轻尾分布的一些充分必要条件

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摘 要:研究了轻尾分布的一些充分必要条件.

关键词:轻尾分布;轻尾函数;危险率函数

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1 引言与介绍

设F是在$[0,\infty)$上的具有无界支撑的分布函数,即对任意$x\geqslant 0$,尾函数$\bar F(x)\equiv F(x,\infty)>0$. 以下分布均指上述分布. 对于$\gamma\geqslant 0$,分布$F$的$\gamma$指数矩为

$$\varphi_F(\gamma)=\int_0^\infty e^{\gamma y}F(dy)\in(0,\infty].$$

令

$$\hat\gamma_F=\sup\{\gamma:\varphi_F(\gamma)<\infty\}\in[0,\infty].$$

根据$\hat\gamma_F$的取值不同,把分布分为两种类型:

若$\hat\gamma_F=0$,则称分布F是重尾的;若$\hat\gamma_F>0$,则称分布F是轻尾的. 文献[1-5]是近期关于轻尾分布的文献,而近期关于属于轻尾的卷积等价分布的文献[6-10]总引用达469次(数据来源于Google学术搜索),其中,文献[7]研究重尾分布的一些充分必要条件,本文研究轻尾分布的一些充分必要条件.

定义A[7] 函数f是轻尾的,如果存在$\gamma>0$使得$\limsup_{x\to\infty}e^{\gamma x}f(x)<\infty$.

2 主要结果的证明

定理1 对任意分布F,下列条件是等价的:

(i)F是轻尾的;

(ii)存在$\gamma>0$使得当$x\to\infty$时,$\bar F(x)=o(e^{-\gamma x})$;

(iii)尾函数$\bar F$是轻尾的;

(iv)对应的危险率函数$R\equiv-\ln\bar F(x)$满足$\liminf_{x\to\infty}R(x)/x>0$;

(v)存在$T>0$,使得函数

$$F(x,x+T]\equiv F(x+T)-F(x)$$

是轻尾的.

证明:(i)⇒(ii). 若分布F是轻尾的,则$\hat\gamma_F>0$,即存在$\gamma>0$使$\varphi_F(\gamma)<\infty$,故对任意给定的$\varepsilon>0$,存在$A>0$使当$x>A$时,

$$\int_x^{+\infty}e^{\gamma y}F(dy)<\varepsilon.$$

于是由

$$e^{\gamma x}\bar F(x)\leqslant\int_x^{+\infty}e^{\gamma y}F(dy)<\varepsilon,$$

得$\lim_{x\to\infty}e^{\gamma x}\bar F(x)=0$,即$x\to\infty$时,$\bar F(x)=o(e^{-\gamma x})$.

(ii)⇒(iii). 显然.

(iii)⇒(iv). 反证. 若$\liminf_{x\to\infty}R(x)/x=0$,则存在序列$x_n\to\infty$,使$\lim_{n\to\infty}R(x_n)/x_n=0$,故对任意$\varepsilon>0$,存在$N>0$使当$n>N$时,$R(x_n)/x_n<\varepsilon$,即有

$$e^{\varepsilon x_n}\bar F(x_n)>1,$$

从而当$n>N$时有

$$e^{2\varepsilon x_n}\bar F(x_n)=[e^{\varepsilon x_n}\bar F(x_n)]e^{\varepsilon x_n}>e^{\varepsilon x_n}.\qquad(2.1)$$

由当$n\to\infty$时,$e^{\varepsilon x_n}\to\infty$及$2\varepsilon$的任意性知(2.1)与(iii)矛盾.

(iv)⇒(v). 由$\liminf_{x\to\infty}R(x)/x>0$知,存在$A>0$

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$A>0$ 与 $\varepsilon>0$，使当 $x>A$ 时，有 $R(x)/x>\varepsilon$，即有 $e^{\varepsilon x}\bar{F}(x)<1$，再由 $F(x,x+T]\leqslant\bar{F}(x)$知(v)成立.

(v)⇒(i). 由存在 $\gamma>0$ 及 $T>0$ 使 $\limsup_{x\to\infty}e^{\gamma x}F(x,x+T]<\infty$知，存在$A>0$与$M>0$，使当$x>A$时，有

$$e^{\gamma x}F(x,x+T]<M,$$

从而当$x>A$时，有

$$\begin{aligned}\int_x^{+\infty}e^{\frac{\gamma}{2}y}F(dy)&=\sum_{n=0}^{\infty}\int_{x+nT}^{x+nT+T}e^{\frac{\gamma}{2}y}F(dy)\\&\leqslant\sum_{n=0}^{\infty}\int_{x+nT}^{x+nT+T}e^{\frac{\gamma}{2}(x+nT+T)}F(dy)\\&=\sum_{n=0}^{\infty}e^{\frac{\gamma}{2}(x+nT+T)}F(x+nT,x+nT+T]\\&=e^{\frac{\gamma T}{2}}\sum_{n=0}^{\infty}e^{\gamma(x+nT)}F(x+nT,x+nT+T]e^{-\frac{\gamma}{2}(x+nT)}\\&<e^{\frac{\gamma T}{2}}\sum_{n=0}^{\infty}Me^{-\frac{\gamma}{2}(x+nT)}\to0,x\to\infty.\end{aligned}$$

故$\varphi_F(\gamma/2)<\infty$，即(i)成立.

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Some Sufficient and Necessary Conditions for the Light-Tailed Distributions

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Abstract: We study some sufficient and necessary conditions for the light-tailed distributions.

Key words: light-tailed distribution; light-tailed function; hazard function