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Automated Construction of Variable Density Navigable Networks in a 3D Indoor Environment for Emergency Response

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2 Indoor Environment for Emergency Response

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8 Abstract

- 9 Widespread human-induced or natural threats on buildings and their users have made
- 10 preparedness and rapid response crucial issues for saving human lives. The ability to identify the
- paths of egress during an emergency is critical for rescue and emergency services. Quality models
- 12 supporting real, or near-real, time decision making and allowing the implementation of automated
- methods are very important. In this paper, we propose a novel automated construction of the
- 14 Variable Density Network (VDN) for determining egress paths in an emergency dangerous
- 15 <u>environments</u>. VND is used for deriving a navigable network in an indoor building environment,
- including a full 3D topological model. The accuracy of the proposed paths prediction tool was
- 17 compared with key methods for navigable network generation, using the actual floor plan of Doha
- 18 World Trade Centre. Findings revealed that in comparison to prevailing approaches, a key benefit
- 19 from this approach is an increased prediction accuracy of egress route planning.
- 20 **Keywords**: navigable networks, indoor navigation, emergency response, 3D modelling, topological
- 21 models

22 1. Introduction

- 23 Destructive disasters caused by events such as fire, storm, or explosion result in damages and
- 24 structural instability of buildings. Such disasters are rare but inevitable, and in the urban
- 25 environment with high-rise and big public buildings they lead to casualties and occupants trapped
- inside. In this type of crisis, the challenge is to assist the search and rescue personnel in quickly
- 27 preparing a plan to locate and rescue the surviving victims. Disasters such as the fire at Villagio
- shopping mall in Doha in May 2012, resulted in 19 people killed including 13 children, are reminders
- 29 that effective preparedness and response might eliminate or reduce human errors and panic leading
- into the loss of lives (Scharfenort, 2012).
- 31 Tashakkori et al. (2015) emphasized the importance of situation awareness available for first
- 32 responders in emergency response systems. The knowledge about indoor structure, occupancy or
- 33 localization of fire utilities is minimal prior their arrival to the emergency scene. Thus, assessment of
- 34 the scene and wrong decisions increase the response time. Therefore, actual and constantly updated
- information about the indoor situation, close outdoor area and optimal routes to selected points
- inside a building is crucial. This information includes, to name a few: floor plans, material types,
- 37 location of hazardous materials, occupancy information, space ownership, location of water
- 38 resources, fire and emergency utilities, etc.

39 In an emergency, up-to-date information is needed for coordination, communication and efficient

40 decision making (DeCapua and Bhaduri, 2007). Availability of information about the indoor structure

- 41 is critical for simulation of emergency movement of people and egress way finding for individual
- 42 pedestrians and groups of people (Nelson and MacLennan, 1995; Pauls, 1995). Usually the focus is
- put on evacuation paths, flow of people and time necessary to leave a building (Kuligowski, 2004).
- 44 However, the opposite paths, from outdoor to trapped people at certain locations in a building are
- 45 taken by search and rescue teams. In such a scenario, a guick location of first responders and
- occupants is of great importance in search and rescue operations (Li et al., 2014). In the case of
- 47 emergency, standard path finding using normal navigation routes may not be sufficient, as they may
- 48 be too risky or not available due to damage (Liu and Zlatanova, 2012). In these circumstances, the
- 49 ability for emergency and rescue personnel to identify alternative and optimal paths becomes
- 50 critical.

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- 51 Zverovich et al. (2016) proposed a new method for calculating safe paths taking hazard proximity
- 52 into consideration. Standard navigable networks are not sufficient to perform hazard analysis. They
- 53 include only connections available for pedestrians, whereas topological relations between adjacent
- rooms without doors in-between are not reflected. This applies not only for adjacency in horizontal
- direction but also vertical, through slabs. However, these connections are important in simulation of
- several hazard types, such as fire spread or explosion impact. Thus, 3D properties included in a
- 57 model, e.g. spatial relationship in all directions, are essential for emergency response applications
- which go beyond evacuation path-finding.
- 59 This paper reports on the development of a new automated method for deriving a navigable
- 60 network in a 3D environment, including full 3D topological model, which may be used for finding
- alternative egress routes and simulating phenomena associated with emergency response situation.
- The motivation of this research is to provide better situation awareness to the first responders prior
- 63 to their arrival to the emergency scene.

1.1. Navigable models

- 65 Available navigable models in emergency response research focus on standard path finding using
- doors and corridors (Goetz and Zipf, 2011; Kwan and Lee, 2005; Liu and Zlatanova, 2012; Vanclooster
- et al., 2014; Yang and Worboys, 2015). Most of these approaches retrieve a navigable model from
- 2D floor plans, while the 3D properties of the original environment are not always reflected in the
- 69 model; even when the original 3D model is available. Very often, adjacent floors are connected only
- by vertical connections representing staircases, and spatial relationships between adjacent rooms
- sharing a ceiling/floor or a wall without a door are not reflected in these models. The resulting
- 72 networks are often simplified in order to reduce the storage cost and improve path-finding
- 73 computation time, which makes them unsuitable for alternative path finding or simulation of
- 74 phenomena related to the emergency.
- 75 Various researchers propose more sophisticated models (Lamarche and Donikian, 2004; Lee, 2004;
- 76 Liu and Zlatanova, 2012; Stoffel et al., 2007; Wenjie and Schneider, 2010), where a complex network
- 77 is generated for special spaces intensively used for pedestrian navigation such as corridors or
- 78 complex open spaces. This improves the navigable network within a building, because the original
- 79 logical network, which reflects connections among adjacent rooms, is not an appropriate
- 80 representation of navigation routes. The proposed automated methods are based on door and

concave corner locations and visibility (Liu and Zlatanova, 2012), Straight Medial Axis Transformation (Lee, 2004), Delaunay Triangulation spatial subdivision (Krūminaitė and Zlatanova, 2014; Lamarche and Donikian, 2004), and convex sub-regions partitioning (Stoffel et al., 2007). These methods often require input models with preserved consistency between the geometry and topology. In the case of incomplete of inaccurate floor plans, a network model may be interactively generated (Luo et al., 2014), but this may not be efficient in the case of emergency situations, when models for decision support should be available in a real or near-real time.

Indoor navigation including emergency navigation models have to meet several requirements (Brown et al., 2013; Wang and Zlatanova, 2013). Indoor spaces and their geometry included in an original building model should be classified based on predefined types and represented in the navigable model. Sub-division of spaces is one of the required functionalities, which makes the model suitable for accurate routing computation. Semantic information defining function, use or occupancy should be also reflected and attached to the relevant spaces. Static and moving objects that are obstacles for navigation are another elements, which should be incorporated in the model. Finally, connectivity among spaces based on their type has to be derived. These requirements are the most fundamental and are usually supplemented by other, more specific rules.

A slightly different approach to navigable network generation is utilized in robotic motion planning (Wallgrün, 2005). It is based on the Voronoi Diagram (VD) (Aurenhammer, 1991), where vertices and edges of the VD are used respectively to represent nodes and links in the network. This method may be used in an unknown environment, where the network is dynamically and automatically updated while the indoor scene is explored. Hierarchical structures are used to store networks in different granularity: from the coarsest to more detailed networks reflecting geometric details of the indoor environment. Different scales are used depending on required precision of path-planning.

Higher level of granularity may be achieved through a network densification similar to mesh densification utilized in the finite element mesh generation and refinement (Du and Gunzburger, 2002; Ho-Le, 1988; Petersen and Martins, 1997). Usually, mesh elements, e.g. triangles, are divided into smaller elements based on their centroids and original edges until the required level of granularity is achieved. Different criteria may be considered: the area of a mesh element or edge length. The size of the element is determined in a process (often iterative) minimizing an error between the exact solution and its finite element approximation (Du and Gunzburger, 2002; Dyck et al., 1992). A denser mesh may be generated locally in the vicinity of selected points (Chedid and Najjar, 1996).

Another field where densification of mesh is applied is geo-information: Triangulated Irregular Network (TIN), which is used to represent a digital model of terrain generated from point clouds and obtained from laser scanning. TIN is often densified using the Progressive TIN Densification (PTD) method (Axelsson, 2000; Zhang and Lin, 2013). New points are interpolated into a mesh representing the terrain which triggers division of a selected mesh triangle into smaller parts. The selection of a new point and the right triangle is based on specialized filters. Point candidates that do not belong to the terrain (e.g. buildings, trees, etc.) are calculated from their distance to TIN, and an angle between a mesh facet and a line connecting the point with the closest vertex of the facet.

122 Other models for indoor navigation systems are presented by Afyouni et al. (2012). The biggest 123 drawback of these methods, in the context of applications other than standard indoor navigation, is 124 that not all spatial relations among adjacent rooms are available in the model. Usually, 2D plans are 125 used to reconstruct a navigable network within an individual building level. A link between two 126 rooms is created only if they share a door. Otherwise, where adjacent rooms share a slab or wall 127 without doors, adjacency relationship is not reflected in a model. Navigable networks for 128 consecutive floors are connected at certain points by additional links representing staircases (Lee 129 and Kwan, 2005). 130 However, some studies sought to fully utilise the 3D properties available from model reconstruction (Becker et al., 2009; Boguslawski et al., 2011). Their methods are based on the Poincaré duality: in 131 132 practice the geometry of a volume is represented in the primal space using the boundary 133 representation (B-Rep) (Stroud, 2006), while a volume object (e.g. a room) is represented as a dual 134 node. A dual representation of a facet (e.g. a wall) is a dual edge bounded by dual nodes 135 representing adjacent rooms – a link. Therefore, the dual structure is a graph of connections among 136 3D objects. Attributes assigned to links may determine the navigability of the connections. These 137 links with attributes form a sub-graph of the initial graph of connections and may be called a 138 network. 139 The dual half-edge (DHE) proposed by Boguslawski (2011) is one of few data structures, which are 140 able to simultaneously store both the primal and the dual graphs. The construction of a model is 141 realized using CAD-like operators - Euler operators - where only the primal structure is explicitly 142 constructed while the dual is automatically updated (Boguslawski and Gold, 2010). Figure 1 provides 143 an example of a building represented with the DHE. In the model, doors and windows are 144 represented as zero-volume objects with associated dual nodes, which are part of the graph; 145 windows, which are embedded in a wall, are connected to the wall boundary using 'bridge' edges in 146 order to fulfil B-Rep requirements and avoid disconnections in the graph. A characteristic property of 147 the DHE model is the external volume enclosing the indoor model, which represents outside space. 148 The indoor model can be combined with the terrain model or transportation network by the 149 external cell (Boguslawski and Gold, 2015). The external volume is also considered in automatic exit 150 detection: a door adjacent to internal and external volumes is marked as an exit.

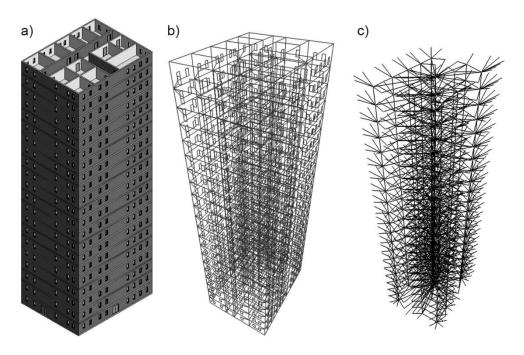


Figure 1. A 3D model of a building represented with the DHE data structure: a) an original BIM model; b) the DHE primal structure; c) the DHE dual graph of connections.

The dual graph, which reflects the adjacency relation between rooms, may be used to determine all the neighbours of a certain room. This graph is a network: rooms are represented by dual nodes; adjacent rooms are connected by a link; links can have different weights (i.e. attributes) associated with each direction. The network may be used for indoor navigation: paths between two locations can be calculated using graph algorithms (Boguslawski, 2011). A significant limitation of this representation is its applicability only to simple models with 'cubical' rooms. Complex rooms need to be manually partitioned into smaller sub-spaces with a specific configuration of doors in order to produce a network suitable for accurate indoor navigation. This issue will be addressed in the following section.

One of the advantages of the DHE representation is that not only links between adjacent rooms in the horizontal direction are present in the model, but all possible adjacency relations are reflected including those in the vertical direction. This is an important property, which is primarily used in hazard propagation simulation. The possibility of vertical navigation through staircases is automatically included in the model. Additionally, different weights for path-finding algorithms may be attached to links going upstairs and downstairs.

Regardless of a method used for navigable network generation, in order to share the resulting network and exchange between different applications, it should be represented in a common schema. The IndoorGML (OGC, 2014) is a new standard designed for indoor navigation applications, which complements other standards such as CityGML, KML and IFC. It consists of two main data models: topology of indoor environment and indoor navigation, which describe a network topology in built environment.

1.2. Problem statement

Figure 2a illustrates a close-up of a single level, the ground floor, in the building shown in Figure 1.

The corridor (the grey area) is connected to adjacent rooms by doors and to the outdoors by exits A-

C; there are three staircases S1-S3. The logical network representing connections among rooms at the ground floor is shown in Figure 2b. Other vertical connections between adjacent rooms at different levels are also included in the logical network (see Figure 1c). In a non-emergency scenario, only standard navigation routes via doors are used. These routes are represented by a sub-network of the logical network shown in Figure 2c.

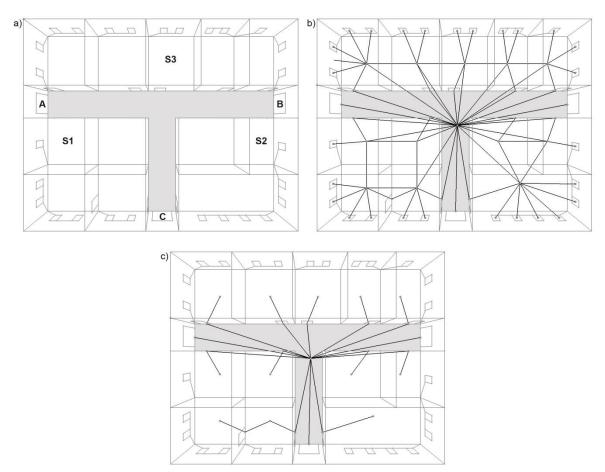


Figure 2. Ground floor of the building represented with the DHE: a) grey area represents a corridor with three exits A-C; S1-S3 are staircases; b) logical network; c) sub-network reflecting navigable connections through doors.

The network can be effectively used for navigation routes in simple models, e.g. when it consists of 'cubical' rooms with one door or two doors located on opposite walls. A problem arises when rooms have a complex shape and many doors (e.g. long corridors, concave rooms, big open spaces). If the shortest path from a room to the closest exit goes through a corridor node, located in the geometric centre, an incorrect exit may be selected (see Figure 3a). In reality, the shortest path between the graph nodes is calculated properly, but navigation routes within a corridor are not represented correctly. In addition, if the path finding algorithm is forced to select the correct closest exit (A in Figure 3b), then the path is not a 'natural' way of navigation – no one goes to the centre of a corridor first and then takes the correct door. A solution to the problem is to enhance the model by providing a better representation of 'natural' ways of indoor navigation.

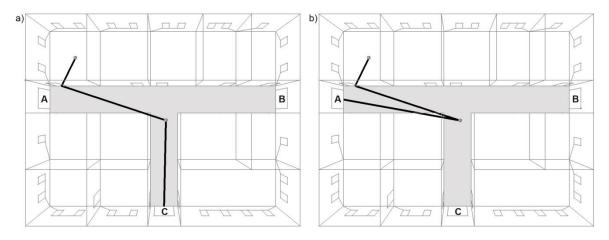


Figure 3. Paths (black lines) from a selected room through the corridor (grey area) to one of the exits (A-C are exits from a building): a) C is wrongly calculated as the closest exit; b) A is the closest exit but the path is not a 'natural' way of navigation.

The closest model is the one proposed by Liu and Zlatanova (2012), where the logical network representing a building structure is combined with navigable networks within single rooms: the network generation algorithm is based on the location of doors and concave corners. However, the navigable network is built from a spatial subdivision, the Voronoi tessellation, which is related to the method proposed by Lee (2004).

In this paper, a novel approach of navigable network generation, the Variable Density Network (VDN), based on the Voronoi Diagram is proposed. This approach allows a new automated method for deriving a navigable network in a 3D indoor environment, including a full 3D topological model, which may be used not only for standard navigation but also for finding alternative egress routes and simulating phenomena associated with disasters such as fire spread and heat transfer. The main application for the proposed network is calculation of egress paths in a dangerous environment to assist rescue teams, thus, evacuation of people and related topics, such as determination of position, route tracking, crowd simulation, human and social behaviour, are not taken into consideration. This can be a part of a bigger emergency response system.

The main focus of this research is a decision support system to assist emergency teams to determine egress paths in the event of emergency, thus, evacuation of people and related topics, such as crowd simulation, human and social behaviour, are not taken into consideration.

3.2. Methodology

In this research, two mock data models and a real-life case study were created in order to illustrate the proposed new method of navigable network generation (VDN). The first model is a high-rise office building with cubical rooms connected to a centrally located corridor (see Figure 4a). It is used to illustrate how the method works in narrow corridors of a complex shape with several doors to adjacent rooms. The second model is a two-storey building with a central big open space (see Figure 4b). It is used to show applicability of the same method to open spaces. The third model represents a floor plan of the Doha World Trade Centre in Qatar. This model is selected to carry out a comparative study between the proposed VDN and other prevailing models. The models were built in the Autodesk Revit design environment and exported to the gbXML data format. A gbXML file was imported and analysed in in-house software developed for testing purposes. Models are

represented using the DHE data structure, which is an integral part of the in-house system. Some models may require validation in order to preserve a consistency between geometry and topology, which in particular concerns detection and fixing two problems: missing faces and overlapping between adjacent rooms. The workflow is shown in Figure 5.

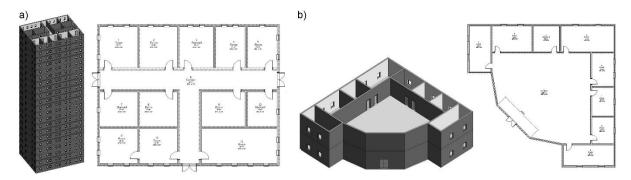


Figure 4. Mock data models – 3D views and ground floor plans: a) high-rise building; b) two-storey building.

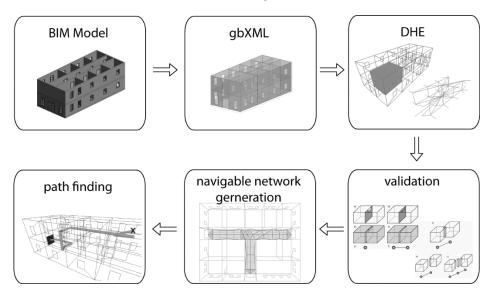


Figure 5. Method workflow.

The model enhancement, proposed in this paper, introduces a novel network for indoor navigation in spaces along navigation routes, such as corridors, big open spaces, rooms with several doors. A floor is tessellated using a VD construction algorithm, where doors and concave corners are initial Voronoi vertices (nucleation points), and additional points are added in order to create a denser tessellation. The nucleation points from adjacent cells are connected by links, thus forming a network, which is better suited for navigation, than the network without such innovation.

There are several advantages to this strategy: 1) path finding algorithms for the additional network give more accurate results within a single room; 2) the additional network is generated using a 2D algorithm and does not affect the original network representing the 3D topology of a building; 3) the same method may be applied for corridors and big open spaces of an arbitrary shape; 4) 'natural' paths for human movement are reflected in the network.

The proposed method consists of several steps: A) spaces (we will call them rooms in the following description) with more than one door are selected; B) the floor of the selected room is detected based on semantic information from the original BIM model or, if this information is not available, then surfaces with the normal vector pointing upwards are considered as a floor; C) dual nodes representing doors are projected on the floor and together with concave vertices of the floor surface are selected as nucleation points for a tessellation algorithm – they are called 'constraint nucleation points' in the following description; D) the VD algorithm is performed iteratively and additional points are added; the nucleation points of neighbour cells are connected by links forming a network; E) the locations of additional nucleation points occurring on constraint edges are moved towards the geometric centre of the associated cell, while the original location of the constraint points is not changed.

The process of the VD construction (see Step D above) requires a detailed description. An iterative algorithm for the constraint VD was implemented based on the Green and Sibson algorithm (Green and Sibson, 1978), see Figure 6.

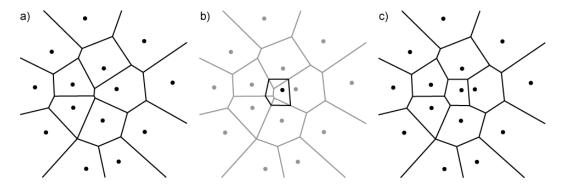


Figure 6. Process of Voronoi Diagram construction: a) existing tessellation; b) new vertex and its Voronoi cell; c) tessellation after point insertion.

The floor is represented by a polygon of an arbitrary shape. The original polygon edges are constraint edges, which are not modified in the tessellation process. In the first iteration dual nodes representing doors, projected on the floor and concave corners, are selected as constraint nucleation points. The locations of the constraint points are not changed later. The nucleation point is always enclosed by a single Voronoi cell. Subsequently, each node is connected by links with nodes from adjacent cells; all these links and nodes form a network. In the next step, new nucleation points are added at the middle of each link if the link is bounded by constraint points, or if both bounding points lie on constraint edges, which are not co-linear. The process is repeated until no new points may be added because they would be too close, or a certain number of iterations have been reached.

An algorithm for VD construction is presented below as Algorithm 1. In the examples presented in this paper, the following input parameters were used: the threshold T_1 =6 (see Figure 7c and Figure 8b), the threshold T_2 =0.7 (see Figure 9) and the maximum tessellation level TL_{max} =10. T_2 is used if an edge e in the tessellation is close to the hazard location. In this paper, e is considered to be close to the hazard location if length(e)×3 > distance(v, v) + distance(v, v), where: v and v are end-points of v and v is the hazard location.

- 283 For the sake of algorithm clarity, only one polygon representing a floor is considered. If a floor is
- represented as a set of adjacent polygons in an original model, they may be merged into one and the
- same algorithm can be used.
- 286 Algorithm 1: Navigable network generation (tessellation of a floor)
- 287 **Input:** The cell representing room *R*.
 - The floor polygon for R.
 - T_1 , T_2 (threshold values for edge length).
- 290 Maximum tessellation level TL_{max}
- 291 **Output:** Tessellation *f*.

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- **1.** Make a copy of the floor polygon, denoted by *f*.
- 293 **2.** Calculate the number of doors in room R, denoted by d. If $d \le 1$, then stop the algorithm (no network generation is necessary).
- **3.** For each door dual node, make its projection on the boundary of *f*.
- 4. Create a list *N* consisting of constraint nucleation points for VD in order of their appearance on the boundary of *f*. The constraint nucleation points are door nodes and concave corner nodes. *f*.
- 5. Create a list V of VD cells and put $V_1=f$. (The floor polygon f is the initial Voronoi cell associated with N_1 .)
- **6.** Put TL=1, where TL is the current tessellation level.
- 302 **7.** Take the first unprocessed element from N, denoted by N_i .
- 303 **8.** Create a list of cells for testing, denoted by CT and add to it a cell from V, which includes node N_i
- **9.** Take the first unprocessed element from *ctest* and denote it by V_k (it is associated with N_k).
- **10.** Calculate a bisector line *bi* of the segment (N_i, N_k) .
- 306 **11.** Calculate intersection points of *bi* with V_k . The set of such points is denoted by *IP*.
- **12.** If the number of points in *IP* is greater than 2, then do the following:
 - **a.** In *I* find two intersection points whose line segment divides the cell V_k into two parts such that each part includes one of the nodes N_i and N_k ;
 - **b.** Remove the rest of the intersection points from *IP*.
- **13.** If the number of points in *IP* is less than 2, then go to Step 9.
- 312 **14.** Create the edge (j,k) bounded by intersection points from IP, and insert it into the existing tessellation of the polygon f. The edge (j,k) divides V_k into two cells: C_j and C_k such that N_j is located in C_j and N_k in C_k .
- 315 **15.** Add C_j to V_j i.e. $V_j = C_j$, and put $V_k = C_k$. (The edge (j,k) is a part of the boundary of the newly created cell for N_j .)
- **16.** Add those cells to the list *CT*, which are adjacent to the edges intersected by the edge (*j*,*k*), taking into account the following:
- 319 **a.** If an intersected edge is a constraint edge (the initial boundary of f), then add all cells adjacent to V_k .
- **b.** If a cell is already in *CT*, then do not add it.
- 322 **17.** If there are unprocessed elements in *CT*, then go to Step 9. (Otherwise the boundary of the new cell for N_i was created.)

- **18.** Remove all edges from inside the boundary of the newly created cell for N_j . (This is illustrated in Figure 6, the transition from b) to c).)
- **19.** If there are unprocessed elements in *N*, then go to Step 7. (Otherwise the current level of tessellation is completed.)
- **20.** If $TL < TL_{max}$, then for each edge e connecting nucleation points of adjacent cells in vd, do the following. Calculate the bisector point B for e only if:
 - **a.** *e* is longer than T₁; or

- **b.** e is close to the hazard location and it is longer than T₂; or
- **c.** e is bounded by two constraint nucleation points (doors or concave corners); or
- d. both end-points of *e* lie on constraint edges, which are not co-linear.Add the point *B* into *N*.
 - **21.** If TL < TL_{max} and there are unprocessed elements in N, then TL=TL+1 and go to Step 7.
 - **22.** If there are nucleation points in np, other than constraint nucleation points, which are located on constraint edges (the initial boundary of f), then move locations of those points towards the centre of the associated cells.
 - **23.** Report *f*. Algorithm stops.

The first iteration of the tessellation (grey cells) and the corresponding network (black edges) are shown in Figure 7a. Because the tessellation is not dense, the network consists of few links. Many of them lie on constraint floor edges. Therefore, several navigation routes go from door to door along walls. The process stops after the second iteration generates shorter links (see Figure 7b). Shorter links are the result of new point insertion in the midpoint of the original link, which leads into generation of a Voronoi cell with associated dual edges (i.e. shorter links) connecting the new cell with neighbour cells. The locations of new nucleation points occurring on constraint edges are modified by moving them towards the centre of the associated cell (see Step E), which overcomes the problem of navigation along walls. However, there are some links in the network, which are perpendicular to the walls, and their bounding points occur on constraint edges. Such links may produce sharp turns on navigation routes. Additional nodes and links are added in the next iterations, which solves this problem. The final result is shown in Figure 7c. The tessellation and corresponding network are attached to the dual node representing the room. The network is used for precise path calculation within a room.

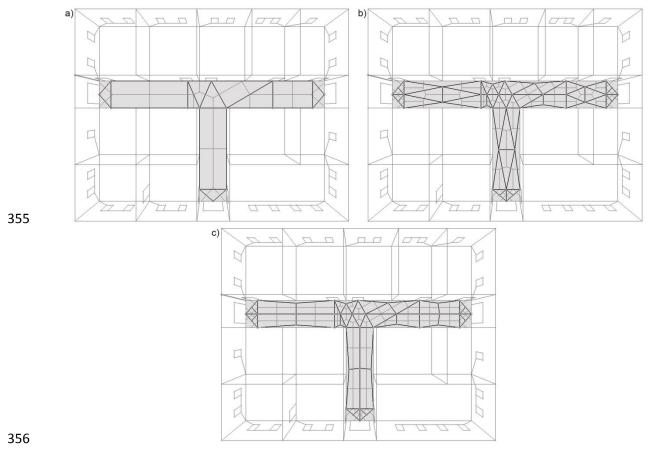


Figure 7. Floor tessellation (grey cells) and associated network (black lines): a) first iteration; b) second iteration; c) final tessellation.

An advantage of the proposed solution is that it can be applied not only to corridors but also to any room shape, e.g. irregular spaces. This is an important property, which helps to avoid the need to classifying a room as a corridor or an open space, and then performing different algorithms for network generation. An example of an indefinable area is a narrow corridor which, at some point, turns into a bigger area (a hub) with many doors or a junction with other corridors.

However, in this scenario, long links in the network may be introduced, because nucleation points in the central area of a room are enclosed by relatively bigger cells and the distance to neighbours is bigger (see Figure 8). This may produce 'wobbling' paths. Consequently, for links that are longer than a specified threshold, new nucleation points are introduced in the midpoint of these links.

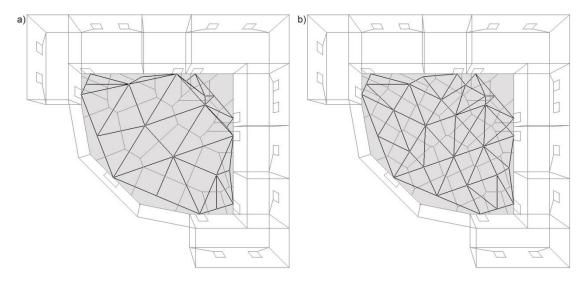


Figure 8. Tessellation of big open space (grey area): a) tessellation without a maximum link length threshold; b) tessellation with a maximum link length threshold (in this case 6 m).

The higher density of the network is helpful for finding alternative or safest paths when a hazardous event takes place within an open space: some nodes may be prohibited for navigation, while the rest of the space, with a lower risk, is still considered as safe. This applies to buildings such as airports or shopping malls.

The network might be densified around a dangerous area using the same method. For links located closer to the hazard source, a smaller threshold for links is introduced. Therefore, new nucleation points are introduced in the midpoints of these links and new shorter links are introduced. This operation provides local network densification (see Figure 9). TODO: (move to introduction???) Densification is commonly applied in the finite element mesh generation and refinement (Du and Gunzburger, 2002; Ho-Le, 1988; Petersen and Martins, 1997).

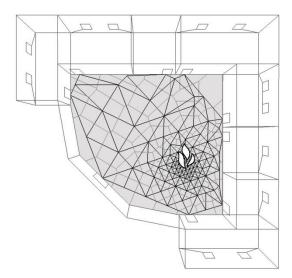


Figure 9. Local densification of the navigable network around a hazardous area.

4.3. Results and discussion

The main reason for introducing additional nucleation points is to improve the network for a path finding application. This increases the number of cells in the tessellation, thus the number of

network links is increased and they are shorter. If the tessellation process is finished after the first iteration, a path between two locations goes through constraint points, which is not a 'natural' route for navigation – it is like walking from door to door along walls (see Figure 10a). The second iteration produces a network better suited for navigation, but the path is slightly 'wobbling' (see Figure 10b), mainly because there are links that cross the corridor – perpendicular to the constraint edges. The path based on the final network is still influenced by 'wobbling' but less than in the previous iterations (see Figure 10c), and it is acceptable in the presented research, since a very precise path calculation is not required.

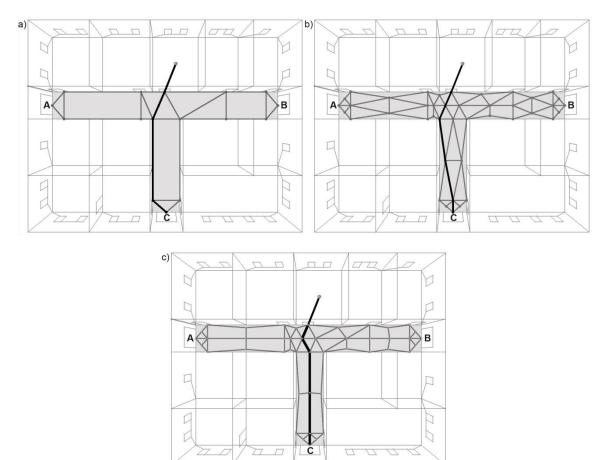


Figure 10. Shortest path from a selected room to door C: a) using the network produced by the first iteration; b) using the network produced by the second iteration; c) using the final network.

The proposed method solves the problem of incorrect paths shown in Figure 3. When the additional network within the corridor is used, the result of the shortest path finding algorithm is correct (see Figure 11). The egress route from the selected room goes through the door straight to the nearby exit.

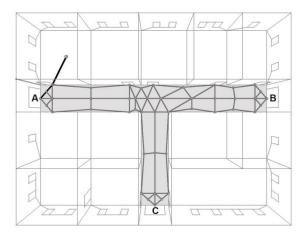


Figure 11. The shortest path from a selected room to exit A (A-C are exits from a building) based on the original network and the network produced by the corridor floor tessellation.

A satisfactory improvement is also achieved in the case of open spaces. All shortest paths from a selected room to all doors of the open space are shown in Figure 12. The network within the room is a better approximation of 'natural' navigation routes than the original logical network.

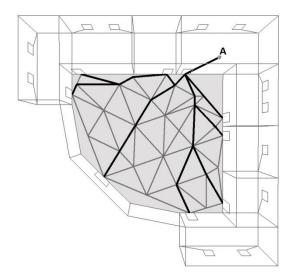


Figure 12. Shortest paths (black lines) from a selected room (A) to all doors of an open space.

The result of Dijkstra's algorithm applied from a selected room in the building is shown in Figure 13. The shortest path goes from the room at the top floor to the ground floor through one of three staircases located near the room. All corridors and rooms with more than one door in the building were automatically tessellated and the navigable network was retrieved before the application of the path finding algorithm.

It should be noted that the building model used here is a mock dataset, where the main navigation routes include corridors and staircases, and lifts are not included in the model. However, lift shafts may be considered for navigation in the same way as staircases but normally they cannot be used in case of emergency situations.

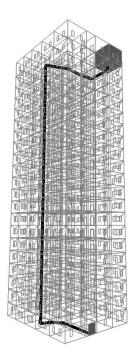


Figure 13. Egress route (black lines) from a selected room (grey cell) to the closest exit (grey door).

The logical network may be used for alternative path finding in the case of an emergency, when the conventional navigation routes are blocked. In the hypothetical scenario shown in Figure 14, a section of the corridor may be on fire. Therefore, it is not safe to use a conventional route to leave the selected room and reach exit A as a result of a fire behind the door. Thus, an alternative path is calculated, which goes through walls, where a hole can be possibly made. Once the room with a door located outside the dangerous area is reached, a conventional egress route to exit B is used.

Alternative paths are found automatically if there is no safe connection from a room through doors. A connection from the logical network with the smallest weight is selected. The weight for each wall in the room may differ depending on a construction material. Therefore, walls easier to demolish are selected as candidates for navigation, e.g. partitions made of plaster panels is favoured over walls made of concrete.

It is important to note that windows may be considered as alternative exits if they are located close above the external ground, for example, windows on the ground floor. In addition, rescue teams are able to get inside the building through windows located high above the ground using ladders. In order to show navigation through walls using the logical network windows were not allowed to be used in the illustrated simulation.

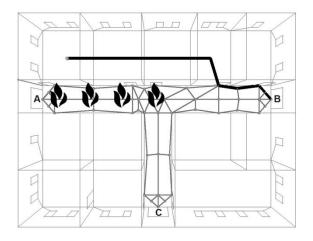


Figure 14. Alternative path finding.

Another type of alternative paths are the safest paths, which avoid dangerous areas using normal navigation ways. Typically they are longer than shortest paths but they are located as far from hazard as possible. In current research, an abstract hazard is manually triggered in selected rooms and is propagated within a building. If information from sensors (e.g. temperature and smoke sensors) is available, a hazardous event may be automatically detected. In hazard simulation, additional weights reflecting hazard spread in the network are calculated based on the number of obstacles, such as walls, and distance from hazard locations. Then, new safest paths are calculated using these new weights.

5.4. Comparison of VDN with prevailing indoor navigable networks

Two state of the art methods of the navigable network reconstruction used in emergency response research (Lee, 2004; Liu and Zlatanova, 2011) were selected for a comparison. The reason for this selection is the close relationships to the VDN method proposed in this paper. Three configurations of rooms shown in Figure 7, Figure 8 and Figure 15 are used as examples for this comparison: T-shaped corridor (also see Figure 16 a), d), g) and j)), open space (also see Figure 16 b), e), h) and k)) and Z-shaped corridor (also see Figure 16 c), f), i) and l)), respectively. Two first examples are mock data, while the latter is a real building – World Trade Centre in Doha, Qatar.

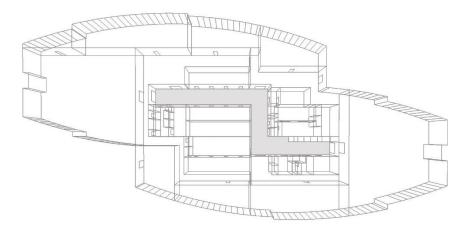


Figure 15. Floor plan – World Trade Centre in Doha, Qatar. Grey-shaded corridor is used for comparison with other methods.

Lee (2004) proposed the Node-Relation Structure (NRS) as the topological model. The Straight
Medial Axis Transformation is applied for generating a navigable network within a corridor. A
skeleton of the corridor floor is calculated; this is based on VD construction. The result is a linear
representation of a polygon – a 'backbone' representing a navigation route through the corridor.
Nodes representing rooms adjacent to the corridor with a door in between are connected to the
skeleton by additional links. The advantage of this method is that it is a simple network with a
relatively small number of links (see Table 1).

However, the applicability to big open spaces was not presented, possibly because of unsatisfactory representation of navigation routes. Consequently, it is suggested that it is not practical to use the aforementioned method for spaces other than simple-shaped corridors. Additionally, because the skeleton is located in the middle of the corridor and the adjacent rooms are connected to the skeleton by perpendicular links, the network does not reflect 'natural' ways of navigation – if two doors are coplanar, i.e. located on the same wall, one must go from the first door to the middle of a corridor, turn 90 degrees, walk in the middle of the corridor towards the second door, turn 90 degrees again, and then get to the second door. This may be not a significant drawback, especially for narrow corridors when the shortest paths are calculated. However, as complexity increases, it may be become significant in computation of the simplest paths, where the sharpness of turn angle is considered.

The door-to-door (DtD) method (Liu and Zlatanova, 2011) is based on a simple idea of navigation from a selected door directly to another door, if two doors are in direct visibility. If there is no direct visibility, then intermediate concave corridor corners are introduced to the path. Taking all possible connections among all the doors within a corridor including concave corners, the navigable network consists of more links than NRS (see Table 1). The network is a non-planar graph in the considered examples. The method may be easily applied for big irregular spaces.

These two methods produce a fixed network depending on the geometry of a room and location of doors. This may be a drawback when alternative paths within a big room are necessary to calculate, for instance to avoid hazard located in the room. This is critical, when big open spaces in a shopping mall or airport are considered. In contrast, the proposed VDN method is more flexible, mainly because the density of tessellation may be easily increased resulting in more links in the network and thus more flexible routes (see Table 1).

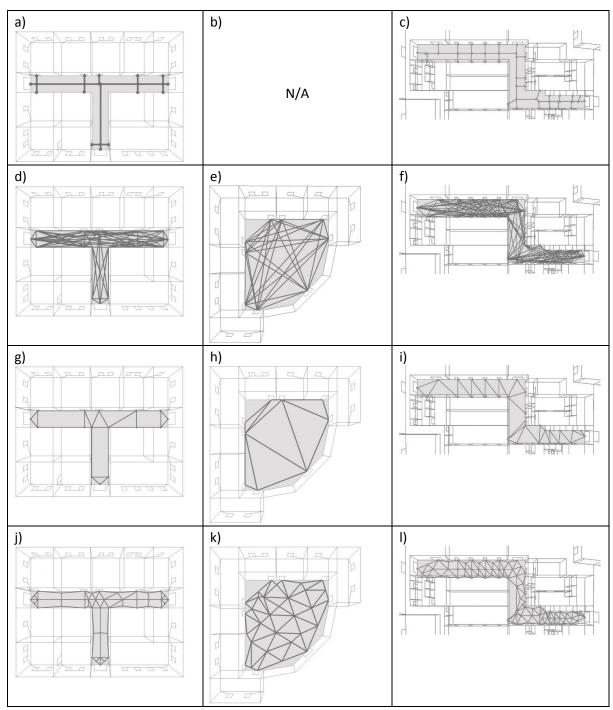


Figure 16. Navigable networks generated for different floor shapes using different methods: a)-c) NRS; d)-f) DtD; g)-i) VDN (1st level); j)-l) VDN (max. level with the threshold).

It should be noted, that the number of links calculated for the DtD method is based on the following assumption: links connecting the same nodes are not duplicated in the network. For example, considering three doors d_1 - d_3 and one concave corner c_1 , if doors d_1 and d_2 are in a direct visibility and d_3 is connected with d_1 and d_2 through c_1 , there are four links: d_1 - d_2 , d_1 - c_1 , d_2 - c_1 and c_1 - d_3 .

It should be also noted that, in the NRS construction, the skeleton of the corridor is calculated based on straight medial axis transformation. Door nodes are connected to the skeleton by line segments, which are perpendicular to the skeleton, and new nodes at an intersection point are added to the skeleton. However, if the distance from a door to an existing node in the skeleton is shorter than the

perpendicular connection, the door node is connected to that node. Conversely, in order to avoid very short links in the network, a door node is connected to an existing node if a new intersection point is located within a range of 10 cm from the existing node.

Table 1. Number of nodes and links in navigable networks generated using different methods.

	NRS	DtD	VDN	VDN	
	(Lee, 2004)	(Liu and	(1 st level)	(max. level with	
		Zlatanova, 2011)		the threshold)	
T-shaped	20 nodes	16 nodes	16 nodes	45 nodes	
corridor	20 links	77 links	26 links	84 links	
Open	N/A	9 nodes	9 nodes	32 nodes	
space		36 links	15 links	74 links	
Z-shaped	47 nodes	30 nodes	30 nodes	85 nodes	
corridor	46 links	187 links	56 links	190 links	
Comments	- simple network with a small	- non-planar	 variable number o densification level 	f links depending on	
	number of links - limited applicability to	graph - minimized number of links on door-to-door	- local network densification possible		
	open spaces	routes			

In order to compare the precision of different methods for tessellation, we consider the main corridor of the actual floor plan of the Doha World Trade Centre (see Figure 15). There are 27 doors in the corridor as depicted in Figure 17. Ten pairs of doors were randomly generated, which represent ten random possibilities for moving from one door to another in the corridor (see Table 2). For each pair of doors, the length of the actual walking path between them was calculated, as well as the lengths of paths generated by the NRS method, the DtD approach and the VDN method for the first and maximal levels of tessellation. The corresponding results are summarised in Table 2.

Examples of path P_6 generated by the aforementioned methods are shown in Figure 18.

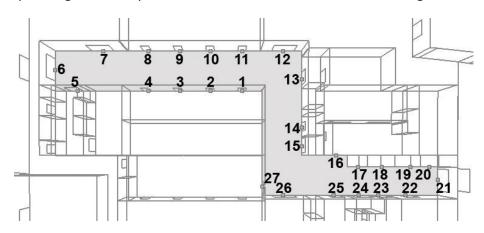


Figure 17. Location of doors 1-27.

Table 2. Ten random routes and their lengths (meters) for different methods

Path	From Room	To Room	Actual Route	NRS	DtD	VDN (1 st level)	VDN (max level with the threshold)
P_1	11	23	18.8	24.1	18.0	19.3	19.5
P_2	7	9	7.4	10.1	6.6	6.6	6.8
P_3	5	16	28.1	34.3	25.4	26.0	27.3
P_4	17	12	15.7	18.8	14.1	14.8	16.1
P_5	15	20	14.1	18.1	12.0	12.2	13.2
P_6	25	8	22.7	28.3	21.4	23.1	23.1
<i>P</i> ₇	10	26	15.7	18.7	14.8	17.1	15.6
<i>P</i> ₈	20	18	5.1	6.2	4.1	4.1	4.2
P 9	9	1	6.8	8.9	6.4	7.1	6.6
P_{10}	22	2	22.9	29.0	20.6	22.0	22.6

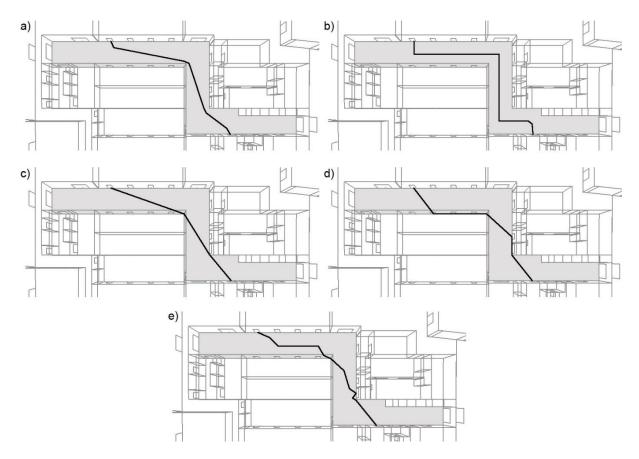


Figure 18. Path P₆ from room 25 to 8 generated by methods: a) actual walking path; b) NRS; c) DtD; d) VDN (1st level); e) VDN (max level with the threshold)

The actual path, which is a point of reference in this comparison, was generated manually considering the following rules: the path should not run closer than 0.8 m to the concave corners (see Figure 19a); a next node after a door node is located 0.8 m from the door frame into direction of the next move (see Figure 19b); the direction of door opening is not taken into consideration.

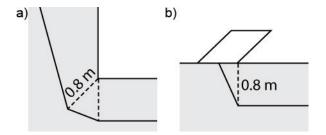


Figure 19. Actual path generation: a) distance from a wall; b) distance from a door.

Furthermore, three standard errors were calculated: the mean error (ME), the mean absolute error (MAE) and the mean squared error (MSE). Note that the former represents the bias, whereas the last two refer to the accuracy of methods. The results are given in Table 3. The NRS method very often overestimates the length of the actual path, which is reflected in the ME equal to -3.9m, whereas the DtD approach tends to underestimate the length with ME=1.4 m. On average, there is some underestimation in the VDN method with the ME=0.5 m and 0.2 m for the first and maximal levels. The MAE figures for the above methods are similar: 3.9 m, 1.4 m, 1.0 m and 0.5 m, respectively. The same numbers expressed as percentages of the MAE to the average length (15.7 m) of actual routes are as follows: 24.8%, 8.8%, 6.4% and 3.3%, thus the 5% standard threshold for errors is achieved by the maximal level of tessellation in the VDN method. Finally, the MSE measure, which penalises large errors, is rather high (18.1) for the NRS method, whereas the DtD and VDN approaches exhibit small MSEs as can be seen in Table 3. Thus, the VDN method with the maximal level of tessellation demonstrates the best characteristics for the bias and accuracy.

Table 3. Errors of the four methods

	NRS	DtD	VDN (1st level)	VDN (max level)
ME	-3.9	1.4	0.5	0.2
MAE	3.9	1.4	1.0	0.5
MSE	18.1	2.4	1.3	0.4

6.5. Conclusions

In this paper, the navigable indoor network reconstruction from an indoor model was presented and compared with other methods <u>applied for route planning</u> in the field of emergency preparedness and response. The network consists of a logical network reflecting the 3D building structure and a detailed navigable network within rooms with many doors and complex geometry. The logical network represents the full 3D topology including spatial relations between rooms at different floors and rooms sharing a wall without doors or other openings. It can be used not only for indoor navigation but also for alternative path finding and potential simulations of various phenomena associated with emergency situations.

DHE data structure was used in this research to represent a 3D indoor scene reconstructed from a BIM model. Two graphs, primal and dual, stored and updated simultaneously allow for the geometry, topology and semantics representation in one consistent model. They are used for direct implementation of Dijkstra's algorithm for path-finding and visualisation of the model, and egress routes.

To demonstrate the robustness of the VDN approach, a static model was presented. This consists of a structure of the building and location of events that are known before the navigable network is generated. In case of a new event, the whole navigable network within an affected room is generated from the start. However, local densification of the network may be performed, when necessary, without recalculating the whole network.

The VDN algorithm may be applied to any space type including corridors and big open spaces, unlike other methods, which focus on corridors only. The tessellation may be easily densified locally or within the whole space by changing the threshold for a link length, which increases the number of links in the navigable network. This may be helpful in finding alternative paths within a single big room with a hazardous event located within the room. Because VDN is generated independently for each space, a different tessellation level and threshold may be defined for different spaces, for example, denser network available within corridors, while sparse in rooms.

It was shown that VDN, in comparison to prevailing approaches, increases accuracy of egress route planning. The length of paths calculated using VDN is closer to the length of 'natural' paths taken by pedestrians. Comparison study was performed in a model of the real building: Doha World Trade Centre in Qatar.

The proposed method uses basic algorithms for network construction (Green and Sibson algorithm for VD generation), densification (new nucleation points added in the middle of links longer than a threshold) and path-finding (Dijkstra's algorithm) in order to present a general idea and its applicability in emergency response applications. It may be improved by using advanced algorithms for better Voronoi point location, e.g. using Lloyd's algorithm (Du et al., 1999), real-time network updates including point deletion (Mostafavi et al., 2003), and network densification utilized in finite element mesh generation. These two-operations, i.e. change of nucleation point location and network densification, may should improve effectiveness of the proposed method by quick recomputation of the network and generation of more natural the shape of for navigable paths in such a way that more natural paths are generated. Enhancement of the navigable network in order to get-obtain natural paths and quick network updates will be considered in future research.

Methods of alternative path-finding are currently under development in presented research. An alternative path may be understood in two ways: i) a path, which runs through a wall if all standard navigation routes through doors are blocked; ii) a path, which is longer than the shortest path and runs through standard navigation routes, but avoiding hazardous areas. In future work it is planned to include information about construction material in models and use it for more advanced egress route finding and simulations of hazard propagation in buildings.

The future work includes development of the decision support system based on VDN and the multi-attribute decision-making technique, which takes shortest, safest and simplest paths as input data and returns 'the best path'. The system will be suited for rescue and emergency teams, who need to get access to trapped people.

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