RHODES UNIVERSITY
Where leaders learn

Department of Physics & Electronics

Analysing emergent time within an isolated Universe through the application of interactions in the conditional probability approach

A thesis submitted in fulfilment of the requirement for the degree of

Doctor of Philosophy

of

Rhodes University

by

KLH Bryan

March 2020

Abstract

Time remains a frequently discussed issue in physics and philosophy. One interpretation of growing popularity is the 'timeless' view which states that our experience of time is only an illusion. The isolated Universe model, provided by the Wheeler-DeWitt equation, supports this interpretation by describing time using clocks in the conditional probability interpretation (CPI). However, the CPI customarily dismisses interaction effects as negligible creating a potential blind spot which overlooks the potential influence of interaction effects. Accounting for interactions opens up a new avenue of analysis and a potential challenge to the interpretation of time. In aid of our assessment of the impact interaction effects have on the CPI, we present rudimentary definitions of time and its associated concepts. Defined in a minimalist manner, time is argued to require a postulate of causality as a means of accounting for temporal ordering in physical theories. Several of these theories are discussed here in terms of their respective approaches to time and, despite their differences, there are indications that the accounts of time are unified in a more fundamental theory. An analytic analysis of the CPI, incorporating two different clock choices, and a qualitative analysis both confirm that interactions have a necessary role within the CPI. The consequence of removing interactions is a maximised uncertainty in any measurement of the clock and a restriction to a two-state system, as indicated by the results of the toy models and qualitative argument respectively. The philosophical implication is that we are not restricted to the timeless view since including interactions as agents of causal interventions between systems provides an account of time as a real phenomenon. This result highlights the reliance on a postulate of causality which forms a pressing problem in explaining our experience of time.

Declaration

I hereby declare that this thesis has not been submitted, either in the same or different form, to this or any other university for a degree and that it represents my own work. I know the meaning of plagiarism and declare that all of the work in this thesis, save for that which is properly acknowledged, is my own.

KLH Bryan

March 2020

Acknowledgements

Firstly I would like to thank my supervisor, Prof. A.J. Medved, without whom this thesis would not have been possible. Thank you for your guidance, continual assistance, and wide-ranging conversations topics.
I would also like to thank Francis Williamson and Pedro Tabensky for their guidance on the philosophical topics and many interesting conversations over coffee.
The following funding contributions which supported this research are also owed a debt of gratitude:

- The Henderson Scholarship, provided by Rhodes University
- The NRF/DAAD Bursary (Grant Numbers: 111616)
- The NRF bursary through Competitive Programme Grant 93595

Incorporated in this is the tireless efforts of Mr John Gillam and the Rhodes University Postgraduate Funding Office to whom I will always be grateful.
I would also like to thank the many members of the Rhodes Physics Department who have supported me throughout this journey.
To my Mom and Dad, thank you for all you have done and continue to do for me. This work represents a culmination of your efforts and love in assisting me in more ways than I can list.
To my sisters, Megan and Laura, thank you for being behind me all the way. I cannot wait to force you to pretend to read this.
To my grandmother, Ganny, thank you for your continued support in this endeavour and believing I could accomplish the task at hand.
Finally, to Michael, thank you, you know why. Regardless of what you may say, this would not have happened without you.

Contents

List of Figures

Chapter 1

Introduction

"Time is what keeps everything from happening at once."

- Ray Cummings

'Killing time' is a phrase typically used to refer to the practice of idling the hours away. However, it has become applicable to physics in a much more literal sense as a description of arguments that are in favour of doing away with time as a fundamental concept.

1.1 Determining a starting position

Part of the motivation in removing time, which is accomplished by relegating it to the arena of 'illusion', comes from the desire to provide conclusive answers to the open questions on the topic of time. One such question involves finding a reason for why we experience limited freedom of movement in time in that physical systems all appear to be compulsively 'pushed' towards the future. So far as we can tell, this is a universal phenomenon affecting all systems in Nature. Yet despite the ubiquity of change in time, we cannot directly access time by experiment. We cannot point to time. We must rely on measurements of physical systems to make indirect observations of time. A second question concerns the implications of combining space and time into one object known as spacetime, a concept we define fully in 2. Can time be explained only when it is not considered as a separate from space? Indeed, can time be considered an entity in its own right which exists independently of any physical matter?

There are also open questions which are perhaps more directly relatable to modern physics. Experiment continues to tell us that the distinction between future and past is an unavoidable feature of Nature and yet most laws of physics are consistently reversible in time; they show no preference to either a 'future' or 'past' direction in time. There are few exceptions to this time-reversal invariance in physics with the most notable being the second law of thermodynamics. [1] How do we reconcile experimental results with our time reversal invariant theories? Related to this issue is another pressing problem for physics: the lack of consensus among the treatments and interpretations of time across the different theories of physics. In order to match a particular treatment of time to the observations available in Nature, we would also need to find a consistent account which could apply to all theories of physics.

Providing definitive answers to questions of this nature can be problematic. In part, the difficulty arises due to the prevalence of interpretations and the level of subjectivity that is present in the discussion. Tied to this lack of objectivity is the problem of definitive definitions, or lack thereof, for the concepts under debate. A given discussion may take the meaning of a phrase or word for granted, only to face difficulties when those interpreting the argument apply their own definitions. Even when care is taken, opinions over the 'correct' definitions do vary. For a variety of investigations, each with their own slightly different approaches in laying out the problems of time, see, for example, [1, 2, 3, 4, 5, 6, 7, 8, 9]. Although a consensus might be found between a couple of viewpoints, an overarching sense of disagreement is still present and, as yet, the available experimental data does not definitively lend credence to any one interpretation over another.

In an attempt to avoid the pitfalls of misrepresentation, we will attempt to outline a set of definitions that will be sufficient for the investigation to follow. The nature of the topic of time, however, is indeed a slippery one. We aim therefore to be economic in our definitions and use the minimal requirements necessary to capture each concept in its simplest form. We also acknowledge that we cannot hope to pin down each and every concept in a manner which satisfies all the aspects of time or treatments of it. Without the benefit of direct access to time, we must base our definitions on the second-hand information provided by our observations of physical objects. The definitions to follow are then not intended as full and complete explanations of the phenomenon in question. Instead they are simply a means of providing context and grounding for the following investigation and can be considered as primitive concepts which have not, or cannot, be subjected to further analysis yet.

[1] Other examples can be found in quantum mechanics. These include the use of decoherence, particularly in open systems, the wavefunction collapse, and the weak interaction which violates charge-parity (and so time) invariance.

1.2 Definitions

1.2.1 Time and change

We begin by considering a definition for 'time' given that we cannot access it directly, as pointed out above. That there exists a feature of Nature which we call time is based on the phenomenon of change in a system's configuration. This appears to be the primary manifestation of time in physical matter and so we use this as a base for our definition: 'Time', which we take here as a primitive notion, is a feature of the Universe which allows change. A change to a system's configuration can be taken to represent an 'experience of time', as does a system's potential to change, where configuration is taken to mean the structure or arrangement of a system's constituent parts. [2] While this might be more commonly referred to as the 'state' of the system, we reserve that particular word for discussions on quantum mechanics where it has particular significance.

Notice the use of the word *allows*, as opposed to *compels*, in the above definition. If we were to incorporate the compulsion for systems to change as part of the definition of time, we would move beyond the requirement that the definition take the simplest form. We wish to be able to distinguish between the potential to change and the compulsion to do so. We can then avoid assuming they both arise from the same source and that compulsive change *must* be how matter experiences time. In order to allow for these distinct phenomena to be produced by potentially different elements of Nature, we separate them conceptually. It is possible that there is some other aspect element of Nature, working in conjunction with time, that is responsible for matter's 'flow of time', a concept we will consider momentarily.

1.2.2 Static versus frozen systems

We can use the above definition to distinguish between two scenarios, both of which involve a system that remains in a single configuration. On the one hand, we have a 'static' system which stays in one single configuration even as it 'moves' through time. [3] Shown in Figure 1.1, a static system has the *potential* to change, since it is 'in' time, but does not do so. Such a system would have no functional dependence on a time parameter and it would not be able to distinguish one moment from the next. [4] For comparison purposes, we have illustrated a

[2] We consider 'change' here as a primitive concept of a single system transitioning between at least two (different) configurations.

[3] We use terminology, such as 'move', which is primarily found in discussions regarding *space*, rather than time, for ease of discussion. This should not be taken as a sign of any particular interpretation of *time* that strictly depends on space.

[4] As used in physics, static is distinct from, and more restrictive, than the notion of *stationary*.

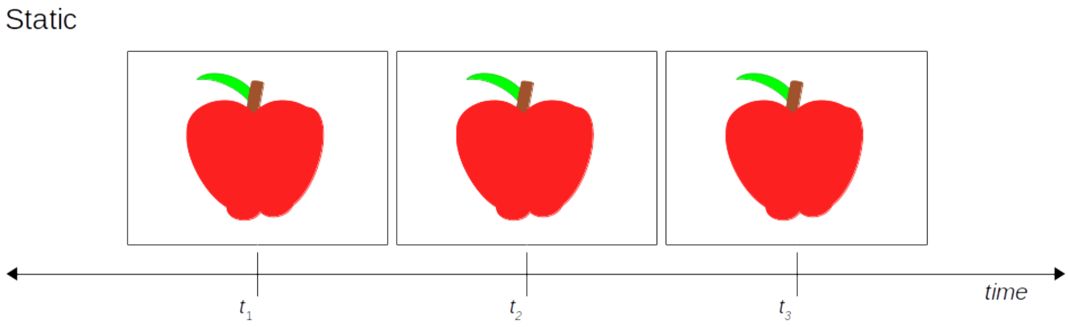

Figure 1.1: An illustration of a static system. This corresponds here to *being*.

changing system in Figure 1.2.

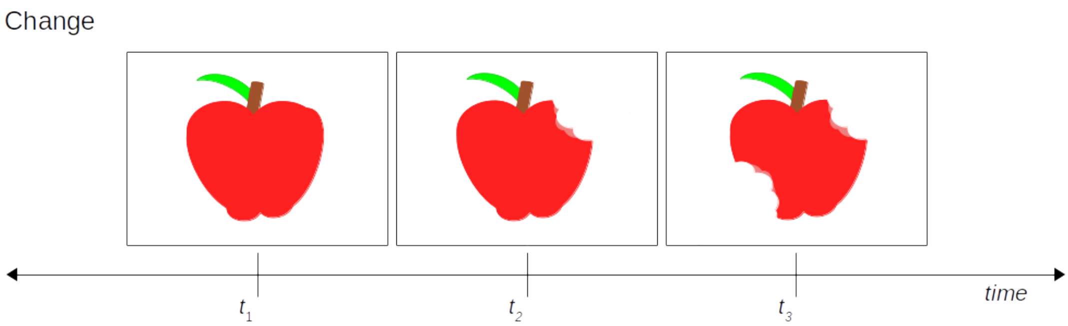

Figure 1.2: An illustration of a system undergoing change, corresponding here to both *being* and *becoming*.

There is also the concept of a 'frozen' system: one which remains is a single configuration as a result of existing in one, single moment of time. In an analogy to a system in space that is stuck in one location, the frozen system is prevented from changing and cannot 'move' into a second moment of time. Conceptually, it is difficult to provide a strict definition of a frozen system, as the depiction in Figure 1.3 implies. If we say 'frozen in time', it implies the existence of a dimension of time. However, it might be argued that no notion of time is necessary for the existence of a frozen system if it not only has no time dependence but also no need for the concept of time at all. One counter argument would be that time is a

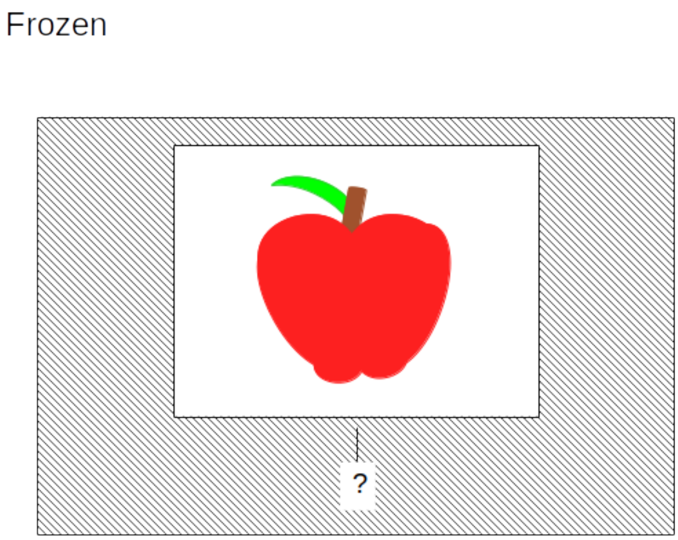

Figure 1.3: A illustration of a frozen system depicting the lack of conclusive definitions surrounding the concept.

necessary component for the existence of all physical systems. From the above definitions, the configuration of a system can be identified as what is affected by time. If time were somehow removed, we can then reasonably expect that this would affect the configuration of the system. For a frozen system, restricting to one 'moment', we might argue that the single moment allows for one configuration to be selected out of all others. In other words, in order for a system to exist in a classical manner, and not in a superposition of all its potential configurations, a moment of time may be required in order to restrict the system to one configuration. This leads us directly to considerations of time (and also space) as entities in their own right and the question of whether they can exist independently of matter. Although related to the topics here, this debate currently lies squarely in the realm of philosophy.

1.2.3 Being versus becoming

There are other concepts used in philosophical discussions of time that are perhaps more directly tied to the debate in physics than those above, such as the contrast between the notions of 'being' and 'becoming'. Here we use 'being' to refer to the physical existence of a system in a given configuration. This is similar to the definition used by Parmenides, as described in [10]. There are more nuanced definitions available that relate more to philosophical concerns and a discussion with a stronger focus on the semantics involved can be found in, for example, [11]. In contrast to being, the notion of 'becoming' corresponds to the idea of change; it is the transition of a single system from one configuration to another and was suggested by Heraclitus in response to the 'eternal time' of Parmenides. An account of the differences between the Heraclitus and Parmenides views is given, for example, in [4]. Essentially, static systems represent a state of being while systems changing through a succession of configurations represent systems becoming. In later chapters we will discuss several arguments which suggest that systems can be classified as being and so experience the illusion of change, with no real transition occurring.

1.2.4 The flow of time

Regardless of whether time is taken to be necessary for existence, or indeed considered an illusion, we still require several definitions. One necessary element that is missing is the previously mentioned appearance of a compulsion for physical systems to experience continual change. Often the experience of this compulsion is referred to as the 'flow of time' and likened to a river which pulls all systems along: the 'river of time'. However, given the above definitions, there is something of a misnomer in this analogy. It is not time which flows but rather the physical systems which are compelled to flow through time by means of changing configurations. To relate this abstract idea to physics, we can identify the flow of time as the record, or history, of the sequence of configurations that a system is compelled to progress through. [5] Although not entirely rigorous, this aligns with the definition of the concept of a flow of time used in [12].

1.2.5 The arrow of time

It is not enough to stop at the definition of a flow of time since most laws of physics can 'point' the flow of time in either direction. In order to distinguish between past and future, as systems appear to do in Nature, we must also define an 'arrow of time': the direction in time along which systems are compelled to change. This is necessary if we hope to recover a description of time that agrees with the experimental observation that Nature directs changes in time towards the future. There is more than one way to define an arrow of time but, at least within physics, the most well known arrow identifies the future as the direction in which entropy always increases as per the second law of thermodynamics. A violation of this has

[5] We refer to the experience of a sequence of configurations in this manner as a 'sense of time'.

never been observed over sufficiently large time (or distance) scales. Indeed Eddington once remarked that "if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation" [13]. We will discuss this thermodynamic arrow of time in more detail later along with other arrows associated with psychological time, cosmological time, and entanglement time. What is of particular interest to the debate in time is why these different arrows of time consistently align with one another.

1.2.6 Causality

We are left with the question of how Nature enforces change down a one-way street through time and we anticipate there should be some mechanism or principle responsible for the consistent behavior of physical systems in time. To account for this phenomenon here, we turn to the causality postulate. This can be stated as the requirement for physical systems to experience a continual compulsion to change (the effect) as a result of an interaction with another system (the cause). [6] This encompasses the idea that a future configuration is dependent on, and indeed exists because of, past configurations.

The use of causality in this manner also allows a consistent history of configurations to be constructed. Each past interaction between two systems, which categorises an 'event', would produce new configurations. That being said, we stress that we are not invoking any notion of determinism by including causality. Although there are those who would argue that causality necessarily implies determinism, we align our view with that presented in [14]. Causality is then viewed as a *process*, in a manner very similar to 'becoming', and we can consider the possibility of applying 'probabilistic causality' where a particular cause does not set a single outcome in stone.

Using this definition of causality, interactions can then be seen as a part of the description of a system's compulsion to evolve in time. Returning to the 'river of time analogy', we modify it to identify 'time' as the riverbank, physical systems as the water, and causality as the current or force driving the water downstream. Invoking causality in this manner, along with our other definitions, does not explain away any of the unanswered question of time but it does allow us to highlight the separate issues at play within each question, issues which might otherwise be conflated into one. Armed with these definitions, we investigate several aspects and interpretations of time that are relevant to physics.

[6] 'Interaction' is taken here as a primitive concept.

1.3 Our Investigation

We begin by considering the different views of time that are presented in the various theories of physics. Although there are apparent contradictions between the different treatments, specifically quantum field theory and general relativity, this issue does not appear insurmountable. By casting the problem into the context of a theory of quantum gravity, several arguments present themselves in favour of the view that the framework which forms the basis for the interpretation of time is in some sense shared by all theories of physics. We do still find it necessary to invoke a postulate of causality, regardless of the theory in question. This is potentially problematic for those interpretations that wish to do away with the notion of change as a real phenomenon since causality, as defined above, would be similarly dismissed as an illusion under such interpretations. To better understand the role of causality and the feasibility of 'timeless' theories, we turn to a model of the Universe which is predominantly argued to enforce the notion that change in time is not real.

The timeless view in question is provided by the conditional probability interpretation (CPI) that was put forward by Page and Wootters in [15]. It is a framework designed to recover a description of evolution within a Universe that is modeled on the Wheeler–DeWitt equation, which we will define in detail later. The primary features of the CPI framework are the use of a subsystem of the Universe as a 'clock' and the recovery of time through the entanglement of this clock with the remainder of the Universe. Thus the CPI does not describe dynamics in the usual sense employed in physics. Rather, it describes every potential pair of correlated states between the clock and the remainder of the Universe, as defined by the superposition of the entangled state. All these correlated pairs can be argued to exist equally, leading some to the interpretation that any experience of change is merely an illusion based the view provided by a succession of paired states.

Our investigation questions the strict adherence to this timeless view, especially when interactions are taken into account. By examining clocks which are allowed to interact with their environment, we can determine the effect interactions have on the CPI framework. In particular, we can examine the efficacy of the clock as a function of the strength of the environmental effect; how much the environment affects the uncertainty related to the clock reading. In doing so, we can determine whether there is any gain to taking interactions seriously within the CPI framework or whether isolated clocks should be considered preferable from both physics and philosophical perspectives.

We lay out the investigation as follows. Chapter 2 presents an account of the various treatments of time found in physics along with the arguments suggesting we might eventually resolve all versions of time into one, unified concept. [7] Chapter 3 summarises the CPI as it is typically presented, along with the timeless interpretation of evolution. Part of this presentation involves a discussion on the difficulties faced by the CPI and the role isolated clocks have in overcoming certain criticisms of the approach. Particular mathematical details pertaining to the resolution of these criticism are given in Appendix A. Once the CPI framework is defined, we attend to the examples of interacting clocks in Chapter 4. The results of the first clock, a damped harmonic oscillator, are supported by a second example which uses a description of an atomic clock as an open system. [8] In both cases, we can rely on the uncertainty associated with a 'clock reading' to indicate the effect of the interactions of the clock with its environment. Appendix B contains mathematical details of the calculation steps, as well as motivations for the approximations used in the analysis of the damped harmonic oscillator. Peripheral aspects of the analysis of the atomic clock can be found in Appendix C, including calculation details as well as a summary of the mechanism of the clock. Contrary to what might be expected, maximising the interaction effects will be shown to minimise the uncertainty of the clock, suggesting that interactions are a necessary component of the CPI. We consider the qualitative aspects of each scenario in Chapter 5. The analysis and comparison of each case leaves us in favour of the interacting clock system since, while both isolated and interacting clocks evade the ambiguity issues raised against the CPI, the latter clock also provides a means for subsystems of the Universe to directly access the clock's time parameter. [9]

In Chapter 6 we turn to some philosophical considerations concerning the two interpretations of time. While the isolated clock maintains the CPI's usual interpretation of change as an illusion, we argue that the inclusion of interactions offers a viable alternative where the experience of time may be interpreted as a real phenomenon. Further considerations of a speculative nature are presented in Chapter 7 where we discuss our results and consider potential avenues for future research. Finally, our conclusions regarding the completed analysis are presented in Chapter 8. A full account of the research outputs related to the investigation to follow can be found in Appendix D.

[7] This argument can be found in [16].
[8] The calculations involving the first and second clock choices can be found in [17] and [18] respectively.
[9] This argument can be found in [16].

Chapter 2

Time in Physics

"It's very hard to talk quantum using a language originally designed to tell other monkeys where the ripe fruit is."
- Terry Pratchett

Building on the definitions laid out in Chapter 1, we begin our investigation by considering the tricky issue of relating disparate accounts of time across physics theories.

2.1 A brief history of the "problem of time"

While multiple questions pertaining to the phenomenon of time remain open and in need of resolution one problem of particular significance to physics is the apparent disagreement between the different theories of how to incorporate and interpret time. Specifically, we are concerned with the two different descriptions of time that are respectively attributed to quantum mechanics and general relativity. These accounts appear to contradict one another, resisting attempts to reach some consensus over how time should be treated.

2.1.1 Quantum gravity

We certainly need the theories of general relativity and quantum mechanics to agree if we hope to explain the phenomena that occur where the theories overlap. An example of one such case where this is necessary can be provided by considering a measurement that is designed to probe a smaller and smaller scale. The uncertainty principle dictates that completing this measurement would require a greater and greater amount of energy. When the scale reaches a small enough threshold, the amount of energy required by the measurement device becomes large enough to cause the system to collapse into a black hole. In that scenario, a high gravitational energy is applied at an infinitesimal scale and a theory of 'quantum gravity', which combines the necessary relativistic and quantum effects, is required along with a consistent version of time. [1]

By quantum gravity, we are specifically referring to a theory which unifies the principles of general relativity with quantum *field theory*. Currently, string theory provides the only example of such a theory as it alone accounts for both general relativity and the quantum fields of the standard model. While there is a class of related theories that provide a means for quantising gravity, such as loop quantum gravity, we do not consider these to be examples of quantum gravity as these descriptions do not account for the standard model description and so do not unify general relativity with quantum field theory. By omitting any account of the principles of quantum field theory, these alternatives do not account for the Weinberg-Witten theorem which states that massless spin-2 particles cannot be consistently coupled to the fields associated with the standard model [19]. [2] It should be kept in mind that a way to circumvent the Weinberg-Witten theorem is suggested by string theory where point-like particles are no longer used. We will return to this point in Section 2.5.

Quantum gravity is frequently considered to be synonymous with the 'theory of everything': an ultimate account of all phenomena. However, in the context of an account of quantum gravity provided by string theory at least, this need not be the case since string theory itself is not a representation of the ultimate description of Nature. Instead, there are five variations of string theory that are all considered to 'emerge' from M-theory, an underlying framework which supposedly provides the 'fundamental theory of everything'. This account is explained in, for example, [20] from which the accompanying illustration in Figure 2.1, which depict the variations of string theory, has been adapted.

2.1.2 Emerging theories

The sense in which one theory emerges from another refers to the construction of an emergent theory from the components contained within a more encompassing and more fundamental theory. As an example, the principles of Newtonian mechanics can all be found within the

[1] For another example of where the regime of quantum gravity is anticipated to apply, we can also consider the birth of the Universe which similarly involves a high energy system limited to an infinitesimal scale.

[2] This is discussed in more detail in Section 2.4.2.

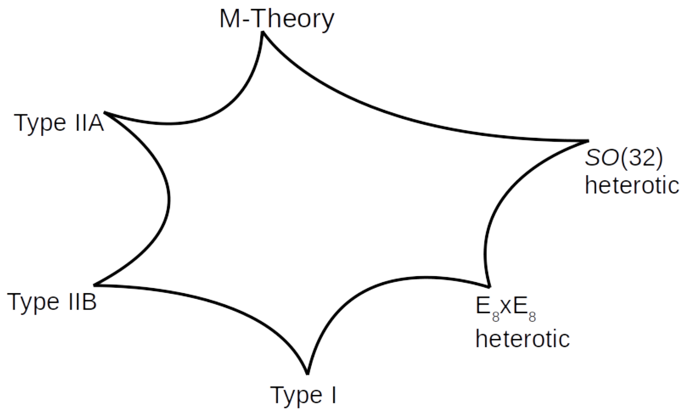

Figure 2.1: An illustration adapted from the depiction in [27]. This represents a 'phase space' showing the five versions of string theory along with the underlying M-theory .

framework of special relativity. The non-relativistic mechanics emerge intact in the limit where the speed of light goes to infinity. Adopting the viewpoint that all theories in physics can be related in this manner allows us to create a hierarchy of theories from most to least fundamental. A schematic representation of this shown in Figure 2.2 . Also illustrated are the limits that connect one theory to another. These will be discussed in more detail later. All theories are then be expected to trace back to and emerge from a single fundamental physical theory.

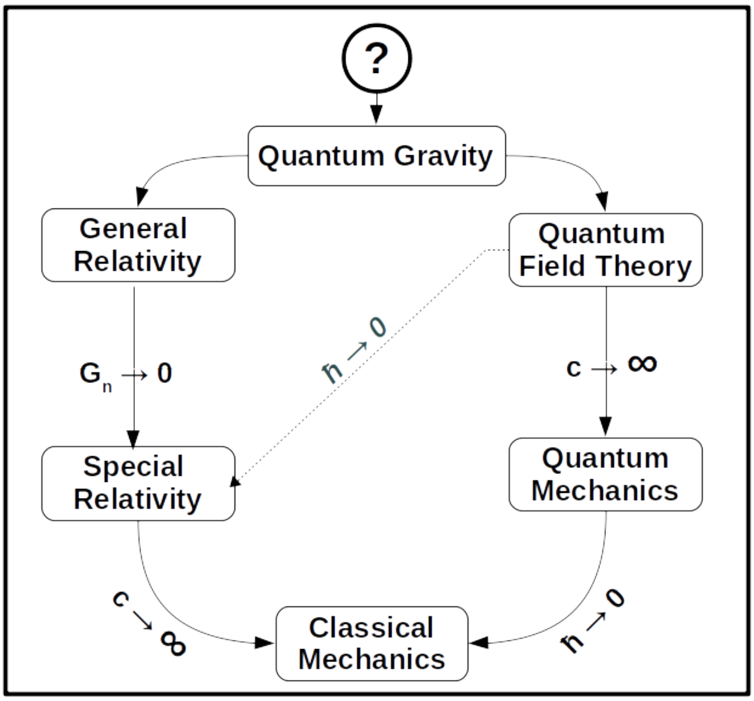

Figure 2.2: A schematic representation of the hierarchy of theories in physics. The question mark represents the as yet unknown 'theory of everything' while c, $\hbar$, and G_N represent speed of light, Plank's constant, and Newton's gravitational constant respectively.

We cannot yet specify the form of this fundamental theory of everything but we can anticipate that it must contain some feature that is ultimately responsible for the various versions of time found in emergent theories. We will refer to this underlying feature as 'blueprint time' but must stress that this should not be taken to imply that blueprint time would resemble a description of time that we are familiar with. Without specifying the exact form or framework of blueprint time, we can still discuss the feasibility of it as a concept in principle. Specifically, it would ensure the consensus between different versions of time in the theories of physics as all these descriptions would have to spring from the same source. In Section 2.5, we will consider several arguments in support of the notion that a consensus of this nature is indeed achievable, despite the apparent difficulties. What these arguments collectively suggest is that the interpretation of time in quantum field theory can be considered a limit of a more general description which utilises metric fields. We will clarify the definitions of these concepts in the discussion to follow.

First, however, we explore absolute time, as it is used in Newtonian and quantum physics. We can then contrast this to the description of time used in the special and general theories of relativity, as well as the treatment of time that is used in quantum field theory. It is this last case which highlights the difference in interpretations since quantum field theory, while utilising features of special relativity, does not agree with general relativity. We will outline these differences in order to discuss the possibility of a theory of quantum gravity that provides a definition of time from which both quantum and relativistic time treatments can emerge.

Lastly, we point out that we will be taking note of the role of causality throughout the discussion in order to highlight the ubiquitous nature of the postulate of causality, regardless of the theory in question. The addition of causality is required as a means of ensuring that the order of events matches up between the physical theories' descriptions and our experimental experience. The definitions summarised in Chapter 1 can then be reinforced in a manner which augments the philosophical assessment presented in Chapter 6.

2.2 Newtonian time

The nature of time has been considered by many people throughout the history of mankind and yet, despite this continued attention, it was only in the seventeenth century that we find a recorded attempt at a strict, scientific definition. This is in reference to the work of Newton who provided a definition of "absolute time" which is, as stated by Newton, an "absolute, true and mathematical time" [7]. As per this description, absolute time is treated as external to all physical systems; It is 'outside' of any system that we might measure via physical access and, as such, it essentially provides a backdrop upon which change occurs.

2.2.1 Absolute time and relative motion

A common visualisation of external time is as a number line that monotonically extends into the future as well as into the past. In a similar manner, space can also be visualised using three numberlines, one for each of the three spatial dimensions. Neither space nor time is directly accessible by experiment. Instead, they are indirectly measured by examining the observable quantities of physical systems with the use of a labeled coordinate system. For space, this might mean using the physical ends of a ruler to determine a distance. Measuring time, on the other hand, requires the periodic change to a system's configuration to define a unit of duration. Considering the manner in which these coordinate frames are used in Newtonian mechanics, a distinction between time and space becomes apparent.

If we wish to shift our description from one coordinate system to the other, we must apply Galilean transformations. These are transformations that describe how to change perspective from one frame of reference to another which involves specifying which objects are in motion and which are stationary, relative to the origin of a given coordinate system. Time, however, remains invariant under these transformations. Simply put, Newtonian mechanics does not allow two separate systems to advance through time at different rates, regardless of their motion through space. Instead, all systems agree on one manner in which they all progress through time, leading to the identification of time in Newtonian mechanics as a 'global time'. Although first explicitly defined by Newton, the notion of a global time also underlies the theory of relativity put forward by Galileo: Motion is relative to a reference frame but time is a global and absolute parameter, regardless of any position or motion in space.

2.2.2 Absolute time and causality

We are left with an interpretation of time as a universally applicable phenomenon, experienced by all systems in the same manner; time 'ticks' the same for all systems. Included in this is the implication of a global 'now', or present, moment that is shared by all physical systems. This is not a statement regarding the existence of a privileged moment which defines the present and moves from past to future. An analysis of that particular concept is reserved for the philosophical discussion in Chapter 6. Rather, the 'now' moment in Newtonian mechanics defines the notion of simultaneity such that different systems, regardless of whether they are in separate frames of reference or not, can be associated with and share the same single, global time.

Selecting a particular moment in time along the time dimension can be easily done with the numberline representation of a coordinate system. This description is commonly termed a 'timeline' in reference to the incremental moments that are mapped onto the numberline. However, the construction of this concept can arguably be said to invoke an overlooked assumption. Specifically, we *assume* that a system's progression through successive configurations must occur in a manner that is consistent with physical laws and that matches with the monotonic ordering of the numberline. Essentially, the assumption is that there is in an order to events, which cannot be contradicted in Nature, that is represented by following the prescribed order as dictated by the labels of our timeline. This ordering, along with an alternative option for comparison, is shown schematically in Figure 2.3. [3]

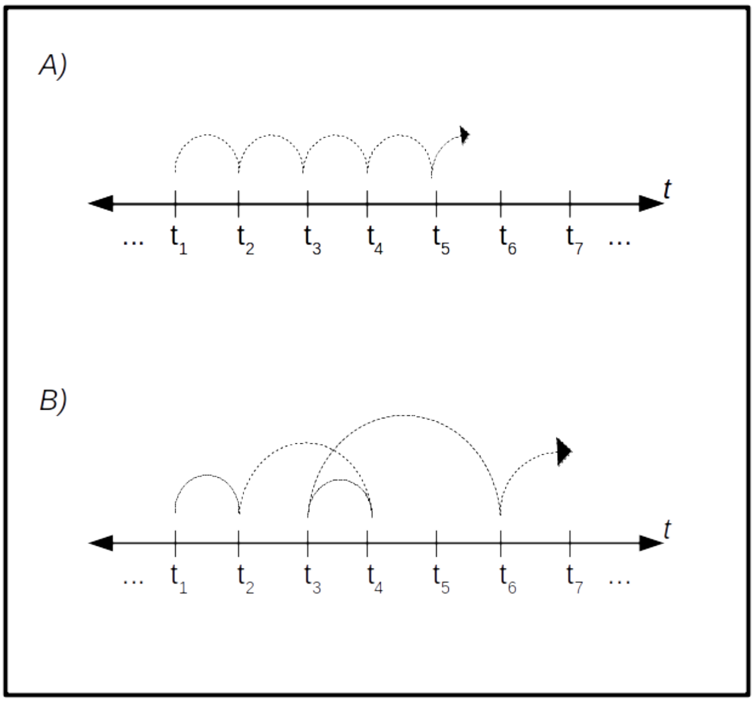

Figure 2.3: Two alternative orders for paths through time.*The dotted line represents the successive order of moments a system passes through as it evolves. Diagram A shows a monotonic order corresponding to the timeline ordering while diagram B shows an example of an alternative ordering that would contradict experiment.*

Imposing this restriction can be interpreted as an assumption of causality. Considering the monotonic increments along the timeline, we presuppose that there will be no 'jumps'

[3] Note this refers to a local, rather than global, causal ordering.

from one point in time to another distant point such that any consecutive points are skipped. It is similar to how we expect events to follow a specific and consistent order in time, such that the causes always precede their effects. To ensure that the laws of physics, such as we understand them, match with our experimental measurements of the world, we require this assumption of causality, or some similar axiom, to impose an order on events.

With this account of time in Newtonian mechanics in hand, we can consider the comparison to quantum mechanics. As illustrated in Figure 2.2, Newtonian mechanics is anticipated to emerge from quantum mechanics as Plank's constant $\hbar$ goes to zero. This does not always represent a well-defined limit but, baring very few special scenarios, quantum behaviour does not survive an application of the condition $\hbar = 0$. The potential link between the two theories is much more stringent in the other direction since there is no manner in which quantum mechanics may be retrieved from classical mechanics by simply inserting $\hbar$. However, we can still expect that the concept of time used in Newtonian physics can be, in some sense, contained within the notion of time that is present in quantum mechanics which represents the more fundamental of the two theories.

2.3 Quantum time

In the quantum realm, we find a notion of time that is similar to the one used in Newtonian physics with all quantum systems operating under a global time coordinate. However, there is an important distinction to make note of.

2.3.1 The troublesome time operator

Time is not directly measurable in quantum mechanics and, although this is also true of Newtonian mechanics, quantum mechanics widens the divide between space and time by the introduction of observables: Variables which are used to define measurable quantities. While a position measurement can be accomplished by acting on a system with the operator associated with the position observable, there is no such observable that can be associated with time. Time intervals may still be inferred indirectly via an observation of a change to the system's configuration, such as its position or energy state, but there is no time operator available. [4] Time only appears as an evolution parameter that we impose on the system.

[4] Specifically, there is no self-adjoint time operator available. For an account of how a self-adjoint time operator can be constructed at the cost of some information see, for example, [21].

To understand why we cannot have a time operator, consider a system's minimum energy state as an indicator of the threshold below which that system's energy cannot decrease. If there were a time operator $\hat{T}$ for such a system, necessarily conjugate to the Hamiltonian $\hat{H}$ of the system, a transformation could then take the system from one representation to the other. In the energy representation the unitary operator $U(\hat{H})$, as the well-known evolution operator, moves the system along the time dimension. The conjugate representation would then be a unitary operator $U'(\hat{T})$ which moves the system along the energy spectrum. While this may initially sound feasible, a problem soon arises as pointed out by Pauli [22]: There is no restriction against $U'(\hat{T})$ from progressing the system along the energy spectrum in either direction and for any distance. The possibility then exists of a system with arbitrarily negative energy, in contradiction of the notion of a stable vacuum. The conclusion is that the existence of a time operator is prohibited. [5]

2.3.2 An emergent notion of time

The quantum view of time is then, as with the previous case, an inaccessible, global coordinate system 'within' which all physical systems evolve homogeneously. However, this is not the end of the line for quantum time. Just as Newtonian mechanics was seen to limit from the quantum theory, quantum mechanics can itself be considered a limit of another, more fundamental account: Quantum field theory. As with the Newtonian–Quantum limit, there is no well-defined limit between quantum field theory and quantum mechanics, a point we clarify in Section 2.4.2. Nonetheless, we anticipate that standard quantum mechanics should be in some sense recoverable from the field theory as the speed of light c is allowed to go to infinity and so any notion of time appearing in the field theory should inform the concept of quantum mechanical time discussed above.

As quantum field theory uses a relativistic description of time, we first introduce the notion by examining the classical theories that share this view on time before returning to an account of quantum field theory.

2.4 Relativistic time

There is more than one theory in physics that utilises relativistic time. We begin this section with a summary of time in the classical relativistic theories of special and general relativity

[5] In Chapter 3 we will see how an effective time operator can be developed by tying it to existing observables of a quantum system. A similar approach can be found, for example, in [23].

after which we briefly discuss quantum field theory and its interpretation of time.

2.4.1 Special and general relativity

In the limit where $c \to \infty$, Newtonian mechanics emerges from the theory of special relativity. Similarly, special relativity itself emerges from the theory of general relativity as the gravitational constant, given by G_N, vanishes. We turn to the progressively more fundamental description of the relativistic theories in an attempt to gain insight into how the phenomenon of time arises.

2.4.1.1 Transformations

As with Newtonian mechanics, special and general relativity rely on coordinate systems to identify and label the dimensions of space and time. The coordinate system is used in the description of the properties associated with a system and its frame of reference, essentially giving the perspective from the standpoint of the system. To relate two different frames which move relative to one other, we once again rely on transformations. However, as we are now dealing with relativistic theories, the Galilean transformations of Newtonian mechanics are no longer sufficient.

In the case of special relativity, we must account for 'boosts'. [6] A boosted frame is one which moves at a constant velocity with respect to another. Boosts can also be viewed as rotations which mix space and time in a 4-dimensional Euclidean space that is obtained by the application of a Wick rotation to imaginary time: $t \to it$. Shifting between the perspectives of these boosted frames requires using Lorentz transformations. Along with boosts, this class of transformations also include the usual rotation in three dimensions that are covered by Galilean transformations. Furthermore, Lorentz transformations, along with translations, form part of the larger set know as the Poincare group.

In aid of the discussion to follow, we briefly consider the manner in which special relativity is constructed. The requirement of Lorentz transformations is often identified as a consequence of the axioms of special relativity: the requirement that light travel at a constant speed in all frames along with the restriction that the laws of physics must remain the same in all frames [24]. It is also possible to arrive at the same theory starting with Lorentz transformations and leading to the framework of special relativity wherein certain

[6] We restrict our use of this terminology to the relativistic case for clarity and note 'boosts' should not be confused with the different form as represented by 'Galilean boosts'.

properties, such as the speed of light, are shown to be 'Lorentz invariant': unchanged by the Lorentz transformations. As will be seen in Section 2.5, a useful perspective can be attained by relating the physical properties which remain invariant under Lorentz transformation with the notion of a 'global symmetry'.

General relativity utilises yet another type of transformation. [7] Unlike special relativity, the general theory accounts for non-inertial frames which are those that accelerate relative to one another. The transformations responsible for relating such frames are known as diffeomorphisms, also referred to as generic transformations which smoothly map from one coordinate system to another. [8] More conceptually, they can be considered as transformations which maintain the consistency of physical laws when we shift between the perspectives of accelerating frames. In the language of tensor algebra, a geometric object which transforms in a manner which preserves the laws of physics is said to transform covariantly. It should be noted that a rank-0 tensor or scalar field is invariant under diffeomorphisms.

2.4.1.2 Spacetime and the metric

A marked difference appears between the Newtonian and relativistic treatments of both space and time as the application of a transformation now affects both the spatial *and* temporal aspects. This led to the amalgamation of space and time into a single entity, known as spacetime. Before continuing our discussion further, we must also introduce another concept that is crucial to special and general relativity: the metric. We begin with a manifold, defined as a collection of points in spacetime, that has a topology; an overall geometric shape. [9] Between any pair of points is an interval and at each point there is a local 'curvature' of spacetime. [10] The description of these properties relies on a rank-2 tensor known as the metric. In special relativity, where the topology is flat, the Minkowski metric is used since there is no curvature to account for.[11] In the case of general relativity, which allows for distorted spacetime, the derivatives of the metric determine the curvature of the manifold. A description of the infinitesimal interval between two points is provided by a line element which is defined as a two-fold contraction of the metric and a differential change in the spacetime coordinates.

[7] We will be restricting to a mostly conceptual description of general relativity. For a full mathematical account of the framework see, for example, [25].

[8] Specifically, we are refering here to local diffeomorphisms.

[9] This can also be described as properties such as simple or multiple connectedness.

[10] More accurately, this refers to a distance between any pair of points on a given curve.

[11] Other metrics can also be used. However, in special relativity, we may adopt the Minkowski metric without any loss of generality.

The four dimensions contained in spacetime are each associated with a component of the metric. Here we are considering the case of general relativity in approximately Minkowskian spacetime. [12] We can then discuss four-dimensional spacetime that has one 'timelike' direction, a notion we will clarify in a moment, as opposed to the other potential configurations of spacetime allowed in general relativity. In such a case, one diagonal component of the metric can be identified as having the opposite sign to the other three. This sign associated with the metric is usually referred to as the signature and it allows us to distinguish time from space.

2.4.1.3 Lightcones

This distinction between space and time can be further illustrated using the geometric construction of nullcones. Choosing to restrict to one spatial dimension, rather than three, we can plot the straight line path taken by light as it travels through spacetime, as is shown in Figure 2.4. The resulting geometric shape is referred to as a 'lightcone' and it divides

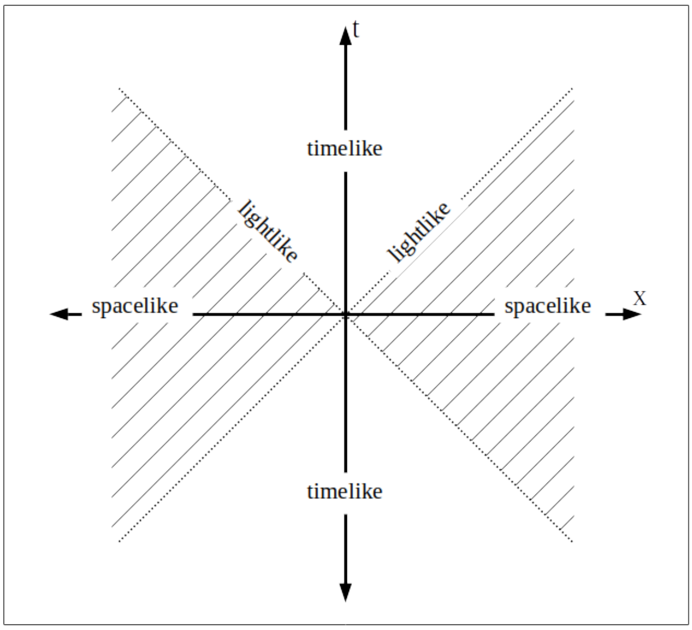

Figure 2.4: A null cone, with one spatial dimension. *Within the cone, paths are timelike while paths outside the cone are spacelike. A path following the cone, representing the path of light, is called lightlike.*

spacetime into three distinct sections. The paths followed by objects within each section can then be labeled as follows: timelike paths fall within the cone, spacelike paths fall outside the cone, and lightlike paths lie along the boundary of the cone. [13]

[12] This refers to the use of a 'background' consisting of a flat spacetime onto which general relativistic corrections are made.

[13] An additional spacial dimension has been used in Figure 2.5 to better illustrate the cone shape.

We now return to our considerations of the differences between the relativistic theories. Special relativity utilises reference frames in a spacetime that is 'flat' in the sense that two

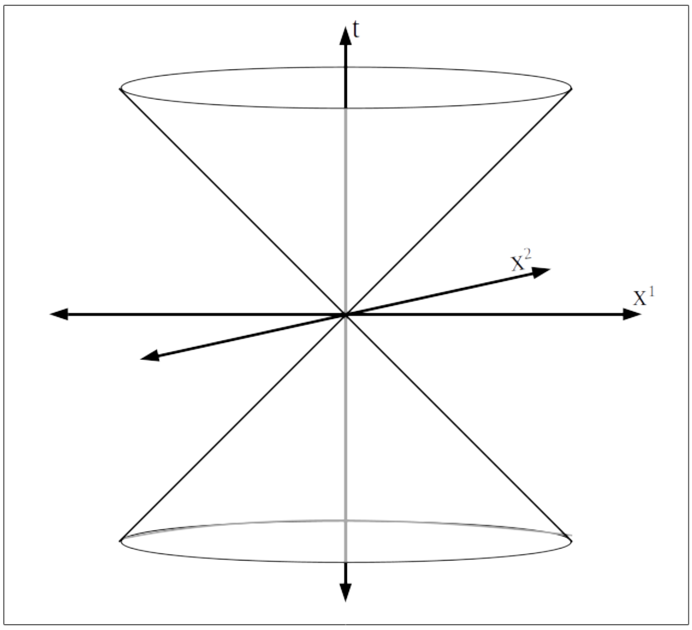

Figure 2.5: A schematic diagram of a lightcone in the flat spacetime of special relativity

free-falling objects each following a straight-line path, the definition of the shortest distance between two points, will remain parallel if they begin parallel. Compared to general relativity, a different picture emerges as a result of the theories capacity for different geometrical structures of spacetime. A free-falling object follows a generally curved path, which is known as a geodesic, the general relativistic analogue of a straight line and whose curve is influenced by the shape spacetime takes. [14] Lightcones constructed under these circumstances must similarly adhere to spacetime's curvature, as illustrated schematically in Figure 2.6. Regardless of this difference, special and general relativity both indicate timelike directions by identifying timelike paths.

2.4.1.4 Proper time

Given the description of lightcones in special relativity, we can discuss a related concept: the Lorentz invariant quantity known as 'proper time'. Proper time, as defined in special relativity, refers to the shortest path between two events in spacetime. It corresponds to the the interval of time measured by a system using a clock 'attached' to the system's co-moving frame of reference. [15] Coordinate time, by comparison, is the interval of time as measured

[14] Note that it is the projection of geodesics onto spacelike slices which produces curved paths and, furthermore, in the case of Minkowski space, the geodesics are straight lines.

[15] Proper time can also be thought of as the interval of time measured by a clock following a strictly timelike path.

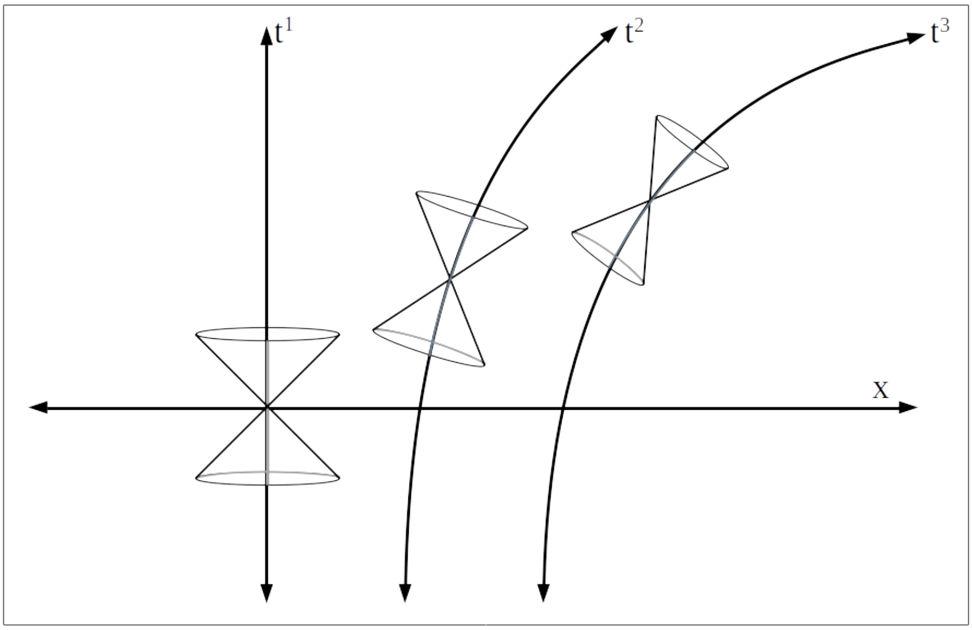

Figure 2.6: A schematic diagram of lightcones in the curved spacetime of general relativity.

by a system that is moving relative to the reference frame of the clock. Two different frames could then disagree over coordinate time intervals since each would measure according to their own perspective but both frames must agree on the proper time interval which is defined through the use a line element, as previously described.

In the case of either relativistic theory, the proper time remains invariant. In special relativity, it is unchanged under Lorentz transformations, ensuring that all observers agree. In general relativity, proper time is represented as a scalar quantity and so will be invariant under diffeomorphisms. It must be stressed that the metric, however, is not constrained to be invariant in general relativity. This point will be important in the discussion in Section 2.5.

2.4.1.5 Causality in special and general relativity

The metric and the lightcones (or null cones) are built-in features of the geometry used to formulate special and general relativity. As it is these features which help identify time, the claim is often made that time *emerges* from the framework of relativity. Indeed, one might argue that time is built-in. However, such an argument omits a crucial (but often hidden) assumption: causality. While causality is often referred to in discussions on relativity, it is typically used in the context of a restriction against faster-than-light travel: the condition that a system's path through spacetime must remain in (or on, in the case of photons) the lightcone's boundary. And yet we also expect a specific order of events which the relativity framework does not include by default.

Mathematically, the same framework which provides lightcones does not restrict against solutions which would appear as discontinuous paths through spacetime. Such a path would, for example, correspond to systems suddenly appearing and disappearing. In order to prevent this, we must always add causality in by hand in much the same way we restrict to a monotonically increasing numberline for the time parameter in Newtonian mechanics. As it is so natural to do so, it is often overlooked. Thus time, as we know it, does not emerge intact from the axioms of special relativity alone, nor does general relativity fix the problem. A causality postulate to dismiss discontinuous solutions is needed if the description is to match with reality.

There now remains one more relativistic theory to consider: Quantum field theory.

2.4.2 Quantum field theory

As mentioned previously, we anticipate that quantum mechanics is obtained as a limit from quantum field theory as $c \to \infty$. This does not represent a well-defined limit as it would require somehow reversing the quantisation which forms the bedrock of quantum field theory. We are referring here to the description of particles as quantised fields, which is known as second quantisation, and is built on the work started by Dirac who developed the formalism to quantise the electromagnetic field [26]. Attempting to recover special relativity from quantum field theory by setting Planck's constant to zero similarly represents a naive limit as it would also require us to 'undo' the quantisation mentioned above.

2.4.2.1 The metric in quantum field theory

What is of importance to our discussion is that quantum field theory and special relativity both interpret and use the Minkowski metric in the same manner. While the metric is used to describe the background spacetime, it remains independent of the spacetime points. Time in quantum field theory, as a feature built on this view of the metric, is interpreted as 'external' to physical systems in a manner which aligns with the interpretation in standard quantum mechanics.

For quantum field theory, there is a strong restriction against the interpretation of the metric as a field. If we consider general relativity as a gauge theory, it then incorporates a gauge field which represents the metric with the background subtracted. This gauge field describes massless spin-2 particles [16] in the same manner that the gauge field in electromagnetism describes a massless spin-1 particles, known as the photon. Herein lies the problem: The Weinberg–Witten theorem prohibits the the existence such particles in quantum field theory. This implies that the gauge field associated with these particles, representing the spacetime dependent part of the metric, must be similarly ruled out. Instead quantum field theory must employ a metric which remains independent of spacetime coordinates. This interpretation of the metric as constant is shared with the Minowski metric of special relativity which is also almost always taken as constant. However, the independence of spacetime coordinates does not constitute a strict requirement of special relativity. [17]

It is possible to argue that the metric in special relativity can be interpreted differently. As a limit of general relativity, special relativity could be argued to utilise the same concepts and, as such, the metric could be taken to be a field which does indeed depend on spacetime coordinates. [18] If the Minkowski metric in special relativity can be interpreted in this manner, adhering to the general relativity view, it would contradict the account in quantum field theory. We would then have two different and incompatible interpretations of time tied to each distinct interpretation of the metric.

To better address the implications of viewing the metric in special relativity in a different manner to quantum field theory, we cast the problem into the language of symmetries in order to highlight how the treatment of the metric is analogous to the treatment of gauge fields. The following section summarises the disagreement between the treatments of the metric, and so time, in terms of the symmetries associated with gauge fields.

2.5 Hope for a resolution through fundamental time

2.5.1 Emerging theories

We can consider two different paths of limits through the theories depicted in Figure 2.2. Starting first with quantum field theory, this leads to quantum mechanics which in turn leads to Newtonian mechanics. On the other hand, from general relativity we can progress to special relativity and finally back to Newtonian mechanics. These 'transitions' are ac-

[16] Particles of this nature are sometimes referred to as gravitons.

[17] Here we maintain a distinction between 'constant' and 'independent' as a metric may depend on spacetime coordinates and yet still be constant, an interpretation we clarify in Section 2.5.

[18] This relies on a special solution of the vacuum Einstein equations.

complished using the appropriate limits although we once again reiterate that these are not always well-defined. They are only being invoked in order to imply that the notion of time in Newtonian mechanics is contained in the more fundamental theories. Although both quantum field theory and general relativity can limit to Newtonian mechanics, this does not mean they can be related to one another following this reasoning. The limits leading to Newtonian mechanics are 'one-way'.

We then consider the other direction, towards a more fundamental theory from which all theories emerge. Quantum field theory and general relativity both utilise metrics as part of the respective formalisms. However each theory takes a different approach as in one case the metric is a field and in the other it is not, potentially influencing the interpretations of time that relate to these metrics. In Section 2.4.1, we examined how the treatments of time are tied to the metric of relativistic theories and so it is the use of the metric that we focus on here to try find some resolution to the disagreement.

2.5.2 Interpreting the metric in relativity

2.5.2.1 The metric and gauge transformations

First, we consider the differences between special and general relativity. As mentioned previously, the process of shifting between coordinate frames requires Lorentz transformations in special relativity while general relativity uses diffeomorphisms. The Lorentz transformations represent a 'global transformation' as they are not restricted to local neighbourhoods but instead apply universally. The diffeomorphisms used in general relativity are, by contrast, local transformation which are dependent on the spacetime coordinates. Although this seems a marked difference between the theories, further examination of these transformations can lead to a shared interpretation of the metric.

Although general relativity can be represented as a geometric theory, with diffeomorphisms transforming the metric, we are not restricted to this position as it may also be viewed as a field theory, in which case the use of diffeomorphisms can be interpreted as gauge transformations. This is exactly analogous to classical electromagnetism where gauge fields are commonly used to alter mathematical descriptions of systems without altering the laws of physics that govern measurable properties. In the case of general relativity, the laws of physics are similarly maintained under the application of gauge transformations. The rules of tensor algebra which govern the transformation ensure that no one observer is privileged and that the laws of physics remain invariant. To reiterate, an object transforming in this manner is said to transform covariantly under diffeomorphisms.

2.5.2.2 The metric and symmetries

Utilising the field theory view, the discussion can be cast into the language of symmetries. In the case of special relativity, the Lorentz invariance of a given property describes a global symmetry. Of importance to our investigation is that the Minkowski metric is a feature which remains invariant under Lorentz transformations. In general relativity, by contrast, we are instead considering diffeomorphisms which represent *local* transformations and these can be viewed, in some sense at least, as generalisations of Lorentz transformations. The transition from the special to the general theory can then be viewed as a breaking of the global symmetries that are associated with various properties of the Minkowski spacetime, including the Lorentz invariance of the metric. In this field theory point of view, gauge fields are included in the general relativistic case in order to maintain the required covariance in systems that are transformed by diffeomorphisms. A similar approach can be seen in quantum electrodynamics when the phase of the electron wavefunction is made spacetime dependent and so a gauge field must be introduced in order to maintain the consistency of the physical laws [27]. In the case of general relativity, the gauge field is the full metric minus the background. [19]

Although the use of gauge fields is typically restricted to the general theory of relativity, the concept can be applied to special relativity and, in particular, allow us to reinterpret the metric. As special relativity is a classical theory, we are free to incorporate a gauge field provided the gauge choice constrains the field to vanish as would be necessary for the continued use of Lorentz transformations. The implication is that there is a 'hidden' dependence on the spacetime coordinates as the more fundamental picture is one of local, not global, symmetry. [20] It may seem superfluous, but a similar logic is used in classical electromagnetism where the inclusion of a gauge field is not done as a necessary step in completing the theory but rather represents a means making our calculations more manageable. Introducing gauge choices into special relativity may not influence the calculations in a similar way but could allow for the interpretation of the use of global transformations and a constant metric as a useful but ultimately incomplete picture.

[19] For a fully general theory, the background can be spacetime dependent.

[20] Note the orthodox interpretation considers the spacetime coordinates are constant without considering any hidden dependence.

2.5.3 Incorporating the quantum field theory metric

We now return to our consideration of the potential differences between interpretations of special relativity and quantum field theory. Certainly the quantum theory incorporates special relativity but if the metric is now associated with a gauge field, we run into a problem. In quantum field theory, the gauge fields are viewed as representations of physical systems. For example, consider the photon, a particle represented by excitations controlled by the electromagnetic field operators. Endowing a metric with gauge freedoms is tantamount to the elevation of this metric to a quantum field operator. The associated excitation of the field would represent the creation of a massless spin-2 particle which, according to the Weinberg–Witten theorem previously mentioned, cannot be coupled to the fields of the standard model. It seems we cannot have the same interpretation of the metric in quantum field theory as we do in general relativity, undermining the argument that these ideas should arise from the same underlying source.

Hope, however, is not lost as we may turn to examples of a theory of quantum gravity where the Weinberg–Witten theorem need not hold sway. Such cases can be argued to indicate that the theory which does ultimately unify quantum field theory with general relativity may do so without necessarily including the Weinberg–Witten theorem. The use of a constant metric in quantum field theory may once again be interpreted as a limit of a more complete picture in which the global symmetry has been broken. We briefly list four arguments which suggest the Minkowski metric in quantum field theory can in fact be interpreted to have precisely this type of hidden dependence on spacetime coordinates.

The first argument is the expectation that, in a general sense, the global symmetries of quantum field theory are expected to be broken [28]. We are referring here to the so-called quantum anomalies [29], where an unavoidable breaking of global symmetries follows the quantisation and renormalisation of a theory. If this is the case, Lorentz invariance should certainly not be excluded from this requirement, provided that any effect of breaking the Lorentz invariance remains hidden as outlined above. Based on this argument, we could anticipate that a theory with a metric independent of spacetime coordinates is only a limit in which symmetries remain unbroken.

The second argument is provided from considerations of string theory. The suggestion from the framework of string theory is that any emergent theory must have a hidden dependence on spacetime coordinates [30]. As a representation of a self-consistent account of a theory containing both general relativity and all the quantum fields of the standard model, string theory provides insight into how such theories might look. Even though the ultimate theory of everything is likely different from string theory, we can at least expect certain features, such as those implied here, to remain a necessary addition. [21] And so once again we find that a lack of global symmetries is implied. This aligns with the suggestion above and corroborates the first argument.

Our third argument centers around the presence of quantum fluctuations. We are again looking for evidence of a dependence on spacetime coordinates and these fluctuations can be used to imply that such a dependence exists. If we consider a (hypothetical) theory of quantum gravity we would anticipate all aspects of this to be influenced by quantum fluctuations. Provided no fundamental restriction against this arose, there should be no reason to prevent these fluctuations from depending on the spacetime coordinates of the manifold. Thus we anticipate that the metric, as a feature of the theory, would also depend on these coordinates as a result of the effect on it from quantum fluctuations. If this is the case in the more fundamental theory, it once again implies a hidden dependence of the metric in an emergent quantum field theory.

As a final argument, we mention a more standard case against global symmetries. We are referring to the implied breaking of globally conserved quantities, for example the number of baryons, in the case where particles fall irretrievably into a black hole [32]. From a classical perspective, this rests on the expectation that all in-falling matter is destroyed by the singularity. Such an argument remains intact even after appealing to the process of black hole evaporation [33]. The Hawking radiation emitted from black holes is predominantly populated by massless particles which would be unable to carry a baryon number and other similarly conserved quantities.

2.5.4 Conclusions

Of the arguments presented above, the second and third invoke the existence of a more fundamental theory which would contain general relativity and quantum field theory. Although we anticipate that such a theory would contain a 'blueprint time', from which all the different treatments of time would emerge, there is no reason to think the fundamental description would resemble time as we know it. Ultimately, our suggestion is simply that all emergent theories are utilising the same underlying feature of the Universe, albeit in different ways. Nonetheless, the current indications are that this ultimate theory would

[21] For arguments countering the notion that string theory is fundamental see, for example, [31].

imply spacetime dependence as the more fundamental viewpoint.

Although we have focused here on arguing that the disparate versions of time can be related in a consistent manner, what we have found most compelling is the continual dependence of our existing theories on a postulate of causality, or some similar notion that is responsible for a consistent order among events in time. Motivating a dependence on such a postulate would presumably form part of the fundamental picture of time, in order to explain the experience we have of an ordered and irreversible progression of moments.

CHAPTER 3

The Isolated Universe

"What we observe is not nature itself, but nature exposed to our method of questioning."
- Werner Heisenberg

Engaging with the topic of time has led physicists to many interesting, and often diverse, conjectures. In one particular case, John Wheeler and Bryce DeWitt assembled an equation describing a Universal state which was notably time independent. The following discussion summarises this equation, the Universe it describes, and the account of dynamics produced in such a Universe. We also consider criticisms of the presented framework along with their associated rebuttals.

3.1 The Wheeler-DeWitt Equation

The Wheeler-DeWitt equation, originally presented without a formal derivation, has been investigated at length since its initial presentation in [34] and is regularly applied as a Universal model. An account of the origin and development of the equation can be found in [35], along with many other sources.

A large part of what generates interest in the Wheeler-DeWitt equation is that it unites aspects of both quantum mechanics and general relativity. To see how this is accomplished, consider the equation in Dirac notation

$$\hat{H} |\Psi\rangle = 0. \tag{3.1}$$

Here $|\Psi\rangle$ is a wavefunction representing the Universe in a pure quantum state. The $\hat{H}$ term, on the other hand, is constructed from the classical Hamiltonian constraint which governs dynamics in general relativity [36]. This classical Hamiltonian is elevated to the status of a quantum operator, thus producing $\hat{H}$ to act on $|\Psi\rangle$. The dynamics prescribed by a general relativistic theory are then applied to a quantum state description, bringing the principles of both of these theories of physics together. To follow, we point out a few important features and implications of equation (3.1), which is deceptively simple in its appearance.

Firstly, consider the Universal state $|\Psi\rangle$. While it cannot be taken for granted that the total Universe can be described by a quantum state, this approach is at the very least considered feasible. Accounts of such descriptions can be found in, for example, [37, 38, 39]. That a definitive description of a Universal wavefunction remains to be settled does not prevent us from using $|\Psi\rangle$ as representative of a Universal state which exists in principle. We do note that if a description of the Universe as a quantum state is shown to be impossible outright, it would provide a strong argument against any reasonable application of the Wheeler-DeWitt equation.

The second notable feature of equation (3.1) is the annihilation of the Universal state by the Hamiltonian operator which provides a description of the Universe as a closed system. The energy of the Universe is restricted to a constant value of zero and has no capacity to change as a function of time under equation (3.1). The conservation of energy in this manner permits the interpretation that the Universe does not exchange energy across its boundary, hence describing a closed system. [1] [2] The possibility of any external influence produced by systems 'outside' the Universe is prohibited from affecting $|\Psi\rangle$, which remains a closed, and indeed isolated, system.

It should be noted that this does not forbid the existence of 'outside' systems in principle. For example, parallel universes or a multiverse may well exist but are prevented from interacting with $|\Psi\rangle$. If such external systems are shown to be capable of interacting with $|\Psi\rangle$, it seems feasible that a boundary could be constructed to include all systems under a new state $|\Psi'\rangle$ and so preserve the principles of equation (3.1). As a brief caveat, we also point out that a total non-zero energy value in equation (3.1) would also provide a picture of an isolated Universe, provided this energy value remained constant. Such a system could

[1] A detailed perspective on the treatment of the Universe as a closed system can be found in, for example, [40].

[2] As an alternative, an open Universe interpretation could be suggested whereby energy enters and leaves the Universe in equal quantities. While this would conserve the total energy value, it requires the additional assumption of systems 'outside' the Universe along with the specific constraints on energy transfers.

easily be related back to equation (3.1) simply by adding a constant term to the Hamiltonian without altering the dynamics.

The last feature of equation (3.1) to discuss is the implications it has on our interpretation of time evolution. Considering the half of the Schrödinger equation not shown in equation (3.1), we have $\frac{d}{dt}|\Psi\rangle = 0$. The Universal state does not have any functional dependence on time and the $|\Psi\rangle$ is unable to experience any change with regard to the time parameter t. A problem is opened up by this result: How does the experience of time by systems arise within a Universe which does not itself experience time?

It would seem our Universe should simply be interpreted as a time-independent quantum state. There is however, a subtle difference between $|\Psi\rangle$ and the states used in standard quantum mechanics. Consider standard time-independent states which, when characterised by a conserved energy value, operate under a similar application of the Schrödinger equation as above. The parameter t influences such systems through the evolution of phase terms. This highlights an implicit assumption in the standard quantum scenario of isolated systems: that they reside within a larger system which does experience time. Thus the smaller isolated system does not provide an account of time itself but can 'import' the notion from the larger time-dependent system. This description corresponds to a 'static' description, persistence without change, and is discussed further in Chapter 5.

If this standard time-independent interpretation is applied to $|\Psi\rangle$ it contradicts the characteristics of an isolated Universal state. A larger system, containing $|\Psi\rangle$, may be postulated such that it provides a notion of time for internal systems of $|\Psi\rangle$, thus allowing the Universe to remain static while subsystems within it can experience time 'inherited' from the larger containing system. There are, however, at least two issues with this resolution.

First, providing time through a larger system implies that there is an external influence on $|\Psi\rangle$ albeit one restrained to influencing phase terms. This nonetheless contradicts the interpretation of the Universe as a completely isolated system free from all external influences. This is a stricter use of isolation than in the standard quantum mechanical sense but, if we relax the definition, we encounter a second issue. The larger system does not account for time itself. The problem of defining time is shifted to a larger system, undermining the method intended to account for the experience of change. Essentially, we cannot label $|\Psi\rangle$ as static without implying a containing (or reference) system which $|\Psi\rangle$ does not change with regard to. Such a system would necessarily influence $|\Psi\rangle$ by 'providing' time while remaining unable to account for how it does so.

The resulting, and generally accepted, interpretation is that the Universe, as modeled by the Wheeler-DeWitt equation, is not simply static in time but “timeless”. The experience of time is considered an emergent phenomenon which arises within the Universe but not one which $|\Psi\rangle$ undergoes. By way of analogy, if all space is contained within the Universe, it is sensible to claim the Universe cannot experience movement through space. In this manner, the Universe somehow ‘contains’ the phenomenon of time, as experienced by systems within it, but $|\Psi\rangle$ cannot ‘move’ through time as a whole by changing its state. This point is considered again in Chapter 6 and Chapter 7.

This reasoning provokes the question: can we reconcile our experience of time as *change* with this timeless model of the Universe? We will now present an account of one answer which suggests it is indeed possible. [3]

3.2 Time in the Page-Wootters Method

A method for describing change, or evolution in time, within the Universal model outlined above was presented by Page and Wootters in [15]. While maintaining the timelessness of equation (3.1), the Page–Wootters method recovers a description of time evolution for subsystems within $|\Psi\rangle$, leaving the Universal state unchanged.

The first iteration of the Page–Wootters method, as presented by the original authors, appeared vulnerable to two serious criticisms. The first is an ambiguity issue related to the choice over how to partition $|\Psi\rangle$ into subsystems [43]. The second issue is concerned with an apparent inability to account for a succession of states, leaving the subsystems of $|\Psi\rangle$ in a single state and unable to account for change [44]. A more detailed description of these concerns follows later.

Both of these criticisms were resolved by a reformulation which had the effect of clarifying the intent, and subsequently the use, of the original authors’ axioms. Here we present the Page-Wootters method in terms of this reformulation, presented in [45] (and later in [12]), and discuss the manner is which this settles the issues raised above. [4] It should be noted that the reformulation does not alter any fundamental features or assumptions, only the way in which these aspects are presented.

[3] For examples of alternative approaches regarding this model of the Universe, see [41, 42].

[4] The arguments presented in [45] employ Rovelli’s ‘evolving constants’ method which represents an alternative but related approach to recovering dynamics from the Wheeler-DeWitt equation.

3.2.1 An outline of the method

The Page–Wootters method calculates probabilities as functions that are conditional on specific states. As such, this approach is frequently referred to as the conditional probability interpretation (CPI). The CPI relies on three key features: the partitioning of $|\Psi\rangle$ into subsystems, entanglement between these subsystems, and a requirement of weak interactions. We discuss these, along with their implications, below.

We begin with the division of the Universe into two subsystems: the clock and the remainder of the Universe labeled as C and R respectively. The Universal state $|\Psi\rangle$ can then be decomposed into $|\phi_C\rangle$ and $|\phi_R\rangle$, with each representing the state of C and R respectively.

Next comes the requirement of maximal entanglement between superpositions of states of the form C_j and R_j. Under this condition, the Universal state can be written as

$$|\Psi\rangle = \sum_j \alpha_j |\phi_C\rangle_j |\phi_R\rangle_j , \tag{3.2}$$

where the coefficients α_j are normalised under the condition $\sum_j |\alpha_j|^2 = 1$. [5]

Dividing the Universe up into this entangled pair has implications for the Hamiltonian as well. Given these two subsystems, the global Hamiltonian $\hat{H}$ must contain the terms $\hat{H}_C$ and $\hat{H}_R$ to govern the dynamics of C and R respectively. As there is no boundary which must necessarily isolate C from R, any potential influence of one subsystem on the other must be accounted for by an interaction Hamiltonian $\hat{H}_I$, bringing us to the third requirement of the CPI.

The last condition stipulates that C and R only interact weakly with one another. As will be seen below, this allows the CPI to relate the dynamics of C and R such that the evolution of one corresponds to the evolution of the other. Under this weak interaction condition, $\hat{H}_I$ is considered negligible and the total Hamiltonian is

$$\hat{H} = \hat{H}_C \otimes \mathcal{I}_C + \mathcal{I}_R \otimes \hat{H}_R + \hat{H}_I = 0 \tag{3.3}$$

$$\hat{H} \approx \hat{H}_C \otimes \mathcal{I}_C + \mathcal{I}_R \otimes \hat{H}_R \approx 0, \tag{3.4}$$

where $\approx$ is the weakly vanishing constraint corresponding to the annihilation of physical states, as defined in [46], and $\mathcal{I}_{C,R}$ is the identity operator with the same dimensionality as C and R respectively.

[5] More strictly, a maximally entangled state requires the reduced density matrix for each subsystem be maximally mixed, ensuring degeneracy of the coefficients.

The relation between the remaining Hamiltonians is then given as

$$\hat{H}_C \approx - \hat{H}_R. \tag{3.5}$$

Using equation (3.5) to relate the dynamics of C and R, any knowledge of how the clock states evolve relates to a correlated evolution in R. The separate Hilbert spaces defined for each of the two subsystems governed by $\hat{H}_C$ and $\hat{H}_R$ are given as $\mathcal{H}_C$ and $\mathcal{H}_R$, which form a tensor product description for the total Hilbert space $\mathcal{H} \sim \mathcal{H}_C \otimes \mathcal{H}_R$.

The three features outlined above form the backbone of the CPI, providing a description of the time evolution of R as a process conditional on the correlated states of C. Specifically, applying the relations in equations (3.2) and (3.5) allows an observable of C, which essentially labels each state, to serve as a 'time' parameter for R. It must be kept in mind that this observable is not a measurement of the standard time parameter t, which remains a parameter that cannot be represented by a Hermitian operator, as per the framework of standard quantum mechanics explained in Chapter 2.

To provide a time parameter for R, an operator that is conjugate to $\hat{H}_C$ is identified and labeled as $\hat{K}$, such that the relation

$$[\hat{H}_C, \hat{K}] = i\hbar, \tag{3.6}$$

holds. [6] Under this condition, the observable associated with $\hat{K}$, which we call κ, can be used as an evolution parameter in place of the usual time parameter t. The operator $\hat{K}$ now serves as an effective time operator on C.

Taking κ in place of t alters the operator which governs changes of state of C. This evolution operator, labeled as U_C, becomes

$$U_C = e^{i\hbar \hat{H}_C \kappa}. \tag{3.7}$$

All dependence on the absolute time t is removed, allowing a description of the change of state of C solely in terms of an observable parameter.

Although initially given as $|\phi_C\rangle$, the clock state can be rewritten as an eigenvector of κ, such that a 'measurement' of the clock is represented by

$$\hat{K} |\kappa\rangle = \kappa |\kappa\rangle. \tag{3.8}$$

The eigenvalues $\{\kappa\}$ can be considered as 'labels' for each state of ϕ_C in much the same way that values of the usual time parameter t are used to differentiate (and order) states in

[6] The Hamiltonian in equation 3.6 is for a subsystem, not the entire system, and so the conjugate does not violate any prohibition against time operators.

standard quantum mechanics. If the spectrum of values for κ approximates a continuous number line, this observable further mimics the usual t parameter and makes it an adequate parameter for distinguishing states of R's evolution.

Taking κ to represent the 'tick' of a clock, each state $|\phi_R\rangle$ can be assigned a tick of C as per the entanglement requirement. A progression of the correlated states of C and R constitutes a description of evolution. Essentially, as C advances through its states, the entanglement ensures R is advancing through a correlated evolution which is similarly associated with κ. Under this setup, the evolution operator for R can be constructed as

$$U_R = e^{\,i\hbar \hat{H}_R \kappa}, \tag{3.9}$$

which becomes

$$U_R \approx e^{-i\hbar \hat{H}_C \kappa}, \tag{3.10}$$

under the application of equation (3.5). Thus the CPI recovers a description of evolution in time for R as a series of states linked to a measurable variable of C which progresses as $\kappa = 1, 2, 3...$.

In this description, there is no need to call upon an external time variable which 'contains' the evolving systems. The usual notion of 'evolution in time' is replaced by the description provided by the entangled subsystems C and R. An account of R's dynamics is not provided by increasing an absolute time variable t monotonically, but by identifying the associated state of C (along with the observable κ). Thus the picture of evolution which emerges is a sequence of correlated C and R states. These subsystems rely on one another, and their entanglement relationship, to describe their sequence of evolution in the absence of any external time. [7]

At this point it is necessary to point out that there is a wide choice of ways to partition $|\Psi\rangle$ and not all choices produce a subsystem which serves well as a time-keeper. A criteria for 'good' clocks needs to be identified over and above the requirement that C must interact minimally with R. It is easy enough to identify the crucial feature as the necessity of having a sufficient amount of distinguishable states for a system to function as a good clock.

The importance of having enough states can be illustrated by considering an example of an analog wall clock with no minute or second hand, only an hour hand which moves in discrete jumps from one hour to the next. Such a system cuts down the usual forty-three thousand two hundred states of a wall clock to only twelve distinguishable states.

[7] The sequence of states can also be described in terms of rotation, as discussed in Chapter 5.

Events which might have been associated with different times are then all lumped together under one label, decreasing more finely grained distinctions. In this way, a degeneracy is introduced by either associating more states of R with fewer states of C or having only a few states of R labelled by ticks of the clock, leading to a less effective clock in either case.

Even if the choice of clock subsystem is restricted to only 'good' clocks which have enough distinguishable states, a problem results from having any choice at all. We discuss this in the following section, along with a second important criticism of the CPI.

3.3 Criticisms and their responses

Here we present the two criticisms mentioned in Section 3.2 and discuss how the resolution of each within refined versions of the Page–Wootters method.

3.3.1 The problem of ambiguous clocks and the non-interacting clock solution

The clock ambiguity problem was first described in [47] and later presented in more detail in [43]. The difficulty in question revolves around the apparently arbitrary choice of how to partition the Universe into the clock C and the remainder R. This leads to a similarly arbitrary description of dynamics. We outline the crux of the argument, as it applies to the Page–Wootters method.

As per equation (3.9) given above, the subsystem R evolves with respect to the observable κ provided by C. Mimicking the usual time parameter t, κ represents a real-valued label for the clock states. The values are given, for simplicity, by a discrete series of values, $\kappa = 1, 2, 3, ..$. This follows the assumption of discreteness used in the analysis given in [47, 43].

What if we were to choose to divide the Universe up differently? As we are free to do under the Page–Wootters method, we might choose a new partition choice associated with the subsystems $\tilde{C}$ and $\tilde{R}$. The dynamics resulting from Hamiltonian operators and the clock observable associated with this new partition can then be arbitrarily different from the dynamics associated with C and R; An apparently arbitrary partition choice results in an equally arbitrary dynamical description.

As alluded to previously, this ambiguity is not cleared up by resorting to the good clock criteria. There are multiple partition choices available with weakly interacting subsystems where the clock provides a sufficient number of distinguishable states. There are any number of potential ways to divide up $|\Psi\rangle$ and, in principle, every possible description of evolution is then possible. Producing a particular dynamical description would simply require determining an appropriate choice of subsystems.

With no clear way to remove more partitioning choices and pick out a particular description of evolution as 'special', the ambiguity over dynamics remains, putting the Page-Wootters method on shaky ground. We observe a Universe in which dynamical systems exhibit a strong preference with regard to their evolution in time. Dynamical descriptions certainly do not appear to satisfy all the variations permitted by the arbitrary choice of which system happens to be the clock and we would expect to see this reflected our physical theories. [8]

We now summarise a solution to the clock ambiguity problem that was developed by Marletto and Vedral [12]. [9] These authors showed that the partitioning choice can be suitably limited if one strengthens the weakly interacting requirement discussed above. In particular, this involves extending it to the limit where no interactions are possible between C and R. Using this non-interacting limit as the criteria for the 'ideal' clock, the partitioning choice can be sufficiently reduced in order to resolve the ambiguity, an argument which we outline below.

We start by partitioning the Universal state into two non-interacting subsystems C and R. We can then consider a second partition choice associated with new subsystems $\tilde{C}$ and $\tilde{R}$. These two partition choices are equivalent if C and $\tilde{C}$ are unitarily equivalent and, similarly, R and $\tilde{R}$ are unitarily equivalent. As discussed below, the partition choices for isolated clocks must then all be the same and, if we restrict to isolated clocks only, any ambiguity is removed.

To see this, consider a partition choice $\tilde{C}$ and $\tilde{R}$ such that this partition is equivalent to C and R. Since C and R are non-interacting, then $\tilde{C}$ and $\tilde{R}$ must be similarly non-interacting. All partition choices restricted to non-interacting subsystems can then be expected to provide the same description of dynamics. If, however, we choose a partition $\tilde{C}$ and $\tilde{R}$ which is *not* equivalent to C and R, then the transformation $C \otimes R \rightarrow \tilde{C} \otimes \tilde{R}$ is not factorisable. The transformed Hamiltonian governing $\tilde{C}$ and $\tilde{R}$ is also not factorisable, leading to the

[8] The arbitrary assignment of partitions differs from, say, coordinate choices

[9] In Chapter 5 we will discuss an alternative resolution based on a different interpretation of time in the isolated Universe.

conclusion that $\tilde{C}$ and $\tilde{R}$ are interacting subsystems. Now consider a partition choice which is related to C and R via non-unitary transformations. In such a scenario, the non-unitary transformations are related to interactions occurring between subsystems.

Consider the unitary operator $\hat{U} = e^{i\hat{X}}$ used to induce a finite partition where U defines an isomorphism, a reversible mapping relating two objects of the same structure, which in this case are H and $H_1 \otimes H_2$. The term $\hat{X}$, which is the generator of the transformations of U, is a Hermitian partition operator defined as a Hermitian matrix which induces an infinitesimal partition of the system. The operator $\hat{X}$ can be mathematically described as $\hat{X} = \hat{X}_C \otimes \mathbb{1} + \mathbb{1} \otimes \hat{X}_R + \hat{X}_{CR}$. If the term $\hat{X}_{CR}$ which mixes the subsystems is present, it ensures that any separable partition acted on by $\hat{U}$ is no longer separable after the transformation. Provided the term which mixes the states of the subsystems is non-vanishing, the partition becomes non-trivial. In particular, consider a separable Hamiltonian given as $\hat{H} = \hat{H}_C \otimes \mathbb{1} + \mathbb{1} \otimes \hat{H}_R$. The transformed Hamiltonian $\tilde{\hat{H}} = \hat{U}^{\dagger}\hat{H}\hat{U}$ can be expected to contain an interaction term $\hat{I}_C \otimes \hat{I}_R$ as a result of $\hat{X}_{CR}$. The conclusion is that the operators $\hat{X}$ and $\tilde{\hat{X}}$, associated with $\hat{H}$ and $\tilde{\hat{H}}$ respectively, cannot be taken to describe comparable partitions because, while the transformation relating them is formally unitary, it restructures the partition in a non-local way. The associated evolution is also non-unitary, an example of which is given in the Lindblad equation [49], the features of which are discussed in more detail in Chapter 4.

The inclusion of a significant interaction term in the Hamiltonian contradicts the relation given in equation (3.5) which is required at least approximately by the CPI. Extending this, any potential ambiguity can be removed by restricting the choice of how to divide up the Universe to non-interacting subsystems only as they are all related by a unitary transformation and so using one isolated clock is essentially the same as using another. This solution appears to fit extremely well within the Page–Wootters method as the restriction to use only isolated clocks is simply an extension of the weakly-interacting condition already in place.

The case of a non-interacting clock does resolve the ambiguity issue but it raises other concerns. One in particular is the interpretation of time as an illusion, a notion that is reinforced by the use of non-interacting clocks. [10] A second issue is the reliance on the idea that interactions can be excluded in principle. As the model of the Universe in question incorporates aspects of general relativity, the effects of gravity must be taken into account since these cannot be shielded. Interactions are then not so easily dismissed and in Chapter 5 we

[10] Two non-interacting systems operate as isolated systems, bringing us back to the initial interpretation of time as an illusion.

further assess the interpretive issues surrounding non-interacting subsystems. Specifically, we argue that interactions are conceptually incompatible with the timeless interpretation of an isolated Universe and that the presence of (weak) gravitational interactions supports interpretation of change as a real phenomenon.

Putting these considerations on hold for the time being, we move on to a second important criticism of the CPI.

3.3.2 Kuchar's criticism: measuring successive times

The most commonly mentioned criticism of the CPI was raised by Kuchar in [44]. The crux of the issue is the CPI's inability to compare successive states. In brief, the formalism presented in [15] calculates a probability associated with one instant; The states of C and R are correlated to one another but not related to other paired states, i.e. C' and R'. Restricting the description to one state in the probability calculation implies that the CPI fails to describe a succession of evolving states. But standard evolution through the Schrödinger equation includes this dynamical aspect where states follow each other as they progress through time. The lack of such a feature then makes the CPI an inadequate description of evolution in the Universe.

Page's response was to dismiss the relevance of Kuchar's claim [50], arguing that the direct comparison of a state with a previous (or later) one is unnecessary. One instantaneous state is all that is ever directly accessible in practice, making tests of any other scenario impossible. There is then no reason to require a description of the progression of states in the probability calculations.

Page's viewpoint has since been superseded as rebuttals to Kuchar's criticism have been presented from multiple sources, including in [45]. One argument of particular relevance here is the solution offered by Dolby [51]. It involves a refinement of the CPI which relies in part on an abstract integration variable, among other features. While the use of this variable assists in resolving Kuchar's criticism, it is typically overshadowed by other features. A similar approach can be found in [52] where the use of the abstract integration variable is emphasised. After presenting Dolby's resolution, we will briefly discuss this second approach.

There is another advantage to the refined CPI. The clock, as previously described, is an abstract system with a similarly abstract observable which provides an account of time in principle. But it need not remain so as realistic clock systems can be employed within the refined CPI framework and, in particular, this includes interacting subsystems. As a more realistic scenario than the isolated case, interacting clocks are better investigated using the refined formalism along with toy models based on real clocks. [11] To assist in this, we focus on the resolution to Kuchar's criticism which makes use of the refined CPI, built for such a scenario, rather than the formulation provided in [45].

3.3.3 The refined CPI solution

The refinement relies on recasting the CPI in terms of physical operators. The description of C and R that then appears in the formalism is one of physically measurable states. The Hilbert space containing all possible physical states is identified as $\mathcal{H}_{ph}$, along with the projection operator $\hat{P}_{ph}$ which is responsible for selecting out physically measurable states. As mentioned above, one advantage of this approach is its ability to investigate real world clocks. The refined CPI not only potentially allows for more application of the framework but also provides a means of analysing scenarios involving interacting clocks, as we will see in Chapter 4.

In order to use this physical representation to resolve Kuchar's concerns, the conditional probability functions required for the CPI are considered. An important distinction is needed here. A typical conditional probability expression resembles the function $P(R \mid C; \rho)$, where ρ represents the density matrix of the Universal state containing C and R. The interpretation is that the probability of the outcome of a state of R is *conditional* on the state of C, the terms after the vertical divider. This can be loosely described as the probability of R *if* C. [12] In the refined CPI, a new probability expression is described: $P(R \textit{ when } C; \rho)$ and, although it is presented as a means of ensuring simultaneity in [51], there is no consensus over this physical interpretation. The mathematical differences between the probability expression are clarified in Appendix A where we also discuss the interpretive issues. As the issue does not prevent the refined CPI from recovering standard quantum mechanics results, we do not discuss it here. Without relying on the interpretation above, the states of C and R are still able to be synchronised as required by the refined CPI formalism.

In order for C to act as a clock for R, the state of one must coincide with the other such that the state of R correlates to the provided time reading. The refined CPI accomplishes

[11] That interacting clocks should be used in principle is explored in more detail in Chapter 5 where we discuss potential shortcomings of the isolated case and the problem with isolation as matter of principle.

[12] In the terminology used in [51], this standard probability is identified as $P(R \textit{ given } C; \rho)$.

this by the use of an abstract integration variable n. This variable replaces the usual time parameter t in the dynamical equations used to evolve C and R according to

$$\int dn\ \lambda_n \left|C_n\right\rangle \left|R_n\right\rangle . \tag{3.11}$$

. It allows a definition of the 'overlap', or 'syncing', or C and R. Crucially, n is not a physically realisable quantity. The role it plays is as a label for the states of C and R. This works much the same way that the monotonically increasing numberline, denoted by t, is used to represent labels for states as they evolve. If two states are labeled by the same value of n, then they are 'synced' and we have a useful way of defining 'simultaneous' states. The two subsystems C and R can be evolved separately using the abstract evolution parameter n. The state of R 'at' n can then be written as a function dependent of the value of the clock observable associated with C 'at' n. This process involves integrating out n, ensuring it remains an abstract variable and is not mistaken for a physical quantity of any kind.

Given a pair of initial states $\phi_C\ (n_0)$ and $\phi_R\ (n_0)$ for C and R respectively, with both contained in $\rho(n_0)$, where n_0 represents an 'initial time'. A final state for each subsystem at a 'later time' n would be given by $\phi_C(n)$ and $\phi_R(n)$. The two types of probability expressions described above can be used to provide the 'two-time' probability description

$$P\Big(\phi_R(n)\ when\ \phi_C(n) \mid \phi_R(n_0)\ when\ \phi_C(n_0); \rho(n_0)\Big). \tag{3.12}$$

This expression allowed Dolby to calculate the probability of finding a specific physical state of R which corresponds to a physical state of C, given a pair of initial (and similarly correlated) states of C and R, thus resolving Kuchar's concern.

A useful feature arises from this refinement of the CPI. We are referring to the definition of an 'ideal clock limit'. It represents the limit in which the time parameter provided by C is indistinguishable from the parameter t in standard quantum mechanics. The identification of this limit relies on the separate evolution of C and R as functions of n. Analysing the clock state as a separate system, the following function can be identified: $f_C(n) \equiv \langle \phi_C\ |\phi_C(n)\rangle = \langle \phi_C(n)|\, e^{-i\hat{H}_C n}\ |\phi_C(n)\rangle$. Within the refined CPI, the effective state of R is functionally dependent on the clock observable as a result of including this 'clock function'; a measure of the overlap of the clock wavefunction at different times. It should be noted that the initial time is taken to be $n_0 = 0$. When $f_C\ (n)$ approaches a delta-function $\delta(n)$, and the overlap is minimal, the clock observable is expected to begin to mimic the usual time parameter t. Under this limit, the evolution of R resembles standard quantum mechanics and the clock can be identified as optimal. This provides a useful way of measuring the effectiveness of a clock, as will be seen in Chapter 4.

This refinement of the CPI, and its use of an abstract integration variable used with physical operators, bears a resemblance to the description used in the investigations carried out by Gambini *et al* in [53, 52, 54]. In both cases, the emphasis is on the use of realistic clocks. This must, by definition, refer to clocks which experience interactions. As pointed out in Section 3.3.1, any realistic scenario which incorporates the effects of gravity must contain unavoidable interactions, however negligible. Adequate analysis tools, such as that provided by the clock function, are required in order to investigate the effect of taking these interaction effects seriously.

3.4 Considering interactions

As pointed out in Section 3.3, the standard approach to the CPI is to assume that the weakly interacting subsystems are, in the 'ideal' case, limited to perfect isolation, a scenario which has been argued to be possible in principle, as in [12]. In a sense, this backs up the timeless interpretation. The isolation of C and R imply they must also be interpreted as timeless, similarly to the isolated Universe, a point explored in more detail in Chapter 5.

By taking the non-interacting limit from the start of the analysis, unknown consequences are possibly being excluded. The result of including interaction has the potential to affect various issues surrounding the interpretations associated with the CPI and the isolated Universe at large. In Chapter 4 we examine these interaction effects with the purpose of revealing any influence they may have on how the CPI is applied and interpreted. This has been done to some extent in [55] where the focus was on the mathematical complexities of including an explicit interaction Hamiltonian ($\hat{H}_I$) in the CPI framework. Here we will focus on developing toy models of the subsystem C to instead investigate any interpretive implications of either ignoring or including interactions between C and R as a matter of principle.

The potential shortcoming of the non-interacting limit and the associated timeless interpretation, briefly mentioned in Section 3.3.1, are further discussed in Chapter 5. There are also philosophical considerations to take into account. If interaction effects are considered a necessary feature of the CPI, this has potential ramifications for the interpretation of time in the isolated Universe. As we will argue in Chapter 6, interactions can be considered incompatible with the timeless view, resulting in an alternative approach to the interpretation of time under the CPI.

CHAPTER 4

Interacting Clocks

"Entropy is the price of structure."

- Ilya Prigogine

Analysing the role of interactions within the CPI framework can be done by considering specific physical systems. After defining a suitable system to serve as our clock, we can examine a chosen observable along with its associated uncertainty. According to the definitions in Chapter 3, the ideal clock is an isolated system. Given that any interaction effects are excluded, the isolated clock may be made arbitrarily accurate and so provide a potential uncertainty of zero. Alternatively, in examining the *optimal* case we might provide a more nuanced analysis of the situation.

The optimal case can be defined as follows. Taking an interacting system as the clock, with a suitably chosen observable, the optimal outcome is one where the uncertainty accompanying the clock observable is *minimised*. If the isolated clock is indeed the optimal choice, we would expect to see a minimum uncertainty when the clock hits the non-interaction limit with zero interactions. However, as will be shown below, it is the presence of weak interactions that ultimately ensures the smallest uncertainty, while still maintaining a functioning formalism for the clock system. In a result which perhaps runs counter to intuition, *weakly* interacting clocks appear to be the optimal choice.

To substantiate this claim, we examine interacting clock systems within a toy model of the Universe, the details of which are supplied below. The first clock system under consideration is a damped harmonic oscillator. The analysis is then reinforced and further developed by an investigation into a second clock system, given by a two-state atom. [1]

4.1 The toy Universe

As per the description in Chapter 3 any potential interaction effects between C and R, the partitioned subsystems of the Universal state $|\Psi\rangle$, would be governed by an interaction term $\hat{H}_I$. The weak interaction condition of the CPI implies that $\hat{H}_I$ is expected to have a minimal contribution to the total Hamiltonian. Thus the approximate relation $\hat{H}_C \,\dot{\approx}\, -\hat{H}_R$ can be considered true, in terms of the weakly vanishing Dirac constraint introduced in the previous chapter. In the case of a strictly non-interacting clock partition with $\hat{H}_I = 0$, this can be extended to the weak equality $\hat{H}_C \,\dot{=}\, -\hat{H}_R$.

Although the approximate relation implied by the weak interaction condition is used commonly in applications of the CPI, formulations exist where $\hat{H}_I$ has been explicitly included, as done in, for example, [55]. As we are interested here in examining general trends of the formalism, our investigation follows the former approximation approach. In aid of this, we define a toy model of the Universe which simplifies the mathematics in favour of the more general examination.

The toy model is based on the approximation that the clock is much smaller than the rest of the Universe: $C \ll R$. [2] This contradicts the CPI framework in the sense that C and R are typically required to be of the same order, a result of enforcing maximal entanglement between the two subsystems as well as the necessity of sharing a sufficient amount of information regarding the states. In this context, 'sufficient' refers to the minimal information R requires in order to have enough knowledge of the clock state to make use of the time parameter. We expand on this issue in Chapter 5 where interactions are considered between subsystems of comparative sizes. For now, we consider clocks within a toy model Universe only.

Regardless of this restriction, we may investigate realistic clocks without dramatically changing the formulation of the CPI. Under the condition $C \ll R$, any interaction effects can be assumed to have a significantly larger effect on C than on R. The CPI's weak interaction requirement further ensures the effect of C on R is so minimal as to be considered negligible. The advantage of this scenario is that it allows us to incorporate the interactions

[1] A published description of the damped harmonic oscillator analysis is given in [17] while the atomic clock investigation can be found in [18].

[2] This assumption regarding the dimensionality of the subsystems is more explicitly stated as $\mathcal{D}im(C) \ll \mathcal{D}im(R)$.

as a component of the clock dynamics and the effect of R on C can be contained within $\hat{H}_C$, negating the need for an explicit expression of $\hat{H}_I$. As we discuss in the analysis of the clock to follow, this also permits a recovery of the CPI. The subsystem dynamics remain related via $\hat{H}_C \doteq\approx -\hat{H}_R$ and so we need not consider an independent and explicit form of $\hat{H}_R$ either.

One last point on the toy model description above: The interaction effects are more appropriately interpreted as the influence of an environment on C. While we continue to use the notation R to represent the system with which C interacts, it must be kept in mind that in this context the 'remainder of the Universe' is synonymous with an environmental system for the analytical examples which follow. We now turn to our first clock system.

4.2 The lightly damped harmonic oscillator

The first clock is a harmonic oscillator. It involves a periodic motion that arguably underlies most mechanical clocks. Interactions are also easily included into such a system in the form of a damping effect imposed by the environment.

4.2.1 The clock state

As per the description laid out in Section 3.3.3, the evolution of the clock is initially described in terms of n: The effective evolution parameter. The series of states through which C progresses are labeled by successive values of n, in the same manner that t conventionally labels sequential moments of time. Ultimately, n is integrated out and replaced by a time parameter based on an observable of the clock. Our goal is to examine how the evolution of successive states of C affects the uncertainty associated with the clock observable and, by extension, the resultant time parameter ultimately used by R. We begin our analysis by selecting an observable of the damped harmonic oscillator.

The optimal choice of observable would be one that has an approximately linear relation to n. This is due to the fact that the linearity ensures the clock observable is 'synchronised' to the sequential values of the evolution parameter n; For each successive value of n, the clock observable will progress similarly, up to a multiplication factor. This will be expanded on and clarified by example with our specifically selected observable once we have defined the linear relation.

For the case of the damped harmonic oscillator, we select the position of the center of mass as our clock observable. We abbreviate this to 'position' and denote it as x in order to simplify the discussion to follow. The position of the oscillator can be interpreted as a measurement of time in a similar manner as how a (physical) measurement of the position of the hands of an analog clock correlates to a time reading. The conversion of a position variable into one of time can be easily accomplished by dividing the position value by c, the speed of light. To utilise this conversion in an efficient manner, we set $c = 1$. The analysis to follow can now use the position as an effective time measurement.

For each change in n to correspond to a similar change to the x, the position must relate to n in a linear fashion as it is this one-to-one relation which allows us to replace n with x as the evolution parameter. To show that a linear relation between x and n is possible, we apply the classical relations connecting position and time for the oscillator. Replacing t with n as the surrogate evolution parameter, we can determine n's relation to the clock's position. This is done explicitly in Section 4.2.2 after the following mathematical description of the clock evolution.

The Universe, as a large system, is expected to behave in a classical manner. To facilitate this, we choose a coherent state description for our clock, based on the propensity of such a state to mimic classical behaviour. Specifically, we use the coherent state wavefunction that is applicable to a damped harmonic oscillator system and which was developed in [56]. Since x has been identified as the clock observable of interest, the wavefunction description is put into the position representation. We outline the development of the wavefunction below but a complete account is available in [57, 56].

The dynamics of the damped oscillator can be described by the Caldirola–Kanai Hamiltonian [58, 59], which is given as

$$\hat{H}_C = e^{-rn}\frac{p^2}{2m} + e^{rn}\frac{1}{2}\omega_0^2 x^2, \tag{4.1}$$

where r, typically referred to as the damping coefficient, is treated as a variable controlling the strength of the damping effect on an oscillator with an undamped frequency ω_0 and mass m. After damping is applied, the clock will oscillate at a frequency given by $\omega = \sqrt{\omega_0^2 - \frac{r^2}{4}}$. Note that we have set $\hbar = 1$ both here and throughout the remaining calculation.

As per the CPI formulation, equation (4.1) evolves as a function of n. The initial value of n, simulating an initial time, is taken to be $n = 0$. An important feature to note regarding the above Hamiltonian is that the mass can be interpreted as the time-dependent term $m(n) = e^{rn}m$. Then, even though a coherent state can maintain a time-independent uncertainty, this adapted description ensures the effect of R on C introduces a time-dependence into the uncertainty, provided a mass term is present.

With equation (4.1) providing a starting point, the coherent state representation can be developed by following the procedure in [57, 56]. To qualify as a description of a coherent state, four properties must be met. First, the wavefunction must be described in terms of the eigenstates of a suitably defined annihilation operator $\hat{a}$. In mathematical terms, this eigenstate relationship is represented as

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle\,, \tag{4.2}$$

where α is the complex parameter of the coherent state. The second condition for the coherent state is that these eigenstates $|\alpha\rangle$ be associated with a minimal uncertainty. Thirdly, the eigenstates must be produced by acting on the vacuum state, given as $|0\rangle$, with $\hat{a}^\dagger$. The fourth and final requirement is that the coherent states form a complete and normalized set.

The annihilation and creation operators, $\hat{a}$ and $\hat{a}^\dagger$ respectively, are defined as follows:

$$\begin{aligned}\hat{a} &= \frac{1}{i}(\eta\hat{x} - \mu\hat{p})\\ \hat{a}^\dagger &= \frac{1}{i}(\mu^*\hat{p} - \eta^*\hat{x}),\end{aligned} \tag{4.4}$$

where η and μ are time-dependent complex coefficients that are defined in detail in Appendix B.1. These coefficients also depend on ω, the damped frequency of the oscillator, as well as on r, and their dimensionality matches that of the position and momentum; Specifically, $[\eta] = [p]$ and $[\mu] = [x]$. Using these definitions for the annihilation and creation operators, equation (4.1) can be rewritten in terms of coherent states. Using the position representation, the wavefunction associated with the resulting Hamiltonian is

$$\phi_C(x,n) = \langle x\,|\alpha(n)\rangle = \mathcal{A}\; exp\Big[-\frac{\eta(n)}{2i\mu(n)}x^2 + \frac{\alpha}{\mu(n)}x - \frac{1}{2}|\alpha|^2 - \frac{\mu^*(n)}{2\mu(n)}\alpha^2\Big], \tag{4.5}$$

where $\mathcal{A}$ represents a normalisation constant defined as $\mathcal{A} = (2\pi\mu\mu^*)^{-\frac{1}{2}}$. We now have a wavefunction for the damped harmonic oscillator in the representation of our selected observable.

The probability density of this clock landing up in a particular position is given by $P(x,n) = |\phi_C(x,n)|^2$. By quantifying the likelihood of finding a particular state, this probability expression serves as a means of analysing the effectiveness of the clock in question. We are specifically interested in the associated uncertainty and the manner in which it changes with respect to successive values of n. This will be explained further in Section 4.2.3 after we examine how the damped harmonic oscillator fits within the CPI framework to provide a description of time.

4.2.2 Recovering time and the 'run time' limit

The CPI is typically formulated to use a clock observable κ, associated with an operator conjugate to the Hamiltonian $\hat{H}_C$. In the case of the damped harmonic oscillator, however, our chosen observable is the position x. As described above, by converting x to a time reading with a suitable multiplication of c, a position measurement can be directly interpreted as a time reading. We now consider the required linear relation between x and the evolution parameter n which is used to recover the CPI. [3] What follows is our motivation that such a relationship can exist for the damped harmonic oscillator described above. [4]

Before relating n and x, we establish some necessary conditions. First, we enforce $\frac{r}{2} < \omega_0$ in order to ensure that the clock remains underdamped, avoiding the critical and overdamped cases. Alone this is not a sufficient restriction. The weak interaction condition of the CPI requires an additional restraint on the exponential term rn in equation (4.1) to constrain the clock to the *light* damping regime. We must include the condition $rn \lesssim \mathcal{O}(1)$ which only permits weak interactions that would not affect the clock in a significant manner over a small 'time' n.

One final restriction must be enforced in order to recover the CPI formalism. Within the domain of light damping, we introduce a 'reset time', n_{reset}, such that $n_{reset}r < 1$. As the discussion to follow shows, the Taylor expansion allowed by enforcing this condition is a necessary step in recovering a linear relationship between n and x. Thus n_{reset} does not represent a physical reset of the clock but should instead be seen as a reset of the formalism to maintain the linearity necessary for the CPI. [5] With the light damping and reset time conditions in place, we can now develop a linear expression for $x(n)$.

Our starting position is the equation for the expectation value which provides a relation between $\langle x \rangle$ and n. A simple rearrangement is then required to attain $n(x)$ which can be used to evaluate whether the relation is in fact linear. The wavefunction in equation (4.5) has an associated expectation value given by $\langle x \rangle = \mu^*\alpha + \mu\alpha^*$ [56]. Employing the condition $rn < rn_{reset} < 1$ discussed above, the calculation can be simplified by considering the leading order terms of rn only, resulting in the expression

$$\langle x \rangle = Ae^{-\frac{rn}{2}}, \tag{4.6}$$

where $A = \sqrt{\frac{2\hbar}{m\omega}\mathcal{R}(\alpha)}$. A more detailed description of this calculation is presented in Ap-

[3] This may be generalised to a monotonic relationship.
[4] For our argument to follow, the linear relationship need only hold piecewise.
[5] The reset time should not be interpreted as a means of ensuring a state of minimum damping but solely as a means of countering the breakdown of linearity between n ad x as n becomes large.

pendix B.2. Inverting equation (4.6) produces

$$n(x) = \frac{2}{r} ln\Big(\frac{A}{< x >}\Big). \tag{4.7}$$

A naturalness argument can be applied at this point to the effect that no one scale is considered to be parametrically larger or smaller than any other. The calculation can essentially be simplified by maintaining that this is a 'one-scale' problem: $n_{reset} \sim \frac{1}{\omega}$. As we are in the light damping regime, we can also assume that there is very little difference between the undamped frequency and the resultant frequency: $\omega_0 \sim \omega$. These substitutions allow us to rewrite the resultant frequency expression $\omega = \sqrt{\omega_0^2 - \frac{r^2}{4}}$ as

$$\omega \sim \omega_0 \sim r \;\rightarrow\; \omega = ar, \tag{4.8}$$

where a is some numerical factor of order one.

Equation (4.7) is then expanded and rewritten as

$$\begin{aligned} \frac{rn}{2} &= ln\Big(\frac{A}{x}\cos(arn)\sqrt{1+arn}\Big) \\ &= ln\Big(\frac{A}{x}\Big) + ln\Big(1 - \frac{a^2r^2n^2}{2} + O(r^4n^4)\Big) + ln\Big(1+arn\Big)^{\frac{1}{2}} \\ &= ln\Big(\frac{A}{x}\Big) + \frac{arn}{2} - \frac{a^2r^2n^2}{2} + O(r^4n^4) \\ \frac{rn}{2}(1-a) &= ln\Big(\frac{A}{x}\Big) - \frac{a^2r^2n^2}{2} + O(r^4n^4). \end{aligned} \tag{4.10}$$

From inspection, equation (4.10) is resolved by either setting $\frac{A}{x} > 0$, $a < 1$ or, vice versa, $\frac{A}{x} < 0$, $a > 1$. We select the first combination of values for A and a but point out that either choice would lead to similar conclusions. Substituting these values into equation (4.10) gives

$$\ln\Big(\frac{A}{x}\Big) \approx rn(1-a) \gg \frac{r^2n^2}{2}, \tag{4.11}$$

where higher-order terms are excluded as per the reset time constraint and the factor of $\frac{1}{2}$ is omitted as allowed by the small enough value of rn. The relation $\frac{rn}{2} \approx ln\Big(\frac{A}{x}\Big)$, once rearranged, serves as an adequate description of $n(x)$.

We have from equation (4.6) that $x \lesssim A$. Using this, expressed as $\frac{A}{x} \gtrsim 1$, we resolve $n(x)$ as

$$\begin{aligned} n &= \frac{2}{r} ln\Big(\frac{A}{A-y}\Big) \\ &\approx \frac{2}{rA} y, \end{aligned} \tag{4.13}$$

where we have used the definition $y = A - x$.

The application of this result within the CPI requires consideration of the evolution operator $U_C = e^{i\hat{H}_C n}$. Given equation (4.13), we can make the substitution $n = \frac{2}{rA}y$ resulting in

$$U_C = e^{\,i\tilde{\hat{H}}_C y}, \tag{4.14}$$

where we have also used the redefinition $\tilde{\hat{H}}_C = \frac{2}{rA}\hat{H}_C$; a simple matter of rescaling given $\hbar = c = 1$ and so the dimensionless relation $[rA] = 1$. The evolution is now a described as a function of the clock observable.

As the weak interaction limit has been enforced, the Hamiltonian $\hat{H}$ governing the total state of C and R is roughly separable. The result is an approximate commutation relation between the position operator on C and any operator on R, defined as $\hat{O} \in R$. The relation $\hat{H}_R \approx -\hat{H}_C$, described previously, implies that the evolution operator for R can be written as $U_R = e^{-i\tilde{\hat{H}}_R y}$, where a similar redefinition $\tilde{\hat{H}}_R = \frac{2}{rA}\hat{H}_R$ is used. Once this is taken into account, along with the description of the decomposition of $|\Psi\rangle$ described in Chapter 3, the evolution description becomes [6]

$$\begin{aligned} \langle x|\, \hat{x}\, (|\phi_R\rangle\, |\phi_C\rangle) &= \langle x|\, \hat{x}\, |\phi_C\rangle\, |\phi_R\rangle \\ &= x\; \phi_C(x)\, |\phi_R(x)\rangle\,. \end{aligned} \tag{4.16}$$

The conclusion is that, from the point of view of R, any x (or $y(x)$) is only seen as a numerical parameter. This is in comparison to the perspective of C where x is seen as the spectral value of an operator.

This use of the measurement of C as a time parameter remains feasible under the reset time constraint

$$rn < rn_{reset} < 1, \tag{4.17}$$

and so a linear relation between x and n can be established. We now begin our analysis of the clock by considering the associated uncertainty.

4.2.3 The uncertainty and the decoherence rate

As laid out in Section 3.3.3, the effectiveness of the clock can be tied to how sharply peaked the probability of finding C at a particular x at n is, with the ideal case being a delta-function. If this probability, denoted $P(x, n)$, is represented as a Gaussian function, the associated width provides us with a measure of how far the distribution is from the desired delta function. Minimising the width in order to approach the sharpest possible peak also affects the related

[6] Specifically, it is the seperable state described in equation 3.2 that we draw on here.

terms; namely, the variance δ_x^2. The value of r which minimises δ_x represents the strength of interactions resulting in the most accurate, and so optimal, clock. Also of interest is the rate of change of δ_x as a measure of how quickly the system decoheres. [7] If the rate of decoherence is too high, the oscillator will not 'run' long enough to provide the required number of distinguishable states necessary for it to function as a clock.

The uncertainty associated with the variance of $P(x, n)$, for the damped harmonic oscillator described above, is given by [56]

$$\delta_x = \sqrt{\langle x\rangle^2 - \langle x^2\rangle} = \sqrt{\mu\mu^*}. \tag{4.18}$$

As our interest remains in the basic qualitative result and so, once again, the calculation is simplified by restricting to the first order limits following the outline in Section 4.2.2.

To the first order of $rn \sim \omega n$, equation (4.18) can be rewritten as

$$\delta_x = e^{\frac{-rn}{2}}\sqrt{\frac{1}{2m\omega}}. \tag{4.19}$$

A more detailed description of this calculation can be found in Appendix B.3.

The rate of change of the uncertainty is established from equation (4.19). This refers to the decoherence rate of an open system in a surrounding environment: How quickly does the environment influence the state. We follow the analysis presented in [60] (also see [61]), where the decoherence rate was defined as $\sigma(n) = \frac{\partial(\delta^2_{x,min})}{\partial n}$ and used in the context of non-unitary Lindblad evolution, which we discuss in detail later. After substituting in the uncertainty from equation (4.19) into the decoherence expression, we have

$$\sigma(n) = \frac{re^{-rn}}{m\omega}. \tag{4.20}$$

We can now examine how the expressions for $\sigma(n)$ and $\delta_x(n)$ might be minimised.

The decoherence rate, as given in equation (4.20), is minimised by $\frac{\partial\sigma(n)}{\partial n} = 0$. This calculation gave

$$rn = 1. \tag{4.21}$$

This reinforces the reset time condition discussed in Section 4.2.2, minimising the decoherence rate under $r = \frac{1}{n}$.

The same minimisation procedure was then applied to the uncertainty from equation (4.19), giving

$$\frac{-n}{2}\sqrt{\frac{1}{m\omega}}e^{-\frac{rn}{2}} = 0. \tag{4.22}$$

[7] Note that this decoherence is a result of the oscillator's interaction with its environment, which we have purposely kept large enough to remain negligibly affected by the clock.

To satisfy equation (4.22), we would need to set $r \to \infty$. However, we are restricted by the condition $rn < rn_{reset} < 1$. As such, setting r to infinity would require setting $n = 0$. Physically this would correspond to a clock which did not run at all! We must limit the expression to a 'maximum' value of r which would still allow the clock to progress through a sufficient number of states. It may well be argued that the alternate limit of $r \to 0$ could be applied in principle as this would simply require the limit $n \to \infty$. However, while this cannot be excluded in the case of the damped harmonic oscillator, the atomic clock system considered in the following section rules out this possibility.

Ultimately then, the weak damping condition results in the need to balance the rn term between two opposing scenarios. On the one hand, increasing the time (or amount of states) available to the clock runs requires setting r to the smallest possible value in order to keep n_{reset} large enough. On the other hand, we require r to be as large as possible in order to maximise the accuracy of the clock. A more detailed optimisation of these factors would depend on further specification of the clock. However, within the context of this investigation, we simply note that a totally isolated clock, which would require setting $r = 0$, does not correspond to the optimally functioning clock. This result is not consistent with the claims made in [12] and so, to verify the validity of our result, we consider another clock system.

4.3 The atomic clock

The next system under investigation is an atomic clock; A real world example of an extremely accurate time-keeping procedure. [8] More importantly, at least with respect to our investigation, the atomic clock system can be shown to eliminate the possibility of satisfying $rn_{reset} < 1$ by setting $r = 0$ and allowing for an infinite run time. Additionally, we can choose a different observable. Specifically, we can exchange position for frequency since the atomic clock relies on the frequency associated with the energy transitions of specific atoms. While this alternative choice should not affect our conclusions in any crucial manner, it nonetheless broadens the scope of our results.

[8] Note that we use a toy model, intended to investigate the importance of interactions in princple rather than represent reality, and so the description of entanglement is limited and not developed into a fully realistic model. For a realistic description see, for example, [55].

4.3.1 Keeping time with atomic clocks

The first use of atomic systems and their associated energy transitions as a means to define time measurements is credited to Rabi [62]. The specific type of repeated transitions described in this method have become known as Rabi oscillations, an explanation of which can be found in most quantum mechanic textbooks, for example [63]. The procedure starts with a two-state system which we denote S. Next, ω_{12} is identified as the specific frequency associated with a transition of S from the ground state to the excited state which have energy levels of E_1 and E_2 respectively. Setting $\hbar = 1$, similarly to the first analysis, the transition frequency can be written as $\omega_{12} = E_2 - E_1$. If an electromagnetic wave is set to this frequency, we can measure time by counting the total number of cycles the wave progresses through and dividing by the frequency.

Our focus is on the general use this procedure and, as such, it is not necessary to specify any one particular atomic system, over the multiple similar systems available, as the one employed here. However, we do provide a brief example in order to give a sense of the precision these clocks can achieve. In the case of a cesium atom, the transition frequency between the two states used by the clock is $9,192,631,770$ Hz. Counting the oscillations of an electromagnetic wave set to this frequency produces a measure of one second that can be separated into a phenomenally large number of discrete intervals. This provides the opportunity for highly precise time measurements.

The electromagnetic wave's frequency, which we denote ω, must be as close to ω_{12} as possible for this procedure to produce the most accurate time measurement. The electromagnetic wave is tested by preparing an atom in its ground state and then exposing said atom to the wave oscillating at frequency ω with the intention of stimulating a transition to the higher energy level. The probability P_{ex} of finding the system in the excited state can then be examined as a function of ω. The maximum chance of seeing a transition is associated with the resonance case, when the wave oscillates exactly at the transition frequency: $\omega = \omega_{12}$. This defines the ideal case with a maximum probability of transition $P_{ex,MAX}$. In practice, the frequency cannot be set with such precision. To account for any deviation from the ideal result, we introduce the quantity $\theta \equiv \omega - \omega_{12}$. For the analysis to follow, the resonance case is then associated with $\theta = 0$.

The probability P_{ex}, plotted as a function of ω, is portrayed in Figure 4.3.1. This representation helps identify a useful parameter: the full width half maximum (FWHM) which measures the spread of P_{ex} across the values of ω satisfying $P_{ex} = \frac{1}{2}P_{ex}$. In terms of our

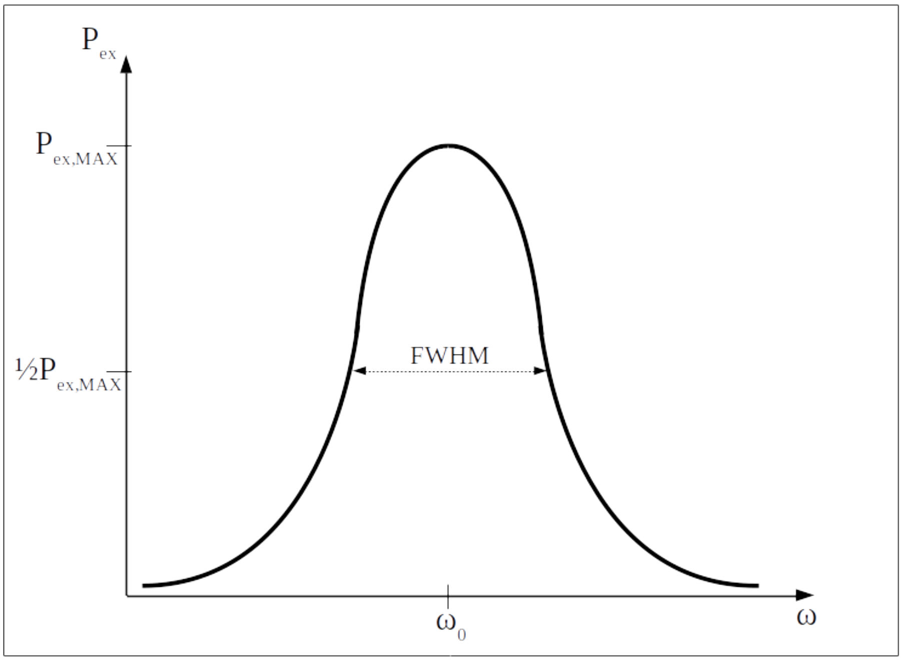

Figure 4.1: Schematic graph showing the probability of finding S in the excited state as a function of ω, adapted from [61]. The maximum probability corresponds to the resonance case where $\omega = \omega_0$. The FWHM, from which we get an expression for the uncertainty, is also shown.

investigation, the FWHM can be used to determine the uncertainty associated with the frequency, which we label δ_ω. The accuracy of the atomic clock relies on setting the frequency ω as precisely as possible and so δ_ω provides a measure of the effectiveness of the clock. The smallest possible uncertainty in the frequency would translate to an optimally set clock system.

If the oscillation of the wave is governed by the potential $V(n) = \lambda e^{i\omega n} + \lambda e^{-i\omega n}$, δ_ω is proportional to the amplitude λ. Our expectation for the calculation to follow is that a greater environmental effect on the clock will result in a greater uncertainty in the frequency. And so the effectiveness of the clock can be measured by analysing this uncertainty This effectiveness is further improved by the change to the atomic clock procedure introduced by Ramsey [64]. Here we mention only the salient features of Ramsey's alterations but a more detailed account of how this, and the Rabi clock system, keep time is laid out in Appendix C.1. Ramsey's method involves exposing the two-state system S to the electromagnetic wave for a short pulse time τ which is then followed by a non-interaction period for a time T, which is referred to as the Ramsey time and is typically much larger than τ. Finally, another pulse is then applied, also for the time τ, to complete the process. As τ and T each refer to a duration, it is tempting to think that the above description contradicts the CPI by including a standard quantum mechanic account of time evolution. However, as in the previous clock example, the 'time' is replaced with the abstract variable n and so here τ and T are taken to be components of this abstract variable. This can be represented as $n = 2\tau + T$ such that the interaction and non-interaction components remain separated.

In the adjusted procedure, the uncertainty is no longer associated with λ. Provided the frequency is close to resonance with $\omega \approx \omega_{12}$, the uncertainty now goes as $\delta_\omega \propto \frac{\pi}{T}$, implying that an increase in T leads to greater precision for the value of ω and ultimately a more accuracy of the clock. It must be noted that it is not possible to take this to the limit where $T \to \infty$. The second pulse must be applied to complete the procedure and this ensures a *finite* T. It is this caveat that is ultimately responsible for ensuring $n \to \infty$ is not a viable solution, an option which remained possible in principle in the case of the damped harmonic oscillator.

With this outline of the atomic clock procedure, we move on to the details of S's evolution through the three phases of Ramsey' method and the inclusion of decoherence interaction effects. The clock system is once again assumed to produce a negligible effect on its environment as compared to the environment's effect on it. Given S is initially prepared in a pure state, we expect the interaction with the environment to manifest as a decoherence effect causing a progressively more mixed state for S. A density matrix description of S can be used to account for said decoherence effects as an effect on the off-diagonal terms. We define this density matrix as ρ_S, which is constructed by considering the typical treatment of systems undergoing Rabi oscillations.

4.3.2 The first pulse: Rabi oscillation

In the wavefunction representation, the initial state of S is typically described by $|\phi_S\rangle = c_1|1\rangle + c_2|2\rangle$, where $|1\rangle$ and $|2\rangle$ represent the ground and excited state with coefficients c_1, c_2 respectively. The first pulse is applied for the duration $\tau \ll T$. This, along with the assumption that the electromagnetic pulse swamps any potential interaction effects induced by the environment, implies there are no decoherence effects to account for in this first stage. The wavefunction representation is then sufficient to a description of the effect of the first pulse on S. We begin with that description and then convert to the density matrix form.

The evolution of S as it is exposed to the potential $V(n)$ for a pulse 'time' τ is given as

$$|\phi(\tau)\rangle = \Big(\cos(\Omega\tau) - \frac{i\theta}{2\Omega}\sin(\Omega\tau)\Big)e^{i\theta\tau/2}|1\rangle + \frac{\lambda e^{-i\theta\tau/2}}{i\Omega}\sin(\Omega\tau)|2\rangle, \tag{4.23}$$

with the Rabi frequency defined as $\Omega = \sqrt{\lambda + \frac{\theta^2}{4}}$, representing the resultant frequency of the oscillator after exposure to the potential $V(n)$. Having determined a description for the effect of $V(n)$ on S, the density matrix is given as

$$\rho_S(\tau) = |\phi_S(\tau)\rangle \langle\phi_S(\tau)| = \begin{bmatrix} 1 - \frac{\lambda^2}{\Omega^2}a^2 & \frac{i\lambda e^{i\theta\tau}}{\Omega}a(b - \frac{i\theta}{2\Omega}a) \\ \frac{-i\lambda e^{-i\theta\tau}}{\Omega}a(b + \frac{i\theta}{2\Omega}a) & \frac{\lambda^2}{\Omega^2}a^2 \end{bmatrix}, \tag{4.24}$$

where $a = \sin(\Omega\tau)$ and $b = \cos(\Omega\tau)$.

The evolution of S, in the density matrix representation, is governed by $\rho(\tau) = U_S(\tau, 0)\rho(0)U_S^\dagger(\tau, 0)$, where $U_S(\tau, 0)$ is the evolution operator applied to S during the pulse time τ. Under the assumption that S is initially in the ground state, $\rho(0) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, a specific form of U_S can be determined from the evolution expression. [9] The explicit manner in which this is used to calculate U_S is presented in Appendix C.2. Here we simply quote the result as

$$U(t, 0) = \begin{bmatrix} b - \frac{i\theta}{2\Omega}a & -\frac{i\lambda e^{i\theta t}}{\Omega}a \\ -\frac{i\lambda e^{-i\theta t}}{\Omega}a & b + \frac{i\theta}{2\Omega}a \end{bmatrix}, \tag{4.25}$$

which is also used in the application of the second pulse to S in the third stage of the procedure. Next we apply the second non-interaction stage of the evolution.

4.3.3 Decoherence during Ramsey time

To evolve $\rho_S(\tau)$ through the Ramsey time T, where decoherence effects are experienced, we utilise a Lindblad equation [49]. [10] The crucial feature that this equation provides for our investigation is the ability to describe the evolution of an open system, such as a clock within an environment.

Given a general density matrix ρ, the term $\dot{\rho}$ represents the 'time' derivative where once again t is replaced with n. The Lindblad equation is given by

$$\dot{\rho} = -i[\hat{H}, \rho(n)] + \sum_{i}^{N^2-1} \left[\hat{L}_i\rho(n)\hat{L}_j^\dagger - \frac{1}{2}\hat{L}_i^\dagger\hat{L}_i\rho(n) - \frac{1}{2}\rho(n)\hat{L}_i^\dagger\hat{L}_i\right], \tag{4.26}$$

where $\hat{L}, \hat{L}^\dagger$ represent the Lindblad operators and N is the dimensionality of the system. [11]

[9] The form of U_S does not depend on the initial, we are only specifying the ground state as the S's starting state.

[10] For a derivation of the Lindblad equation see, for example, [65, 66].

[11] The term N^2 is set proportionally to the identity matrix. This term is then excluded from the summation, which runs up to $N^2 - 1$, as the N^2 term components will vanish. For a motivation of this claim, along with a detailed derivation of equation (4.26), see, for example, [67].

In order to apply this equation to our clock system, we utilise the application of the Lindblad equation as it is laid out in [68]. [12] Replacing the general description, we identify $\rho(n)$ as the density matrix representation of our two-state clock system prior to any application of equation (4.26). We also identify the ground and excited states as $|g\rangle$ and $|e\rangle$ respectively, representing the eigenvectors of a Hamiltonaian where the Linblad operators are included. The energy values associated with these two states, given by E_g and E_e respectively, represent the eigenvalues for $\hat{H}$. As equation (4.26) is the diagonalised Lindblad equation, the operators $\hat{L}_j$ and $\hat{L}_j^\dagger$ also have eigenvalues. These are identified here as $l_{j,g}, l_{j,g}^*$ for the ground state and $l_{j,e}, l_{j,e}^*$ for the excited state.

Equation (4.26) evolves the two-state system ρ_{eg} from an initial state, where $n = 0$, to a final state, corresponding to n, to give

$$\rho(n)_{eg} \propto e^{i(E_e - E_g)n - \gamma_{eg} n} = e^{-i(E_g - E_e)n - \gamma_{ge}^* n}. \tag{4.27}$$

where the decoherence component is re-expressed as

$$\gamma_{eg} = \alpha + i\beta = (\alpha - i\beta)^* = \gamma_{ge}^*, \tag{4.28}$$

with the substitution $\alpha = \sum_i \frac{1}{2}|l_{i,g} - l_{i,e}|^2$ and $\beta = \sum_i \mathcal{I}(l_{i,g} l_{i,e}^*)$.

The diagonal terms of $\rho(\tau)$ are not affected by the evolution described in equation (4.27) given that the exponential terms in equation (4.28) vanish for $e = g$. After the Ramsey time T, our clock system S is then

$$\rho(\tau + T) = \begin{bmatrix} b^2 + \frac{\theta}{4\Omega^2}a^2 & \frac{i\lambda e^{i\theta} e^{(i\omega_{21} - \gamma)T}}{\Omega} a(b - \frac{i\theta}{2\Omega}a) \\ \frac{-i\lambda e^{-i\theta} e^{(-i\omega_{21} - \gamma^*)T}}{\Omega} a(b - \frac{i\theta}{2\Omega}a) & b^2 + \frac{\theta}{4\Omega^2}a^2. \end{bmatrix} \tag{4.29}$$

This concludes the second stage of the atomic clock procedure and marks the end of any environmental effect on S. We move on the the final stage, where the second pulse is applied.

4.3.4 The second pulse time

Now S interacts with $V(n)$ a second time in order to complete the Ramsey process. Importantly, the electromagnetic wave still maintains in own oscillations during the non-interaction time T. A phase term is included to account for this effect which, based on the phase term already present in $V(n)$, is given as $e^{\pm i\omega T}$. This only affects the off-diagonal terms of the density matrix, as will be seen below. With this new phase term taken into account, the effect of the second pulse on S can be determined using the evolution operator from equation (4.25).

[12] The analysis in [68] also examines the decoherence of atomic clocks, although in a different context. As such, the method we employed here bears some similarity to this previous analysis.

The goal of this process is ultimately to determine the probability of finding S in the excited state and so quantify the effectiveness of the clock in terms of the associated uncertainty. If the system were at resonance, where $\omega = \omega_{12}$, the probability of a transition to the excited state would be exactly $P_{ex} = 1$. We need an expression for P_{ex} as a function of the decoherence terms in order to calculate which values (or limits) will maximise the probability of finding the excited state after the second pulse, providing the optimal scenario with the closest match between the frequency of the potential $V(n)$ and the transition frequency.

To simplify this calculation, we only concerned ourselves with the density matrix element corresponding to the probability P_{ex}, rather than the entire matrix. The state of S after the second interaction time τ is defined as

$$\begin{bmatrix} \rho(2\tau+T)_{11} & \rho(2\tau+T)_{12} \\ \rho(2\tau+T)_{21} & \rho(2\tau+T)_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ -B^* & A^* \end{bmatrix} \begin{bmatrix} \rho(\tau+T)_{11} & \rho(\tau+T)_{12} \\ \rho(\tau+T)_{21} & \rho(\tau+T)_{22} \end{bmatrix} \begin{bmatrix} A^* & -B \\ B^* & A \end{bmatrix}, \tag{4.31}$$

where $A = b - \frac{i\theta}{2\Omega}a$ and $B = -\frac{i\lambda e^{i\theta t}}{\Omega}a$ are defined, up to a phase, in the detailed derivation of equation (4.25) in Appendix C.2.

Identifying the matrix element $\rho(\tau + T)_{22}$ as representative of the probability of finding the density matrix in the excited state, we can calculate

$$\begin{aligned} P_{ex}(2\tau+T) = \rho(2\tau+T)_{22} = & \frac{4\lambda^2}{\Omega^2}\sin^2(\Omega\tau)\Big[2\cos(\Omega\tau) + \frac{\theta^2}{2\Omega^2}\sin^2(\Omega\tau) \\ & + e^{-\alpha T}\Big(2\cos^2(\Omega\tau)\cos((\theta-\beta)T) \\ & - \frac{\theta}{2\Omega^2}\sin^2(\Omega\tau)\cos((\theta-\beta)T) \\ & - \frac{2\theta}{4\Omega}\sin(2\Omega\tau)\sin((\theta-\beta)T)\Big)\Big]. \end{aligned} \tag{4.33}$$

This expression can be simplified by considering the substitution of a particular value for τ. Considering the ideal outcome, that of resonance, we can identify values for P_{ex}, θ, and T. Substituting these values into equation (4.33), we can solve for τ and, in so doing, identify the optimal value for this pulse time. [13] This optimal value for τ can then be substituted back into equation (4.33).

The resonance case corresponding to the probability $P_{ex} = 1$ gives the system in the excited state with certainty, matching the frequency of the wave with the transition frequency. Restricting to this case, $\theta = \omega - \omega_{12} = 0$ is also set. Now we turn to the Ramsey time

[13] Note that, although we are considering the final expression, the value of τ determined here applies to both the first and second pulse; the terms from each pulse time have been grouped in equation (4.33).

T which was introduced as a means of lowering the uncertainty. At resonance, however, the non-interaction stage cannot increase the accuracy which would already be maximised. Qualitatively, this justifies setting $T = 0$ for the resonance case. Substituting these values of T and Θ into equation (4.33) gives a resulting value of $\tau = \frac{\pi}{4\lambda}$. This represents the optimal pulse time which we substitute back into equation (4.33).

If we assume the system is operating close to resonance, we have $\omega \approx \omega_{12}$. [14] The implication of this assumption is that θ must be very small and must adhere to the relation $\theta \ll \lambda$. While we cannot guarantee the frequencies will match exactly, we can at least restrict to cases close enough to resonance to enforce the above relation and further simplify equation (4.33) to

$$P_{ex}(2\tau + T) = \frac{1}{2}\Big[1 - e^{-\alpha T}\cos((\theta - \beta)T)\Big], \tag{4.34}$$

where we have applied $\frac{\theta}{\lambda} \to 0$.

4.3.5 Analysing the uncertainty

The expression defining the probability of finding the system in the excited state is given by $P_{ex}(2\tau + T)$. We determine the associated uncertainty in order to analyse it in the same manner as the damped oscillator. From equation (4.34), the probability P_{ex} can be considered as a function of ω. This frequency's uncertainty expression is determined by the FWHM, as was done for the Rabi oscillation scenario, produces the uncertainty expression $\delta_\omega = \frac{\pi}{T}$. The precision of the clock is seen to rely on the Ramsey time, rather than the amplitude of the oscillator as in the previous case. But there appears to be no dependence on the decoherence effects. One might conclude that the decoherence does not affect the time measurement under the Ramsey method.

However, this is not the end of the analysis. We anticipate that there should be an influence of decoherence on the system and this is corroborated by an examination of the probability expression in equation (4.34) which shows that the maximum value is smaller in the case where decoherence is included. To illustrate this, we set $\alpha = \beta = 0$ to restrict to the case of no decoherence effects, and substitute these values into equation (4.34). Comparing the outcome to the original expression for equation (4.34), we have

$$P_{ex}(\alpha = \beta = 0) = \frac{1}{2}\Big[1 - \cos((\theta T)\Big] > P_{ex}(\alpha, \beta) = \frac{1}{2}\Big[1 - e^{-\alpha T}\cos((\theta - \beta)T)\Big], \tag{4.35}$$

[14]The resonance case was only used to set specific variable to their 'ideal' values and not applied for the continued analysis which may then use the near resonance limit.

where the left-hand side remains larger than the right-hand side since $\alpha \geq 1$, as per equation (4.28). [15]

It is clear that the decoherence has some effect on the clock. Certainly a decreased efficiency might be expected when the system is exposed to environmental effects. While this is not apparent above, we can investigate further by converting the uncertainty δ_ω to an uncertainty in P_{ex} using the expression

$$\delta_{P_{ex}} = \sqrt{\Big(\frac{\partial P_{ex}}{\partial \omega}\Big)^2 \delta\omega^2}. \tag{4.36}$$

From equation (4.36), the relative uncertainty $\delta_{P_{rel}}$ can then be calculated as

$$\delta_{P_{rel}} = \frac{\delta_{P_{ex}}(T)}{P_{ex}(T)} = \frac{\pi}{2} e^{-\alpha T} \sin(\Theta T)\Big(1 + e^{-\alpha T}\cos(\Theta T)\Big)^{-1}, \tag{4.37}$$

where we have re-expressed $\Theta = \theta - \beta$.

Setting $T \to \infty$ minimises the expression in equation (4.37) with respect to the Ramsey time. While at first this may seem to be applicable in principle, it would in fact be inconsistent with the clock procedure outlined above. An infinite Ramsey time implies the second pulse, a crucial feature, is never applied. In order to maintain the use of the system as a clock, the Ramsey time must have an upper limit to ensure the procedure is completed by the final pulse.

For the case without decoherence effects, the uncertainty expression becomes$\delta_\omega = \frac{\pi}{T}$, setting the relation

$$T \sim \frac{1}{\delta\omega}. \tag{4.38}$$

This similarly reinforces the restriction of a finite T.

To determine the expression which does minimise $\delta_{P_{rel}}$ under these conditions, and to allow us to determine the optimal amount of decoherence, we can introduce the Lagrange multiplier Λ to calculate

$$\frac{\partial}{\partial T}\left[\frac{\delta_{P_{rel}}}{P_{ex}} - \Lambda(\delta\omega - \frac{\pi}{T})\right] = 0. \tag{4.39}$$

Substituting the terms from equation (4.37) into equation (4.39), the resulting expression can be simplified to

$$e^{-2\alpha T}\Big(\Theta T^2 - 2\Lambda\cos^2(\Theta T)\Big) + e^{-\alpha T}\Big((\Theta T^2 - 4\Lambda)\cos(\Theta T) - \alpha T^2 \sin(\Theta T)\Big) - 2\Lambda = 0. \tag{4.40}$$

Next we enforce the near resonance condition on the clock as $\theta \approx 0$ when ω closely approaches ω_{12} in a similar manner as before. Following this, we then also have $\Theta \approx -\beta$. A

[15] Furthermore, the entanglement with the environment ensured the clock was no longer in a pure state and a second pulse would not undo this.

naturalness argument can be used to motivate that the relation $\beta \sim \pm\alpha$ should hold, which we can then enforce as $|\beta| \approx \alpha$. We acknowledge that this does not constitute a rigorous calculation but we argue that it is sufficient in determining the relationship between α, which represents the strength of the decoherence effect, and T. A more rigorous calculation, while useful, is not expected to contradict our conclusions. [16]

The minimisation of the relative uncertainty using the Lagrange multiplier method results in one of two cases, both of which lead to $\Lambda = 0$. The first is given by $\alpha T \gg 1$ and the second by $\alpha T \ll 1$. In either scenario there is a contradiction. If Λ vanishes, it would imply that the condition enforcing a finite Ramsey time is no longer valid and so contradict the requirements of the atomic clock procedure. This restriction must affect the term αT which can not be either infinitesimal nor infinite. Rather, αT must be some finite number, a condition captured schematically by [17]

$$\alpha T \sim \mathcal{O}(1). \tag{4.41}$$

This result can be considered from the perspective of the clock system. An increase in T implies an increase in precision. However, T must remain finite. By equation (4.41), the decoherence effect governed by α cannot vanish since allowing $\alpha \to 0$ would require setting $T \to \infty$. The two finite values for T and α must be continually balanced against one another in order for the system to function as intended. As the interaction of the clock with its environment is responsible for the decoherence effect, equation (4.41) can be interpreted as a weakly interacting condition represented by

$$\alpha T \lesssim \mathcal{O}(1). \tag{4.42}$$

Unlike the previous clock example in Appendix 4.2, the isolated clock limit can no longer apply, even as a matter of principle.

4.4 Implications for interactions

Removing the interaction effects while still achieving maximal certainty was stymied in the clock examples presented here. When the ideal clock limit with zero interaction terms is applied, it ultimately runs into the restrictions imposed by equation (4.17) and (4.42). The standard interpretation of the CPI is that while interactions may be necessary in practice, the ideal case should not include them as a matter of principle. But the above results do

[16] We briefly discuss other ways this calculation can be made more rigorous in Chapter 7.
[17] More precisley, the condition enforces $\alpha T \not\ll 1$, $\alpha T \not\gg 1$.

not agree with this interpretation and, once interactions are considered, there is no way to remove them by taking the ideal limit and still maintain maximum precision,

The previously raised concerns are reinforced by the results here: Interaction represent an unavoidable and necessary feature. Ignoring interactions from the outset allows certain issues to be overlooked. To further the new considerations raised here regarding the importance of interactions, we next consider a qualitative view of the limits of isolated clocks and the potential shortcomings they fall prey to.

CHAPTER 5

Isolation Versus Interaction

"The irreversibility of time is the mechanism that brings order out of chaos."

- Ilya Prigogine

No restriction against the use of interacting clocks was implied from the results in Chapter 4. On the contrary, a review of the importance of including interactions within the CPI appears to be necessary. We turn to more qualitative arguments and matters of principle, examining the implications of choosing either isolated or interacting clocks.

5.1 The limitations of isolated clocks

There are three key issues which arise when using strictly isolated clocks within the CPI framework. First, there are the interpretive implications which affect our view of time. Specifically, we argue that the view of time as an illusion is reinforced by the use of isolated clocks. The second issue we examine is the notion that a subsystem can be isolated at all. We argue that complete isolation is impossible as a matter of principle, rather than simply in practice, which raises concerns for isolated clocks. Lastly, the isolated clock appears to restrict the CPI to a two-system description. This apparently inevitable outcome of ignoring all interactions limits the potential of the CPI to be used as an adequate description of the Universe.

5.1.1 The 'timelessness' conclusion

In Chapter 3 the model of the Universe provided by the Wheeler-DeWitt equation was described as timeless. Restricting to this interpretation relies on the annihilation of the physical state $|\Psi\rangle$ by the Hamiltonian H, and so the removal of any functional dependence on the time parameter t as far as the Universal state is concerned. This should not be seen as a purely quantum issue since, in the classical limit, the Hamilton-Jacobi formulation of general relativity features the same timelessness. [1] The interpretation in the CPI is then consistent with the similarly timeless view found in general relativity.

An alternate, but related, perspective on the timeless nature of $|\Psi\rangle$ is also available. Given the conservation of energy implied by the Wheeler-DeWitt equation, the Universe in this model must be an isolated system. No exchange of energy is allowed across its boundary and this state of constant energy implies $|\Psi\rangle$ is isolated from any 'external' influences. Without any permitted change, the state must remain the same resulting in a scenario we might interpret as timelessness.

Yet another perspective is granted by considering other systems which are similarly time independent. In standard quantum mechanics, a closed system with a constant energy value is described by the time-independent Schrödinger equation. In more accurate terms, this scenario describes the 'evolution' of a stationary state solution of the time-dependent Scrödinger equation. While systems that are isolated in this manner bear a strong resemblance to $|\Psi\rangle$, we do not typically interpret such system as timeless in standard quantum mechanics. Instead, they maintain a sense of time which can be seen in the phase terms from the time-dependent perspective.

To understand the difference between the isolated Universe and a standard isolated quantum system, consider the surrounding environment. If a system is isolated, as per standard quantum mechanics, it can nonetheless be assumed to exist within a larger system which *does* change with time. Thus the sense of time can be 'inherited' by the isolated system. The Universe cannot rely on the same logic. Firstly, the timelessness is not simply a result of considering $|\Psi\rangle$ as an isolated quantum system. It is the inclusion of the classical Hamiltonian, and its related criteria, which enforce the energy constraints resulting in the negation of any time dependence. Secondly, as mentioned in Chapter 3, the boundary of the Universe includes all systems, ensuring there are no 'exterior' systems available which can influence $|\Psi\rangle$. A larger system which might provide a sense of time by 'containing' the

[1] This classical view of timelessness in relevant to the Block Universe interpretation which is discussed in more detail in Chapter 6.

Universe is then excluded. Ultimately, if a sense of time is present, it must emerge from within the Universal boundary.

In light of the above discussion regarding isolated system of constant energy, we argue that there is neither an inherent sense of time nor an inherited time available to the isolated clocks; With no change in energy permitted, they remain independent of any inherent time. As for an inherited time, the larger system in this case is $|\Psi\rangle$ which must remain timeless. There is then no sense of time is available for the clock to inherit.

The sense of time in the CPI is provided by the entanglement between C and R. Although the energy value remains constant, the clock system can still provide distinguishable states which correlate to states in R. An account of C and R's history is provided by these states as each pair of states essentially represents a 'moment' of time within the Universe. These consequences are summarised in, for example, [12]. While this view appears internally consistent, adopting the interpretation does imply that particular features which form part of our experience of time must be excluded.

Firstly, there is no sense of a 'now' moment; A privileged instance of time which constitutes the present. [2] Rather, each and every pair of entangled states of C and R are on equal footing and considered to exist 'simultaneously'. A consequence of this perspective is the removal of the experience of the 'flow of time' as a system's change from one state to another, recorded as a continual history. To claim that the states of a system must all exist as equally real can be argued to be synonymous with claiming that a system exists in all states. While this is an acceptable position for a quantum system to adopt, we do not expect classical systems, or semi-classical in the case of the CPI, to behave this way. Thus to maintain the existence of every state, we must postulate a distinct system 'at each instant'. The collection of these systems, each in a distinct configuration, is what we recognise as an object undergoing change.

A useful analogy to explain this perspective is given by equating each instant of time to a frame from a film. The system in a particular state can be represented within a frame as an image. A succession of images then gives the illusion of change but each frame, and the system within it, is an inherently static object which does not move from one frame to another frame. Proponents of the timeless interpretation can then argue that a description of our experience of time is still recoverable under this view. This brings us to the last issue under discussion in Section 5.1.1.

[2] This is discussed in more detail in Chapter 6.

In order for the timeless account of Nature to match up with the physical measurements in experiment, the frames must line up in a very particular order. In other words, the laws of physics as experienced require a consistent history of states to be maintained. However, there is as yet no inherent reason to order the states of an isolated clock in a manner consistent with our expectations. Rather than a film where the frames are all stuck one after the other, we can think of the clock states as forming a collection of frames which cover every possible eventuality and are in no particular order. The very specific order necessary to recover a description of an 'arrow of time' is still missing.

One approach to resolve the ordering issue is to add an auxiliary system which may serve as a 'memory'. A history of previous states can be recorded by this memory system such that any given state is assigned a position in the 'timeline'. Regardless of the system in any particular frame, the record would only hold information regarding 'past' states. This would maintain the illusion that a system evolves in a privileged direction; there would be a perceived difference between past and future enforced by the information in the memory system. Such a system is used, for example, in [12]. We discuss this approach, and the related interpretation, from a more philosophical standpoint in Chapter 6 while here we argue that the construction of a history in this manner is largely meaningless under the following interpretation on the recovery of time.

We take the entangled state of the Universe, the Hamiltonians , and the unitary operators responsible for the evolution of each system, $\hat{U}_C$ and $\hat{U}_R$, as defined in Chapter 3. However, rather than viewing illusion of change as the generation of successive pairs of C and R in correlated states, another interpretation is available. The application of the unitary operator $\hat{U}_C$ can be seen as the rotation of R through the superposition of entangled states. This amounts to a change of basis for R with no physically realisable change occurring. Thus the construction of a 'history' of states by the addition of an auxiliary memory system is a somewhat pointless exercise as it does not represent any physically real phenomenon, leaving the order as an ad hoc addition imposed on the timeless states. [3]

Ultimately, restricting to isolated clocks removes the sense of directed evolution in time. A description of change as a real process cannot be recovered if we start from the isolated clock perspective. We are left with the interpretation that time, as it emerges within the CPI, must be an illusion and this is carried through to all systems. [4]

[3] Without changing the outcomes of the framework, this interpretation highlights how the flow of time is a physical reality under the CPI.

[4] The view of time described here can be interpreted as a quantum version of the Block Universe, as defined in Chapter 6.

To assess whether isolated clocks, which lead to treating time as illusion, is a viable approach, we consider two potential shortcomings: the limit to a two-state Universe and the ability to restrict to isolated systems. Following this, we present the alternative perspective of time afforded by using interacting clocks.

5.1.2 The two-system Universe

Let us consider a clock C and remainder R which are perfectly isolated from one another. As pointed out above, the sense of time recovered in the CPI is provided by the maximal entanglement between C and R. In the absence of any interaction effects, the entanglement correlations are the only available avenue for information about each system to be shared with the other. Essentially, the systems in question can only 'know' about each other through their shared entangled state. The implication is that R can only access the clock parameter when the description of C and R is viewed as a whole state, rather than any one part.

There is another consideration to take into account: The respective sizes of C and R. Instead of discussing the systems' physical sizes, we are interested here in the relative sizes of the Hilbert spaces associate with each subsystem. If R's configuration space was larger than C's, there would be a degeneracy in the entangled state $|\Psi\rangle$. This leads to an ineffective clock since more than one state of R would be associated with a single state of C and the ordering of correlated states would become ambiguous. Similarly, if C is much bigger than R, the problem arises that multiple clock states could be associated with a single state of R. The subsystems C and R must then be of comparable dimensions if the time parameter provided by the clock is to be useful in the description of R's evolution. [5]

Given the above account, how do subsystems within R (or C) account for a sense of time? Consider a subsystem of R, which we label R', which is significantly smaller than the whole system: $R' \ll R$. This is illustrated in Figure 5.1. Since R' is not correlated to the entire state of C, only a portion of the clock state, and so only part of the entanglement information, is available to the subsystem R'.

Since the entanglement is the only means of sharing information between C and R in the isolated scenario, there is no other recourse for R' to learn the clock state. The same is true from the perspective of the clock. It is only aware of R as a whole state, rather than as a collection of constituent parts. From the limited access, R' only has part of the picture and

[5] The requirement that every state of R be entangled with a state of C results from the CPI's requirement for a maximally entangled state.

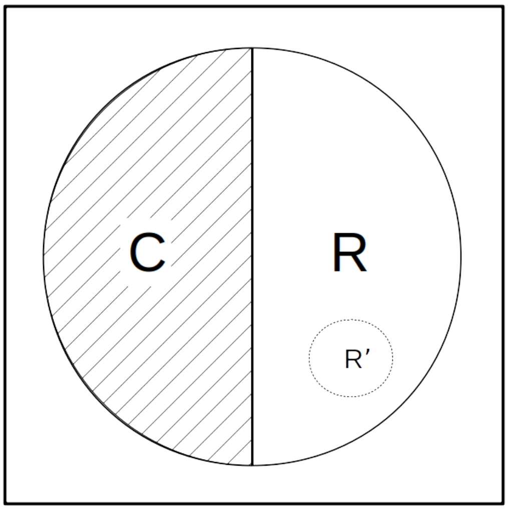

Figure 5.1: A schematic representation of the Hilbert space of the maximally entangled systems C and R which are perfectly isolated.

the result is an ineffective time parameter, from R''s perspective, which cannot successfully convey a sense of time in the manner accomplished for the total R state. Even if we were to consider R' to be entangled with C', a subsystem of C, we could not guarantee that that R' and C' would maintain a maximally entangled state and the necessary correlations. The time description between such a pair could differ sharply from that provided by C and R. The conclusion is that C and R provide a sense of time, and so a description of evolution, only when taken as a whole. The subsystems C and R contain are not privy to the information regarding the overall entangled state nor time parameter measured by C.

The cost of recovering time using isolated clocks seems to be a restriction to a two-system Universe. This is not the only disadvantage of the isolation constraint as can be seen when we consider whether interactions can be entirely excluded.

5.1.3 Interactions as a matter of principle

The introduction of the weak interaction limit in the CPI formalism is commonly used as a motivation to discount *all* interaction effects. Specifically, this argument is used in [12] to limit the discussion to these features of the CPI which are necessary as a matter of principle. The assumption is that perfectly isolated systems are required by the framework of the CPI and that interacting systems, while important in practice, are not considered relevant in principle.

If we examine the manner in which interactions occur, however, an argument arises which is in favour of their inclusion as a matter of principle rather than in practice. Consider two subsystems of the Universe, similar to (but not necessarily) C and R. We anticipate that, even if they are isolated from all other interaction effects, there will still be a gravitational force between them. This effect, however negligible, cannot be shielded and, while it might remain a very small, insignificant effect, it would nonetheless be present as a matter of principle.

If interaction effects must be included in the manner described above, an alternative interpretation of the weak interaction limit presents itself. Following a much more literal definition of 'weak', we suggest that the interactions effects should be included as negligible but *not zero*. Small gravitational effects are ideally suited to this perspective. They become negligible over large distances, leaving the framework of the CPI largely unchanged but they cannot be zero, even in principle, and so must be taken into account in order to provide a more complete picture than the current view.

The notion that interactions are involved as a matter of principle supports the analysis presented in Chapter 4. The influence of interactions on the accuracy of the clock would be missed entirely in the isolated clock approach since interactions are excluded from the start. The analytic results confirm that the interactions should be kept close to zero which reinforces the use of the gravitational force as a useful method of facilitating interaction effects between C and R. In the discussion to follow, we will build on this in order to argue that the subsystems within R rely on interactions for their sense of time.

5.2 The consequences of including interactions

The inclusion of interactions in principle ultimately lead us to question whether time and change, as emergent phenomena of an isolated Universe, *must* be interpreted as an illusion. A discussion of this nature naturally leads to philosophical issues which we will discuss in Chapter 6. [6] Here we present an argument to show that the inclusion of weak interactions allows the subsystems of R to 'retrieve' a sense of time without contradicting the CPI framework. We will also argue that restricting to weakly interacting clocks does mean renewed trouble with the clock ambiguity problem discussed in Chapter 3.

[6] Briefly, interactions constitute a process and so are incompatible with a timeless Universe in which such change is not 'real'.

5.2.1 Multi-system description

From a conceptual viewpoint, the inclusion of interactions provides a way around the two-system Universe restriction. If the subsystems of R are permitted to interact directly with C, as illustrated in Figure 5.2, the result is a measurement of the clock which grants direct access to the time parameter. The time parameter, and an accompanying sense of time, can be distributed throughout any number of subsystems of R in this manner. [7] We will illustrate

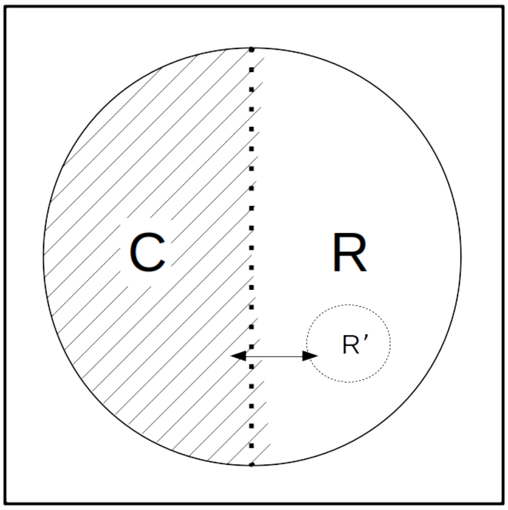

Figure 5.2: A schematic representation of the Hilbert space of the maximally entangled systems C and R when interaction effects are permitted.

this with an example shortly, but first two important points must be noted regarding the inclusion of interactions.

First, the interactions can provide a direct measurement of the clock, while still adhering to the restriction that the effect between systems is very small. Provided the systems remain weakly interacting in this manner, the CPI framework will continue to function. This is not to say that interactions can occur indefinitely without affecting the formalism and we will discuss the outcome of allowing the interaction effects to build up after the presentation of an example of an interacting clock.

The second point is the continued importance of the entangled state. Even though R', and other subsystems of R, rely on interactions with the clock to provide a time parameter, the entanglement between C and R is still necessary in order to recover a sense of time. If we were to describe C and R as separable, we would lose the correlation between specific states of C and R as provided by the entanglement. Any state of R could then be associated with any state of C, nullifying the use of C as a clock. Maintaining the CPI thus requires the continued use of the entangled state. As alluded to above, the build up of interactions does

[7] As with the isolated case, the systems C and R are taken to have similar dimensions.

present a threat to this entanglement but, as we will explain following the example, there is a conceptually consistent interpretation available which resolves the concern.

We now turn to our example and select a suitable clock. The expansion of the Universe serves as a useful system in this regard and has in fact been used previously in a time-keeping capacity as can be seen, for example, in [39]. Although the expansion does not resemble a familiar clock system, we nonetheless assume that the expansion is governed by some mechanism which we can identify as C, a subsystem of the Universal Hilbert space. This type of clock was also used in [69] where interaction effects were once again assumed to be zero.

We are interested in how time emerges for subsystems of R and so we identify two galaxies, labeled R' and R'', to serve as (comparatively) small subsystems of R. The gravitational force between R' and R'' provides a means by which one galaxy may influence another. The

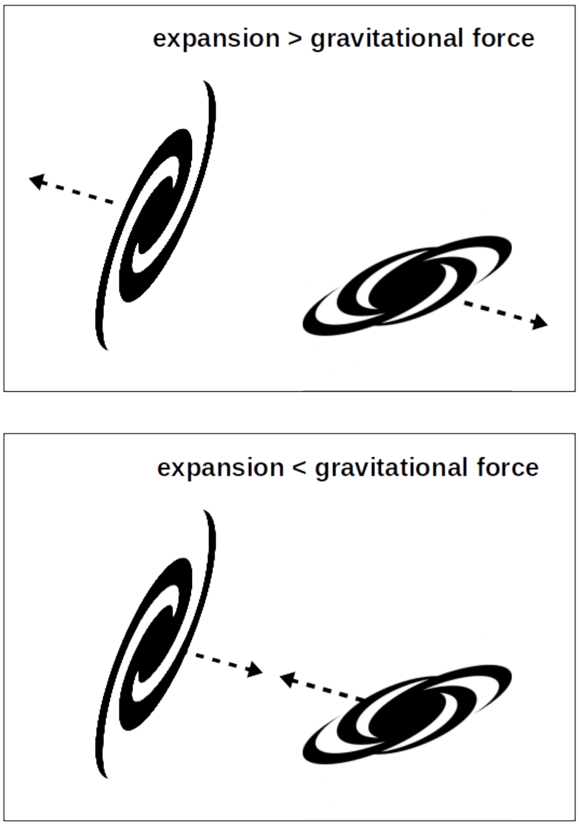

Figure 5.3: A schematic representation of two interacting galaxies. *In the first scenario, the expansion of the Universe overcome the gravitational force while in the second scenario the gravitational force overcomes the expansion.*

resulting 'measurement' of the expansion of the Universe can be separated into two cases which are illustrated in Figure 5.3. In one scenario, the expansion of the Universe overcomes the gravitational force between R' and R''. In this case the two subsystems continue to separate further from one another in space. On the other hand, R' and R'' might have a large enough gravitational force between them to overcome the effect of the expansion meaning they would eventually collide. [8]

In either case, the gravitational forces allow an 'interaction' with the clock and, however weak the effect of the expansion and gravitational forces might be, the effect cannot be shielded. [9] In examining galaxies today, we collect the photons emitted from distant systems in the Universe in order to determine properties such as their mass and their velocity relative to the Earth [70, 71]. To measure the expansion of the Universe as an influence on the strength of the gravitational attraction between them, we may similarly rely on light emitted by R' and subsequently collected by R'' (and *vice versa*). The respective masses of two galaxies determine the gravitational force which, in the absence of any other influence, would cause the two systems to move towards each other at some calculable velocity. The expansion of space would produce a discrepancy between this calculated velocity and the velocity determined by measuring the emitted light.

Overall, we argue that this scenario serves well as a description of time when investigating interacting clocks within the CPI. Indeed the expansion factor can easily be interpreted as a natural choice for a 'cosmic' time parameter since it influences the entire measurable Universe. Utilising weak gravitational effects as the interactions is similarly useful since these are also experienced by all systems across the Universe. Any subsystem of R can then access the clock along with its associated time parameter.

Another benefit is that, as with the examples analysed in Chapter 4, an arrow of time results from the inclusion of interactions. To see how this arrow both manifests and aligns with others, we first consider the psychological arrow of time which, under a certain description, can be closely related to the thermodynamic arrow of time [72, 73]. Specifically, we can discuss the psychological arrow in terms of the continual process of recording new memories. This is essentially the accumulation of information and, as such, represents the extension of the psychological arrow beyond conscious systems such as humans or even the closely related computer systems. If we consider the configuration of a system as its 'memory', then any change to this configuration that results from an external influence can be interpreted

[8] One might also argue for a third option in which the gravitational attraction is exactly balanced by the push of expansion. We do not discuss this in any detail as this unlikely scenario is not expected to affect the outcome of our argument.

[9] Here, we are attributing the expansion of the Universe solely to gravitational forces and not to some form of exotic energy or matter source associated with so-called dark energy.

as the recording of information. [10] Continually recording new information in this manner can then be tied to the thermodynamic arrow of time. In order to 'write' a newly gathered piece of information, the previous configuration of the 'memory' system must be erased. In computer jargon, it must be overwritten. The process of erasing information produces heat [74] and must then be accompanied by an increase in entropy, as per the description given by the Clausius inequality. Thus the psychological arrow can be argued to point in direction of an increase in entropy and in doing so align with the thermodynamic arrow of time.

What about our 'cosmic' time? Certainly the interactions between the subsystems, which are no longer closed, are expected to result in an increase in entropy. For the arrow of time between C and R, an interpretation similar to that given above is available. We can consider the measurement of one system by another as a change to the system's configuration. A direct result of the external influence, this represents a record of new information and the erasure of the previous configuration. Heat must also be produced in this process and the result is an increase in entropy. We can reasonably anticipate that the cosmic arrow of time, as directed by this entropy increase, should also align with the thermodynamic and psychological arrows as described here. While this recovery of an arrow of time aids our description of change in time, weak interactions over a long enough time period raise concerns which we now address. [11]

The cumulative effect of the interactions becomes significant as the evolution of C and R continues. [12] Eventually we would reach a stage where we can no longer approximate to the weak interaction limit, prohibiting any further use of the CPI. In terms of our example, the expansion continues and R' and R'' will either form one system, if gravity overcomes expansion, or be separated by the expansion to such a distance that any interaction effects vanish and no further communication is possible. In either case, neither system can access the clock state via the gravitational effects.

This certainly seems problematic for the CPI but it need not be. Extending this scenario to its conclusion, we would reach a maximum entropy state for the Universe. In this case, the subsystems of R could no longer record the state of C, even if the clock was still accessible. We might then argue that the inability to measure C coincides with the "heat death" of the Universe. This would leave the CPI perfectly functional up to this expected point of

[10] We are referring specifically to *irreversible* change.

[11] For a contrary but related perspective on how the entropy in the above scenario leads to an arrow of time, see [75].

[12] Although the interactions may remain weak indefinitely, the effect will become significant given a long enough time.

no return, a state of maximally coarse-grained entropy after which no evolution would be possible.

There is one more issue to consider before concluding the discussion on interacting clocks. As suggested by the above discussion, isolated clocks may have certain shortcomings but they were successfully used to resolve the clock ambiguity. It might appear that invoking interacting clocks as a matter of principle reopens the problem, an issue we discuss below.

5.2.2 Ambiguity as superficial issue

The ambiguity problem, as explained in Chapter 3, arises as a result of our ability to partition the Universe in any number of ways. Every different partition corresponds to a different clock, a different Hamiltonian, and different clock dynamics. Under this view, the behaviour of C and R depends on an arbitrary partition choice and there is no inherent reason to expect consistency between different choices. By restricting to isolated clocks (or equivalent systems) only, the choice over how to partition the Universe became superficial and the ambiguity was removed. While it may appear that allowing for interacting clocks leaves us back with the ambiguity problem, we argue this is not necessarily the case.

Let us first consider the subsystems within R which each access C directly in order to gain a sense of time. Each subsystems only measures part of the clock state, as per the description above. The perspective a subsystem of R has of the time parameter might then differ significantly when compared to what is measured by another subsystem of R. There is then an ambiguity over dynamics as the subsystems C and R depend on what part of the clock they arbitrarily happen to access. This ambiguity can be resolved if we appeal to the feature ultimately responsible for a sense of time in R: the entangled state. It is only through the entanglement of C and R that a sense of time is recovered in the CPI at all. As such, any measurement of the clock performed by a subsystem of R must ultimately relate back to, and so align with, the 'entanglement time'. [13] All the subsystems of R must then ultimately be using the same time, regardless of which part of C they access directly.

What about the entanglement time? While we can argue that the subsystems of R (and similarly C) are all accessing the same time, the ambiguity remains over the partition choice which is initially used to select C and R. The entanglement time, and related dynamics, certainly depend on a choice but this need not mean the descriptions are entirely arbitrary.

[13] This refers to the time associated with the entanglement between C and R, as per the CPI framework discussed in Chapter 3.

As C and R are interacting with one another, they share more than just their entangled state. The continued interactions ensure that the entropy of the pair increases. If the second law of thermodynamics is adhered to, we can expect *every* partition involving an interacting clock to describe an entanglement time which also aligns to the thermodynamic arrow of time. [14] In effect, all the clocks 'point' the same way. [15] It must be stressed that this relies strongly of the implementation of *weak* interactions to ensure that the increase in entropy, associated with a decrease of entanglement, is gradual enough to provide a sufficient number of correlated states of C and R.

The clock ambiguity need not be seen as means of prohibiting against the use of interacting clocks. While a choice over how to partition $|\Psi\rangle$ does remain, the ambiguity over which direction in time the systems evolve is removed. Realistic clocks may still be governed by different dynamical laws, for example a relativistic versus a Newtonian clock as used in the application of the CPI as per [51], but these clocks would nonetheless agree on which direction in time is toward the past and which is toward the future. We conclude that the inclusion of interactions not only avoids breaking the CPI framework, but also continues to reinforce the phenomenon of an arrow of time as an in-built feature of the isolated Universe. The implications of this outcome can now be assessed with reference to the interpretive and, indeed, philosophical considerations of time in physics.

[14] In discussing interacting clocks, we are discussing open systems undergoing irreversible processes.
[15] Specifically, clocks share the same arrow even while maintaining a partition-dependent entanglement.

CHAPTER 6

Philosophical Implications

"Time is an illusion, lunchtime doubly so."

- Douglas Adams

Beset by differences in opinion, the debate over the nature of time extends at least as far back as the early Greek philosophers and has continued over the centuries. Modern considerations of time are prolific and we while we can point to the examples in [3, 8, 9, 76, 77], this is by no means a complete list. Irrespective of the extent of this debate, two opposing camps which were laid out in the philosophical discussion of the ancient Greeks can still be identified today.

On the one hand, there is the 'eternal' Universe described by Parmenides [10]. Under this interpretation, any perceived change is taken to be an illusion while the Universe remains an unchanging and, indeed, static place. The opposing view offered by Heraclitus argues that change must be a real process and not an illusion. It is worth pointing out that these two views typically use the shorthand term 'time' in a sense which encompasses our perception of change in time. Here, following the definitions laid out in Chapter 1, we maintain the distinction between the two concepts.

The focus of this discussion is to see whether including interactions as a matter of principle offers any further insight into the debate on the opposing interpretations of time mentioned above. This centuries old debate, however, contains many nuanced definitions and arguments which involve notions that are much more philosophical in nature than those typically addressed in physics. We will therefore try to distill the most relevant elements of the interpretations, specifically in terms of how they pertain to the theories of physics discussed in the preceding chapters as the aim is simply to place our results into the philosophical context. [1]

6.1 The experience of time as an illusion

We first consider the view that change, as our perception of time, must be an illusion. While it may have its roots in ancient Greece, this notion of 'eternal existence' without change can often be found in modern physics as well; It is the view we encountered in the timeless interpretation of the CPI discussed in Chapter 5.

6.1.1 The A-series and B-series of time

In examining this view on change, or lack thereof, we turn to an argument presented in [78], known commonly as McTaggart's Paradox. By considering two ways of describing events in time, the resulting paradox implies that interpreting change as real is an invalid approach. McTaggart's description allows us to neatly differentiate between two interpretations of time and so we sketch the argument below in order to the relate its conceptual pictures of time to the interpretations found in physics. However, the validity of the argument, as well as the conclusions drawn from it, remain under debate. [2]

The two descriptions of time are as follows. We have the A series of time that describes the transition of an event in time and requires a privileged 'now' moment to represent the present. An event then undergoes a transition from future, through to 'now', and finally to past as the present moment progresses along in time. This describes our perception of time; essentially the continual change of the configuration labeled 'present'. The B series, on the other hand, assigns only one time 'label' to each event and they remain in that position. Events are then related to one another in terms of where they fall along the dimension of time, rather than by their relation to a moving present moment. These two views are illustrated for comparison in Figure 6.1.

McTaggart's argument is that the inclusion of the A-series presents a contradiction, although there is some disagreement on exactly how the problem manifests. Here we present one view of the paradox in which the labels of 'future', 'present', and 'past' cannot apply

[1] Note that this context involves arguments previously established in the literature as opposed to the more speculative discussion which is reserved for Chapter 7.

[2] A summary of the treatment of time in physics that touches on these issues can be found in [4].

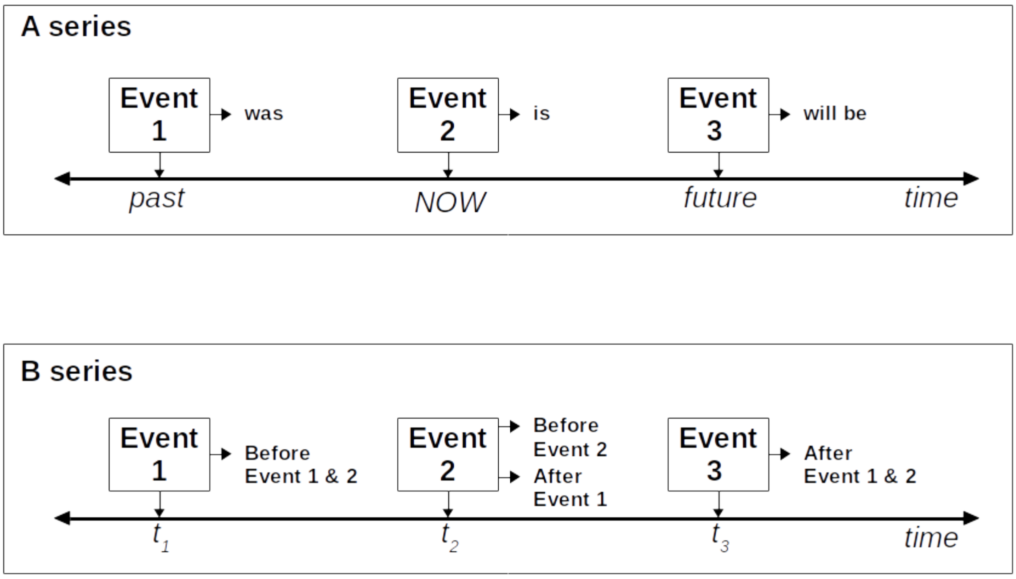

Figure 6.1: A representation of the two ways events may be ordered in time. The A series describes how events change from future to past while the B-series prescribes each event a fixed placement in time.

'simultaneously' to a single event without causing a paradox. It must be explained how an event is *successively* labeled future, then present, and finally past. This requires explaining how the labels are assigned at different times, as dictated by a present moment which is held responsible for the transition of events from 'future' to 'past'. But the specification of 'where' the present moment is, and so what is considered future or past, must also be defined. As argued by McTaggart, this requires a second A series which defines the 'future', 'present', and 'past' status of the present moment itself in order to locate it along the B series. This essentially involves an appeal to a higher-order temporal series, which we have schematically represented in Figure 6.2, but, if the argument holds, we would be stuck in an infinite regress. Each attempt to explain the application of 'past', 'present', and 'future' labels relative to a present moment results in the use of yet another A series.

There have been many concerns raised over the definition and use of the A and B series of time and the arguments which utilise them are by no means conclusive. However, they do provide two useful representations of time which aid our discussion. If a consideration of the A series, as a representation of the experience of time, concludes that the very notion of time is an inconsistent one then change must be an illusion. The Universe in such an interpretation would exist is a stationary state; a permanent 'being' in the sense defined earlier following [10]. This view arises in general relativity, where the Block Universe can

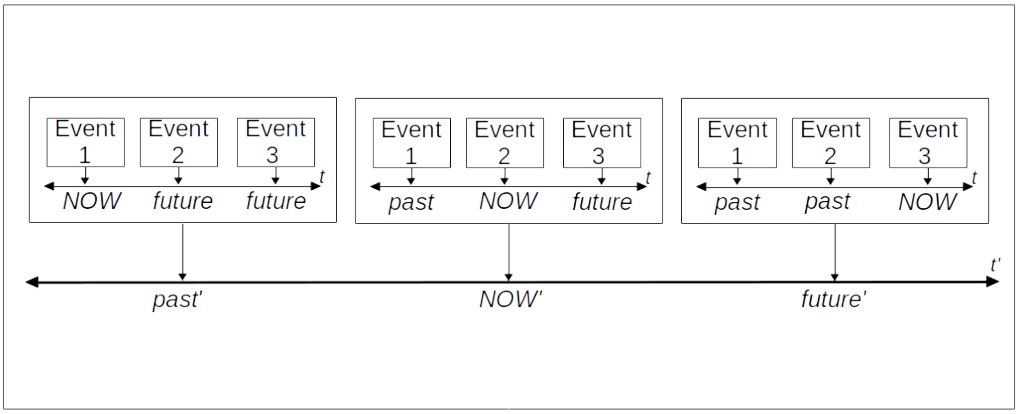

Figure 6.2: A representation of a secondary A series ordering. This would in turn present the same problem as the initial A series and so itself require invoking yet another A series.

be used to describe just such a scenario of permanent existence.

6.1.2 The Block Universe in physics

The Block Universe is an interpretation of time which can be applied to general relativity. [3] The conceptual picture resembles the B series in that it fixes all events in relation to one another in a permanent location in space and time.

There are two key features of general relativity which make it suited to such a view of time. First is the elimination of simultaneity and, with it, the possibility of a privileged present moment. Without the ability to pinpoint 'now', the notions of 'past' and 'present' become meaningless, except as relations between events in spacetime. The second feature is the deterministic nature of general relativity. If one had enough information about the configuration of the Universe for a single moment, all moments before and after could be essentially set in stone.

This leads us to the conceptual picture of the block Universe which is illustrated in Figure 6.1.2. There is no privileged present which can be used to pick out a special single slice of time as 'now'. Furthermore, all events can be described in a deterministic fashion following the classical equations of motion. The entire history of the Universe can then be laid out in a four dimensional block of spacetime. Each event is then assigned a location within that block. [4]

Within the Block Universe, there can be no real sense of change. A system in a particular

[3]The idea is not limited to general relativity and is more broadly considered in philosophy as 'eternalism'. For a summary see, for example, [79].

[4]There are different variations on this idea one of which is the 'evolving Block Universe' in which the block continually grows as successive moments of time are added. This is described in, for example, [5].

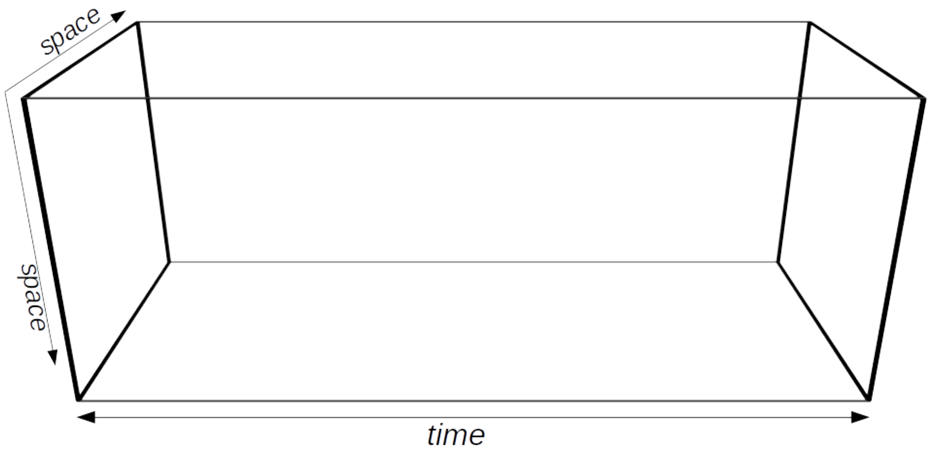

Figure 6.3: A schematic representation of the Universe as a block, with two dimensions of space and one of time.

configuration is permanently set at its assigned spacetime coordinates and what we might perceive as change to a system should rather be interpreted as the sum of several parts. These

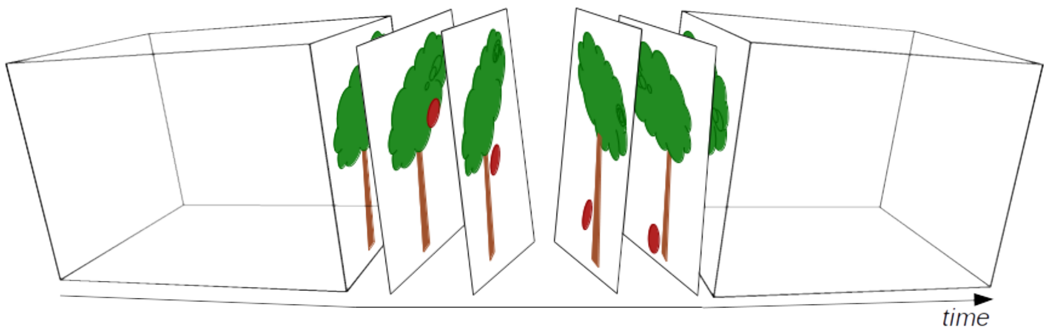

Figure 6.4: A representation of the Block Universe's perspective of a falling apple within the spacetime block which has been expanded to provide a view of successive slices.

parts each represent a particular configuration of the system within a 'slice' of time and it is the presentation of these slices as a progression which produces an illusion of change. This is conceptually illustrated in Figure 6.4 where an apple falling from a tree is viewed from the Block Universe perspective.

We cannot identify any real physical difference between the future and the past under this approach. *All* the configurations of the falling apple exist equally and together; none can claim a privileged status of existence. The description of the series of events can be considered entirely reversible in a manner consistent with the classical equations of general relativity. It would appear that it is a quirk of Nature, rather than a requirement, that we remember the past and not the future since, under current Block Universe descriptions, we could construct alternative representations where the illusion of causal order need not hold.

What is lacking from this interpretation then is an explanation for why we should perceive the illusion of change at all; why should measurement differentiate between the past and future? We certainly see a very specific and consistent order that appears to correspond to a cause and effect chain which leads from past to future. To ensure that the Block Universe picture is consistent with our experience, there must be some additional feature or postulate which explains how and why the Universe goes through the trouble of constructing the illusion of an irreversible causal ordering rather than any other number of potential constructions. [5] [6]

Another potential problem for the Block Universe is the introduction of quantum mechanics. The use of probability and uncertainty present a possible contradiction to the deterministic nature of the Block Universe. [7] However, while the original form of the block Universe might not survive standard quantum mechanics, the interpretation of change as an illusion certainly does.

6.1.3 Quantum timelessness

In Chapter 5 the notion that there is no physically real sense of change within the CPI was introduced. The use of isolated clocks led to an interpretation with neither a privileged present nor an inbuilt arrow of time. Rather, an ad hoc memory system was introduced in order to provide a history of states and, in so doing, an arrow of time.

The requirement to put in an arrow of time by hand and the interpretation that time must be an illusion aligns with the Block Universe approach. However, there is an important difference. Rather than a single, privileged order which aligns with a time dimension, the

[5] If microdynamics are considered, the second law can be invoked to retain an order of events but this does not explain the irreversibility we experience.

[6] This line of thinking brushes against the vast collection of philosophical thought on reality and our perception of it. While there are many varied and nuanced positions to be found, we do not discuss them in detail as the primary focus remains on interpretations applicable to physics.

[7] It should be pointed out that this issue is avoided under deterministic descriptions of quantum mechanics. For example, see [80, 81].

systems in different configurations are 'scattered', in an abstract sense. This is conceptually illustrated in Figure 6.5, where we have again used the example of an apple falling from a tree.

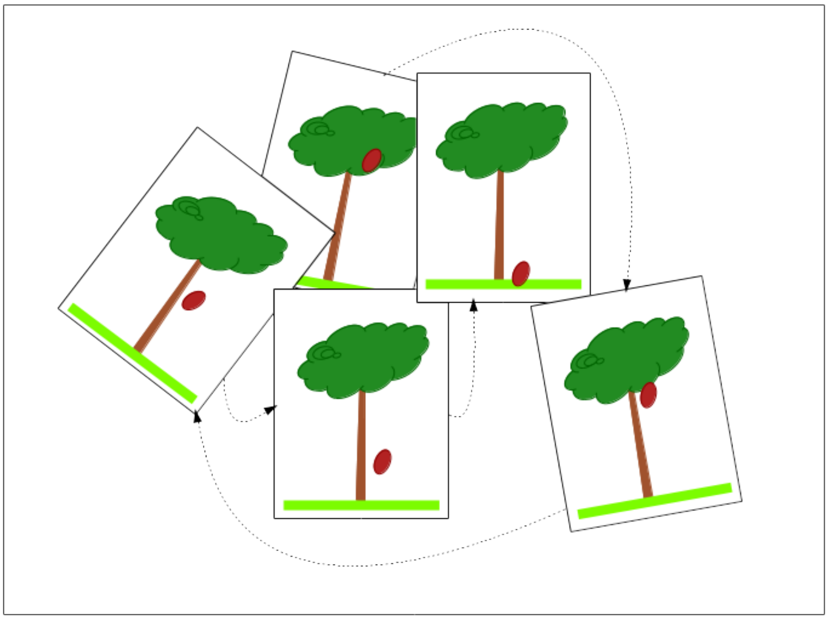

Figure 6.5: Conceptual representation of 'moments of time' which collectively create the illusion of change and where the arrows illustrate only one potential ordering of events.

This 'quantum Block Universe' also appears in other interpretation where it is argued to be a truly time*less* interpretation that effectively removes the time dimension altogether. Championed by Barbour [82], this interpretation claims to remove time ordering at the most fundamental level of reality. Rather, the laws of physics are responsible for the illusion of ordered change as they group systems, such as those represented in Figure 6.5, in terms of the similarity of the configurations. Specifically, the principle of least action is called upon to provide a consistent ordering. For a more nuanced summary of this approach see, for example, [83]. This sentiment which regards time as inessential to the foundational description of reality are echoed in other research as can be seen, for example, in [84] and also in [85] where an alternative approach is taken.

Whether in general relativity or quantum mechanics, or indeed a semi-classical theory such that the CPI provides, it appears that change may be treated as an illusion. However, in either case, there must be some additional feature, or in some cases a postulate, in order to recover the one-way experience of time as a process which appears to be ubiquitous throughout all physical systems. We now contrast this with the alternative approach to argue that, if change is taken to be a real process, there is a consistent interpretation of time with no more cost of additional postulates than what we have seen in these timeless interpretations.

6.2 The experience of time as a physical reality

There are arguments in favour of the notion that time, and our experience of it, should be interpreted as a real phenomenon. Examples of these types of arguments can be found in [86, 87] where the necessity of interpreting general relativity to have a Block Universe-type structure is questioned.

Following along a similar line, recent arguments have also been presented against the inclusion of timeless interpretations in quantum theories. One such example, given in [88], calls to question the assumption that an abstract mathematical description must in all cases stand for physical reality; while the theory's mathematical framework may describe systems as existing perpetually, this need not necessarily represent a view of fundamental reality. This echoes the arguments laid out in [89] which similarly caution against assuming every element of the abstract mathematics of physical theories must correspond to physical reality; a point that has also been argued from the other perspective in [90], who states that current physics' lack of an account of the experience of time should not mean we discard the phenomenon. Others have raised the question of irreversibility, arguing that there are features of quantum theory which align more with an interpretation of time where the future is distinguishable from the past. See, for example, [91, 92, 93]. Still others question the value of the timelessness interpretations, suggesting that, even if they can be shown to be logically self-consistent, they do not allow for any further understanding [94].

What of the alternative view of change as a real phenomenon? Considering change as the result of one system's interaction effect on another, is the result really more costly in terms of assumptions and unresolved issues? As per the definitions laid out previously, such interactions can be considered to be a process by which two (or more) system exchange information regarding their configurations with one another. Under such a view, each system exists in a series of successive configurations but is only ever in one configuration at a time, rather than existing equally in all configurations. With this interpretation in hand, we turn once more to the CPI.

6.2.1 Interactions and interpretations of time in the CPI

The interpretation of time in the CPI is typically taken to align with the timelessness philosophies described in Section 6.1 and, as discussed in Chapter 3 and Chapter 5, this treatment of time as an illusion takes the notion of change with it. Including interactions, however, forces us to reconsider this position. As pointed out above, interactions can be taken to represent the influence of one system on another. If these interactions are to be responsible for transferring information from one system to another, as with the measurement of the clock in Chapter 5, it must arguably be a real physical process. In other words, if we are taking interactions seriously and as a means to transmit information, we are also taking change to be a real, physical process responsible for getting information from one system to another. If the process is not real, but instead taken to be an illusion, we return to the same problem as the timelessness interpretations: We must introduce an auxiliary memory system or ad hoc postulate to account for the ordered experience of time and the 'knowledge' of systems on their, or other systems', previous configurations.

There are several benefits to the interpretation of change as real. The irreversible nature of an interaction between two systems allows us to distinguish between the future and the past since one direction can be picked out as corresponding to the continual increase of entropy, given a sufficiently large scale. From this we gain an arrow of time. Similarly, under the interpretation that an interaction imparts information, we can identify a historical record without including an auxiliary memory system. Rather, we can argue that the current configuration of systems in the present moment represents a record which encodes the past. [8] This incorporates a dependence of present (and future) configurations on past configurations and interactions. By investigating a system and its interactions with others at one time, we may extrapolate the past configurations. In this sense, we recover an account of the flow of time as a historical record which is built into the current configuration of the system that has experienced that history, as opposed to a record kept by a separate 'memory' system.

The importance of irreversible process in the examination of time, along with the treatment of such change as real, has been advocated for previously in physics. Notably, Prigogine argued that a sufficient account of time would require not just the description of a system in a series of configurations (being) but also the process by which it changes from one configuration to another (becoming) [95]. [9] The role of irreversiblity continues to be discussed

[8] While this resembles Barbour's use of 'time capsules', taking change as a real physical process implies these capsules cannot exist equally and so diverges from the timeless picture.

[9] For a detailed account of this type of process philosophy, see [96].

in current accounts of time. As a recent example, the use of irreversible process has been argued to help resolve the gap between the relativistic and quantum theories [91]. There are, however, arguments which are presented against the necessity of incorporating irreversibility at a fundamental level. For example, one suggestion is that reversible processes need not be forbidden provided they remain unobservable to physics' experimental procedures [97]. [10] Nonetheless, even if such an argument were applicable, we would still require an account of how (and why) irreversible procedures remain ever-present in experiment at all.

In this investigation, we find that including interactions and attempting to avoid additional caveats leads us to conclude that the experience of time is a real phenomenon. It may be troubling that the Universe as a whole remains incapable of experiencing time in the manner described here, as there is no 'outside' system for it to interact with. However a possible interpretation exists whereby real change may be considered an emergent feature of a Universe that remains timeless when taken as a whole. For an account of our speculations on such an interpretation, as well as potential path to resolving the A series paradox, see Chapter 7.

It must be noted that we are not arguing that the timeless approach is in any sense beaten out by the interpretation of change as real. While we need not add the additional axioms of the timeless models, we still rely on the postulate of causality which was pointed out to be a requirement in accounting for change in Chapter 2. [11] By enforcing the interpretation that a future system depends on a past system, we can argue that we are invoking a cause and effect relationship of the type described in the introductory definitions. Interpreting interactions as real phenomena which produce new configurations in a consistent way can then be considered as tantamount to including a postulate of causality.

It may be taken as a mark against the 'change is real' interpretation that it too relies on an as yet unexplained postulate, in a similar manner to the timeless interpretations. Nonetheless, taking the process of change as real is no more expensive than assuming change is an illusion. It certainly appears that the alternative interpretation is consistent, and perhaps unavoidable, even within the 'timeless' CPI. We might then consider the task of accounting for causality and its role in the process of change.

[10] For an example of alternative approach in which an arrow of time is discussed in the context of irreversibility, see [98].

[11] Examples of additional axioms include the priinciple of least action and, in some cases, the order of events is simply taken as axiomatic.

6.3 The role of causality

Causality, as a physical phenomenon worth discussing, was thrown out of discussions of physics largely as a result of Hume who argued that there is no possible method by which we may prove cause and effect relationships [99]. His argument focused on the inability of our senses to provide any empirical evidence for causality, arguing that at best we could point out correlations but that this would never be sufficient, according to empiricism, to determine causal links between events.

Hume's approach has since been countered by others who have argued that there are benefits to considering causal relationships. A significant contribution was provided by Popper, who argued that science requires an assumption of causality in order to produce theories which corresponded to the ordered state of the Universe [100]. [12] The argument is that the scientific method relies on causal relations and, while these might not be provable, we continue to use them and in so doing refine our theories which each successive experimental measurement of Nature. In spite of this continued reliance on causality in the scientific method, the concept came to be considered 'folk' science by many. Indeed, Russell had stated that the "law of causality, I believe, like much that passes muster among philosophers, is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm" [101]. And so the use of causality as an explanatory tool, or even a concept worth investigation by physics, fell somewhat by the wayside while nonetheless remaining a crucial feature of our current models.

Regardless of this general opinion, arguments have been put forward that are in favour of treating causal reasoning as a crucial part of the framework of physics and examples can be found in [102, 103, 104]. In recent years, there has been a renewed interest in the role causality has to play in understanding the Universe. Some credit the advent of computers for this stating that the cause and effect relationship is crucial when determining why a computer program is not functioning as expected [14]. [13] Others have advocated for the use of causality specifically as a resolution to issues in quantum mechanics [108]. We can then add the interpretation of the CPI presented here as an additional reason for further investigation into the role of causality.

Certainly adding causality is not a silver bullet to the problem of time. There are multiple open questions which still require explanation. Not least of these is the problem of infinite

[12]Similar considerations can also be found in [89].

[13]Alternative accounts of causality can be found in, for example, [105, 106] which are reviewed, along with [14], in [107].

regress; An explanation of the starting point for the chain of cause and effect which has seemingly been discussed for as as long as time itself. We would also have to address the concerns first put forward by Hume regarding our inability to prove causal relationships.

In terms of the interpretation of time in the CPI, we would have to consider the interplay of interactions and causality. Specifically, if interactions as causes are to be considered external influences as in the case of the environment influencing the clock, we must ensure this is consistent with the closed Universe model used within the CPI framework. The Universe as whole could not be said to experience interactions since all influences would be internal. If cause and effect processes are tied to interactions, the Universal state would not experience them, relating this to the description of change as an emergent phenomenon that is only experienced by subsystems of the Universe but not the Universe as a whole. Following this line of reasoning, a description of emergent causality might be pursued; a postulate which would be incapable of applying to the Universal state but crucial to the account of time nonetheless.

Once we have the postulate of causality in place, our treatment of interactions as a real process leads to a description of change which aligns with experiment. Similarly, the theories discussed in Chapter 2 appeal to some additional feature to provide an ordering that is consistent with cause and effect relationships. If the issues surrounding time are resolved in this manner, leaving only the postulate of causality to be added by hand, it implies that it is this postulate which requires further investigation. Not only should causality be included in the discussions on time, it should be a focal point.

CHAPTER 7

Discussion and Speculations

"Time changes everything except something within us which is always surprised by change."
- Thomas Hardy

All of the interpretations of time that we have discussed up to this point lack conclusive experimental evidence, a result that is mostly due to the inherent interpretive and philosophical nature of the topic. But this need not remain the permanent status quo. History has shown that concepts which begin as purely philosophical considerations can be later related to physical theories, and in some cases even physical measurements, as our knowledge and technological capabilities progress. If we want to try to bring interpretive issues within reach of experimental procedures with the potential to probe the phenomenon of time, further research into the topic of time is warranted. Attempts to do just this can be found in [109, 110]. Here we consider the directions future research might take within the context of the outcomes of the our investigation.

The model of the Universe used in the CPI has had ramifications for the problem of time. The standard interpretation, associated with the Wheeler–DeWitt equation, that this isolated Universe is timeless has led some to the conclusion that our emergent experience of time must ultimately be an illusion. However, this interpretation of change is brought into question by our investigation into the use of interacting systems within the CPI. Certainly timelessness, as associated with isolated clocks, no longer appears to be the only available option. Before discussing these interpretive aspects further, we consider the quantitative results of our clock examples in Chapter 4.

The focus throughout our analysis has been on the general features of the CPI. This leaves open areas where the approach could be refined by, for example, further examining the influence of interactions on the CPI in more detail. We accounted for the influence of interactions on the clock but no specifics were provided about the environment that is responsible for inducing the damping (or decoherence) effect in the clock system. It may be worthwhile to carry out a more rigorous analysis in which the environment is explicitly described in order to examine if this affects the results reported here. We might, for instance, consider the damped harmonic oscillator to be coupled to a secondary oscillator as was done in, for example, [111, 112]. Adapting this description to fit the context of interacting clocks within the CPI, the secondary oscillator would represent the environment and a comparison could be made to other approaches in which the interactions are considered, such as was done in [45, 55]. While we might anticipate that an explicit calculation would deliver similar results, a rigorous example provides the opportunity for verification of the conclusions in Chapter 4. We could also ensure that the two subsystems of the Universe are of a comparable scale, as per the requirement discussed in Chapter 5, to better align with the CPI framework.

Along this same line of thought, we might consider a more rigorous investigation into the use of the Universal expansion as a cosmic clock system. Such a ubiquitous phenomenon throughout the Universe certainly serves well as a clock and the expansion's accessibility by physical systems makes it particularly applicable to the CPI formalism. A clock based on the expansion is only used here in a schematic sense but it could be developed into a more explicit calculation, similar to the investigation in [69]. For example, we might consider how long such a clock could be efficiently used by the physical systems of the Universe. Quantifying a description of the entanglement between the expansion, or the systems responsible for it, and the rest of the physical Universe could provide a more concrete assessment of this clock's ability to function within the CPI framework. Certainly the entanglement must be maintained as a necessary feature of the CPI, as per the definitions in Chapter 3 and the following discussion in Chapter 5. Given a sufficiently detailed account of the entangled state, we might be able to calculate the rate of decoherence resulting from the continual, but weak, interaction effects that are ultimately anticipated to lead to a state of maximum entropy for the Universe. The timescale provided by such a calculation, along with the specification of the entangled state, could be examined with respect to potentially measurable predictions.

The potential connection between the expansion of the Universe and cosmic time is one which has been pointed out before in previous investigations, such as those conducted in [113, 69]. This line of reasoning can be taken further as our proposal, outlined in Chapter 5, may be taken to suggest a connection between entropy, a result of interaction effects, and gravity, which is incorporated into the description via the expansion. Although we will not go into further details of this idea here, as it remains highly speculative, we would nonetheless point out that research conducted in the field of high-energy physics shows indications of a close relationship between entropy and gravity. One such example is the derivation of Einstein's field equation's via the first law of thermodynamics [114]. Others build on the gauge-gravity duality, a description of which can be found in [115], to show a connection between entanglement entropy and null surface [116] and this same holographic framework was also shown to allow a recovery of the second law of thermodynamics [117].

On the philosophical side of the issue, future investigations might also allow us to further refine the interpretive issues surrounding time and change. In particular, the difference between the description of time provided by isolated clocks and that currently used in standard quantum mechanics could be examined. The CPI certainly differs from standard quantum mechanics as it introduces a 'mechanism' to account for time: the entangled state. However, the isolated clock remains inaccessible from the point of view of systems within the rest of the Universe. In this sense, the CPI provides an interpretation of time similar to absolute time: A feature of the Universe which remains 'external' to the systems that it affects. A clock of this nature, which is inaccessible from all other physical systems, is for all intents and purposes 'outside' of the (remainder of the) Universe. One might argue the CPI essentially recovers absolute time only with more steps and the additional interpretation of change as an illusion.

After considering the philosophical aspects, our conclusion in Chapter 6 is that interpreting change as real is a feasible viewpoint, even if it is an emergent phenomenon. This may seem inconsistent at first since the Universal state as a whole continues to have no functional dependence on time and yet its internal subsystems are time dependent. One potential way forward is to consider an analogy to motion through space. It would seem straight forward to claim that the phrase 'move the Universe to the left' makes little sense; The Universe taken as a whole system would not functionally depend on space but rather, in some sense, would contain all space. An internal subsystem, however, may move in space. Applying a similar argument to time, the question of a 'Universe two minutes into the future', or a 'Universe yesterday', might make as little sense as a 'Universe moved to the left'. Under this interpretation, a physical system could experience time by changing while the Universe as a whole could not. More concrete arguments could be pursued along these lines to support the interpretation of emergent time as a real phenomenon within a Universe which *contains* space and time but does not, as a system, experience either. One important consideration would be the role of the Universal boundary and whether it must would depend on space and time in a manner which contradicts the above idea.

The analogy of space to time is certainly useful in providing some insight. Just as movement can be described as relative, time can also be treated as a relative concept as can be seen in, for example, [118, 119] where space and time are both considered primarily as relations between physical systems. For examples of other approaches that use similar reasoning, see, for example, [120]. Another point of comparison where the analogy between time and space may be useful is in considerations of the expansion of the Universe. This is often discussed in terms of the spatial intervals between objects and yet, since time and space are connected as spacetime, we could also look for expansion effects or similar phenomena in time. Indeed, the use of the expansion as a clock might be used to shed light on a potential connection between the expansion of space and the continual and compulsive experience of change.

It does seem necessary to re-evaluate the role of a 'now' moment, or privileged present, in our descriptions of Nature. The introduction of relativity challenged the notion of a Universal present as a matter of principle with the inability of systems to agree on the simultaneity of events as the primary piece of evidence. However, this restriction on defining global simultaneity applies to systems *within* the Universe and the Universal state as a whole need not be so restricted. A means of defining a present moment which is populated by all physical systems already forms part of ongoing research as can be seen, for example, in [88] which describes relative now moments for individual systems. Collectively, these continually advancing now moments might be taken to represent an 'expansion' in time.

The concepts above could also be related to the expanding (or growing) Block Universe interpretations. These are models of the Universe that are characterised by the continual addition of moments of time onto an ever-growing block of spacetime. A detailed account can be found, for example, in [5]. However, if we were attempting to describe an interpretation where change is taken as real, the account would have to differ from the Block Universe in that physical matter would not be 'pinned down' to a particular location in spacetime. If a reasonable description could be provided of physical systems which continually move forward along with the expansion of time it could be taken as support for the notion that a present moment can be consistent with modern physics. This remains a speculative line of thought and an account of physical systems changing both their positions in space and time in a physically meaningful way is yet to be found or ruled out. A more detailed investigation into such descriptions would be necessary before assertions could be made regarding the usefulness, or even the feasibility, of such a view of the Universe. Nonetheless, as we have acknowledged the lack of concrete arguments and calculations in this respect, we indulge in some wild speculation.

Assume for a moment that the notion of a present could be developed into a consistent picture: the Universe as a whole could be taken to contain all spacetime, and so would be 'everywhere' and 'everywhen', while the physical systems inside are only ever in the 'present', each at a single local spacetime location. Now consider motion in space. We naturally posit a force, or some similar mechanism, to set systems in motion. Pressing the analogy between space and time even further, we might look for a similar description for 'movement' in time. This is similar to the idea of 'entropic force' which has been discussed in investigations such as [121]. If the present moment 'moves' with the expansion of spacetime, it could be represented by a 'wavefront': a distortion of some area in spacetime denoting the present. Alternatively, we may consider physical matter to be pressed against the 'edge' of the expanding spacetime and so continually pushing through time as it expands. Although heavily speculative, these suggestions represent possible descriptions of change which have not, to the best of our knowledge, been conclusively ruled out. Without engaging with these and similar descriptions at the thought experiment level at the very least, we may miss potential ways to account for the experience of change as a physically meaningful process simply because we are accustomed to a more standard interpretation. We need to at least look for an account of change that explains the compulsion experienced by physical systems, a continual 'forward momentum' in time, to better understand the irreversiblity measured by experiment.

One benefit of the interpretation that physical systems move through time in the manner described above is that it provides a potential resolution to the A series paradox described in Chapter 6. The A series, and a moving present moment in general, falls prey to the problem that it would require a reference point for the present to move with respect to. A timeless Universal state is capable of providing just such a reference point while still allowing the moving present moment to be tied the expansion of spacetime (or some other suitable clock). The timeless Universal state remains static and the present indicates where change occurs. A meaningful distinction between the future and the past is then maintained with a present moment defined as the area in time where all matter is located.

Throughout the above discussion and speculation, we have found the analogy to space useful in trying to understand the phenomenon of time. As a brief caveat, we point out that it may be worthwhile to consider the parallels between space and time in a more rigorous manner. Many issues that are listed as part of the 'problem of time' have an analog in space where they do not present any difficulty. Thus an investigation into the 'problem of space' might allow us to reconsider what might be misconceptions or unrecognised assumptions in our attempt to understand spacetime as the backdrop of the Universe. That being said, care must be taken, as pointed out in [88], over the temptation of reading too much into the 'spatialisation' of time.

Our last discussion point is the postulate of causality. We have found the theories of time discussed here inevitably invoke an axiom that is responsible for ordering events in a manner consistent with experiment. This is perhaps most clearly seen in the timeless theories which tend not to rely directly on the causality postulate. Instead they posit other axioms, such as memory systems or a manifestation of the principle of least action, which perform the same role of placing events in an order consistent with observation. We would argue that investigating this ubiquitous requirement of Nature should form a focus point of future research into time. Such investigation could inform the discussion on time and change and so may provide insight into the mechanism responsible for our experience of the 'natural order', and perhaps even shed light on time as concept in its own right. Simply discarding causality as 'unprovable' folk science does a disservice to the importance of causal order as displayed in experiment. While the alternative options might provide reason to order events consistently, they do not get very far on explaining the irreversibility of such an order. Regardless of what we call it, there is a need to explain why Nature insists on invoking compulsive and continual change in such a regular and structured manner only one way on time.

While there is some renewed interest in causality as a topic worth investigation in modern physics, for example, in [108, 14], it is still mostly excluded from considerations of time. We would argue that future research into time should include causality in a much more explicit manner. The 'problems of time', specifically the arrow of time and its irreversiblity, are predominantly issues relating to our experience of time, to change. If this is identified as the area in which the postulate of causality plays a role, then we might be better served considering the 'problem of causality'.

CHAPTER 8

Conclusion

"Nothing endures but change."

- Heraclitus

Getting to one single, definitive interpretation remains a challenge in the attempt to quantify time. The disparate versions of time found in quantum field theory and general relativity continue to make this problem difficult and yet there are indications that a resolution is feasible, even if the full solution remains to be identified. While these theories certainly differ in their treatments of time, as based on their interpretations of the metric, the underlying theory of quantum gravity from which both theories are expected to emerge, can be expected to resolve these differences into one cohesive picture. The metric may be required to be treated as constant and independent of spacetime coordinates in quantum field theory but, as per the arguments discussed in Chapter 2 this can be interpreted as a limit of the more complete description to be found in a theory of quantum gravity. Time, as a feature based on the description of the metric, only *appears* differently in quantum field theory and general relativity. What remains to be resolved is the addition of a causality postulate, or some similar axiom, which acts to continually order configurations in a manner consistent with our observations of Nature.

The investigation of interactions within the CPI provides a means of challenging the standard interpretation of time in the isolated Universe described by the Wheeler–DeWitt equation. Although perfectly isolated clocks have been advocated for as the ideal choice for use in the CPI, the interacting clock are also consistent with the framework. Furthermore, the results in Chapter 4 indicate that a small amount of interaction effects would act to decrease the uncertainty of the clock observable. It appears that the weakly interacting condition associated with the CPI should be enforced much more literally: We need negligible interaction effects that are *not zero*.

The arguments in Chapter 5 suggest that incorporating the interactions into the CPI has implications for the interpretation of time. While one is free to choose to partition the Universe into either isolated or interacting subsystems, the isolated clock is at a disadvantage when compared to the clocks which are allowed to interact. Over and above the issue of interactions being included as a matter of principle, due to gravitational effects which cannot be shielded, choosing isolated clocks leads to a restriction on the CPI such that it can only describe a two-system Universe, limiting the effectiveness of the framework. Allowing for weak interactions between subsystems, on the other hand, provides all the subsystems of the Universe a means of directly accessing the clock state while still maintaining the requisite entangled state for a time. Other benefits of interacting clocks include the alignment of all the arrows of time along the direction of increasing entropy. This further supports the use of the interaction picture over the isolation one in describing our experience of time.

The conclusion is that interactions have a role to play in the CPI and has ramifications on the philosophical issues, which we discussed in Chapter 6. The use of the isolated clock is associated with the view that change must be taken to be an illusion within the timeless Universe model. However, the inclusion of interactions allows us to challenge this view. Emergent time, and the experience of it as change, can be interpreted as a real phenomenon when interactions are taken into account; a viewpoint which comes at the cost of the postulate of causality.

The CPI does not restrict us to either interpretation of time, illusion or real, and definitive experimental evidence is still to be found. The benefits of the interaction picture, however, appear to outweigh the alternative choice of treating change as an illusion. The arrow of time, along with a flow of time recorded as a history of states, are provided as a built in feature once interactions are included. The cost is the inclusion of a postulate of causality; a feature so ubiquitous in Nature that we often forget we have invoked it in our descriptions at all. Adhering to the view that time is an illusion does not side step the causality issue, although it may appear under a different name. The theory would still need to provide a reason for the causally consistent ordering of configurations in Nature, which is often done in such theories by adding ad hoc constraints or rules. By taking interactions seriously, we can better identify this issue over causality as a central concern in the problem of time.

Our conclusion is that we need to take a closer look at the role of causality not only in physics but in the philosophical interpretations of time as well. Causality has been somewhat cast aside in such discussions for some time and, while it is experiencing a revival of sorts, it remains distinctly separated from most debates involving time. As suggested by the definitions used in this investigation, incorporating causality allows us to more finely grain our description of the phenomena surrounding our experience of change; time allows change but causality might be identified as the feature of the Universe which compels change in an orderly fashion. Placing causal ordering front and center in the debate about time might allow us to gain insight into this phenomenon that continues to perplex and provoke us.

Appendix A

Contrasting the conditional probability expressions

In Chapter 3 we present a summary of the refined CPI as laid out by Dolby in [51]. Part of the framework of this refined CPI involves the distinction between two different types of conditional probabilities. These are represented as

$$\begin{aligned} &P(a \text{ given } b; \hat{\rho}), \\ &P(a \text{ when } b; \hat{\rho}), \end{aligned} \tag{A.2}$$

where a and b are eigenvalues associated with the observables X_A and X_B. These observables are themselves respectively associated with two subsystems A and B that are both represented in the density matrix $\hat{\rho}$. Here we give a mathematical description of the two different probability expressions, using the same terminology as that defined in Chapter 3.

To begin with, we identify the probability of measuring a (for subsystem A) among the spectrum of states of $\hat{\rho}$. The associated probability expression is

$$P(a;\hat{\rho}) \equiv \frac{Tr(\hat{P}_a\hat{\rho})}{Tr(\hat{\rho})}, \tag{A.3}$$

where $\hat{P}_a$ is the projection operator that selects out any physical states of A which correspond to a value of a for the observable X_A. Equation (A.3) represents a trivial mathematical rearrangement of the standard probability expression used to calculate the likelihood of a particular outcome, in this case a, by summing the individual probabilities associated with each occurrence of a in $\hat{\rho}$.

Next we introduce the second system B and construct the conditional probability of measuring a given that the observable X_B was measured to be b. This is represented as

$$P(a \mid b; \hat{\rho}) \;=\; \frac{Tr(\hat{P}_a \hat{P}_b \hat{\rho} \hat{P}_b)}{Tr(\hat{P}_b \hat{\rho} \hat{P}_b)}, \tag{A.4}$$

where $\hat{P}_B$ is similarly a projection operator selecting out the physical states corresponding to a measurement of b. The variables to the right of the delimiter '|' represent the conditional parameters that are taken into account in calculating the probability of a occurring.

Up to this point, there has been no deviation from the standard descriptions used in quantum mechanics. Next, however, the refined CPI introduces another type of conditional probability denoted as

$$P(a \; when \; b; \hat{\rho}) \;\equiv\; \frac{Tr(\hat{P}_a \hat{P}_b \hat{\rho})}{Tr(\hat{P}_b \hat{\rho})}. \tag{A.5}$$

Although the same projection operators are used, this is not a trivial rearrangement of equation (A.4). We can identify this expression as one which calculates probability of a occurring as being in some sense dependent on b but there is no clear cut physical interpretation of equation (A.5). We can, however, assess the suggested interpretations.

The account in [51] presents interpretations of the above probability expressions in term of paths in configuration space. Equation (A.3) is taken to represent the fraction of $\hat{\rho}$'s path in configuration space where the outcome a is true. Equation (A.4), on the other hand, calculates the likelihood of a occurring as conditional on the fact that b also occurs in some region, but not necessarily the same section, of $\hat{\rho}$'s path in configuration space. The interpretation of equation (A.5) is taken to be a more specific case that corresponds to the case where the outcomes a and b both occur over the *same* portion of the path in configuration space. By taking this to mean a occurring "when" b occurs, the refined CPI attempts to invoke a sense of simultaneity between the measurements of a and b that is not apparent in equation (A.4).

Our concern is that the offered interpretation is somewhat problematic in the following way. If we anticipate that equation (A.5) defines simultaneity, we would expect it to be *more* restrictive than equation (A.4) which only requires that both outcomes occur at some point along the path. This expectation is not verified by an examination of the equations in question since the projection operators appear to be applied more restrictively in equation (A.4) than in equation (A.5). Other investigations have also raised the problem of how to motivate a specific physical interpretation of $P(a \; when \; b; \hat{\rho})$ as can be seen, for example, in [122].

While the interpretation of equation (A.5) remains unresolved, we point out that this does not produce a problem for the refined CPI since the probability expressions that are ultimately used are shown to agree with the expectations of standard quantum mechanics. While there is a distinction between equation (A.4) and equation (A.5) that may be subject to further clarification, this does not negate the final result. Certainly the simultaneity between the clock and the rest is maintained through the use of the abstract integration variable n, which is also discussed in Chapter 3.

APPENDIX

B

The damped harmonic oscillator

B.1 The clock state

The analysis in Section 4.2.1 considers the damped harmonic oscillator in the coherent state representation as defined in [56]. Here we provide details of the wavefunction representation, including explicit descriptions of the terms η and μ. As in the main text, we have set $\hbar = 1$.

The terms remains as previously defined, such that the Caldirola–Kanai Hamiltonian [58, 59] is

$$H = e^{-rn}\frac{p}{2m} + e^{rn}\frac{1}{2}\omega_o^2 x^2, \tag{B.1}$$

and the annihilation and creation operators are

$$\begin{aligned} a &= \frac{1}{i}(\eta x - \mu p) \\ a^\dagger &= \frac{1}{i}(\mu^* p - \eta^* x). \end{aligned} \tag{B.3}$$

The explicit description of η and μ is

$$\begin{aligned} \eta(n) &= \frac{1}{2\sqrt{\mathcal{R}(A)}} e^{i\cot^{-1}(\frac{r}{2\omega}+\cot(\omega n))} \\ \mu(n) &= \sqrt{2} i\hbar \frac{A}{D} e^{i\cot^{-1}(\frac{r}{2\omega}+\cot(\omega n))}, \end{aligned} \tag{B.5}$$

where we have retained the use of n as the evolution parameter.

It remains to identify the terms in equation (B.5). These are given by a collection of inter-dependent definitions:

$$
\begin{aligned}
N &= \frac{m\omega}{\pi}^{\frac{1}{4}} \frac{e^{-\frac{1}{2}rn}}{\zeta(n)\sin^{\frac{1}{2}}(\omega n)} \\
\zeta(n) &= \frac{r^2}{4\omega^2} + \frac{r}{\omega}\cot(\omega n) + \csc^2(\omega n) \\
A(n) &= \frac{m\omega}{2}e^{rn}\Big(\frac{1}{\zeta(n)^2\sin^2(\omega n)} + i\Big(\frac{r}{2\omega} - \cot(\omega n) + \frac{\frac{r}{2\omega} + \cot(\omega n)}{\zeta(n)^2\sin^2(\omega n)}\Big)\Big) \\
D(n) &= \sqrt{m\omega}\frac{e^{\frac{1}{2}rn}}{\zeta(n)\sin(\omega n)},
\end{aligned}
\tag{B.7}
$$

where ω is the damped frequency given by $\omega = \sqrt{\omega_0^2 - \frac{r^2}{4}}$. The variables in equation (B.7) are produced from the wavefunction expression for the damped harmonic oscillator in [56] prior to the transformation to the coherent state representation.

B.2 The expectation value

In Section 4.2.2, the expectation value $\langle x \rangle$ is used to relate the position to n. Here we show the calculation of this expression to leading order in ωn.

The expectation value depends on $\mu = \frac{1}{2}(ReA)^{-\frac{1}{2}} \exp\left[i\cot^{-1}\left(\frac{\gamma}{2\omega} + cot(\omega n)\right)\right]$ along with other factors. To simplify, we take the first order limit to get

$$
\begin{aligned}
\cot^{-1}\left(\frac{\gamma}{2\omega} + \cot(\omega n)\right) &\approx \cot^{-1}\left(\frac{\gamma}{2\omega} + \frac{1}{\omega n}\right) \\
&\approx \cot^{-1}\left(\frac{1}{\omega n}\right) \\
&\approx \cot^{-1}\left(\cot(\omega n)\right) \\
&\approx \omega n \\
\Rightarrow \mu &= \frac{1}{2}(ReA)^{-\frac{1}{2}} \exp^{i\omega n} \\
&= \frac{1}{2}(ReA)^{-\frac{1}{2}}\left[\cos(\omega n) + i\sin(\omega n)\right]
\end{aligned}
\tag{B.9}
$$

Using this result along with $\alpha = Re(\alpha) + iIm(\alpha)$ allows us to write

$$
\begin{aligned}
\langle x \rangle &= \mu^*\alpha + \mu\alpha^* \\
&= \frac{1}{2}(ReA)^{-\frac{1}{2}}\left[\cos(\omega n) - i\sin(\omega n)\right](Re(\alpha) + iIm(\alpha)) \\
&\quad + \frac{1}{2}(ReA)^{-\frac{1}{2}}\left[\cos(\omega n) + i\sin(\omega n)\right](Re(\alpha) - iIm(\alpha)) \\
&= \frac{1}{2}(ReA)^{-\frac{1}{2}}\left[2\cos(\omega n)(Re(\alpha) + 2\sin(\omega t)Im(\alpha)\right] \\
&= (ReA)^{-\frac{1}{2}}\left[\cos(\omega n)(Re(\alpha) + \sin(\omega n)Im(\alpha)\right].
\end{aligned}
\tag{B.11}
$$

Looking at the first term $(ReA)^{-\frac{1}{2}} = \Big[\frac{m\omega}{2}e^{\gamma n}\Big(\frac{1}{\xi^2 sin^2(\omega n)}\Big)\Big]^{-\frac{1}{2}}$ we notice the same term from the uncertainty calculation which we have already resolved to the first order limit: $\xi^2 sin^2(\omega n) \approx 1$. Therefore we have $(ReA)^{-\frac{1}{2}} = e^{-\frac{\gamma n}{2}}\sqrt{\frac{2}{m\omega}}$ and equation (B.11) becomes

$$\langle x \rangle = e^{-\frac{\gamma n}{2}}\sqrt{\frac{2}{m\omega}}\Big[\cos(\omega n)(Re(\alpha) + \sin(\omega n)Im(\alpha)\Big]. \tag{B.13}$$

Restricting to the first order in rn, we can substitute $\cos(\omega n) = 1$ and $\sin(\omega n) = \omega n$, giving

$$\langle x \rangle = e^{-\frac{\gamma n}{2}}\sqrt{\frac{2}{m\omega}}\Big[(Re(\alpha) + \omega n Im(\alpha)\Big]. \tag{B.15}$$

The naturalness argument, essentially the claim that neither the real nor imaginary parts are much smaller or greater than one another, is then used to argue that $Re(\alpha) \sim Im(\alpha)$. As such, we can enforce the relation $Re(\alpha) \gg (\omega n)Im(\alpha)$, allowing us to simplify equation (B.15) to

$$\langle x \rangle = \mathcal{A}e^{-\frac{\gamma n}{2}}, \tag{B.16}$$

where $\mathcal{A} = \sqrt{\frac{2\hbar}{m\omega}}Re(\alpha)$

B.3 Uncertainty calculation

As per the description in Section 4.2.3, the variance, and related uncertainty, are of interest as measures of the effectiveness of the time parameter provided by the clock. Here we provide explicit expressions for the uncertainty δ_x, calculated from the variance associated with a probability distribution $P(x, n)$.

From [57], we have the definition of the variance as

$$\begin{aligned}
(\Delta x)^2 =& \mu\mu^* \\
=& \frac{1}{2}(ReA)^{-\frac{1}{2}}exp\Big[i\ \cot^{-1}\Big(\frac{\gamma}{2\omega} + \cot(\omega n)\Big)\Big]\times \\
& \frac{1}{2}(ReA)^{-\frac{1}{2}}exp\Big[-i\ \cot^{-1}\Big(\frac{\gamma}{2\omega} + \cot(\omega n)\Big)\Big] \\
=& \frac{1}{4}(ReA)^{-1} \\
=& \frac{1}{4}\Bigg[Re\Big[\frac{m\omega}{2}e^{\gamma n}\Big(\frac{1}{\xi^2\sin^2(\omega n)} + i\Big(\frac{\gamma}{2\omega} - \cot(\omega t) + \frac{\frac{\gamma}{2\omega} + \cot(\omega n)}{\xi^2\sin^2(\omega n)}\Big)\Big)\Big]\Bigg]^{-1} \\
=& \frac{1}{4}\Bigg[\frac{m\omega}{2}e^{\gamma n}\Big(\frac{1}{\xi^2\sin^2(\omega n)}\Big)\Bigg]^{-1} \\
=& \frac{1}{4}\frac{2}{m\omega}e^{-\gamma n}\xi^2\sin^2(\omega n) \\
=& \frac{e^{-\gamma n}\sin^2(\omega n)}{2m\omega}\xi^2
\end{aligned} \tag{B.18}$$

As per the small time limit, imposed by the run time constraint, we resolve equation (B.18) to the first order approximation in rn. This gives

$$\begin{aligned} sin^2(\omega n)\xi^2 &= \sin^2(\omega n)\Big[\frac{\gamma^2}{4\omega^2} + \frac{\gamma}{\omega}\cot(\omega n) + \csc^2(\omega n)\Big] \\ &\approx \omega^2 n^2\Big[\frac{\gamma^2}{4\omega^2} + \frac{\gamma}{\omega}\Big(\frac{1}{\omega n}\Big) + \frac{1}{\omega^2 n^2}\Big] \\ &= \frac{\gamma^2 n^2}{4} + \gamma n + 1 \\ &= 1 + \mathcal{O}(\gamma n) \end{aligned} \tag{B.20}$$

To the first order then, $sin^2(\omega n)\xi^2 \approx 1$ and we have

$$\begin{aligned} (\Delta x)^2 &= \frac{e^{-\gamma n}}{2m\omega} \\ \Delta x &= \sqrt{\frac{1}{2m\omega}} e^{-\frac{\gamma n}{2}}, \end{aligned} \tag{B.22}$$

as given in Section 4.2.3.

Appendix C

The atomic clock

C.1 The atomic clock

In Chapter 4 the second clock system we investigate is an atomic clock. Here we sketch the basic layout of such a device to clarify how it used as a time-keeping system.

In the method first laid out in [62], the harmonic oscillations of an electromagnetic wave are used as the 'tick' of a clock. If the oscillations are maintained at a constant frequency, we have a consistent measure of time. However, in order to produce a time reading, we need to know how many oscillations of our electromagnetic wave equate to one second. To set a standard, we can turn to the transitions found in atomic systems.

Although other elements may be used, Cesium is most commonly called upon for this task. An atom of Cesium, prepared in its ground state, will transition to an excited state if it is placed in an appropriate electromagnetic field. In this case, appropriate refers to a field which oscillates at a frequency similar to the energy level difference between the two states of the atom. The closer the frequency of the electromagnetic wave matches the transition frequency of the atom, the higher the probability of a transition. Thus the frequency of the oscillating wave can be tested by measuring how likely it is to cause a transition of the Cesium atoms.

A diagram showing a rough representation of how such a clock incorporates the frequency provided by Cesium transitions is shown in Figure C.1. A collection of Cesium atoms, prepared in the ground state at A, are passed through the electromagnetic field at B.

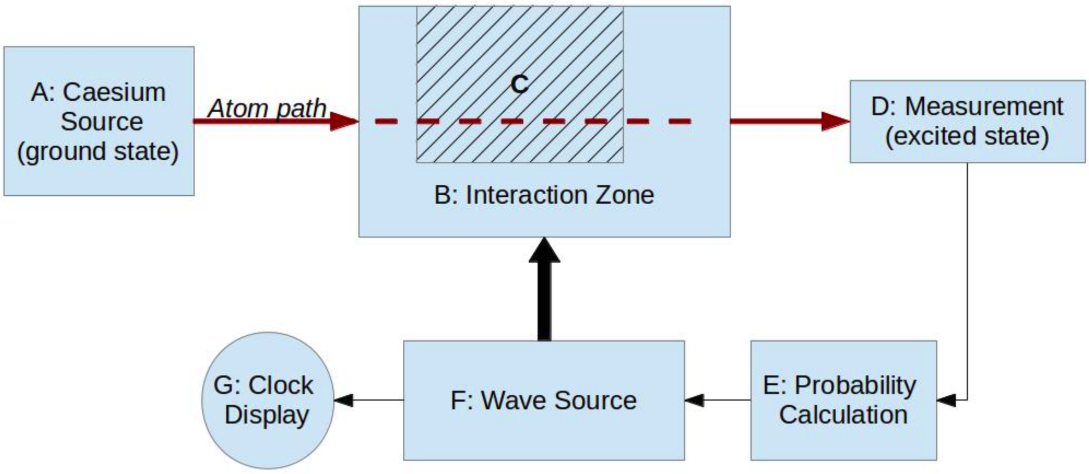

Figure C.1: Schematic representation of the components necessary to the Cesium atomic clock. The area labeled C represents the step added by Ramsey.

For the Rabi method, the interaction zone B permeates the entire block as the restriction of a non-interaction zone at C is only introduced later by Ramsey. After passing through B, the atoms are measured at D.

From the measurements of the atoms, the probability of a transition to the excited state can be calculated at E. By repeating this process, with the aim of maximising the number of transitions, the electromagnetic wave can be tuned to the transition frequency. As this frequency is calculable from theory, we can determine how many oscillations correspond to one second. In the case of Cesium, this is 9192631770 cycles per second, allowing the clock to be set by this number at G. The feedback loop involving the application of the electromagnetic wave to the atoms in order to set the frequency of the wave provides a continual check that the correct frequency is maintained. What emerges from the Rabi method, is that the precision to which the wave frequency can be set is determined by how long the atoms spend interacting with the electromagnetic wave, up to the limit discussed in Chapter 4.

Ramsey determined a way of increasing the precision by inserting a non-interaction zone, within the interaction zone, as labeled in Figure C.1 as C. This reduced the interaction with the wave to two 'pulses'. The resulting probability of a transition was now found to be proportional to the length of time the atoms spent in the non-interaction zone. By comparison to the Rabi method, this provided a more effective way on minimising the uncertainty, thus leading to a better clock.

C.2 The evolution operator

Here, evolution utilised in Section 4.3.2 is derived in detail.

Let us consider a two-level system at time $t = 0$ in its energy basis, $|\psi(0)\rangle = c_1 |1\rangle + c_2 |2\rangle$. If the external potential oscillates according to $V(t) = \lambda e^{i\omega t} + \lambda e^{-i\omega t}$ with λ real, then the state at a later time t is $|\psi(t)\rangle = \left(\cos(\Omega t) - \frac{i\theta}{2\Omega}\sin(\Omega t)\right)e^{i\theta t/2}|1\rangle + \frac{\gamma e^{-i\theta t/2}}{i\Omega}\sin(\Omega t)|2\rangle$ and its density matrix is

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)| = \begin{bmatrix} 1 - \frac{\gamma^2}{\Omega^2}a^2 & \frac{i\gamma e^{i\theta t}}{\Omega}a\left(b - \frac{i\theta}{2\Omega}a\right) \\ -\frac{i\gamma e^{-i\theta t}}{\Omega}a\left(b + \frac{i\theta}{2\Omega}a\right) & \frac{\gamma^2}{\Omega^2}a^2 \end{bmatrix}, \tag{C.1}$$

where $a = \sin(\Omega t)$, $b = \cos(\Omega t)$ and $\Omega = \sqrt{\lambda^2 + \frac{\theta^2}{4}}$. Alternatively, the evolution can be described by $\rho(t) = U(t,0)\rho(0)U^{\dagger}(t,0)$.

Assuming that the system starts in its ground state, $\rho(0) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, one finds an evolution matrix of the form

$$\begin{bmatrix} \rho(t)_{11} & \rho(t)_{12} \\ \rho(t)_{21} & \rho(t)_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ -B^* & A^* \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} A^* & -B \\ B^* & A \end{bmatrix}. \tag{C.2}$$

where

$$\begin{aligned} |A|^2 &= b^2 + \frac{\theta^2}{2\Omega^2}a^2 , \\ |B|^2 &= \frac{\lambda^2}{\Omega^2}a^2 , \\ -AB &= \frac{i\lambda e^{i\theta t}a}{\Omega}(b - \frac{i\theta}{2\Omega}a) , \\ -A^*B^* &= \frac{-i\lambda e^{-i\theta t}a}{\Omega}(b + \frac{i\theta}{2\Omega}a) . \end{aligned} \tag{C.4}$$

Since $A = b - \frac{i\theta}{2\Omega}a$ is true up to a phase, the evolution matrix can be resolved into

$$U(t,0) = \begin{bmatrix} b - \frac{i\theta}{2\Omega}a & -\frac{i\lambda e^{i\theta t}}{\omega}a \\ -\frac{i\lambda e^{-i\theta t}}{\omega}a & b + \frac{i\theta}{2\Omega}a \end{bmatrix}. \tag{C.5}$$

The non-decohering limit of this outcome agrees with the final form of Ramsey's result [64].

APPENDIX D

Research Outputs

We present here a list of the research output generated by this work.

- Publications
  - KLH Bryan and AJM Medved, *"Realistic clocks for a Universe without time."* Foundations of Physics 48.1 (2018)
- Preprint manuscripts
  - KLH Bryan and AJM Medved, *"Requiem for an ideal clock."* arXiv:1803.02045 (2018)
  - KLH Bryan and AJM Medved, *"The problem with 'The Problem of Time'."* arXiv:1811.09660 (2018)
- Conference outputs
  - KLH Bryan, *"No time for isolated clocks in the timeless Universe."* Talk given at the Ninth International Workshop DICE2018: Spacetime - Matter - Quantum Mechanics (2018)
  - Forthcoming in the conference proceedings publication:
    KLH Bryan and AJM Medved, *"No time for isolated clocks in the timeless Universe."* Journal of Physics: Conference Series

- Popular articles
  - KLH Bryan *"Time marches on, but in which direction?"* Science Today https://sciencetoday.co.za/2016/11/14/55/ (2016)
  - KLH Bryan and AJM Medved, *"Finding Time For Evolution."* Science Trends https://sciencetrends.com/finding-time-evolution/ (2018)

Bibliography

[1] D. Z. Albert, *Time and chance*. AAPT, 2001.

[2] E. Anderson, *The problem of time*, *Fundamental Theories of Physics* **190** (2017), no. 1.

[3] J. Barbour, *The nature of time*, *arXiv preprint arXiv:0903.3489* (2009).

[4] B. Dainton, *Time and space*. Routledge, 2016.

[5] G. F. Ellis, *On the flow of time*, *arXiv preprint arXiv:0812.0240* (2008).

[6] J. Halliwell, *The interpretation of quantum cosmology and the problem of time*, *arXiv preprint gr-qc/0208018* (2002).

[7] S. Hawking, *A Brief History of Time*. Bantam Dell Publishing Group, 1988.

[8] H. Price, *Time's arrow & Archimedes' point: new directions for the physics of time*. Oxford University Press, USA, 1997.

[9] C. Rovelli, *The order of time*. Penguin, 2018.

[10] K. S. Popper, *The world of Parmenides: essays on the Presocratic enlightenment*. Routledge, 2012.

[11] M. Nelson, *Existence*, *The Stanford Encyclopedia of Philosophy* (2012).

[12] C. Marletto and V. Vedral, *Evolution without evolution and without ambiguities*, *Physical Review D* **95** (2017), no. 4 043510.

[13] A. Eddington, *The nature of the physical world: Gifford lectures (1927)*. Cambridge University Press, 2012.

[14] J. Pearl, *Causality*. Cambridge university press, 2009.

[15] D. N. Page and W. K. Wootters, *Evolution without evolution: Dynamics described by stationary observables*, *Physical Review D* **27** (1983), no. 12 2885.

[16] K. Bryan and A. Medved, *The problem with 'the problem of time'*, *arXiv preprint arXiv:1811.09660* (2018).

[17] K. Bryan and A. Medved, *Realistic clocks for a universe without time*, *Foundations of Physics* **48** (2018), no. 1 48–59.

[18] K. Bryan and A. Medved, *Requiem for an ideal clock*, *arXiv preprint arXiv:1803.02045* (2018).

[19] S. Weinberg and E. Witten, *Limits on massless particles*, *Physics Letters B* **96** (1980), no. 1-2 59–62.

[20] J. Polchinski, *M-theory and the light cone*, *Progress of Theoretical Physics Supplement* **134** (1999) 158–170.

[21] J. Halliwell, J. Evaeus, J. London, and Y. Malik, *A self-adjoint arrival time operator inspired by measurement models*, *Physics Letters A* **379** (2015), no. 39 2445–2451.

[22] W. Pauli, *Encyclopedia of physics*, *Berlin/Singapore* (1958).

[23] J. Leon and L. Maccone, *The Pauli objection*, *Foundations of Physics* **47** (2017), no. 12 1597–1608.

[24] W. Rindler, *Introduction to special relativity 2nd Edition*. Clarendon Press, 1991.

[25] R. A. d'Inverno, *Introducing Einstein's relativity*. Clarendon Press, 1992.

[26] P. A. M. Dirac, *The quantum theory of the emission and absorption of radiation*, *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character* **114** (1927), no. 767 243–265.

[27] F. Mandl and G. Shaw, *Quantum field theory*. John Wiley & Sons, 2010.

[28] E. Witten, *Symmetry and emergence*, *Nature Physics* **14** (2018), no. 2 116.

[29] G. Hooft, *Symmetry breaking through Bell-Jackiw anomalies*, in *Instantons In Gauge Theories*, pp. 226–229. World Scientific, 1994.

[30] T. Banks and L. J. Dixon, *Constraints on string vacua with spacetime supersymmetry*, *Nuclear Physics B* **307** (1988), no. 1 93–108.

[31] J. Polchinski, *M theory: Uncertainty and unification*, in *Fundamental Physics—Heisenberg and Beyond*, pp. 157–166. Springer, 2004.

[32] J. Bekenstein, *Nonexistence of baryon number for static black holes*, *Physics Review D* **5** (1972) 1239.

[33] D. N. Page, *Particle emission rates from a black hole: massless particles from an uncharged, nonrotating hole*, *Physical Review D* **13** (1976), no. 2 198.

[34] B. S. DeWitt, *Quantum theory of gravity. I. The canonical theory*, *Physical Review* **160** (1967), no. 5 1113.

[35] C. Rovelli, *The strange equation of quantum gravity*, *arXiv preprint arXiv:1506.00927* (2015).

[36] C. W. Misner, K. S. Thorne, J. A. Wheeler, and D. I. Kaiser, *Gravitation*. Princeton University Press, 2017.

[37] J. B. Hartle, *The state of the universe*, in *The future of theoretical physics and cosmology: Celebrating Stephen Hawking's 60th birthday. Proceedings, Workshop and Symposium, Cambridge, UK, January 7-10, 2002*, pp. 615–620, 2002. `arXiv gr-qc/0209046`.

[38] S. W. Hawking, *The quantum state of the universe*, *Nuclear Physics B* **239** (1984), no. 1 257–276.

[39] A. Vilenkin, *Quantum cosmology and the initial state of the universe*, *Physical Review D* **37** (1988), no. 4 888.

[40] J. B. Hartle, *The quantum mechanics of closed systems*, *Directions in General Relativity* **1** (1993) 104–124.

[41] J. J. Halliwell, *The interpretation of quantum cosmological models*, in *Proceedings of the 13th International Conference on General Relativity and Gravitation*, pp. 63–80, IOP Publishers Bristol, UK, 1993.

[42] C. Rovelli, *Time in quantum gravity: An hypothesis*, *Physical Review D* **43** (1991), no. 2 442.

[43] A. Albrecht and A. Iglesias, *Clock ambiguity and the emergence of physical laws*, *Physical Review D* **77** (2008), no. 6 063506.

[44] K. V. Kuchar, *Time and interpretations of quantum gravity*, in *4th Canadian conference on general relativity and relativistic astrophysics*, p. 211, 1992.

[45] V. Giovannetti, S. Lloyd, and L. Maccone, *Quantum time*, *Physical Review D* **92** (2015), no. 4 045033.

[46] P. A. M. Dirac, *Lectures on quantum mechanics*, vol. 2. Courier Corporation, 2001.

[47] A. Albrecht, *The theory of everything vs the theory of anything*, in *Birth of the Universe and Fundamental Physics*, pp. 321–332. Springer, 1995.

[48] J. S. Bell and J. S. Bell, *Speakable and unspeakable in quantum mechanics: Collected papers on quantum philosophy*. Cambridge University Press, 2004.

[49] G. Lindblad, *On the generators of quantum dynamical semigroups*, *Communications in Mathematical Physics* **48** (1976), no. 2 119–130.

[50] D. N. Page, *Clock time and entropy*, in *Physical Origins of Time Asymmetry* (J. J. Halliwell, J. Perez-Mercader, and W. H. Zurek, eds.). Cambridge University Press, 1996.

[51] C. E. Dolby, *The conditional probability interpretation of the hamiltonian constraint*, *arXiv preprint gr-qc/0406034* (2004).

[52] R. Gambini, R. A. Porto, and J. Pullin, *A relational solution to the problem of time in quantum mechanics and quantum gravity: a fundamental mechanism for quantum decoherence*, *New Journal of Physics* **6** (2004), no. 1 45.

[53] R. Gambini, R. A. Porto, and J. Pullin, *Loss of quantum coherence from discrete quantum gravity*, *Classical and Quantum Gravity* **21** (2004), no. 8 L51.

[54] R. Gambini, R. A. Porto, J. Pullin, and S. Torterolo, *Conditional probabilities with Dirac observables and the problem of time in quantum gravity*, *Physical Review D* **79** (2009), no. 4 041501.

[55] A. R. Smith and M. Ahmadi, *Quantizing time: Interacting clocks and systems*, *arXiv preprint arXiv:1712.00081* (2017).

[56] K. Yeon, C.-I. Um, and T. F. George, *Coherent states for the damped harmonic oscillator*, *Physical Review A* **36** (1987), no. 11 5287.

[57] C.-I. Um, K.-H. Yeon, and T. F. George, *The quantum damped harmonic oscillator*, *Physics Reports* **362** (2002), no. 2-3 63–192.

[58] P. Caldirola, *Forze non conservative nella meccanica quantistica*, *Il Nuovo Cimento (1924-1942)* **18** (1941), no. 9 393–400.

[59] E. Kanai, *On the quantization of the dissipative systems*, *Progress of Theoretical Physics* **3** (1948), no. 4 440–442.

[60] Y. J. Ng and H. van Dam, *Limitation to quantum measurements of space-time distances*, *Annals N. Y. Acad. Sci.* **755** (1995) 579–584, [hep-th/9406110].

[61] V. Corbin and N. J. Cornish, *Semi-classical limit and minimum decoherence in the conditional probability interpretation of quantum mechanics*, *Foundations of Physics* **39** (2009), no. 5 474–485.

[62] I. I. Rabi, *Radiofrequency spectroscopy*, 1945. Richtmyer Memorial Lecture, delivered at Columbia University in New York.

[63] J. J. Sakurai and E. D. Commins, *Modern quantum mechanics, revised edition*, 1995.

[64] N. F. Ramsey, *A molecular beam resonance method with separated oscillating fields*, *Physical Review* **78** (1950), no. 6 695.

[65] C. A. Brasil, F. F. Fanchini, and R. d. J. Napolitano, *A simple derivation of the Lindblad equation*, *Revista Brasileira de Ensino de Física* **35** (2013), no. 1 01–09.

[66] P. Pearle, *Simple derivation of the Lindblad equation*, *European Journal of Physics* **33** (2012), no. 4 805.

[67] H.-P. Breuer, F. Petruccione, *et. al.*, *The theory of open quantum systems*. Oxford University Press on Demand, 2002.

[68] S. Weinberg, *Lindblad decoherence in atomic clocks*, *Physical Review A* **94** (2016), no. 4 042117.

[69] S. Stupar and V. Vedral, *Was inflation necessary for the existence of time?*, *arXiv preprint arXiv:1710.04260* (2017).

[70] H. Karttunen, P. Kröger, H. Oja, M. Poutanen, and K. J. Donner, *Fundamental astronomy*. Springer, 2016.

[71] M. Zeilik and E. van Panhuys Smith, *Introductory astronomy and astrophysics*, *Philadelphia: Saunders College Pub., c1987. 2nd ed.* (1987).

[72] S. W. Hawking, *The direction of time*, tech. rep., PRE-30559, 1987.

[73] J. B. Hartle, *The physics of now*, *American Journal of Physics* **73** (2005), no. 2 101–109.

[74] R. Landauer, *Irreversibility and heat generation in the computing process*, *IBM journal of research and development* **5** (1961), no. 3 183–191.

[75] C. Rovelli, *Is time's arrow perspectival?*, *K. Chamcham, J. Silk, JD Barrow and S. Saunders* (2017) 285–96.

[76] S. Hawking and R. Penrose, *The nature of space and time*. Princeton University Press, 2010.

[77] H. Price, *Cosmology, time's arrow, and that old double standard*, *Time's Arrows Today: Recent Physical and Philosophical Work on the Direction of Time* (1997) 66–96.

[78] J. E. McTaggart, *The unreality of time*, *Mind* (1908) 457–474.

[79] T. Maudlin, *On the passing of time*, *The metaphysics within physics* (2007) 104–142.

[80] A. Jabs, *A conjecture concerning determinism, reduction, and measurement in quantum mechanics*, *Quantum Studies: Mathematics and Foundations* **3** (2016), no. 4 279–292.

[81] G. 't Hooft, *Determinism beneath quantum mechanics*, *Quo vadis quantum mechanics?* (2005) 99–111.

[82] J. Barbour, *The end of time: The next revolution in physics*. Oxford University Press, 2001.

[83] J. Butterfield, *Julian barbour the end of time?*, *British Journal for the Philosophy of Science* **53** (2002), no. 2 289–330.

[84] M. Tamm, *Time's arrow in a finite universe*, *International Journal of Astronomy and Astrophysics* **5** (2015), no. 02 70.

[85] C. Rovelli, *Forget time*, *Foundations of Physics* **41** (2011), no. 9 1475.

[86] C. Bouton, *Is the future already present? the special theory of relativity and the block universe view*, in *Time of Nature and the Nature of Time*, pp. 89–121. Springer, 2017.

[87] J. Erasmus, *Can cosmology justify belief in an eternal universe?*, in *The Kalām Cosmological Argument: A Reassessment*, pp. 129–157. Springer, 2018.

[88] L. Smolin, *Temporal relationalism*, *arXiv preprint arXiv:1805.12468* (2018).

[89] A. N. Whitehead and D. W. Sherburne, *Process and reality*. Macmillan New York, NY, 1957.

[90] J. D. Norton, *Time really passes*, *HUMANA.MENTE Journal of Philosophical Studies* **4** (2010), no. 13 23–34.

[91] A. C. Elitzur and S. Dolev, *Becoming as a bridge between quantum mechanics and relativity*, in *Endophysics, Time, Quantum And The Subjective: (With CD-ROM)*, pp. 589–606. World Scientific, 2005.

[92] A. C. Elitzur and S. Dolev, *Quantum phenomena within a new theory of time*, in *Quo vadis quantum mechanics?*, pp. 325–349. Springer, 2005.

[93] A. Schlatter, *On the reality of quantum collapse and the emergence of space-time*, *Entropy* **21** (2019), no. 3 323.

[94] S. M. Carroll, *What if time really exists?*, *arXiv preprint arXiv:0811.3772* (2008).

[95] I. Prigogine and E. N. Hiebert, *From being to becoming: Time and complexity in the physical sciences*, *Physics Today* **35** (1982) 69.

[96] P. Coveney and R. Highfield, *The arrow of time: A voyage through science to solve time's greatest mystery*. Fawcett Columbine, 1992.

[97] L. Maccone, *Quantum solution to the arrow-of-time dilemma*, *Physical review letters* **103** (2009), no. 8 080401.

[98] G. t. Hooft, *Time, the arrow of time, and quantum mechanics*, *arXiv preprint arXiv:1804.01383* (2018).

[99] D. Hume, *An enquiry concerning human understanding*, in *Seven Masterpieces of Philosophy*, pp. 191–284. Routledge, 2016.

[100] K. Popper, *The logic of scientific discovery*. Routledge, 2005.

[101] B. Russell, *On the notion of cause*, in *Proceedings of the Aristotelian society*, vol. 13, pp. 1–26, JSTOR, 1912.

[102] N. Cartwright, *Causal laws and effective strategies*, *Noûs* (1979) 419–437.

[103] J. Dupre and N. Cartwright, *Probability and causality: Why Hume and indeterminism don't mix*, *Nous* **22** (1988), no. 4 521–536.

[104] M. Frisch, *Causal reasoning in physics*. Cambridge University Press, 2014.

[105] S. L. Morgan and C. Winship, *Counterfactuals and causal inference*. Cambridge University Press, 2015.

[106] S. Sloman, *Causal models: How people think about the world and its alternatives*. Oxford University Press, 2005.

[107] A. Gelman, *Causality and statistical learning*, *American Journal of Sociology* **117** (2011), no. 3 955–966.

[108] G. M. D'Ariano, *Causality re-established*, *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* **376** (2018), no. 2123 20170313.

[109] S. Lloyd, L. Maccone, R. Garcia-Patron, V. Giovannetti, Y. Shikano, S. Pirandola, L. A. Rozema, A. Darabi, Y. Soudagar, L. K. Shalm, *et. al.*, *Closed timelike curves via postselection: theory and experimental test of consistency*, *Physical review letters* **106** (2011), no. 4 040403.

[110] E. Moreva, M. Gramegna, G. Brida, L. Maccone, and M. Genovese, *Quantum time: Experimental multitime correlations*, *Physical Review D* **96** (2017), no. 10 102005.

[111] R. Brustein and A. Medved, *Semiclassical black holes expose forbidden charges and censor divergent densities*, *Journal of High Energy Physics* **2013** (2013), no. 9 108.

[112] I.-S. Yang, *Secret loss of unitarity due to the classical background*, *Physical Review D* **96** (2017), no. 2 025005.

[113] A. Allahverdyan and V. Gurzadyan, *Time arrow is influenced by the dark energy*, *Physical Review E* **93** (2016), no. 5 052125.

[114] T. Jacobson, *Thermodynamics of spacetime: the Einstein equation of state*, *Physical Review Letters* **75** (1995), no. 7 1260.

[115] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, *Large N field theories, string theory and gravity*, *Physics Reports* **323** (2000), no. 3-4 183–386.

[116] S. Ryu and T. Takayanagi, *Holographic derivation of entanglement entropy from the anti–de sitter space/conformal field theory correspondence*, *Physical review letters* **96** (2006), no. 18 181602.

[117] N. Engelhardt and S. Fischetti, *Losing the IR: a holographic framework for area theorems*, *Classical and Quantum Gravity* (2018).

[118] C. Rovelli, *Quantum mechanics without time: a model*, *Physical Review D* **42** (1990), no. 8 2638.

[119] C. Rovelli, *Relational quantum mechanics*, *International Journal of Theoretical Physics* **35** (1996), no. 8 1637–1678.

[120] F. Vidotto, *Relational quantum cosmology*, *The Philosophy of Cosmology* (2017) 297.

[121] E. Verlinde, *On the origin of gravity and the laws of Newton*, *Journal of High Energy Physics* **2011** (2011), no. 4 29.

[122] F. Hellmann, M. Mondragon, A. Perez, and C. Rovelli, *Multiple-event probability in general-relativistic quantum mechanics*, *Physical Review D* **75** (2007), no. 8 084033.