

# Localization Dynamics in a Binary Two-dimensional Cellular Automaton: the Diffusion Rule

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We study a two-dimensional cellular automaton (CA), called *Diffusion Rule* (DR), which exhibits diffusion-like dynamics of propagating patterns. In computational experiments we discover a wide range of mobile and stationary localizations (gliders, oscillators, glider guns, puffer trains, *etc*), analyze spatio-temporal dynamics of collisions between localizations, and discuss possible applications in unconventional computing.

*Keywords:* Cellular automata, Diffusion Rule, semi-totalistic rules, particle collisions, mean field theory, reaction-diffusion, unconventional computing.

## 1 INTRODUCTION

In our previous studies on minimal cellular automaton (CA) models of reaction-diffusion chemical system we constructed [4] a binary-cell-state eight-cell neighborhood 2D CA model of a quasi-chemical system with one substrate, state 0, and one reagent, state 1. In that model chemical reactions

were represented by semi-totalistic transition rules. Every cell switches from state 0 to state 1 depending whether sum of neighbors in state 1 belongs to some specified interval or not. A cell remains in state 1 if sum of neighbors in state 1 belongs to another specified interval.

From 1296 cell-state transition rules, we selected a set of rules with complex behavior [4]. Amongst the complex rules, namely in *G*-class of morphological classification [4], we located so-called Diffusion Rule. CA governed by this rule often exhibits slowly non-uniformly growing patterns, resembling diffusive patterns in chemical systems with non-trivial coefficients of diffusion, or reaction-dependent diffusion coefficients, so the name of the rule.

The rule simulates sub-excitable [36] medium-like mode of perturbation propagation—cell in state 0 takes state 1 if there are exactly two neighbors in state 1, otherwise the cell remains in state 0, and, conditional inhibition—cell in state 1 remains in state 1 if there are exactly seven neighbors in state 1, otherwise the cell switches to state 1.

In present paper we are trying to answer the following questions. Is there a reaction-diffusion binary-state CA that express complex dynamic? Can we demonstrate that CA exhibits non-stationary growth of reaction-diffusion patterns? Do stationary or mobile generators of localizations, glider guns, exist in binary-state reaction-diffusion CA? Can the reaction-diffusion CA simulate an effective procedure and therefore be universal?

CA with space-time dynamics similar to that in spatially extended chemical systems are studied from early days of CA theory and applications [35], however most rules discovered so far lack minimality (some of the rules employ dozens of cell-states). Methods of selecting the rules also widely vary depending on theoretical frameworks, e.g. probabilistic spaces [20, 38] and genetic algorithms [14, 32]. Therefore, we envisage a strong need for a systematic analysis of propagating patterns like those observed in the Diffusion Rule. The propagating patterns are of utmost importance in modern computer science because such patterns play a vital role in developing novel and emerging computing paradigms and architectures, particularly collision-based computing [1, 3, 23].

We must mention that various authors have already obtained pioneering results in the studied rule. Magnier *et al.* discovered three primary gliders [33]. David Eppstein found four gliders known, already incorporated in our framework, and four new gliders which were novel for us (Fig. 4 (q) (t) and (u)).<sup>1</sup> The glider traveling along diagonals of the lattice (Fig. 4 (v)) was firstly recorded by Amling in 2002 (see Eppstein's web site). Finally, a glider gun and three puffer trains were discovered by Wótowicz.<sup>2</sup>

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<sup>1</sup> See Eppstein's findings at <http://fano.ics.uci.edu/ca/rules/b2s7/>

<sup>2</sup> [http://www.mirwoj.opus.chelm.pl/ca/rules/life\\_2.gif](http://www.mirwoj.opus.chelm.pl/ca/rules/life_2.gif)

Diffusion Rule CA is just one of many complex CA<sup>3</sup> exhibiting mobile localizations. Other famous examples include semi-totalistic rules as the Game of Life [10, 17], Brain's-brain and Critters rules [34], High Life [9], Life 1133 [22], Life Without Death [18], and Life variant *B35/S236*.<sup>4</sup> Other variant is with Larger than Life [16] and the Beehive and Spiral rules hexagonal CA [5, 6, 39], more recent candidates were proposed by George Maydwell with Hexagonal Life and Hexagonal Long Life rules.<sup>5</sup> Amongst 3D binary state CA supporting gliders Life 4555 and Life 5766 by Carter Bays [7, 8] are most widely known. In 1D there are Rule 110 [13, 26, 30, 37] and Rule 54 [11, 21, 25] CA, which support an impressive range of mobile localizations.

Our paper is structured as follows. In Sect. 2 we introduce basic concepts of CA model under investigation. Section 3 introduces results of statistical analysis of the Diffusion Rule using mean field theory. In Sect. 4 we present basic structures discovered in the Diffusion Rule CA. Section 5 compiles a catalogs of non-trivial interactions between mobile localizations, which could be used to designing basic elements of collision-based computers. In Sect. 6 we highlight our achievements in analysis of the Diffusion Rule and prospects for future studies.

## 2 BASIC NOTATIONS

We study family of 2D binary-state cellular automaton (CA) defined by tuple  $\langle \mathbb{Z}^2, \Sigma, u, f \rangle$ , where  $\mathbb{Z}$  is the set of integers, every cell  $x \in \mathbb{Z}^2$  has eight neighbors, orthogonal and diagonal (i.e. classical Moore's neighborhood)  $u(x) = \{y \in \mathbb{Z} : x \neq y \text{ and } |x - y| \leq 1\}$ ,  $\Sigma = \{0, 1\}$  is the set of *states*, and  $f$  is a local transition function defined as follows:

$$\begin{aligned} x^{t+1} &= f(u(x^t)) \\ &= \begin{cases} 1, & \text{if } (x^t = 0 \text{ and } \sigma_x^t \in [\theta_1, \theta_2]) \text{ or } (x^t = 1 \text{ and } \sigma_x^t \in [\delta_1, \delta_2]) \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (1)$$

where  $\sigma_x^t = |\{y \in u(x) : y^t = 1\}|$ , and  $\theta_1, \theta_2, \delta_1, \delta_2$  are some fixed parameters such that  $0 \leq \theta_1 \leq \theta_2 \leq 8$  and  $0 \leq \delta_1 \leq \delta_2 \leq 8$ .

We can write the rule as  $R(\delta_1\delta_2\theta_1\theta_2)$  or like  $B\theta_1 \cdots \theta_2/S\delta_1 \cdots \delta_2$  more traditional code. Also, the rule can be interpreted as a simple discrete model

<sup>3</sup><http://uncomp.uwe.ac.uk/genaro/otherRules.html>

<sup>4</sup><http://www.ics.uci.edu/~eppstein/ca/b35s236/>

<sup>5</sup>Several interesting candidates in hexagonal representation are proposed by Andrew Wuensche using *DDLAB* at <http://www.ddlab.com/> and by Maydwell using *SARCASim* at <http://www.collidoscope.com/ca/>

of a quasi-chemical system with substrate ‘0’ and reagent ‘1’, where  $[\theta_1, \theta_2]$  is an interval of reaction, or association between substrate and reagent, and  $[\delta_1, \delta_2]$  is an interval of dissociation. The family of rules includes Conway’s Game Life, when intervals  $[\delta_1, \delta_2]$  and  $[\theta_1, \theta_2]$  are interpreted as intervals of survival and birth, respectively.

In our previous paper [4] we morphologically classified all 1296 rules, and studied how changes in parameters  $R(\delta_1\delta_2\theta_1\theta_2)$  of cell-state transition rule influence space-time dynamics. For example, we discovered [4] a small subset of rules *Life 2c22*,<sup>6</sup>  $2 \leq c \leq 8$ , which could be interpreted as quasi-chemical precipitating systems. For parameter set  $[\theta_1, \theta_2] = [22]$  [1, 4], the system is transformed into  $2^+$ -medium, CA model of excitable system in sub-excitable mode.

We have found a cluster of semi-totalistic rules supporting structures of the Diffusion Rule. They are  $B2/S2 \dots 8$  called *Life dc22*<sup>7</sup> where  $d$  and  $c$  take values between 2 and 8, and  $d \leq c$ . Therefore, we found that the rule  $B2/S7$  or  $R(7722)$  exhibits most reach dynamics of localized patterns amongst all the rules studied by us. Rules of the local transition are simple:

1. A cell in state 0 will take state 1 if it has exactly two neighbors in state 1, otherwise cell remains in state 0.
2. A cell in state 1 remains in state 1 if it has exactly seven neighbors in state 1, otherwise cell takes state 0.

### 3 MEAN FIELD APPROXIMATION

Mean field theory is a proved technique for discovering statistical properties of CA without analyzing evolution spaces of individual rules [12, 20, 28]. The method assumes that elements of the set of states  $\Sigma$  are independent, uncorrelated between each other in the rule’s evolution space. Therefore we can study probabilities of states in neighborhood in terms of probability of a single state (the state in which the neighborhood evolves), thus probability of a neighborhood is the product of the probabilities of each cell in the neighborhood. Using this approach we can construct mean field polynomial for a semi-totalistic evolution rule [24] as follow:

$$p_{t+1} = \sum_{v=\delta_1}^{\delta_2} \binom{n-1}{v} p_t^{v+1} q_t^{n-v-1} + \sum_{v=\theta_1}^{\theta_2} \binom{n-1}{v} p_t^v q_t^{n-v} \quad (2)$$

where  $n$  represents the number of cells in neighborhood,  $v$  indicates how often state 1 occurs in Moore’s neighborhood,  $n - v$  shows how often state 0

<sup>6</sup>[http://uncomp.uwe.ac.uk/genaro/Diffusion\\_Rule/life\\_2c22.html](http://uncomp.uwe.ac.uk/genaro/Diffusion_Rule/life_2c22.html)

<sup>7</sup>[http://uncomp.uwe.ac.uk/genaro/Life\\_dc22.html](http://uncomp.uwe.ac.uk/genaro/Life_dc22.html)

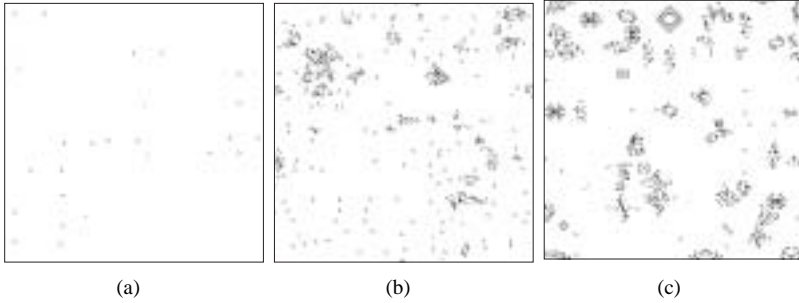


FIGURE 1  
Three random initial densities for the Diffusion Rule: (a) 0.004, (b) 0.013 and (c) 0.995 respectively, on lattices of  $200 \times 200$  to 18 generations.

occurs in the neighborhood,  $p_t$  is a probability of cell being in state 1,  $q_t$  is a probability of cell being in state 0.

On the basis of outcomes of computational experiments we can suggest intervals of extreme densities of initial random configurations which leads to the emergence of localizations in the Diffusion Rule. In the lower limit best densities  $d$  are  $0.004 < d < 0.015$  and in the upper limit they are  $0.992 < d < 0.997$  for the first 15–20 steps of evolution (Fig. 1). CA starting its evolution in random configuration with lower density of 1-states exhibit stationary or mobile self-localizations (like gliders or oscillators) at the beginning of the evolution, however in many cases collisions between mobile localizations leads to catastrophes, when 1-state patterns spread all over the lattice. Random initial configurations with higher (upper limit) density of 1-states produce either vanishing reactions between localizations or symmetrical growing patterns emerged as unions of two or more gliders.

Thus, the mean field polynomial for the Diffusion Rule is following:

$$p_{t+1} = 8p_t^8 q_t + 28p_t^2 q_t^7 \quad (3)$$

The fixed point is 0.236 that represents configurations with large density of 1-states emerging from any random initial condition (we should note that the fixed point for Conway's Game of Life is 0.37); this represents global density of 1-states necessary for evolution dynamics to stabilize. Also, we can see an unstable fixed point 0.05 (Fig. 2), that implies the existence of regions with unpredictable behavior or complex dynamic [28].

Thus,  $p = 0$  is a super-stable point, although it is quite close to the unstable point. The super-stable is important, as it means a quiescent substrate, i.e., the state where live all the structures. Since mean field theory is just an initial approximation, it ought to be worthwhile to gather up some Monte Carlo approximations for a few generations just to see if the first estimate maintains itself, more or less.

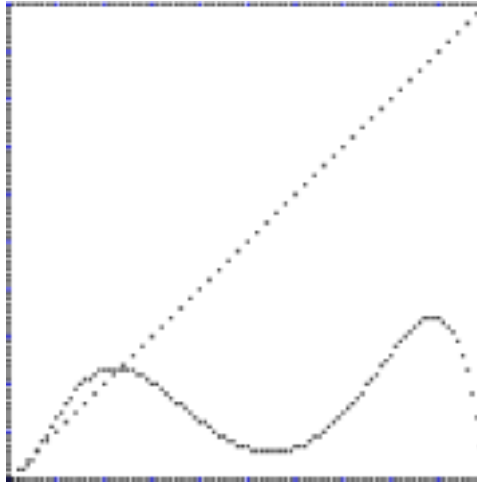


FIGURE 2  
Diagram of mean field polynomial for the Diffusion Rule.

We must also mention that the probability to find ‘interesting’ behavior is very low, about 0.05. Perhaps, this may be the reason why the Diffusion Rule was not studied before – when observing evolution from random configuration one more likely (with probability 0.3) to encounter a catastrophe (e.g. when placing three cells anywhere in Moore’s neighborhood, just not in one line) than stable mobile localization.

## 4 THE DIFFUSION RULE UNIVERSE

In present section we uncover a range of basic structures, stationary and mobile localizations, generators of localizations and polymer-like structures formed of the mobile localizations.

### 4.1 Mobile self-localizations

In computational experiments with the Diffusion Rule CA we discovered 26 mobile self-localizations—gliders or particles—traveling orthogonally or diagonally in the lattice. Properties of the gliders, including volume, speed, direction of motion are listed in Table 1.

Configurations of minimal gliders and compound gliders are shown in Figs. 3 and 4, respectively. From Table 1, we can see that 96% of gliders move orthogonally, and that *g23*, *g24*, *g25* and *g26* are the largest gliders in the family of mobile localizations. First column in the table gives names of gliders. Second column in Table 1 represents glider’s volume calculated as number of cells occupied by the glider. Third and fourth columns are translation and

glider	volume	translation	period	speed	weight	move
g1	8	1	1	$c/1$	4	orthogonal
g2	12	1	1	$c/1$	4	orthogonal
g3	12	1	1	$c/1$	4	orthogonal
g4	12	1	1	$c/1$	4	orthogonal
g5	30	4	4	$c/1$	7	orthogonal
g6	30	4	4	$c/1$	7	orthogonal
g7	45	4	4	$c/1$	14	orthogonal
g8	45	4	4	$c/1$	14	orthogonal
g9	56	4	4	$c/1$	14	orthogonal
g10	56	4	4	$c/1$	14	orthogonal
g11	70	4	4	$c/1$	24	orthogonal
g12	72	4	4	$c/1$	14	orthogonal
g13	75	4	4	$c/1$	18	orthogonal
g14	84	4	4	$c/1$	24	orthogonal
g15	96	4	4	$c/1$	18	orthogonal
g16	96	4	4	$c/1$	22	orthogonal
g17	96	4	4	$c/1$	26	orthogonal
g18	96	4	4	$c/1$	30	orthogonal
g19	112	4	4	$c/1$	26	orthogonal
g20	126	4	4	$c/1$	26	orthogonal
g21	144	4	4	$c/1$	26	orthogonal
g22	144	4	4	$c/1$	30	orthogonal
g23	210	4	4	$c/1$	38	orthogonal
g24	338	2	4	$c/2$	52	orthogonal
g25	405	2	4	$c/2$	79	orthogonal
g26	576	2	8	$c/4$	75	diagonal

TABLE 1  
Properties of gliders in the Diffusion Rule CA

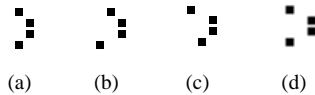


FIGURE 3  
Configurations of minimal, or primary, gliders in the Diffusion Rule: (a) g1 glider, (b) g2 glider, (c) g3 glider and (d) g4 glider.

period. Fifth column shows the speed =  $c/\text{period}$ , where  $c$  is the maximum speed. Sixth column is the weight that represents the number of cells with state 1 within glider's volume. The last column indicates whether or not glider moves along columns and rows, or diagonals.

There are two types of gliders—*primary* and *compound* [26, 38]: a primary glider can not be decomposed into smaller mobile localizations, a compound glider is made of at least two primary gliders.

## 4.2 Oscillators

The Diffusion Rule CA exhibits five types of stationary localizations known as oscillators. The two most common flip-flops and three blinker patterns are shown in Fig. 5.

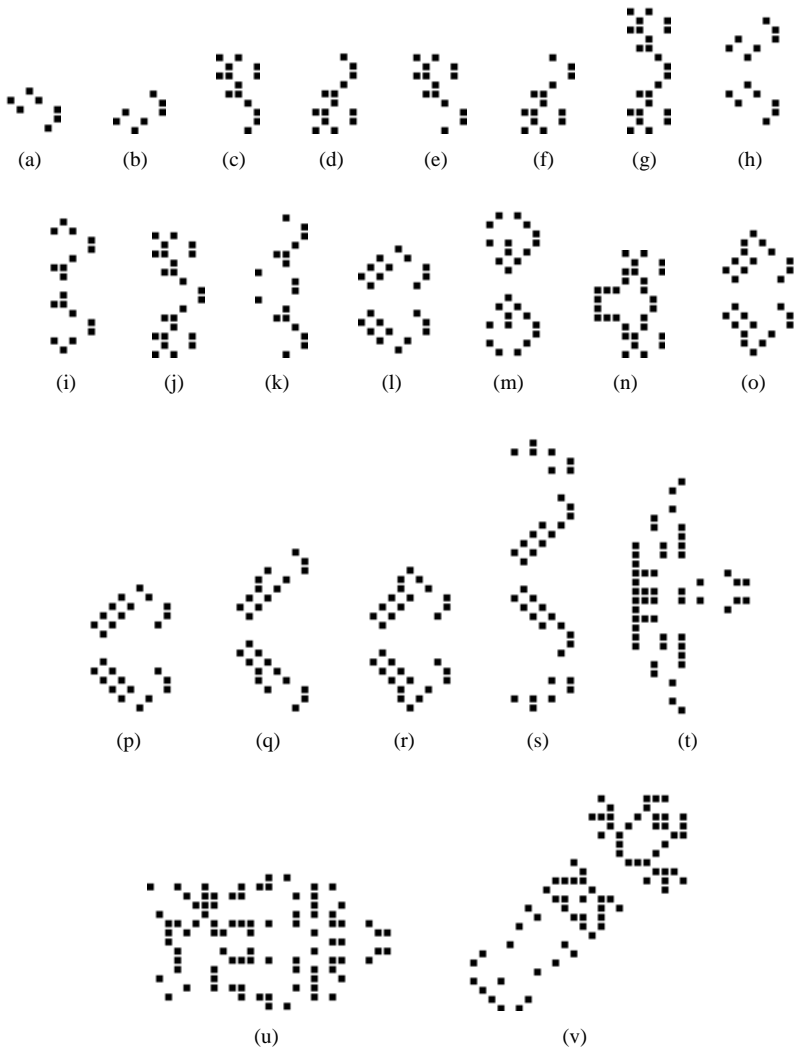


FIGURE 4

Twenty two compound gliders in the Diffusion Rule CA: (a) g5, (b) g6, (c) g7, (d) g8, (e) g9, (f) g10, (g) g11, (h) g12, (i) g13, (j) g14, (k) g15, (l) g16, (m) g17, (n) g18, (o) g19, (p) g20, (q) g21, (r) g22, (s) g23, (t) g24, (u) g25 and (v) g26 gliders, respectively.

Flip-flop configurations are o1 and o2 oscillators both structures flipping at  $45^\circ$ . Blinkers of period four are o3, o4 and o5 oscillators. Table 2 shows basic characteristics of each oscillator.

Some of the oscillators and their assembles act as eaters, i.e. stationary patterns that annihilate gliders colliding to them (see examples of collisions in Sect. 5.3).



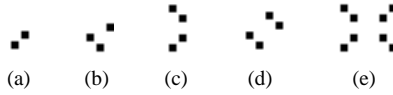


FIGURE 5  
Five oscillators in the Diffusion Rule CA: (a) o1 and (b) o2 are flip-flops, (c) o3, (d) o4 and (e) o5 are blinkers, respectively.

oscillator	volume	period	weight
o1	4	2	2
o2	9	2	3
o3	10	4	4
o4	16	4	4
o5	30	4	8

TABLE 2  
Properties of oscillators in the Diffusion Rule CA

### 4.3 Avalanches

Avalanches are novel structures, have not described before in related studies. They are assembles of adjacent gliders that cause explosive growth of rhomboid shaped patterns with deterministic edges and quasi-chaotic interior. Avalanches can be constructed from various compositions of adjacent gliders.

Figure 6 shows an avalanche produced in composition of two g1 gliders, adjacent at 90°; this avalanche pattern grows diagonally inside the third quadrant. The minimal volume of an avalanche is 4 × 4 with eight cells in state 1.

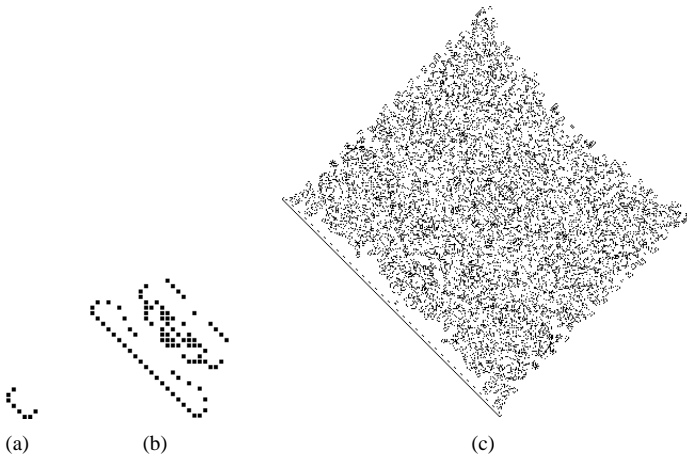


FIGURE 6  
Two g1 gliders produce an avalanche pattern: (a) initial condition, (b) configuration after 15 steps of evolution and (c) configuration after 200 steps of evolution.

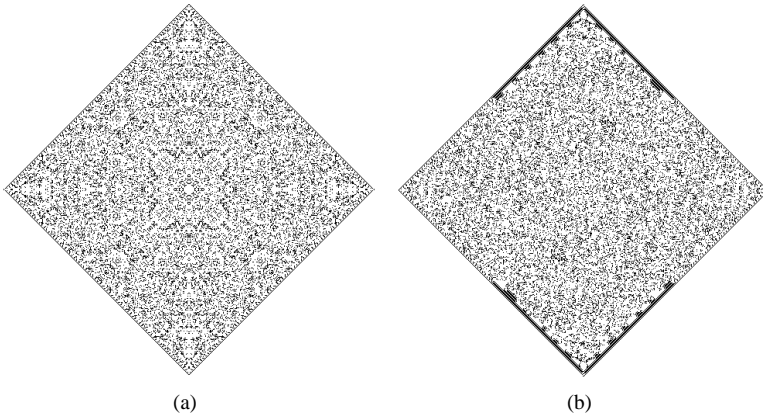


FIGURE 7  
Construction of the symmetrical avalanches: (a) symmetrical growth initiated by two  $g_4$  gliders, configuration at 237th step of evolution, (b) avalanche pattern with non-trivial internal symmetries produced by assembly of two  $g_2$  glider and two  $g_3$  gliders, configuration at 237th step of evolution.

One can use even number of gliders to construct symmetrical avalanches, two examples are shown in Fig. 7.

#### 4.4 Puffer trains

A puffer train is a mobile localization which generate (leaves traces) of stationary localizations along its motion path. There are 16 known types of stable puffer trains (which produce oscillators) in the Diffusion Rule CA. Basic properties of puffer trains are shown in Table 3.

puffer train	produce	volume	translation	period	speed	weight	move
p1	o3	30	4	4	$c/1$	7	orthogonal
p2	o1	35	4	4	$c/1$	8	orthogonal
p3	o1	42	4	4	$c/1$	9	orthogonal
p4	o1	42	4	4	$c/1$	9	orthogonal
p5	o1	42	4	4	$c/1$	10	orthogonal
p6	o1	54	4	4	$c/1$	13	orthogonal
p7	o1	56	4	4	$c/1$	9	orthogonal
p8	$2 \times o1$	63	4	4	$c/1$	14	orthogonal
p9	o1	63	4	4	$c/1$	15	orthogonal
p10	$2n \times o1$	65	4	4	$c/1$	14	orthogonal
p11	$2 \times o1$	80	4	4	$c/1$	17	orthogonal
p12	o1	84	4	4	$c/1$	11	orthogonal
p13	$2 \times o1$	90	4	4	$c/1$	11	orthogonal
p14	$2 \times o1$	105	4	4	$c/1$	15	orthogonal
p15	$2n \times o1$	120	4	4	$c/1$	28	orthogonal
p16	$o1 \vee \epsilon$	242	4	4	$c/1$	22	orthogonal

TABLE 3  
Characteristics of puffer trains in the Diffusion Rule CA

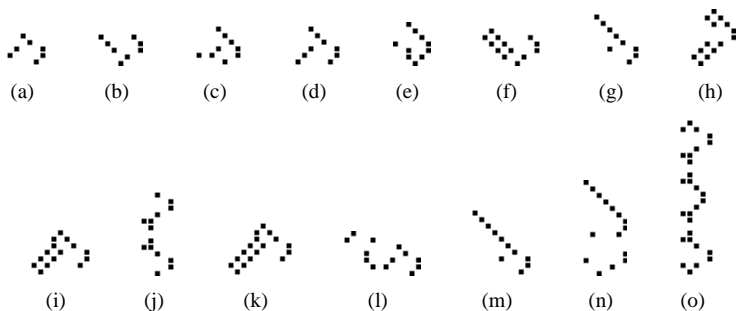


FIGURE 8  
Fifteen puffer trains observed in evolution of the Diffusion Rule CA: (a) p1, (b) p2, (c) p3, (d) p4, (e) p5, (f) p6, (g) p7, (h) p8, (i) p9, (j) p10, (k) p11, (l) p12, (m) p13, (n) p14 and (o) p15 puffer trains, respectively.

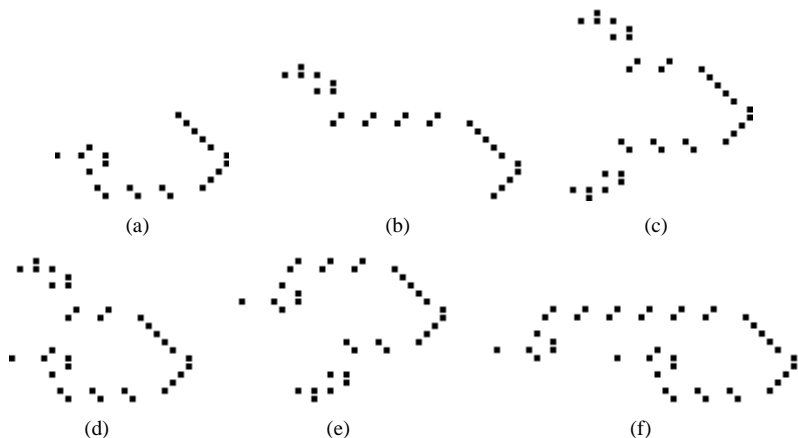


FIGURE 9  
Special p16 puffer train similar to spaceships.

A particular case of puffer train is shown in Fig. 9. This puffer bears fragments of g4 glider. All configurations of the puffer are displayed in Fig. 9, and there we can see that configurations shown in Fig. 9(c) (d) (e) and (f) can be interpreted as spaceships.

Moreover, the Diffusion Rule CA exhibits dozens of non-stable puffer trains. In the Fig. 10 we see five non-stable puffer trains, which produce asymmetrically growing, or quasi-chaotic, patterns. All discovered non-stable puffer trains have speed  $1/c$  and period four.

### 4.5 Mobile glider guns

Glider guns are localized patterns that periodically lose their stability and give birth to traveling mobile localizations, gliders. In computational experiments we discovered twelve types of mobile glider guns in the Diffusion Rule CA.

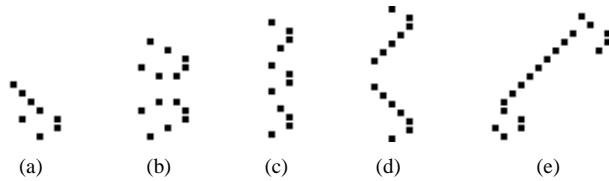


FIGURE 10  
Some non-stable puffer trains.

gun	produce	volume	translation	period	speed	weight	move
gun1	g1	30	4	4	$c/1$	7	orthogonal
gun2	g4	50	4	4	$c/1$	9	orthogonal
gun3	g4	50	4	4	$c/1$	9	orthogonal
gun4	g4	60	4	4	$c/1$	9	orthogonal
gun5	g4	60	4	4	$c/1$	9	orthogonal
gun6	g4	72	4	4	$c/1$	15	orthogonal
gun7	$g2 \wedge g3$	72	4	4	$c/1$	16	orthogonal
gun8	$2 \times g4$	80	4	4	$c/1$	14	orthogonal
gun9	$2 \times g4$	90	4	4	$c/1$	14	orthogonal
gun10	$g1 \wedge g4$	143	4	4	$c/1$	15	orthogonal
gun11	$2 \times g1$	154	4	4	$c/1$	24	orthogonal
gun12	$(2 \times g4) \vee g4$	176	4	4	$c/1$	19	orthogonal

TABLE 4  
Characteristics of glider guns in the Diffusion Rule CA. Second column of the table shows what type of glider each glider gun produces

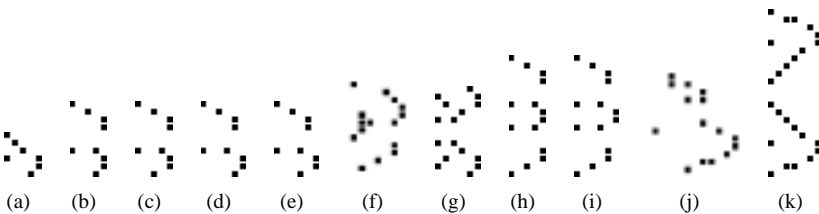


FIGURE 11  
Configurations of glider guns in the Diffusion Rule CA.

Basic parameters of the twelve glider guns are shown in Table 4 and gun's configurations in Fig. 11. The most remarkable feature is that all primary gliders can be produced by glider guns. Some glider guns can generate two types of gliders at once, thus **gun7** (Fig. 11f) generates **g2** and **g3** gliders at the same time, however both gliders travel coupled in pairs.

The generator **gun12** can produce one or two **g4** gliders, see examples in Fig. 12. This gun is also extendable, the extension is determined by the number of **o1** oscillators (which should be more than two). Positions of oscillators determine number of glider streams generated by **gun12**.

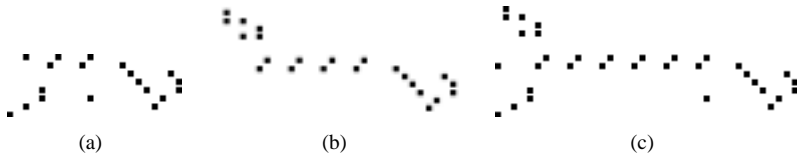


FIGURE 12  
Extendable glider gun in the Diffusion Rule CA.

So far we did not find glider guns which produce diagonally-moving streams of gliders, neither guns generating compound gliders or stationary guns.

**4.6 Glider gun and puffer train**

There is at least one special mobile structure that combines in itself properties of both glider gun and puffer train. Figure 13 shows glider gun producing **g1** glider and **O1** oscillator each 4th step of CA evolution. This puffer-gun moves orthogonally, has a volume of 70 cells and weights 12 units.

**4.7 Avalanche gun**

Avalanche gun is another remarkable example of mobile generators (Fig. 14). The mobile gun produces an avalanche every 4th step of CA evolution.

However, life-time of the gun producing each avalanche is short: when avalanche produced by the gun it grows and then destroys the next avalanche produced.

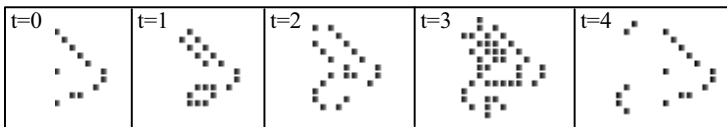


FIGURE 13  
Configurations of puffer-gun in the Diffusion Rule CA.

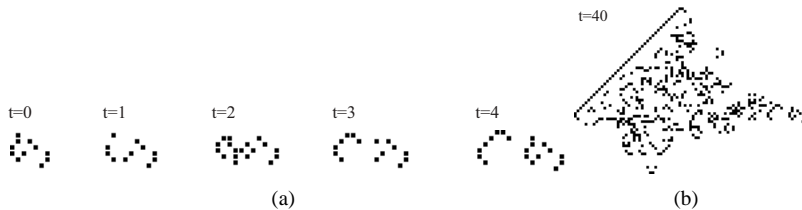


FIGURE 14  
Configuration of an avalanche gun in the Diffusion Rule CA. (a) production of two **g1** gliders to 90°, (b) development of quasi-chaotic reaction that destroys next avalanche conserving the avalanche gun.

## 5 COLLISIONS BETWEEN LOCALIZED PATTERNS

The Diffusion Rule CA combines high-degree unpredictability with enormously rich dynamics of collisions between mobile and stationary objects. This section studies the outcomes of the collision reactions.

### 5.1 Forming diffusing patterns by collisions

Gliders colliding in the Diffusion Rule CA can produce an explosively growing diffusion-like pattern, the diffusive patterns do usually have non-stationary boundaries and they exhibit quasi-chaotic internal dynamics.

Two most distinct examples are shown in Fig. 15. In first example (Fig. 15(a)), **g4** glider collides with **g3** and **g2** gliders, diffusion pattern produced is ‘lead’ by three gliders and puffer train. In second example (Fig. 15(b)), two **g4** gliders collides with two **g1** gliders. The reaction produces multiple gliders, puffer trains, oscillators, and even vertical glider guns during their collision dynamics.

### 5.2 Reactions between propagating patterns

#### 5.2.1 Soliton-like reaction

In certain initial conditions gliders collide similarly to solitons, namely they restore their structure and velocity vector after collisions. In Fig. 16 we can see snapshots of the head-on collision dynamics between two **g4** gliders (begin at even distance before collision). When the gliders collide they temporarily lose their stability, produce varieties of transient structures. After few steps of evolution the gliders are restored and transient structures are annihilated.

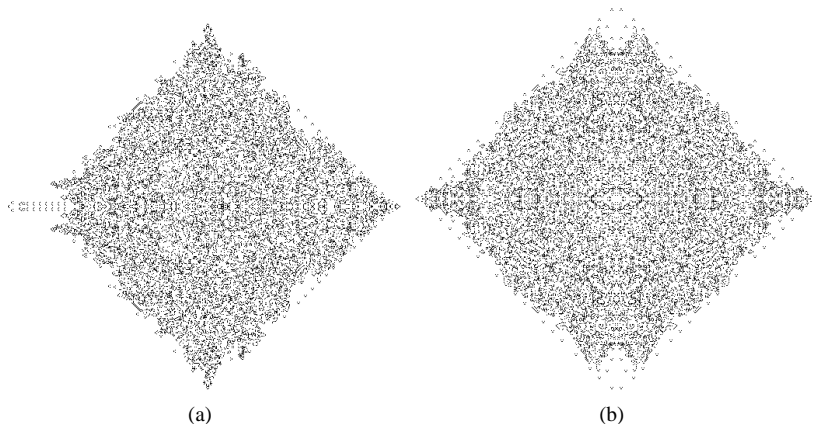


FIGURE 15  
Examples of diffusion-like patterns produced in collision between gliders in the Diffusion Rule CA. (a) pattern produced at 260th step of evolution after collision between **g4**, **g2** gliders and **g3** glider, (b) pattern produced at 260th of evolution after collision between two **g4** gliders and two **g1** gliders.

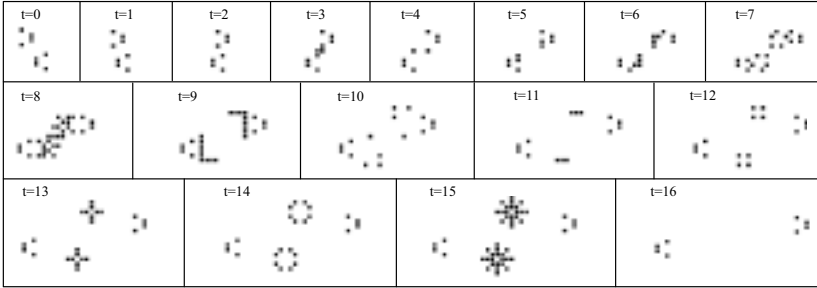


FIGURE 16  
Soliton-like reaction between two  $g_4$  gliders.

At the moment, there is only a reaction soliton-like perhaps some others examples exist with more bigger gliders but initially have not more examples.

### 5.2.2 Eater reaction

Eaters are stationary localizations which destroy gliders colliding into them. The most simple eater is built with  $o_1$  oscillator. This eater destroys  $g_1$  gliders but is shifted four cells along the glider’s initial direction of motion. Both  $g_1$  and  $g_2$  gliders are destroyed when they collide with the eater built of  $o_3$  oscillator. In Fig. 17(a) you can see an eater made of two  $o_3$  oscillators destroying  $g_1$  and  $g_4$  gliders. A ‘universal’ eater, which destroys all types of primary gliders is illustrated in Fig. 17(b).

Using eaters one can control glider streams emitted by glider guns, thus in Fig. 18 we can see how the eater eliminates  $g_1$  gliders produced by  $gun_1$ . There is at least one mobile eater of gliders, this is  $g_{24}$  glider that eats  $g_1$  gliders.

### 5.2.3 Delay reaction

The basic delay reaction can be implemented when  $g_3$  glider collides with  $g_4$  glider and changed to  $g_4$  glider in the result of collision, and the original

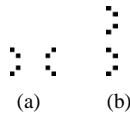


FIGURE 17  
Examples of eater configurations in the Diffusion Rule CA.

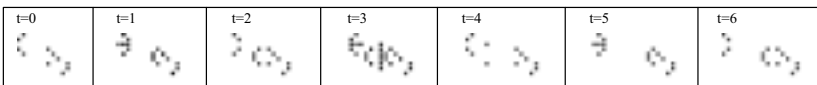


FIGURE 18  
Eater destroys gliders emitted by glider gun.



FIGURE 19

Delay reaction in the Diffusion Rule CA: (a) initial position of gliders before collision, (b) final result of delay operation, the glider traveling west delayed and translated southward.

**g4** glider delayed for two time steps but also translated southward one cell per collision (Fig. 19).

### 5.2.4 Multiplication and reduction reactions

A typical reaction of glider multiplication is shown in Fig. 20(a): two **g4** gliders are involved in head-on collision, with odd distance between glider heads before collision, four new **g4** gliders are produced in result of the collision. Reduction is implemented as multiplication of gliders, where gliders in the multiplied columns are in proximity of each other. As shown in Fig. 20(b), when we collide two rows of **g4** gliders, four gliders in each row (eight gliders in total) are involved in the collision, then just four new gliders are produced. Adjusting distance between gliders in colliding columns of gliders we can achieve almost any (but odd) result of multiplication (Fig. 20(c)).

Using multiplication reactions we can also construct arbitrary packages of gliders. For example, to construct a stream of packages, six gliders per package, we collide stream of four-glider **g4** packages traveling west with a pair of **g4** gliders traveling east (Fig 21(a)). Sequentially, all four-glider packages are transformed to six-glider packages (Fig. 21(b)), the operation is halted by pair **g3** gliders.

### 5.2.5 Reflection reaction

We discovered nine types of reflection-type collisions between stationary and mobile self-localizations. Let us discuss some examples shown in Fig. 22. When two **g1** gliders collide with each other (head-on collision, even distance, with slight shift between gliders along south-north axis) two **g4** gliders are

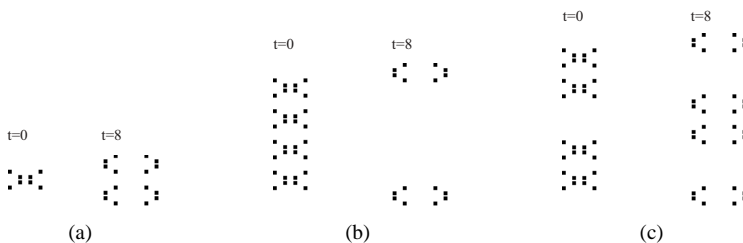


FIGURE 20

Multiplication of gliders: (a) multiplication  $1 \times 1 \rightarrow 4$  reaction of gliders in binary collision, (b) reduction  $4 \times 4 \rightarrow 4$  reaction in glider collision, (c) conservative collision reaction.



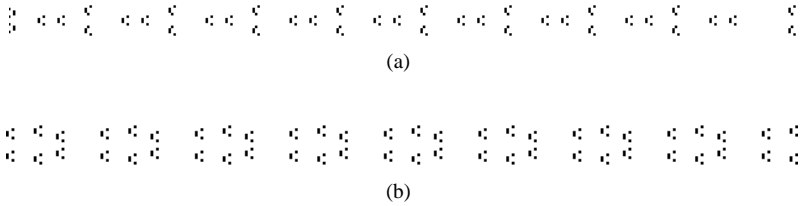


FIGURE 21  
Constructing packages of **g4** gliders by multiplication reaction. (a) initial configuration, (b) final configuration, recorded after 186th steps of evolution.

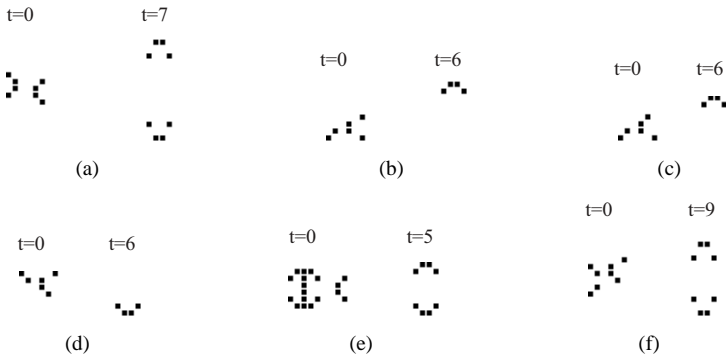


FIGURE 22  
Collisions leading to reflections: (a) two **g1** gliders collide with each other, (b) **g4** glider collides with **o1** oscillator, (c) **g2** glider collides with **o1** oscillator, (d) **g3** glider collides with **o1** oscillator, (e) **g1** glider collides with **o5** oscillator, (f) **g3** glider collides with **o3** oscillator.

generated; these **g4** gliders move in the direction perpendicular to original trajectories of colliding gliders (Fig. 22(a)). Glider colliding with **o1** oscillator is reflected at the angle  $90^\circ$ , as shown for collision of **g2**, **g4** and **g3** gliders with **o1** oscillator (Fig. 22(b) (c) (d)).

Gliders can also be derived in addition of reflections, when colliding with stationary localizations. Thus, when a **g1** glider collides with a **o5** oscillator, two **g1** gliders (going in opposite directions to each other) are generated and follow trajectories perpendicular to original trajectory of collided **g1** gliders (Fig. 22(e)).<sup>8</sup> Similarly, when **g3** glider collides with **o3** oscillator, two **g4** gliders are produced (Fig. 22(f)).

### 5.2.6 Annihilation reaction

Significant amount of collisions between localizations in the Diffusion Rule CA leads to annihilation of colliding patterns. Few examples of initial

<sup>8</sup>This reaction can synchronize multiple collisions as you can see in our example FANOUT.rle available from [http://uncomp.uwe.ac.uk/genaro/Diffusion\\_Rule/diffusionLife.html](http://uncomp.uwe.ac.uk/genaro/Diffusion_Rule/diffusionLife.html)

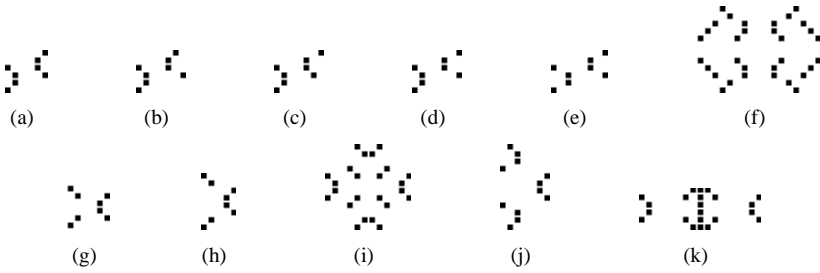


FIGURE 23

Initial positions of colliding gliders and oscillators leading to annihilation reaction. (a)–(f) binary collisions, (g)–(k) multiple collisions.

configurations of colliding objects (leading to annihilation) are shown in Fig. 23, for binary collisions between gliders (Fig. 23(a)–(f)) and multiple collisions between gliders and oscillators (Fig. 23(g)–(k)).

### 5.3 Computation in the Diffusion Rule

Basic operations necessary to implement a functionally complete set of logical gates can be derived from collision dynamics presented in Fig. 22. Following paradigms of collision-based computing [2] we encode logical TRUTH by presence of a glider or an oscillator, while absence of mobile or stationary objects corresponds to logical FALSITY.

Namely, Fig. 22(b) (c) (d) demonstrate that stationary localizations, oscillators, can play a role of mirrors thus deflecting gliders from their original trajectory. The mirrors can be used to route signals. Signals can be deleted, erased by placing eaters along trajectories of gliders, representing the signals. Signals can be also delayed in collisions with other gliders or oscillators.

Collisions used to construct FANOUT gate are shown in Fig. 22(e) (f), a glider collides to stationary localization, and two new gliders are produced in result of the collision.

Dynamics displayed in Fig. 22(a) shows a typical collision gate, where inputs  $x$  and  $y$  are represented by trajectories of gliders traveling East and West, respectively. While trajectories of new gliders, traveling South and North, encode value of  $x$  AND  $y$ . Constant TRUTH is made up of ceaseless stream of gliders, generated by glider guns.

#### 5.3.1 Constructing a memory device

A memory in the Diffusion Rule CA can be constructed using basic interactions between  $g1$  gliders and  $o1$  oscillator, as illustrated in Fig. 24. A bit of information is represented in the memory unit by shaded  $2 \times 2$  cells square in Fig. 24. The bit can be read by sending a  $g1$  glider to the memory unit (top configurations in Fig. 24, glider travels East). When  $g1$  glider collides with oscillator  $o1$  oscillator, both glider and oscillator are annihilated (and one

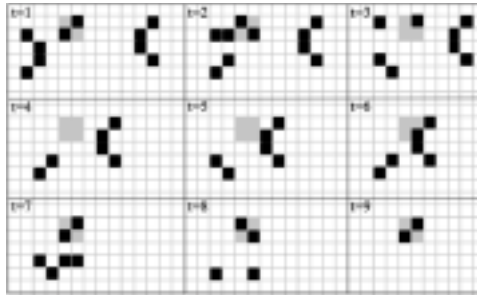


FIGURE 24  
 Constructing a memory device in the Diffusion Rule CA. Snapshots of the configurations. Time increases from right to left and from top to bottom. The domain where the bit of information (01 oscillator) is written to is represented by a shaded zone.

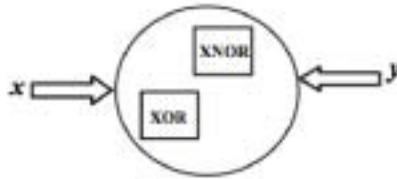


FIGURE 25  
 Scheme of XNOR and XOR gate.

new 01 oscillator is formed at some distance from the memory unit). Then the bit is erased as a result of the reading operation. To write down the bit one can send another g1 glider (in Fig. 24 this ‘writing’ glider travels West) toward now empty memory unit and associated 01 oscillator. Both g1 glider and associated oscillator are destroyed, however 01 oscillator is restored in memory unit (shaded region in Fig. 24), i.e., we write a bit again.

**5.3.2 Asynchronous XNOR and XOR gate**

Exploiting some features of the interaction between g1 glider and 01 oscillator (Fig. 24) we can implement an asynchronous device which calculates XNOR and XOR operation at once (Fig. 25). Such a gate is designed by a scheme similar to that outlined in [6]. Oscillator in position shaded by gray in Fig. 24 represents logical value TRUE and absence of the oscillator—value FALSE of logical operation XNOR, exclusive NOR operation. An auxiliary oscillator (generated when oscillator in shade position is annihilated) represents results of XOR operation, exclusive OR operation.

Assuming that signals on inputs  $x$  and  $y$  can be generated at any time (no synchronization) except not at the same time, we obtain the following

dynamics of the device (Fig. 25):

$x$	$y$	XNOR	XOR
0	0	o1	0
0	g1	0	o1
g1	0	0	o1
g1	g1	o1	0

where o1 and g1 stay for oscillator and glider, 0 means absence of the objects.

## 6 DISCUSSION

Findings discussed in the paper are based on computational experiments with the Diffusion Rule CA and, particularly, exhaustive search of mobile and stationary localizations emerged in spatio-temporal dynamics of the automaton.

Amongst known 2D CA supporting localizations, the Diffusion Rule CA is the minimal model because cell-state transitions depend not on intervals of ‘cell sensitivity’ but on singletons, i.e. transition  $0 \rightarrow 1$  occurs if there is exactly two neighbors in state 1, and transition  $1 \rightarrow 1$  if there exactly seven neighbors in state 1. Moreover, we are not aware of any other CA which exhibits so large variety of mobile localizations (gliders) and high diversity of outcomes of collisions between mobile and/or stationary localizations. Despite

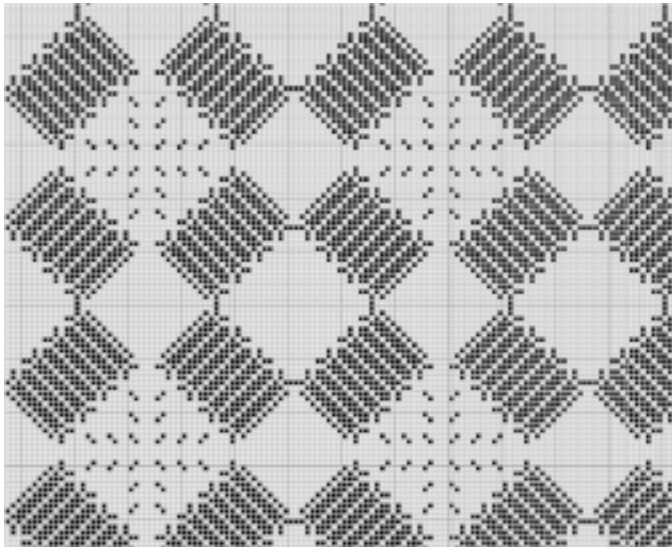


FIGURE 26  
Simulating a luminescence pattern with the Diffusion Rule.

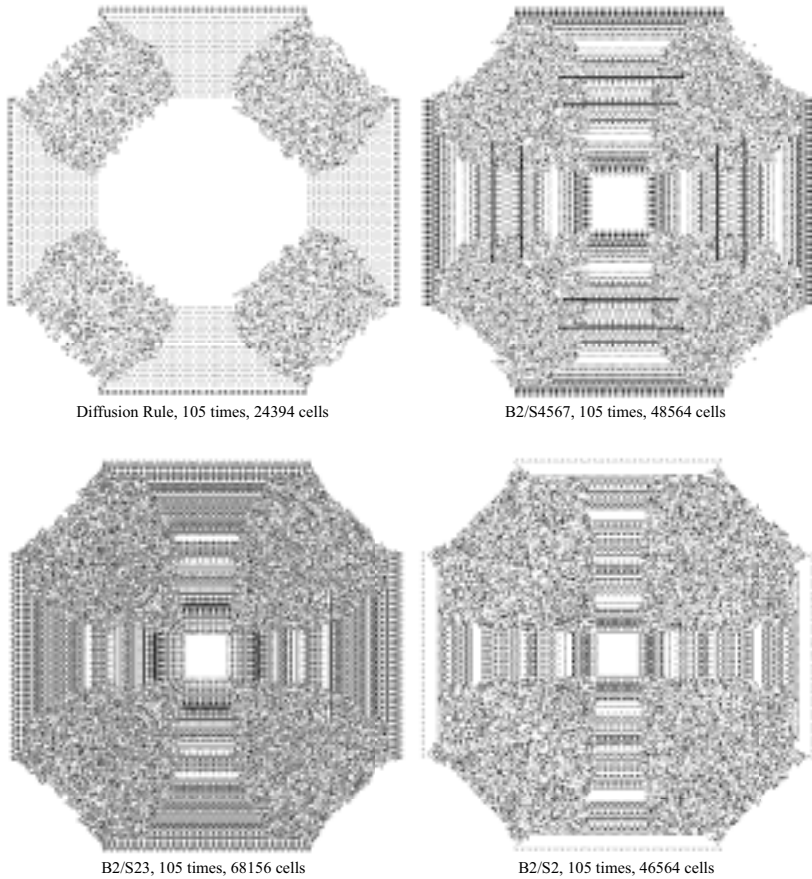


FIGURE 27

Virus propagation in the Diffusion Rule and three mutations in the same initial condition. Evolution rules and data are given below snapshots.

of trying to undertake exhaustive study of localization dynamics we nevertheless missed several important points that could form objectives of future studies.

A simple but interesting physical simulation was made setting a reaction like luminescence. Using packages of both diagonal lines of 50 cells everyone. The luminescence phenomenon was obtained over the evolution in densities of cells in small intervals as illustrated the Fig. 26. The final state is dominated by blinkers or oscillators. This simulations are developed by the group of researchers iGEM-México at the MIT.<sup>9</sup>

<sup>9</sup><http://www.fenomec.unam.mx/pablo/igem/>

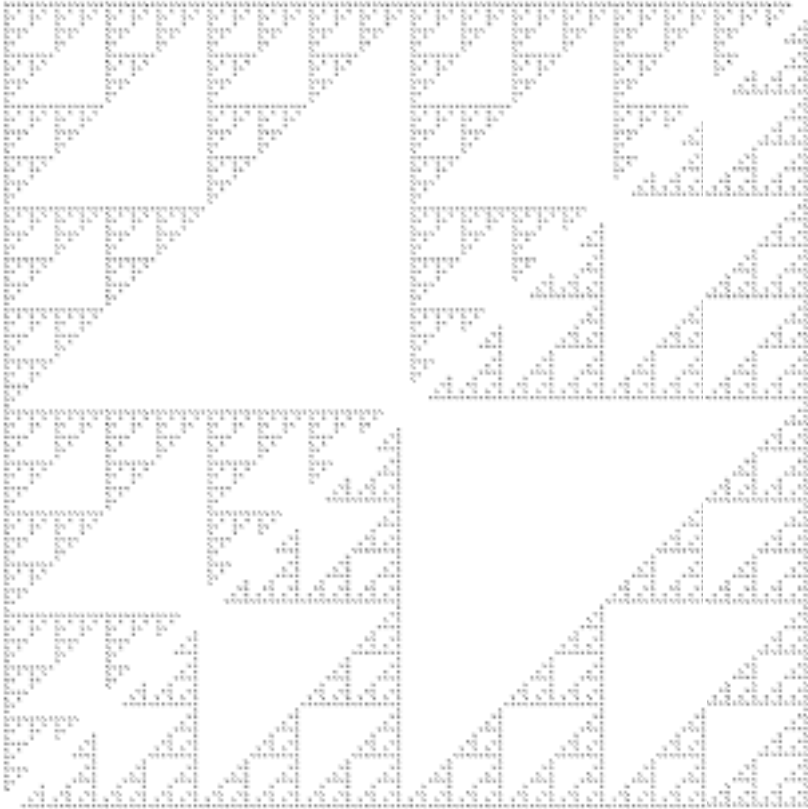


FIGURE 28  
 Simulating the evolution of a cell of the ECA Rule 18 or Rule 90 with the Diffusion Rule.

It is not a trivial problem to find large stable patterns constructed with big complex structures. Cell colonies damaged by a virus is one of such configurations (Fig. 27).

Diffusion Rule can simulate the evolution of a cell like the elemental cellular automata (ECA) Rule 18 or Rule 90. For example, we take a diagonal line with 503 cells in the initial condition. During the evolution original line is multiplied in lines less and less small producing oscillators. Finally, the evolution space is dominated by oscillators that represent exactly a cell alive in 1D case. However, there is generated a second evolution of the same type as illustrated in Fig. 28. In this example, it took 512 steps of evolution to reach the configuration of 7,596 living cells.

The existence of stable configurations seems difficult to find in a rule which is generally chaotically producing super-nova explosions. Nevertheless, we have an example where four glider guns are synchronized to annihilate the

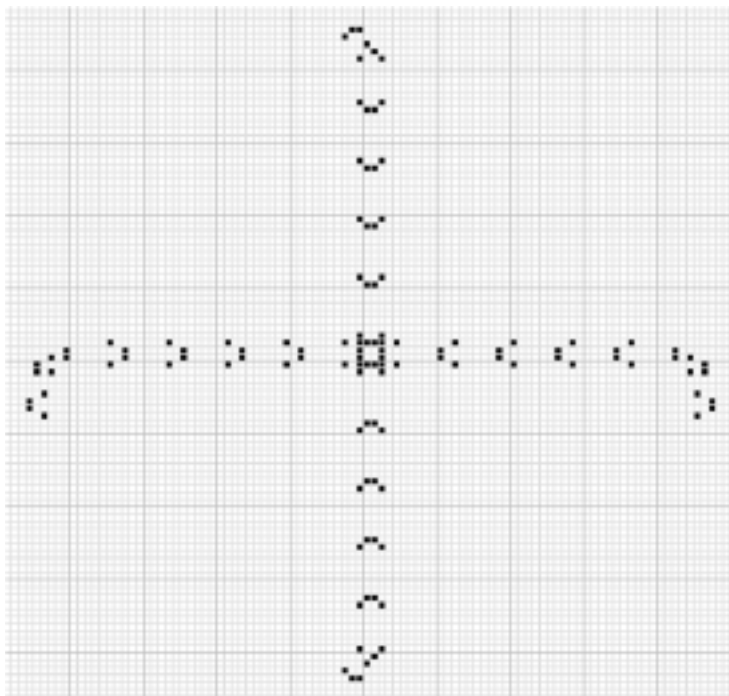


FIGURE 29  
 Simultaneous annihilation across of four particles produced by four glider guns in the Diffusion Rule. The evolution shows a global configuration in 36 steps with 118 live cells.

gliders. In this case, two  $g1$  gliders and two  $g4$  gliders come into quadruple collision shown in Fig. 29.

We envisage that important open problems to be solved include implementation of quasi-chemical reactions between gliders, studies of grammars derived and implementation of a full effective decision procedures based on glider collisions. It will be also worth to demonstrate intrinsic universality and self-reproduction. Another project would be to use de Bruijn diagrams [29] to check if there are any still undiscovered gliders with velocity one or still life configurations, and besides to use algorithms specialized in automatic search for complicated or big gliders [15], oscillators, glider guns or more configurations. Also we are planning to make an exploration of the cluster of semi-totalistic rules originated by the Diffusion Rule.<sup>10</sup> Finally, the last but not least open problem is to decide if all types of gliders can be constructed in collision with other gliders, a closure property with respect to set of gliders.

<sup>10</sup>[http://uncomp.uwe.ac.uk/genaro/Life\\_dc22.html](http://uncomp.uwe.ac.uk/genaro/Life_dc22.html)

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## REFERENCES

- [1] Adamatzky, A. (2001) *Computing in Nonlinear Media and Automata Collectives*, Institute of Physics Publishing, Bristol and Philadelphia.
- [2] Adamatzky, A. (Ed.) (2003) *Collision-Based Computing*, Springer.
- [3] Adamatzky, A., De Lacy Costello B., & Asai T. (2005) *Reaction-Diffusion Computers*, Elsevier.
- [4] Adamatzky, A., Martínez, G.J., & Seck Tuoh Mora, J.C. (2006) Phenomenology of reaction-diffusion binary-state cellular automata, *Int. J. Bifurcation and Chaos* **16** (10) 1–21.
- [5] Adamatzky, A., Wuensche, A. & De Lacy Costello, B. (2005) Glider-based computing in reaction diffusion hexagonal cellular automata, *Chaos, Solitons & Fractals*.
- [6] Adamatzky, A. & Wuensche, A. (2006) Computing in spiral rule reaction-diffusion cellular automaton, *Complex Systems* **16** (4).
- [7] Bays, C. (1987) Candidates for the game of life in three dimensions, *Complex Systems* **1** 373–400.
- [8] Bays, C. (1991) New game of three-dimensional life, *Complex Systems* **5** 15–18.
- [9] Bell, D.I. (1994) High Life—An interesting variant of life, <http://www.tip.net.au/~dbell/>.
- [10] Berlekamp, E.R., Conway, J.H., & Guy, R.K. (1982) *Winning Ways for your Mathematical Plays*, Academic Press, (vol. 2, chapter 25).
- [11] Boccara, N., Nasser, J., & Roger, M. (1991) Particle like structures and their interactions in spatio-temporal patterns generated by one-dimensional deterministic cellular automaton rules, *Physical Review A* **44** (2) 866–875.
- [12] Chaté, H. & Manneville, P. (1991) Evidence of collective behavior in cellular automata, *Europhysics Letters* **14** 409–413.
- [13] Cook, M. (2004) Universality in elementary cellular automata, *Complex Systems*, **15** (1) 1–40.
- [14] Das, R., Mitchell, M. & Crutchfield, J.P. (1994) A genetic algorithm discovers particle-based computation in cellular automata, *Lecture Notes in Computer Science* **866** 344–353.
- [15] Eppstein, D. (2002) Searching for spaceships, *MSRI Publications* **42** 433–452.
- [16] Evans, K.M. (2003) Replicators and larger-than-life examples, (in ref. [19]).
- [17] Gardner, M. (1970) Mathematical Games—The fantastic combinations of John H. Conway's new solitaire game Life, *Scientific American* **223** 120–123.
- [18] Griffeath, D. & Moore, C. (1996) Life Without Death is P-complete, *Complex Systems* **10** 437–447.
- [19] Griffeath, D. & Moore, C. (Eds.) (2003) *New constructions in cellular automata*. (Santa Fe Institute Studies on the Sciences of Complexity) Oxford University Press.
- [20] Gutowitz, H.A. & Victor, J.D. (1987) Local structure theory in more than one dimension, *Complex Systems* **1** 57–68.
- [21] Hanson, J.E. & Crutchfield, J.P. (1997) Computational mechanics of cellular automata: an example, *Physics D* **103** (1–4) 169–189.



- [22] Heudin, J.-K. (1996) A new candidate rule for the game of two-dimensional life, *Complex Systems* **10** 367–381.
- [23] Jakubowski, M.H., Steiglitz, K., & Squier, R. (2001) Computing with Solitons: A Review and Prospectus, *Multiple-Valued Logic*, Special Issue on Collision-Based Computing **6** (5–6).
- [24] Martínez, G.J. (2000) Teoría del Campo Promedio en Autómatas Celulares Similares a The Game of Life, Tesis de Maestría, CINVESTAV-IPN, México.
- [25] Martínez, G.J., Adamatzky, A. & McIntosh, H.V. (2006) Phenomenology of glider collisions in cellular automaton Rule 54 and associated logical gates, *Chaos, Fractals and Solitons* **28** 100–111.
- [26] Martínez, G.J., McIntosh, H.V., & Seck Tuoh Mora, J.C. (2006) Gliders in Rule 110, *Int. J. Unconventional Computing* **2** (1) 1–49.
- [27] McIntosh, H.V. (1988) Life's Still Lives, <http://delta.cs.cinvestav.mx/~mcintosh>
- [28] McIntosh, H.V. (1990) Wolfram's Class IV and a Good Life, *Physica D* **45** 105–121.
- [29] McIntosh, H.V. (1994) Phoenix, <http://delta.cs.cinvestav.mx/~mcintosh/oldweb/pautomata.html>
- [30] McIntosh, H.V. (1999) Rule 110 as it relates to the presence of gliders, <http://delta.cs.cinvestav.mx/~mcintosh/oldweb/pautomata.html>
- [31] Minsky, M. (1967) *Computation: Finite and Infinite Machines*, Prentice Hall.
- [32] Mitchell, M. (2001) Life and evolution in computers, *History and Philosophy of the Life Sciences* **23** 361–383.
- [33] Magnier, M., Lattaud, C., & Heudin, J.-K. (1997) Complexity Classes in the Two-dimensional Life Cellular Automata Subspace, *Complex Systems* **11** (6) 419–436.
- [34] Tommaso, T. & Norman, M. (1987) *Cellular Automata Machines*, The MIT Press, Cambridge, Massachusetts.
- [35] von Neumann, J. (1966) *Theory of Self-reproducing Automata* (edited and completed by A. W. Burks), University of Illinois Press, Urbana and London.
- [36] Sediña-Nadal, I., Mihaliuk, E., Wang, J., Pérez-Mufuzuri, V., & Showalter, K. (2001) Wave propagation in subexcitable media with periodically modulated excitability, *Phys. Rev. Lett.* **86** 1646–49.
- [37] Wolfram, S. (2002) *A New Kind of Science*, Wolfram Media, Inc., Champaign, Illinois.
- [38] Wuensche, A. (1999) Classifying cellular automata automatically, *Complexity* **4** (3) 47–66.
- [39] Wuensche, A. (2004) Self-reproduction by glider collisions: the beehive rule, *Alife9 Proceedings*, 286–291, MIT Press.

