 The Mosaic: Simpler, Separable, and Narratable

Under Humean Unification, the Humean mosaic becomes simpler, separable, and narratable. In standard quantum theories, it is plausible that the quantum state represents something in the mosaic. This leads to a non-separable entity or relation that violates one of the tenets of Humean supervenience: that the mosaic must be separable. The Humean Unification lifts the quantum state from the mosaic into the best system (without adding too much complexity to the best system). It simplifies the mosaic by removing the quantum state ontology and postulating local matters in spacetime—particles, matter density, or flashes. The separability of the mosaic is now restored.

This result provides a response to Teller (1986)'s and Maudlin (2007)'s influential argument that Humean supervenience is incompatible with quantum mechanics. It does so without incurring new costs (see contrast with rival proposals in §6) and with many new benefits (as we see in this section).

An under-appreciated consequence of this result is that it also helps with Albert (2015)'s recent argument that quantum entanglement is incompatible with full Lorentz invariance.[33] This is best seen in W_{IPH}-Sm, which aspires to be a fully Lorentz-invariant theory. Albert (2015) shows that the conjunction of Lorentz invariance and entanglement is inconsistent with narratability, the idea that the full history of the world can be narrated in a single temporal sequence, and other ways of narrating it will be its geometrical transformations. To describe a narratable mosaic temporally, it suffices to list facts along one foliation (with the rest being geometrical transformations); to describe a non-narratable mosaic such as the one that contains entanglement relations, we need to list facts along every foliation, which is infinitely more complex. Thus, one can understand narratability as something akin to descriptive parsimony, which is a desirable but defeasible virtue to be balanced with other considerations. (After all, one can consistently insist on specifying states only directly on the mosaic and not through any temporal sequences picked out along some foliation.) In so far as one finds narratability a plausible principle, there is tension between Lorentz invariance and quantum entanglement (which is a purely kinematic notion).

However, Albert's argument presupposes that the entanglement relations are in the mosaic. However, if we remove the quantum state from the mosaic, the trouble-maker is gone, and the mosaic can be both Lorentz invariant and narratable. This avoids the narratability failure mentioned earlier. (A similar result holds for W_{IPH}-GRWm and W_{IPH}-GRWf.) By allowing the mosaic to be fully narratable and allowing the laws to be fully Lorentz invariant, Humean unification could lead to further simplification of the mosaic and the best system. By removing the conflict between Lorentz invariance and narratability, the Humean Unification removes some tension between quantum mechanics and special relativity.[34]

In summary, the Humean unification strategy simplifies the mosaic by removing the quantum entanglement relations, leaving with a fundamental ontology that is separable and narratable that has a better chance of reconciling quantum theory with fully Lorentz invariance (for those theories that have such aspirations). The quantum entanglement relations and the quantum state would be absorbed into the best system, which then supervenes on the new mosaic.

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[33] I am indebted to discussions with Sheldon Goldstein and Ezra Rubenstein on this point.

[34] To be sure, there is still the issue of quantum non-locality.

5.2 The Best System: Simpler, Less Vague, and More Unified

The Humean unification leads to some important modifications of the Humean best system, making it simpler, less vague, and more unified. By lifting the quantum state from the mosaic to the best system, *prima facie* we increase the complexity of the best system by exactly the amount of complexity of the quantum state. If the quantum state is highly complex, then the resultant best system will be complex as well. Even if the mosaic becomes simpler and separable, the costs may still be too high for the Humeans. (That may be the case for some of our rivals. See §6.) However, given the analysis in §4 about the simplicity of the Past Hypothesis and the Initial Projection Hypothesis, the initial quantum state is simple enough to be nomological. Making $\hat{W}_{IPH}(t_0)$ on a par with Humean laws does not overburden the best system. In fact, since the Wentaculus best system already contains IPH, and IPH completely specifies the values of $\hat{W}_{IPH}(t_0)$, there is no added complexity when we lift $\hat{W}_{IPH}(t_0)$ to the best system. That is a significant advantage over other versions of quantum Humeanism where the focus is on the quantum wave function that is likely much more complex than the initial density matrix.

The Humean unification simplifies the best system in another way: it eliminates fundamental statistical mechanical probabilities in the best system. In the quantum Mentaculus, there are two kinds of probabilities: the quantum probabilities prescribed by the Born rule (or the quantum equilibrium distribution in BM, the collapse probabilities in GRW, and the non-physical decision-theoretic or *de se* probabilities in EQM) and the statistical mechanical probabilities prescribed by the Statistical Postulate (SP). It would be desirable to unify the two sources of probabilities in the theory.Albert (2000)§7 attempts to do it in the GRW framework, relying on a plausible conjecture about Gaussian width during GRW collapses. Wallace (2011) proposes we replace the SP by something like a simplicity constraint, relying on a conjecture about simplicity and reversibility. On Humean unification, however, we have a completely general way of getting rid of SP. By choosing a unique and natural initial density matrix, we no longer have an infinity of possible initial microstates. There is just one state to choose from and we no longer need any probability distribution over possible initial microstates. The Humean unification provides a simple and general way to avoid the dualism of probabilities. This is achieved by making SP simply unnecessary.[35]

In contrast to the quantum Mentaculus (and the classical Mentaculus), Humean unification contains less vagueness. The Past Hypothesis and the Statistical Postulate are exact only when we choose some arbitrary coarse-graining variables in nature. On the Mentaculus, which exact set of microstates counts as the low-entropy subspace, after a certain level of precision, will be entirely arbitrary. There is nothing in nature that pins it down. Beyond a level of precision, the exact boundary of the subspace makes no difference to how things are behaving in physical space and what their probabilities are. The same is true for the Statistical Postulate: the

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[35] For W_{IPH}-Everettian theories, since the Born rule is not supposed to be fundamental, the elimination of SP means that there are no objective probabilities in the world. The actual world is nomologically necessary. Hence, every physical fact would follow from the law statements.

support of the probability distribution becomes exact only after we impose some arbitrary choices, and the exact values do not matter after a certain level of precision. So it is best to think of PH and SP as vague postulates and vague Humean laws in the best system of the Mentaculus.[36] The situation is different under the Wentaculus picture. The exact values of the initial density matrix makes a difference to the world just as importantly as constants of nature—different values will typically lead to different microscopic behaviors and probabilities. The exact values of the initial quantum state is by no means arbitrary—$\hat{W}_{IPH}(t_0)$ plays a central role in the fundamental micro-dynamics; it makes a difference to the exact Bohmian velocity field, GRW collapse probabilities, and configuration of matter densities, etc. That is why we can eliminate the vagueness of IPH without objectionable arbitrariness.

Another advantage of Humean unification is that it leads to more unity in the best system. First, it provides mathematical unity between quantum mechanics and quantum statistical mechanics. Quantum statistical mechanics makes extensive use of density matrices. Quantum mechanics, on the other hand, has often been formulated in terms of wave functions. From the perspective of Humean unification, density matrix is the central object in both theories. A wave function only arises in special circumstances when the density matrix is pure. Second, there is an increase in the dynamical unity in some theories. In BM with spin, there does not exist a conditional wave function since the particles have only positions but no spin degrees of freedom. Hence, in general, for the subsystems in a W-BM universe, there is only a conditional density matrix instead of a conditional wave function. As a result, the guidance equation for many subsystems (that are suitably isolated from the environment) will be the W-guidance equation (5) that refers to a conditional density matrix even in standard BM, while the guidance equation for the whole universe will be the usual guidance equation that refers to a wave function.[37] Therefore, the dynamics for the universe will be importantly different from the dynamics for the subsystems for standard BM. In contrast, in W_{IPH}-BM, the guidance equation for the whole universe and that for the subsystems (that are suitably isolated from the environment) will be the same—(5). Whether dynamical unity holds for W_{IPH}-GRWm and W_{IPH}-Sm will require a further analysis of the subsystem dynamics in those theories. But in any case, those theories also witness an increase in kinematic unity: both the universe and typical subsystems will be in mixed states described by density matrices. This is in contrast to the Mentaculus picture, where most subsystems are in mixed states (due to entanglement) while the universe is in a pure state.

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[36] I discuss the notion of "nomic vagueness" in more details in Chen (2020).

[37] See Dürr et al. (2005) for more details. The standard guidance equation in BM under a universal wave function is:

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\mathrm{Im}\frac{\psi^* \nabla_i \psi}{\psi^* \psi} \tag{12}$$

6 Comparisons

In this section, we discuss two other versions of quantum Humeanism that are motivated by the problem of quantum entanglement but not the problem of time asymmetry. We compare and contrast them with the Humean unification we introduced in this paper.

6.1 Wave Function Humeanism in a High-Dimensional Space

One of the earliest proposals of reconciling Humeanism with quantum entanglement is that of Loewer (1996). Loewer argues that if we adopt David Albert's high-dimensional space fundamentalism, the idea that the fundamental space of the world is the "configuration space," then the quantum state (represented by a wave function) is entirely separable in that fundamental space. We can reify that as the Humean mosaic. Entanglement and non-locality are merely manifestations of our perception in the low-dimensional space, which is not fundamental.

The move from a low-dimensional space to a high-dimensional space is a radical move. It is revisionary from the Humean perspective. On Lewis's original formulation, the Humean mosaic are facts about the physical space-time (or some low-dimensional manifold). But it is also revisionary from the ordinary scientific perspective. There are important reasons to take something like the physical space-time to be fundamental, as it underlies many important symmetries in physics, including Lorentz invariance. They will be difficult to recover from the high-dimensional point of view.[38] In contrast, the Humean Unification does not require such radical revisions and even offers additional theoretical benefits.

6.2 Wave Function Humeanism in a Low-Dimensional Space

The second class of proposals of reconciling Humeanism with quantum entanglement is inspired by Hall (2015), and discussed in Miller (2013), Esfeld (2014), Bhogal and Perry (2015), Callender (2015), Esfeld and Deckert (2017). On this proposal, the quantum state of the universe represented by a wave function is part of the Humean best system. There are three ways to interpret this proposal.

First interpretation: the wave function itself represents a simple and informative Humean law of nature (like the classical Hamiltonian). Miller and Callender are close to endorse this view. However, it requires that the universal wave function to be extremely simple, or at least simpler than the complete facts about local beables through all time. Otherwise the system containing the wave function would not win the competition for being the best system. It faces the *prima facie* problem that the universal wave function may not be simple enough to be nomological.

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[38] See Allori (2013) for related reasons. Chen (2017) argues that the low-dimensional view provides better explanation of the Symmetrization Postulate than the high-dimensional view does. Emery (2017) provides reasons to think that other things being equal we should prefer the more commonsense view of physical reality.

Second interpretation: the wave function supervenes on the mosaic and participates in the dynamical laws of nature. On this view, the Humean laws refer to the wave function, but the wave function itself is not "nomological" in the usual sense. Instead, the wave function is another state of the world—albeit a non-fundamental one. As such, both the wave function and the dynamical laws supervene on the mosaic. This interpretation seems close to the views developed by Bhogal and Perry. The natural question arises again: does the wave function have to be relatively simple? They seem to think so and suggests that the universal wave function (together with the dynamical laws) will be simpler than the mosaic itself. Notwithstanding their careful analysis, we are not convinced: even when the wave function helps systematize the mosaic (as their examples suggest), it is no trivial task to establish the simplicity of the universal wave function (relative to the mosaic). If, on the other hand, we need to relax our criterion of simplicity, we may ask whether it is worth the benefits. Perhaps there is a system that contains the simplest wave function over all rivals; still in that case the wave function could be, given the standard scientific criterion of simplicity, too complex to be on par with laws. That would count as a (defeasible) consideration against the interpretation. The worry is dissolved if none of the alternatives is better. However, as we argued in §4 and §5, there exists another system (which is compatible with the evidence we have) that is simpler than this one.

Third interpretation: the wave function is merely a variable that we introduce into the best system to describe the mosaic of local beables. Esfeld and Deckert (2017) hold this view. On their proposal, only point particles are fundamental, and we can interpret every other bit of the theory as in the best system. It is an open question whether there are any principled constraints on the proposal. If there are no principled constraints, then (in classical physics) we could follow their procedure and regard the electromagnetic fields as part of the best system. But the electromagnetic fields are usually highly complicated. Given the standard view that the electromagnetic fields are part of the material ontology, tying Humeanism to the radical view (that allows us to Humeanize the electromagnetic field) is theoretically costly. Moreover, it seems much more instrumentalist than the original Humean proposal, which *aspires* to be realist. Humeanism requires delicate balances between objectivism and pragmatism, but at least the original Humean proposal has principled constraints on what goes into the best system and what goes into the mosaic, and such constraints are generally compatible with scientific practice.

In short, there are *prima facie* problems facing these versions of wave function Humeanism. In contrast, the Humean Unification avoids these problems, as we know that the initial quantum state is simple and unique, and the proposal is fully realist. Moreover, the Humean Unification has many other theoretical virtues listed in §5.

We do not think of our view as completely opposed to the views discussed above. In fact, there is much common ground; our proposal can be seen as friendly extension to some of the views above. For example, towards the end of her important essay, Miller (2013) is rightly worried about the potential slippery slope in the strategy she proposes, and she suggests we find principled limits to curb over-Humeanization or the"narcissism" if one tries to "Humeanize" away everything one does not like in the ontology, including all particles outside one's brain. Our framework can be seen as implicitly suggesting such a limit: carry out Humean unification *only when* you can show that the best system will not be over-burdened by extra complexity *and* that there will be additional fruits of unification beyond solving the original problems. Moreover, our framework can be combined with Bhogal and Perry (2015)'s proposal to simplify the supervenient L-state, and with Esfeld and Deckert (2017)'s proposal to further simplify their "super-Humean" best-system. We leave that to future work.

7 Conclusion

It has been argued that the origin of time asymmetry in our universe lies in its boundary condition—a low-entropy state now called the *Past Hypothesis*. In this paper, we have used the Past Hypothesis to construct a new class of quantum theories—W_{IPH}-theories. We then showed that they allow the Humeans to use the best system to specify a simple and unique initial quantum state. This led to the Humean Unification that combined the origins of quantum entanglement and time asymmetry. The data in the Initial Projection Hypothesis, with the help of the density-matrix dynamics, gives rise to both time asymmetry and quantum phenomena. The result is a new theory with a separable and narratable mosaic as well as a simpler, less vague, and more unified law system.

Can a non-Humean appreciate our new theory? We think so. The Humean strategy for regarding the low-entropy initial condition (PH) as nomological does not sit well with a governing or dynamical conception of laws (although some non-Humeans would disagree). However, non-Humeans may be less opposed to our theory. The low-entropy initial condition plays a dynamical role via the initial projection density matrix and its dynamics. The initial density matrix directly determines the Bohmian velocity field, the GRW collapse probabilities, and the Everettian branching structure. Hence, it plays an analogous role as the classical Hamiltonian function, which can be given a non-Humean interpretation.[39]

Is our new theory the best theory? Given its empirical equivalence to many other theories including the quantum Mentaculus, we have to appeal to super-empirical virtues to settle the question. It is unlikely, however, we will ever be able to conclusively settle it. What we can do, at this stage, is to build and refine, to the best of our abilities, different models, theories, and frameworks. Only after that can we meaningfully compare them side by side as complete packages. In the meantime, however, we hope the Humean unification outlined above will provoke new ideas in solving the two problems that we began with.

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[39] Can a quantum state monist appreciate our new theory? We think so. She can at least appreciate the W_{IPH}-quantum theories without local beables: W_{IPH}-GRW and W_{IPH}-EQM. The crucial step in the Humean Unification—§4—would not be possible. The quantum state will have to be in the mosaic. Such theories still retain some novel virtues: they make statistical mechanical probabilities unnecessary and they are more unified and less vague than their wave-function counterparts.

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