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Nature/Nurture and the Origin of Individual Differences in Mathematics: Evidence from Infant and Behavioural Genetics Studies

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Abstract

In this Chapter, we discuss empirical evidence addressing the nature-nurture debate from two different perspectives: infant studies and behavioural genetics. Current evidence suggests that there are two cognitive systems for encoding numerical information, and perhaps core systems for geometry. However, questions remain about whether these systems are both present at birth and hence the degree of determinism and the mechanisms by which they connect to later mathematics are still far from established. Behavioural genetics studies offer a valuable way to assess the origin of individual differences in mathematical cognition and to discriminate between genetic and environmental contributions. We thus review relevant evidence on core quantitative knowledge, mathematical abilities and cross-domain relations from twin studies. We conclude by suggesting that while there is convincing evidence of nature's general and specific role in mathematics, it is clear that environment plays a fundamental role too. The real question for the future is not whether mathematics has a natural core but how to optimise the interaction between nature and nurture so that differential domain-specific and domain-general predispositions can meet an ideal environment to blossom into competent mathematics.

Keywords: mathematics, numeracy, neonates, infants, cognitive development, individual differences, twins, behavioural genetics

If you asked a typical 6-year-old child, or a typical adult, to calculate the square root of 13,689 you would not expect them to actually do it. The former because has not yet been introduced to related concepts and procedures, the latter because they have not committed to memory either the procedure that they once learnt in school, or the fact that the square root of 13,689 is 117. To answer correctly, they will have to use an aid (i.e. a calculator). This suggests that mathematics is neither natural nor intuitive. On the other hand, if you showed a 5-month old infant a doll, which is then hidden by a screen, and to which another doll is visibly added, the infant will expect to see exactly 2 – not 1 and not 3 dolls, when the screen is removed. Although infants have not had a chance to receive formal mathematics education yet, they can do simple arithmetic. Thus, mathematics is natural and intuitive.

While our first example suggests a prominent role for educational opportunities to the making of a mathematician, the second suggests that we come into the world with some intuitive mathematics. Professional mathematicians and educators may tend to emphasize the richness and complexity of mathematics, whereas psychologists (and as a consequence the greatest portion of the available empirical literature on whether mathematics is natural) tend to focus on foundational concepts as a representative part for the whole of mathematics. The necessity to partition complex constructs into more elementary components for rigorous experimentation may thus cause a disconnect between the aims and claims of basic research in mathematics cognition and the world of learners and practitioners. By necessity, here we will adopt operational definitions of “mathematics” that emerge from psychological studies, with the caveat that mathematics involves much more than what we psychologists, in our empirical efforts and reports, often seem to believe it can be reduced to.

The question whether mathematics is natural is associated with one of the central themes in cognitive developmental psychology, nature vs. nurture. On one side of the debate, nativists argue that certain skills are hard-wired to the brain even before birth, whereas empiricists propose that while human brains may have inborn capacities for learning, they are not skill specific. Associated with the nature vs. nurture debate is the question about whether skills, hard-wired or not, are observable from birth, or if they appear later in the developmental trajectory. Mathematical development can draw parallels with language acquisition, where much more research has been conducted. For example, nativists suggest that we have a specific language acquisition device that maps onto specific brain regions and enables us to develop these skills quickly, indeed much quicker than would be expected through normal learning (e.g. Chomsky, 1965). However, empiricists suggest that we are able to harness more general cognitive skills and use these to extract information on language (e.g. Piaget, 1959). In addition, language researchers have posited that we can draw upon more resources as the brain develops during infancy and childhood; it is these changes that allow more complex skills to develop. An additional issue is whether individual differences in complex human skills and traits have a counterpart in genetic variability, and whether genetic variability is necessary and sufficient to explain individual differences. Knowing in what proportion genetic variability vs. environmental factors contribute to differences in mathematical abilities and whether their effects are specific to mathematics has obvious consequences for pathway models to numeracy, whereby education may be a moderating factor or the decisive factor behind individual attainment.

This chapter will draw evidence about the origins of mathematics from psychological sources. We will review neonatal research as this may offer insights about what number skills humans enter the world with and describe how these number skills change during the first years of life. We will also present evidence from the data-rich field of behavioural genetics, which examines the heritability of mathematics, the weight of environmental factors in inter-individual differences and the interplay between genes and environment.

Early Mathematical Knowledge

In this section we will review the evidence from infants within the first few days of birth, also known as neonates. Henceforth we shall use the term neonates to indicate any research that has been conducted with infants less than one month, and infants for those older than one month. The most common methods are habituation/dishabituation and violation of expectation paradigms. In the habituation/dishabituation paradigm, the assumption is that when an infant becomes habituated to a stimulus, they lose interest and will look at it for less and less time. Of interest is what the infant does when a new stimulus is shown after habituation. If they can discriminate that the stimulus has changed, they will show longer looking times, expressing interest. Similarly, in the violation of expectation paradigm, if an infant looks longer at an event that is inconsistent with reality, this suggests that they have formed an expectation from their perceptions about what should occur. While this kind of evidence from neonatal research is important to the question of whether we are born with numerical abilities, it should also be interpreted cautiously. The neonate is not the ideal research participant. For example, they may be affected by the birthing process and can sleep for long periods of time. Indeed many neonatal studies have had to base their findings on approximately half of the number tested and list non-inclusion for fussiness from the infant or that they have fallen asleep. There are also problems interpreting the results from the necessary methodologies. It may be the case that all differential viewing times tell us is that neonates know that something is different or wrong, and this may not be the precise variable that experimenters have attempted to manipulate. In addition, differences between infants in the times for habituation may be affected by the timing of the trials. In general, neonate research has become sensitive to the perceptual changes, but individual differences remain an underexplored issue.

Much of the mathematics research conducted with neonates has explored two proposed core systems of non-symbolic number: The Approximate Number System (ANS), a system for representing large and approximate numerosities that does not rely on language or symbols; and the Object Tracking System (OTS)[1], a system that can precisely keep track of small numbers of items up to 3-4 (Feigenson, Dehaene, & Spelke, 2004). Research on these systems has focused on the abilities of neonates and explored when they are able to discriminate between small sets of quantities (≤4) or larger sets (≥4), the size of the ratio between sets for accurate discrimination, and where the boundary between small and large changes. The ability to discriminate objects is crucial as it suggests that neonates have the capacity to represent discrete amounts (Carey, 2009) however questions remain about whether the cognitive resources harnessed to succeed at numerical discrimination tasks are specific to number, and how much do they tell us about later mathematical development.

The earliest research on neonatal perception of number was investigated using a visual habituation/dishabituation paradigm. Antell and Keating (1983) showed neonates visual arrays with either 2 vs. 3 or 4 vs. 6 black dots. In the habituation phase, they were shown cards with the same number of dots but that varied in the length of the line or the density between the dots. In the post-habituation phrase, the array was manipulated so that the number of dots changed (either 2 to 3, 3 to 2, 4 to 6, or 6 to 4), but either the line length or the dot density was maintained. They found that the neonates were able to discriminate between 2 and 3 dots but not the 4 and 6 dot arrays, and suggested that neonates do have the ability to abstract numerical invariance but only for small-set visual arrays. Later works addressed possible confounding factors and reached either consistent (Turati et al., 2013) or inconsistent conclusions (Clearfield & Mix, 1999) with the original study.

While studies focusing on OTS have emphasised the visual dimension, a series of other studies have probed cross-modal quantity representations in neonates. For example,

[1] There are different expressions for these two core number systems. We have chosen to use the terms described in Feigenson et al. (2004).

Izard, Sann, Spelke, and Streri (2009) measured looking times to geometric shapes after hearing auditory syllables. This research focused on large number sets at two different ratios. There were two sets with a 3:1 ratio (4 vs. 12 and 6 vs. 18), and one set with a 2:1 ratio (4 vs. 8), and syllable duration was kept constant in each set. After familiarisation, the neonates were presented with images of coloured smiley geometric shapes animated with a stroboscopic motion to maintain attention. Each neonate completed trials where the number of shapes was congruent with the number of syllables, and trials where it was incongruent. They found that neonates consistently looked longer at the displays that were congruent in number when the ratio was 3:1 but that this was not the case for the 2:1 ratio. Izard et al. interpreted these findings as evidence that humans are able to represent abstract, or amodal, numerical information from birth using the ANS but as this is not evident for ratios of 2:1, that this ability must improve during development.

More recently, Coubart et al. (2014) have used a cross-modal matching task to explore whether infants have two separate systems for encoding numerical information, in particular whether the system proposed for small numerosities, the OTS, is present from birth. In a series of experiments based on Izard et al. (2009), neonates listened to sequences of 2 or 3 syllables before being exposed to numerosity displays that were either congruent or incongruent with the sequence. They found that there was no difference in the length of time that the infants looked at the congruent and incongruent images; they were not able to match small numerosities. Further studies explored whether neonates have two separate systems by looking at their ability to dissociate between small and large numerosities. If neonates only have one system, the ANS, they should not be able to discriminate between numbers differentiated on a 3:1 ratio, even if one of these numbers is small. In two experiments, neonates matched small (2) vs. large (6) numbers of syllables to congruent or incongruent numbers of geometric shapes. In both studies, they looked longer at the 6 visual stimuli, even when the cumulated area of the stimuli was controlled. However, a final study showed that neonates were able to discriminate between sets of 3 vs. 9 where the 3:1 ratio remained the same, but the smaller number was increased. This suggests that neonates are able to discriminate items with a 3:1 ratio when the smaller number is 3 or 4 but not when it is 2, and thus that at this age, small numbers may be ≤2 and the boundary between small and large numbers could be situated between 2 and 3. Overall the evidence for a separate OTS system at birth is weak but without data exploring 1 vs. 2 numerosities, it cannot be ruled out.

Although the focus of studies on neonate ability has been on the core systems of number, mathematics is not limited to this area and the ability to have abstract representations of number is linked to other dimensions that may play a role in the development of number discriminations; for example representations of space, time and geometry (de Hevia et al., 2014; Spelke, Lee, & Izard, 2010). Spelke, Lee, and Izard (2010) hypothesised that analogous to the number systems, we also have two core systems of geometry. Both systems are limited but nevertheless assume innate cognitive mechanisms that allow a more complex knowledge to develop. The first core system, navigation, deals with the world on a large scale, and is believed to include a mechanism to represent distances and directional relationships. The second core system is a separate system applied to small-scale objects and forms and focuses on form perception and object shape description. The evidence for these two systems is mostly from comparative studies and older infants. However, within the small-scale system, there is evidence that neonates have an ability to detect angles. In a series of habituation studies, Slater, Mattock, Brown, and Bremner (1991) showed that neonates were sensitive to variations of angle. In contrast, research that has investigated infants' sensitivity to length and sense has only been conducted with much older infants (e.g. Lourenco & Huttenlocher, 2008; Newcombe, Huttenlocher, & Learmonth, 1999). And the youngest participants in research investigating abilities in 2D geometry were even older at 18 months of age (Huttenlocher &

Lourenco, 2007). Thus, the evidence that these mechanisms are present from birth relies on the developmental trajectory observed as children's spatial skills change throughout the first few years of life.

However, there has been evidence that neonates can integrate numerical abilities with other mathematical domains. For example, de Hevia et al. (2014) explored neonates' representations of the connections between number, time and space in a series of 3 cross-modal preferential looking experiments. In experiment 1, the infants were familiarised to a visual line that either matched a number of syllables (6 syllables-short line; 18 syllables-long line), or mismatched (6 syllable-long line; 18 syllables-short line). The experimenters measured the neonate's sensitivity to whether number and time were mapped onto spatial length by examining their performance when only one dimension was manipulated (i.e. the number of syllables whilst viewing two lines, a familiar one and a novel one), and when both dimensions were manipulated. Importantly in the two dimension change trials, infants that had been familiarised with lines that matched the number of syllables, the changes went in the same direction (i.e. for infants familiarised in the 6 syllable-short line condition, the syllables increased to 18 and the novel line was longer); for those with mismatched lines and syllables, the dimensions changed in opposite directions (i.e. for infants familiarised in the 6-syllable-long line condition, the syllables increased to 18 and the novel line became shorter). In both trials, the time that the neonates looked at each line was measured. They found that when the line changes matched the new number of syllables (i.e. more syllables, longer line) neonates would look to the new lines regardless of whether the number had increased or decreased. However, if there was a mismatch, they looked at each line for equal lengths of time. These results suggest that neonates can map auditory sequential numerical information to a spatial length and expect new information to match these mappings. However, the time to voice 18 syllables was longer in duration than the time for 6 syllables. Therefore they devised two further studies where either time was kept constant (the syllables in the 6-syllable condition were lengthened so that the overall duration matched the 18-syllable condition), or number was kept constant (only one tone was played, but its duration matched the time of the 6- and 18-syllable conditions). Using a similar procedure to the previous study, they found that in each study, the neonates looked longer to the novel line when it matched the change in either number or duration. There were no differences in looking time for the infants exposed the mismatch conditions. Taken together, the studies suggest that neonates are able to build an expectation of number, time and space from a short familiarisation phase. However little is known about the mechanisms used for these mappings.

Overall, Izard et al. (2009), Coubart et al. (2014), and de Hevia et al. (2014) provide powerful demonstrations of cross-modal numerical abilities in neonates, but to provide further support for a domain specific predisposition, we would expect to observe a developmental trajectory that shows how these skills assist numerical development in the early years. One problem is that research conducted with older infants has not found a consistent pattern. For example, Mix, Levine, and Huttenlocher (1997) attempted to replicate a study by Starkey, Spelke and Gelman (1990) that had used cross-modal auditory-visual number matching and found that 6-8 month-old infants show correspondence between hearing drumbeats and seeing pictures of everyday household objects. Mix et al. replicated the preferential looking task with two differences. First they showed dots instead of household objects to eliminate visual interest from the objects, and second they randomised the trials so that infants did not hear blocks of 2 or 3 drumbeats. In contrast to Starkey et al., they found that the infants were not more likely to look longer at the picture that represented the number of beats. Others have had more success with cross-modal correspondence when they have used natural pairings. For example, Jordan and Brannon (2006) found that 6-8 month-olds could accurately match 2 and 3 voices to pictures of people, but there remains inconsistency amongst the findings.

In contrast, although the research investigating the development of number and space mapping in older infants is sparse, the results are much more consistent. For example, de Hevia and Spelke (2010) found that 7- and 8-month old infants were able to map the number of dots (4 vs. 16 vs. 64) displayed with line length when they were habituated with positive pairings (i.e. more dots = longer line length). Furthermore, Lourenco and Longo (2010) showed that 9-month olds are able to transfer associative learning across magnitudes dimensions. In their habituation studies, infants were familiarised with stimuli that associated larger objects to be black with stripes, and smaller objects to be white with dots. They found that infants then transferred this relationship to number and duration. In each case, the infants looked longer at the incongruent display (i.e. if objects in the larger array, or the objects presented for the longest duration, were white with dots). These studies provide evidence that there may be some shared representations between these domains.

A significant amount of studies have explored how this core number knowledge develops further in the first few years. For example, Xu and Spelke (2000) used a visual habituation paradigm to show that 6-month old infants could discriminate between quantities with a 1:2 ratio (e.g. 8 vs. 16 and 16 vs. 32), but not quantities with a 2:3 ratio (e.g. 8 vs. 12) (see also Xu, Spelke & Goddard, 2005). Furthermore, the same results are found with auditory stimuli (van Marle & Wynn, 2009). After this age, infants and young children generally show an increasing sensitivity to number discrimination. Sensitivity to the 2:3 ratio is present by 9 months and by age 3, young children can discriminate dot displays on 4:3 ratios although there are some inconsistencies for small set comparisons, and comparisons across large and small sets appear to show more difficulties (see Cantrell & Smith, 2014; Seigler & Lortie-Forgues, 2014).

It also appears that the two proposed mechanisms for small and large number sets are relevant to simple arithmetic. In a series of studies, Wynn (1992) showed 5-month old infants a sequence of events that represented 1 + 1 = 2 or 1, 2 – 1 = 1 or 2, and 1 + 1 = 2 or 3. In her addition task, the infants viewed a single item that was then occluded by a screen. A hand was then seen to add another item behind the screen. Thus, if an infant could add, they should then believe that there were now 2 items behind the screen. She found that when the screen was removed, infants would look longer when there was only one item, suggesting that they had been able to register that adding another item would result in a change to the display. And in fact, because infants would also look longer if 1 + 1 = 3, Wynn was able to go further and suggest that infants were able to manipulate numerical values, and she initially suggested that this could be via magnitude estimation. However, evidence by Feigenson, Carey, and Hauser (2002), who found that 10-12-month olds could track the number of graham crackers being placed in to containers (1 vs. 2 or 2 vs. 3) suggested that infants may be able to complete these simple calculations via the OTS. Therefore, McKrink & Wynn (2004) investigated whether infants were able to perform numerical computations with numbers outside the limits of the OTS. In their study, 9-month old infants were shown a sequence of events that represented 5 + 5 = 5 or 10, and 10 – 5 = 5 or 10. Thus, they suggested that infant numerical computation could be supported by a magnitude-based estimation system alongside an OTS.

In general, human infants appear to possess core representations related with numerosity, and so far the evidence is that there may be different systems for small and larger numbers, and that these can be used across different modalities. A discussion of the extent to which performance in these simple tasks relates to later mathematical achievement is provided elsewhere in this book (Geary, 2016). In the following section we provide a discussion of whether individual differences in similar and more complex mathematical tasks can be attributed to nature or nurture.

Quantitative and molecular genetics studies: looking for the origins of individual differences in mathematical cognition

According to Turkheimer (2000), the general question whether genes (nature) or environment (nurture) determine variations in human complex behaviours has been resolved in favour of the nature side. Indeed, genetic variance is shown as an influential component of the normal and pathological variation continuum in an ever-increasing range of complex traits and behaviours. Mathematics abilities make no exception. Kovas et al. (2013), for example, reported that at primary school age numeracy is as heritable as literacy, and both numeracy and literacy are more heritable than general cognitive ability (i.e. intelligence). Children at ages 7 and 9 show average heritability in the .60 - .70 range for numeracy and literacy and in the .38 - .41 range for intelligence. At age 12, heritability settles around the .50 - .60 range for all three variables. In other words, individual differences in numeracy and literacy for a current sample of UK children are more strongly related to nature than nurture during primary school. Further, Davis et al. (2014) reported that, at age 12, a large proportion of the correlation between reading and mathematics abilities ($r = .60$) may be due to a wide range of genes affecting several learning abilities. How should this evidence be interpreted and what does it tell about the nature and nurture of mathematics?

Quantitative genetics theory of complex behavioural traits is the theoretical framework that allows researchers to infer “the extent to which observed differences among individuals are due to genetic differences of any sort and to environmental differences of any sort without specifying what the specific genes or environmental factors are”, Plomin et al., 2013). *Heritability* is the term used to describe the proportion of observed variation among individuals in a population that can be attributed to underlying genetic differences. It is a characteristic of a population rather than an individual (at the individual level the relative importance of genetic influences and environment can vary widely), and heritability estimates apply only to the population under study at a particular time and under a particular set of environmental circumstances; the relative influence of genetic and environmental factors can also significantly differ at different ages. *Environment* is the term used to describe the proportion of observed variation among individuals in a population that can be attributed to shared (e.g. within a family) or idiosyncratic (i.e. based on individual-specific experiences) external influences. Classical quantitative genetics research relies on the occurrence of two quasi-experimental manipulations: one introduced by nature (i.e. twinning), the other introduced by nurture (i.e. adoption). Because the twin method is much more common than adoption studies in the field of mathematical cognition, we will only discuss the former. The *twin study method* builds on identical twins being, on average, twice as similar genetically as non-identical twins; moreover, half of non-identical twin pairs are of the same sex and thus offer a very good term of comparison with identical twins. The study rationale posits that if genetic factors are important for a trait, then identical twins will be significantly more similar on that trait than non-identical twins. It rests on the assumption that environmental influences will be roughly the same for identical and non-identical twins reared in the same family. Naturally, any violation of the equal environment assumption will lead to an underestimation or an overestimation of genetic influences. The fact that identical twins may experience increased differences in their prenatal environment than non-identical twins may lead to a net underestimation of genetic influences. On the other hand, the post-natal environment may treat identical twins more similarly than non-identical twins thus leading to an overestimation of genetic influences. Empirical tests suggest that - at least after birth - the equal environment assumption is tenable (Plomin et al., 2013). A related and complementary issue is whether identical genes may not lead to a higher similarity of experiences. In this case, which does not precisely challenge the equal environment assumption, the differences between identical and non-identical twins would be caused by an interaction between genes and environment rather than by the environment itself.

Heritability estimates can theoretically vary between 0 and 1 and are calculated by doubling the difference between the correlation of the level at which the trait of interest is observed in identical and in non-identical twins. This is based on the consideration that identical twins have identical genes and non-identical twins are 50% similar genetically; the difference in their correlations, therefore, reflects about half of the genetically influenced variance. We have recently witnessed an increase in the number of twin studies looking to estimate the heritability of learning abilities, several of them with very large sample sizes and providing robust estimates for the ages and populations under study. However, the majority of studies focused on mathematics have used tasks that require skills and notions acquired during formal education, rather than non-symbolic tasks comparable to the ones that are used in neonate and infant research. An interesting exception is a study by Tosto et al. (2014a) assessing non-verbal number acuity in 16-year old UK nationals with a web-based test. Participants were shown sets of yellow and blue dots varying in size and numerosity, similar to the displays used by Halberda et al. (2008), for 400ms and asked to judge whether there were more blue or yellow dots. Although individual differences specific to numerosity discrimination emerge very early in life (Libertus & Brannon, 2010) and may be expected to relate to inborn predispositions, Tosto et al.'s (2014a) quantitative genetics analyses revealed only modest heritability (.32) for non-verbal number acuity. This was somewhat counterintuitive, given that moderate-to-high heritabilities are typically found in numeracy twin studies (see e.g. Kovas et al., 2007a,b; 2013) and that non-verbal number acuity is significantly related with mathematical abilities (Halberda et al., 2008). Thus Tosto et al.

(2014a) proposed that, because non-verbal number acuity may be an evolutionarily preserved basic trait (Butterworth, 1999), variability is kept to a minimum within the core set of genes that provides a blueprint for its development across species; individual differences would stem from genes involved in controlling other task-specific functions such as visuo-motor speed and perceptual processes. However, because heritability estimates may significantly vary with age, it is also possible that higher heritability could be found at younger ages than 16, with environmental influences taking the lion's share only by the end of obligatory education.

In quantitative genetics theory, the proportion of variance that is not explained by heritability is explained by *environmentality*, including shared (non-genetic influences that make family members similar to one another) and non-shared environmental influences (non-genetic influences that are independent across family members). If non-identical twins are as similar as identical twins, then shared environmental influences may play an important role. Dissimilarities within pairs of identical twins are attributed to non-shared environmental influences (e.g. differential treatment in the family, different friends outside the family), including measurement errors. Tosto et al. (2014a) estimated that non-shared environmental influences explained the largest proportion (.68) of individual differences in non-verbal number acuity, whereas shared environmental influences played no role. Turkheimer (2000) notices that whereas twinning provides a methodological shortcut to study heritability (even though the exact mechanisms of genetic influence may be unknown), no such a shortcut is naturally available to study environmentality, because there is no such occurrence as identical environmental twins who share 100% of their experience and non-identical environmental twins who share 50% of their experience. Moreover, it would be unethical to bring complex developmental processes under full experimental control. Therefore, behavioural genetics studies typically provide strong evidence in favour of the importance of environment (especially non-shared experiences) but fail to unveil the mechanisms by which it influences individual differences. Tosto et al. (2014a) speculated that although heritability may be expected to explain a larger proportion of individual differences in numerical skills at a young age, it is also likely that by age 16 factors like exposure to symbolic numbers and self- or other-directed practice with number-related activities (i.e. non-shared environmental differences) exert a bigger influence on non-verbal number acuity.

In Tosto et al.'s (2014a) study it is not possible to know whether genetic influence on non-verbal number acuity is shared for example with genetic influence on intelligence, due to the lack of control tasks and general cognitive ability measures. However other studies on mathematical cognition have directly addressed this issue. Based on twin data on early scholastic achievement at 7-9 years of age for English, mathematics and science, and on a series of mathematics curriculum-related tasks at 10, Kovas et al. (2007b, 2007c) concluded that "genes are generalists and environments are specialists" (Kovas et al., 2007a, p.1). Indeed they found that genes largely explain consistency across domains or tasks as well as in general cognitive ability measures, and environment mostly contributes to performance differences (see also Thompson et al., 1991; Plomin & Kovas, 2005). Based on longitudinal analyses, Kovas et al. (2007c) found that whilst genetic influences explained consistency over time, environmental influences explained changes. Recently, Tosto et al. (2014b) tested whether common genetic influences may subtend the moderate behavioural correlation ($r = .43$) between spatial abilities (operationalised as reasoning about the properties of shapes and their relations, mental transformations) and mathematical performance (operationalised as curriculum-related tasks including calculations, problem solving and geometrical concepts) they found at age 12. Although spatial ability and mathematics resulted only moderately heritable (.27 and .43 respectively), genetic factors contributing to variation in these traits were highly correlated with an average correlation between their genetic components of r =

.75. No intelligence measures were available but it is possible to speculate that the correlation may be largely due to generalist genes (Plomin & Kovas, 2005). Moderate heritability suggests that the link between space and mathematics performance may be moulded via environmental pathways (e.g. teaching practices). Lukowski et al. (2014) tested the relation between working memory components, general cognitive ability and different aspects of mathematics in 12-year olds. While both phonological loop and visuospatial sketchpad performance correlated with all the tested aspects of mathematical ability (mathematics story problem solving, timed and untimed calculation), only performance in the visuospatial sketchpad task shared substantial genetic influences with all the mathematics tasks, most of which could not be attributed to generalist genes. Instead, performance in the phonological loop shared genetic influences with performance in problem solving, which were also shared with general cognitive ability. The relation between different working memory components and mathematics could thus have different origins. Although the use of a phonological loop task taken from a standardised intelligence test may have partly confounded the results, the fact that several measures of mathematics abilities shared specific genetic influence with the visuospatial sketchpad (as assessed with the Corsi block tapping task; Corsi, 1972) is very interesting and calls for further studies to clarify the relation between spatial skills, spatial working memory and mathematics abilities/disabilities.

A relatively underexplored topic relating working memory and mathematics is mathematics anxiety (i.e. an affect management issue rather than an information processing issue, which is thought to disrupt working memory functioning in mathematics-related activities). In particular, the question whether mathematics anxiety shares genetic and environmental risk factors with mathematics abilities/disabilities and general anxiety. Recently, Wang et al. (2014) investigated genetic and environmental contributions to individual differences in mathematics anxiety at age 12. They found moderate genetic influence (.40) on mathematics anxiety, with non-shared environmental influences accounting for all the remaining variability. Unique genetic and non-shared environmental factors independent of both general anxiety and mathematics problem solving accounted for .20 and .53 of the variability. Further analyses showed an influence on mathematical anxiety by genetic and non-shared environmental risk factors associated with general anxiety; additional independent genetic influences were found to be associated with math-based problem solving but not with reading. In conclusion, the development of mathematical anxiety may involve not only exposure to negative experiences with mathematics but is also likely to involve genetic risks that are partly shared with general anxiety and mathematics cognition. This suggests that child-specific risk factors may be identified early on and that a multidimensional approach to mathematical anxiety will be more effective than individual interventions aimed to reduce anxiety, strengthen mathematics abilities or improve mathematics learning experiences.

In general terms, the lack of shared environmental influence appears to be in sharp contrast with robust empirical evidence linking mathematics achievement with socio-economic status (SES; e.g. Jordan et al., 2007) and reporting significant differences between low- and middle-income children in mathematics-related skills before children even begin school (e.g. Klibanoff et al., 2006). SES is typically defined by family income, the level of poverty in the child’s neighbourhood and educational attainment by parents. It would thus correspond to “shared environment” in behavioural genetics, a component that does not appear to explain significant portions of performance variance in most of the reviewed twin studies. Kovas et al. (2007c) for example state that “having the same parents, the same SES, and going to the same school does not contribute to the similarity between the two children beyond the similarity due to their shared genes” and “being in the same class does not contribute to children’s similarity in mathematical performance. That is, classroom environments affect mathematical ability in different children (even twins and even [identical] twins) in different ways; these effects are subsumed under the non-shared environmental estimate" (p.12; Kovas et al., 2007). To solve this apparent contradiction, behavioural geneticists propose that a proportion of variance in SES may have a genetic basis (e.g. via intelligence and other personality traits; Krapohl et al. 2014) with pivotal role in the correlation between SES and educational achievement (e.g. Krapohl & Plomin, 2015; Plomin & Deary, 2014; see below). On the other hand, a few quantitative genetics studies also report significant shared environmental influence on mathematical abilities (e.g. Thompson et al., 1991; Hart et al., 2009; Petrill et al., 2012; Wang et al., 2014), although this appears to be less replicable than the findings regarding genetic and non-shared environmental influences. Hart et al. (2009) suggest that, in the absence of a national education curriculum (i.e. in the absence of homogeneous practices/objectives across their U.S. twin cohort) variability between schools becomes more influential than in studies conducted in the UK where practices/objectives are homogeneous (Oliver et al., 2004 and Kovas et al., 2007b,c). Mathematical abilities that are typically acquired at school would thus unveil significant shared environmental influence, because twins from the same family are usually enrolled in the same school.

Frequent findings are the lack of differences between sexes (e.g. Kovas et al., 2007c; Tosto et al., 2014a) and of qualitative and quantitative differences in genetic/environmental contribution for typical and atypical performance (e.g. Alarcón et al., 1997; Docherty et al., 2010). The former finding is taken to indicate that the origin of number skills is essentially the same for both sexes; as a consequence, both sexes may benefit from the same educational interventions. The latter suggests that the same mechanisms underlying individual differences in the general population are also involved in mathematical difficulties; these can thus be conceptualized as the lower end of a normal spectrum of variation (Plomin et al., 2009).

Classic quantitative genetics methods cannot identify the specific genes and environments that contribute to estimates. Behavioural genetics studies have started to address this limitation by providing heritability estimates and genetic correlations obtained via Genome-wide Complex Trait Analysis (GCTA) and/or molecular genetics techniques in addition to those obtained with the quantitative method. GCTA requires a large number of unrelated individuals to have been screened for a series of known genetic markers. Another common approach looks for associations between complex traits and specific variants of the same genes (or their associated markers) and is known as Genome-Wide Association (GWA) technique. This generally captures very small fractions of genetic variance for complex traits. However, arrays of trait-associated markers may be usefully combined to obtain a genome-wide polygenic score (GPS) that captures a more significant proportion of variance.

Using GCTA and GPS, Krapohl and Plomin (2015) reported substantial genetic influence on children's educational achievement at age 16 and on its association with family SES. Moreover, children's intelligence accounted for about one-third of the genetic link between family SES and educational achievement. More directly related to mathematics is the GWA study by Docherty et al. (2010), that identified multiple loci associated with individual differences in mathematical ability at age 10 and proposed that overall genetic influence (accounting for 2.9% of the observed variance) is caused at multiple loci with small effect. Their study raises the question whether the multiple effective loci that were isolated for mathematics may be also related with intelligence and other abilities and disabilities. Docherty et al. (2010) tested precisely this hypothesis and found that the set of genetic variants identified on the basis of their associations with ability at age 10 are as strongly associated with reading and intelligence. Further, they showed significant longitudinal association with mathematical ability (i.e. they resulted still associated with it from the age of 7 through to the age of 12). More clarity about the genetic sources of mathematical ability and disability, as made possible by molecular genetics, will allow for targeted testing of gene-environment interactions. If interactive mechanisms are identified, it will be possible to tailor environmental intervention strategies to individual genotypes. Moreover, it may be possible to clarify the reason why certain environmental factors may positively or negatively affect some individuals but not others.

Docherty et al. (2011) examined whether the association between mathematical ability and the set of genetic variants they had previously identified changes depending on environmental variables such as mathematical environment (e.g. how frequently a child engages in mathematics-related activities at school), household organization, parental feelings towards the child, parental discipline, classroom organization, classroom peer context, children’s perceptions of their teacher and SES. They found that two environmental variables, household organization and parental feelings, significantly interacted with the set of genetic variants, the set being more strongly related with mathematical ability in disorganized homes and with negative parents. Given the small effect sizes, the authors cautiously suggested that their finding supports the diathesis-stress model, according to which individuals at genetic risk have worse-than-expected outcomes when subjected to environmental risk (Asbury et al., 2005). In the presence of highly chaotic households and parent negativity, the genetic effect of a set of genetic variants associated with low performance would thus result amplified. With more comprehensive sets to include additional candidate DNA markers for mathematical ability (see e.g. Greven et al., 2014) and the use of larger sample sizes, it will perhaps be possible to increase the explanatory power of these gene-environment interaction studies.

Conclusive considerations

This chapter reviewed the origins of mathematics by exploring what numerical skills are present at birth and before formal schooling, as well as looking for clues from behavioural genetics to genetic and environmental influences. The evidence from infant, and, in particular, neonatal research, provide some evidence that the two cognitive systems proposed for non-symbolic representations are present even from birth, but more robust data may be needed, especially for the OTS. Similarly, the hypothesised core systems of geometry are lacking in empirical evidence. Nevertheless, the evidence so far suggests that humans may have natural predisposition to processing quantitative knowledge, but more research is needed to explore how these core systems influence later mathematic achievement. In contrast, behavioural genetics studies offer evidence for both genetic and environmental influences on the origin of individual differences in non-symbolic number acuity, mathematical abilities and disabilities. While part of this influence may be shared with other cognitive abilities and individuals, a significant part of it is mathematics-specific and idiosyncratic. Our overall conclusion points to the dynamic interplay between nature and nurture. The standardization of teaching practices can highlight individual differences due to genetic makeup and individual reactivity. A better understanding of the origin of such complex interplay between individual differences and environment may lead to building more flexible educational approaches and perhaps better capitalize on nature’s contribution to mathematics cognition. Mathematics is natural and therefore schooling should keep it natural.

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