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Form-field Gauge Symmetry in M–theory

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Abstract: We show how to cast an interacting system of M–branes into manifestly gauge-invariant form using an arrangement of higher-dimensional Dirac surfaces. Classical M–theory has a cohomologically nontrivial and noncommutative set of gauge symmetries when written using a "doubled" formalism containing 3-form and 6-form gauge fields. We show how the arrangement of Dirac surfaces allows an integral subgroup of these symmetries to be preserved at the quantum level. The proper context for discussing these large gauge transformations is relative cohomology, in which the 3-form transformation parameters become exact when restricted to the five-brane worldvolume. This structure yields the correct lattice of M-theory brane charges.

1 Introduction

The bosonic sector of $D = 11$ supergravity is derived from the action

$$I_{11}^{\text{bulk}} = \int_X (R * \mathbb{1} - \tfrac{1}{2} G_4 \wedge *G_4 - \tfrac{1}{6} C_3 \wedge G_4 \wedge G_4) \ , \tag{1}$$

where $G_4 = dC_3$ and the relative coefficients are fixed by the requirement of $D = 11$ local supersymmetry once the fermions have been included. The $D = 11$ spacetime X is taken here to be without boundary.

The form-field C_3 has the field equation

$$d * G_4 + \tfrac{1}{2} G_4 \wedge G_4 = 0 \ ; \tag{2}$$

this is manifestly invariant under the gauge transformation $\delta C_3 = \Lambda_3$, where $d\Lambda_3 = 0$. This allows "large" gauge transformation if Λ_3 is taken to be closed but not exact; δC_3 is "small" if $\Lambda_3 = d\lambda_2$, for λ_2 globally defined.

Rewriting the C_3 field equation as $d(*G_4 + \frac{1}{2} C_3 \wedge G_4) = 0$, notice that one can introduce a dual field strength

$$\tilde{G}_7 = d\tilde{C}_6 - \tfrac{1}{2} C_3 \wedge G_4 \tag{3}$$

and impose the duality condition

$$\tilde{G}_7 = *G_4 \ ; \tag{4}$$

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then the C_3 field equation becomes a Bianchi identity:

$$d\tilde{G}_7 = -\tfrac{1}{2}G_4 \wedge G_4. \qquad (5)$$

In this way, the C_3 field equation may be replaced by a duality condition for the "doubled" $(C_3, \tilde{C}_6)$ system.

This doubled system has a noncommutative ring of large gauge transformations:

$$\begin{aligned} \delta C_3 = \Lambda_3 \ , \qquad & \delta\tilde{C}_6 = \Lambda_6 - \tfrac{1}{2}\Lambda_3 \wedge C_3 \\ [\delta_{\Lambda_3}, \delta_{\Lambda_3'}] = \delta_{\Lambda_6} \quad \text{with} \quad & \Lambda_6 = \Lambda_3 \wedge \Lambda_3' \\ [\delta_{\Lambda_3}, \delta_{\Lambda_6}] \ = \ & [\delta_{\Lambda_6}, \delta_{\Lambda_6'}] = 0 \ . \end{aligned} \qquad (6)$$

This is the cohomology ring for 3-forms and 6-forms on the underlying spacetime. These cohomologies are taken for the time being to be defined over the real numbers, but they will soon be restricted to integral cohomologies when we consider the corresponding Dirac quantization conditions.

In the rest of this article, which is based on [1], we will investigate the way in which the algebra (6) is preserved *a)* in the presence of 2–branes and 5–branes, and *b)* at the quantum level.

2 Current couplings to 2–branes and 5–branes

For an M_2–brane worldvolume W_3 ending on an M_5–brane worldvolume W_6, one has $\partial W_3 \neq 0$, so the basic M_2 coupling $\int_{W_3} C_3$ fails to be gauge invariant even for small gauge transformations $\Lambda_3 = d\lambda_2$. The cure for this problem is provided by a form-field that exists on the W_6 worldvolume: the self-dual 3–form h_3, which has a potential b_2. Using the latter, one can take the combination $\int_{W_3} C_3 - \int_{\partial W_3} b_2$, which is invariant under small gauge transformations when taken together with a compensating Green-Schwarz mechanism, $\delta C_3 = d\lambda_2$, $\delta B_2 = \lambda_2$.

Note that a self-dual 3–form field strength is precisely what is needed in order to complete the bosonic part of the $(2,0)$ worldvolume fluctuation-field supermultiplet. The transverse oscillations of the 5–brane provide 5 worldvolume scalar bosonic degrees of freedom, while the 16 broken supersymmetries contribute 8 worldvolume fermionic degrees of freedom (taking into account that the fermionic equations of motion are of first-order). Thus, in order to have a bose–fermi balance on the W_6 worldvolume, one needs to have an additional 3 bosonic degrees of freedom. This is what is contributed by the self-dual 3-form, which contributes precisely $\frac{1}{2}(4 \cdot 3/2) = 3$ degrees of freedom.

The gauge-invariant field strength for the C_3 gauge field is accordingly the bulk $\oplus$ worldvolume combination

$$h_3 = i^*C - db_2 \qquad (7)$$

where i^* is the pullback to the W_6 worldvolume effected by the $i : W_6 \hookrightarrow X$ embedding map.

The action for the M_2, M_5 system [2] can be written

$$I_{\text{branes}} = I_{\text{kinetic}} + I^{\text{brane}}_{\text{forms}} + I_{\text{WZ}} + I_{\text{counterterms}} \qquad (8)$$

in which the various terms are

$$I_{\rm kinetic} = T_3\int_{W_3} d^3\xi\sqrt{-\det(\gamma_{\mu\nu})} + T_6\int_{W_6}\sqrt{-\det(G_{ij})} \tag{9}$$

$$I^{\rm brane}_{\rm forms} = \tfrac{1}{2}\int_{W_6} h\wedge *h + b\int_{W_6}\tilde{C} + e\int_{W_6} h\wedge C + a\int_{W_3} C + k\int_{W_2} b \tag{10}$$

$$I_{\rm WZ} = f\int_{W_7} C\wedge G + \frac{q}{2}\int_{W_8} G\wedge G\ ; \tag{11}$$

the values of the coefficients a, b, e, k, f, q shall be determined by requiring gauge invariance. The selection of possible terms in (9–11) is made by taking all the relevant products of operators, integrated over spaces of appropriate dimensionality. Clearly, the $I_{\rm WZ}$ terms are unusual, and their inclusion will need to be explained. The final term, $I_{\rm counterterms}$, is required for anomaly cancellation and will involve both worldvolume and bulk contributions.

In order to determine the values of the coefficients in (9–11), one has to impose the requirements of gauge invariance but also take into account the fact that the "magnetically" charged 5–brane and the string boundary of the 2–brane give rise to violations of the normal Bianchi identities for the corresponding bulk and worldvolume form-fields.

3 Relative homology and cohomology, Bianchi identities and gauge invariance

In order to express the violated Bianchi identities compactly, it is convenient to use the language of relative cohomology.[1] Consider a pair of form-fields of adjacent rank, (C_k, C_{k-1}). The first element in a pair is taken to be valued in the bulk spacetime, but the second element is taken to be valued in the subspace with respect to which the relative cohomology is being defined, in this case the 5–brane worldvolume W_6. Using the pull-back i^* from the bulk spacetime to the worldvolume W_6 as above, one can define the relative exterior derivative (or coboundary operator) as

$$d(C_k, C_{k-1}) := (dC_k, i^*C_k - dC_{k-1})\ . \tag{12}$$

The group of forms closed in this sense, taken for now over the real numbers, is denoted $H^k(X, W_6; \mathbb{R})$.

The pair of field strengths (G_4, h_3), valued respectively in the bulk and in the worldvolume W_6, is thus given locally in terms of the exterior derivative (12): $(G_4, h_3) = d(C_3, b_2)$, so the naïve Bianchi identity is $d(G_4, h_3) = 0$. In the presence of magnetically charged sources, however, this must be violated on the corresponding source loci:

$$d(G_4, h_3) = (\kappa T_6\delta(W_6), T_{2\hookrightarrow 6}\delta(W_2))\ , \tag{13}$$

where T_6 and $T_{2\hookrightarrow 6}$ are real coefficients; κ is the gravitational coupling constant, needed on dimensional grounds. T_6 has been chosen to equal the 5–brane tension, as is necessary in order for the 5–brane to be a $\frac{1}{2}$ supersymmetric BPS soliton.

After some analysis [1], the various conditions for small gauge invariance, taken together with the form (13) of the violated Bianchi identities plus use of the field equations

[1]For some original applications of relative cohomology to branes, see [3, 4].

yield the following relationships between the coefficients in (9–11):

$$\begin{aligned} T_3 &= -a = 2k && (14)\\ T_6 &= -2e = 6f && (15)\\ T_{2\hookrightarrow 6} &= -\frac{k}{e} = \frac{T_3}{T_6} && (16)\\ b &= 0\ . && (17) \end{aligned}$$

These relations are fully consistent with the "brane surgery" relations [5, 6] expected on the basis of charge conservation for a $(q-1)$ brane intersecting a $(p-1)$ brane over a $(k-1)$ brane,

$$T_{k\hookrightarrow q}T_q = T_{k\hookrightarrow p}T_p\ , \qquad (18)$$

where $T_{k\hookrightarrow q}$ and $T_{k\hookrightarrow p}$ represent the tension of the $(k-1)$ brane as seen within the $(q-1)$ brane or as seen within the $(p-1)$ brane. The relations (14–17) fit this rule for $T_{2\hookrightarrow 6} = T_3/T_6$ if one takes $T_{2\hookrightarrow 3} = 1$.

In addition to the coefficient relations (14–17), one obtains also the following homology relations from the gauge invariance requirements:

$$W_2 = \partial W_3\ , \qquad W_6 = \partial W_7\ ; \qquad (19)$$

the first of these is of course expected since the string is always located at the boundary of the 2–brane on the 5–brane; the second establishes W_7 as a "Dirac surface" for the 5–brane worldsheet W_6.

Homology relations are of course dual to cohomology relations, and so one has an appropriate boundary relation that is dual to the relative exterior derivative/coboundary operator defined in (12). For cycles (W_k, W_{k-1}) in (X, W_6) one has the relative boundary relation

$$\partial(W_k, W_{k-1}); = (W_{k-1} - \partial W_k, \partial W_{k-1})\ . \qquad (20)$$

Thus in relative homology, the statement that a pair has no boundary means

$$\partial(W_k, W_{k-1}) = 0 \Rightarrow W_{k-1} = \partial W_k \subset W_6\ , \quad \partial W_{k-1} = 0\ . \qquad (21)$$

The group of homology chains in this sense, again taken for the time being over the reals, is denoted $H_k(X, W_6; \mathbb{R})$.

Pairs of cycles and pairs of forms can be integrated as follows:

$$\int_{(W_k, W_{k-1})} (C_k, C_{k-1}) := \int_{W_k} C_k - \int_{W_{k-1}} C_{k-1}\ . \qquad (22)$$

The duality of relative homology and cohomology is then expressed *via* Stokes' Theorem:

$$\int_{(D_{k+1}, D_k)} d(C_k, C_{k-1}) = -\int_{\partial(D_{k+1}, D_k)} (C_k, C_{k-1})\ . \qquad (23)$$

One can also give the following meaning to integrals over linear combinations of spaces:

$$\int_{\alpha W + \beta U} C := \alpha \int_W C + \beta \int_U C\ , \alpha, \beta \in \mathbb{R}\ . \qquad (24)$$

Relative cohomology language can now be used to describe the sense in which the gauge transformations (6) remain "large" in the presence of 2–branes and 5–branes. The transformation parameters (Λ_3, λ_2) for the form-fields (C_3, b_2) are taken to be elements of $H^3(X, W_6; \mathbb{R})$. Thus, Λ_3 may remain cohomologically nontrivial on $X - W_6$, but it must reduce to an exact form $d\lambda_2$ when restricted to the 5–brane worldvolume W_6.

4 Dirac-Schwinger-Zwanziger quantization relations

The above real relative homology and cohomology groups become restricted to integral subgroups when quantum effects are taken into account. The basic requirement is that adiabatic deformations of a dual pair of electric and magnetic solitons through a closed deformation path should not produce any change in the quantum generating functional path-integral. Thus, variations of the action I by amounts $2\pi k$, $k \in \mathbb{Z}$ are allowed since the integrand $\exp(\mathrm{i}I)$ becomes multiplied in that case just by $\exp(2\pi \mathrm{i}k) = 1$. A Wu-Yang style argument [1] then shows that if an M_2 brane worldvolume W_3 is deformed through a closed path Σ_4 around an M_5 brane worldvolume W_6, one must for quantum consistency have at most a phase change

$$T_3 \int_{W_3^{\mathrm{final}}} C - T_3 \int_{W_3^{\mathrm{initial}}} C = T_3 \int_{\Sigma_4} G \stackrel{!}{=} 2\pi\ell \ , \quad \ell \in \mathbb{Z} \ , \tag{25}$$

i.e. the class

$$\frac{T_3}{2\pi}\,[G_4] \tag{26}$$

must be integral when integrated over closed manifolds Σ_4 that do not intersect the 5–brane worldvolume W_6 itself.

Taking then $\Sigma_4 = \partial D_5$ and using the violated Bianchi identity $dG = \kappa T_6 \delta(W_6)$ yields the M_2-M_5 brane Dirac quantization rule for the cell units of the brane charge/tension lattice:

$$\kappa T_3 T_6 \stackrel{!}{=} 2\pi \ . \tag{27}$$

In addition, one has a quantization relation for the self-dual (*i.e.* dyonic) string on the 5–brane worldvolume W_6. Firstly, note that the dimension here is in the sequence $d = 4n + 2$, $n \in \mathbb{Z}$, for which the Dirac-Schwinger-Zwanziger quantization condition for dyons with (electric,magnetic) charges (e_i, g_i) is *symmetric* [7]:

$$e_1 g_2 + e_2 g_1 = 2\pi\ell \ , \quad \ell \in \mathbb{Z} \ . \tag{28}$$

Thus, for dyons with $e = g$, one has the charge relation $e_1 e_2 = \pi\ell$, $\ell \in \mathbb{Z}$. This might appear to give a unit cell that is out by a factor of $\frac{1}{2}$ with respect to the unit expected, but one needs to recall that the coefficient k of $\int_{W_2} b$ in (10) also contained a factor of $\frac{1}{2}$ in the coefficient relations (14). Taking this factor into account and performing an adiabatic closed deformation similar to those above, one obtains

$$T_3 \int_{(W_3,W_2)^{\mathrm{final}}} (C_3, b_2) - T_3 \int_{(W_3,W_2)^{\mathrm{initial}}} (C_3, b_2) = T_3 \int_{(\Sigma_4,\Sigma_3)} (G_4, h_3) \stackrel{!}{=} 2\pi\ell \ , \ell \in \mathbb{Z} \ . \tag{29}$$

Then taking $(\Sigma_4, \Sigma_3) = \partial(D_5, D_4)$ and using the violated Bianchi identities (13), one finds out that of the two terms, only the D_4 integral contributes because the D_5 integral of $\delta(W_6)$ vanishes since $\dim(D_5 \cap W_6^{\perp}) = 1$ only. Thus one obtains $T_3 \int_{(\Sigma_4,\Sigma_3)} (G_4, h_3) = T_3 \int_{D_4} T_{2\hookrightarrow 6}\,\delta(W_6)$ and accordingly there is a second quantization rule for the charge/tension lattice-cell units:

$$T_3 T_{2\hookrightarrow 6} \stackrel{!}{=} 2\pi \ . \tag{30}$$

Combining this with the relation (17) for $T_{2\hookrightarrow 6}$, one obtains the quadratic cell unit rule

$$(T_3)^2 \stackrel{!}{=} 2\pi T_6 \ , \tag{31}$$

The quantization rules (27) and (31) yield the correct charge lattice for M–theory solitons [8, 9, 10, 11, 12].

5 $D = 12$ Formulation and Dirac Surfaces

The bulk plus brane-source action discussed so far for M–theory needs to be completed by gravitational counterterms in order to cancel diffeomorphism anomalies on the 5–brane worldvolume W_6 that arise from loops of worldvolume chiral fermion modes [13]. In order to write these, it is convenient to introduce a $D = 12$ spacetime Y such that

$$X = \partial Y \ . \tag{32}$$

Using this, the Chern-Simons term in the bulk action can be conveniently written $-\frac{1}{6\kappa}\int_Y G_4 \wedge G_4 \wedge G_4$.

The main advantage of the $D = 12$ formulation, however, is in the way it gives to express the Dirac surfaces needed to maintain manifest gauge invariance. For this purpose, we need to introduce two surfaces bounded by W_6: V_7, which extends into Y in such a way that

$$i^*\delta_Y(V_7) = \delta_X(W_6) \ , \tag{33}$$

and another surface $W_7 \subset X$; for both one has the boundary relation

$$\partial V_7 = \partial W_7 = W_6 \ . \tag{34}$$

Since V_7 and W_7 share a boundary, one may flip the orientation of V_7 and glue it onto W_7 in order to make a closed surface $W_7 \cup (-V_7) = W_7 - V_7$:

$$\partial(W_7 - V_7) = W_6 - W_6 = 0 \ . \tag{35}$$

Hence, one can find a ball $V_8 \subset Y$ such that

$$\partial V_8 = W_7 - V_7 \ . \tag{36}$$

A similar construction can be made on the 5–brane worldvolume for surfaces bounded by W_2. Letting $i : W_6 \hookrightarrow W_7$, introduce $U_3 \subset W_7$ with $\partial U_3 = W_2$ such that

$$i^*\delta_{W_7}(U_3) = \delta_{W_6}(W_2) \ . \tag{37}$$

Since also one has $\partial W_3 = W_2$, one can flip the orientation of W_3 and glue it onto U_3 along W_2 to produce a closed surface $U_3 \cup (-W_3) = U_3 - W_3$ which can similarly be taken to be the boundary of a ball U_4:

$$\partial U_4 = U_3 - W_3 \ . \tag{38}$$

Taken all together, one has the following relative homology relations for the Dirac surfaces:

$$\begin{aligned} \partial(V_8, W_7) &= (V_7, W_6) \\ \partial(U_4, U_3) &= (W_3, W_2) \ . \end{aligned} \tag{39}$$

At this point, we can also state the gauge invariance requirement for the integration domain in the remaining term $\frac{q}{2}\int_{W_8} G_4 \wedge G_4$ in the action (11), with a coefficient to be understood in the sense of Eq. (24):

$$qW_8 = -\frac{T_6}{3} V_8 \ . \tag{40}$$

Taken all together, the gauge-field part of the action is then

$$
\begin{aligned}
I_{\rm gauge} \;=\;& \int_X \frac{1}{2\kappa}\, G_4\wedge *G_4 + T_3 G_4\wedge\Omega_7(R) - \int_Y \frac{1}{6\kappa} G_4\wedge G_4\wedge G_4 \\
& -\tfrac{1}{2}T_6\int_{(V_8,W_7)} \Big(G_4\wedge G_4 - 2\kappa T_3\ \Omega_8,\ h_3\wedge i^*G_4\Big) \\
& +\frac{T_6}{4}\int_{W_6} h_3\wedge *h_3 + T_3\int_{U_4} G_4 - \frac{T_3}{2}\int_{U_3} h_3\ ,
\end{aligned}
\tag{41}
$$

where the integration domains satisfy the relations (32,39). The terms involving Ω_8 and Ω_7 are parts of the counterterm structure needed to cancel the diffeomorphism anomalies on the W_6 worldvolume. $\Omega_8 = d\Omega_7$ generates the anomaly compensator $\mathcal{A}_6$ by transgression:

$$
\delta_{\rm diff}\Omega_7 = d\mathcal{A}_6\ . \tag{42}
$$

$\mathcal{A}_6$ then cancels a part of the anomaly from loops of chiral fermions and of the self-dual h_3 field in the 5–brane's W_6 $(2,0)$ supersymmetric worldvolume theory.

6 Dependence on Dirac surfaces

The presence of terms like $\int_{V_8} G_4\wedge G_4$ in the action (41) may give rise to concern whether the action as presented describes correctly the 2–brane/5–brane system, or whether extra degrees of freedom have sneaked in *via* the dynamics of surfaces like V_8.

The answer is "no." One has just the required degrees of freedom and nothing more. This is demonstrated by showing that variations of the Dirac surfaces in the action (41) produce effects that vanish modulo 2π as a result of appropriate integrality and relative homology conditions.

Independence from variations of Y and V_8 under shifts by closed surfaces boils down to requiring that the classes

$$
\frac{1}{2\pi\kappa}\left[\frac{1}{3!}G_4^3\right] \qquad \text{and} \qquad \frac{T_6}{2\pi}\left[\frac{1}{2!}G_4^2\right] \tag{43}
$$

be integral. From the flux integrality condition (26) that

$$
\left[\frac{T_3}{2\pi}G_4\right] \in H^4(Y;\mathbb{Z})\ , \tag{44}
$$

one sees that the needed integrality conditions follow from the charge-lattice unit conditions

$$
\frac{(2\pi)^2}{\kappa} = (T_3)^3 \qquad \text{and} \qquad 2\pi T_6 = (T_3)^2\ , \tag{45}
$$

which follow from (27,31).

Independence from variations of W_7, V_7, U_4 and U_3 is more complicated because some of these surfaces are subsurfaces of others that can be varied, and so are carried along. Thus, varying Y and W_7 by closed surfaces ∂Z and ∂D_8, one induces variations

$$
\begin{aligned}
Y' = Y + \partial Z \qquad &\Rightarrow \qquad V_7' = V_7 + \partial Z_8 \\
W_7' = W_7 + \partial D_8 \qquad &\Rightarrow \qquad U_3' = U_3 + \partial D_4\ .
\end{aligned}
\tag{46}
$$

At the same time, one should also consider the variations

$$\begin{array}{rcl} V_8' & = & V_8 + T_8 \\ U_4' & = & U_4 + T_4 \ . \end{array} \qquad (47)$$

The shift in the action (41) under these Dirac surface variations is [1]

$$\frac{T_6}{2}\int_{D_8-T_8-Z_8} G_4\wedge G_4 + 2\pi\int_{T_8}\Omega_8 + T_3\int_{T_4-D_4} G_4 + \frac{T_3}{2}\int_{(D_4,\partial D_4)}(G_4,h_3) \ . \qquad (48)$$

Using again the flux integrality condition (44), one finds that this shift is in $2\pi\mathbb{Z}$ provided one has the boundary conditions for the integration domains

$$\partial(-D_8+T_8+Z_8) = \partial T_8 = \partial(T_4-D_4) = 0 \ , \qquad (49)$$

which follow from the Dirac surface relative homology relations (39). In addition, one needs to require that

$$[\Omega_8]\in H^8(Y;\mathbb{Z}) \ . \qquad (50)$$

This condition is known to be required also by membrane tadpole cancellation requirements [14, 15].

7 Lattice of Large Gauge Transformations

Finally, we return to the large gauge transformations. Since the various Dirac and DSZ quantization conditions restrict the M–theory charges to lie on the charge lattice determined by (27,30,31), the large gauge transformations are also restricted. From the flux integrality condition (44), it follows that gauge transformations relating the gauge fields on different hemispheres must also lie on an integral lattice,

$$\left[\frac{T_3}{2\pi}\Lambda_3\right]\in H^3(Y;\mathbb{Z}) \ . \qquad (51)$$

Similarly, the flux integrality condition

$$\left[\frac{T_3}{2\pi}(G_4,h_3)\right]\in H^4(Y,W_6;\mathbb{Z}) \qquad (52)$$

requires the gauge transformation integrality condition

$$\left[\frac{T_3}{2\pi}(\Lambda_3,\lambda_2)\right]\in H^3(Y,W_6;\mathbb{Z}) \qquad (53)$$

and for the 6-form transformations one likewise finds the requirement

$$\left[\frac{T_6}{2\pi}\Lambda_6\right]\in H^6(Y;\mathbb{Z}) \ . \qquad (54)$$

Remembrance

The work of Ref. [1] on which this article is based was significantly aided by early penetrating discussions with Sonia Stanciu, who is sadly no longer with us. Her gentleness and her keen intelligence will be much missed.

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