

# On half-adders based on fusion of signal careers: excitation, fluidics, electricity

Andrew Adamatzky  
University of the West of England, Bristol, UK

May 19, 2016

## Abstract

Likely outcomes of a collision between two objects are annihilation, reflection or fusion. We show how to construct a one-bit adder with pattern that fuse on impact. A fusion gate has two inputs and three outputs. When a signal is generated on a single input the object propagates along its own output trajectory. When both inputs are active the objects collide at a junction of input trajectories, fuse and propagate along dedicated output trajectory. Thus two outputs produce conjunction of one signal with negation of another signal; and, third output produces conjunction of input signals. By merging two outputs in one we make a one-bit half adder: one output is the conjunction of input signals, another output is the exclusive disjunction of the signals. We discuss blue-prints of the half-adders realised with two types of physical signal careers — wave-fragments in excitable medium and high-velocity jet streams. We also propose an electrical circuits analogous of a fusion half-adder. By running fusion half-adders in reverse we find that, despite realising the same functions when in a straight mode, all devices implement different functions when their inputs swapped with outputs.

## 1 Introduction

Unconventional computing is 99% conventional theory and only 1% novel computing substrates and devices. Majority of the novel computing devices are logical gates and circuits. Most circuits implemented in laboratory experiments so far are one-bit half-adders. Half-adders and their component gates are realised using enzymatic networks [38, 33, 10, 30, 27], photonic molecular devices [7, 8, 25], quantum logic inside a single molecule [18], molecular structural circuits [19, 32], acellular slime mould [40, 3, 28, 41], nuclear magnetic resonance [36], ribosomes and mRNAs [35], peptide networks [9], and excitable chemical media [23, 21, 16, 22]. In excitable chemical media signal careers are excitation waves; the signals are modified via interaction between the excitation waves or wave-fragments. The wave-fragments colliding head-on annihilate. Not an-

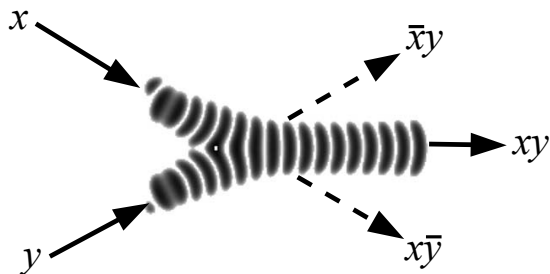


Figure 1: Time lapse overlays of the fusion of two excitation wave-fragments. One fragment is travelling from north-west to south-east, another fragment from south-west to north-east. The wave-fragments collide and fuse into a new localised excitation travelling east. Note that these are not trains of waves but two single-wave-fragments recorded at regular time intervals and superimposed on the same image.

nihilation but merging happens when the wave-fragments collide at an acute angle.

While designing reaction-diffusion computers, we found that when the excitation wave-fragments in a thin-layer Belousov-Zhabotinsky (BZ) medium collide at an acute angle they fuse into a single wave-fragment which propagates along bisector of the collision angle (Fig. 1). Thus a fusion gate came into play [1]. If one of the wave-fragments was not present another wave-fragment would move along its original trajectory. We interpret the presence and absence of wave-fragments at given site at given time as TRUE ('1') and FALSE ('0') values of Boolean variables. Let wave-fragment travelling South-East represent value  $x$  and wave-fragment travelling North-East represent value  $y$ . If  $y = 0$ , the corresponding wave-fragment is not present. Then the wave-fragment  $x$  continues its travel undisturbed. Thus its output trajectory represents  $x\bar{y}$  (Fig. 1). The output trajectory of undisturbed wave-fragment  $y$  represents  $\bar{x}y$ . When both input variables are TRUE the wave-fragments  $x$  and  $y$  collide and merge into a single wave-fragment. This newly born wave-fragment represents conjunction  $xy$ . The design works well not just with BZ but also with swarms of simulated or even living creatures, as we experimentally demonstrated in the prototype of the fusion gate made with soldier crabs [24].

If we go back into 1950-1960s we find the fusion gate (Fig. 1) was actually a rediscovery, indeed implemented in a novel substrate, of a jet stream AND-gate, a key component of fluidic circuits [34, 15]. The fact that two gates discovered with a forty-years interval prompted us to look closer at the potential implementations of the fusion gate and binary arithmetic circuits made from it. This is what the present paper about. In Sect. 2 we discuss design of gates made with excitable chemical medium. Fluidic implementation of an adder is given in Sect. 3. Section 8 shows electrical circuit analogous to BZ and fluidic adders. Several design of half-adders proposed produce different from each other

results when they are reversed, inputs swapped with outputs, this is discussed in Sect. 5.

## 2 Excitable half-adder

The Belousov-Zhabotinsky (BZ) medium [12, 43, 42, 29], a thin layer of a reaction-diffusion chemical system, can be non-excitable, sub-excitable and excitable states. In an excitable thin layer BZ system a localised perturbation leads to a formation of omni-directional target or spiral excitation waves. A sub-excitable BZ medium responds to asymmetric local perturbations by producing travelling localised excitation wave-fragments [37, 17]. The size and life span of an excitation wave-fragment depend on a degree of medium’s excitability. The degree of excitability can be controlled by light [31, 37, 2]. Under the right conditions the wave-fragments conserve their shape and velocity vectors for extended periods of time.

Interaction of wave-fragments shown in Fig. 1 happens in a free space. Advantage of this is that literally any loci of space can be a signal conductor, cause ‘wires’ are momentary. Disadvantage is that, due to instability of wave-fragments [37] which either expand or collapse, we must continuously monitor a size of the wave-fragment and adjust excitability of the medium to conserve the fragment’s shape. A compromise can be achieved by geometrically constraining excitation waves to excitable channels and allowing wave-fragments to interact at sub-excitable junctions [4].

To illustrate interaction between wave-fragments we use two-variable Oregonator equation [20] adapted to a light-sensitive Belousov-Zhabotinsky (BZ) reaction with applied illumination [11]:

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{1}{\epsilon}(u - u^2 - (fv + \phi)\frac{u - q}{u + q}) + D_u \nabla^2 u \\ \frac{\partial v}{\partial t} &= u - v\end{aligned}\tag{1}$$

The variables  $u$  and  $v$  represent local concentrations of an activator, or an excitatory component of BZ system, and an inhibitor, or a refractory component. Parameter  $\epsilon$  sets up a ratio of time scale of variables  $u$  and  $v$ ,  $q$  is a scaling parameter depending on rates of activation and inhibition,  $f$  is a stoichiometric coefficient. Constant  $\phi$  is a rate of inhibitor production. In a light-sensitive BZ  $\phi$  represents the rate of inhibitor production proportional to intensity of illumination (1). We integrate the system using Euler method with five-node Laplace operator, time step  $\Delta t = 0.001$  and grid point spacing  $\Delta x = 0.25$ ,  $\epsilon = 0.02$ ,  $f = 1.4$ ,  $q = 0.002$ . The parameter  $\phi$  characterises excitability of the simulated medium. To generate excitation wave-fragments we perturb the medium by square solid domains of excitation,  $10 \times 10$  sites in state  $u = 1.0$ ; if different shape of perturbation was used we indicate this in the captions of the figures. The medium is excitable and exhibits ‘classical’ target waves when  $\phi = 0.05$  and the medium is sub-excitable with propagating localizations, or

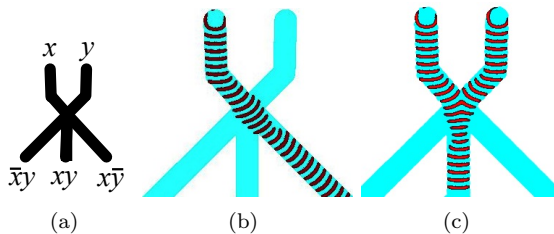


Figure 2: Fusion gate  $F$ . Channels are excitable. Junctions are sub-excitable. (a) Scheme: inputs are  $x$  and  $y$ , outputs are  $xy$ ,  $\bar{x}y$ ,  $x\bar{y}$ . (bc) Time lapsed overlays of excitation waves. Note that these are not trains of waves but two single-wave-fragments recorded at regular time interval and superimposed on the same image. (b)  $x = 1, y = 0$ . (c)  $x = 1, y = 1$ .

wave-fragments, when  $\phi = 0.0766$ . Time lapse snapshots provided in the paper were recorded at every 150 time steps, we display sites with  $u > 0.04$ . The model has been repeatedly verified by us in experimental laboratory studies of BZ system, and the satisfactory match between the model and the experiments has been demonstrated in [6, 17, 39, 5].

Geometry of the fusion gate  $F$  is shown in Fig. 2a. Channels are excitable. That is if the channels' width and length were infinite the excitation wave would be a growing circle. Thus we do not waste resources by controlling excitability when signals are in transit. If we kept a junction excitable then excitation would propagate to all output channels. We keep the junction sub-excitable and therefore wave-fragments either propagate across the junction, along their original trajectories, without spreading into branching channels (just one input is '1', Fig. 2c) or collide and merge into a single wave-fragment entering the central channel (two inputs are '1', Fig. 2e)

By merging two lateral output channels  $x\bar{y}$  and  $\bar{x}y$  into a single output channel we get a one-bit half-adder, gate  $A$ . A one-bit half-adder is a device with two inputs  $x$  and  $y$  and two outputs  $xy$  (Carry out) and  $x \oplus y$  (Sum) (Fig. 3a). It consists of two input channels  $a$  and  $b$  and two output channels  $g$  and  $h$ . When gate  $A$  is filled with BZ medium we call this device  $A_{BZ}$ . Presence/absence of a wave-fragment in an input/output channel of  $A_{BZ}$  symbolises logical TRUE/FALSE state of the input variable assigned to the channel. Synchronisation of signal wave-fragments is achieved geometrically:  $|a| + |e| + |g| = |b| + |d| + |g| = |a| + |f| + |h|$ , where  $|\cdot|$  is a length of a channel (Fig. 3a). Functioning of the half-adder  $A_{BZ}$  is shown in (Fig. 3bcd). Two half-adders can be cascaded into a one-bit full adder, see details in [4].

### 3 Flueric half-adder

BZ fusion gate proposed in [1] was in fact a re-discovery of the fluidic, or rather flueric, fusion gate. Signals in fluidic and flueric devices are represented by

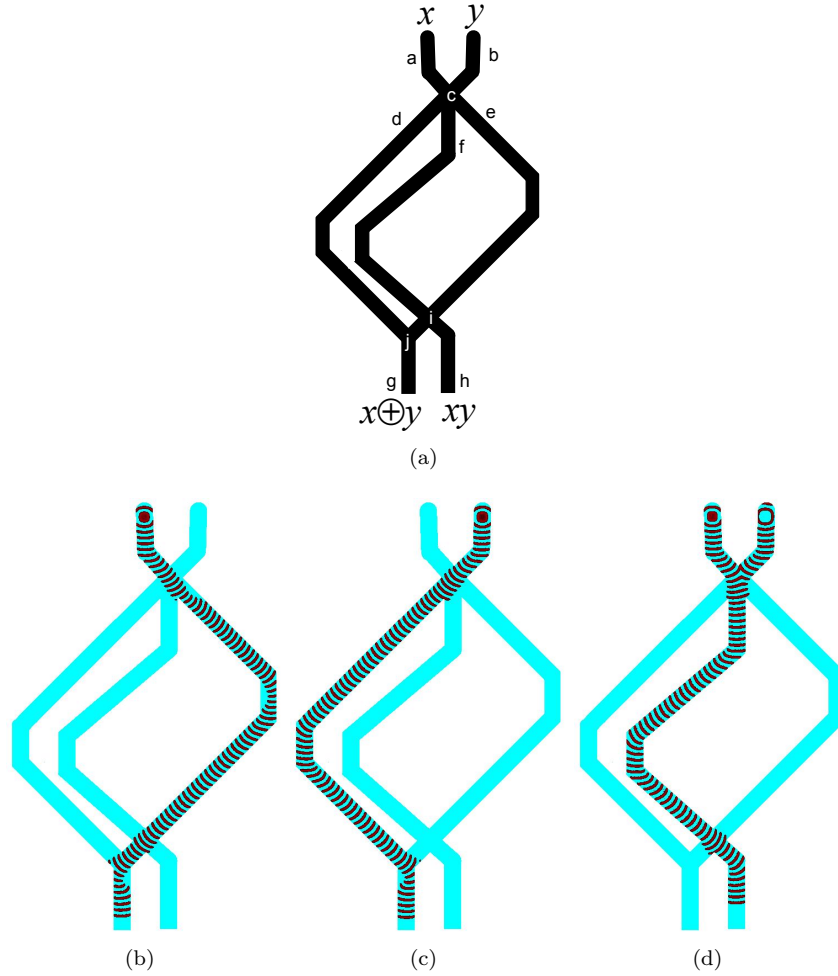


Figure 3: A one-bit half-adder,  $A_{BZ}$ -gate, filled with excitable chemical medium. (a) Scheme of the half-adder, gate  $A$ :  $a$  and  $b$  are input channels,  $g$  and  $h$  are output channels,  $d$ ,  $e$ ,  $f$  are internal channels, and  $c$ ,  $i$  and  $j$  are junctions. Input variables  $x$  and  $y$  are fed into channels  $a$  and  $b$ , results  $x \oplus y$  and  $xy$  are read from channels  $g$  and  $h$ . (bcd) Time lapsed overlays of excitation waves propagation for inputs (b)  $x = 1$ ,  $y = 0$ , (c)  $x = 1$ ,  $y = 0$ , (d)  $x = 1$ ,  $y = 1$ . Sites of initial perturbation are visible as discs. Grid size is  $500 \times 790$  nodes. These are time lapsed snapshots of a single wave (bc) or two waves merging into a single wave (d) recorded every  $150^{th}$  step of numerical integration.

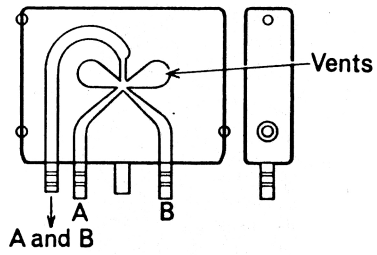


Figure 4: Diagram of an AND gate from [34].

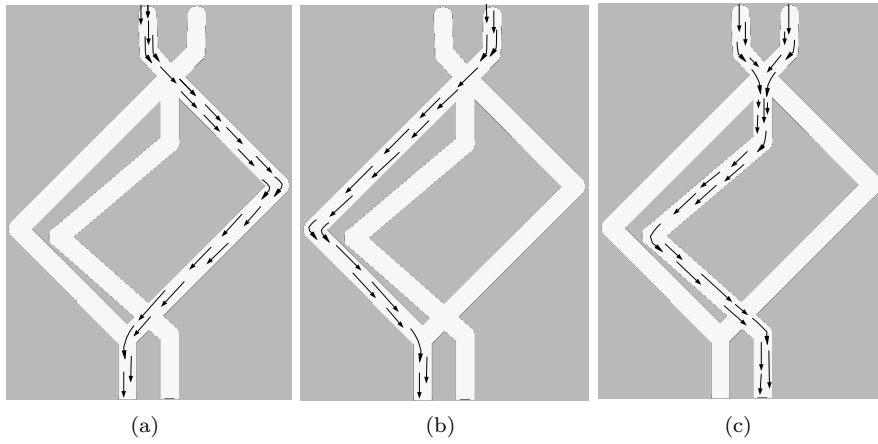


Figure 5: Flueric half-adder  $A_F$  based on geometry of  $A_{BZ}$ . (a)  $x = 1, y = 0$ , (b)  $x = 0, y = 1$ , (c)  $x = 0, y = 1$ . Jet streams are shown by arrows.

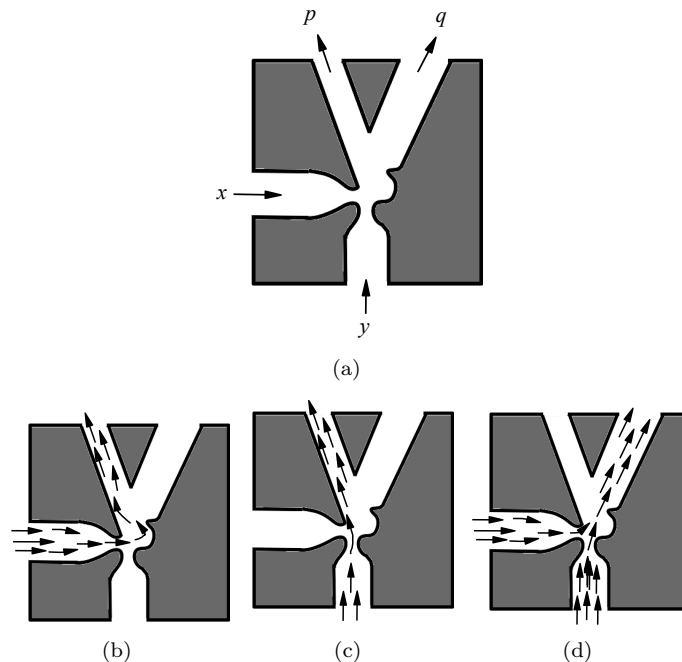


Figure 6: Hobbs gate  $H$ . (a) Structure of a hook type fluidic half-adder (a). (bcd) Dynamics of fluid streams for inputs (b)  $x = 1$  and  $y = 0$ , (c)  $x = 0$  and  $y = 1$ , (d)  $x = 1$  and  $y = 1$ . Redrawn from [26].

high-velocity jets issuing into the vented interaction region. Flueric devices are passive fluidic devices. They do not have any moving parts and do not change their shape to control a fluid [34, 15]. The flueric gates employ phenomena of a wall attachment, jet interaction and inertia [13]. The wall attachment of a jet happens due to a difference in space from the jet to the channel walls jet attracts air in the space between itself and one wall, and makes a vacuum in the space between itself and another wall [34].

The jet fusion gate shown in Fig. 4 is amongst first ever fluidic devices made in 1960s [34, 15]. It has two input ports ‘A’ and ‘B’, one output port and two vents. If pressure is applied only to one input the jet goes into vent. If both inputs are TRUE, pressure applied to both input channels, the opposing streams collide and merge into a single stream. The combined stream flows along the line that bisects the angle between the two intersecting jets and exits through the output port ‘A’ AND ‘B’ (Fig. 4). By converting vents into output channels one can get a classical geometry of the fusion gate  $F$  (Fig. 2a). By joining the vents into a single channel one can produce a one-bit half-adder, gate  $A_F$ , similarly to Fig. 3a. See scheme of gate  $A_F$  functioning in Fig. 5.

Joining vents seem to be unnecessary as Hobbs demonstrated in 1963 [26]. Hobbs gate  $H$  is shown in Fig. 6a. Logical values are encoded into presence

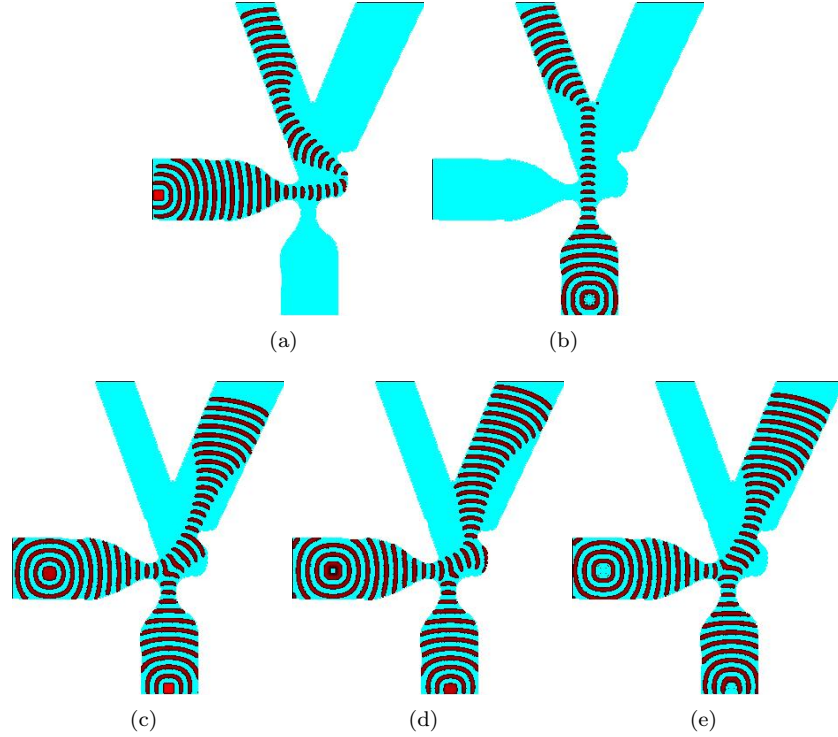


Figure 7: BZ in Hobbs gate, the  $H_{BZ}$  gate (Fig. 6a). (a–e) Time lapsed overlays of excitation waves for inputs (a)  $x = 0, y = 1$ . (b)  $x = 1, y = 0$ . (cde)  $x = 1, y = 1$ , non-perfect collision of wave-fragments.

of streams at specified channels. When only input  $x = 1$  a power jet stream enters the gate via channel  $x$ . The stream is turned by the hook and follows the channel  $p$  (Fig. 6b). When only input  $y = 1$  the power jet stream entering the gate via channel  $y$  locks on the left boundary wall of its channel, and propagates along the channel  $p$  (Fig. 6c). If both inputs are TRUE streams entering  $x$  and  $y$  merge and follow the channel  $q$  (Fig. 6d). Thus the stream exiting the channel  $p$  represents  $p = x \oplus y$  and the stream exiting the channel  $q$  represents  $q = xy$ .

Functioning of the Hobbs gate filled with BZ medium, instead of fluid, the  $H_{BZ}$  gate, is shown in Fig. 7. For inputs  $x = 1$  and  $y = 0$  an excitation is initiated at the input lateral channel (Fig. 7a). The wave-front propagates till the junction, enters the hook, reflects and travels into North-West output channel  $p$ . There the wave-fragment is reflected again by the channel's wall and proceeds to the output port. To represent input  $x = 0$  and  $y = 1$  we excite BZ medium at the South input channel (Fig. 7b). The excitation wave-fragment travels straight, collides into the separation between output channels, and is reflected into the North-West output channel  $p$ . Excitation waves are generated in both input channels to represent inputs  $x = 1$  and  $y = 1$  (Fig. 7cde). The



wave-fragments collide at the junction and merge into a single wave-fragment which propagates into North-East output channel  $q$ . Even when wave-fragments arrive at the collision site not exactly at the same moment the resultant wave-fragment travels into the channel  $q$ . An example of ‘perfect’ timing of the collision is shown in Fig. 7c. The wave-fragment produced in a collision of wave-fragments  $x$  and  $y$  partially collides with northern part of the hook. A segment of the wave-fragment collided into the hook gets extinguished while the intact segment of the wave-fragment continues its travel and propagates along the middle of the channel  $q$ . In the example shown in Fig. 7d the wave-fragment  $x$  arrives at the junction earlier than the wave-fragment  $y$ . The delayed wave-fragment  $y$  causes a substantial part of the newly formed wave-fragment to collide into the hook; remaining part is reflected into the northern wall of the channel  $q$  (Fig. 7d). Late arrival of wave-fragment  $y$  also aberrates trajectory of the output wave-fragment (Fig. 7e).

## 4 Electrical half-adder

Electrical current takes the path of the least resistance. When designing a one-bit half-adder  $E$  where logical values are represented by electrical current we take two input terminals and two output terminals. We should direct current along dedicated  $x \oplus y$  terminal when only one input terminal is connected to a current source (inputs  $x = 1$  and  $y = 0$  or  $x = 0$  and  $y = 1$ ), and along  $xy$  terminal when both input terminals are connected to current sources (input  $x = 1, y = 1$ ). This can be done with a fuse which normal current rating is higher than the current going through one input terminal but less than a sum of currents via two input terminals. A scheme of  $E$  is shown in Fig. 8a. We use ideal electrical constant current sources  $s_1$  and  $s_2$ . The sources have infinite source impedance and drive current through their terminals to the value of the specified current level  $e$ . Ideal resistor  $R$  has no parasitic effects. Fuse  $f$  has resistance much smaller than  $R$  and nominal rated current  $e < i_f < 2e$ . We assume logical input or output is TRUE (‘1’) when current through input or output terminal is non-zero. In practical circuits that would mean the current exceeds some small threshold value  $\epsilon$ .

If only switch  $x$  or switch  $y$  is ON ( $x = 1$  and  $y = 0$  or  $x = 0$  and  $y = 1$ ) the current driven through the circuit is  $e$ . This current does not melt the fuse  $f$  and, because a resistance of the fuse  $f$  is much less than that of  $R$ , the current travels along a path from  $x$  or  $y$  to  $f$  to ammeter  $i_1$ . When both switches  $x$  and  $y$  are ON, the current through the circuit is  $2e$ . The current melts the fuse  $f$  and then travels along the route  $(s_1, s_2)$  to  $R$  to  $i_2$ . Thus current on the output terminal  $i_1$  represents disjunction  $x \oplus y$  of input variables and the output terminal  $i_2$  — conjunction  $xy$ . The analogous half-adder can be interfaced with digital circuits. Switches  $x$  and  $y$  are binary. Terminals  $i_1$  and  $i_2$  can be connected to a converter from a continuous quantity input to a one-bit digital output, e.g. standard IEEE 1164, by specifying the conversion threshold of the current.

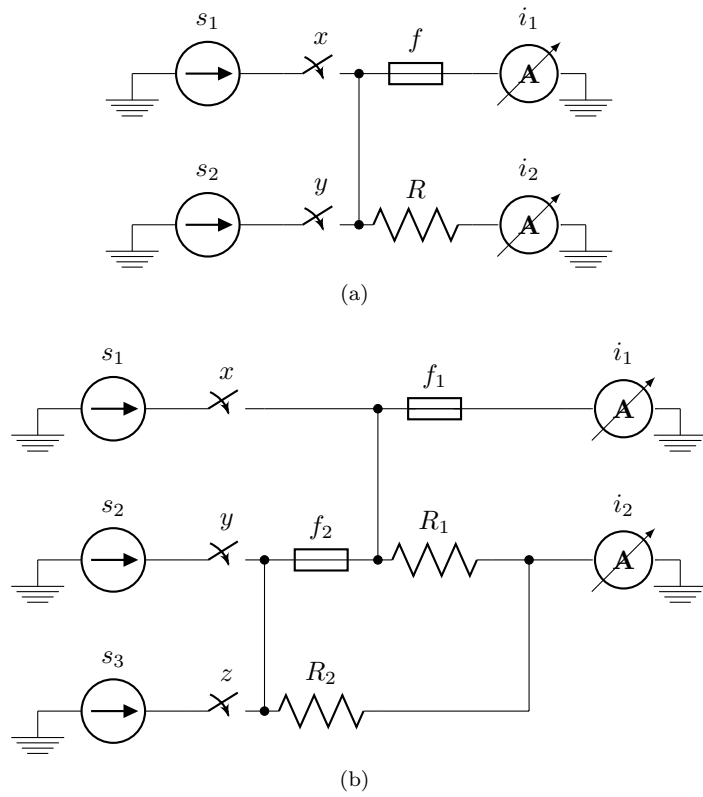


Figure 8: Schematic of (a) one-bit half adder,  $E$ -gate and (b) full adder implemented with constant current sources  $s_1$ ,  $s_2$ ,  $s_3$ , resistors  $R$ , fuses  $f$ , and switches  $x$  and  $y$ . Both circuits have been verified in simulation in SystemVision [14].

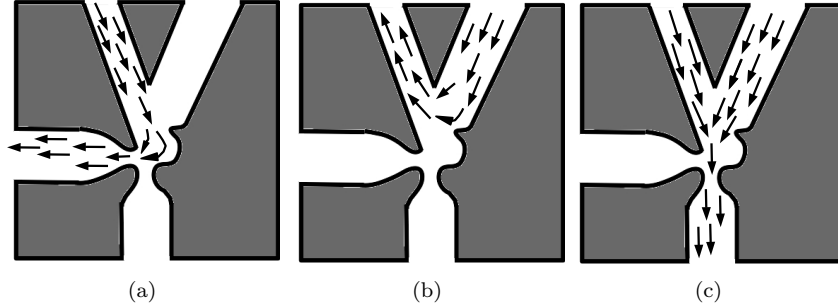


Figure 9: Reversed Hobbs gate, or  $H^*$  gate, for inputs (a)  $p = 1, q = 0$ , (c)  $p = 0, q = 1$ , and (d)  $p = 1, q = 1$ .

The half-adder  $E$  is modified to a full adder as shown in Fig. 8b. If only one input is TRUE the current goes along a path including only fuses, e.g. for input  $x = 0, y = 0, z = 1$  the current path is  $s_3$  to  $x$  to  $f_2$  to  $f_1$  to  $i_1$ . In case of two input terminals are TRUE the current goes from each source along its unique shortest path till merger. For example, for input  $x = 1, y = 0, z = 1$  current propagates from  $s_1$  to  $x$  and from  $s_3$  to  $z$  via fuse  $f$  to junction point. Since the junction point the current becomes  $2e$  and the fuse is blown and then current follows the path via  $R_1$  to  $i_2$ . Ammeter  $i_1$  shows non-zero current  $e$  only if exactly one of the switches is ON. Ammeter  $i_2$  shows non-zero current if two or three switches are ON, the current shown is  $n \cdot e$ , where  $n$  is a number of switches in position ON. Terminal  $i_1$  represents  $x \oplus y \oplus z$ . Terminal  $i_2$  represents  $xy + z(x \oplus y)$ .

## 5 Reversing circuits

What would happen if we swapped inputs with outputs in the BZ, flueric and electrical half-adders? We call a gate  $G$  reversed,  $G^*$ , if its original inputs become outputs and outputs become inputs.

The reversed BZ gate (Fig. 3a) always produces FALSE outputs,  $A_{BZ}^*(p, q) = \langle 0, 0 \rangle$ . This is because a medium at the junctions  $j$  and  $c$  is sub-excitable (see labels in Fig. 3a). Thus, when input is generated only in channel  $g$  the wave-fragment collides into the ‘groynes’ at junction  $j$  and gets extinguished. Even if the excitation wave did not die at junction  $j$  but split into two waves, propagating along channels  $d$  and  $e$ , the waves would collide with each other at junction  $c$  and annihilate. If input is generated only in channel  $h$  then excitation wave propagates along channel  $f$  and reaches junction  $c$ . The medium in junction  $c$  is sub-excitable: the waves collide into the separation between channels  $a$  and  $b$  and extinguish. When inputs are activated in both channels  $g$  and  $h$  the wave travelling from  $g$  becomes extinguished at junction  $j$  and the wave travelling along channels  $h$  and  $f$  becomes extinguished at junction  $c$ .

Reversed Hobbs gate  $H^*$  is shown in Fig. 9. Assume ports  $p$  and  $q$  (Fig. 6a)

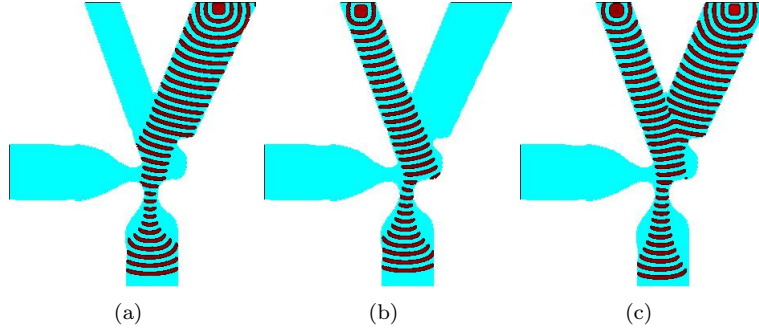


Figure 10: Reversed BZ implementation of Hobbs gate,  $H_{BZ}^*$  gate, for inputs (a)  $x = 0, y = 1$ , (b)  $x = 1, y = 0$ , (c)  $x = 1, y = 1$ .

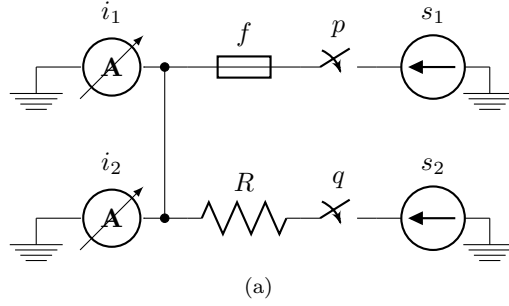


Figure 11: Reversed electronic circuit,  $E^*$ -gate.

are inputs. When  $p = 1$  and  $q = 0$  jet stream entering channel  $p$  interacts with cavity of the hook structure and gets diverted into the output port  $x$  (Fig. 9a). When  $p = 0$  and  $q = 1$  jet stream entering channel  $q$  is deflected by upper part of the hook structure and diverted into channel  $p$  (Fig. 9b). When both inputs are activated ( $p = 1$  and  $q = 1$ ) the jet streams merge into a single stream which enters output channel  $y$  (Fig. 9d). Thus, the reversed Hobbs gate implements operation  $H^*(p, q) = \langle p\bar{q}, pq \rangle$ .

Swapping input and output ports in flueric half-adder (Fig. 5),  $A_F^*$ , makes mapping indeterministic. This is because when only one input is activated the stream meets a binary branching point at least once. At that branching site the stream either splits into two streams, if jet energy is moderate, and then follows both branches, or attaches itself arbitrary to one of the output branches, if jet energy is high.

Hobbs gate with reversed inputs and filled with BZ medium implements operation  $H_{BZ}^*(p, q) = \langle 0, p + q \rangle$  because for all non-zero inputs the resultant wave-fragments travel into output channel  $q$  as demonstrated by modelling results in Fig. 10.

The reversed circuit  $E$ , gate  $E^*$ , is shown in Fig. 11. If switch  $p$  is ON, the

Table 1: Reversed half-adders.

Gate	Operation	Medium
$A_{BZ}^*(p, q) =$	$\langle 0, 0 \rangle$	excitable media
$A_F^*(p, q) =$	$\langle ?, ? \rangle$	fluid streams
$H^*(p, q) =$	$\langle p\bar{q}, pq \rangle$	fluid streams
$H_{BZ}^*(p, q) =$	$\langle 0, p + q \rangle$	excitable media
$E^*(p, q) =$	$\langle p + q, p + q \rangle$	electricity

current flows along the fuse  $f$ , spreads along the short, and reaches ammeters  $i_1$  and  $i_2$ . If switch  $q$  is ON, the current flows along the resistor  $R$ , spreads along the short, and reaches both ammeters. If both switches are ON the current again propagates towards both outputs. Thus, we have  $E^*(p, q) = \langle p + q, p + q \rangle$ , each output of the reversed electrical half-adder is a disjunction of inputs. Operations implemented by all reversed gates are summarised in Tab. 1.

## 6 Discussion

Half-adder implementation via fusion of signals in three different substrates — excitable chemical media, fluidic system and electrical circuit — show the same degree of structural complexity. Gate  $A$  has two junctions (intersection  $i$  of channels does not count because signals never meet there). Gate  $H$  has one junction and one reflector (the hook structure). Circuit  $E$  consists of one fuse and one resistor. The gates with excitable system and jet streams employ physical interaction of patterns representing signals and their reflection by impenetrable barriers. The electrical circuit uses a link which becomes non-conductive when strength of input signal increases. The hook structure in Hobbs gate  $H$  is functionally analogous to a fuse in circuit  $E$ : the fuse blows when both sources of current are ON and the hook does not affect jet streams when they merge in a single stream.

The half-adders implement different sets of functions when they are reversed, i.e. inputs are swapped with outputs. Reversed gate  $A$  produces constant FALSE or indeterministic outputs. Reversed gate  $H$  working on jet streams implements conjunction of one input with negation of another on one output and conjunction of inputs on another output. Reversed gate  $H$  filled with excitable chemical medium produces constant FALSE on one output and disjunction of inputs on another. Reversed electrical circuit  $E$  produces conjunction of input signals on both outputs.

To make fusion based gates cascable we must preserve physical quantity of signals. Gates implemented in excitable chemical medium preserve the quantity of signals: when two wave-fragments fuse the newly formed wave-fragment, when it leaves output port, has the same shape and level of excitation as each of the input wave-fragments. Fluidic and electrical gates do not preserve quantity of signals. Two merging jet streams are stronger than each of the input streams. Electrical current on the output terminal doubles (subject to resistance of the

fuse) when both input terminals are active. These issues are not critical though because by using technology converters the signals can be normalised to their original states.

## References

- [1] Andrew Adamatzky. Collision-based computing in Belousov-Zhabotinsky medium. *Chaos, Solitons & Fractals*, 21(5):1259–1264, 2004.
- [2] Andrew Adamatzky. If BZ medium did spanning trees these would be the same trees as Physarum built. *Physics Letters A*, 373(10):952–956, 2009.
- [3] Andrew Adamatzky. Slime mould logical gates: exploring ballistic approach. *arXiv preprint arXiv:1005.2301*, 2010.
- [4] Andrew Adamatzky. Binary full adder, made of fusion gates, in a sub-excitable Belousov-Zhabotinsky system. *Physical Review E*, 92(3):032811, 2015.
- [5] Andrew Adamatzky, Ben De Lacy Costello, Larry Bull, and Julian Holley. Towards arithmetic circuits in sub-excitable chemical media. *Israel Journal of Chemistry*, 51(1):56–66, 2011.
- [6] Andrew Adamatzky and Benjamin de Lacy Costello. Binary collisions between wave-fragments in a sub-excitable Belousov-Zhabotinsky medium. *Chaos, Solitons & Fractals*, 34(2):307–315, 2007.
- [7] Joakim Andréasson, Gerdenis Kodis, Yuichi Terazono, Paul A Liddell, Subhajit Bandyopadhyay, Reginald H Mitchell, Thomas A Moore, Ana L Moore, and Devens Gust. Molecule-based photonicly switched half-adder. *Journal of the American Chemical Society*, 126(49):15926–15927, 2004.
- [8] Joakim Andréasson, Stephen D Straight, Gerdenis Kodis, Choong-Do Park, Michael Hamburger, Miguel Gervaldó, Bo Albinsson, Thomas A Moore, Ana L Moore, and Devens Gust. All-photonic molecular half-adder. *Journal of the American Chemical Society*, 128(50):16259–16265, 2006.
- [9] Gonen Ashkenasy and M Reza Ghadiri. Boolean logic functions of a synthetic peptide network. *Journal of the American Chemical Society*, 126(36):11140–11141, 2004.
- [10] Ronan Baron, Oleg Lioubashevski, Eugenio Katz, Tamara Niazov, and Itamar Willner. Elementary arithmetic operations by enzymes: A model for metabolic pathway based computing. *Angewandte Chemie International Edition*, 45(10):1572–1576, 2006.
- [11] Valentina Beato and Harald Engel. Pulse propagation in a model for the photosensitive Belousov-Zhabotinsky reaction with external noise. In *SPIE's First International Symposium on Fluctuations and Noise*, pages 353–362. International Society for Optics and Photonics, 2003.

- [12] Boris P Belousov. A periodic reaction and its mechanism. *Compilation of Abstracts on Radiation Medicine*, 147(145):1, 1959.
- [13] Charles A Belsterling. *Fluidic systems design*. John Wiley & Sons, 1971.
- [14] SystemVision Cloud. <https://www.systemvision.com/>, 2016.
- [15] Arthur Conway. *Guide to Fluidics*. Macdonald & Cp. (Publishers) Ltd, 1971.
- [16] Ben De Lacy Costello, Andy Adamatzky, Ishrat Jahan, and Liang Zhang. Towards constructing one-bit binary adder in excitable chemical medium. *Chemical Physics*, 381(1):88–99, 2011.
- [17] Ben de Lacy Costello, Rita Toth, Christopher Stone, Andrew Adamatzky, and Larry Bull. Implementation of glider guns in the light-sensitive belousov-zhabotinsky medium. *Physical Review E*, 79(2):026114, 2009.
- [18] I Duchemin and C Joachim. A quantum digital half adder inside a single molecule. *Chemical physics letters*, 406(1):167–172, 2005.
- [19] James C Ellenbogen and J Christopher Love. Architectures for molecular electronic computers. I. Logic structures and an adder designed from molecular electronic diodes. *Proceedings of the IEEE*, 88(3):386–426, 2000.
- [20] Richard J Field and Richard M Noyes. Oscillations in chemical systems. IV. Limit cycle behavior in a model of a real chemical reaction. *The Journal of Chemical Physics*, 60(5):1877–1884, 1974.
- [21] Joanna N Gorecka, Jerzy Gorecki, and Yasuhiro Igarashi. On the simplest chemical signal diodes constructed with an excitable medium. *International Journal of Unconventional Computing*, 5(2), 2009.
- [22] J Gorecki, JN Gorecka, and Andrew Adamatzky. Information coding with frequency of oscillations in Belousov-Zhabotinsky encapsulated disks. *Physical Review E*, 89(4):042910, 2014.
- [23] J Gorecki, K Yoshikawa, and Y Igarashi. On chemical reactors that can count. *The Journal of Physical Chemistry A*, 107(10):1664–1669, 2003.
- [24] Yukio-Pegio Gunji, Yuta Nishiyama, and Andrew Adamatzky. Robust soldier crab ball gate. *Complex systems*, 20(2):93, 2011.
- [25] Xuefeng Guo, Deqing Zhang, Guanxin Zhang, and Daoben Zhu. Monomolecular logic: “half-adder” based on multistate/multifunctional photochromic spiropyrans. *The Journal of Physical Chemistry B*, 108(32):11942–11945, 2004.
- [26] E. V. Hobbs. Fluid amplification. 9. Logic elements. Technical report, Harry Diamond Laboratories TR-1114, 1963.

- [27] Masato Ikeda, Tatsuya Tanida, Tatsuyuki Yoshii, Kazuya Kurotani, Shoji Onogi, Kenji Urayama, and Itaru Hamachi. Installing logic-gate responses to a variety of biological substances in supramolecular hydrogel–enzyme hybrids. *Nature chemistry*, 6(6):511–518, 2014.
- [28] Jeff Jones and Andrew Adamatzky. Towards Physarum binary adders. *Biosystems*, 101(1):51–58, 2010.
- [29] Raymond Kapral and Kenneth Showalter. *Chemical waves and patterns*, volume 10. Springer Science & Business Media, 2012.
- [30] Evgeny Katz and Vladimir Privman. Enzyme-based logic systems for information processing. *Chemical Society Reviews*, 39(5):1835–1857, 2010.
- [31] L Kuhnert. A new optical photochemical memory device in a light-sensitive chemical active medium. 1986.
- [32] Steven J Langford and Trevor Yann. Molecular logic: A half-subtractor based on tetraphenylporphyrin. *Journal of the American Chemical Society*, 125(37):11198–11199, 2003.
- [33] Harvey Lederman, Joanne Macdonald, Darko Stefanovic, and Milan N Stojanovic. Deoxyribozyme-based three-input logic gates and construction of a molecular full adder. *Biochemistry*, 45(4):1194–1199, 2006.
- [34] Michael J Moylan. *Fluid logic in simple terms*. The Machinery Publishing Co. Ltd., 1968.
- [35] Oliver Rackham and Jason W Chin. Cellular logic with orthogonal ribosomes. *Journal of the American Chemical Society*, 127(50):17584–17585, 2005.
- [36] Marta Roselló-Merino, Matthias Bechmann, Angelika Sebald, and Susan Stepney. Classical computing in nuclear magnetic resonance. *International Journal of Unconventional Computing*, 6, 2010.
- [37] Irene Sendiña-Nadal, Eugene Mihaliuk, Jichang Wang, Vicente Pérez-Muñuzuri, and Kenneth Showalter. Wave propagation in subexcitable media with periodically modulated excitability. *Physical review letters*, 86(8):1646, 2001.
- [38] Milan N Stojanovic and Darko Stefanovic. Deoxyribozyme-based half-adder. *Journal of the American Chemical Society*, 125(22):6673–6676, 2003.
- [39] Rita Toth, Christopher Stone, Ben de Lacy Costello, Andrew Adamatzky, and Larry Bull. Simple collision-based chemical logic gates with adaptive computing. *Theoretical and Technological Advancements in Nanotechnology and Molecular Computation: Interdisciplinary Gains: Interdisciplinary Gains*, page 162, 2010.



- [40] Soichiro Tsuda, Masashi Aono, and Yukio-Pegio Gunji. Robust and emergent physarum logical-computing. *Biosystems*, 73(1):45–55, 2004.
- [41] James GH Whiting, Ben PJ de Lacy Costello, and Andrew Adamatzky. Slime mould logic gates based on frequency changes of electrical potential oscillation. *Biosystems*, 124:21–25, 2014.
- [42] Arthur T Winfree. The prehistory of the Belousov-Zhabotinsky oscillator. *Journal of Chemical Education*, 61(8):661, 1984.
- [43] Anatol M Zhabotinsky. Periodical oxidation of malonic acid in solution (a study of the belousov reaction kinetics). *Biofizika*, 9:306–311, 1964.