Control of Complex Nonlinear Dynamic Rational Systems

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Abstract: Nonlinear rational systems/models, also known as total nonlinear dynamic systems/models, in expression of a ratio of two polynomials, has roots in describing general engineering plants and chemical reaction processes. The major challenge issue in control of such system is the control input embedded in its denominator polynomials. With extensive searching, it could not find any systematic approach in designing this class of control systems directly from its model structure. This study expands U-model based approach to establish a platform for the first layer of feedback control and the second layer of adaptive control of the nonlinear rational systems, which, in principle, separates control system design (without involving a plant model) and controller output determination (with solving inversion of the plant U-model). This procedure makes it possible to achieve closed loop control of nonlinear systems with linear performance (transient response and steady state accuracy). For the conditions using the approach, this study presents the associated stability and convergence analyses. Simulation studies are performed to show off the characteristics of the developed procedure in numerical tests, and to give the general guidelines for applications.

Keywords: Rational systems/models, total nonlinear systems/models, U-model based control(U-control), control complex dynamics, adaptive control, parameter estimation/optimisation

1. Introduction

This section justifies the reasons for designing controllers for rational models by introducing model expression and representations, achieved results in model identification, and a critical review of controller designing approaches.

1.1. Nonlinear dynamic rational systems

Definition [1]: Assign a triplet (X, f, h), X is an irreducible real affine variety, (f, h) are mapping functions. A system $\sum$, with input $U \in \mathbb{R}^m$ and output $Y \in \mathbb{R}^r$, is defined as polynomial/rational, while the functions $f = \{ f_\alpha | \alpha \in U \}$ and $h: X \to \mathbb{R}^r$ both on X are, mappings from input space to state space and from state space to output space respectively, polynomial/rational. That is for polynomial systems $h_i \in A$ for all $i = 1, \cdots, r$ where A is the algebra of all polynomials on the variety X and for rational systems $h_i \in Q$ for all $i = 1, \cdots, r$ where Q is the algebra of all rational functions on the variety X .

For a single-input and single-output (SISO) nonlinear dynamic rational system, it can be generally modelled with a ratio of two polynomials [1, 2].

$$
\begin{aligned}
y(k) &= \frac{N_p(k)}{D_p(k)} + e(k) = \frac{N_p(Y_{k-1}, U_{k-1}, E_{k-1})}{D_p(Y_{k-1}, U_{k-1}, E_{k-1})} + e(k) \\
&= \frac{\sum_{j=1}^{num} p_{nj}(k)\theta_{nj}}{\sum_{j=1}^{den} p_{dj}(k)\theta_{dj}} + e(k)
\end{aligned} \qquad (1.1)
$$

where $y(k) \in \mathbb{R}$, $u(k) \in \mathbb{R}$, and $e(k) \in \mathbb{R}$ denote measured output, input and model error/noise/uncertainties at time instant $k (= 1, 2, \cdots)$ respectively. $N_p(k)$ and $D_p(k)$ are real valued and smooth numerator and denominator polynomials respectively. $Y_{k-1} \in \mathbb{R}^n \supset y(k-1), \ldots, y(k-n)$, $U_{k-1} \in \mathbb{R}^n \supset u(k-1), \ldots, u(k-n)$, and $E_{k-1} \in \mathbb{R}^n \supset e(k-1), \ldots, e(k-n)$ denote the delayed outputs inputs, and model noises respectively. $p_{nj}(k) \in \mathbb{R}$ and $p_{dj}(k) \in \mathbb{R}$ for regression terms, $\theta_{nj} \in \mathbb{R}$ and $\theta_{dj} \in \mathbb{R}$ are the coefficients, and *num* and *den* for numbers of total regression terms of the polynomials respectively. The major properties of the rational model (1.1) are summarised below:

It is also defined as a total nonlinear model [2] as it covers many different linear and nonlinear models as its subsets (such as NARMAX (Nonlinear AutoRegressive Moving Average with eXogenous input) models [3] and intelligent models for neuro-fuzzy systems [4]. Rational systems have been observed in general engineering, chemical processes, physics, biological reactions, and econometrics, for example, rational models are a class of mechanistic models in describing catalytic reactions in chemical kinetics [5, 6], metabolic, signal and genetic networks in systems biology [1], and movement of satellite in earth orbit [1]. There have also been reports of rational modelling applications [7-9].

This is more concise in structure than a polynomial; the example below uses a Taylor series expansion to approximate a simple rational model below.

$$y(k) = \frac{\sin(u(k-1))}{1+y^2(k-3)} = \sin(u(k-1))[1-y^2(k-3)+y^4(k-3)\cdots] \tag{1.2}$$

The other characteristic of the rational models is the power in quick change of the model output while input has small variations. Consider a simple system output below

$$y(k) = \frac{1}{1+u(k-2)} \tag{1.3}$$

Clearly the model output will be dramatically increased, as the input $u(k-2)$ approaches –1. This comes from function of the denominator.

Introducing a denominator polynomial, makes model concise in describing complexity and add more functions in describing nonlinearities. On the other side, contrast to polynomial systems, this makes identification and control system design noticeably different and more difficult with the inherent nonlinear parameters and control inputs [2]. Therefore, it requests comprehensive studies of this class of systems in theoretical and application aspects. This study takes the pioneer step towards to the control of the rational systems

1.2. Model identification

Model identification has been relatively mutual to some extent. So far, the identification aspect has gone through data-driven model structure detection, parameters estimation and model validation from noise contaminated input and output data. The major work on rational model identification is summarised in the following categories. Linear Least Squares (LLS) algorithms for parameter estimation: Extended LLS estimator [10]; Recursive LLS estimator [11]; Orthogonal LLS structure detector and estimator [12]; Fast orthogonal algorithm [13]; Implicit least squares algorithm [14]. Nonlinear least square algorithms: Prediction error estimator [15]; Globally consistent nonlinear least squares estimator [16]. Other algorithms include the following categories. Back Propagation (BP) algorithm [17]; Enhanced Linear Kalman Filter (EnLKF) [18].

Model validation: Higher order correlation tests [19]. Omni-directional cross-correlation tests [20].

A summary of the representative publications till 2015 can be found in a survey of rational model identification [2].

1.3. Controller design

As surveyed above, rational models have been increasingly used to represent nonlinear dynamic plants. Consequently, the control system design should have been considered on agenda in the follow up studies. However, up to now, there is no reference found for designing such controllers directly referred to the model analytical knowledge. The paramount difficulty is that part of the controller output is embedded in the denominator polynomial $D_p(k)$. For example: $y(k) = \frac{0.5y(k-1)-y(k-2)u(k-1)+0.1u^5(k-1)}{1+y^2(k-2)+0.2u^2(k-1)}$. With extensive investigations through major academic publication searching engines, it can be concluded that this study is the first trial with analytical approaches to design a controller for rational systems.

Regarding controller design approaches possibly referred to the rational systems, these could be the reduction of rational model structure complexity, which are neural network models, linear approximation models, linearization, and iterative learning control, and U-model enhanced control. A brief critical review of the approaches is presented below.

Neural controller [21]: this is probably the first publication relating to control of rational models. However, the design approach has merely used rational models as extreme nonlinear examples, it has not designed controllers by taking the model structure into consideration (even known in advance), except taking the models as the representatives of complex nonlinear dynamic systems.

Piecewise linearization [22, 23] around operating points has been widely studied to simplify controller designed procedures when plants are subject to mild nonlinear dynamics. It should be mentioned that a group of piecewise linear models can be admitted as a linear model, with varying order and parameters in different operating intervals. The promising property is using linear control design strategies directly. However, it could induce inaccuracy and dynamic uncertainty because of ignoring some inherent nonlinearities from their original nonlinear representations. Further this method may also increase computational burden/complexity while over barrowing piecewise linear intervals to match severe nonlinearities.

Point-wise linearisation has been claimed by neural network based control and/or adaptive control, which uses linear models to approximate predominant dynamics around an operating point or every input output dynamic gain at each time instance, and then employs a neural network to determine the error induced by the linearisation [24, 25]. Once again, it uses linear control systems design to construct nonlinear control systems. However, this involves on-line neural network learning or online model iden parameter estimation, and therefore the constructed nonlinear control system is operated under adaptive principles (the controller parameters are updated with the neural network output), even for deterministic nonlinear plants. The other related issue is the selection of neural network topology, which has no systematic procedure available to find the best fitted neural network representative.

Feedback linearization is a well-developed subject [26]. A general SISO nonlinear system is described as

$$\begin{aligned} \dot{x} &= f(x) = g(x)u \\ y &= h(x) \end{aligned} \tag{1.4}$$

where x is the state vector, u and y are input and output respectively. $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ are real valued and smooth mapping functions. With this model structure a series of analogies with some fundamental features of linear control systems have been established, which provides a very useful concept in design of nonlinear control systems using linear design methodologies. Obviously the model has u in an explicit position. The studied nonlinear rational model has no such explicit expression for input u to be designed, and this immediately reveals that the methodologies rooted in the approach, although useful references, are not directly applicable in designing control of nonlinear rational systems. The other input-output linearization techniques [27] have had similar requirements for an explicit u expression and special skills for state variable transformation.

Iterative learning/data-driven control/Model-free control is another possible control system design methodology in avoiding model structure complexity. The approaches do not require clear plant model structure, but still need plants with some mild conditions in control [28, 29]. Again, if a rational model is available, it is wasteful without using the model information in the control system design. It is believed, particularly for man-made engineering systems/products (built up by rules/models), that any repetitive process and motion has model exist in operation even though the model is yet to identify.

U-model based control has claimed to radically relieve the dependence of plant model oriented design foundation. The use of the plant model is effectively reduced as a reference for converting to U-model and accordingly to work out the control output [30]. U-model based control assumes the feasibility of using linear system design procedures to design the control of nonlinear dynamic plants with assigned response performances. The U-model control platform is illustrated in Fig. 1.

The U-model systematically converts smooth (polynomial and extended including transcendental functions) models, derived from principles or identified from measurements, into a type of u-based model to equivalently describe plant input/output relationship, so that it establishes a general platform to facilitate control system design and dynamic inversion. It should be mentioned that there is nothing lost with the derived U-models from the original nonlinear models. The difference between the two types of model expressions is that those original nonlinear models could be obtained from principles, such as Newton's law, or identified directly from measured data, the U-models are derived from the original models in control design oriented expressions. Regarding the U-control (U-model based control) research status, Zhu and Guo [31] have brought forward a fundamental framework in terms of pole placement control for nonlinear systems. More recently, U-control has been expanded to General Predictive Control [32] and Sliding Mode Control [30]. To accommodate U-control of state space models, a Backstepping algorithm is being expanded to extract the controller output within multi loop U-models. With the nature of separating control system design (specifying closed loop performance) and controller output calculation (by resolving plant dynamic inversion through U-model), it can be forecast that the other classical control issues could be similarly formulated within a general and concise framework.

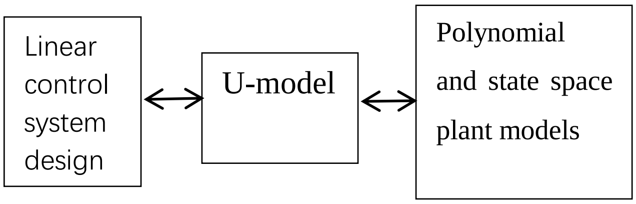


Fig. 1.*U-model based control systems design*

1.4. Organisation of the study

The remaining study is organised in five major sections. Section 2 is used to define a generic framework of control oriented U-model for representing smooth nonlinear dynamic plants. It is then expanded with including rational model and transcendental functions as its subsets to lay a basis for applying linear control system design techniques. Section 3 proposes a general pole placement controller for nonlinear rational systems within the U-model framework. Section 4 shows design of adaptive UPPC for the control of stochastic nonlinear rational systems. Section 5 tests a number of typical rational systems with the developed procedures and show the exemplary procedures for potential users.

2. U-model --- a generic framework of control oriented nonlinear plant models

2.1. U-model foundation --- polynomial [30]

Consider a general polynomial description of.

$$\begin{aligned} y(k) &= N_p(Y_{k-1}\ U_{k-1}) \\ &= \sum_{i=0}^{L} p_i(k)\theta_i \end{aligned} \tag{2.1}$$

where $y(k) \in \mathbb{R}$ and $u(k) \in \mathbb{R}$ denote the plant output and input at time instant $k(=1,2,\cdots)$ respectively. $N_p(\cdot) \in \mathbb{R}$ is a real valued and smooth polynomial function, $Y_{k-1} \in \mathbb{R}^n \supset y(k-1)$,..., $y(k-n)$ and $U_{k-1} \in \mathbb{R}^n \supset u(k-1)$,..., $\mathrm{u}(k-n)$ denote the delayed outputs and inputs respectively. $p_i(k) \in \mathbb{R}$ denotes the model structure variables, e.g. $u(k-2)y^3(k-1)$, $u(k-1)u^2(k-3)$, $y(k-2)y(k-3)$, and $\theta_i \in \mathbb{R}$ denote the coefficients. To convert the above polynomial into U-model, which is a polynomial with argument of control input $u(k-1)$ (also called controller output while talking about control system design), it gives [30]

$$y(k)=\sum_{j=0}^{M}\lambda_j(k)u^j(k-1) \qquad (2.2)$$

where degree M is of controller output $u(k-1)$, $\lambda(k)=[\lambda_0(k) \quad \cdots \quad \lambda_M(k)]\in\mathbb{R}^{M+1}$ is the time varying parameter vector, a function of absorbing past inputs U_{k-2}, outputs Y_{t-1}, and parameters θ_{nj} in the original polynomial. An example illustrating the conversion to U-model from an ordinary polynomial is shown here. Consider a polynomial,

$$y(k)=0.2y(k-1)y(k-3)+0.5u(k-1)u(k-3)-0.9y(k-2)u^2(k-1) \qquad (2.3)$$

Rearrange polynomial (2.3) with

$$y(k)=\lambda_0(k)+\lambda_1(k)u(k-1)+\lambda_2(t)u^2(k-1) \qquad (2.4)$$

where $\lambda_0(k)=0.2y(k-1)y(k-3)$, $\lambda_1(k)=0.5u(k-3)$, and $\lambda_2(k)=-0.9y(k-2)$.

Clearly, the time varying $\lambda_j(k)$ is absorbing the past inputs/outputs and parameters of the original polynomial, associated with $u^j(k-1)$.

Property 1: Assign φ: $\mathbb{R}^{L+1}\to\mathbb{R}^{M+1}$ a U-mapping to convert classical polynomial expression of (2.1) to its U-expression of (2.2) and the inverse be φ^{-1}, that is

$$f(p_i,\ \theta_i)\xrightarrow{\varphi} f(u^j,\ \lambda_j) \qquad (2.5)$$

Thus it has good mapping properties [30].

2.2. U-mode --- rational

With reference to (1.1), its deterministic parametric rational expression is given below.

$$y(k)=\frac{N_p(*)}{D_p(*)}=\frac{\sum_{j=1}^{num}p_{nj}(k)\theta_{nj}}{\sum_{j=1}^{den}p_{dj}(k)\theta_{dj}} \qquad (2.6)$$

Its U-model realisation can be determined by removing the denominator to the left hand side of (2.6), it gives

$$y(k)D_p(*)=N_p(*) \qquad (2.7)$$

Convert (2.7) into U-model form to yield

$$y(k)\sum_{i=0}^{L}\gamma_i(k)u^i(k-1)=\sum_{j=0}^{M}\lambda_j(k)u^j(k-1) \qquad (2.8)$$

where $\lambda_j(k)\in\mathbb{R}$ is a function of past inputs U_{k-2} and outputs Y_{k-1}, and parameters θ_{nj} in the numerator polynomial. Similarly, $\gamma_i(k)\in\mathbb{R}$ is a function of past inputs U_{k-2} and outputs Y_{k-1}, and parameters θ_{dj} in the denominator polynomial. M and L are the degrees of the model input $u(k-1)$ in numerator and denominator respectively. Here is a simple example to show the conversion of

$$y(k)=y(k-1)\frac{1}{u(k-1)} \qquad (2.9)$$

Inspection of (2.8), it gives

$$y(k)\gamma_1(k)u(t-1)=\lambda_0(k) \qquad (2.10)$$

where $\gamma_1(k)=1 \quad \lambda_0(k)=y(k-1)$.

In the following sections of the controller design, it is required making dynamic inversion of (2.8) in way of root-solving.

There are many standard root-solving algorithms for such polynomial equations [30].

Remark 1: Compared with polynomial U-realisation, it can be noted that rational model U-realisation is an implicit expression of $y(k)$ due to the multiplicative item $y(k)D_p(k)$.

2.3. U-model --- extended

To describe more general nonlinear terms including those transcendental functions, define the extended U-model below

$$y(k)f_b(u(k-1))=f_a(u(k-1)) \qquad (2.11)$$

where $f_b(u(k-1))\in\mathbb{R}$ and $f_a(u(k-1))\in\mathbb{R}$ are smooth functions. In general, these can be expressed as

$$\begin{aligned} f_b(u(k-1))&=\sum_j f_{bj}(u(k-1)) \\ f_a(u(k-1))&=\sum_j f_{aj}(u(k-1)) \end{aligned} \qquad (2.12)$$

Here is a simple example to show its U-model representation, consider

$$y(k)=\frac{y(k-1)\sin(u(k-1))}{1+\cos^2(u(k-1))} \qquad (2.13)$$

For its U-model of (2.11), it gives

$$\begin{aligned} f_b(u(k-1))&=\gamma_0(k)+\gamma_1(k)\cos^2(u(k-1)) \\ f_a(u(k-1))&=\lambda_1(k)\sin(u(k-1) \end{aligned} \qquad (2.14)$$

where $\gamma_0(k)=1 \quad \gamma_1(k)=1 \quad \lambda_1(k)=y(k-1)$

3. Pole placement controller – A show case of the design procedure

The control objective is, for a desired trajectory $v(k)$, determine a control input $u(t)$ to drive the underlying system output $y(k)$ to follow the desired trajectory $v(k)$ with an acceptable performance (such as transient response and steady-state error), while all the inputs and outputs of the control system are bounded within the permitted ranges.

3.1. U-control system design

In general, there are three steps in the U-control system design routine:

Form a proper linear feedback control system structure, as shown in Fig. 2. The controller, in the dashed line block, is consist of two functions, invariant controller G_{c1} and dynamic inverter G_p^{-1}. The plant model is G_p.

Design invariant controller G_{c1} by linear control system approach. By letting $G_p = 1$, therefore, $G_p^{-1} = 1$, and specifying the desired closed loop transfer function G, it gives $G_{c1} = \dfrac{G}{1-G}$ and the invariant controller output $v(t)$ is the desired output while the plant model is a unit constant.

Determine dynamic inverter G_p^{-1} to work out the controller output $u(k-1)$. Assuming the plant model is bounded-input/bounded-output (BIBO) stable and the inverse of G_p exist, expressing the plant model G_p in forms of U-model, letting $y(k) = v(k)$ in the U-model, it gives model (2.11) in expression of $v(k)\,f_b(u(k-1)) = f_a(u(k-1))$. To determine control input $u(t-1)$ is to find the inverse by resolving the equation of $v(k)\,f_b(u(k-1)) - f_a(u(k-1)) = 0$.

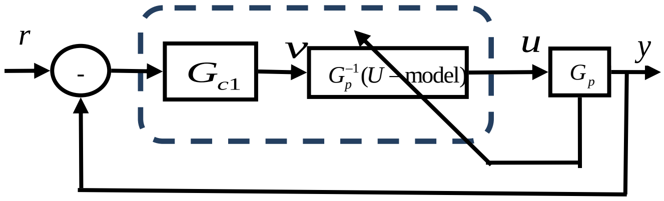


Fig. 2.*U-model control system*

It should be noted that the arrow line from the plant to the dashed line block represents the U-model update from the plant model at each time instance.

Proposition 1: Generality, U-model based control allows once-off design for all linear and nonlinear polynomial models. This is due to the controller G_{c1} design being independent of model G_p.

Proposition 2: Simplicity, U-model based control requires no repeated computation if a plant model is changed. Again, this is due to the controller G_{c1} design being once-off and independent of model G_p, and changes to plant model G_p only changing the U-model to resolve different roots. In comparison, almost all classical and modern control approaches are plant model based designs, that is, the controller design is a function of both system performance and plant, accordingly if the plant model is changed, and controller must be redesigned.

Proposition 3: Feasibility for controller design of rational systems, this is can be proved directly from proposition 1 and U-realisation of the rational model in (2.8).

In formality, U-adaptive control is very similar to deterministic U-model control. The difference is that the plant model is required to be estimated or updated online in adaptive control.

For simplicity, but not losing generality, in formulation of U-model --- polynomial, once the invariant controller is designed, the real controller output can be determined by letting

$$v(k) = \sum_{j=0}^{M} \lambda_j(k) u^j(k-1) \qquad (3.1)$$

Then resolving one of the roots from

$$v(k) - \sum_{j=0}^{M} \lambda_j(k) u^j(k-1) = 0 \qquad (3.2)$$

3.2. Stability and robust analysis of U-model control systems

There are two typical situations: Ideal case: deterministic systems without modelling error and disturbance. Non-ideal case: deterministic systems with modelling errors and/or disturbance.

Theorem 1: Bounded-input, bounded-output (BIBO) stability of deterministic U-model control systems
Regarding the U-model control system shown in Fig. 2, it is BIBO stable and tracks the bounded reference signal r properly while the following conditions are satisfied:

(i) Invariant controller G_{c1} is closed-loop stable, that is, all poles of the closed loop are located with the unit circle.

(ii) Plant model G_p is a bounded-input/bounded-output (BIBO).

(iii) The inverse of the plant model G_p^{-1} exits.

Proof: With reference to Fig. 2, it has $G_p^{-1}G_p = 1$ from the conditions (ii) and (iii). Accordingly, the closed loop transfer function is given in terms of $\dfrac{G_{c1}}{1+G_{c1}}$, which is stable from (i) and thus the tracking performance is given by $\dfrac{rG_{c1}}{1+G_{c1}}$. □

Remark 2: This establishes a framework for designing control for both linear and nonlinear dynamic plants. It is feasible, simple, general, and with no repetition of controller design on changes to the plant model, except the computation of the inversion of the changed plant U-model polynomial. In the other words, this is a new methodology for minimising the complexity induced by the plant model in control system design, which is particularly important for nonlinear plants. U-model, as a universal dynamic inverter, is the key to achieve the gaols.

Theorem 2: BIBO Stability of uncertain U-model control systems

Regarding the U-model control system structured in Fig. 2, modelling error and/or disturbance $\varepsilon_U(t)$ can be treated as an external disturbance as shown in Fig. 3. It is BIBO stable and tracking the reference signal with a bounded error while the following conditions are satisfied:

(i) Invariant controller G_{c1} is closed loop stable.

(ii) Plant model G_p is a bounded-input/bounded-output (BIBO).

(iii) The inverse of the plant model G_p^{-1} exits.

(iv) The upper bound of modelling error and/or disturbance $\varepsilon_U(t)$ is satisfied with the conditions of small gain robust stability [41].

Proof: In Fig. 3, $G_p^{-1}G_p = 1$ this gives $y = \frac{rG_{c1}}{(1+G_{c1})} + \frac{\varepsilon_U}{(1+G_{c1})}$.

Then the stability of Fig. 3 is the same as in Fig. 2 while the upper bound $\varepsilon_U(t)$ is satisfied with the small gain robust stability criterion.

Remark 3: It should be noted that the tracking error is determined by $\frac{\varepsilon_U}{(1+G_{c1})}$; therefore, a properly designed G_{c1} will have a degree of robustness against uncertainties/disturbance.

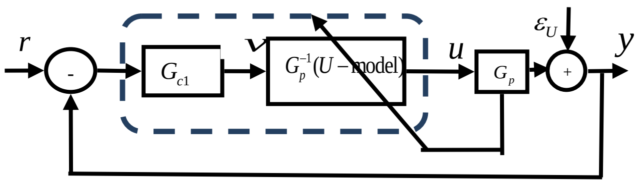


Fig. 3. *Uncertain U-model control system*

3. Design of pole placement controller

A classical approach [33] has been selected to formulate the U-model enhanced pole placement controller (UPPC) [30, 31]. Here a further refined version of UPPC is presented. Within U-model framework, closed loop control system performance is independently specified without involving the plant model. Therefore, the classical version involving plant model can be simplified as below.

$$Rv(k) = Tr(k) - Sy(k) \tag{3.1}$$

and

$$\begin{aligned} R &= z^n + r_1 z^{n-1} + \cdots r_n \\ T &= t_0 z^m + t_1 z^{m-1} + \cdots t_m \\ S &= s_0 z^l + s_1 z^{l-1} + \cdots s_l \end{aligned} \tag{3.2}$$

with $r(k)$ for reference, $v(k)$ for invariant controller G_{c1} output, and $y(k)$ for plant output. The polynomials *R*, *S*, and *T*, with backward shift operator z^{-1} and proper orders (*n*, *m*, and *l*), are used to specify closed loop control system performance.

To guarantee the control system realistically implementable, specify

$$O(S) < O(R) \Leftrightarrow l < n \quad O(T) \le O(R) \Leftrightarrow m \le n \tag{3.3}$$

where the operator $O(*) = Order(*)$ denotes the order of the concerned linear polynomial.

With reference to (3.1), two control roles can be assigned with negative feedback $-R/S$ for stabilising closed loop system with requested dynamics and feedforward T/R for reducing steady state errors. The structured control system is shown in Fig. 4.

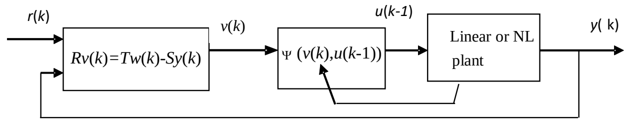


Fig. 4. *Structured UPPC control system*

For designing an invariant controller, let $v(t) = y(t)$ in (3.1), thus it gives the closed loop transfer function

$$y(k) = \frac{T}{R+S} r(k) = \frac{T}{A_c} r(k) \tag{3.4}$$

Accordingly, the required design task is to assign the closed loop denominator polynomial A_c and the numerator polynomial T.

It should be noted that after A_c specified (by customers and/or designers), a routine for resolving Diophantine is needed to workout the parameters of polynomials R and S from the following relationship

$$R + S = A_c \tag{3.5}$$

To achieve zero steady state, *T*can be designed with

$$T = A_c(1) \tag{3.6}$$

The detailed design procedure and examples can be refereed to [31].

Remark 4: Compared with classical pole placement control design procedures [33], the UPPC is more concise and independent of the plant model, which results in the UPPC being generalised to any plant model structure and once off designed. For each different plant model, this task is merely the resolving of the U-model to obtain one of the roots as the operational controller output. The relevant comparison details can be referred in [30].

4. U-model based pole placement control with adaptive parameter estimation

U-model based adaptive control schematic diagram is shown in Fig. 5. Again, this U-model adaptive control is different from those classical adaptive/self-tuning control approaches in terms of control structure. The feedback controller parameters are not tuned and thereafter fixed: the only adaptation is to update U-model parameters to accommodate the plant model parameter variation and/or external disturbance, which is consistent with propositions 1, 2 and 4.

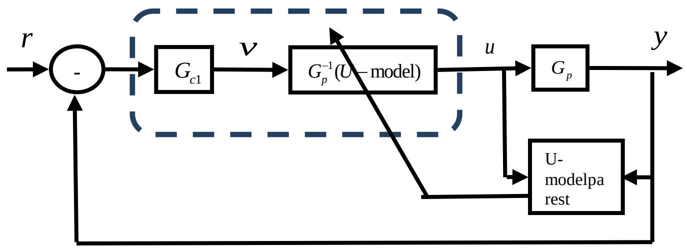


Fig. 5. *Adaptive U-model control system*

In general, an adaptive control system can be considered as a two layer system, that is,

Layer 1: conventional feedback control;

Layer 2: adaptation loop.

In this study, the UPPC presented in section 3 is selected to form conventional feedback control. Thus this section mainly develops this adaptation loop formulation.

In recursive formulation, there are two ways to estimate the U-model parameters in the adaptation loop.

Indirect parameter estimation: Estimate the original rational model parameters ($\theta_{nj}(k)$, $\theta_{dj}(k)$) first and then convert into U-model parameters $\lambda_j(k)$. The challenging issue is that classical recursive least squares estimation algorithms give biased estimates and recursive rational model estimators need noise variance information in advance [11, 18].

Direct parameter estimation: Estimate the U-model parameters $\lambda_j(k)$ directly. The challenging issue is that the parameters $\lambda_j(k)$, while converted from a rational model, are time varying at every sampling time. It has been proved [34] that for time-varying stochastic models, the parameter estimation errors (PEE) with the well-known forgetting factor least-squares (FFLS) algorithm are bounded and the FFLS is capable of reducing the squared measurement error (the difference between measured output and model predicted output) even the time-varying parameter estimate are not converged to their real values.

In this study a FFLS estimator [35] is selected with the following formulations

$$\begin{aligned}
&\varepsilon_U(k) = y(k) - \Psi^T(k)\hat{\lambda}(k-1)\\
&K(k) = \frac{P(k-1)\Psi(k)}{\rho + \Psi^T(k)P(k-1)\Psi(k)}\\
&\hat{\lambda}(k) = \hat{\lambda}(k-1) + K(k)\varepsilon_U(k)\\
&P(k) = [\mathrm{I} - K(k)\Psi^T(k)]P(k-1)
\end{aligned} \tag{4.1}$$

where vector $\hat{\lambda}(k) = \begin{bmatrix}\hat{\lambda}_0(k) & \hat{\lambda}_1(k) & \cdots & \hat{\lambda}_M(k)\end{bmatrix}^T \in \mathbb{R}^M$ is the estimate of $\lambda(k)$, $\varepsilon(k) \in \mathbb{R}$ is the error, that is, the difference between the measured output and the model predicted output, $K(t) \in (M+1)\times 1$ is the weighting factor vector indicating the effect of $\varepsilon(t)$ to change the parameter vector, $\Psi(k) = \begin{bmatrix}1 & u(k-1) & \cdots & u^M(k-1)\end{bmatrix}^T \in \mathbb{R}^M$ is the input vector at time k-1, ρ is forgetting factor (a number less than 1, e.g. 0.99 or 0.95, represents a trade-off between fast tracking and noisy estimate), the smaller value of ρ , the quicker the information in previous data will be forgotten, and $P(k) \in \mathbb{R}^{(M+1)\times(M+1)}$ is the covariance matrix.

In presenting the stability of the proposed adaptive U-control, expand the virtual equivalent system (VES) concept and methodology [36] for the analysis, which is an alternative insight and judgement of the stability/convergence for adaptive control systems. Following the similar arguments as shown before, we assume $G_p^{-1}G_p = 1$, and the invariant controller G_{c1} is well defined to stabilise conventional feedback control systems and track the bounded reference signal in terms of mean squares. Then for a slow time varying parameter model (because it is converted from its original time invariant parameter model referred to (2.1) and (2.2)), the U-model parameter estimation errors $\varepsilon_U(t)$ are bounded with FFLS or the other recursive algorithms [34, 37]. In this case, using Fig. 3 again, knowing $\varepsilon_U(t)$ includes U-model parameter estimation errors as. Hence, in terms of VES, the adaptive control system can be treated as a summation of two sub-systems of

$$y = y_1 + y_2 = \frac{rG_{c1}}{(1+G_{c1})} + \frac{\varepsilon_U}{(1+G_{c1})} \tag{4.2}$$

As $\varepsilon_U(t)$ is bounded, the adaptive control system is stable and the tracking control error will converge to a bounded compact set around zero, whose size depends on the ultimate bounds of estimation error ε_U .

Remark 5: U-model provides a platform for simplifying control system design and VES provides a platform for simplifying the analysis of stability and convergence of general adaptive control systems.

5. Simulation studies

Four case studies have been conducted to initially validate the new design procedure. It should be made clear that there is no other comparison result can be provided as this is the first study in controlling of such nonlinear rational systems.

As described before, the design is split into two stages, design invariant control G_{c1} (thus $v(k)$) by pole placement) and determination of the controller output $u(k-1)$ by resolving plant U-model equation.

To design the pole placement controller, assign the characteristic equation

$$A_c = z^2 - 1.3205z + 0.4966 \tag{5.1}$$

Factorisation of (5.1) gives the closed loop poles as $0.6603 \pm 0.2463i$, this gives a decayed oscillatory response ($\omega_n = 1 \quad \zeta = 0.7$), which is a commonly used dynamic response index. For steady state error performance, making its error zero gives

$$T = A_c(1) = 1 - 1.3205 + 0.4966 = 0.1761 \qquad (5.2)$$

From the causality condition, specify the structures of R and S with

$$\begin{aligned} R &= z^2 + r_1 z + r_2 \\ S &= s_0 z + s_1 \end{aligned} \qquad (5.3)$$

Form a Diophantine equation with polynomials $A_c \quad R \quad S$ [30] to yield

$$\begin{aligned} r_2 + s_1 &= 0.4966 \\ r_1 + s_0 &= -1.3205 \end{aligned} \qquad (5.4)$$

To make polynomial R stable and having the requested response, assign $r_1 = -0.06$, $r_2 = 0.0005$, which gives two poles $(z - 0.05) \quad (z - 0.01)$. Then the coefficients of polynomial S are resolved in the Diophantine equation of (5.4) as follows.

$$s_0 = -1.2605 \quad s_1 = 0.4961 \qquad (5.5)$$

Consequently controller (3.1) can be recursively implemented to calculate the virtual controller output $v(t)$

$$\begin{aligned} v(k+1) &= 0.06v(k) - 0.0005v(k-1) + 0.1761r(k-1) \\ &\quad + 1.2605y(k) - 0.4961y(k-1) \end{aligned} \qquad (5.6)$$

5.1. Case 1 --- feasibility test of U-control of nonlinear rational systems

Consider a rational system modelled by

$$y(k) = \frac{0.5y(k-1)u(k-1) + u^3(k-1)}{1 + y^2(k-1) + u^2(k-1)} \qquad (5.7)$$

where $y(k)$ is the model output, $u(k)$ is the input of the model or controller output. This is used to test deterministic feedback control. The model structure has been typically investigated in system identification. Accordingly, its U-realisation can be expressed as

$$y(k)\left(1 + y^2(k-1) + u^2(k-1)\right) = 0.5y(k-1)u(k-1) + u^3(k-1) \qquad (5.8)$$

To obtain the dynamic inverter G_p^{-1} output, that is, the controller output $u(t)$, let $y(k) = v(k)$, then it gives rise to

$$v(k)\left(1 + y^2(k-1) + u^2(k-1)\right) = 0.5y(k-1)u(k-1) + u^3(k-1) \qquad (5.9)$$

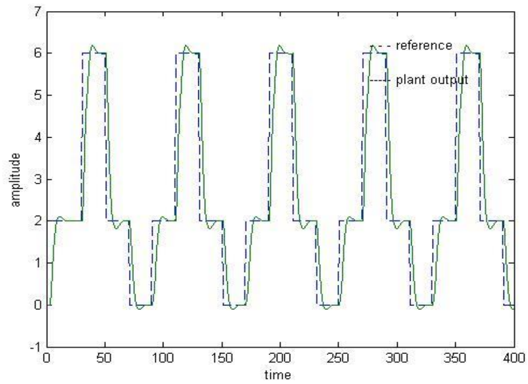
To determine the control input $u(k\text{-}1)$, form a U-model equation from (5.9) as

$$\lambda_0(k) - \lambda_1(k)u(k-1) + \lambda_2(k)u^2(k-1) - \lambda_3(k)u^3(k-1) = 0 \qquad (5.10)$$

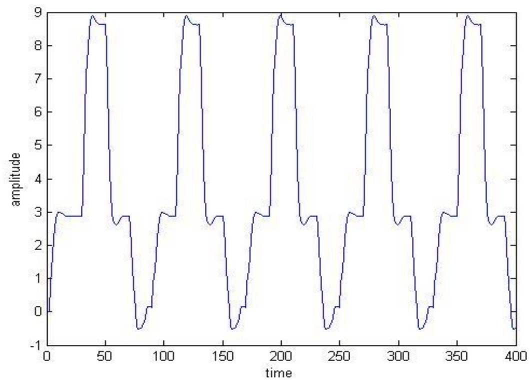
where

$$\begin{aligned} \lambda_0(k) = v(k)(1 + y^2(k-1)) &\quad \lambda_1(k) = 0.5y(k-1) \\ \lambda_2(k) = v(t) &\quad \lambda_3(k) = 1 \end{aligned} \qquad (5.11)$$

In this simulation, the operation time length was configured with 400 sampling points, and reference was a sequence of multi-amplitude steps. The achieved output response and controller output are shown in Fig. 6(a) and Fig. 6(b) respectively.



(a) Plant output response



(b) Control input

Fig. 6. *Plant output and control input*

5.2. Case 2 --- test of external disturbance

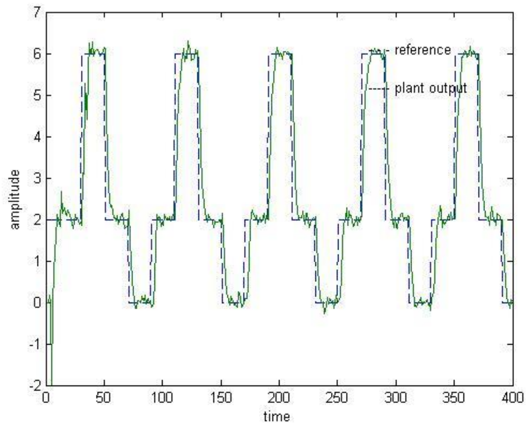
Consider a stochastic rational system modelled by

$$y(k) = \frac{0.5y(k-1)u(k-1) + u^3(k-1)}{1 + y^2(k-1) + u^2(k-1)} + e(k) \qquad (5.12)$$

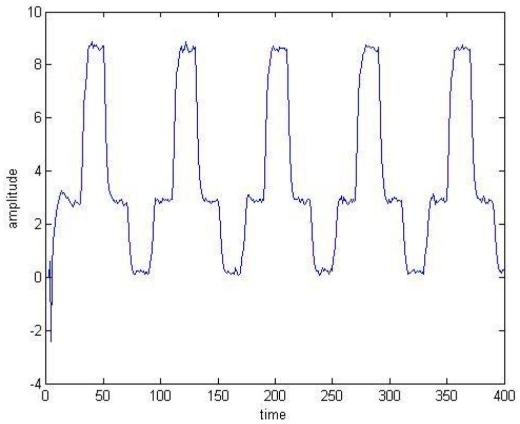
where $y(k)$ is the model output, $u(k)$ is the input of the model or controller output respectively, and $e(k)$ is Gaussian noise representing unknown an disturbance acting on the controlled plant output.

This case study was used to test adaptive feedback control. The feedback control loop has been designed as in

Case 1, that is, all configurations for feedback control were kept as those used in Case 1. For the adaptation loop, the disturbance was configured with $e(k) \sim N(0, 0.01)$, the initial covariance matrix with $P(k) = 10^6 I_4$, the forgetting factor with $\rho = 0.95$ to deal with fast time varying parameter estimation, the initial parameter vector was randomly assigned with $\hat{\lambda}(0) = \left[\hat{\lambda}_0(0) \quad \hat{\lambda}_1(0) \quad \hat{\lambda}_3(0) \quad \hat{\lambda}_4(0)\right]^T = \left[0.3 \quad 0.2 \quad 0.1 \quad 0.1\right]^T$, and the input vector was specified with $\Psi(k) = \left[1 \quad u(k-1) \quad u^2(k-1) \quad u^3(k-1)\right]^T$. The achieved output response and controller output are shown in Fig. 7(a) and Fig. 7(b) respectively.



(a) Plant output



(b) Control input

Fig. 7. *Plant output and control input*

5.3. Case 3 --- test of internal parameter variation

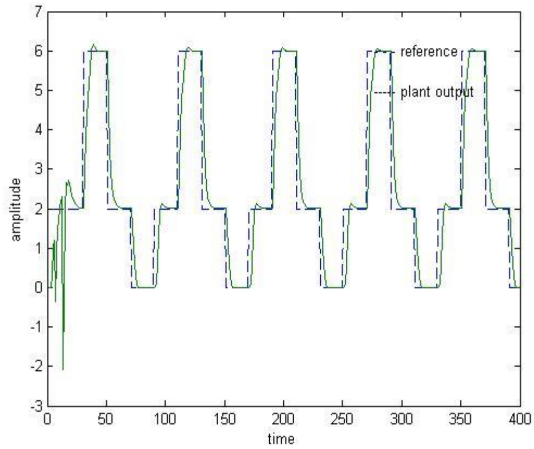
The same model structure as Case 1 is used, but the parameter associated with $y(k\text{-}1)\ u(k\text{-}1)$ is time varying representing internal parameter disturbances, such as worn parts in mechanical and electrical systems.

$$y(k) = \frac{a(k)y(k-1)u(k-1) + u^3(k-1)}{1 + y^2(k-1) + u^2(k-1)} \tag{5.13}$$

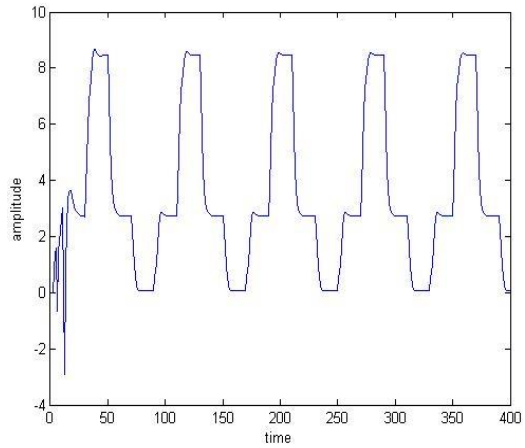
In simulation, all the setups were the same as those used in Case 1. The parameter variation was configured as

$$a(k) = \begin{cases} 0.9 & 120 \le k \le 250 \\ 0.5 & otherwise \end{cases} \tag{5.14}$$

The adaption loop, specified as in Case 2, was used to follow the plant model internal structure variation. The achieved output response and controller output are shown in Fig. 8(a) and Fig. 8(b) respectively. Inspecting the simulation results, the output of the systems are seen to track the reference signals after a short transient phase. U-model parameter estimation is shown in Fig. 9. It should be noted that this estimated parameter vector is to achieve smaller squared error between measured output and model predicted output. Therefore the estimates are not converged to those real time varying parameters in the U-model. In the future studies to deal with time varying parameter estimation will be conducted in terms of reducing both squared measurement errors and squared dynamic errors [40].



(a) Plant output



(b) Control input

Fig. 8. *Plant output and control input*

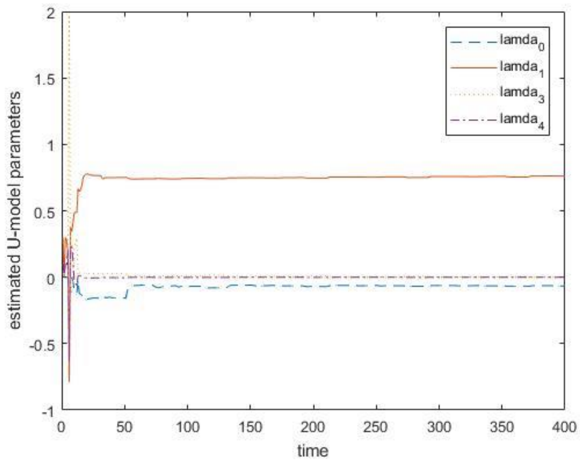


Fig. 9. *U-model parameter estimates*

5.4. Case 4 --- feasibility test of U-control of extended nonlinear rational systems

This study is used to test the U-control of extended rational systems with transcendental input and delayed output.

$$y(k)=\frac{0.5y(k-1)+\sin(u(k-1))+u(k-1)}{1+\exp(-y^2(k-1))} \tag{5.15}$$

where $y(k)$ is the model output, $u(k)$ is the input of the model or controller output. Accordingly, the extended U-model can be expressed as

$$y(k)\left(1+\exp(-y^2(k-1))\right)=0.5y(k-1)+\sin(u(k-1))+u(k-1) \tag{5.16}$$

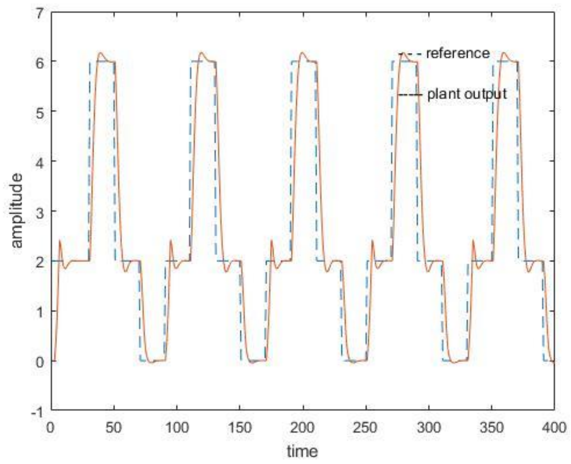
With the same controller designed in (5.16) above, assign the output $y(k)$ of (5.16) with the desired output $v(k)$ of (5.6) gives

$$v(k)\left(1+\exp(-y^2(k-1))\right)=0.5y(k-1)+\sin(u(k-1))+u(k-1) \tag{5.17}$$

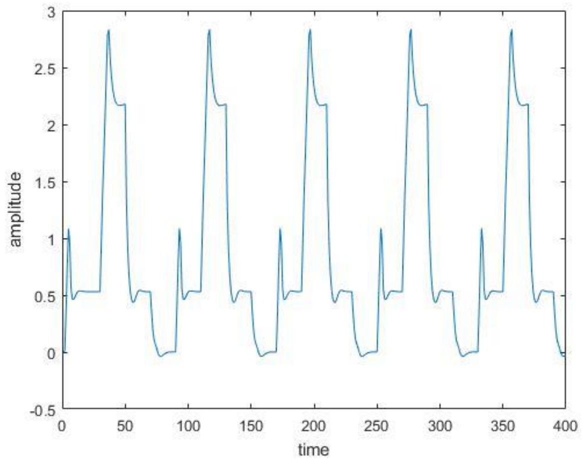
Therefore the control input $u(k-1)$ can be solved by

$$v(k)\left(1+\exp(-y^2(k-1))\right)-0.5y(k-1)-\sin(u(k-1))-u(k-1)=0 \tag{5.17}$$

The achieved output response and controller output are shown in Fig. 10(a) and Fig. 10(b) respectively. Once again the computational experiment confirms the feasibility of U-control.



(a) Plant output



(b) Control input

Fig. 10. *Plant output and control input*

6. Conclusions

A fundamental question is raised in this study and those for the other U-model enhanced controls: after two generations of plant model (polynomial and state space) centered control system design research/applications, what is the next generation of development? Should the research for new model structures continue, or should control systems be designed without such plant model requirements (possibly implying separation of control system design and controller output determination)?

One of the feasible choices in the future progression could be the U-control design methodology, which radically reduces the complexity of plant model oriented design methods. The proposed U-control method provides a platform with 1) a universal control oriented structure to represent existing models; 2) separating closed control system design from plant model structure (no matter linear or nonlinear, polynomial or state space); 3) all well-developed linear control system design methods can be expanded in parallel to nonlinear plant models, 4) a supplementary to all existing control design methods. Accordingly, this study is a show case using the U-model framework to design the control of the nonlinear rational systems with classical linear design approaches. Further study on the rational model control could be deriving concise algorithms for robust and adaptive control with reference to the recent research development [38-39].

This foundation work has put an emphasis on formulation of structure in a systematic approach. Rigorous mathematical considerations should be followed to establish a comprehensive description and explanation.

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