Dynamic feedback control through wall suction in shear flows

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This is a dedicatory.

Flow control is of interest in many open and wall-bounded shear flows in order to reduce drag or to avoid sudden large fluctuations that may lead to material failure. An established means of control is the application of suction through a porous wall. Here, we combine suction with a feedback strategy whereby the suction velocity is adjusted in response to either the kinetic energy or the shear stress at the bottom wall. The control procedure is then used in an attempt to stabilize invariant solutions and to carry out direct numerical simulations with a prescribed value of the friction coefficient.

1 Suction velocity control

We consider a plane Couette setup with a fluid located in the gap between two infinitely extended parallel plates at a distance $2d$. The bottom plate is stationary and the fluid is driven by the motion of the top plate with velocity U_0 in x-direction. The flow is assumed to be incompressible and isothermal such that the density can be regarded as constant. For large gap widths and a constant suction velocity $-V_s$ in the wall-normal y-direction through the bottom plate, this system is often used to emulate the asymptotic suction boundary layer (ASBL). The aim here is to study the impact of variations in the suction velocity on the global properties of the flow.

The stationary laminar solution of the Navier-Stokes equations is given by

$$\boldsymbol{U} = \begin{pmatrix} \left(1 - e^{-yV_s/\nu}\right) \\ -V_s/U_0 \\ 0 \end{pmatrix} , \tag{1}$$

in units of U_0, with ν the kinematic viscosity. The deviations $\boldsymbol{u}$ from the laminar flow then obey the dimensionless equations

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \boldsymbol{U} \nabla \boldsymbol{u} + \boldsymbol{u} \nabla \boldsymbol{U} + \nabla p - \frac{1}{\mathrm{Re}_0} \Delta \boldsymbol{u} = 0 , \tag{2}$$

$$\nabla \cdot \boldsymbol{u} = 0 , \tag{3}$$

where p is the pressure divided by the constant density ρ and $\mathrm{Re}_0 = U_0 d/\nu$ the Reynolds number based on the velocity of the top plate, the half-height of the domain and the kinematic viscosity of the fluid.

The feedback control procedure is based on a control variable R, here related to the suction velocity V_s by $V_s/U_0 = R/R_0^2$, and an observable A, e.g. the L_2-norm $\|\boldsymbol{u}\|_2$ or the friction factor $C_f = \tau_w/(\rho U_0^2)$, where τ_w is the shear stress at the bottom wall. The dynamics of the control variable is given by

$$\dot{R} = -\gamma \left(R - R_0\right) - \gamma \mu (A - A_0) , \tag{4}$$

where (A_0, R_0) are the values of R and A at the desired operating point and γ and μ are free parameters. The first parameter defines the time scale on which the control is adjusted and, and second one regulates the sensitivity of the control variable to changes in the observable. In order to achieve stabilization, the reaction of the control variable has to be faster than the internal dynamics with which the dynamics runs away from the operating point, and the amplitude has to be sufficiently large to switch between stable and unstable regions (details will be given in [4]).

2 Numerical aspects and simulation data

Equations (2)-(4) are solved numerically with an adapted version of the open-source code `channelflow2.0` [1], using a Fourier-Chebyshev-Fourier discretisation for the velocity field on a rectangular domain with periodic boundary condition in stream- and spanwise directions and no-slip boundary conditions in the wall-normal direction for the stream- and spanwise velocity component and a nonzero vertical suction velocity for the wall-normal velocity component. We note that an adjustment in the suction velocity also results in a change in the laminar profile through Eq. (1).

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control variable	observable	Re_0	V_{s0}/U_0	μ	γ	perturbation type	$\|\delta\boldsymbol{u}_0\|_2/\|\boldsymbol{u}_0\|_2$
V_s	L_2-norm	1600	0.00058	-1.5×10^7	0.1	random	0.01
V_s	L_2-norm	1600	0.00058	-1.5×10^3	0.5	mean flow	4.5
V_s	C_f	1600	0.00058	-10^4 - -10^5	1.0	random	1.5

Table 1: Simulation parameters and observables, with V_s the suction velocity, C_f the friction coefficient, $\mathrm{Re}_0 = U_0 d/\nu$ the Reynolds number, V_{s0} the suction velocity at the operating point, μ and γ the free parameters in Eq. (4) and $\delta\boldsymbol{u}_0$ a perturbation of the invariant solution $\boldsymbol{u}_0$.
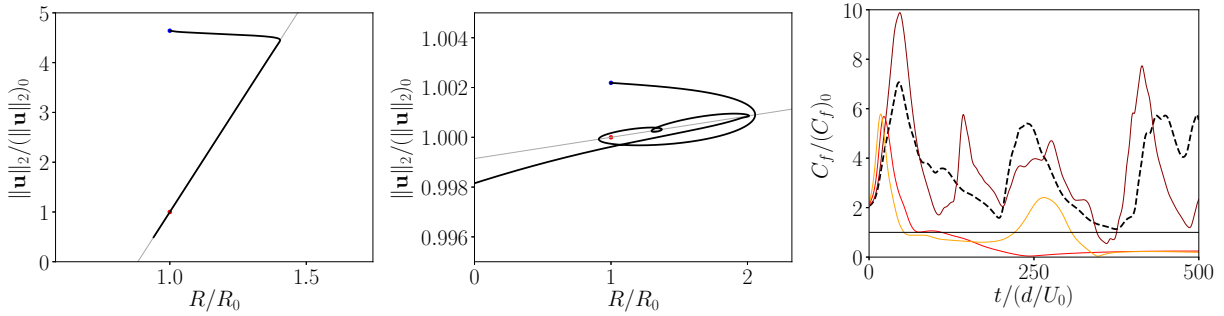
Fig. 1: Left and middle: Controlled dynamics of an invariant solution subject to perturbations of the mean flow (left) and random perturbations (middle). The initial data is indicated in blue and the operating point in red. The control line with slope $1/\mu$ is shown in grey. Right: Controlled dynamics for different values of μ (dark red: $\mu = -10^4$, red: $\mu = -5 \times 10^4$, orange: $\mu = -10^5$) with a prescribed value of the friction coefficient (solid black line). The dashed line corresponds to the uncontrolled dynamics.

The initial conditions are perturbations of an invariant solution $\boldsymbol{u}_0$ of Eqs. (2)-(3). Here we choose $\boldsymbol{u}_0$ to be a travelling wave with one unstable direction in a subspace that enforces the symmetries of the invariant solution [2]. All simulations have been carried out with spanwise, wall-normal and streamwise dimensions $L_x \times L_y \times L_z = 4\pi \times 8 \times 2\pi$ using $N_x \times N_y \times N_z = 32 \times 65 \times 32$ grid points. Further simulation parameters are summarized in table 1.

3   Results

Applying a linear feedback control to an invariant solution with one unstable direction can result in stabilisation thereof, if (i) the control overlaps with the unstable direction, and (ii) if the stable directions are not destabilised. Unstable invariant solutions have indeed been stabilised in pipe flow through an instantaneous feedback control strategy where the Reynolds number is adjusted in response to an observable connected with deviations from laminar flow [3]. Here, we attempt to stabilise an unstable travelling wave through the suction control procedure specified in Eqs. (2)-(4). Since the feedback procedure is based on the suction velocity, which is translationally invariant in all directions, perturbations of the mean flow should be easiest to control. The left and middle panels of Fig. 1 show phase-space representations of the controlled dynamics, starting from either a strong perturbation of the mean flow (left panel) or a weak random perturbation (middle panel). One notes that the controlled dynamics approaches the operating point in both examples. However, on longer times the systems slowly drifts away from the target state, due to the adverse effects of the control on the stable directions of the invariant solution [4]. A stable control hence requires a feedback that is orthogonal to the stable directions.

As a second application of the feedback control, we use it to target a global constraint on the flow. Specifically, we pick a target value for the drag that is lower than the average drag $C_f = 5 \times 10^{-3}$ at a Reynolds number of $\mathrm{Re}_0 = 1600$. The right most panel in Fig. 1 shows time series of the controlled for three different values of μ and the free dynamics, where the target value of the shear stress was set to $(C_f)_0 = 1.25 \times 10^{-3}$. The uncontrolled dynamics shows large fluctuations. The controlled dynamics shows much smaller fluctuations if μ is large enough, approaches the target value and stays its vicinity for up to 200 time units.

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