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THE ANGULAR BROADENING OF RADIO SOURCES BY SCATTERING IN THE INTERSTELLAR MEDIUM

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SUMMARY

The variation with direction of interstellar scattering at 81·5 MHz has been determined by two independent methods based on interplanetary scintillation measurements at 151·5 MHz, presented here, and at 81·5 MHz, already published. The scattering angle, which increases at low galactic latitudes, has a value of $0''\cdot15 \pm 0''\cdot05$ at 81·5 MHz for the lines of sight perpendicular to the galactic plane. The effect of the scattering on the apparent angular diameters of OH and H_2O maser sources is discussed.

1. INTRODUCTION

The apparent angular sizes of radio sources may be significantly increased by scattering in the interstellar medium; the effect is particularly severe for sources observed at low galactic latitudes, but even at high latitudes it imposes limits on the angular resolution obtainable. The scattering angle, θ_S, is defined to be the diameter (to $1/e$) of the angular spectrum of the scattered radiation. It is a function of frequency and direction and can be inferred from pulsar scintillation data (Falgarone & Lequeux 1973), or more directly from measurements at low frequencies of the angular sizes of extragalactic sources. Readhead & Hewish (1972) have used interplanetary scintillation data on ~400 radio sources for a preliminary investigation of the directional dependence of the scattering at 81·5 MHz. In this paper we present the results of scintillation observations of 32 radio sources at 151·5 MHz and from them, together with measurements at 81·5 MHz (Readhead & Hewish 1974), determine the variation with galactic latitude of the interstellar scattering at 81·5 MHz. We have also extended the investigation by Readhead & Hewish, using their survey of ~1500 sources at 81·5 MHz (1974). There is good agreement between the results of the two methods and with those derived from pulsar scintillation measurements.

It has been suggested that the observed diameters of OH and H_2O maser sources are due to scattering in the interstellar medium rather than to the intrinsic diameters of the sources themselves. This scattering may take place in the general interstellar medium (Burke *et al.* 1968; Johnston *et al.* 1971; Litvak 1971; Moran *et al.* 1973) or in intervening H II regions (Boyd & Werner 1972; Little 1973). We conclude, as did the latter authors, that the scattering in the general interstellar medium is not sufficient to produce the observed angular diameters and that there must be additional scattering for these sources.

2. THE OBSERVATIONS AT 151·5 MHZ

The angular diameters of 32 strongly-scintillating sources were measured at Cambridge during 1973 January to December using the 10 000-m^2 parabolic-trough antenna at 151·5 MHz. The half-power beamwidths were 30′ in right ascen-

TABLE I

The results of the survey at 151·5 MHz

Source*	Apparent angular diameter θ_A (″)	Error ± (″)	Percentage of total flux scintillating
2	0·4	0·2	100
43	0·5	0·2	100
48	0·25	0·05	100
53	0·5	0·2	100
67	0·3	0·2	100
CTA 21	0·25	0·1	100
99	0·4	0·2	70
119	0·2	0·1	100
138	0·3	0·05	100
147	0·3	0·1	100
161	0·5	0·3	50
4C 33.21	0·5	0·3	100
181	0·2	0·2	35
186	0·5	0·1	85
196	1·0	0·2	50
216	0·6	0·2	55
222	0·3	0·1	100
225	0·4	0·2	54
230	0·5	0·3	80
237	0·4	0·2	100
238	0·4	0·2	60
241	0·2	0·1	80
255	0·1	0·1	100
256	0·4	0·2	70
263.1	0·5	0·3	50
267	0·6	0·2	60
273	0·5	0·2	40
275	0·7	0·1	100
279	0·5	0·2	60
286	0·4	0·2	80
287	0·4	0·3	100
298	0·4	0·2	60
445	0·4	0·2	25
446	0·1	0·1	100
454	0·1	0·2	50
454·3	0·3	0·2	100
459	0·4	0·3	60

* 3C numbers are quoted unless otherwise specified.

sion and 4° in declination, δ. The two halves of the antenna were operated as a phase-switched interferometer and both the 'sin' and 'cos' outputs were used so that sources could be observed for 2 sec δ min at transit. The bandwidth was 1 MHz and the post-detector time-constant was 0·1 s. Each source was observed at weekly intervals while its solar elongation ϵ was in the range $20° < \epsilon < 80°$, and its scintillation index, defined as $F = \Delta S/S$ where S is the total flux and ΔS is the rms variation due to scintillations, was measured in the manner described by Hewish & Burnell (1970). Analogue records of the sin and cos receiver outputs, together with their respective filter and integrator outputs, were processed by hand so that interference could be eliminated. Both ΔS and S for each source were measured from the same chart thereby avoiding the necessity of daily calibrations.

The apparent angular diameters were determined by comparing (Readhead 1971) the observed variations of F as a function of ϵ with the variations expected for different values of the source diameter. The curves were calculated by assuming that the irregularities of electron density have a gaussian spectrum with the characteristic scale measured by Houminer (1973) as a function of ϵ. This scale size was slightly different from that originally used by Readhead, but substitution for his value made less than 0″·05 difference to the angular diameters derived. The results are presented in Table I where the 151·5 MHz diameters are given with their standard errors, which range from ±0″·05 to ±0″·3. Each set of measurements was differently affected by such things as interference, solar activity, signal : noise ratio, and the prevailing conditions in the solar plasma, so that the angular diameters could not all be measured with the same accuracy.

3. THE ANGULAR BROADENING OF RADIO SOURCES AT 81·5 MHZ AND 151·5 MHZ

Since θ_S varies inversely as the square of the frequency, its value may be derived by comparing the apparent angular diameters, θ_A, of an extragalactic source measured at two frequencies. If $\theta_I(\nu)$ is the intrinsic angular diameter (to $1/e$) at frequency ν MHz of the scintillating component (assumed to have a gaussian profile), then in an extragalactic source the apparent angular diameter of the scintillating component is given by

$$\theta_A(\nu) = [\theta_I^2(\nu) + \theta_S^2(\nu)]^{1/2}$$

(Readhead & Duffett-Smith 1975). The equivalent expression for a galactic source is $\theta_A \simeq [\theta_I^2 + (\theta_S/2)^2]^{1/2}$ as explained in the Appendix. Thus, for an extragalactic source, $\theta_A(81{\cdot}5)$ is given by $\theta_A(81{\cdot}5) = [\theta_I^2(81{\cdot}5) + \theta_S^2(81{\cdot}5)]^{1/2}$. The assumption that θ_I is not very different at the two frequencies leads to

$$\theta_S(81{\cdot}5) \simeq [\theta_A^2(81{\cdot}5) - \theta_A^2(151{\cdot}5)]^{1/2}.$$

We have used the new measurements of angular diameters at 151·5 MHz, together with those of Readhead & Hewish (1974) at 81·5 MHz, to compute $\theta_S(81{\cdot}5)$ directly for 32 extragalactic radio sources.

Our assumption that θ_I does not vary significantly with frequency needs justification. The evidence is as follows:

(i) Measurements at 81·5 MHz and 151·5 MHz of angular diameters for most of the sources listed in Table I agree within the errors except at low latitudes where interstellar scattering is large.

(ii) A few of the larger sources in our 151·5 MHz sample have been observed by the 5-km telescope at 5 GHz; the angular sizes at 5 GHz were inferred from the fringe visibilities at the largest spacing and agree within the errors. Examples are 3C 67 and 3C 196 (Pooley & Henbest 1974), and 3C 186 and 3C 263.1 (J. M. Riley, private communication).

(iii) The effects of self-absorption are unlikely to matter since, if $B \gtrsim 10^{-6}$ gauss, they are only important for sizes $< 0''{\cdot}03$ at our frequencies. (We have omitted 3C 273 which has a complex spectrum.)

(iv) The bulk of our sample consists of steep-spectrum quasars which have structure at low frequencies essentially independent of frequency (Wilkinson, Richards & Bowden 1974; Readhead & Hewish 1975, in preparation).

The derived values of $\theta_s^2(81{\cdot}5)$ are shown in Fig. 1(a), together with their standard errors. θ_s^2 is given rather than θ_s since for some sources the apparent 81·5 MHz diameters were smaller than those at 151·5 MHz, as is to be expected

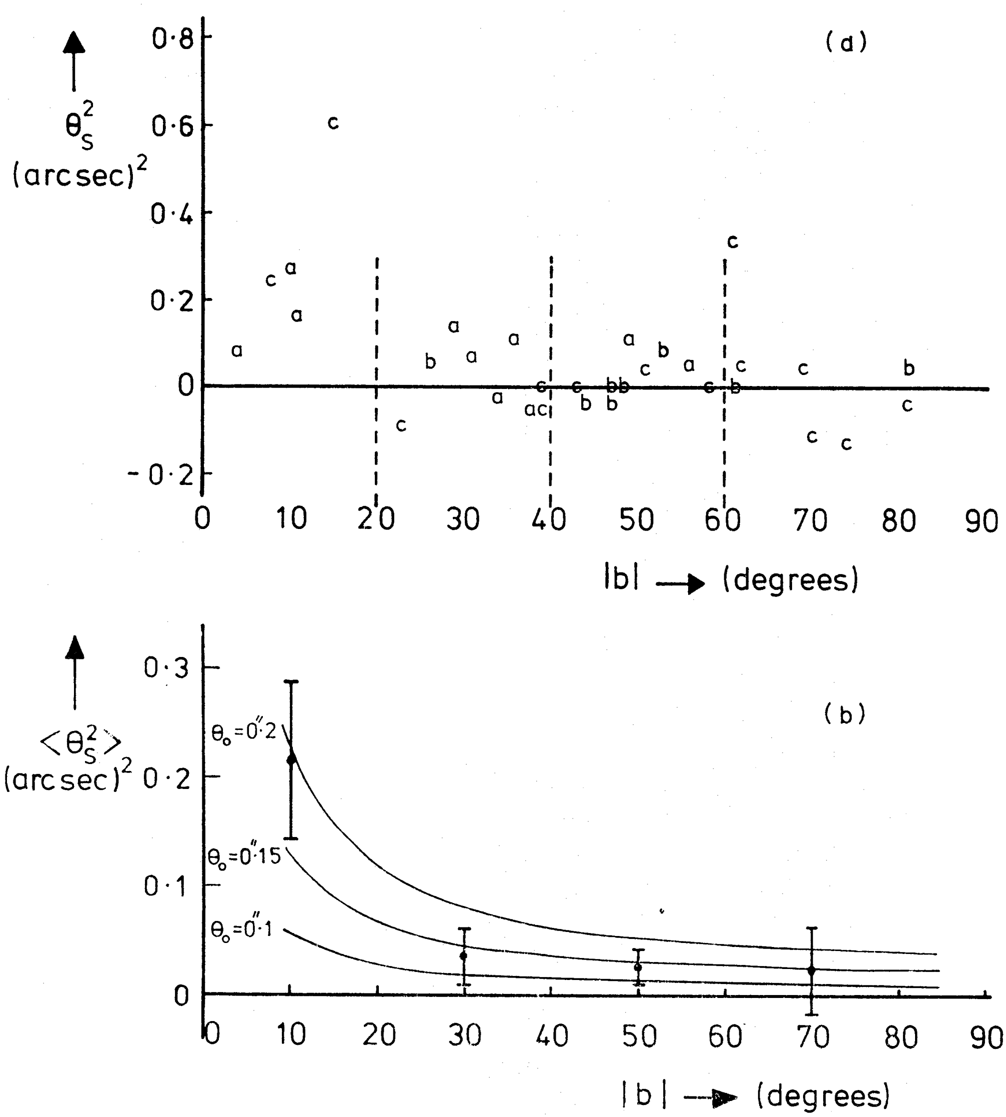

FIG. 1. (*a*) *The derived values of* θ_s^2 (81·5 *MHz*) *for the* 32 *sources. Standard errors are indicated by the letters as follows:* $a = \pm 0{\cdot}1$, $b = \pm 0{\cdot}2$, $c = \pm 0{\cdot}3$ *arcsec*^{2}. (*b*) *Values of* $\langle\theta_s^2\rangle$ *for the four ranges of galactic latitude. The individual values of* θ_s^2 *have been given weights as follows:* $a = 3$, $b = 2$, $c = 1$.

when the errors are large. In the determination of the mean scattering for each latitude range it is then more accurate to use $\langle\theta_s^2\rangle^{1/2}$ than to ascribe $\theta_s = 0$ to the cases for which the lower-frequency diameters are smaller and then take $\langle\theta_s\rangle$.

The sample of 32 sources was divided into four ranges of galactic latitude, $0° \leqslant |b| < 20°$, $20° \leqslant |b| < 40°$, $40° \leqslant |b| < 60°$, $|b| \geqslant 60°$ and $\langle\theta_s^2\rangle$ was computed within each range. The individual values of θ_s^2 were given weights according to the uncertainties in their values as indicated in Fig. 1(b). The results are shown in Fig. 1(b) where $\langle\theta_s^2\rangle$ is plotted against $|b|$, together with the variation expected from a model in which the interstellar gas is stratified in layers parallel to the galactic plane so that $\theta_s = \theta_0/(\sin b)^{1/2}$ where θ_0 is a constant. Curves for $\theta_0 = 0''\cdot10$, $0''\cdot15$ and $0''\cdot20$ are drawn. It is clear that at high galactic latitudes the mean value of θ_0 lies in the range $0''\cdot10 < \theta_0 < 0''\cdot20$. However, at low latitudes there is an indication that θ_0 is larger which would be consistent with increased scattering due to H II regions.

4. THE OBSERVED DISTRIBUTION OF SCINTILLATING SOURCES AT 81·5 MHZ

The measurements by Readhead & Hewish (1974) in their survey of ~1500 radio sources at 81·5 MHz were analysed to determine the variation of θ_s, using their statistical method (1972). The larger sample of sources enabled us to make a more accurate assessment of θ_s and to investigate the difference between the scattering in the centre and anticentre directions. Sources whose angular diameters are greater than about 2″ do not exhibit interplanetary scintillations, so that the interstellar scattering reduces the proportion of strongly to weakly scintillating sources. When the sample is divided into intervals of galactic latitude, and the fraction of strongly scintillating sources in each interval is plotted against galactic latitude, there is a marked deficiency of these sources near the galactic plane where interstellar scattering is greatest, even after correction for the increase in system noise near the plane (Readhead & Hewish 1972).

The 1500-source sample was divided into two groups according to the fraction, R, of the total flux at 81·5 MHz in the scintillating component of each source. Those with $R \geqslant 0\cdot4$ were classified as strongly scintillating sources, while those with $R < 0\cdot4$ were classified as weakly scintillating sources. Maximum scintillation at 81·5 MHz occurs at a solar elongation of 30°, while at 50° the scintillation has dropped to half its maximum value. Sources with $R < 0\cdot4$ and a minimum elongation greater than 50° were carefully examined to ensure that possible scintillation was not overlooked, and any doubtful cases were omitted. We may therefore have slightly underestimated the number of sources in this category, the effect being most pronounced at low galactic latitudes and towards the galactic centre. Since there are few reliable measurements at 81·5 MHz we have, for consistency, used the 178 MHz 4C flux densities extrapolated to 81·5 MHz assuming a spectral index of 0·75, and corrected upwards by 22 per cent as suggested by Scott & Shakeshaft (1971).

Fig. 2 shows a map in galactic coordinates of the 1500 sources, in which crosses mark those with $R \geqslant 0\cdot4$ and circles mark those with $R < 0\cdot4$. The boundaries of the map are defined by the area of sky ($\delta > -10°$) visible to the 18 000-m^2 array at Cambridge. In Fig. 3 the fraction of strongly scintillating sources in each 10° latitude range is plotted. Curves for three values of θ_0 are shown and it is clear that

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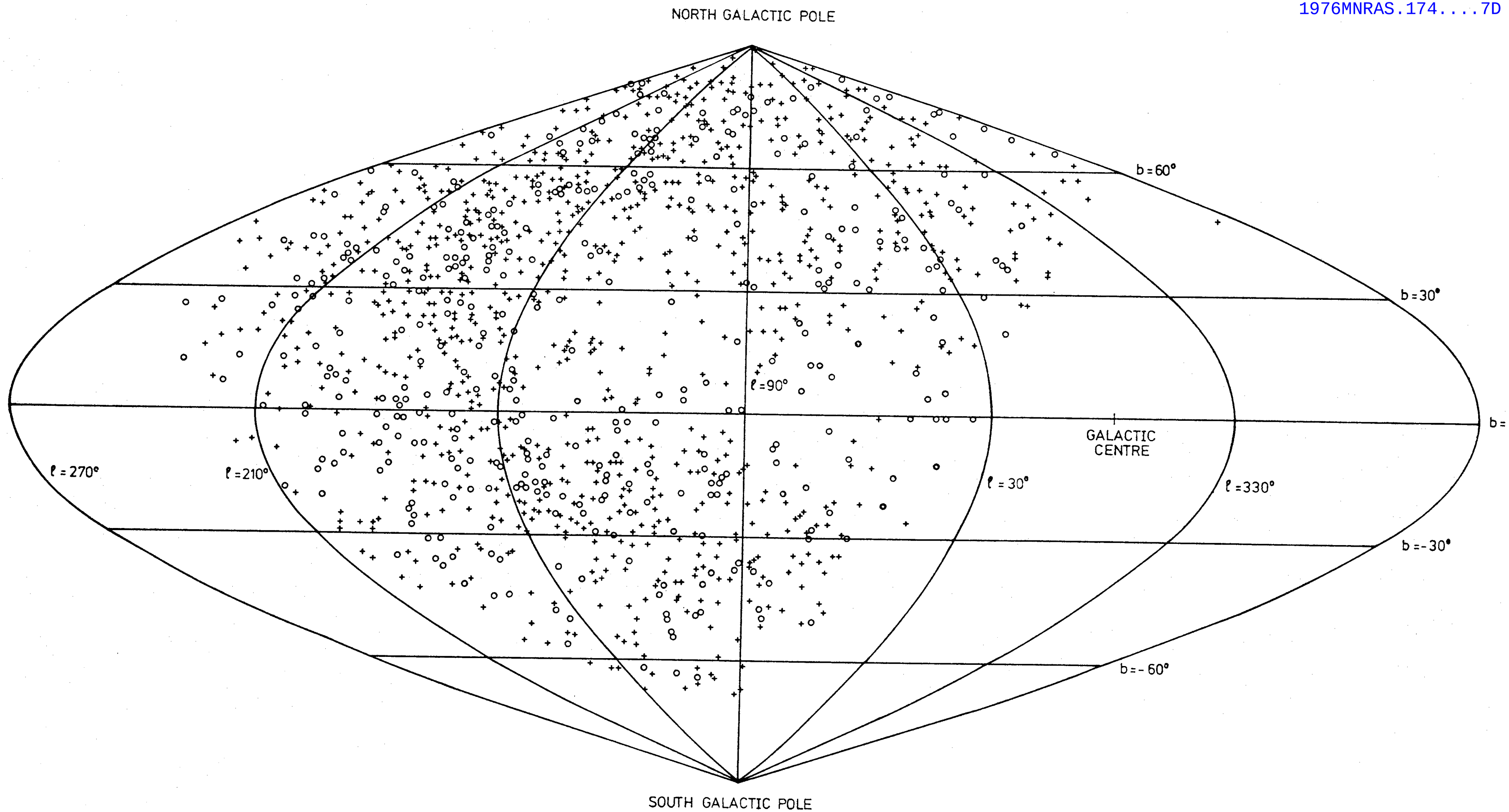

FIG. 2. *The distribution of the 1500 sources at 81·5 MHz: $R \geqslant 0{\cdot}4$ (+) and $R < 0{\cdot}4$ (○).*

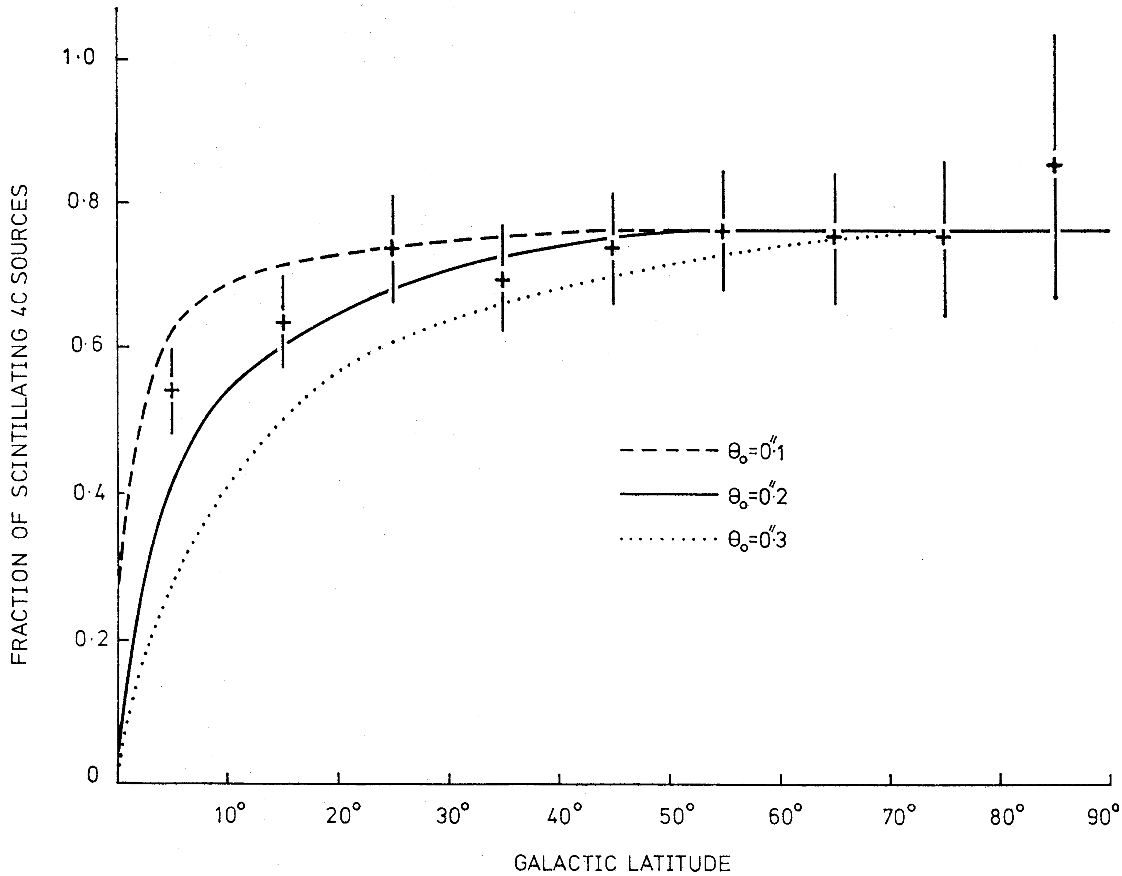

FIG. 3. *The fraction of strongly scintillating 4C sources in each 10° latitude range at 81·5 MHz.*

the best fit lies in the range $0''\cdot1 < \theta_0 < 0''\cdot2$ at 81·5 MHz. The marked increase in θ_0 for $|b| < 10°$ suggested by the preliminary analysis of Readhead & Hewish (1972) is not confirmed. In Fig. 4 the sample has been divided between galactic longitude ranges $30° \leqslant l < 120°$ and $120° \leqslant l < 300°$. The data are not sufficiently accurate to show any significant difference between the two ranges for latitudes greater than about 10°, but the larger scattering exhibited in the range $30° \leqslant l < 120°$ for lower latitudes may be due to the increased chance of the line of sight intersecting an H II region (Prentice & ter Haar 1969). On a few sources we find remarkably little interstellar scattering which suggests that the scattering medium is far from uniform. Examples are 3C 119 and 4C 21.53, both of which are close to the galactic plane.

The two methods used to derive θ_s are independent of each other except in so far as they both rely on interplanetary scintillation measurements. In addition, the second method does not depend on accurate measurements of angular diameters, but only on the ability to distinguish between strongly and weakly scintillating sources. Yet there is good agreement, both in the variation of θ_s with galactic latitude and in the value of θ_0. Taken alone, the results of the first method do not rule out the possibility that there are two effects: interstellar scattering which becomes prominent at low latitudes, and a small dependence of source size on frequency. However, the second method is not affected by a variation of source size with frequency, and the agreement between the methods in the value of scattering predicted at high latitudes suggests that this effect, if it exists, is much

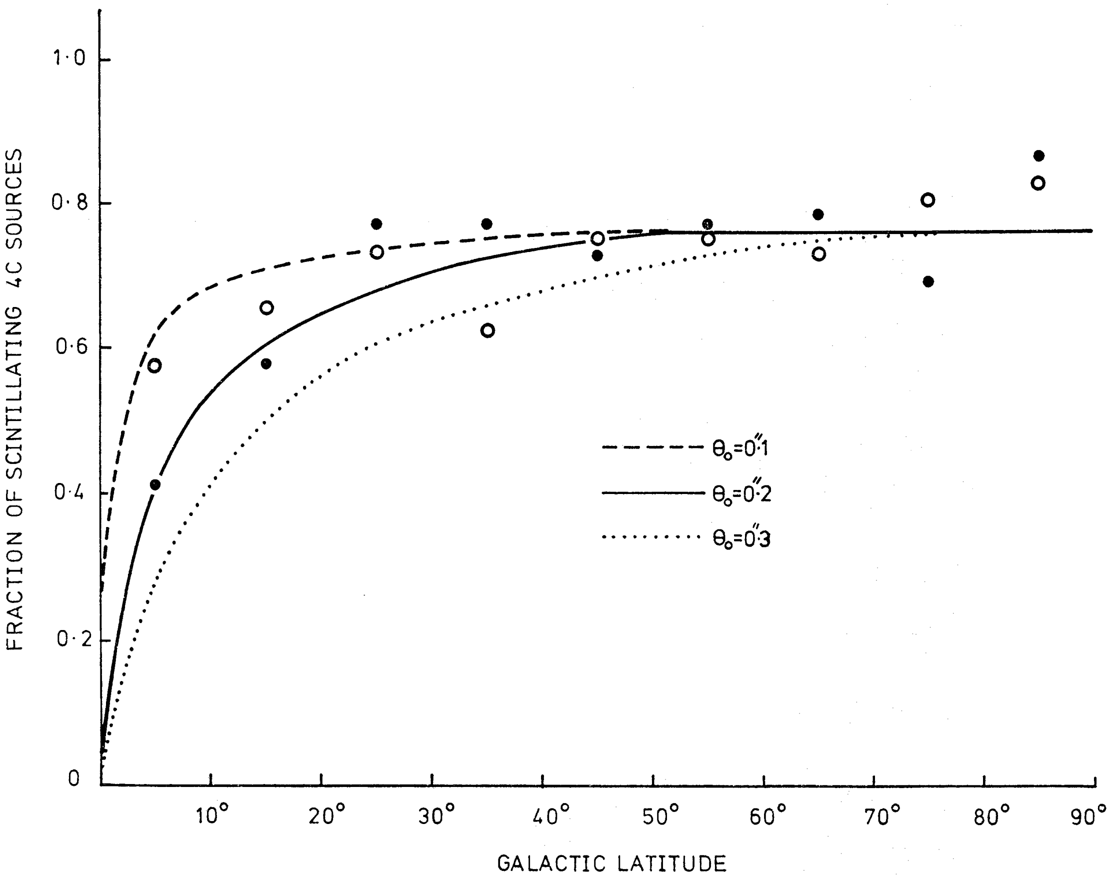

FIG. 4. *The difference in the scattering between the longitude ranges* $30° \leqslant l < 120°$ (●) *and* $120° \leqslant l < 300°$ (○).

smaller than that due to interstellar scattering. From Sections 3 and 4 we conclude that:

$$\theta_0 = 0''\cdot15 \pm 0''\cdot05 \text{ at } 81\cdot5 \text{ MHz}, \quad |b| > 10°.$$

5. COMPARISON WITH PULSAR DATA

It is well known that scintillations in pulsar radiation are caused by scattering in the interstellar medium (Scheuer 1968). Measurements on pulsars can therefore be used to determine θ_s provided that several assumptions are made (Harris, Zeissig & Lovelace 1970). Simple scattering theory, applied to the situation where all the scattering takes place in a thin screen situated halfway between the pulsar and the Earth, shows that θ_s is related to the decorrelation bandwidth $\Delta\nu$ of a pulsar by $\theta_s^2 = 16c/L\pi\,\Delta\nu$, where L is the distance to the pulsar and c is the velocity of light. This expression assumes the consistency relationship $2\pi\,\Delta t\,\Delta\nu = 1$, where Δt is the pulse broadening, which Sutton (1971) has shown is not well observed in practice. The observations are approximately consistent with theory, however, in that both $\Delta\nu$ and Δt^{-1} scale as $(DM)^{-2}$, where

$$DM = \int_0^L N_e\,dl$$

is the dispersion measure, and we adopt the line shown in Sutton's Fig. 1 as the best fit to the available data, giving $\Delta\nu = 10^7 (DM)^{-2}$ at 318 MHz. On the assumption that $\langle \Delta N_e \rangle$ is proportional to $\langle N_e \rangle$, we obtain $\theta_s^2 = 4.8 \times 10^{-2} (DM)\langle N_e \rangle$ at 81.5 MHz. The values of DM for pulsars at high latitudes lie in the range 3–30 cm^{-3} pc. It is likely that the high values are due to scattering in intervening H II regions, while the low values are for pulsars which are nearby. Following Falgarone & Lequeux (1973) we choose a value of $DM = 13\ cm^{-3}$ pc. Then, adopting $\langle N_e \rangle = 0.03\ cm^{-3}$, we find $\theta_0 = 0''.08$ at 81.5 MHz, which is in fair agreement with our value in view of the assumptions made in its derivation.

6. THE OBSERVED DIAMETERS OF OH SOURCES

The observed diameters of OH and H_2O maser sources may either be intrinsic or they may be dominated by scattering in the interstellar medium. The scattering may take place in the H II region associated with a source, in intervening H II regions, or in the general interstellar medium. If we assume that our value of θ_0, the interstellar scattering angle for the lines of sight perpendicular to the galactic plane, is due entirely to the general interstellar medium we can estimate the angular broadening expected for a point source in the plane if there are no intervening clouds which produce enhanced scattering.

The apparent angular diameter at frequency ν for a point source lying at distance L from the Earth and at galactic latitude b is given by:

$$\theta_A(\nu) \simeq \frac{\theta_0(\nu)}{2(\sin b)^{1/2}} \left\{1 - \exp\left[-\frac{2L \sin b}{z_0}\right]\right\}^{1/2}$$

if all the scattering takes place in the general interstellar medium (see Appendix for the derivation). We here assume an exponential electron distribution with height above the plane, of scale-height z_0 taken as 500 pc (Readhead & Duffett-Smith 1975). Table II shows the observed and predicted diameters for four OH sources and four H_2O sources. It is clear that the observed diameters are far larger than can be accounted for by the scattering and we conclude that scattering in the general interstellar medium is *not* sufficient to produce the required broadening. The observed angular diameters are either intrinsic, or there must be additional scattering along the lines of sight to these sources.

TABLE II

The observed and predicted diameters of maser sources

Source	Distance (kpc)	Galactic latitude (°)	Predicted diameter (10^{-3} ″)	Observed diameter (10^{-3} ″)	(Observed diameter) / (Predicted diameter)
OH sources					
NGC 6334	0.7	0.6	0.3	<15	<47
W3	2.6	1.2	0.6	4	7
W24	10	−0.1	1.2	90	75
W49	14	−0.2	1.4	50	37
H_2O sources					
W49N	15	−0.2	8	300	38
W3(OH)	2.5	1.2	3	<2000	<619
Ori A	0.5	−19.2	1	800	800
VY CMa	1	−4.9	2	<200	<100

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APPENDIX

THE APPARENT ANGULAR DIAMETER OF A GALACTIC POINT SOURCE

We consider the passage of radiation travelling along an axis through a scattering medium. Let the rms value of the angular spectrum of the radiation incident on an element dx be $\langle\theta^2(x)\rangle^{1/2}$, and let the increment added by the element be $d\theta$. Then

$$\frac{\partial(\langle\theta^2(x)\rangle)}{\partial x}\,dx = (d\theta)^2.$$

Scattering theory shows that $(d\theta)^2 = k_1(\Delta N_e)^2\,dx$ where ΔN_e is the rms deviation of the electron density in the medium and k_1 is a constant. If $\Delta N_e \propto \langle N_e\rangle$, then

$$\int_0^{\theta_s^2} d(\langle\theta^2\rangle) = k_2\int_0^x \langle N_e(x)\rangle^2\,dx.$$

We choose a model of the galaxy in which the interstellar electron density varies only with height z above the plane, with $\langle N_e\rangle = N_0 \exp(-z/z_0)$ where z_0 is the scale height. For a galactic source at distance L from the Earth and at galactic latitude b we have

$$\int_0^{\theta_S^2} d(\langle\theta^2\rangle) = k_2 \int_0^{L \sin b} N_0^2 \exp(-2z/z_0).\, dz/\sin b$$

giving

$$\theta_S = \frac{\theta_0}{(\sin b)^{1/2}} \left\{1 - \exp\left[-\frac{2L \sin b}{z_0}\right]\right\}^{1/2}$$

where θ_0 is the total scattering along the lines of sight perpendicular to the plane. If all the scattering may be presumed to take place in a thin layer halfway between the source and the observer, the apparent angular diameter (see Fig. A1) is given

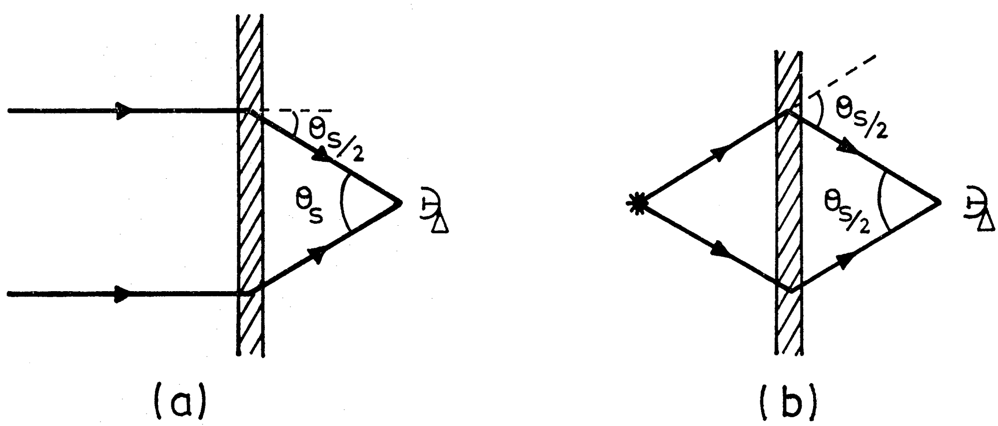

FIG. A1. *Scattering in a thin screen for an extragalactic source (a) and a galactic source (b).*

by $\theta_A = \theta_S/2$. In practice the situation is more complicated, but to a sufficiently good approximation we have

$$\theta_A(\nu) \simeq \frac{\theta_0(\nu)}{2(\sin b)^{1/2}} \left\{1 - \exp\left[-\frac{2L \sin b}{z_0}\right]\right\}^{1/2}.$$