Journal of Advances in Science and Engineering, ISSN: 2636-607X
Volume 2 Number 1 (2019): 25-35

Journal of Advances in Science and Engineering (JASE)

Unavailability of K-out-of-N: G Systems with non-identical Components Based on Markov Model

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Abstract

The process industry has always been faced with the challenging tasks of determining the overall unavailability of safety instrumented systems (SISs). The unavailability of the safety instrumented system is quantified by considering the average probability of failure on demand. To mitigate these challenges, the IEC 61508 has established analytical formulas for estimating the average probability of failure on demand for K-out-of-N (KooN) architectures. However, these formulas are limited to the system with identical components and this limitation has not been addressed in many researches. Hence, this paper proposes an unavailability model based on Markov Model for different redundant system architectures with non-identical components and generalised formulas are established for non-identical k-out-of-n and n-out-of-n configurations. Furthermore, the proposed model incorporates undetected failure rate and evaluates its impact on the unavailability quantification of SIS. The accuracy of the proposed model is verified with the existing unavailability methods and it is shown that the proposed approach provides a sufficiently robust result for all system architectures.

Keywords

Failure rates;

K-out-of-N system;

Markov model;

Non-identical components;

Safety instrumented system;

Unavailability of System architectures.

Received 13 March 2019; Revised 12 May 2019; Accepted 15 May 2019; Available online 17 May 2019.

1. Introduction

The ever-increasing complexity in processing of hydrocarbon in refineries and petrochemical industries had imposed substantial industrial hazards. Despite the process automation and intervention of skilled operators in ensuring process safety, the industrial disaster is still alarming and inevitable. Functional safety, safety system that is independent of the process control system has received a considerable attention over a couple of decades in process industry because of the tremendous industrial disasters witnessed over few decades. These remind us of an explosion in BASF plant in Oppau, Germany, 1921 in which inappropriate mixture of an ammonium sulphate and ammonium nitrate fertilizer claimed the lives of about 4300 people. In 1932-1968, the Minamata disaster in Bay, Japan led to the severe degree of deformities and death of Bay's inhabitants.

Recently, explosion in Tianjin, China, 2015 claimed over one hundred lives and left hundreds of people in disability position and these incidents necessitate safety instrumented system (SIS) to provide safe isolation of flammable or potentially toxic material in the event of disaster. In response to this hazardous incident in process industries, government continue to enact legislation and impose fines focused on reducing the likelihood of the future event. To ensure a balance between availability and safety, the international standard IEC61508 provides a quantitative approach based on the estimation of the average probability of failure on demand (PFD_{avg}) safety related systems that associate a Safety Integrity Level (SIL) to a Safety Instrumented System (SIS). In quantifying the SIL of a safety related system, IEC61508 defines two modes of SIS, the low demand mode, which is widely used in the process industry and the high demand mode.

Article citation: Okubanjo, A. A., Olasunkanmi, O. A., Oyetola, O. K., Olaluwoye, O. O., & Alao, O. P. (2019). Unavailability of K-out-of-N: G Systems with non-identical Components Based on Markov Model. *Journal of Advances in Science and Engineering, 2*(1): 25-35. Retrieved from https://www.sciengtexopen.org/index.php/jase/article/view/45

Nomenclatures	
τ	Proof test interval
a_{kj}	Transition rate from state k to j
P_{ij}	Probability from state i to j
P_j	State probabilities
U(t)	Safety Unavailability of SIF
PFD_{avg}	Average probability of failure on demand
PFD_{KooN}	Total probability of failure on demand for a KooN system
$\lambda_{DU,i}$	Dangerous undetected failure rate for i components
Abbreviations	
KooN	K-out-of-N system
SIL	Safety Integrity Level
SIS	Safety Instrumented System
SIF	Safety Instrumented Function
MA	Markov Model
IEC	International Electrotechnical Commission

For low demand mode, the failure rate measured is related to both the hardware fault tolerance (HFT) and the PFD_{avg}, while high demand mode is quantify based on the probability of failure per hour (PFH) (Torres-Echeverría *et al*, 2011). On the other hand, PFD_{avg} represents the safety unavailability of an SIS, and is more difficult to determine. It is a norm in practice to vote each part of the SIS in redundancy, which is generally built in K-out-of-N (KooN) system to enhance safety availability and reliability (Wang *et al.*, 2012; Wang & Rausand, 2014). A KooN is a redundant system where at least K-out-of-N components (or channels) must be functional for the redundant system to be successful (Tang *et al.*, 2014b).

In related research (Seop *et al.*, 2016), Markov model simplification has been studied to obtain the average probability of failure on demand for quantitative safety assessment and to determine the safety integrity level of safety instrumented functions (SIF). The reliability block diagram (RBD) has been proposed by Biyanto *et al.* (2015), Catelani *et al.* (2013) and Seop *et al.* (2016), the fault tree analysis (FTA) by Long *et al.* (2002) and Markov analysis by El-damcese *et al.* (2016), Hyungju *et al.* (2014), Khatab *et al.* (2009) and Shu & Zhao (2014). In a similar way, Liu *et al.* (2012) proposed a hybrid method combining the Markov and petri method. Mechri *et al.* (2015) and Redutskiy (2017) suggested a simplified formula based on an approximation to obtain PFD_{avg} of SIF and this method was extended to generalised KooN configurations (Chebila & Innal, 2015; Tang *et al*, 2014a). This generalised formula has not only gained acceptance in the process industry but also frequently used because of the conservative result in calculating the probability of failure on demand of identical and independent components of KooN architecture. Okubanjo (2016) and Okubanjo *et al.* (2018) had formulated a Markov model to obtain PFD_{avg} for burner management system (BMS) of non-identical subsystem configuration. They further proposed the lowest failure rate and maximum beta factor contrary to the pragmatic choice of existing beta-factor to evaluate the commonality of the failure in the BMS. Karimi *et al.* (2014) proposed novel availability approach for hybrid M-out-of-N system and Khatab *et al.* (2009) further extend the availability to K-out-of-N: G systems for non-identical component subjected to repair priorities.

IEC 61508 has established analytical formulas for estimating the average probability of failure on demand for KooN architectures. However, these formulas are limited to the system with identical components and this limitation has not been addressed in many researches. So, publications on unavailability of the system architecture for non-identical components are rarely reported. This paper aims to address this limitation by proposing a unique Markov chain to model the unavailability of redundant SIS for non-identical components.

2. Materials and Methods

2.1 Markov Modelling of Safety Instrumented System

A Markov model (MA) is mathematical model whose dynamic behaviour is such that the probability distribution for its future state is independent of the previous state history. The main assumption in the Markov model is that the system is memory less, that is, the transition probabilities are determined only by the current state and not on the past history. The model uses state transition diagrams to represent system states and the possible transition paths between the states and transition rate. The solution of a Markov model for a system with N components involve M system states, where $M \leq 2^N$. The probability of being in a particular state at some time t is called a state probability. To be able to evaluate the unavailability of SIS on demand, the state probabilities are developed by a set of first-order differential equation named after a famous Russian mathematician Chapman-Kolmogorov (1903-1987), which is presented in (Rausand & Høyland, 2004) as:

$$\dot{p}_{ij}(t) = \sum_{\substack{k=0 \\ k\neq j}}^{r} a_{kj} P_{ik}(s) - \alpha_j P_{ij}(t) = \sum_{k=0}^{r} a_{kj} P_{ik} \tag{1}$$

Hence, this study aims to evaluate the SIS performance, and it is assumed that the Markov process system has a state i at time o, that is $X(0)=i$ and the probability of

$$P_i(0) = P(X(0)=i) = 1$$

and

$$P_k(0) = P(X(0)=k) = 0 \quad \text{for } k \neq i \tag{2}$$

This permits us to simplify the notation by writing P_{ij} as P_j, hence, equation (1) is further simplified using this assumption as

$$\dot{P}_j(t) = \sum_{k=0}^{r} a_{kj} P_k \tag{3}$$

2.2 Assumptions

The following assumptions were made:

(a) all the *N* components in a KooN system are non-identical and dependent;
(b) the SIS components are limited to operational or failed states;
(c) the failure rates of the *N* components of the KooN system are different but constant;
(d) the repair is assumed to be impossible because the failure rates are dangerous undetected;
(e) the mean time to repair is ignored since repair is not possible; and
(f) each system state is a function of all components states, and is either an operational system state or a failed system state.

2.3. Markov Model for Unavailability of 1-out-of-3 Non-identical Subsystem or System

The 1-out-of-3 (1oo3) configuration for non-identical system can be well understood by considering three components arranged in parallel and the state for such configuration is detailed in Table 1 and the Markov state transition diagram is illustrated in Figure 1.

Table 1. System state and its description for 1oo3 configuration

State Probability	State	State Description
P_0	0	All the components 1, 2, and 3 are operational
P_1	1	Components 2 and 3 are operational while component 1 failed
P_2	2	Components 1 and 3 are operational while component 2 failed
P_3	3	Components 1 and 2 are operational while component 3 failed
P_4	4	Component 3 is operational while components 1 and 2 failed
P_5	5	Component 1 is operational while components 2 and 3 failed
P_6	6	Component 2 is operational while components 1 and 3 failed
P_7	7	All the three components 1, 2, and 3 failed

The Chapman-Kolmogorov set of differential equations are given as:

$$\frac{dP_0(t)}{dt} = (\lambda_{Du,1} + \lambda_{DU,2} + \lambda_{DU,3})P_0(t) \tag{4}$$

$$\frac{dP_1(t)}{dt} = \lambda_{DU,1}P_0(t) - (\lambda_{DU,2} + \lambda_{DU,3})p_1(t) \tag{5}$$

$$\frac{dP_2(t)}{dt} = \lambda_{DU,2}P_0(t) - (\lambda_{DU,1} + \lambda_{DU,3})P_2(t) \tag{6}$$

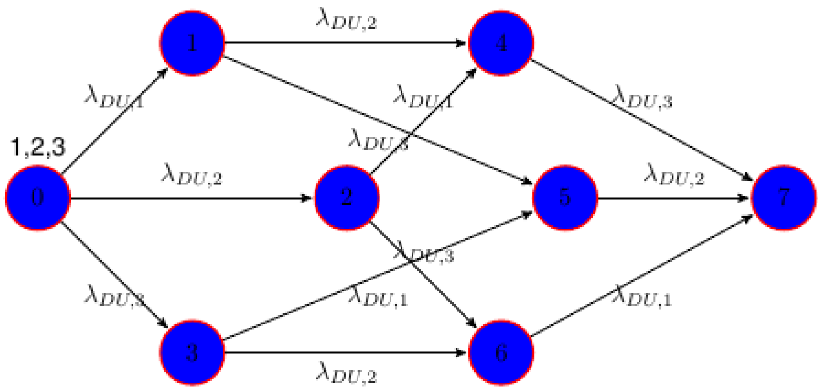

Figure 1. Markov State transition diagram for 1oo3 non-identical configuration

$$\frac{dP_3(t)}{dt} = \lambda_{DU,3}P_0(t) - (\lambda_{DU,1} + \lambda_{DU,2})P_3(t) \tag{7}$$

$$\frac{dP_4(t)}{dt} = \lambda_{DU,2}P_1(t) + \lambda_{DU,1}P_2(t) - \lambda_{Du,2}P_4(t) \tag{8}$$

$$\frac{dP_5(t)}{dt} = \lambda_{DU,3}P_1(t) + \lambda_{DU,1}P_3(t) - \lambda_{DU,2}P_5(t) \tag{9}$$

$$\frac{dP_6(t)}{dt} = \lambda_{DU,2}P_3(t) - \lambda_{DU,1}P_6(t) + \lambda_{DU,2}P_3(t) \tag{10}$$

$$\frac{dP_7(t)}{dt} = \lambda_{DU,3}P_4(t) - \lambda_{DU,3}P_5(t) + \lambda_{DU,1}P_6(t) \tag{11}$$

By solving chapman-Kolmogorov system of equation presented in (4)-(11) with the inclusion of the boundary initial conditions of each state, an analytical solution is obtained for probability of each state as:

$$P_0(t) = e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} \tag{12}$$

$$P_1(t) = e^{-(\lambda_{DU,2}+\lambda_{DU,3})t} - e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} \tag{13}$$

$$P_2(t) = e^{-(\lambda_{DU,1}+\lambda_{DU,3})t} - e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} \tag{14}$$

$$P_3(t) = e^{-(\lambda_{DU,2}+\lambda_{DU,2})t} - e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} \tag{15}$$

$$P_4(t) = e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} - e^{-(\lambda_{DU,2}+\lambda_{DU,3})t} - e^{-(\lambda_{DU,1}+\lambda_{DU,3})t} + e^{-(\lambda_{DU,3})t} \tag{16}$$

$$P_5(t) = e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} - e^{-(\lambda_{DU,2}+\lambda_{DU,3})t} - e^{-(\lambda_{DU,1}+\lambda_{DU,2})t} + e^{-(\lambda_{DU,2})t} \tag{17}$$

$$P_6(t) = e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} - e^{-(\lambda_{DU,1}+\lambda_{DU,2})t} - e^{-(\lambda_{DU,1}+\lambda_{DU,3})t} + e^{-(\lambda_{DU,1})t} \tag{18}$$

$$P_7(t) = 1 - e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} + e^{-(\lambda_{DU,1}+\lambda_{DU,2})t} + e^{-(\lambda_{DU,1}+\lambda_{DU,3})t} + e^{-(\lambda_{DU,2}+\lambda_{DU,3})t} - e^{-(\lambda_{DU,1})t} - e^{-(\lambda_{DU,2})t} - e^{-(\lambda_{DU,3})t} \tag{19}$$

The 1oo3 configuration will fail to perform its intended function on demand if **all** the components fail on demand, hence, $P_7(t)$ denotes the probability that the SIF component is not able to perform the safety function at time t, that is, the unavailability of the 1oo3 configuration. Since, we are more concerned about the SIF unavailability on demand over average proof test interval, there is need to compute average probability of failure on demand of the system configuration over the time period to a full system test τ :

$$PFD_{avg} = \frac{1}{\tau}\int_0^{\tau} P_7(t)dt \tag{20}$$

The exponential function is approximated using Taylor series and the first five terms are substituted in (20)

$$PFD_{avg} = \frac{(\lambda_{DU,1}.\lambda_{DU,2}\lambda_{DU,3})(\lambda_{DU,1}\lambda_{Du,2})(\lambda_{DU,1}\lambda_{DU,3})(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})\frac{1}{4!}(6\lambda_{DU,1}\lambda_{DU,2}\lambda_{DU,3})\tau^4}{(\lambda_{DU,1}.\lambda_{DU,2}\lambda_{DU,3})(\lambda_{DU,1}\lambda_{Du,2})(\lambda_{DU,1}\lambda_{DU,3})(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})\tau} \tag{21}$$

After simplification the average probability of failure on demand for 1oo3 non-identical component architecture is given as:

$$PFD_{avg}^{(1oo3)} = \frac{(\lambda_{DU,1}.\lambda_{DU,2}.\lambda_{DU,3})\tau^3}{4} \tag{22}$$

2.4. Markov Model for Unavailability of 3-out-of-3 Non-Identical Subsystem or System

The state and its description for 3oo3 configuration for non-identical components is shown in Table 2 and the Markov diagram for this configuration is shown in Figure 2.

Table 2. System state and its description for 3oo3 configuration

State Probability	State	State Description
P_o	0	All the components 1,2, and 3 are operational
P_1	1	Component 1 while components 2 and 3 are operational
P_2	2	Component 2 failed while components 1 and 3 are operational
P_3	3	Component 2 while components 1 and 2 are operational

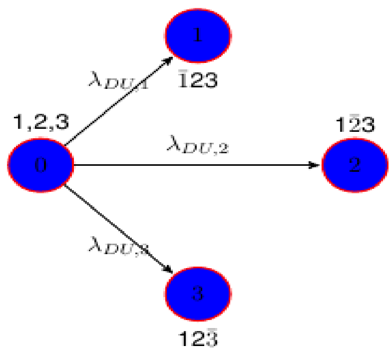

Figure 2. Markov State transition diagram for 3oo3 non-identical configuration

Similarly, The Chapman-Kolmogorov set of differential equations are given as:

$$\dot{P}_0 = -(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})P_0(t) \tag{23}$$

$$\dot{P}_1(t) = \lambda_{DU,1}P_0(t) \tag{24}$$

$$\dot{P}_2(t) = \lambda_{DU,2}P_0(t) \tag{25}$$

$$\dot{P}_3(t) = \lambda_{DU,3}P_0(t) \tag{26}$$

Hence, the initial condition of the state probability are inserted, then the analytical solutions results

$$P_0(t) = e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} \tag{27}$$

$$P_1(t) = \frac{-\lambda_{DU,1}}{(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})}e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} + \frac{-\lambda_{DU,1}}{(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})} \tag{28}$$

$$P_2(t) = \frac{-\lambda_{DU,2}}{(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})}e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} + \frac{-\lambda_{DU,2}}{(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})} \tag{29}$$

$$P_3(t) = \frac{-\lambda_{DU,3}}{(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})}e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} + \frac{-\lambda_{DU,3}}{(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})} \tag{30}$$

The 3oo3 configuration fails to perform its intended function on demand if one of the components fail on demand, the unavailability of in state 1, 2, 3 and the probability on demand is computed by summing the probabilities in the failed state. Hence,

$$PFD_{avg} = 1 - e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t} \tag{31}$$

Similarly, the average probability of failure on demand is evaluated based on (20)

$$PFD_{avg} = \frac{1}{\tau}\int_0^{\tau} (P_1(t) + P_2(t) + P_3(t))dt$$

$$= \frac{1}{\tau}\int_0^{\tau} \left(1 - e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t}\right)dt$$

$$\frac{\left[\left((\lambda_{DU,1} + \lambda_{DU,2} + \lambda_{DU,3}) + e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t}\right)\tau - 1\right]}{(\lambda_{DU,1} + \lambda_{DU,2} + \lambda_{DU,3})\tau} \tag{32}$$

The first three terns of Taylor series for $e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t}$ is substituted in equation (32) and simplified to cancel out equal terms. Hence, the average probability on demand for 3oo3 configuration is expressed as

$$PFD_{avg}^{(3oo3)} = \frac{(\lambda_{DU,1} + \lambda_{DU,2} + \lambda_{DU,3})\tau}{2} \tag{33}$$

2.5 Markov Model for Unavailability of 1oo2 and 2oo2 non-identical system

The 1oo2 and 2oo2 non-identical subsystem or system is best explained with two parallel non-identical components. The system is considered to be in one of the four states at any time as detailed in Table 3. Since, the probability that the system undergoes a transition from one state to another is memoryless. We can use Markov model to compute $P_i(t)$, the probability that the system is in state i at time t and the system unavailability. The state transition diagram is depicted in Figure 4.

Table 3. System state and its description for 1oo2 and 2oo2 configurations for non-identical component

State Probability	State	Component 1	Component 2
P_3	3	operational	operational
P_2	2	operational	failed
P_1	1	failed	operational
P_0	0	failed	failed

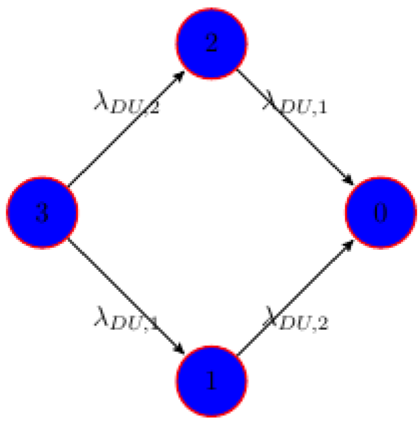

Figure 4. Markov State Transition Diagram for 1oo2 an 2oo2 non-identical configuration

Based on (3), the chapman Kolmogorov differential equations are written as:

$$\frac{dP_0(t)}{dt} = \lambda_{DU,2}P_1(t) + \lambda_{DU,2}P_2(t) \tag{34}$$

$$\frac{dP_1(t)}{dt} = \lambda_{DU,1}P_3(t) - \lambda_{DU,2}P_1(t) \tag{35}$$

$$\frac{dP_2(t)}{dt} = \lambda_{DU,2}P_3(t) - \lambda_{DU,1}P_2(t) \tag{36}$$

$$\frac{dP_3(t)}{dt} = -(\lambda_{DU,1} + \lambda_{DU,2})P_3(t) \tag{37}$$

After substitution of initial condition of the state probability couple with integrating factor method result in these set of solutions

$$P_0(t) = 1 - e^{-\lambda_{DU,1}t} - e^{-\lambda_{DU,2}t} + e^{-(\lambda_{DU,1}t + \lambda_{DU,2}t)} \tag{38}$$

$$P_1(t) = e^{-\lambda_{DU,2}t} - e^{-(\lambda_{DU,1}t + \lambda_{DU,2}t)} \tag{39}$$

$$P_2(t) = e^{-\lambda_{DU,1}t} - e^{-(\lambda_{DU,1}t + \lambda_{DU,2}t)} \tag{40}$$

For series 1oo2 configuration, the system is unavailable in state 0, that is, the 1oo2 configuration will fail to perform it intended function upon demand in state 0, hence, the Unavailability of the system is related to average probability on demand in reliability content, so, the PFD_{avg} for 1oo2 is computed as:

$$PFD_{avg} = \frac{1}{\tau}\int_0^\tau (P_0(t))dt$$

$$PFD_{avg} = \frac{1}{\tau}\int_0^\tau \left(1 - e^{-\lambda_{DU,1}t} - e^{-\lambda_{DU,2}t} + e^{-(\lambda_{DU,1}t + \lambda_{DU,2}t)}\right)dt \tag{41}$$

This is further simplified by approximating the four terms of exponential function with Taylor series and higher orders are negligible such that the expression in (41) becomes

$$PFD_{avg} = \frac{\left[\begin{array}{l} (\lambda_{DU,1} + \lambda_{DU,2})\lambda_{DU,1}.\lambda_{DU,2} + \lambda_{DU,2}(\lambda_{DU,1} + \lambda_{DU,2})\left[1 - \lambda_{DU,1}\tau + \frac{(\lambda_{DU,1}\tau)^2}{2!} - \frac{(\lambda_{DU,1}\tau)^3}{3!}\right] + \\ \lambda_{DU,1}(\lambda_{DU,1} + \lambda_{DU,2})\left[1 - \lambda_{DU,2}\tau + \frac{(\lambda_{DU,2}\tau)^2}{2!} - \frac{(\lambda_{DU,2}\tau)^3}{3!}\right] \\ -\lambda_{DU,1}\lambda_{DU,2}\left[1 - (\lambda_{DU,1}\lambda_{DU,2}\tau) + \frac{(\lambda_{DU,1} + \lambda_{DU,2}\tau)^2}{2!} - \frac{(\lambda_{DU,1} + \lambda_{DU,2}\tau)^3}{3!}\right] \\ -\lambda_{DU,2}(c) - \lambda_{DU,1}(\lambda_{DU,1} + \lambda_{DU,2}) + ((\lambda_{DU,1} + \lambda_{DU,2})) \end{array}\right]}{\lambda_{DU,1}\lambda_{DU,2}(\lambda_{DU,1} + \lambda_{DU,2})} \tag{42}$$

After cancellation of equal terms, the expression for 1oo2 configuration is

$$PFD_{avg} = (\lambda_{DU,1}\lambda_{DU,2}\tau^2)\left(\frac{2\lambda_{DU,1}\lambda_{DU,2}(\lambda_{DU,1} + \lambda_{DU,2})\tau}{6\lambda_{DU,1}\lambda_{DU,2}(\lambda_{DU,1} + \lambda_{DU,2})\tau}\right)$$

$$PFD_{avg}^{(1oo2)} = \frac{(\lambda_{DU,1}\lambda_{DU,2})\tau^2}{3} \tag{43}$$

Similarly, for 2oo2 configuration is unavailable if at least one component fails. The unavailability states are 0, 1, 2 and the average probability of failure on demand for 2oo2 is computed as:

$$PFD_{avg} = \frac{1}{\tau}\int_0^{\tau}\left(P_0(t)+P_1(t)+P_2(t)\right)dt$$

$$= \frac{1}{\tau}\int_0^{\tau}\left(1-e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t}\right)dt$$

$$= \frac{\left[\left((\lambda_{DU,1}+\lambda_{DU,2})+e^{-(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})t}\right)\tau-1\right]}{(\lambda_{DU,1}+\lambda_{DU,2})\tau}$$

$$PFD_{avg} = \frac{(\lambda_{DU,1}+\lambda_{DU,2})^2\tau^2}{2!(\lambda_{DU,1}+\lambda_{DU,2})} \quad (44)$$

$$PFD_{avg}^{(2oo2)} = \frac{(\lambda_{DU,1}+\lambda_{DU,2})\tau}{2!} \quad (45)$$

The derivation for all other voting configurations considered in this paper from 1oo1 to 3oo3 non-identical components follow the same procedure. Table 4 presents the analytical solutions for the PFDavg of the derived voting Architectures and generalized formula for NooN and KooN voting configurations.

Table 4. Equation for PFDavg for undetected failures from PFD formula method and Markov Model

Voting architecture	PFD_{avg} as per PFD formula (Identical Components)	PFD_{avg} Markov Model (non-identical components)
1oo1	$\lambda_{DU}\cdot\frac{\tau}{2}$	$\lambda_{DU,1}\cdot\frac{\tau}{2}$
1oo2	$\frac{(\lambda_{DU}.\tau)^2}{3}$	$\frac{(\lambda_{DU,1}.\lambda_{DU,2})\tau^2}{3}$
1oo3	$\frac{(\lambda_{DU}.\tau)^3}{4}$	$\frac{(\lambda_{DU,1}.\lambda_{DU,2}.\lambda_{DU,3})\tau^3}{4}$
2oo2	$\frac{2.(\lambda_{DU}.\tau)}{2}$	$\frac{(\lambda_{DU,1}+\lambda_{DU,2})\tau}{2}$
2oo3	$(\lambda_{DU}.\tau)^2$	$\frac{(\lambda_{DU,1}.\lambda_{DU,2}+\lambda_{DU,1}.\lambda_{DU,3}+\lambda_{DU,2}.\lambda_{DU,3})\tau^2}{3}$
3oo3	$\frac{3.(\lambda_{DU}.\tau)}{2}$	$\frac{(\lambda_{DU,1}+\lambda_{DU,2}+\lambda_{DU,3})\tau}{2}$
NooN N=1,2, 3…	$\frac{N.(\lambda_{DU}.\tau)}{2}$	$\sum_{i=1}^{N}\left(\frac{\lambda_{DU,1}.\tau}{2}+\frac{\lambda_{DU,2}.\tau}{2}+\frac{\lambda_{DU,3}.\tau}{2}+\ldots+\frac{\lambda_{DU,N}.\tau}{2}\right)$
KooN K<N; N=2, 3…	$\frac{N!(\lambda_{DU}.\tau)^{N-K+1}}{(N-K+2)!(K-1)!}$	$\frac{\sum_{i=1}^{k}\left(\prod_j \varepsilon C_i \lambda_{DU,j}\right)\tau^{N+K+1}}{(N-K+2)}$ where k is the number of minimal cutset and is expressed as C_{N-K+1}^{N}

2.6 Model Real-Life Application, Implementation and Validation

This paper is different from the previous related works in terms of parameters and approach adopted. In the most real-life scenario, the repair is not taken into consideration, it is assumed that the repair rate is impossible because the failure rates are dangerous undetected. Also, this paper formulated a new average probability of failure on demand (PFD_{avg}) formulae for evaluating all K-out-of-N and N-out-of-N configurations. These formulae have not been achieved over years even in the developed world. It was a joint program sponsored by HAN University of Applied Sciences and Beldick Automation International. The research outcome has been adopted to solve a five years pending safety issues at Beldick Automation International.

The derived models have been applied at Beldick Automation Inc. to solve safety issues such as a Burner Management System, nuclear reactor etc. Additionally, the data is taken from Beldick Automation Inc. the Netherlands with a period of 2 years January 2014 to December 2016. The validation technique is the Markov model and the proposed model is experimentally validated on a burner management system. Hence, the results of the research work have been validated with the previous research work and this has been reflected in the text.

4 Results and Discussion

This paper has presented a new unavailability model for a redundant safety instrumented system using Markov model approach. The main objective of this paper was to develop a new model for unavailability of K-out-of-N non-identical components in low demand operation. The present paper focuses on comparing architectures noting that the proof test coverage, repair rate and common cause failure are omitted and leading to conservative result. The PFDavg formula for non-identical components of a redundancy system is derived with MA model under slightly different assumptions as compared to commonly and widely used simplified formula of KooN voting architectures for identical components in the process industry.

In Table 4, it can be seen that the only differences between the result of PFD method and the proposed Markov model are the failure rates that resulted in different voting configurations. However, it is remarkable to note that the results of the methods will be the same if the failure rates are assumed to be equal and constant. It can also be deduced from Table4 that the Markov model permits the possibility of formulating a generalized formula for NooN and KooN system architectures. Figs. 5 and 6 present PFD_{avg} values of the different architectures according to the proof test interval of 10 years for identical and non-identical components respectively. It curves indicated that the PFD_{avg} of all architectures increases with increasing proof test interval and when compared with 1oo3 configuration in both cases the 1oo1,1oo2,2oo2,2oo3, and 3oo3 are more influenced by the increasing proof test. For further work, it would be of great interest to study the effects of common cause failure on the safety unavailability with non-identical components.

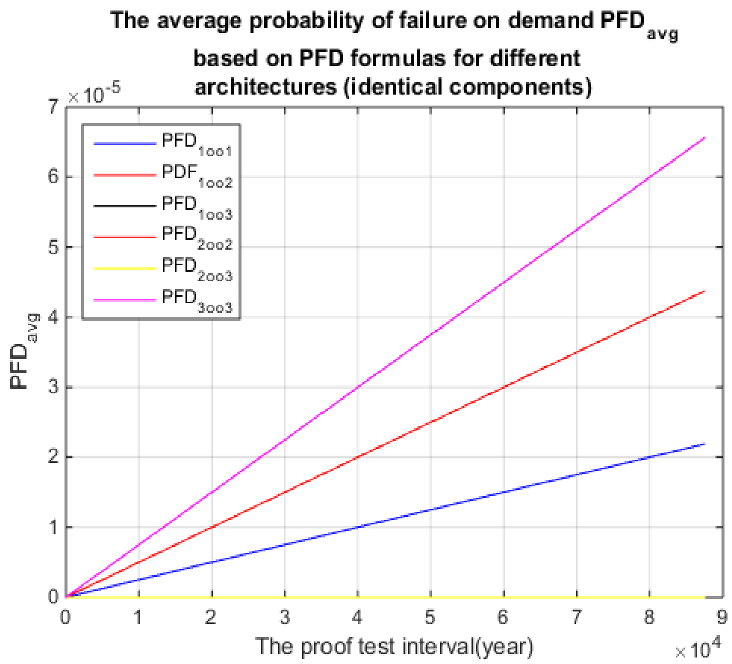

Figure 5. PFDavg for different architectures (identical components)

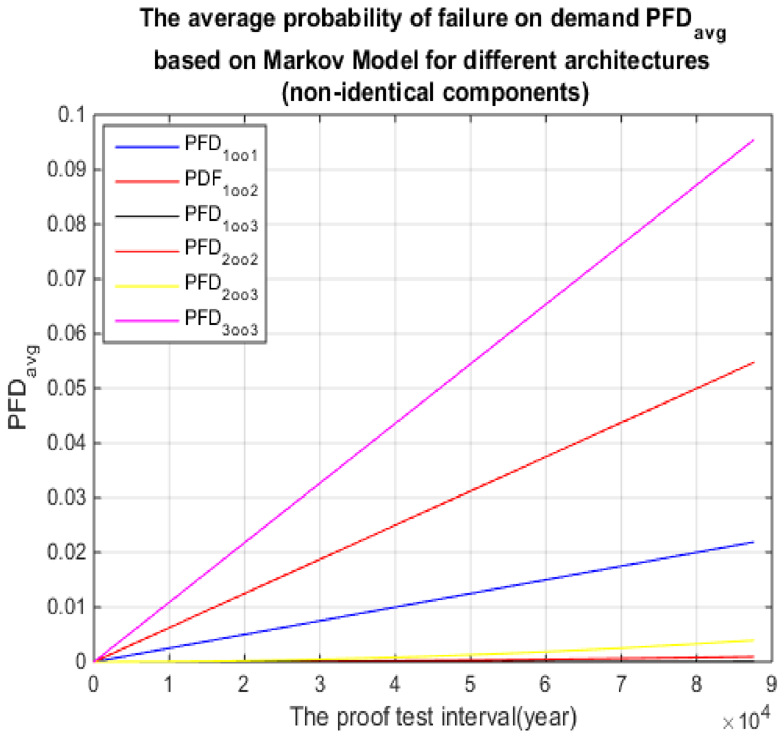

Figure 6. PFDavg for different architectures (non-identical components)

4 Conclusion

In this paper, Markov models are formulated to modify the weakness of simplified formula in quantifying the probability of failure on demand for KooN architectures with different failure rates. It is a known fact that the Markov model shows great advantages in flexibility and ability to describe the time-dependent probability of failure on demand. Hence, the Markov models are derived analytically for PFDavg for different voting architectures of 1oo1, 1oo2, 1oo3, 2oo2, 2oo3, and 3oo3 for non-identical redundancy system. This is further extended to NooN configuration and a generalized formula is also proposed for KooN voting architectures of non-identical components.

A comparison analysis of the existing PFD formula for voting architectures of identical components and the proposed Markov models formula are examined. The results are shown graphically by the aid of the MATLAB program. Results indicated that the PFDavg of the system configurations increases by increasing the proof test interval for both methods. The proposed generalized formulas are expedient in process industry especially in a burner management system with different SIF architectures. In the future, the contribution of common cause failure for non-identical components and the optimal approach to quantify the commonality should be the focus of interest.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work was supported by Beldick Automation International, the Netherlands and in conjunction with HAN University of Applied Sciences, the Netherlands.

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