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Distributed resilient filtering of large-scale systems with channel scheduling

DISTRIBUTED RESILIENT FILTERING OF LARGE-SCALE SYSTEMS WITH CHANNEL SCHEDULING

Lili Xu, Sunjie Zhang and Licheng Wang

This paper addresses the distributed resilient filtering for discrete-time large-scale systems (LSSs) with energy constraints, where their information are collected by sensor networks with a same topology structure. As a typical model of information physics systems, LSSs have an inherent merit of modeling wide area power systems, automation processes and so forth. In this paper, two kinds of channels are employed to implement the information transmission in order to extend the service time of sensor nodes powered by energy-limited batteries. Specifically, the one has the merit of high reliability by sacrificing energy cost and the other reduces the energy cost but could result in packet loss. Furthermore, a communication scheduling matrix is introduced to govern the information transmission in these two kind of channels. In this scenario, a novel distributed filter is designed by fusing the compensated neighboring estimation. Then, two matrix-valued functions are derived to obtain the bounds of the covariance matrices of one-step prediction errors and the filtering errors. In what follows, the desired gain matrices are analytically designed to minimize the provided bounds with the help of the gradient-based approach and the mathematical induction. Furthermore, the effect on filtering performance from packet loss is profoundly discussed and it is claimed that the filtering performance becomes better when the probability of packet loss decreases. Finally, a simulation example on wide area power systems is exploited to check the usefulness of the designed distributed filter.

1. INTRODUCTION

In the past few years, large-scale systems (LSSs) have attracted an increasing research attention due to their practical application insights in various engineering fields, such as wide area power systems, automation processes, biological systems as well as transportation networks, see [1,7,12,15,23,32,38,39] and the references therein. In comparison with traditional centralized networked control systems, LSSs usually consist of lots of subsystems geographically distributed in a certain area with a topology, which is predetermined to govern the connection both in physics and in communication [2]. For instance, the new energy, energy storage units and users are integrated with the distribution network of power systems through the multi-level transmission network [1, 7]. In the operation

monitoring of LSSs, a straightforward solution is to design a centralized filter by utilizing traditional filtering approaches, such as Kalman filtering and its extended version [18], H_∞ filtering [13, 20], or moving horizon estimation [24]. Unfortunately, such a solution is usually dependent on the assumptions that there exists a calculation centre to carry out the parameter design and the information collection via affiliated sensor networks. There is no doubt that this kind of centralized paradigms is inevitably exposed to the challenges of the high dimension, the high computation and communication cost.

It is worth noting that, up to now, there are two kinds of representative approaches to deal with the above mentioned shortages: the decoupling technologies [12, 19] and the distributed design paradigms [4, 34]. For the first one, the existence condition of filter gains for addressed LSSs can be decoupled into a series of matrix inequalities by utilizing the well-known small gain theory or vector passivity theory and these matrix inequalities can be effectively handled to obtain the desired parameters in an off-line scenario [6]. In other words, it is still incapable to recursive filtering with the on-line implementation. As such, distributed filtering has been developed with the help of the own measurement and the estimation from its neighboring subsystems, and some interesting results have been published in the literature [2, 4, 19, 26]. For instance, a distributed Kalman filtering has been investigated in [4] for discrete sequential systems, and the corresponding stability condition has been provided to guarantee the boundedness of estimated error dynamics. Furthermore, a distributed filtering algorithm has been developed in [2] to solve the monitoring of power systems subject to denial-of-service attacks.

When the filtering issue is a concern, a sensor network with the same topology of LSSs usually need to be deployed to collect and process the information of subsystems. Different from traditional distributed ones applying to centralized systems [9, 14, 36], filtering structures emerge complex dynamical coupling due to the inherently physical coupling of LSSs. It is a pity that this kind of coupling cannot be completely removed in order to guarantee the essential filtering function, and therefore greatly limits the robustness and the scalability of developed distributed filters. In other words, most existing filtering algorithms do not effectively deal with the design challenges of addressed filtering issues of LSSs. Besides, the rounding of implemented filtering algorithms in digital platforms inevitably results in unpredictable uncertainties or gain perturbations, which arouses a new research concern on nonfragility or resilience, see [21, 22, 35] and the references therein. As such, a seemingly natural requirement is that the designed distributed filter of LSSs is with the advantage of insensitivity against possible uncertainties or gain perturbations, which constitutes one of main research motivations in this paper.

On the other hand, the deployed sensor nodes or subsystems could be powered by energy-limited batteries and the main energy is costed in communication especially in wireless communication. It should be pointed out that batteries could not be replaceable or recharged in complex environments and hence reducing the energy consumption is of crucial importance to prolong the service life [11, 37, 40]. In other words, one of the most important researches is how to save the energy of information transmission while ensuring the necessary filtering quality [41]. To this end, an effective approach is to design a reasonable scheduling strategy to govern the information exchange, see [10, 16, 27, 31] for examples. Among literature, the scheduling is usually controlled by a remote estimator

and the scheduling signal need to been transformed in real time, which is not suitable for the application scenario of distributed filtering of LSSs. As a promising alternative, the data can be exchanged via preallocated channels with different energy requirements among sensor nodes or subsystems. Usually, channels with enough energy allocation can realize the reliability of data transmission and channels with low energy allocation could lead to packet dropouts with some certain probability [5, 27]. For instance, a recursive distributed fusion scheme with the form of Kalman filtering has been developed in [5] by resorting to developed strategies compensating the no transmitted components. So far, to the best of the authors' knowledge, the distributed filtering for discrete-time LSSs with energy constraints is still an open yet challenging issue. It is, therefore, the main purpose of this paper to shorten such a gap.

According to the above analysis, it would be interesting to propose an effective design framework to satisfy the requirements of energy constraint and low calculation burden. For this purpose, we aim to initiate a study on the the distributed resilient filtering of discrete-time LSSs with energy constraints. The main contributions of this paper can be highlighted as follows:

1) for a class of discrete-time LSSs modeling some typical engineering practice, a novel distributed filter subject to gain perturbations is designed by fusing the compensated neighboring estimation;

2) two matrix-valued functions are derived to obtain the bounds of the covariance matrices of one-step prediction errors and the filtering errors, which are further minimized to analytically obtain the desired gain matrices;

3) the impact on filtering performance from packet loss is profoundly disclosed in light of properties of conditional expectation; and

4) a wide area power systems is employed to check the usefulness of the designed distributed filter.

Notation. The notations used throughout the paper are fairly standard except where otherwise stated. $\mathbb{R}^{n \times m}$ and $\mathbb{R}^n$ denote, respectively, the set of all $n \times m$ real matrix space and the n-dimensional Euclidean space. For given matrix X, $\mathrm{Tr}\{X\}$ and X^T represent the operation of the trace and the transpose, respectively. $\mathbb{E}\{\xi\}$ denotes the expectation of stochastic variable ξ, and $\mathbb{P}\{\xi\}$ represents the occurrence probability of event ξ.

2. STATEMENT OF THE PROBLEM

Consider the following LSSs consisting of M interconnected subsystems, each of which is described by:

$$\begin{cases} x_{i,k+1} = A_{ii}x_{i,k} + \displaystyle\sum_{j \in \Gamma_i} A_{ij}x_{j,k} + w_{i,k} \\ y_{i,k} = C_i x_{i,k} + v_{i,k}, \end{cases} \tag{1}$$

where $x_{i,k} \in \mathbb{R}^n$ and $y_{i,k} \in \mathbb{R}^m$ are, respectively, the state vector and the measurement output of subsystem i. A_{ii}, A_{ij}, and C_i are known system matrices of suitable dimensions. Furthermore, assume that $w_{i,k} \in \mathbb{R}^n$ and $v_{i,k} \in \mathbb{R}^m$ are zero-mean white noises with $\mathbb{E}\{w_{i,k}w_{j,k}^T\} = Q_i\delta_{ij}$ and $\mathbb{E}\{v_{i,k}v_{j,k}^T\} = R_i\delta_{ij}$, where covariance matrices $Q_i > 0$ and $R_i > 0$ are known. If $A_{ij} \neq 0$, subsystem j is called as a neighbor of subsystem i. In addition, for subsystem i, the set of neighboring subsystems is denoted as Γ_i and the number of neighboring subsystems is denoted as $\lfloor \Gamma_i \rfloor$, which is equal to the number of $A_{ij} \neq 0$ for any j with $j \neq i$. The topology matrix is defined as $\mathcal{A} = [a_{ij}]_{M \times M}$ where $a_{ii} = 0$ and $a_{ij} = 1$ if $A_{ij} \neq 0$ otherwise $a_{ij} = 0$.

Considering the distributed filtering issue, we denote $\hat{x}_{i,k|k}$ as the estimation of state $x_{i,k}$ at current instant k. It must be transmitted to the neighboring subsystem via some common communication channels to calculate the estimation $\hat{x}_{j,k+1|k+1}$. Notice that the communication bandwidth and the battery energy are usually limited, and therefore two kinds of channels are employed to implement the information transmission: the one (named as a reliable channel) has the merit of high reliability by sacrificing energy cost, and the other (called as a general channel) could be subject to packet loss due to the application of low energy cost.

Assume that, at a particular time k, r_s $(1 \leqslant r_s < m)$ components of state $\hat{x}_{i,k}$ are admitted to be transmitted to neighbors via a reliable channel and the rest is transmitted via a general channel with data missing phenomenon. Usually, the missing phenomenon can be detected in time with the help of well-known data coding techniques, such as transmission control protocols (TCPs). In this scenario, we introduce a Bernoulli-distributed white sequence $\{\alpha_{i,k}\}$ and a diagonal matrix

$$H_{i,k} = \mathrm{diag}\{\gamma_{i1,k}, \gamma_{i2,k}, \cdots, \gamma_{im,k}\}, \tag{2}$$

where $\gamma_{ij,k}$ $(j = 1, 2, \cdots, m)$ are the binary variables satisfying

$$\sum_{j=1}^{n} \gamma_{ij,k} = r_s, \quad i \in \{1, 2, \cdots, M\}. \tag{3}$$

Obviously, $\gamma_{ij,k} = 1$ stands for that the jth component of local estimation $\hat{x}_{i,k|k}$ is selected to send to its neighbors via the reliable channel.

According to above discussion, the received estimation $\hat{x}_{i,k|k}^r$ is described by

$$\hat{x}_{i,k|k}^r = H_{i,k}\hat{x}_{i,k|k} + \alpha_{i,k}(I - H_{i,k})\hat{x}_{i,k|k}. \tag{4}$$

Note that the time-varying matrix $H_{i,k}$ is usually known via the communication coding protocol, and is regarded as a communication scheduling matrix.

In the case of limited communication channels and energy supply constraints, a set of distributed filters are designed for the addressed power systems. Note that there may be a large amount of mathematical calculations or tuning uncertainties in practical applications. Obviously, the performance of the filters may be degraded by the disturbed gain parameter, and therefore the design considering filter gain variations has important engineering significance.

For the above spatially distributed power systems, a two-step distributed filter is designed as follows:

$$
\begin{cases}
\hat{x}_{i,k+1|k} = A_{ii}\hat{x}_{i,k|k} + \sum_{j\in\Gamma_i} A_{ij}\hat{x}^r_{j,k|k} \\
\hat{x}_{i,k+1|k+1} = \hat{x}_{i,k+1|k} + (L_{i,k+1} + \Delta L_{i,k+1})(y_{i,k+1} - C_i\hat{x}_{i,k+1|k}),
\end{cases}
\tag{5}
$$

where $\hat{x}_{i,k+1|k}$ is the one-step prediction at time $k+1$ and $L_{i,k+1}$ is the filter gain to be determined. The term $\Delta L_{i,k+1} \in \mathbb{R}^{n_x \times n_y}$ models the computation or implementation error associated with the filter gain, and is assumed to have zero mean and a bounded moment, i. e.

$$
\mathbb{E}\{\Delta L_{i,k+1}\} = 0, \ \mathbb{E}\{\Delta L_{i,k+1}, \Delta L^T_{i,k+1}\} \le \delta_i I,
$$

where δ_i is a known positive scalar. Moreover, $\Delta L_{i,k}$, $w_{i,k}$ and $v_{i,k}$ are mutually independent.

Remark 2.1. For filtering algorithms performed in networked environment [25, 28], the microprocessor with limited types plays an irreplaceable role, and therefore the implementation or computation of theoretical filter gain $L_{i,k+1}$ is inevitably subject to the distortion error, which is regarded as the filter gain perturbation $\Delta L_{i,k+1}$. As such, it is of great practical value to investigate the effect from such a perturbation.

Remark 2.2. Taking the structure of LSSs into consideration, a Kalman-type distributed filter is constructed by fusing the received information from neighboring subsystems. In other words, such a distributed filter keeps the hierarchical structure of plant systems, and the function of information is performed via the inherent topology of plant systems. In comparison with existing results obtained via augmented approaches, the adopted filtering structure is very crucial to the development and the implementation of filtering strategy to be designed in the next section.

For the convenience of subsequent analysis, the error vectors of local prediction and local estimation are respectively defined as follows

$$
\tilde{x}_{i,k+1|k} = x_{i,k+1} - \hat{x}_{i,k+1|k}
$$
$$
\tilde{x}_{i,k+1|k+1} = x_{i,k+1} - \hat{x}_{i,k+1|k+1}.
$$

It follows from (1) and (5) that

$$
\begin{cases}
\tilde{x}_{i,k+1|k} = A_{ii}\tilde{x}_{i,k|k} + \sum_{j\in\Gamma_i} A_{ij}\big(\tilde{x}_{j,k|k} \\
\qquad\qquad + (1-\alpha_{j,k})(I - H_{j,k})\hat{x}_{j,k|k}\big) + w_{i,k} \\
\tilde{x}_{i,k+1|k+1} = \big(I - (L_{i,k+1} + \Delta L_{i,k+1})C_i\big)\tilde{x}_{i,k+1|k} \\
\qquad\qquad - \big(L_{i,k+1} + \Delta L_{i,k+1}\big)v_{i,k}.
\end{cases}
\tag{6}
$$

The aim of this paper is to develop a recursive filtering algorithm for a class of power systems with energy constrained channels. Specifically, we plan to design Kalman-type filters capable of online calculating such that, for all possible filter gain perturbations, the filtering error covariance satisfies the following condition in finite-horizon:

$$
\mathbb{E}\{\tilde{x}_{i,k|k}\tilde{x}^T_{i,k|k}\} \le \Phi_{i,k|k}, \ k \in \{0,1,2,\cdots,N\},
$$

where $\{\Phi_{i,k|k}\}$ is a set of positive-definite matrices.

3. MAIN RESULTS

In this section, two Riccati-like difference equations are first derived to describe the upper bound of error covariance matrices in light of the framework of classical Kalman filtering. Then, the desired filter gains are obtained by resorting to the minimum of such an upper bound. To this end, two lemmas are needed for the following analysis.

Lemma 3.1. Assume that there are two matrix-value function $Q_k^h(X) = Q_k^{hT}(X) \in \mathbb{R}^{n \times n}$ and $S_k^h(X) = S_k^{hT}(x) \in \mathbb{R}^{n \times n}$ for the given matrix $X \in \mathbb{R}^{n \times n}$. If there exists $Y \geq X > 0$ such that

$$Q_k^h(X) \leq Q_k^h(Y), \ Q_k^h(Y) \leq S_k^h(Y),$$

then the recursive solutions Φ_k and Ψ_k associated with these two matrix value functions

$$\Phi_{k+1} = Q_k^h(\Phi_k), \ \ \Psi_{k+1} = S_k^h(\Psi_k), \ \ \ \Phi_0 = \Psi_0 > 0,$$

satisfy $\Phi_k \leq \Psi_k$.

Lemma 3.2. For matrices R, S, X and P with appropriate dimensions, one has the following differential operations on trace functions

$$\frac{\partial(RXS)}{\partial(X)} = R^T S^T, \ \ \frac{\partial(RX^T S)}{\partial(x)} = RS,$$

$$\frac{\partial(RXSX^T L)}{\partial(X)} = R^T L^T X S^T + LRXS.$$

Define the covariance matrix $P_{i,k|k-1}$ of one-step prediction errors and the covariance matrix $P_{i,k|k}$ of filtering errors

$$P_{i,k|k} = \mathbb{E}\{\tilde{x}_{i,k|k}\tilde{x}_{i,k|k}^T\}, \ P_{i,k|k-1} = \mathbb{E}\{\tilde{x}_{i,k|k-1}\tilde{x}_{i,k|k-1}^T\}.$$

By resorting to stochastic analysis and matrix operation, one has the following two theorems.

Theorem 3.3. For the designed distributed filter (6) with given gains $L_{i,k}$ and any positive scalars $\varepsilon_{0i,k}$ and $\varepsilon_{1i,k}$, the covariance matrices $P_{i,k+1|k}$ and $P_{i,k+1|k+1}$ obey:

$$P_{i,k+1|k} \leq \Phi_{i,k+1|k}, \ \ P_{i,k+1|k+1} \leq \Phi_{i,k+1|k+1}, \tag{7}$$

where

$$\Lambda_{i,k} = (1 + \varepsilon_{1i,k})\Omega_{i,k} + (1 + \varepsilon_{1i,k}^{-1})\aleph_{i,k}\aleph_{i,k}^T, \tag{8}$$

$$\Phi_{i,k+1|k} = (1 + \varepsilon_{0i,k})A_{ii}\Phi_{i,k|k}A_{ii}^T + Q_i + (1 + \varepsilon_{0i,k}^{-1})\Lambda_{i,k}, \tag{9}$$

$$\Phi_{i,k+1|k+1} = (I - L_{i,k+1}C_i)\Phi_{i,k+1|k}(I - L_{i,k+1}C_i)^T$$
$$+ \delta_i \lambda_{\max}(R_i + C_i\Phi_{i,k+1|k}C_i^T)I + L_{i,k+1}R_i L_{i,k+1}^T, \tag{10}$$

with

$$\aleph_{i,k} = \sum_{j\in\Gamma_i}(1 - \alpha_{j,k})A_{ij}(I - H_{j,k})\hat{x}_{j,k|k}, \ \Omega_{i,k} = \lfloor\Gamma_i\rfloor\sum_{j\in\Gamma_i}A_{ij}\Phi_{j,k|k}A_{ij}^T.$$

P r o o f. Due to the detectability of sequences of $\{\alpha_{i,k}\}$, we first define $\Upsilon_{i,k} = \{\alpha_{j,k}|j \in \Gamma_i\}$ and then denote the covariance of prediction error dynamics as

$$P_{i,k+1|k} = \mathbb{E}\{\tilde{x}_{i,k+1|k}\tilde{x}_{i,k+1|k}^T|\Upsilon_{i,k}\}.$$

Then, along with the trajectory (6), one has

$$
\begin{aligned}
&P_{i,k+1|k} \\
&= A_{ii}P_{i,k|k}A_{ii}^T + Q_i + \mathbb{E}\Big\{\Big(\sum_{j\in\Gamma_i} A_{ij}\big(\tilde{x}_{j,k|k} + (1-\alpha_{j,k})(I-H_{j,k})\hat{x}_{j,k|k}\big)\Big) \\
&\quad \times \Big(\sum_{j\in\Gamma_i} A_{ij}\big(\tilde{x}_{j,k|k} + (1-\alpha_{j,k})(I-H_{j,k})\hat{x}_{j,k|k}\big)\Big)^T\Big|\Upsilon_{i,k}\Big\} \\
&\quad + \mathbb{E}\Big\{A_{ii}\tilde{x}_{i,k|k}\Big(\sum_{j\in\Gamma_i} A_{ij}\big(\tilde{x}_{j,k|k} + (1-\alpha_{j,k})(I-H_{j,k})\hat{x}_{j,k|k}\big)\Big)^T\Big|\Upsilon_{i,k}\Big\} \\
&\quad + \mathbb{E}\Big\{\sum_{j\in\Gamma_i} A_{ij}\Big(\tilde{x}_{j,k|k} + (1-\alpha_{j,k})(I-H_{j,k})\hat{x}_{j,k|k}\Big)\tilde{x}_{i,k|k}^T A_{ii}^T\Big|\Upsilon_{i,k}\Big\}, \quad (11)
\end{aligned}
$$

is rewritten as follows

$$
\begin{aligned}
&P_{i,k+1|k} \\
&= A_{ii}P_{i,k|k}A_{ii}^T + Q_i \\
&\quad + \mathbb{E}\Big\{\Big(\sum_{j\in\Gamma_i} A_{ij}\tilde{x}_{j,k|k} + \aleph_{i,k}\Big)\Big(\sum_{j\in\Gamma_i} A_{ij}\tilde{x}_{j,k|k} + \aleph_{i,k}\Big)^T\Big|\Upsilon_{i,k}\Big\} \\
&\quad + \mathbb{E}\Big\{A_{ii}\tilde{x}_{i,k|k}\Big(\sum_{j\in\Gamma_i} A_{ij}\tilde{x}_{j,k|k} + \aleph_{i,k}\Big)^T\Big|\Upsilon_{i,k}\Big\} \\
&\quad + \mathbb{E}\Big\{\Big(\sum_{j\in\Gamma_i} A_{ij}\tilde{x}_{j,k|k} + \aleph_{i,k}\Big)\tilde{x}_{i,k|k}^T A_{ii}^T\Big|\Upsilon_{i,k}\Big\}. \quad (12)
\end{aligned}
$$

On the other hand, in light of some element matrix inequalities, one has

$$\mathbb{E}\Big\{\sum_{j\in\Gamma_i} A_{ij}\tilde{x}_{j,k}\Big(\sum_{j\in\Gamma_i}\tilde{x}_{j,k}^T A_{ij}^T\Big)\Big|\Upsilon_{i,k}\Big\} \leq \tilde{\Omega}_{i,k}, \quad (13)$$

where $\tilde{\Omega}_{i,k} = \lfloor\Gamma_i\rfloor\sum_{j\in\Gamma_i} A_{ij}P_{j,k|k}A_{ij}^T$. Then, we further obtain

$$
\begin{aligned}
&\mathbb{E}\Big\{\Big(\sum_{j\in\Gamma_i} A_{ij}\tilde{x}_{j,k|k} + \aleph_{i,k}\Big)\Big(\sum_{j\in\Gamma_i} A_{ij}\tilde{x}_{j,k|k} + \aleph_{i,k}\Big)^T\Big|\Upsilon_{i,k}\Big\} \\
&= \mathbb{E}\Big\{\sum_{j\in\Gamma_i} A_{ij}\tilde{x}_{j,k}\Big(\sum_{j\in\Gamma_i}\tilde{x}_{j,k}^T A_{ij}^T\Big)\Big|\Upsilon_{i,k}\Big\} \\
&\quad + \mathbb{E}\Big\{\sum_{j\in\Gamma_i} A_{ij}\tilde{x}_{j,k|k}\aleph_{i,k}^T\Big|\Upsilon_{i,k}\Big\} + \mathbb{E}\Big\{\sum_{j\in\Gamma_i}\aleph_{i,k}\tilde{x}_{j,k|k}^T A_{ij}^T\Big|\Upsilon_{i,k}\Big\} + \aleph_{i,k}\aleph_{i,k}^T
\end{aligned}
$$

$$\leq (1 + \varepsilon_{1i,k})\tilde{\Omega}_{i,k} + (1 + \varepsilon_{1i,k}^{-1})\aleph_{i,k}\aleph_{i,k}^T := \tilde{\Lambda}_{i,k}. \tag{14}$$

Taking (14) into consideration, we further deal with (12) as follows

$$P_{i,k+1|k} \leq (1 + \varepsilon_{0i,k})A_{ii}P_{i,k|k}A_{ii}^T + Q_i + (1 + \varepsilon_{0i,k}^{-1})\tilde{\Lambda}_{i,k}$$
$$:= Q_k^h(P_{i,k|k}). \tag{15}$$

In what follow, let us calculate the covariance matrix $P_{i,k|k}$ of filtering errors. According to the updated dynamics, one has

$$
\begin{aligned}
&P_{i,k+1|k+1}\\
&= \mathbb{E}\big\{ \big(I - (L_{i,k+1} + \Delta L_{i,k+1})C_i\big)^T P_{i,k+1|k}\\
&\quad \times (I - (L_{i,k+1} + \Delta L_{i,k+1}C_i))\\
&\quad + (L_{i,k+1} + \Delta L_{i,k+1})R_i(L_{i,k+1} + \Delta L_{i,k+1})^T \big\}\\
&= (I - L_{i,k+1}C_i)P_{i,k+1|k}(I - L_{i,k+1}C_i)^T\\
&\quad + \mathbb{E}\big\{ \Delta L_{i,k+1}C_i P_{i,k+1|k}C_i^T \Delta L_{i,k+1}^T \big\}\\
&\quad + L_{i,k+1}R_i L_{i,k+1}^T + \mathbb{E}\big\{ \Delta L_{i,k+1}R_k \Delta L_{i,k+1}^T \big\}\\
&\leq (I - L_{i,k+1}C_i)P_{i,k+1|k}(I - L_{i,k+1}C_i)^T\\
&\quad + \delta_i \lambda_{\max}(R_k + C_i P_{i,k+1|k}C_i^T)I + L_{i,k+1}R_i L_{i,k+1}^T. \tag{16}
\end{aligned}
$$

Finally, select the matrix value function

$$S_k^h(X) = (1 + \varepsilon_{0i,k})A_{ii}X A_{ii}^T + Q_i + (1 + \varepsilon_{0i,k}^{-1})\Lambda_{i,k}.$$

When $P_{i,0|0} \leq \Phi_{i,0|0}$, one has $P_{i,1|0} \leq \Phi_{i,1|0}$ with the help of Lemma 3.1. Furthermore, it is not difficult to find that

$$
\begin{aligned}
&P_{i,1|1} - \Phi_{i,1|1}\\
&\leq (I - L_{i,1}C_i)(P_{i,1|0} - \Phi_{i,1|0})(I - L_{i,1}C_i)^T\\
&\quad + \delta_i \lambda_{\max}(R_i + C_i(P_{i,1|0} - \Phi_{i,1|0})C_i^T)I\\
&\leq 0.
\end{aligned}
$$

Assuming that $P_{i,k|k} \leq \Phi_{i,k|k}$ is true, we can also derive that $P_{i,k+1|k} \leq \Phi_{i,k+1|k}$ and $P_{i,k+1|k+1} \leq \Phi_{i,k+1|k+1}$ hold via Lemma 3.1. According to the mathematical induction, we deduce that (7) is true for any instant k and hence the proof is complete. $\qquad\square$

Up to now, the upper bound of the covariance of the filtering error has been given. Next, we use a gradient-based method to obtain the desired filter gain and meanwhile minimize such an upper bound.

Theorem 3.4. For the addressed discrete-time LSSs (1) with energy constraints, the upper bound of estimation error variance $\Phi_{i,k+1|k+1}$ is locally minimized when the gain of constructed resilient filter (5) is

$$L_{i,k+1} = \Phi_{i,k+1|k}C_i^T(C_i\Phi_{i,k+1|k}C_i^T + R_i)^{-1}, \tag{17}$$

where $\Phi_{i,k+1|k}$ and $\Phi_{i,k+1|k+1}$ are the same with that of Theorem1 3.3.

P r o o f. Firstly, taking the trajectories on both sides of (10) leads to

$$\mathrm{Tr}\{\Phi_{i,k+1|k+1}\}$$
$$= \mathrm{Tr}\{\Phi_{i,k+1|k} - L_{i,k+1}C_i\Phi_{i,k+1|k} - \Phi_{i,k+1|k}C_i^T L_{i,k+1}^T$$
$$+ L_{i,k+1}(C_i\Phi_{i,k+1|k}C_i^T + R_i)L_{i,k+1}^T\} + \delta_i\lambda_{\max}\{R_i + C_i\Phi_{i,k+1|k}C_i^T\}. \qquad (18)$$

Via Lemma 3.2, taking the partial derivative of $\mathrm{Tr}(\Phi_{k+1|k+1})$ with respect to $L_{i,k+1}$, one has

$$\frac{\partial \mathrm{Tr}(\Phi_{i,k+1|k+1})}{\partial L_{i,k+1}} = 2L_{i,k+1}(C_i\Phi_{i,k+1|k}C_i^T + R_i) - 2\Phi_{i,k+1|k}C_i^T. \qquad (19)$$

To minimize the upper bound of error covariance, let (19) be equal to zero:

$$2L_{i,k+1}(C_i\Phi_{i,k+1|k}C_i^T + R_i) - 2\Phi_{i,k+1|k}C_i^T = 0,$$

which implies

$$L_{i,k+1} = \Phi_{i,k+1|k}C_i^T(C_i\Phi_{i,k+1|k}C_i^T + R_i)^{-1},$$

and there the proof is complete. $\square$

To evaluate the impact on filtering performance from the missing probability due to energy constraints, assuming that statistical features are identified, then the following results are obtained.

Theorem 3.5. Suppose that the probabilities of two information missing sequences $\{\alpha_{1j,k}\}$ and $\{\alpha_{2j,k}\}$ $(j \in \Gamma_i)$ are identified and satisfy $\mathbb{P}\{\alpha_{1j,k} = 0\} < \mathbb{P}\{\alpha_{2j,k} = 0\}$. For the prefixed $\varepsilon_{0i,k} = \varepsilon_{0i}$ and $\varepsilon_{1i,k} = \varepsilon_{1i}$ for any instant k, the following relationship is satisfied:

$$\mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{1j,k}}\} \leq \mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{2j,k}}\}, \mathbb{E}\{\Phi_{i,k+1|k+1}^{\alpha_{1j,k}}\} \leq \mathbb{E}\{\Phi_{i,k+1|k+1}^{\alpha_{2j,k}}\}$$

where $\Phi_{i,k+1|k}^{\alpha_{*j,k}}$ (or $\Phi_{i,k+1|k+1}^{\alpha_{*j,k}}$) stands for the minimized upper bound of $\Phi_{i,k+1|k}$ (or $\Phi_{i,k+1|k+1}$) under the given sequence $\{\alpha_{*j,k}\}$.

P r o o f. Without loss of generality, denote $\tilde{\Gamma}_i = \Gamma_i/\{j\}$, $\tilde{\Upsilon}_{i,k} = \{\alpha_{j,k}|j \in \tilde{\Gamma}_i\}$ and $\mathbb{P}\{\alpha_{j,k} = 1\} = \bar{\alpha}_j$. For any sequence $\{\alpha_{j,k}\}$ $(j \in \Gamma_i)$, one has

$$\mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{j,k}}\}$$
$$= \mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{j,k}}|\tilde{\Upsilon}_{i,k}, \{\alpha_{j,k} = 0\}\}\mathbb{P}\{\tilde{\Upsilon}_{i,k}, \alpha_{j,k} = 0\}$$
$$+ \mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{j,k}}|\tilde{\Upsilon}_{i,k}, \{\alpha_{j,k} = 1\}\}\mathbb{P}\{\tilde{\Upsilon}_{i,k}, \alpha_{j,k} = 1\}$$
$$= (1 - \bar{\alpha}_j)\mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{j,k}}|\tilde{\Upsilon}_{i,k}, \{\alpha_{j,k} = 0\}\}\mathbb{P}\{\tilde{\Upsilon}_{i,k}\}$$
$$+ \bar{\alpha}_j\mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{j,k}}|\tilde{\Upsilon}_{i,k}, \{\alpha_{j,k} = 1\}\}\mathbb{P}\{\tilde{\Upsilon}_{i,k}\}. \qquad (20)$$

For the second term in the right side of (20), we have

$$\bar{\alpha}_j \mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{j,k}} | \tilde{\Upsilon}_{i,k}, \{\alpha_{j,k} = 1\}\} \mathbb{P}\{\tilde{\Upsilon}_{i,k}\}$$
$$= \bar{\alpha}_j\big((1 + \varepsilon_{0i})A_{ii}\Phi_{i,k|k}A_{ii}^T + Q_i + (1 + \varepsilon_{0i}^{-1})\hat{\Lambda}_{i,k}\big)\mathbb{P}\{\tilde{\Upsilon}_{i,k}\},$$

where

$$\tilde{\aleph}_{i,k} = \sum\nolimits_{j \in \tilde{\Gamma}_i} (1 - \alpha_{j,k})A_{ij}(I - H_{j,k})\hat{x}_{j,k|k}$$
$$\hat{\Lambda}_{i,k} = (1 + \varepsilon_{1i})\Omega_{i,k} + (1 + \varepsilon_{1i}^{-1})\tilde{\aleph}_{i,k}\tilde{\aleph}_{i,k}^T.$$

Substituting it into (20), one has

$$\mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{j,k}}\}/\mathbb{P}\{\tilde{\Upsilon}_{i,k}\}$$
$$= \bar{\alpha}_j\big((1 + \varepsilon_{0i})A_{ii}\Phi_{i,k|k}A_{ii}^T + Q_i + (1 + \varepsilon_{0i}^{-1})\hat{\Lambda}_{i,k}\big)$$
$$+ (1 - \bar{\alpha}_j)\big((1 + \varepsilon_{0i})A_{ii}\Phi_{i,k|k}A_{ii}^T + Q_i + (1 + \varepsilon_{0i}^{-1})\Lambda_{i,k}\big).$$

Taking the derivation of $\mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{j,k}}\}/\mathbb{P}\{\tilde{\Upsilon}_{i,k}\}$ on the probability $\bar{\alpha}_j$ leads to

$$\frac{d}{d\bar{\alpha}_j}\mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{i,k}}\}/\mathbb{P}\{\tilde{\Upsilon}_{i,k}\}$$
$$= (1 + \varepsilon_{0i}^{-1})(\hat{\Lambda}_{i,k} - \Lambda_{i,k})$$
$$= (1 + \varepsilon_{0i}^{-1})(\tilde{\aleph}_{i,k}\tilde{\aleph}_{i,k}^T - \aleph_{i,k}\aleph_{i,k}^T)$$
$$= -(1 + \varepsilon_{0i}^{-1})(1 + \varepsilon_{1i}^{-1})$$
$$\times (A_{ij}(I - H_{j,k})\hat{x}_{j,k|k})(A_{ij}(I - H_{j,k})\hat{x}_{j,k|k})^T$$
$$\leq 0.$$

On the other hand, denote

$$\Im_{i,k}^0 = \mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{j,k}} | \tilde{\Upsilon}_{i,k}, \{\alpha_{j,k} = 0\}\}$$
$$= (1 + \varepsilon_{0i})A_{ii}\Phi_{i,k|k}A_{ii}^T + Q_i + (1 + \varepsilon_{0i}^{-1})\Lambda_{i,k}$$
$$\Im_{i,k}^1 = \mathbb{E}\{\Phi_{i,k+1|k}^{\alpha_{j,k}} | \tilde{\Upsilon}_{i,k}, \{\alpha_{j,k} = 1\}\}$$
$$= (1 + \varepsilon_{0i})A_{ii}\Phi_{i,k|k}A_{ii}^T + Q_i + (1 + \varepsilon_{0i}^{-1})\hat{\Lambda}_{i,k},$$

and have $\Im_{i,k}^0 \geq \Im_{i,k}^1$.

Then, it follows from (10) and (17) that

$$\Phi_{i,k+1|k+1} = (\Phi_{i,k+1|k}^{-1} + C_i^T R_i C_i)^{-1} + \delta_i \lambda_{\max}\{R_i + C_i\Phi_{i,k+1|k}C_i^T\}I.$$

Furthermore, it follows that

$$\mathbb{E}\{\Phi_{i,k+1|k+1}^{\alpha_{j,k}}\}/\mathbb{P}\{\tilde{\Upsilon}_{i,k}\}$$
$$= (1 - \bar{\alpha}_j)\mathbb{E}\{\Phi_{i,k+1|k+1}^{\alpha_{j,k}} | \tilde{\Upsilon}_{i,k}, \{\alpha_{j,k} = 0\}\} + \bar{\alpha}_j\mathbb{E}\{\Phi_{i,k+1|k+1}^{\alpha_{j,k}} | \tilde{\Upsilon}_{i,k}, \{\alpha_{j,k} = 1\}\}$$

$$= (1 - \bar{\alpha}_j)\big(((\Im_{i,k}^0)^{-1} + C_i^T R_i C_i)^{-1} + \delta_i \lambda_{\max}\{R_i + C_i \Im_{i,k}^0 C_i^T\}I\big)$$
$$+ \bar{\alpha}_j\big(((\Im_{i,k}^1)^{-1} + C_i^T R_i C_i)^{-1} + \delta_i \lambda_{\max}\{R_i + C_i \Im_{i,k}^1 C_i^T\}I\big).$$

Finally, taking the derivation of above equation on the probability $\bar{\alpha}_j$ leads to

$$\frac{d}{d\bar{\alpha}_j} \mathbb{E}\{\Phi_{i,k+1|k+1}^{\alpha_{j,k}}\}/\mathbb{P}\{\tilde{\Upsilon}_{i,k}\}$$
$$= ((\Im_{i,k}^1)^{-1} + C_i^T R_i C_i)^{-1} - ((\Im_{i,k}^0)^{-1} + C_i^T R_i C_i)^{-1}$$
$$+ \delta_i \lambda_{\max}\{R_i + C_i(\Im_{i,k}^1 - \Im_{i,k}^0)C_i^T\}I$$
$$\leq 0.$$

Therefore, it conclude that the filtering performance becomes better when the probability $\bar{\alpha}_j$ increases, which completes the proof.   □

Remark 3.6. In Theorem 3.3, the introduced positive scalars can be selected as

$$\varepsilon_{1i,k} = \sqrt{\frac{\mathrm{Tr}(\Omega_{i,k})}{\mathrm{Tr}(\aleph_{i,k}\aleph_{i,k}^T)}}, \quad \varepsilon_{0i,k} = \sqrt{\frac{\mathrm{Tr}(A_{ii}\Phi_{i,k|k}A_{ii}^T)}{\mathrm{Tr}(\Lambda_{i,k})}}$$

such that the trace of covariance matrices is small as much as possible. However, the analysis of filtering performance in this case is nontrivial due to the complex calculation of $\varepsilon_{0i,k}$ and $\varepsilon_{1i,k}$ and therefore it constitutes our future research topic.

4. SIMULATION RESULTS

In this section, the developed filtering algorithm will be applied into the representative power systems in order to illustrate its effectiveness. To this end, we employ a power system, which consists of 4 coupled power generation areas shown in Figure 1. In this figure, the predetermined topology $a_{12} = a_{21} = a_{23} = a_{24} = a_{32} = a_{34} = a_{42} = 1$. Note that it is just the scenario 1 of the Hycon2 Project proposed in [30].
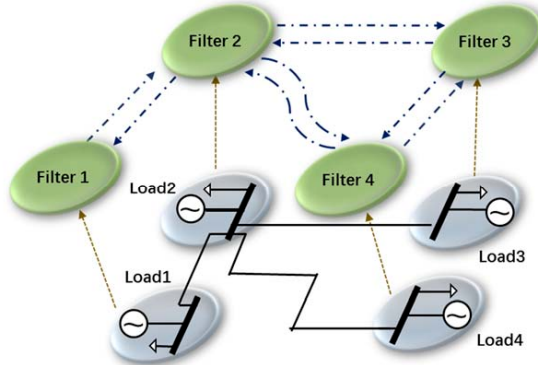

Fig. 1. A framework of distributed resilient filtering for a power system.

$\Delta\theta_i$	Deviation of the angular displacement of the rotor with respect to the stationary reference axis on the stator.
Δw_i	Speed deviation of rotating mass.
ΔP_{m_i}	Deviation of the mechanical power.
ΔP_{v_i}	Deviation of the steam valve position.
ΔP_{ref_i}	Deviation of the reference set power.
ΔP_{L_i}	Deviation of the nonfrequency-sensitive load change.
H_i	Inertia constant defined as

$$H_i = \frac{\text{Kinetic energy at rated speed}}{\text{Machine rating}}.$$

| R_i | Speed regulation. |
| D_i | Defined as |

$$D_i = \frac{\text{Percent change in load}}{\text{Change in frequency}}.$$

T_{t_i}	Prime mover time constant.
T_{g_i}	Governor time constant.
P_{ij}	Slope of the power angle curve at the initial operating angle between area i and area j.

Tab. 1. System variables of power system.

	Area1	**Area2**	**Area3**	**Area4**
H_i	12	10	8	10
K_i	0.05	0.0625	0.08	0.05
D_i	0.7	0.9	0.9	0.86
T_{t_i}	0.65	0.4	0.3	0.8
T_{g_i}	0.1	0.1	0.1	0.15

Tab. 2. Parameters of the four-area power systems.

P_{ij}	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$		4		
$i = 2$	4		2	
$i = 3$		2		2
$i = 4$			2	

Tab. 3. Parameters of the four-area power systems.

Time (k)	1	2	3	4	5	6	$\cdots$
$\gamma_{i1,k}$	1	1	1	1	1	1	$\cdots$
$\gamma_{i2,k}$	1	1	0	1	1	0	$\cdots$
$\gamma_{i3,k}$	1	0	1	1	0	1	$\cdots$
$\gamma_{i4,k}$	0	1	1	0	1	1	$\cdots$

Tab. 4. Parameters of the four-area power systems.

In what follows, the dynamic of each power generation region in continuous-time case is modeled as follows

$$\dot{x}_i(t) = A'_{ii}x_i(t) + \sum_{j \in \mathcal{N}_i} A'_{ij}x_j(t) + L'_i\Delta P_{L_i} + B'_i u_i,$$

where $x_i = [\ \Delta\theta_i \quad \Delta w_i \quad \Delta P_{m_i} \quad \Delta P_{v_i}\ ]^T$ is the state of area i, $u_i = \Delta P_{ref_i}$ represents the control effort of area i, and ΔP_{L_i} is the local power load. Furthermoer, the definitions of elements (i. e. system variables of power systems) are shown in Table 4 and Table 4 and their values are listed in Table 4. Finally, the system matrices are further described as

$$A'_{ii} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{\sum_{j \in \mathcal{A}_i} P_{ij}}{2H_i} & -\frac{D_i}{2H_i} & \frac{1}{2H_i} & 0 \\ 0 & 0 & -\frac{1}{T_{t_i}} & \frac{1}{T_{t_i}} \\ 0 & -\frac{1}{R_iT_{g_i}} & 0 & -\frac{1}{T_{g_i}} \end{bmatrix},$$

$$B'_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_{g_i}} \end{bmatrix}, \quad A'_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{P_{ij}}{2H_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$L'_i = \begin{bmatrix} 0 \\ -\frac{1}{2H_i} \\ 0 \\ 0 \end{bmatrix}, \quad C_i = \begin{bmatrix} 1 & 0 & 0 & 0.6 \\ 0 & 1 & 0 & 0.3 \\ 0 & 0 & 1 & 0.5 \end{bmatrix}.$$

According to the filtering issure, the knwon inputs do not affect the filtering performance and thereby the known constants ΔP_{ref_i} and ΔP_{L_i} are removed from the system addressed above. Similar to the method of reference [29], we discretize the continuous system that the covariance matrices of Gaussian noise sequences $w_{i,k}$ and $v_{i,k}$ are selected as $Q_i = 0.1I$ and $R_i = 0.1I$ for any subsystem i.

In order to save the energy, two kinds of channels are employed to implement the information transmission where the scalar r_i is predetermined as 3 and the selection schmem of reliable channels $H_{i,k} = \text{diag}\{\gamma_{i1,k}, \gamma_{i2,k}, \gamma_{i3,k}, \gamma_{i4,k}\}$ are shown in Table 4 for all i. Furthermore, the bound of gain variation is $\delta_i = 0.001$ for all subsystem i.

In the simulation, the corresponding discrete-time model is not difficult to be obtained by selecting the sampling period $T = 1s$, and the initial conditions of power systems

and filtering error covariance are chosen as

$$x_{1,0} = [5.6,\ 6,\ 5.2,\ 5.6]^T, x_{2,0} = [4.4,\ 4.8,\ 7.2,\ 5.2]^T,$$
$$x_{3,0} = [4.4,\ 11.2,\ 11.2,\ 6.4]^T, x_{4,0} = [5.2,\ 5.2,\ 4.4,\ 5.6]^T,$$
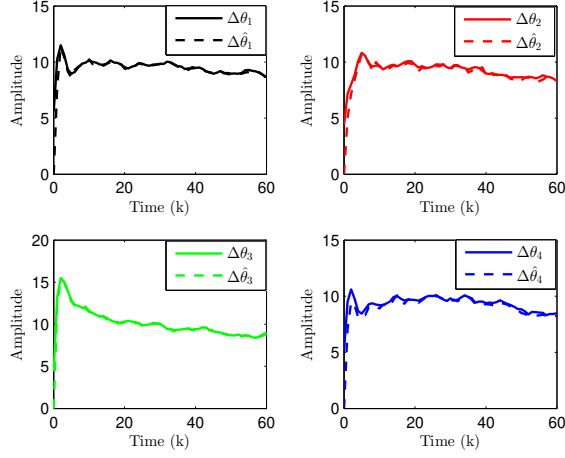$$\Psi_{i,0|0} = I, \quad (i = 1, 2, 3, 4).$$

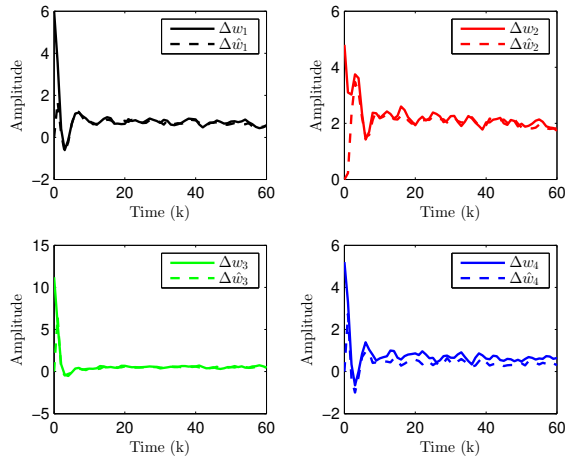**Fig. 2.** The true value θ_i and its estimation $\hat{\theta}_i$ $(i = 1, 2, 3, 4)$.

Fig. 3. The true value w_i and its estimation $\hat{w}_i$ $(i = 1, 2, 3, 4)$.
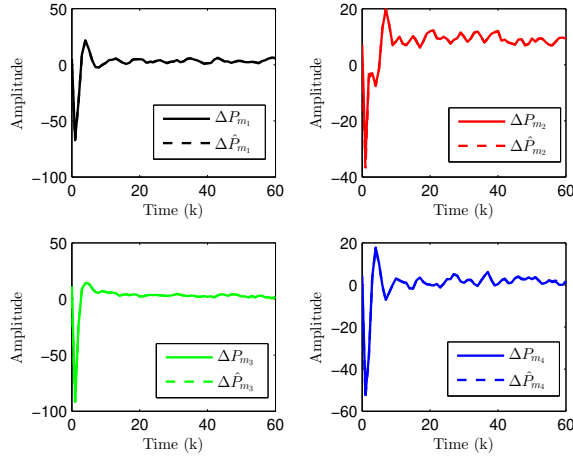
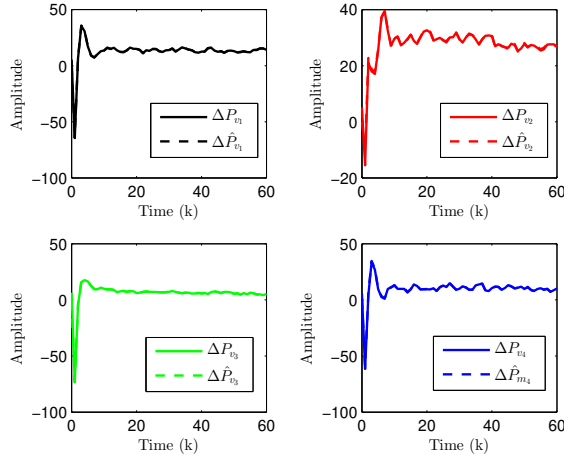**Fig. 4.** The true value P_{m_i} and its estimation $\hat{P}_{m_i}$ $(i = 1, 2, 3, 4)$.

Fig. 5. The true value P_{v_i} and its estimation $\hat{P}_{v_i}$ $(i = 1, 2, 3, 4)$.

Furthermore, the square of filtering errors, defined by $\mathrm{SE}_{i,k} = e_{i,k|k}^T e_{i,k|k}$, is utilized to evaluate the filtering performance. The proposed scheme is tested in MATLAB (R2014a). The simulation results are depicted in Figures $2-6$, where Figures $2-5$ describe the trajectories of the true states $x_{i,k}(i = 1, 2, 3, 4)$ and its estimation $\hat{x}_{i,k|k}$. Figure 6 plots the logarithmic trace of matrix $\Phi_{i,k|k}$ and the logarithmic square of estimation error $(\mathrm{SE}_{i,k})$. The simulation example illustrates that the developed filtering algorithm performed well.
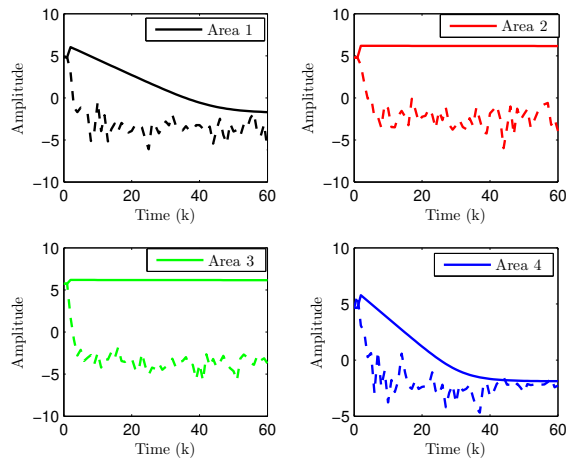
Fig. 6. The upper bound of $\Omega_{i,k|k}$ and $\mathrm{SE}_{i,k}$.

5. CONCLUSIONS

In this paper, we have investigated a distributed resilient filtering for discrete-time LSSs, where energy constraints have been governed by two kinds of channels with different energy allocation. For the addressed issue, a novel distributed filter subject to gain perturbations has been designed with the help of compensated neighboring estimation. The bounds of the covariance matrices of one-step prediction errors and the filtering errors have been obtained and then minimized to analytically obtain the desired gain matrices. The proposed distributed filtering algorithm satisfies the requirements of low calculation burden and scalability although it is suboptimal. Furthermore, it has been disclosed that the filtering performance becomes better when the probability of packet loss decreases. Further research topics will focus on distributed resilient filtering of large-scale systems under more complex scenarios, such as cyber attacks [2, 17, 33], communication protocols [3], as well as performance optimization [8].

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