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2 **When optimal foragers meet in a game theoretical conflict: A model of kleptoparasitism**

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1

2 **Abstract**

3 Kleptoparasitism can be considered as a game theoretical problem and a foraging tactic at the
4 same time, so the aim of this paper is to combine the basic ideas of two research lines:
5 evolutionary game theory and optimal foraging theory. To unify these theories, firstly, we take
6 into account the fact that kleptoparasitism between foragers has two consequences: the
7 interaction takes time and affects the net energy intake of both contestants. This phenomenon
8 is modeled by a matrix game under time constraints. Secondly, we also give freedom to each
9 forager to avoid interactions, since in optimal foraging theory foragers can ignore each food
10 type (we have two prey types: either a prey item in possession of another predator or a free prey
11 individual is discovered). The main question of the present paper is whether the zero-one rule
12 of optimal foraging theory (always or never select a prey type) is valid or not, in the case where
13 foragers interact with each other?

14 In our foraging game we consider predators who engage in contests (contestants) and those who
15 never do (avoiders), and in general those who play a mixture of the two strategies. Here the
16 classical zero-one rule does not hold. Firstly, the pure avoider phenotype is never an ESS.
17 Secondly, the pure contestant can be a strict ESS, but we show this is not necessarily so. Thirdly,
18 we give an example when there is mixed ESS.

19

20 **Keywords:** ESS, food stealing, matrix game, time constraints, zero-one rule

21

1 **1. Introduction**

2 Kleptoparasitism is the stealing of already procured food by one individual from another
3 (Brockmann and Barnard 1979), and it is observed across several taxonomic groups, including
4 spiders (Coyle et al. 1991), insects (Erlandsson 1988), mammals (Janson 1985; Carbone et al.
5 2005), and birds (Barnard 1990). The advantage of kleptoparasitic behavior is that it allows
6 individuals to avoid some of the costs of the foraging cycle (searching for, acquiring and
7 handling food items) by exploiting food discovered by another individual's effort (Giraldeau
8 and Caraco 2000). Clearly, kleptoparasitism can be considered as a game theoretical problem
9 and a foraging tactic at the same time. Starting from this point, the aim of this paper is to
10 combine the basic ideas of two research lines.

11 The first research line is optimal foraging theory (Stephens and Krebs 1986). The main
12 assumptions of optimal foraging theory are the following:

- 13 a) the focal forager has all necessarily information about its prey (cf. omniscient forager
14 e.g. Schmidt and Brown 1996, and Garay and Móri 2010);
- 15 b) the focal forager has absolute control of its own food preferences, i.e. the forager freely
16 accepts or ignores any of its prey types (food items);
- 17 c) energy collection by a forager does not depend on the food preferences of other foragers,
18 and finally;
- 19 d) an individual's fitness is its net energy intake rate, which is given by the functional
20 response (Holling 1959, Jeschke et al. 2002). The overwhelming majority of the
21 derivation of functional responses (see e.g. Garay 2019) are based on the assumption
22 that either the prey density is renewed after each killing (Cressman et al. 2014,
23 McNamara et al. 2006) or the predators have no (or only a negligible) effect on prey
24 density during the duration of the foraging time (Garay and Móri 2010, Holling 1959),
25 thus classical optimal foraging theory assumes that the prey density is fixed.

1 In the prey choice model (where each forager has different prey types providing different energy
2 intakes and with different handling times), the basic result of optimal foraging theory is the
3 zero-one rule, which claims that a predator accepts a given prey type if its energy / handling
4 time ratio is bigger than the average intake rate on the whole foraging process (Charnov 1976).
5 In other words the predator either ignores or accepts a given prey type, so it never uses a mixed
6 prey preference.

7

8 The second research line is evolutionary game theory (Maynard Smith and Price 1973) focusing
9 on the fitness consequences of interaction between conspecifics, when individuals' behavior
10 have effects on the fitness of others, often through direct contests. In such contests, they
11 assumed that when two individuals encounter each other then they always play a game. Observe
12 that the latter assumption is not in harmony with the basic view of the optimal foraging, see
13 assumption c) above, where the individual can ignore any interaction with its prey types. In this
14 paper we concentrate on the case where each individual has freedom to interact or not to interact
15 with others it encounters, and each activity needs a period of time.

16 There are three points, which offer us a way to make a connection between the above two
17 research lines. Firstly, the functional response can take account of the interference between
18 predators, which has an effect on the functional response, since this interference takes time. De
19 Angelis' (1975) and Beddington's (1975) functional response takes account of the time duration
20 of the interactions between predators, but these interactions have no effect on the energy intake
21 of predators. In this paper we consider the case when this interference has an effect on the net
22 energy intake of predators as well, i.e. there are game theoretical conflicts between predators
23 for prey. Secondly, in the classical matrix model of evolutionary game theory, Maynard Smith
24 (1982) included a positive basic fitness, which is independent from the phenotypes (i.e. the
25 strategy of players), in order to avoid a negative total fitness. But "*There is no such thing as a*

1 *free lunch*". In biology, the collection of basic fitness at least needs time, as in optimal foraging
2 theory. Thus, the concept of time constraints gives us a way to introduce the "time cost" of
3 collecting the basic fitness of Maynard Smith (1982). Thirdly, the Nash principle can make a
4 bridge between game theory and optimal foraging theory, namely the zero-one rule and the
5 Nash-equilibrium condition are connected by the rule of time averages (Garay et al. 2015),
6 claiming that "*the optimal predator behavior involves those activities that ensure larger time*
7 *average intake than the time average of all activities*".

8 Furthermore, there are game theoretical models, which are related to the present paper. Firstly,
9 kleptoparasitism is modeled by ecological games with time constraints (e.g., Broom and
10 Ruxton, 1998; Broom et al., 2004, 2008, 2009, 2010; Broom and Rychtář, 2013; Sirot, 2000).
11 The models of Broom and colleagues are compartmental, where individuals follow a Markov
12 transition process between searching, handling and contesting states, with each behavior taking
13 (an exponential amount of) time. Unlike in the present paper, strategic decisions are made at
14 the transition stage, so a searching individual can decide whether to challenge a handler for a
15 food item, after which the handler decides whether to defend it, the winner being decided at
16 random, with no further decisions. The game is thus a type of sequential game. The model of
17 Sirot (2000) had a similar basis, but here individuals made simultaneous decisions when
18 contesting a food item. Secondly, the present paper builds on a general game-theoretical
19 modeling methodology, namely a matrix game under time constraints (Křivan and Cressman
20 2017, Garay et al. 2018a), when each interaction between players has a time duration. Matrix
21 games under time constraint are then characterized by two matrices, the intake matrix $A =$
22 $(a_{i,j})_{n \times n}$ and the time constraint matrix $T = (t_{i,j})_{n \times n}$, i.e. when the focal individual uses the
23 i -th pure strategy and its opponent the j -th one, the focal individual's payoff is $a_{i,j}$, and the focal
24 individual cannot play the next game during an average time duration $t_{i,j} \geq 0$. If this time
25 duration depends on the strategies that the players use in the interaction, then the matrix game's

1 evolutionary outcome is no longer given solely through its payoff matrix. Instead, an
2 individual's payoff is given at the stationary distribution of a Markov chain that depends on the
3 time constraint matrix. A similar process is followed for the more complex kleptoparasitism
4 model developed that follows.

5 The aim of this paper is to combine the basic ideas of optimal foraging theory and evolutionary
6 game theory with time constraints. A good combination of two theories should get back these
7 theories as special cases. Clearly, for this aim, we have to keep as many basic assumptions of
8 these theories as possible. From optimal foraging theory we keep the following three
9 assumptions:

- 10 1. The predators have no (or a negligible) effect on prey density during the foraging
11 time duration, so the prey density is fixed. In other words, we use one of the basic
12 assumptions of optimal foraging theory: prey renewal, see assumption d) above.
- 13 2. The predator is searching for food, and there are two types of food: (i) free food
14 means that there is no other predator nearby; (ii) not free food means that the
15 predator finds the food of a conspecific, but the acquired food has still not been
16 consumed by the killer. Here we assume the interaction is symmetric, i.e. there is no
17 ownership. In other words, when a predator kills a prey, then the "ownership" has
18 no effect on the behavior of the killer. The difference between a symmetric game,
19 e.g. hawk-dove game, and an asymmetric version of this game, the hawk-dove-
20 bourgeois game (Maynard Smith 1982), is well known.
- 21 3. As in optimal foraging theory, each forager can neglect all types of food. In other
22 words, when two predators have only one food item, the interaction between them
23 is not a must, as in the basic evolutionary matrix game model. If an individual can
24 evade the interactions, then this kind of individual has two extreme behaviors: either

1 it evades the interactions (thus collects “basic fitness” alone, i.e., only looking for
2 free food), or it interacts with others, i.e. plays a game.
3
4 Thus we will introduce a situation dependent sequential game with time constraints. The first
5 level gives the ratio of the materialization of the interaction. When two foragers encounter each
6 other and one of them has killed but not eaten a prey individual, then they either interact for
7 this killed prey (we call an individual playing this strategy a *contestant*) or they do not interact
8 (we call an individual playing this strategy an *avoider*). The avoider (non- contesting) strategy
9 means that before any interaction the avoider predator leaves the place, thus it has neither payoff
10 nor extra time cost. The second level of our sequential game describes the situation when both
11 foragers use the contestant strategy, and we consider the hawk-dove game as a mathematical
12 description of the interaction between predators, when they find the same food item. So the
13 hawk-dove game is a subgame in the sequential game introduced here. Now let us make clear
14 the difference between a non-contest and a non-fight. The contest but not-fight behavior is the
15 dove strategy, needing some extra time when interacting with a hawk and it has extra time and
16 some payoff when interacting with another dove strategy user. We will assume a symmetric
17 situation where all individuals can evade the interactions, so interaction takes place if and only
18 if both individuals want to play the game. We emphasize that this situation is a combination of
19 the basic problem of the optimal foraging theory (where the forager has a free decision on the
20 acceptance of any type of prey) and the matrix game under time constraints, since both methods
21 take account of the time constraints of different activities. In the present paper we will
22 investigate this combined model. The main question of the present paper is whether the zero-
23 one rule remains valid when the foragers interact with each other and assumption c) of optimal
24 foraging theory does not hold.

25

1 **2. Optimal foragers face game theoretical conflicts with others: a general monomorphic** 2 **model**

3 **2.1. Model description**

4 We start from an optimal foraging model (e.g. Stephens and Krebs 1986, Garay and Móri 2010),
5 but now we consider two types of food: free food that has not been found by a forager and food
6 in the possession of another forager (called discovered food). A focal individual forager begins
7 in the searching stage, the average time duration of which will be denoted by τ_s . During this
8 time, the focal forager can either find free food or discovered food. The model described here
9 concentrates on the following question: which foraging behavior is optimal, engaging in a
10 contest with the other forager over discovered food or avoiding contests by focusing only on
11 free food.

12 Firstly, consider the case where a searching focal forager has found free food without another
13 forager. Then it starts to handle the food item without consuming it (e.g. killing, transporting
14 the food, etc). We call this period the *vulnerable stage*, the average time duration of which will
15 be denoted by τ_v . This is the only stage where there is the possibility for the interaction with
16 another forager, one result of which may be the theft of the food item. During the vulnerable
17 stage, either the focal individual does not encounter a searching forager, or such an intruder
18 arrives from the whole population and these two individuals will or will not interact. If there is
19 no encounter, the focal forager passes to the *digestive stage*, the average time duration of which
20 will be denoted by τ_d . If there is an encounter, there are the following four conditional events.
21 (i) The focal individual does not retire and the intruder leaves. (ii) The focal individual retires
22 and the intruder does not. In both these cases, there is no interaction between them and the
23 forager who does not retire starts to digest the food in the digestive stage and the other returns
24 to the searching stage. (iii) Both the focal individual and the intruder retire, in which case there

1 is no interaction and each gets the food item with probability $\frac{1}{2}$. Finally, when (iv) neither the
 2 focal nor the intruder retire, they interact in a contest, called the subgame, which is modelled
 3 as a symmetric matrix game with time constraints. In the interaction in this subgame between
 4 the two foragers, one of them possesses the food item and digests it before returning to the
 5 searching stage while the other returns to the searching stage. Note that we split the standard
 6 notion of “handling time” into two stages, the vulnerable stage and the digestive stage (cf.
 7 Jeschke 2002). Moreover, all time durations are assumed to be independent and exponentially
 8 distributed.

9 Here we assume that the subgame is based on the classical hawk-dove game where pairs of
 10 foragers are engaged in a contest over the food item (i.e. the resource) of value B . Prior to the
 11 contest, neither forager has any information concerning the behavior (i.e. strategy) of the other
 12 forager. Moreover, we assume that this contest is symmetric (i.e., there is no ownership, so the
 13 winning probabilities of the contestants can only depend on the strategies they use, and not on
 14 which one discovered the food item and which is the intruder). The subgame is then specified
 15 as a matrix game under time constraints characterized by the following intake and time
 16 constraint matrices:

$$17 \quad A := \begin{pmatrix} \frac{B-C_{HW}-C_{HL}}{2} & B \\ 0 & \frac{B}{2} \end{pmatrix} \quad \text{and} \quad T := \begin{pmatrix} \frac{\tau_{HW}+\tau_{HL}}{2} & 0 \\ 0 & \frac{\tau_{DW}+\tau_{DL}}{2} \end{pmatrix},$$

18 where the entries of A (respectively, T) are the intake (respectively, time duration) of the row
 19 player when interacting with the column player. When two hawks interact, they engage in an
 20 escalated fight with one of them winning without getting injured and the other losing with
 21 injuries. This is reflected in matrix A where C_{HW} is the winner’s cost and C_{HL} is the losing
 22 hawk’s cost (including the cost of fighting and the cost of recovery). Moreover, τ_{XW}
 23 (respectively, τ_{XL} , $X = D, H$) is the time duration for the winner (respectively, loser) that is

1 associated with this interaction, including fighting and recovery time. When a hawk and dove
2 interact, the hawk gets the food item immediately (i.e. the time duration is 0), which accounts
3 for the off diagonal terms in matrices A and T . Finally, when two doves interact, there is no
4 fight (one wins the food item and the other loses) and the time duration is τ_{DW} for the winner
5 and τ_{DL} for the loser (they can differ, e.g., in the time of digestion). We emphasize that, from
6 the game theoretical perspective, the subgame is symmetric. Indeed, in hawk-hawk and dove-
7 dove interactions, both contestants win with the same probability (i.e. who wins the contest
8 does not depend on who discovered the food). Since all time durations are exponentially
9 distributed, the matrix T contains the means of these independent exponential random variables.

10 We note that here we follow the basic modelling methodology of our earlier paper (Garay et al.
11 2017) on matrix games with time constraints. Namely, the intake matrix A and the time
12 constraint matrix T are independent parameters and the time constraints decrease the number
13 of interactions between individuals. In essence, we build our model in two distinct steps. After
14 setting up a continuous time Markov chain, first we look for the stationary distribution of the
15 chain. This depends on the time constraint matrix. Then we calculate the average payoff
16 determined by the intake matrix at this equilibrium. Thus our model is a static one, similar to
17 the basic model of Maynard Smith and Price (1973), since we are interested in the set of
18 conditions under which a sufficiently rare mutant cannot invade the resident population, but we
19 are not interested in the dynamical frequency change of different phenotypes. In particular, we
20 do not use replicator dynamics (cf. Garay et al. 2018b, Varga et al. 2019).

21 Secondly, consider the case where the focal forager finds discovered food (i.e. food with
22 another forager who is in the vulnerable stage). In this case, the focal forager is the intruder,
23 and these two individuals will or will not interact, leading to a similar “story” to the one above.
24 If the focal individual leaves, then it starts a new search. If the focal individual does not leave
25 and the other forager retires, then the focal individual gets the food and enters the digestive

1 stage. (For the sake of simplicity, we assume throughout that at most one intruder can find a
2 given food item that is with a forager in the vulnerable stage; i.e., no sequence of encounters
3 can occur among foragers over the same food item.) When the focal individual does not leave
4 and the other forager does not retire, then the above subgame (a matrix game under time
5 constraints) takes place.

6 In this model, each forager has two types of decision. When a forager in the vulnerable stage
7 and an intruder encounter each other, they can choose to interact or not to interact. Their
8 strategies can be characterized by a real number $\sigma \in [0,1]$; namely, a σ -strategist is willing to
9 interact with probability σ . Observe that the subgame will be realized if and only if both foragers
10 are willing to interact. Furthermore, in the subgame under time constraints, an individual
11 forager can use a mixed strategy that can be described by a discrete probability distribution $p =$
12 (p_1, p_2) where p_1 (respectively p_2) is the probability that the forager plays hawk (respectively,
13 dove) in the subgame. A forager's phenotype is then characterized by its choice of σ and p .

14

15 **2.2 Mathematical model**

16 Suppose there are m phenotypes in the forager population with y_i the number of foragers with
17 phenotype i ($i = 1, \dots, m$). Then $y = y_1 + y_2 + \dots + y_m$ is the total number of foragers. An
18 individual forager, labelled as the ordered pair (i, j) , corresponds to the j -th forager (where $1 \leq$
19 $j \leq y_i$) of the phenotype i . An individual can be in one of the following stages at any moment:

- 20 • *searching stage*, denoted by s ,
- 21 • *vulnerable stage*, denoted by v ,
- 22 • *subgame stage*, denoted by $g(u, w)$ or simply by g . This means that the forager, using
23 pure strategy u in the subgame, is interacting with another forager who is using pure

1 strategy w . The duration of this stage depends on the strategies used and may differ for
2 the two contestants.

3 • *digestive stage*, denoted by d .

4 We emphasize that in our model the subgame stage includes digestion and hence it is not
5 followed by staying in the digesting stage. This is because the duration of digestion may depend
6 on the amount of food, and in a subgame we allow the contestants to share the food in an
7 undetermined proportion. Therefore, separating digestion from the subgame would make the
8 mathematical model significantly more complicated. In all other cases, i.e. when food is
9 acquired outside of a subgame, digestion always presumes a digestive stage. Note that the
10 subgame stage may include recovery from injuries, which can also be different for the
11 contestants.

12 Here we assume that there are n possible pure strategies a forager can use in the subgame (in
13 the model description of Section 2.1, $n = 2$). If a forager uses the pure strategy u against an
14 opponent using pure strategy w , its intake is $a_{u,w}$, and the average time it spends in the subgame
15 stage is $t_{u,w}$. Thus, following Garay et al. (2017), the subgame is characterized by the intake
16 matrix $A = (a_{i,j})_{n \times n}$ and the time constraint matrix $T = (t_{i,j})_{n \times n}$. Phenotype i is then
17 determined by the probability σ_i that such a forager is willing to interact in the subgame
18 combined with the strategy distribution vector $p_i = (p_{i1}, \dots, p_{in})$, where p_{iu} is the probability
19 that this phenotype uses the pure strategy u in the subgame; thus $\sum_{u=1}^n p_{iu} = 1$.

20 Further notations: Let x denote the number of food items in the habitat. Food is assumed to
21 regenerate at the same rate as it is consumed, thus x is assumed constant in time, in other words,
22 we assume food renewal. We introduce

23
$$\theta_i = \frac{y_i}{x}, 1 \leq i \leq m, \quad \theta = \frac{y}{x} = \sum_{i=1}^m \theta_i; \quad (1)$$

1 here θ_i is the number of foragers of phenotype i per one food item, and θ is the same quantity
2 with respect to all foragers, regardless of the phenotype. Let $\rho_{s,i}, \rho_{v,i}, \rho_{g,i}, \rho_{d,i}$ denote the
3 proportions of phenotype i in the searching, vulnerable, subgame, and digestive stages,
4 respectively. Moreover, let $\rho_s, \rho_v, \rho_g, \rho_d$ be the equivalent proportions for the whole population.
5 Clearly, $\rho_s = \sum_{i=1}^m \frac{y_i}{y} \rho_{s,i}$, and analogous equations can be established for the vulnerable,
6 subgame, and digestive stages.

7 The state of the population can be described with a vector of the form

$$8 \quad z = (z_{(1,1)}, \dots, z_{(1,y_1)} | z_{(2,1)}, \dots, z_{(2,y_2)} | \dots | z_{(m,1)}, \dots, z_{(m,y_m)}),$$

9 each coordinate being an element of the stage set $\{s, v, d\} \cup \{g(u, w) : u, w = 1, \dots, n\}$. Here
10 $z_{(i,j)}$ is the stage of individual (i, j) . Thus, the cardinality of the state space \mathcal{S} is $(3 + n^2)^y$,
11 since we have searching, vulnerable and digestive stages, and, in addition, the subgame stage
12 can be realized in n^2 different ways (pure strategy pairs). Let us introduce the following Markov
13 dynamics on the state space \mathcal{S} . In the state transitions we only indicate the coordinates that
14 change. An individual searching for food finds it with constant rate $\frac{1}{\tau_s}$, i.e. spends an average
15 time τ_s searching. In our Markov process all transitions occur at a constant rate, so all of our
16 events have durations that follow an exponential distribution with means corresponding to the
17 stated times, equivalently transitions out of these states occur at rates 1 divided by this time.
18 The possible transitions from the searching stage (listed in the first three following bullet points)
19 depend on whether the food is free or already discovered. The remaining bullet points describe
20 transitions from the other stages.

- 21 • $z_{(i,j)} : s \mapsto v$ with transition rate $\frac{x - \rho_v y}{x \tau_s} = \frac{1 - \rho_v \theta}{\tau_s}$
22 — individual (i, j) finds free food. Note that $1/x$ is the probability that a given searcher
23 finds a *prescribed* food item, thus $1/x \tau_s$ is the rate of this transition. There are $x - \rho_v y$

1 free food items, thus the probability that the food item found by the searcher is still free
 2 is $1 - \rho_v \theta$. (We keep the basic assumption of optimal foraging theory, namely, that the
 3 food density is fixed.)

4 • $z_{(i,j)}: s \mapsto g(u, w)$ and $z_{(k,\ell)}: v \mapsto g(w, u)$ with rate $\frac{1}{x\tau_s} \sigma_i \sigma_k p_{iu} p_{kw}$, where $(i, j) \neq$
 5 (k, ℓ)

6 — individual (i, j) finds food discovered by forager (k, ℓ) , both are willing to interact,
 7 and they use game strategies u and w , respectively.

8 • $z_{(i,j)}: s \mapsto d$ and $z_{(k,\ell)}: v \mapsto s$ with rate $\frac{\sigma_i(1-\sigma_k)}{x\tau_s} + \frac{1}{2} \frac{(1-\sigma_i)(1-\sigma_k)}{x\tau_s} = \frac{(1+\sigma_i)(1-\sigma_k)}{2x\tau_s}$

9 — the first term corresponds to the case where individual (i, j) finds food discovered
 10 by forager (k, ℓ) , the former is willing to interact but the latter is not. If both retire,
 11 then each has probability $\frac{1}{2}$ to win the food, thus the second term in the rate stands for
 12 the case where chance favors individual (i, j) . Only phenotype i receives an intake
 13 (which we will denote by G_i): $G_i = B$.

14 • $z_{(i,j)}: v \mapsto d$ with rate

$$22 \quad \frac{1}{\tau_v} + \frac{1}{x\tau_s} \sum_{k=1}^m \rho_{s,k} \gamma_k (1 - \sigma_k) \left(\sigma_i + \frac{1 - \sigma_i}{2} \right) = \frac{1}{\tau_v} + \frac{(1 - \bar{\sigma})(1 + \sigma_i)}{2\tau_s},$$

15 where $1 - \bar{\sigma} = \sum_{k=1}^m \rho_{s,k} \theta_k (1 - \sigma_k)$

16 — the first term corresponds to the case where no forager in the searching stage
 17 encounters individual (i, j) during its vulnerable stage. For the second term, a
 18 searching forager (the intruder) encounters individual (i, j) in the vulnerable stage but
 19 the intruder is not willing to interact. Then individual (i, j) moves to the digestive
 20 stage if either it is willing to interact or, if not, with probability $\frac{1}{2}$ it retains the food
 21 item. Intake: $G_i = B$.

- 1 • $z_{(i,j)}: d \mapsto s$ with rate $\frac{1}{\tau_d}$
- 2 — digestion is over.
- 3 • $z_{(i,j)}: g(u, w) \mapsto s$ with rate $\frac{1}{t_{u,w}}$
- 4 — a game played with strategies u and w is over. Intake: $G_i = a_{u,w}$

5 It is easy to see that this Markov chain is irreducible as every state communicates with the state
6 (s, \dots, s) , hence it has a unique stationary distribution. Similarly to as in Garay et al. (2017),
7 one can show that the random proportions $\rho_{s,i}, \rho_{v,i}, \rho_{g,i}, \rho_{d,i}$ converge to constants as the size
8 y of the population and the amount x of food increase to infinity in such a way that the ratios
9 θ_i converge.

10 This result allows us to define a game among the m phenotypes where the payoff is taken as
11 the intake rate at the stationary equilibrium and to do this we need to consider cycles.

12 In what follows, we will focus on a sufficiently large equilibrium population with a single
13 (resident) phenotype, where a mutant phenotype appears. In this general monomorphic model,
14 we then take $m = 2$ in the above mathematical model. Consider an arbitrary focal forager,
15 resident or mutant. We will distinguish its strategy parameters, σ^* and p^* , by asterisks. The
16 population parameters then have no asterisks. Let us call a sequence of consecutive stages a
17 *cycle* if it lasts from the beginning of a searching stage to the next searching. What's going on
18 during a cycle?

- 19 • 1) The focal individual is searching until it finds food, and the average searching time
20 is τ_s . At the end of searching
- 21 • 2a) The focal individual finds free food with probability $1 - \rho_v \theta$. Then it moves to the
22 vulnerable stage. Its average time length is

9
$$\frac{1}{\frac{1}{\tau_v} + \frac{\rho_s \theta}{\tau_s}} = \frac{\tau_s \tau_v}{\tau_s + \rho_s \theta \tau_v}, \quad (2)$$

1 because the length of the vulnerable stage is the minimum of two independent
 2 exponential time spans, one of them is the length of the uninterrupted vulnerable
 3 period, and the other one is the time needed by the fastest searcher to find the focal
 4 individual. As is well-known, the minimum of two independent, exponentially
 5 distributed random variables is also exponential, with expectation being half of the
 6 harmonic mean of the two expectations (equivalently, with hazard rate being the sum
 7 of the two hazard rates). According to this, at the end of vulnerable stage there are two
 8 possibilities.

- 10 ○ Either the focal individual starts digesting, with probability

12
$$\frac{\frac{1}{\tau_v}}{\frac{1}{\tau_v} + \frac{\rho_s \theta}{\tau_s}} = \frac{\tau_s}{\tau_s + \rho_s \theta \tau_v},$$

11 average time τ_d , and intake B ,

- 13 ○ or it meets an intruder with probability

14
$$1 - \frac{\tau_s}{\tau_s + \rho_s \theta \tau_v} = \frac{\rho_s \theta \tau_v}{\tau_s + \rho_s \theta \tau_v}.$$

15 Note that the occurrence of these possibilities is (stochastically) independent of
 16 the length of the vulnerable period, i.e. knowing the length of any occurrence of
 17 the period (as opposed to its expectation τ_v) provides no information on which
 18 event will occur.

19 Then, from the point of view of the focal individual, the following outcomes are
 20 possible.

- 1 ▪ The focal individual is not willing to interact but the intruder is. This has
2 probability $\sigma(1 - \sigma^*)$ and leads to no additional time, and zero intake.
- 3 ▪ The focal individual is willing to interact but the intruder is not, which
4 happens with probability $\sigma^*(1 - \sigma)$. Then the focal individual receives
5 intake B and moves to the digestive stage with average time τ_d .
- 6 ▪ Neither the focal nor the intruder are willing to interact. Such a case
7 occurs with probability $(1 - \sigma^*)(1 - \sigma)$. Here the whole food item is
8 taken by one of them, with equal probability for each. The luckier one
9 moves to the digestive stage, the other to the searching stage. Thus the
10 average time left for the focal individual in the cycle is $\tau_d/2$, and its
11 average intake is $B/2$.
- 12 ▪ Both the focal individual and the intruder are willing to interact,
13 occurring with probability $\sigma^*\sigma$. The average time for the game is p^*Tp ,
14 and the average intake is p^*Ap .
- 15 • 2b) Alternatively, the focal individual finds previously discovered food with
16 probability $\rho_v\theta$. Then the following scenarios are possible.
- 17 ○ The focal individual is not willing to interact but the intruder is. This has
18 probability $(1 - \sigma^*)\sigma$. There is no additional time and zero intake.
- 19 ○ The focal individual is willing to interact but the intruder is not, with probability
20 $\sigma^*(1 - \sigma)$. The focal individual starts digesting with average time τ_d and intake
21 B .
- 22 ○ Neither the focal individual nor the intruder are willing to interact, with
23 probability $(1 - \sigma^*)(1 - \sigma)$. The focal individual spends average time $\tau_d/2$
24 digesting, with average intake $B/2$.

- 1 ○ Both the focal individual and the intruder are willing to interact. The probability
2 of this possibility is $\sigma^* \sigma$, and the average time and average intake are $p^* T p$, and
3 $p^* A p$, respectively.

4 After all of the above, the cycle starts over again. Let τ^* denote the average time of the focal
5 individual's cycle. It has the following components.

- 6 • searching stage with average length τ_s ,
7 • vulnerable stage with average length $\tau_v \pi_d$, where π_d is the probability that free food
8 is found and no intruders arrive, namely,

$$9 \quad \pi_d = \frac{\tau_s}{\tau_s + \rho_s \theta \tau_v} (1 - \rho_v \theta),$$

- 10 • subgame stage with average length $\pi_c \sigma^* \sigma p^* T p$, where

$$16 \quad \pi_c = \frac{\rho_s \theta \tau_v}{\tau_s + \rho_s \theta \tau_v} (1 - \rho_v \theta) + \rho_v \theta = \frac{\theta (\rho_s \tau_v + \rho_v \tau_s)}{\tau_s + \rho_s \theta \tau_v}$$

11 can be interpreted as the probability of getting into a contest situation (i.e., where two
12 individuals, one with food and another without it, meet): the first term stands for the
13 case where an intruder appears, and the second one for the case where the searching
14 focal individual finds previously discovered food. A contest situation leads to a
15 subgame if and only if $\sigma \sigma^* \neq 0$.

- 17 • digesting stage of average length $\tau_d \left(\pi_d + \pi_c \frac{(1-\sigma)(1+\sigma^*)}{2} \right)$. The first term corresponds to
18 the case where free food is found and no intruders come, and the second term stands for
19 the case where in an encounter food is taken without a contest. The multiplier of π_c in
20 the above is $\frac{(1-\sigma)(1+\sigma^*)}{2} = (1-\sigma)\sigma^* + \frac{(1-\sigma)(1-\sigma^*)}{2}$, where the first term comes from the
21 case where the focal would fight but the intruder would not, and the second term comes
22 from the case where both retire and the food is awarded randomly. Neither of these cases
23 correspond to a subgame.

1 Thus

$$2 \quad \tau^* = \tau_s + \tau_v \pi_d + \pi_c \sigma^* \sigma p^* T p + \tau_d \left(\pi_d + \pi_c \frac{(1-\sigma)(1+\sigma^*)}{2} \right).$$

3 The average amount of food taken by the focal individual during one cycle is

$$4 \quad G^* = \left(\pi_d + \pi_c \frac{(1-\sigma)(1+\sigma^*)}{2} \right) B + \pi_c \sigma^* \sigma p^* A p.$$

5 In order to characterize the equilibrium, let the focal individual belong to the resident
6 population, i.e., there is no need for asterisks, as all quantities tagged with asterisks are equal
7 to their unmarked counterparts. Then the proportions of individuals in searching, vulnerable,
8 subgame, or digestive stages, respectively, are equal to the proportions of time spent in those
9 stages during one cycle. Thus, in equilibrium we have

$$10 \quad \tau_s = \rho_s \tau, \quad \tau_v \pi_d = \rho_v \tau, \quad \pi_c \sigma^2 p T p = \rho_g \tau.$$

11 The fourth equation is omitted, because it follows from the preceding three. In detail, we obtain
12 the following system of quadratic equations in the two variables ρ_s and ρ_v ;

$$13 \quad \tau_s^2 + \rho_s \theta \tau_s \tau_v = \rho_s \tau_s (\tau_s + \tau_v + \tau_d) \\ 14 \quad + \rho_s^2 \theta \tau_v \left(\tau_s + \sigma^2 p T p + \frac{1-\sigma^2}{2} \tau_d \right) \\ 15 \quad + \rho_s \rho_v \theta \tau_s \left(-(\tau_v + \tau_d) + \sigma^2 p T p + \frac{1-\sigma^2}{2} \tau_d \right), \quad (3a)$$

$$16 \quad \tau_s \tau_v - \rho_v \theta \tau_s \tau_v = \rho_v \tau_s (\tau_s + \tau_v + \tau_d) \\ 17 \quad + \rho_v^2 \theta \tau_s \left(-(\tau_v + \tau_d) + \sigma^2 p T p + \frac{1-\sigma^2}{2} \tau_d \right) \\ 18 \quad + \rho_s \rho_v \theta \tau_v \left(\tau_s + \sigma^2 p T p + \frac{1-\sigma^2}{2} \tau_d \right). \quad (3b)$$

19 After this system is solved, the third equation provides us an explicit formula for ρ_g in terms of
20 ρ_s and ρ_v . Since an irreducible, continuous time, finite state space Markov chain always has a
21 stationary distribution, this system does have a feasible solution. Though the stationary

1 distribution is unique, it does not necessarily imply the uniqueness of the solution of our system
 2 of equations.

3 To illustrate our model, we use the game tree method introduced earlier by Cressman et al.
 4 (2014). Game trees provide a way to describe the forager's behavior in detail, based on the
 5 sequence of its choices at different decision points. The game tree describes all possible
 6 foraging situations, which start from the beginning of the search for food (the root of the tree),
 7 and end at different random events (the leaves of the tree). In the illustration, we will consider
 8 two types of focal foragers, i.e. we consider polymorph model for visualization.¹ The first type,
 9 called an avoider (see Figure 1), is a forager who is never willing to interact (i.e. $\sigma = 0$). The
 10 second type, called a contestant (see Figure 2), is a forager who is always willing to interact
 11 (i.e. $\sigma = 1$). Accordingly, we will use the notations $\rho_{s,a}, \rho_{v,a}, \rho_{g,a}, \rho_{d,a}, \theta_a$ (respectively $\rho_{s,b},$
 12 $\rho_{v,b}, \rho_{g,b}, \rho_{d,b}, \theta_b$) instead of $\rho_{s,1}, \rho_{s,2}, \rho_{v,1}, \rho_{v,2}, \theta_1, \theta_2$ etc. for the avoider (respectively,
 13 contestant). We call the reader's attention to the fact that it is not assumed here that at least one
 14 of these phenotypes is arbitrary rare.

15 In *Figure 1* we consider a focal individual that adopts the avoider strategy in all foraging turns.
 16 This individual encounters a food item discovered by another avoider with rate $\rho_{v,a}\theta_a$, see (1).
 17 Similarly, the focal avoider encounters a food item discovered by a contestant and free food
 18 with rates $\rho_{v,b}\theta_b$ and $1 - \rho_{v,a}\theta_a - \rho_{v,b}\theta_b$, respectively. The time spent in the vulnerable stage
 19 is the minimum of two independent, exponentially distributed random variables, as in (2), so it
 20 has mean

$$21 \quad \tau_m = \frac{1}{\frac{1}{\tau_v} + \frac{\rho_{s,b}\theta_b + \rho_{s,a}\theta_a}{\tau_s}}$$

¹ Our model is monomorphic, since each individual can use a mixed strategy, i.e. each one can use all pure strategies with a genetically fixed probability.

1 There is no interaction when no intruder arrives during the vulnerable stage of the focal avoider,
2 that is, when the focal individual can pass to the digestive stage before meeting a searcher. This
3 happens with probability τ_m/τ_v . In this case the focal avoider starts digesting its free prey, so
4 in this particular foraging turn, the focal avoider spends time $\tau_s + \tau_m + \tau_d$ and gets benefit B .

5 Next, consider the possibilities of interactions. Firstly, let us start with the case where the focal
6 avoider is in the vulnerable stage and another individual arrives in the meantime. This happens
7 with probability $1 - \frac{\tau_m}{\tau_v}$. The intruders must be in searching stage. The probabilities that the
8 intruder plays the subgame or uses the avoider strategy are proportional to the frequencies of
9 the corresponding phenotypes, that is, a contestant individual arrives with probability
10 $\frac{\rho_{s,b}\theta_b}{\rho_{s,b}\theta_b + \rho_{s,a}\theta_a}$. If a contestant arrives, it takes the focal avoider's prey, thus the focal individual
11 is left without prey and in this particular foraging turn the focal individual spends $\tau_s + \tau_m$ time
12 on average. On the other hand, an avoider individual arrives with probability $\frac{\rho_{s,a}\theta_a}{\rho_{s,b}\theta_b + \rho_{s,a}\theta_a}$, and
13 after the encounter, without a subgame occurring, one of them gets the prey and starts digestion,
14 each with probability $1/2$, so the average time duration and benefit are $\tau_s + \tau_m + \frac{\tau_d}{2}$ and $B/2$
15 . When a focal avoider finds prey with a contestant, the focal individual retires and immediately
16 starts a new search, thus the time duration of this kind of foraging turn is just τ_s . Finally, when
17 a focal avoider finds prey with another avoider, no contest follows, and both parties have the
18 same chance to take the whole prey. Thus the focal individual spends time $\tau_s + \frac{\tau_d}{2}$ and gets
19 $B/2$ on average.

20 In *Figure 2* we consider a focal individual that follows the contestant strategy ($\sigma = 1$) in all
21 foraging turns. Differences only appear on the leaves of the tree. The leftmost leaf (no intruder
22 arrives) is the same as in the case of a focal avoider. When the focal contestant is in the
23 vulnerable stage and another contestant arrives, they start to play the matrix game with time

1 constraints, so in this particular foraging turn the focal contestant spends time $\tau_s + \tau_m + pT\tau$
 2 on average and its average intake is pAp . If the intruder is an avoider, then no contest (subgame)
 3 begins: the focal contestant gets the prey and starts digesting, so the average time duration and
 4 benefit are $\tau_s + \tau_m + \tau_d$ and B , respectively. Similarly, when a focal contestant finds a prey
 5 with another contestant, they start to play the game immediately, so this particular foraging turn
 6 takes an average $\tau_s + pT\tau$ of the focal contestant's time, and the focal individual gets pAp .
 7 Finally, when the discovered prey is with an avoider, the focal contestant takes the prey and
 8 starts to digest it, so it only spends time $\tau_s + \tau_d$ and gets benefit B .

9

10 2.3 Strict ESS

11 We say that the resident phenotype is strictly evolutionarily stable if for an arbitrary focal
 12 different from the resident we have

$$13 \quad \frac{G^*}{\tau^*} < \frac{G}{\tau},$$

14 that is, the resident phenotype maximizes the average intake per time unit among all possible
 15 phenotypes, and this maximum is unique. This is equivalent to maximizing the long-term payoff
 16 of the individual, the standard measure of evolutionary success. We note that an alternative way
 17 of approaching this problem was developed in Krivan and Cressman (2017). The fact that these
 18 two methods are actually equivalent was shown in Broom et al. (2019).

19 **Claim.** *If the resident phenotype is strictly evolutionarily stable, then $\sigma = 1$.*

20 *Proof.* Let $p^* = p$. Then the focal individual's average intake per time unit can be written in
 21 the following form:

$$22 \quad \frac{G^*}{\tau^*} = \frac{Q_1 + Q_2\sigma^*}{Q_3 + Q_4\sigma^*} =: f(\sigma^*),$$

1 where the coefficients are positive, namely

$$\begin{aligned}
2 \quad Q_1 &= \left(\pi_d + \pi_c \frac{1-\sigma}{2}\right) B, & Q_2 &= \pi_c \left(\frac{1-\sigma}{2} B + \sigma p^* A p\right), \\
3 \quad Q_3 &= \tau_s + (\tau_v + \tau_d) \pi_d + \tau_d \pi_c \frac{1-\sigma}{2}, & Q_4 &= \pi_c \left(\frac{1-\sigma}{2} \tau_d + \sigma p^* T p\right).
\end{aligned} \tag{4}$$

3 This is a linear rational function of σ^* , hence monotone. Thus, if $0 < \sigma < 1$, there exists a
4 mutant with $\sigma^* \in \{0, 1\}$ which is at least as good as the resident. This is excluded by
5 supposition. If $\sigma = 0$, then $Q_2 Q_3 - Q_1 Q_4 = \frac{1}{2} B \pi_c (\tau_s + \tau_v \pi_d) > 0$, so the function $f(\sigma^*)$ is
6 strictly increasing, therefore the resident can be outperformed by choosing $\sigma^* = 1$. ■

7

8 Next we show an example of a strictly evolutionarily stable phenotype.

9 **Example 1.** Suppose the matrices T and A have unique smallest and largest elements, resp.,
10 at the same diagonal position, say

$$11 \quad t_{11} = t < \min \{t_{ij}: (i, j) \neq (1, 1)\}, \quad a_{11} = a > \max \{a_{ij}: (i, j) \neq (1, 1)\}.$$

12 Then $p = (1, 0, \dots, 0)$ is optimal: $p^* T p > t = p T p$ and $p^* A p < a = p A p$ for every $p^* \neq p$.

13 Let the resident phenotype be defined by $p = (1, 0, \dots, 0)$ and $\sigma = 1$. Then

$$\begin{aligned}
14 \quad G\tau^* - G^*\tau &= [\pi_d B + \pi_c p A p][(\tau_s + (\tau_v + \tau_d) \pi_d) + \pi_c \sigma^* p^* T p] \\
15 \quad &\quad - [\pi_d B + \pi_c \sigma^* p^* A p][(\tau_s + (\tau_v + \tau_d) \pi_d) + \pi_c p T p].
\end{aligned}$$

16 This is a linear function of σ^* , thus it suffices to check its positivity at $\sigma^* = 0$ and $\sigma^* = 1$.

17 If $\sigma^* = 1$, then

$$\begin{aligned}
18 \quad G\tau^* - G^*\tau &= \pi_c [(\tau_s + (\tau_v + \tau_d) \pi_d)(p A p - p^* A p) + \pi_c (p A p p^* T p - p^* A p p T p) \\
19 \quad &\quad + \pi_d B (p^* T p - p T p)] \geq 0,
\end{aligned}$$

20 and equality holds if and only if $p^* = p$. If $\sigma^* = 0$, then

$$1 \quad G\tau^* - G^*\tau = \pi_c[(\tau_s + (\tau_v + \tau_d)\pi_d)pAp - \pi_d B pTp].$$

2 This is obviously positive if $(\tau_v + \tau_d)pAp > B pTp$, which can be achieved by suitably
 3 choosing a and t . Then the resident phenotype is evolutionarily stable.

4

5 2.4 Mixed ESS

6 Of course, a strictly evolutionarily stable phenotype does not necessarily exist. For example, if
 7 the matrices T and A are given in such a way that $a_{ij} = a$ and $t_{ij} = t$ for every $i, j \in \{1, 2, \dots, n\}$,
 8 then game strategy p is indifferent. Therefore, two phenotypes are equivalent if they have the
 9 same contesting probability σ . Thus no phenotype can be strictly evolutionarily stable. Apart
 10 from this trivial case, if the duration of the game is very long, and the reward is small, it is not
 11 worth contesting. Our second example presents a case where a strictly evolutionarily stable
 12 phenotype cannot exist.

13

14 **Example 2.** Consider a model where $\theta < 1$, that is, there is more food than individuals. It is
 15 easy to see that

$$16 \quad \pi_d = \frac{\tau_s}{\tau_s + \rho_s \theta \tau_v} (1 - \rho_v \theta) \geq \frac{\tau_s(1 - \theta)}{\tau_s + \theta \tau_v}.$$

17 Though π_d depends on σ through ρ_s and ρ_v , this estimate does not. Let T and A be defined as
 18 in Example 1. Clearly, if $p \neq (1, 0, \dots, 0)$, then the phenotype given by $\sigma = 1$ and p cannot be
 19 evolutionarily stable, as the mutant with $\sigma^* = 1$ and $p^* = (1, 0, \dots, 0)$ is better. If $p = (1, 0, \dots, 0)$
 20 and $\sigma = 1$, then for $p^* = p$ and $\sigma^* = 0$ we have already shown that

$$21 \quad G\tau^* - G^*\tau = \pi_c[(\tau_s + (\tau_v + \tau_d)\pi_d)pAp - \pi_d B pTp].$$

22 Recalling the lower estimate for π_d we can see that $G\tau^* - G^*\tau < 0$, if

1
$$[\tau_s(\tau_s + \theta\tau_v) + (\tau_v + \tau_d)\tau_s(1 - \theta)]pAp - \tau_s(1 - \theta)B pTp < 0,$$

2 that is,

3
$$(\tau_s + \tau_v)pAp < (1 - \theta)(B pTp - \tau_d pAp). \quad (5)$$

4 Suppose

5
$$(\tau_s + \tau_v + \tau_d)a < Bt, \quad \theta < \frac{Bt - (\tau_s + \tau_v + \tau_d)a}{Bt - \tau_d a}. \quad (6)$$

6 Then $(\tau_s + \tau_v)a < (1 - \theta)(Bt - \tau_d a)$, that is, (5) holds, therefore a mutant with $p^* = p$ and
 7 $\sigma^* = 0$ is strictly better than the resident. Thus, in this model there does not exist a strictly
 8 evolutionarily stable phenotype.

9

10 2.5 Weak ESS

11 We can also define the weak evolutionary stability property of phenotype (p, σ) . It means that
 12 for an arbitrary focal with (p^*, σ^*) we have $\frac{G^*}{\tau^*} \leq \frac{G}{\tau}$. In Example 2, though there exist no strictly
 13 evolutionarily stable phenotypes, still there may be one in the weaker sense. Again, $p^* = p =$
 14 $(1, 0, \dots, 0)$ can be assumed, thus $p^*Ap = pAp = a$ and $p^*Tp = pTp = t$. For a weakly
 15 evolutionarily stable σ one has to solve the equation $Q_1Q_4 - Q_2Q_3 = 0$. It looks quadratic, but
 16 in fact it is not, because π_d and π_c also depend on σ through ρ_s and ρ_v , which are only implicitly
 17 given. Fixing $a, t, B, \tau_s, \tau_v, \tau_d$ so that the conditions of Example 2 are satisfied one solves the
 18 equation numerically, by computing $Q_1Q_4 - Q_2Q_3$ for σ running from 0 to 1. We shall see that
 19 for suitably choosing $a, t, B, \tau_s, \tau_v, \tau_d$ the existence of a weakly evolutionarily stable σ can be
 20 realized.

21

1 **Example 3.** For the sake of simplicity, we suppose $\tau_s = \tau_v = \tau_d = 1$, that is, all time durations
2 are identically distributed, namely, exponential with mean 1, and let $a = 1$. Set the positive
3 parameters t, B, θ , such that they satisfy (6) (i.e. $Bt > 3$ and $\theta < \frac{Bt-3}{Bt-1}$). With $C = \sigma^2 t + \frac{1-\sigma^2}{2}$,
4 equations (3a) and (3b) take the form:

$$(3 - \theta)x + \theta(C + 1)x^2 + \theta(C - 2)xy = 1 ,$$

$$(3 + \theta)y + \theta(C - 2)y^2 + \theta(C + 1)xy = 1 ,$$

7 where x and y stand for ρ_s and ρ_v respectively. For σ fixed between 0 and 1, we find numerically
8 the unique positive solution of this system of two quadratic equations in x and y that satisfy $x +$
9 $y < 1$. We then plot the function

$$\begin{aligned} \varphi(\sigma) = & \left(1 - \theta y + \theta \frac{1-\sigma}{2}(x+y)\right) B \left(\frac{1-\sigma}{2} + \sigma t\right) \\ & - \left(3 + \theta x - 2\theta y + \theta \frac{1-\sigma}{2}(x+y)\right) \left(\frac{1-\sigma}{2} B + \sigma\right), \end{aligned}$$

12 which is equal to a positive multiple $\frac{(1+x\theta)^2}{\theta(x+y)}$ of $Q_1 Q_4 - Q_2 Q_3$. When $t = 1, B = 4$ and $\theta =$
13 0.1, Figure 3 shows that there is a mixed solution $\sigma \cong 0.81$ that satisfies the weak evolutionary
14 stability property.

15 By Example 3, the zero-one rule is not valid in general since the ESS phenotype is ready to
16 contest with probability 0.81 (i.e. the expected outcome is a mixed ESS).

17

18

19 3. Conclusion

1 Through considering the functional response, we can see that kleptoparasitism is a special
2 interference between foragers, which does not only take time but also has an effect on the net
3 energy intake of both forager individuals. Thus, kleptoparasitism is an excellent example for a
4 foraging game (e.g. Filippi and Nomakuchi 2016, Sirot 2000, Spencer and Broom 2018).
5 Furthermore, it is also a good example for the game with time constraints, for instance the
6 victim not only lost its acquired food item but also the time it has spent to get this food item
7 before it was stolen. That is, although we only formally introduce time constraints in the
8 subgame, it is clear that time constraints also play an important part in other stages of the
9 sequential game.

10 The novelty of the present work is that we make a bridge between two theoretical research lines:
11 optimal foraging theory and a sequential evolutionary game theory with time constraints. One
12 of the basic ideas of optimal foraging theory is that the densities of different prey types
13 determine the optimal foraging tactics. In our game theoretical model a similar effect takes
14 place. For instance, in Example 2 we found that if there is more food than individuals (i.e. the
15 free food is abundant enough) then there is no strict ESS. Furthermore, in our model, although
16 there is only one prey species, there are different prey types (as a free food item, but also as a
17 food item at the vulnerable stage of foragers, moreover according to which phenotype acquired
18 the prey). During our investigation the relative frequency of these different food items implicitly
19 determines the evolutionary stability (see the role of θ_i in the main text and Examples). In this
20 sense, the above basic ideas of optimal foraging theory are transferred to the game theory.
21 Moreover, the Nash solution concept in our game (where the payoff is the ratio of average
22 intake to the average time duration of one of the foraging cycle) is equivalent with the rule of
23 time averages (Garay et al. 2015), claiming that “*the optimal predator behavior involves those*
24 *activities that ensure larger time average intake than the time average of all activities*”. Thus

1 the time durations of different activities also have effect on the optimal behavior in the game
2 with time constraints.

3 In our game, where the interactions between predators have an effect on their net energy
4 intake and need extra time, we found that the classical zero-one rule is not valid. Firstly, the
5 avoider phenotype ($\sigma = 0$) is never an ESS, since if there are only avoiders in the resident
6 population (that is, e.g. the resident never contests), the mutant always get an advantage by
7 stealing the resident's food. Secondly, we point out that the contestant ($\sigma = 1$) can be a strict
8 ESS, but is not necessarily one. Thirdly, the contestant ($\sigma = 1$) is sometimes not an ESS,
9 since if the average time duration of the game is very long, and the reward is small, it is not
10 worth contesting. Fourthly, we give an example where a mixed ESS does exist. The
11 kleptoparasitism models of Broom and colleagues did not generally produce mixed solutions,
12 as have been produced here. A key reason for this was the sequential nature of decisions in
13 that model. The challenger decides their choice first and then the defender responds, and if the
14 challenger decided not to challenge then the defender automatically keeps their food. We note
15 that simple sequential games generally have only pure solutions (see Broom and Rychtář,
16 2013). These food stealing games (see e.g., Broom and Ruxton, 1998; Broom et al., 2004) are
17 not simple but affected by population density. However, the effect of density is destabilizing
18 for mixtures. If all individuals fight then the effective foraging rate is low, meaning the value
19 of any given food item is effectively higher, making it more attractive to fight for. Thus more
20 than one ESS was common. The exceptions that produced mixed strategies were Broom et al.
21 (2008, 2009), where individuals which did not attempt to search for conspecifics had a higher
22 rate of finding free food than others, whereas in the other models the efficiency of food
23 finding was assumed the same for all individuals. In the present paper individuals make
24 simultaneous decisions, and they do it without making a distinction in whether they are the
25 challenger or the defender (since our model is symmetric without ownership), in a similar way

1 to Sirot (2000), and so can similarly obtain a mixed solution. We note that there are a number
2 of differences in the current model and the Sirot (2000) model. In the latter Dove versus Dove
3 contests took no time (as in Broom et al., 2004, although there it arose naturally as there was
4 no contest), whereas in the current paper it does, in the spirit of the “war of attrition” game
5 (see Maynard Smith, 1982). Sirot (2000) also effectively had a simplifying assumption for the
6 payoffs, where the value of a reward compared to the cost of a fight was independent of the
7 population strategy, which is not made here (or in the Broom et al. (2004) models).

8 Although we concentrate on a theoretical symmetrical selection situation, we think the game-
9 tree method (Cressman et al. 2014) can handle other biological situations, as well. For instance,
10 two different types of asymmetry occur in kleptoparasitism. The first one takes place within the
11 same species, namely ownership, which may have effect on the behavior of owner, like the
12 bourgeois strategy (Maynard Smith 1982). The effects of ownership on the evolutionary
13 outcome when, unlike kleptoparasitism, it is only interaction times that are strategy dependent,
14 were investigated by Cressman and Krivan (2019). The second one is when kleptoparasitism
15 occurs between different species (e.g. Balme et al. 2017, Garthe and Hüppop 1998). These types
16 of asymmetry (ownership and/or multispecies interactions) can be modelled by the game-tree
17 method, but the analysis of these asymmetric games will need more investigation and is left to
18 future research.

19

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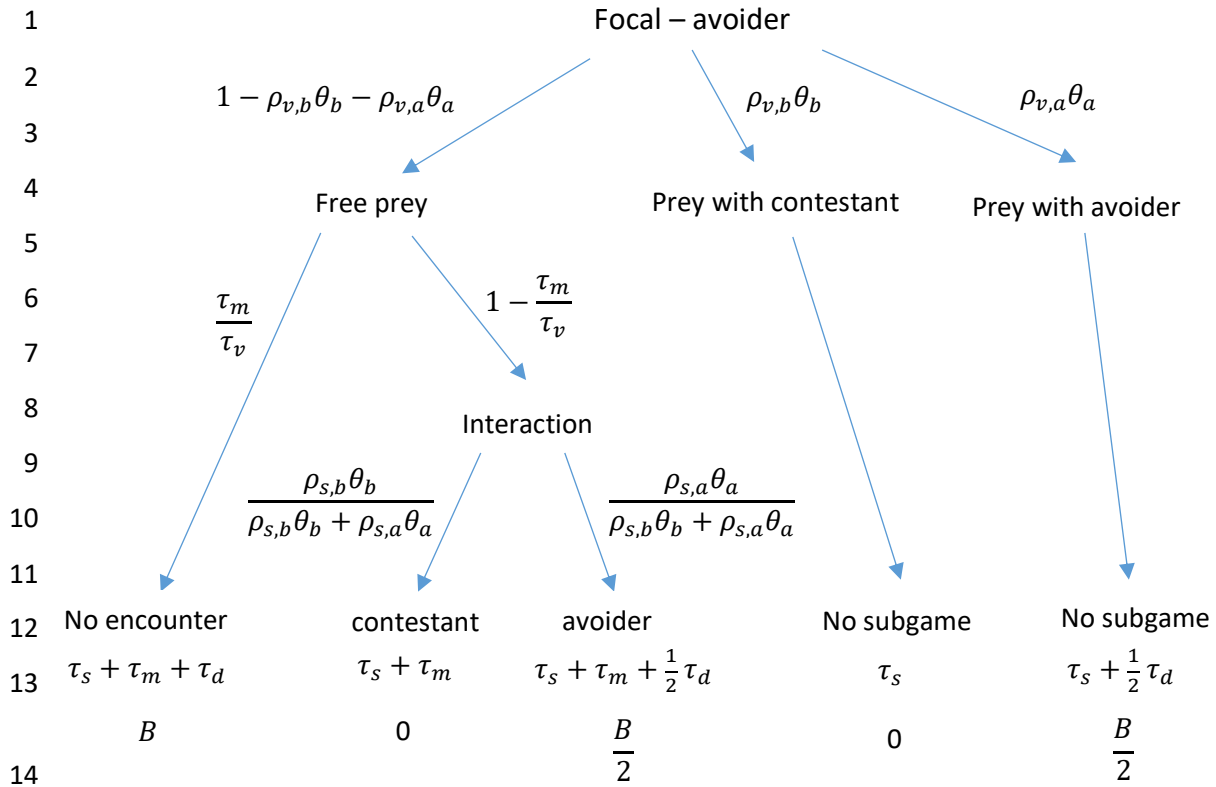
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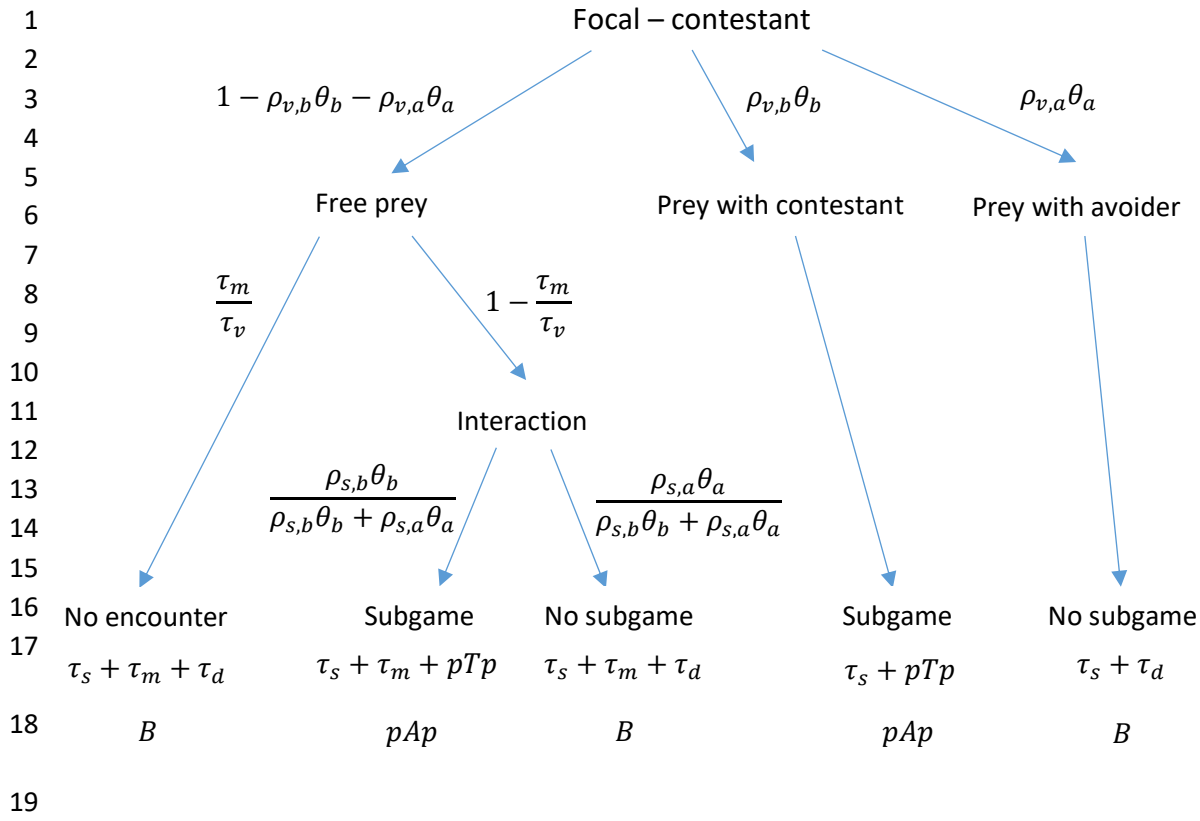
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- 21



15 *Figure 1.* Game tree of a focal individual following the avoider strategy. On the leaves the
 16 average time durations of the corresponding foraging turns (upper row), and the average
 17 intakes (lower row), are exhibited. For the notations see the main text.

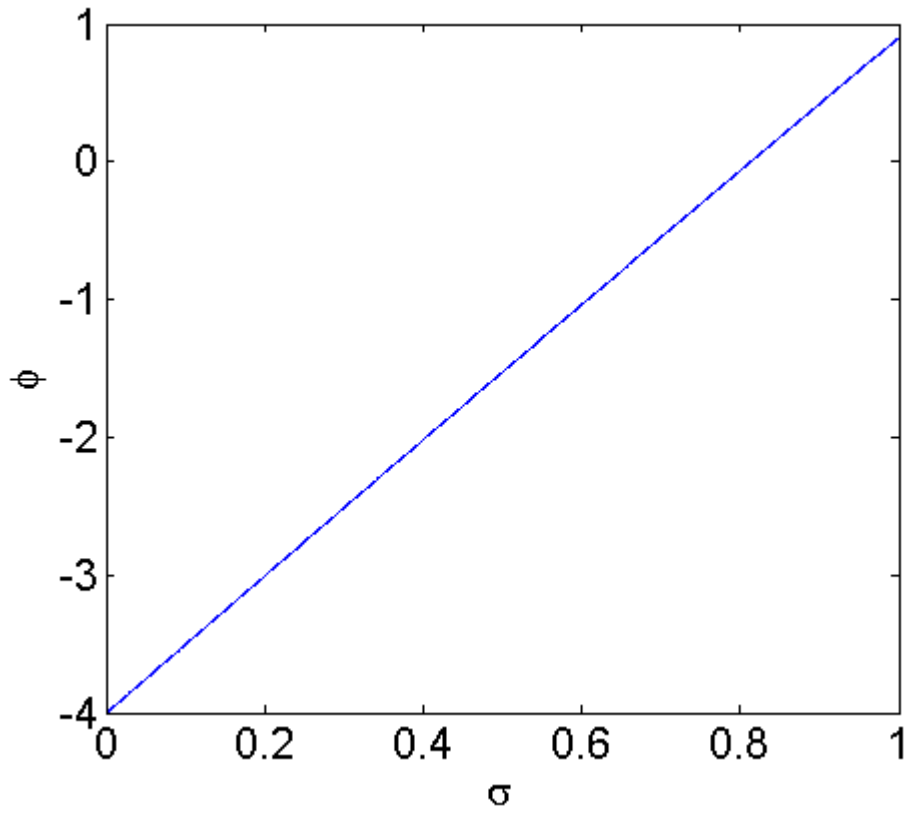
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20 *Figure 2.* Game tree of a focal individual following the contestant strategy. On the leaves the
 21 average time durations of the corresponding foraging turns (upper row), and the average intakes
 22 (lower row), are exhibited. For the notations see the main text.

23



1

2

3 *Figure 3.* (see Example 3) By setting $t = 1, B = 4$ and $\theta = 0.1$, the graph shows $\varphi(\sigma) = 0$ at
4 approximately $\sigma = 0.81$.

5