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VARIANCE-CONSTRAINED H_∞ FINITE-HORIZON FILTERING FOR MULTI-RATE TIME-VARYING NETWORKED SYSTEMS BASED ON STOCHASTIC PROTOCOLS

Ming Lyu, Jie Zhang, and YuMing Bo

In this paper, the variance-constrained H_∞ finite-horizon filtering problem is investigated for a class of time-varying nonlinear system under muti-rate communication network and stochastic protocol (SP). The stochastic protocol is employed to determine which sensor obtains access to the muti-rate communication network in order to relieve communication burden. A novel mapping technology is applied to characterize the randomly switching behavior of the data transmission resulting from the utilization of the SP in muti-rate communication network. By using relaxation method, sufficient conditions are derived for the existence of the finite-horizon filter satisfying both the prescribed H_∞ performance and the covariance requirement of filtering errors, and the solutions of filters satisfying the above indexes are obtained by using linear matrix inequalities. Finally, the validity and effectiveness of the proposed filter scheme are verified by numerical simulation.

Keywords: H_∞ finite-horizon filtering, muti-rate communication, stochastic protocol (SP), time-varying systems

Classification: 93C10, 93C55, 93E11, 60G35

1. INTRODUCTION

Filtering or state estimation problems have long been of far-reaching significance in signal processing, communication, control, and other fields and received many attentions in past few decades [1, 2, 8, 18, 22, 31]. The celebrated Kalman filtering approach is one of the most popular ways to deal with the state estimation of a dynamic system [3, 5, 9]. It is noted that the design of the Kalman filter relies on the exact knowledge of the statistics of the external disturbance. However, in practical applications, the statistics of the external noises may not be known accurately and the uncertainties of the system may also be included in the modeling, hence the H_∞ filtering approach has gained considerable research attentions [11, 13, 14, 19, 36]. It has been shown that the H_∞ filtering approach provides not only a guaranteed noise attenuation level but also robustness against unmodeled dynamics. So far, there has been a rich body of research results available in the literature concerning the analysis of the H_∞ filtering problem,

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see, e. g., [13,19]. On the other hand, it is usually required that the estimation or filtering error variance of design for estimator or filter is below a certain upper bound [16, 27]. The two upper bounds mentioned above may not be the smallest, but be able to meet certain engineering criteria. At present, the theory based on variance constraints has been widely used in solving multi-objective control problems and filtering problems [25].

With the rapid development of network technologies, recently, there has been a lot of interests in networked control systems whose components are interconnected through the communication networks, see [4, 6, 7, 10, 12, 32, 34, 35]. In particular, the control and filtering problems of NCSs have received a great deal of research attentions. As we all know, the networks with limited bandwidth could lead to data collisions. One of the valid ways to avoid the data collisions is to schedule data transmissions according to a set of protocols. More specifically, protocols should be designed to determine a subset of the sensors that can access the communication network at each time. Generally, the typical scheduling protocols include the periodic protocols, the stochastic protocols, and the quadratic protocols. Up to now, some preliminary results have been reported in the literature concerning the stability analysis, control, filtering, and fault detection problems subject to various communication protocols, see, e. g., [15,20,28]. Among these protocols, the stochastic communication protocol (SCP) has been widely applied in the automation industry, see, e. g., [15, 23].

Many studies have shown that the faster the update frequency of the sensor or controller signal, the better the filtering performance of the network control system. On the other hand, because the network control system has the characteristics of limited network bandwidth, computing power, and other resources, it is obviously unrealistic to use infinite update frequency. In order to balance the contradiction between limited resources and system performance, the multi-rate sampling method has attracted many attentions, see, [17, 24, 26, 30]. A great number of achievements have been made in the research of time-invariant multi-rate systems in recent years. [26] studies the multi-rate fusion estimation problem under random nonlinearity and colored measurement noise based on event triggering mechanism. The problem of state estimation for multi-rate systems with probabilistic sensor faults and measurement quantization is studied in [33]. In [21], the finite time domain H_∞ filtering problem for multi-rate time-varying systems is studied by using stochastic communication protocol (SCP). Most of the above articles on multi-rate network systems only consider the time-invariant situation [26, 33]. Because of the complexity and uncertainty of the network environment, the time-varying network system model is closer to reality. A few papers on time-varying multi-rate network systems [15, 21] only consider H_∞ constrained performance index, it should be noted that it is more meaningful to consider the variance of time-varying systems in a limited time range in order to provide better transient performances.

In this paper, we aim to deal with the finite-horizon H_∞ filtering problem for the time-varying networked system under multi-rate communication network and stochastic protocol(SP). Specifically, the main contributions of this paper lie in the following aspects:

(i) The H_∞ filtering problem is investigated for stochastic discrete-time systems with the SP constraints;

(ii) The communication process under consideration is comprehensive that takes both the scheduling protocol and the high-rate network into account;

(iii) The filter gain matrices are obtained by solving a set of recursive linear matrix inequalities (LMIs).

The rest of this paper is organized as follows. In Section 2, the variance-constrained H_∞ filtering problem over finite-horizon is formulated for the time-delayed networked system under multi-rate communication network and the stochastic protocol(SP). The conditions for H_∞ performance and state variance are expressed in terms of LMI in Section 3. Based on the developed conditions, the desired finite horizon filter is then designed. In Section 4, a numerical example is provided to demonstrate the effectiveness of the proposed filtering schemes. Finally, the conclusion is drawn in Section 5.

Notation. The notations used throughout the paper are fairly standard except where otherwise stated. $\mathbb{R}^{n\times m}$ and $\mathbb{R}^n$ denote, respectively, the set of all $n \times m$ real matrix space and the n-dimensional Euclidean space. For given matrix X, $\mathrm{tr}\{X\}$ and X^T represent the operation of the trace and the transpose, respectively. $\mathbb{E}\{x\}$ denotes the expectation of stochastic variable x, and $\mathbb{P}\{x\}$ represents the occurrence probability of event x. The notation $\mathbf{0}$ represents a matrix or vector of appropriate dimensions, where each element is zero.

2. PROBLEM FORMULATION

Consider the following class of discrete time-varying systems:

$$\begin{cases} x(t_{k+1}) = A(t_k)x(t_k) + f(t_k) + B(t_k)\omega(t_k) \\ y(t_k) = C(t_k)x(t_k) + g(t_k) + D(t_k)\omega(t_k) \\ z(t_k) = L(t_k)x(t_k) \end{cases} \tag{1}$$

where t_k $(k \in \{0,1,2,\cdots\})$ is the sampling instants, $x(t_k) \in \mathbb{R}^{n_x}$ represents the state vector, $y(t_k) \in \mathbb{R}^{n_y}$ is the measured output vector, $z(t_k) \in \mathbb{R}^{n_z}$ is the signal to be estimated, $\omega(t_k) \in \mathbb{R}^{n_\omega}$ is a zero mean Gaussian white noise sequence with covariance $W > 0$. $A(t_k)$, $B(t_k)$, $C(t_k)$, $D(t_k)$ and $L(t_k)$ are known time-varying matrices.

The nonlinear stochastic functions $f(t_k)$ and $g(t_k)$ are assumed to have the following statistical characteristics:

$$\mathbb{E}\left\{\begin{bmatrix} f(t_k) \\ g(t_k)\end{bmatrix} \Big| x(t_k)\right\} = \mathbf{0}, \tag{2}$$

$$\mathbb{E}\left\{\begin{bmatrix} f(t_k) \\ g(t_k)\end{bmatrix} \begin{bmatrix} f^T(t_j) & g^T(t_j)\end{bmatrix} \Big| x(t_k)\right\} = \mathbf{0},\ (k \neq j) \tag{3}$$

$$\begin{aligned}\mathbb{E}&\left\{\begin{bmatrix} f(t_k) \\ g(t_k)\end{bmatrix} \begin{bmatrix} f^T(t_k) & g^T(t_k)\end{bmatrix} \Big| x(t_k)\right\} \\ &= \sum_{\ell=1}^{q} \Pi_\ell(t_k)\left(x^T(t_k)\Gamma_\ell(t_k)x(t_k)\right),\end{aligned} \tag{4}$$

where $\Pi_\ell(t_k)$ and $\Gamma_\ell(t_k)$ $(\ell = 1,2,\cdots,q)$ are known positive definite matrices with compatible dimensions.

Next, we discuss the effect induced by high-rate communication network. Before proceeding further, we introduce the following assumptions.

Assumption 2.1. The system state $x(t_k)$ and its estimation $z(t_k)$ are updated at instants t_k, and the sampling period satisfies $t_{k+1} - t_k = h$ for any $k = 0, 1, 2, \cdots$.

Assumption 2.2. The signal transmissions are implemented periodically between the sensors and the filter with a constant period h_c satisfying $h_c = h/d$, where d is a positive integer.

Assumption 2.3. The initial transmission instant equals to the initial sampling instant, i. e., $\bar{t}_0 = t_0$.

In this part, the stochastic protocol is employed to determine which sensor obtains access to the high-rate communication network. According to the SP scheduling, only one sensor is allowed to get access to communication network at each transmission instant. For the sake of examining the influence of the SP constraints, let $\sigma(\bar{t}_k) \in S \triangleq \{1, 2, \cdots, n_y\}$ be the selected sensor obtaining access to the communication network at transmission instant $\bar{t}_k$. Under the SP scheduling, $\sigma(\bar{t}_k)$ can be governed by a Markov chain with the transition probability matrix $\Lambda = [\rho_{ij}]$, which is defined as follows:

$$\rho_{ij} = \text{Prob}\left(\sigma(\bar{t}_{k+1}) = j \,|\sigma(\bar{t}_k) = i\right), \forall i, j \in S \tag{5}$$

where ρ_{ij} is the transition probability from the state i to the state j satisfying $\sum_{j=1}^{n_y} \rho_{ij} = 1$ $(\forall i \in S)$.

Let $\bar{y}(\bar{t}_k) = \left[\bar{y}_1^T(\bar{t}_k)\ \bar{y}_2^T(\bar{t}_k)\ \cdots\ \bar{y}_{n_y}^T(\bar{t}_k)\right]^T$ be the measurement output after transmitted through the communication network at time $\bar{t}_k$. The updating rule for $\bar{y}_i(\bar{t}_k)$ $(i = 1, 2, \cdots, n_y,\ t_k \le \bar{t}_k \le t_{k+1})$ is set to be

$$\bar{y}_i(\bar{t}_k) = \begin{cases} y_i(t_k) + E_i(\bar{t}_k)\nu(\bar{t}_k), & \text{if } i = \sigma(\bar{t}_k) \\ \bar{y}_i(\bar{t}_{k-1}), & \text{otherwise} \end{cases} \tag{6}$$

where $\nu(t_k) \in \mathbb{R}^{n_\nu}$ is a zero mean Gaussian white noise sequence with covariance $V > 0$, and $E_i(\bar{t}_k)$ is a known, real, time-varying matrix.

Denoting

$$E(\bar{t}_k) = \left[E_1^T(\bar{t}_k)\ E_2^T(\bar{t}_k)\ \cdots\ E_{n_y}^T(\bar{t}_k)\right]^T,$$
$$\Xi(\sigma(\bar{t}_k)) = \text{diag}\left\{\delta(\sigma(\bar{t}_k) - 1), \delta(\sigma(\bar{t}_k) - 2), \cdots, \delta(\sigma(\bar{t}_k) - n_y)\right\},$$

we have

$$\bar{y}(\bar{t}_k) = \Xi(\sigma(\bar{t}_k))(y(t_k) + E(\bar{t}_k)\nu(\bar{t}_k)) + (I - \Xi(\sigma(\bar{t}_k)))\bar{y}(\bar{t}_{k-1}). \tag{7}$$

where $\delta(\cdot)$ is the Kronecker delta function.

Considering the measurement (7) in the time interval $(t_{k-1}, t_k]$, one has

$$\bar{y}(t_k - (d-1)h_c) = \Xi(\sigma(t_k - (d-1)h_c))E(t_k - (d-1)h_c)\nu(t_k - (d-1)h_c)$$

$$
\begin{aligned}
&+\Xi\left(\sigma\left(t_k-(d-1)\,h_c\right)\right)y\left(t_{k-1}\right)\\
&+\left(I-\Xi\left(\sigma\left(t_k-(d-1)\,h_c\right)\right)\right)\bar{y}\left(t_{k-1}\right)\\
\bar{y}\left(t_k-(d-2)\,h_c\right)=&\ \Xi\left(\sigma\left(t_k-(d-2)\,h_c\right)\right)E\left(t_k-(d-2)\,h_c\right)\nu\left(t_k-(d-2)\,h_c\right)\\
&+\Xi\left(\sigma\left(t_k-(d-2)\,h_c\right)\right)\left(y\left(t_{k-1}\right)\right.\\
&+\left(I-\Xi\left(\sigma\left(t_k-(d-2)\,h_c\right)\right)\right)\bar{y}\left(t_k-(d-1)\,h_c\right)\\
&\ \vdots\\
\bar{y}\left(t_k-h_c\right)=&\ \Xi\left(\sigma\left(t_k-h_c\right)\right)E\left(t_k-h_c\right)\nu\left(t_k-h_c\right)\\
&+\Xi\left(\sigma\left(t_k-h_c\right)\right)\left(y\left(t_{k-1}\right)\right.\\
&+\left(I-\Xi\left(\sigma\left(t_k-h_c\right)\right)\right)\bar{y}\left(t_k-2h_c\right)\\
\bar{y}\left(t_k\right)=&\ \Xi\left(\sigma\left(t_k\right)\right)\left(y\left(t_k\right)+E\left(t_k\right)\nu\left(t_k\right)\right)+\left(I-\Xi\left(\sigma\left(t_k\right)\right)\right)\bar{y}\left(t_k-h_c\right)
\end{aligned}
$$

which implies

$$
\begin{aligned}
\bar{y}\left(t_k\right)=&\ \Xi\left(\sigma\left(t_k\right)\right)y\left(t_k\right)+\psi_1\left(\bar{\sigma}\left(t_k^{d-1}\right)\right)y\left(t_{k-1}\right)\\
&+\psi_2\left(\bar{\sigma}\left(t_k^{d-1}\right)\right)\bar{y}\left(t_{k-1}\right)+\psi_3\left(\bar{\sigma}\left(t_k^{d-1}\right)\right)\bar{\nu}\left(t_k\right)
\end{aligned}
\tag{8}
$$

where, for $\ell=1,2,\cdots,d-1$

$$
\begin{aligned}
\bar{\nu}\left(t_k\right)=&\ \left[\nu^T\left(t_k\right)\ \ \nu^T\left(t_k-h_c\right)\ \ \cdots\ \ \nu^T\left(t_k-(d-1)\,h_c\right)\right]^T,\\
\bar{\sigma}\left(t_k^{\ell}\right)=&\ \left(\sigma\left(t_k\right),\sigma\left(t_k-h_c\right),\cdots,\sigma\left(t_k-\ell h_c\right)\right),\\
\psi_1\left(\bar{\sigma}\left(t_k^{d-1}\right)\right)=&\sum_{i=1}^{d-1}\left(\prod_{j=0}^{i-1}\left(I-\Xi\left(\sigma\left(t_k-jh_c\right)\right)\right)\Xi\left(\sigma\left(t_k-ih_c\right)\right)\right),\\
\psi_2\left(\bar{\sigma}\left(t_k^{d-1}\right)\right)=&\prod_{i=0}^{d-1}\left(I-\Xi\left(\sigma\left(t_k-ih_c\right)\right)\right),\\
\psi_3\left(\bar{\sigma}\left(t_k^{d-1}\right)\right)=&\left[\Xi\left(\sigma\left(t_k\right)\right)E\left(t_k\right)\ \ \psi_{31}\left(\bar{\sigma}\left(t_k^{1}\right)\right)\ \ \cdots\ \ \psi_{3(d-1)}\left(\bar{\sigma}\left(t_k^{d-1}\right)\right)\right],\\
\psi_{3\ell}\left(\bar{\sigma}\left(t_k^{\ell}\right)\right)=&\prod_{j=0}^{\ell-1}\left(I-\Xi\left(\sigma\left(t_k-jh_c\right)\right)\right)\Xi\left(\sigma\left(t_k-\ell h_c\right)\right)E\left(t_k-\ell h_c\right).
\end{aligned}
$$

In what follows, denoting

$$
\begin{aligned}
&\tilde{x}\left(t_k\right)\triangleq\left[x^T\left(t_k\right)\ \ y^T\left(t_{k-1}\right)\ \ \bar{y}^T\left(t_{k-1}\right)\right]^T,\\
&d\left(t_k\right)=\begin{bmatrix}\omega\left(t_k\right)\\ \bar{\nu}\left(t_k\right)\end{bmatrix},\ \bar{n}\left(x\left(t_k\right)\right)=\begin{bmatrix}f\left(x\left(t_k\right)\right)\\ g\left(x\left(t_k\right)\right)\\ g\left(x\left(t_k\right)\right)\end{bmatrix},
\end{aligned}
$$

the system (1) combined with (8) can be reformulated as follows:

$$
\left\{
\begin{aligned}
\tilde{x}\left(t_{k+1}\right)=&\ \bar{A}\left(t_k,\bar{\sigma}\left(t_k^{d-1}\right)\right)\tilde{x}\left(t_k\right)+\bar{M}\left(t_k\right)\bar{n}\left(x\left(t_k\right)\right)+\bar{B}\left(t_k,\bar{\sigma}\left(t_k^{d-1}\right)\right)d\left(t_k\right)\\
\bar{y}\left(t_k\right)=&\ \bar{C}\left(t_k,\bar{\sigma}\left(t_k^{d-1}\right)\right)\tilde{x}\left(t_k\right)+\bar{E}\left(t_k\right)\bar{n}\left(x\left(t_k\right)\right)+\bar{D}\left(t_k,\bar{\sigma}\left(t_k^{d-1}\right)\right)d\left(t_k\right)\\
z\left(t_k\right)=&\ \bar{\bar{L}}\left(t_k\right)\tilde{x}\left(t_k\right)
\end{aligned}
\right.
\tag{9}
$$

where

$$\begin{aligned}
\bar{A}\left(t_k,\bar{\sigma}\left(t_k^{d-1}\right)\right) &= \begin{bmatrix} A\left(t_k\right) & 0 & 0 \\ C\left(t_k\right) & 0 & 0 \\ \Xi\left(\sigma\left(t_k\right)\right)C\left(t_k\right) & \psi_1\left(\bar{\sigma}\left(t_k^{d-1}\right)\right) & \psi_2\left(\bar{\sigma}\left(t_k^{d-1}\right)\right)\end{bmatrix}, \\
\bar{B}\left(t_k,\bar{\sigma}\left(t_k^{d-1}\right)\right) &= \begin{bmatrix} B\left(t_k\right) & 0 \\ D\left(t_k\right) & 0 \\ \Xi\left(\sigma\left(t_k\right)\right)D\left(t_k\right) & \psi_3\left(\bar{\sigma}\left(t_k^{d-1}\right)\right)\end{bmatrix}, \\
\bar{C}\left(t_k,\bar{\sigma}\left(t_k^{d-1}\right)\right) &= \begin{bmatrix}\Xi\left(\sigma\left(t_k\right)\right)C\left(t_k\right) & \psi_1\left(\bar{\sigma}\left(t_k^{d-1}\right)\right) & \psi_2\left(\bar{\sigma}\left(t_k^{d-1}\right)\right)\end{bmatrix}, \\
\bar{D}\left(t_k,\bar{\sigma}\left(t_k^{d-1}\right)\right) &= \begin{bmatrix}\Xi\left(\sigma\left(t_k\right)\right)D\left(t_k\right) & \psi_3\left(\bar{\sigma}\left(t_k^{d-1}\right)\right)\end{bmatrix}, \\
\bar{M}\left(t_k\right) &= \mathrm{diag}\{I,I,\Xi\left(\sigma(t_k)\right)\},\quad \bar{E}\left(t_k\right) = \Xi\left(\sigma\left(t_k\right)\right), \\
\bar{\bar{L}}\left(t_k\right) &= \begin{bmatrix}L\left(t_k\right) & 0 & 0\end{bmatrix}.
\end{aligned}$$

In addition, $d\left(t_k\right)$ satisfies

$$\begin{aligned}
&\mathbb{E}\left\{d\left(t_k\right)d^T\left(t_j\right)\right\} = 0,\ \left(k \neq j\right),\ \ \mathbb{E}\left\{d\left(t_k\right)\right\} = 0, \\
&\mathbb{E}\left\{d\left(t_k\right)d^T\left(t_k\right)\right\} = \mathrm{diag}\left\{W,\bar{V}\right\} = R\left(t_k\right)
\end{aligned}$$

where $\bar{V} = \mathrm{diag}\underbrace{\{V,\cdots,V\}}_{d}$.

For the convenience of performance analysis, we now reformulate the system (9) via a mapping method.

Proposition 2.4. The SP constraints governed by $\bar{\sigma}\left(t_k^{d-1}\right)$ in the augmented system (8) can be mapped to the sequence $r\left(t_k\right) \in \mathbb{S} \triangleq \left\{1,2,\cdots,n_y^d\right\}$ by the mapping $\mathscr{R}\left(\cdot\right)$

$$r\left(t_k\right) = \mathscr{R}\left(\bar{\sigma}\left(t_k^{d-1}\right)\right) \triangleq \sum_{i=0}^{d-1}\left(n_y\right)^i\left(\sigma\left(t_k - ih_c\right) - 1\right) + 1. \tag{10}$$

Moreover, if $r\left(t_k\right)$ is given, the values of $\sigma\left(t_k - ih_c\right)$ can be derived by $\phi_i\left(r\left(t_k\right)\right)$,

$$\sigma\left(t_k - ih_c\right) = \phi_i\left(r\left(t_k\right)\right) \triangleq \mathrm{mod}\left(\left\lfloor\frac{r\left(t_k\right) - 1}{\left(n_y\right)^i}\right\rfloor, n_y\right) + 1,\ \left(i = 0,1,\cdots,d-1\right). \tag{11}$$

Proposition 2.5. The stochastic process $r\left(t_k\right) \in \mathbb{S}$ is a Markov chain, and the transition probability matrix $\overline{\Lambda} = \left[\bar{\rho}_{ij}\right]$ is obtained as follows:

$$\bar{\rho}_{ij} = \mathrm{Prob}\left(r\left(t_{k+1}\right) = j\left|r\left(t_k\right) = i\right.\right) = \rho_{\phi_0(i)\phi_{d-1}(j)}\prod_{s=1}^{d-1}\rho_{\phi_s(j)\phi_{s-1}(j)} \tag{12}$$

where ρ_{ij} has been defined in (5).

Based on Propositions 2.4 and 2.5, the augmented system (9) can be rewritten as follows:

$$\begin{cases} \tilde{x}(t_{k+1}) = \bar{\bar{A}}_i(t_k)\tilde{x}(t_k) + \bar{M}(t_k)\bar{n}(x(t_k)) + \bar{\bar{B}}_i(t_k)d(t_k) \\ \bar{y}(t_k) = \bar{\bar{C}}_i(t_k)\tilde{x}(t_k) + \bar{E}(t_k)g(x(t_k)) + \bar{\bar{D}}_i(t_k)d(t_k) \\ z(t_k) = \bar{\bar{L}}(t_k)\tilde{x}(t_k) \end{cases} \tag{13}$$

where

$$\bar{\bar{A}}_i(t_k) = \bar{A}\left(t_k, \bar{\sigma}\left(t_k^{d-1}\right)\right), \bar{\bar{B}}_i(t_k) = \bar{B}\left(t_k, \bar{\sigma}\left(t_k^{d-1}\right)\right),$$

$$\bar{\bar{C}}_i(t_k) = \bar{C}\left(t_k, \bar{\sigma}\left(t_k^{d-1}\right)\right), \bar{\bar{D}}_i(t_k) = \bar{D}\left(t_k, \bar{\sigma}\left(t_k^{d-1}\right)\right).$$

In this paper, consider the following filter for the discrete time-varying system (13):

$$\begin{cases} \hat{x}(t_{k+1}) = G_i(t_k)\hat{x}(t_k) + H_i(t_k)\bar{y}(t_k) \\ \hat{z}(t_k) = K_i(t_k)\hat{x}(t_k) \end{cases} \tag{14}$$

where $\hat{x}(t_k)$ is the state estimate of $x(t_k)$ and $\hat{z}(t_k)$ denotes the estimate of $z(t_k)$. $G_i(t_k), H_i(t_k)$ and $K_i(t_k)$ are the filter gain parameters to be determined.

Letting

$$\eta(t_k) = \left[\tilde{x}^T(t_k) \;\; \hat{x}^T(t_k)\right]^T,$$
$$e_z(t_k) = z(t_k) - \hat{z}(t_k),$$
$$n(x(t_k)) = \left[\bar{n}^T(x(t_k)) \;\; g^T(x(t_k))\right]^T,$$

one has

$$\begin{cases} \eta(t_{k+1}) = \tilde{A}_i(t_k)\eta(t_k) + \tilde{N}_i(t_k)n(x(t_k)) + \tilde{D}_i(t_k)d(t_k) \\ e_z(t_k) = \tilde{L}_i(t_k)\eta(t_k) \end{cases} \tag{15}$$

where

$$\tilde{A}_i(t_k) = \begin{bmatrix} \bar{\bar{A}}_i(t_k) & 0 \\ H_i(t_k)\bar{\bar{C}}_i(t_k) & G_i(t_k) \end{bmatrix}, \quad \tilde{D}_i(t_k) = \begin{bmatrix} \bar{\bar{B}}_i(t_k) \\ H_i(t_k)\bar{\bar{D}}_i(t_k) \end{bmatrix},$$

$$\tilde{N}_i(t_k) = \begin{bmatrix} \bar{M}(t_k) & 0 \\ 0 & H_i(t_k)\bar{E}(t_k) \end{bmatrix}, \quad \tilde{L}_i(t_k) = \begin{bmatrix} \bar{\bar{L}}^T(t_k) \\ -K_i^T(t_k) \end{bmatrix}^T.$$

The main purpose of this paper is to design the H_∞ finite-horizon filter in the form of (14) such that the following requirements are satisfied simultaneously:

$R1)$: For the given positive scalar γ, positive definite weighted matrices U, S and the initial state $x(t_0)$, the following performance index is guaranteed

$$J_1 \triangleq \mathbb{E}\left\{\sum_{k=0}^{N-1} \|e_z(t_k)\|^2 - \gamma^2\|d(t_k)\|_U^2 - \gamma^2\|x(t_0)\|_S^2\right\} < 0 \tag{16}$$

where $\|d(t_k)\|_U^2 = d^T(t_k)Ud(t_k)$, $\|x(t_0)\|_S^2 = x^T(t_0)Sx(t_0)$.

$R2)$: The estimation error covariance satisfies the following constraints:

$$J_2 \triangleq \Upsilon(t_k) = \mathbb{E}\left\{(x(t_k) - \hat{x}(t_k))(x(t_k) - \hat{x}(t_k))^T\right\} \le \Theta(t_k) \tag{17}$$

where $\Theta(t_k)$ a sequence of given matrices specifying the acceptable covariance upper bounds obtained from the engineering requirements.

3. MAIN RESULTS

3.1. H_∞ performance

In this subsection, the objective (H_∞ performance) will be considered for the augmented system.

Theorem 3.1. Let the disturbance attenuation level $\gamma > 0$, the positive definite weighted matrices $U > 0, S > 0$, and the filter gain matrices $\{G_i(t_k), H_i(t_k), K_i(t_k)\}_{0\le k\le N-1}$ be given. For the augmented system (15), the performance criterion (16) is guaranteed for all nonzero $d(t_k)$ if there exists a family of positive definite matrices $\{P_i(t_k)\}_{0\le k\le N-1}$ satisfying

$$\begin{bmatrix} \mho_{11i}(t_k) & \mho_{12i}(t_k) \\ * & -I \end{bmatrix} < 0 \tag{18}$$

with the initial condition

$$P_i(t_0) - \gamma^2 \hat{S} < 0 \tag{19}$$

where

$$\mho_{11i}(t_k) = \begin{bmatrix} \mho_{111i}(t_k) & * & * \\ 0 & -\gamma^2 U & * \\ \bar{P}_i(t_{k+1})\tilde{A}_i(t_k) & \bar{P}_i(t_{k+1})\tilde{D}_i(t_k) & -\bar{P}_i(t_{k+1}) \end{bmatrix},$$

$$\mho_{111i}(t_k) = -P_i(t_k) + \sum_{\ell=1}^{q} \widetilde{\Gamma_\ell}(t_k)\,\mathrm{tr}\left[\tilde{N}_i^T(t_k)\bar{P}_i(t_{k+1})\tilde{N}_i(t_k)\Pi_\ell(t_k)\right],$$

$$\mho_{12i}(t_k) = \begin{bmatrix} \tilde{L}_i(t_k) & 0 & 0 \end{bmatrix}^T, \quad \widetilde{\Gamma_\ell}(t_k) = \mathrm{diag}\{\Gamma_\ell(t_k), 0, 0, 0\}.$$

P r o o f . Consider the following Lyapunov function for the system(15) :

$$\Psi(t_k) = \eta^T(t_k) P_{r(t_k)}(t_k)\eta(t_k) \tag{20}$$

where $P_{r(t_k)}(t_k)$ is a symmetric positive definite matrix with appropriate dimensions.

Letting $\mathbb{E}\{\triangle\Psi(t_k)\} = \mathbb{E}\{\triangle\Psi(t_{k+1}) - \triangle\Psi(t_k)\}$, we can obtain that:

$$\begin{aligned} \mathbb{E}\{\triangle\Psi(t_k)\} &= \mathbb{E}\left\{\eta^T(t_{k+1})\bar{P}_i(t_{k+1})\eta(t_{k+1}) - \eta^T(t_k)P_i(t_k)\eta(t_k)\right\} \\ &= \mathbb{E}\Big\{\left[\tilde{A}_i(t_k)\eta(t_k) + \tilde{N}_i(t_k)n(x(t_k)) + \tilde{D}_i(t_k)d(t_k)\right]^T \\ &\quad \times \bar{P}_i(t_{k+1})\left[\tilde{A}_i(t_k)\eta(t_k) + \tilde{N}_i(t_k)n(x(t_k)) + \tilde{D}_i(t_k)d(t_k)\right] \end{aligned}$$

$$
\begin{aligned}
&- \eta^T(t_k) P_i(t_k) \eta(t_k) \Big\} \\
&= \mathbb{E}\Big\{ \eta^T(t_k) \Big\{ \tilde{A}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{A}_i(t_k) - P_i(t_k) \Big\} \eta(t_k) \\
&\quad + 2\eta^T(t_k) \tilde{A}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{D}_i(t_k) d(t_k) \\
&\quad + n^T(x(t_k)) \tilde{N}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{N}_i(t_k) n(x(t_k)) \\
&\quad + d^T(t_k) \tilde{D}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{D}_i(t_k) d(t_k) \Big\}. \qquad (21)
\end{aligned}
$$

where $\bar{P}_i(t_{k+1}) = \sum_{j\in\mathbb{S}} \bar{\rho}_{ij}(t_k) P_j(t_{k+1})$.

Taking (4) into consideration, we obtain that:

$$
\begin{aligned}
& \mathbb{E}\Big\{ n^T(x(t_k)) \tilde{N}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{N}_i(t_k) n(x(t_k)) \Big\} \\
= \;& \mathbb{E}\Big\{ \mathrm{tr}\Big[n(x(t_k)) n^T(x(t_k)) \tilde{N}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{N}_i(t_k) \Big] \Big\} \\
= \;& \mathbb{E}\Big\{ \mathrm{tr}\Big[\tilde{N}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{N}_i(t_k) n(x(t_k)) n^T(x(t_k)) \Big] \Big\} \\
= \;& \mathrm{tr}\Big[\tilde{N}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{N}_i(t_k) \mathbb{E}\left\{ n(x(t_k)) n^T(x(t_k)) \right\} \Big] \\
= \;& \mathrm{tr}\left[\tilde{N}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{N}_i(t_k) \sum_{\ell=1}^{q} \Pi_\ell(t_k) \left(x^T(t_k) \Gamma_\ell(t_k) x(t_k) \right) \right] \\
= \;& \mathrm{tr}\left[\tilde{N}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{N}_i(t_k) \sum_{\ell=1}^{q} \Pi_\ell(t_k) \left(\eta^T(t_k) \widetilde{\Gamma_\ell}(t_k) \eta(t_k) \right) \right] \\
= \;& \eta^T(t_k) \sum_{\ell=1}^{q} \widetilde{\Gamma_\ell}(t_k) \, \mathrm{tr}\Big[\tilde{N}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{N}_i(t_k) \Pi_\ell(t_k) \Big] \eta(t_k) \qquad (22)
\end{aligned}
$$

In what follows, combing (21) and (22) result in

$$
\mathbb{E}\left\{ \triangle\Psi(t_k) \right\} = \mathbb{E}\Big\{ \tilde{\eta}^T(t_k) \hat{\mho}_{1i}(t_k) \tilde{\eta}(t_k) \Big\} \qquad (23)
$$

where

$$
\begin{aligned}
\tilde{\eta}(t_k) &= \begin{bmatrix} \eta^T(t_k) & d^T(t_k) \end{bmatrix}^T, \\
\hat{\mho}_{1i}(t_k) &= \begin{bmatrix} \hat{\mho}_{11i}(t_k) & \tilde{A}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{D}_i(t_k) \\ * & \tilde{D}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{D}_i(t_k) \end{bmatrix}, \\
\hat{\mho}_{11i}(t_k) &= - P_i(t_k) + \tilde{A}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{A}_i(t_k) \\
&\quad + \sum_{\ell=1}^{q} \widetilde{\Gamma_\ell}(t_k) \, \mathrm{tr}\Big[\tilde{N}_i^T(t_k) \bar{P}_i(t_{k+1}) \tilde{N}_i(t_k) \Pi_\ell(t_k) \Big].
\end{aligned}
$$

Adding the zero term to (23) leads to

$$
\mathbb{E}\left\{ \triangle\Psi(t_k) \right\} = \mathbb{E}\left\{ \tilde{\eta}^T(t_k) \mho_i(t_k) \tilde{\eta}(t_k) - \left(e_z^T(t_k) e_z(t_k) - \gamma^2 d^T(t_k) U d(t_k) \right) \right\} \qquad (24)
$$

where

$$\mho_i(t_k) = \hat{\mho}_{11i}(t_k) + \mathrm{diag}\left\{\tilde{L}_i^T(t_k)\tilde{L}_i(t_k), -\gamma^2 U\right\}.$$

Now, Summing (24) from 0 to $N-1$ with respect to t_k leads to

$$\begin{aligned}\sum_{k=0}^{N-1}\mathbb{E}\{\triangle\Psi(t_k)\} &= \mathbb{E}\{\Psi(t_N) - \Psi(t_0)\}\\ &= \mathbb{E}\left\{\sum_{k=0}^{N-1}\tilde{\eta}^T(t_k)\mho_i(t_k)\tilde{\eta}(t_k)\right\}\\ &\quad - \mathbb{E}\left\{\sum_{k=0}^{N-1}\left(e_z^T(t_k)e_z(t_k) - \gamma^2 d^T(t_k)Ud(t_k)\right)\right\}\end{aligned}$$

which can be rewritten as follows:

$$\begin{aligned}J_1 = \mathbb{E}\Big\{&\sum_{k=0}^{N-1}\tilde{\eta}^T(t_k)\mho_i(t_k)\tilde{\eta}(t_k)\\ &- \eta^T(t_N)P_i(t_N)\eta(t_N) + \eta^T(t_0)\left(P_i(t_0) - \gamma^2\hat{S}\right)\eta(t_0)\Big\} < 0.\end{aligned} \tag{25}$$

The above-mentioned inequality is equivalent to (16). Finally, by using the Schur Complement Lemma, it is straightforward to see that $\mho_i(t_k) < 0$ is equivalent to (18) which ends the proof. □

Theorem 3.2. Let the disturbance attenuation level $\gamma > 0$, the positive definite weighted matrices $U > 0, S > 0$, and the filter gain matrices $\{G_i(t_k), H_i(t_k), K_i(t_k)\}_{0\le k\le N-1}$ be given. For the augmented system (15), the performance criterion (16) is guaranteed for all nonzero $d(t_k)$ if there exists families of positive definite matrices $\{P_i(t_k)\}_{0\le k\le N-1}$, a sequence of real-valued matrices $\{Z_i(t_k)\}_{0\le k\le N-1}$ satisfying

$$\begin{bmatrix}\bar{\mho}_{11i}(t_k) & \mho_{12i}(t_k)\\ * & -I\end{bmatrix} < 0 \tag{26}$$

with the initial condition

$$P_i(t_0) - \gamma^2\hat{S} < 0 \tag{27}$$

where

$$\bar{\mho}_{11i}(t_k) = \begin{bmatrix}\mho_{111i}(t_k) & * & *\\ 0 & -\gamma^2 U & *\\ Z_i(t_k)\tilde{A}_i(t_k) & Z_i(t_k)\tilde{D}_i(t_k) & \bar{\mho}_{33i}(t_k)\end{bmatrix},$$

$$\bar{\mho}_{33i}(t_k) = \bar{P}_i(t_{k+1}) - Z_i(t_k) - Z_i^T(t_k).$$

P r o o f. Compared with Theorem 3.1, we just need to verify that the inequality (26) can be derived from the inequality (18). To this end, from the inequality (26), we know that

$$\bar{P}_i(t_{k+1}) - Z_i(t_k) - Z_i^T(t_k) < 0.$$

According to $\bar{P}_i(t_{k+1}) > 0$, we have $Z_i(t_k)+Z_i^T(t_k) > 0$, which means that $Z_i(t_k)$ are nonsingular matrices. On the other hand, for arbitrary $Z_i(t_k)$, the following inequality is true:

$$\left(\bar{P}_i(t_{k+1}) - Z_i(t_k)\right)^T \bar{P}_i^{-1}(t_{k+1}) \left(\bar{P}_i(t_{k+1}) - Z_i(t_k)\right) > 0$$

which is equivalent to

$$\bar{P}_i(t_{k+1}) - Z_i(t_k) - Z_i^T(t_k) > -Z_i^T(t_k)\bar{P}_i^{-1}(t_{k+1}) Z_i(t_k).$$

Based on the above analyses combing with inequality (26), we have

$$\begin{bmatrix} \mho_{111i}(t_k) & * & * \\ 0 & -\gamma^2 U & * \\ Z_i(t_k)\tilde{A}_i(t_k) & Z_i(t_k)\tilde{D}_i(t_k) & \tilde{\mho}_{33i}(t_k) \end{bmatrix} < 0 \tag{28}$$

where $\tilde{\mho}_{33i}(t_k) = -Z_i^T(t_k)\bar{P}_i^{-1}(t_{k+1}) Z_i(t_k)$.

Applying the congruence transformation diag $\left\{I, I, Z_i^{-1}(t_k)\bar{P}_i(t_{k+1}), I\right\}$ to (28) , we can obtain inequality (18) from (26). This proof is completed. □

3.2. Variance analysis

In this subsection, we shall proceed to investigate the variance analysis for the augmented system. In order to facilitate the proof and derivation later in this subsection, we define

$$\mathscr{E}(t_k) \triangleq \mathbb{E}\left\{\eta(t_k)\eta^T(t_k)\right\}. \tag{29}$$

The following theorem can be accessed to disclose the variance analysis.

Theorem 3.3. Let the filter gain matrices $\{G_i(t_k), H_i(t_k), K_i(t_k)\}_{0\le k\le N-1}$ be given. For the augmented system (15), we have $\Upsilon(t_k) \le \Theta(t_k)$ with the initial condition $\mathscr{E}(t_0) = Q(t_0)$ if there exists families of positive matrices $\{Q(t_k)\}_{1\le k\le N+1}$ satisfying

$$\mathscr{F}(Q(t_k)) \le Q(t_{k+1}), \quad \bar{I}Q(t_k)\bar{I}^{\mathrm{T}} \le \Theta(t_k), \tag{30}$$

where

$$\begin{aligned} \mathscr{F}(Q(t_k)) =& \tilde{A}_i(t_k) Q(t_k)\tilde{A}_i^T(t_k) + \tilde{D}_i(t_k) R(t_k)\tilde{D}_i^T(t_k) \\ &+ \sum_{\ell=1}^{q} \tilde{N}_i(t_k)\Pi_\ell(t_k)\tilde{N}_i^T(t_k)\,\mathrm{tr}\left[\widetilde{\Gamma_\ell}(t_k) Q(t_k)\right], \\ \widetilde{\Gamma_\ell}(t_k) =& \mathrm{diag}\{\Gamma_\ell(t_k), 0, 0, 0\}, \ \ \bar{I} = \begin{bmatrix} I & 0 & 0 & -I \end{bmatrix}. \end{aligned}$$

Proof. From (15) and (29), we have:

$$\begin{aligned} \mathscr{E}(t_{k+1}) &= \mathbb{E}\left\{\eta(t_{k+1})\eta^T(t_{k+1})\right\} \\ &= \mathbb{E}\left\{\left[\tilde{A}_i(t_k)\eta(t_k) + \tilde{N}_i(t_k) n(x(t_k)) + \tilde{D}_i(t_k) d(t_k)\right]\right. \\ &\quad \left. \times \left[\tilde{A}_i(t_k)\eta(t_k) + \tilde{N}_i(t_k) n(x(t_k)) + \tilde{D}_i(t_k) d(t_k)\right]^T\right\} \end{aligned}$$

$$
\begin{aligned}
&= \tilde{A}_i(t_k)\mathscr{E}(t_k)\tilde{A}_i^T(t_k)+\tilde{D}_i(t_k)R(t_k)\tilde{D}_i^T(t_k)\\
&\quad +\mathbb{E}\left\{\tilde{N}_i(t_k)n(x(t_k))n^T(x(t_k))\tilde{N}_i^T(t_k)\right\}\\
&= \tilde{A}_i(t_k)\mathscr{E}(t_k)\tilde{A}_i^T(t_k)+\tilde{D}_i(t_k)R(t_k)\tilde{D}_i^T(t_k)\\
&\quad +\mathbb{E}\left\{\sum_{\ell=1}^{q}\tilde{N}_i(t_k)\Pi_\ell(t_k)\tilde{N}_i^T(t_k)\left(x^T(t_k)\Gamma_\ell(t_k)x(t_k)\right)\right\}\\
&= \tilde{A}_i(t_k)\mathscr{E}(t_k)\tilde{A}_i^T(t_k)+\tilde{D}_i(t_k)R(t_k)\tilde{D}_i^T(t_k)\\
&\quad +\mathbb{E}\left\{\sum_{\ell=1}^{q}\tilde{N}_i(t_k)\Pi_\ell(t_k)\tilde{N}_i^T(t_k)\left(\eta^T(t_k)\widetilde{\Gamma_\ell}(t_k)\eta(t_k)\right)\right\}\\
&= \tilde{A}_i(t_k)\mathscr{E}(t_k)\tilde{A}_i^T(t_k)+\tilde{D}_i(t_k)R(t_k)\tilde{D}_i^T(t_k)\\
&\quad +\mathbb{E}\left\{\sum_{\ell=1}^{q}\tilde{N}_i(t_k)\Pi_\ell(t_k)\tilde{N}_i^T(t_k)\,\mathrm{tr}\left[\widetilde{\Gamma_\ell}(t_k)\eta(t_k)\eta^T(t_k)\right]\right\}\\
&= \tilde{A}_i(t_k)\mathscr{E}(t_k)\tilde{A}_i^T(t_k)+\tilde{D}_i(t_k)R(t_k)\tilde{D}_i^T(t_k)\\
&\quad +\sum_{\ell=1}^{q}\tilde{N}_i(t_k)\Pi_\ell(t_k)\tilde{N}_i^T(t_k)\,\mathrm{tr}\left[\widetilde{\Gamma_\ell}(t_k)\mathscr{E}(t_k)\right]\\
&= \mathscr{F}(\mathscr{E}(t_k))
\end{aligned}
\tag{31}
$$

Obviously, one has $Q(t_0) \geq \mathscr{E}(t_0)$.

Now, in the framework of introduction, we first assume that $Q(t_k) \geq \mathscr{E}(t_k)$, then we have:

$$Q(t_{k+1}) \geq \mathscr{F}(Q(t_k)) \geq \mathscr{F}(\mathscr{E}(t_k)) \geq \mathscr{E}(t_{k+1}).$$

Based on the definitions of (17) and (29), we have

$$\bar{I}\mathscr{E}(t_k)\bar{I}^T = \Upsilon(t_k),$$

which means $\Upsilon(t_k) \leq \bar{I}Q(t_k)\bar{I}^T$. This proof is completed. □

According to Theorem 3.2 and Theorem 3.3, we have the following results on H_∞ performance and variance constraints.

Theorem 3.4. Let the disturbance attenuation level $\gamma > 0$, the positive definite weighted matrices $U > 0, S > 0$, the filter gain matrices $\{G_i(t_k), H_i(t_k), K_i(t_k)\}_{0\leq k\leq N-1}$ be given. For the augmented system (15), the performance criterion (16) and the variance constrains $\Upsilon(t_k) \leq \Theta(t_k)$ are guaranteed if there exists families of positive scalars $\epsilon_\ell(t_k)_{0\leq k\leq N}$ and positive definite matrices $\{P_i(t_k), Q_i(t_k)\}_{1\leq k\leq N+1}$ satisfying the following recursive matrix inequalities

$$\begin{bmatrix} -\epsilon_\ell(t_k) & \pi_\ell^T(t_k)\tilde{N}_i^T(t_k)\bar{P}_i(t_{k+1}) \\ * & -\bar{P}_i(t_{k+1}) \end{bmatrix} < 0, \tag{32}$$

$$\begin{bmatrix} \bar{\tilde{\mho}}_{11i}(t_k) & \mho_{12i}(t_k) \\ * & -I \end{bmatrix} < 0, \tag{33}$$

$$\begin{bmatrix} -Q(t_{k+1}) & \tilde{A}_i(t_k)Q(t_k) & \Psi_{13i}(t_k) & \tilde{D}_i(t_k) \\ * & -Q(t_k) & 0 & 0 \\ * & * & \Psi_{33i}(t_k) & 0 \\ * & * & * & -R^{-1}(t_k) \end{bmatrix} < 0, \tag{34}$$

$$\bar{I}Q(t_k)\bar{I}^{\mathrm{T}} \leq \Theta(t_k) \tag{35}$$

with the initial condition

$$P_i(t_0) \leq \gamma^2 \hat{S}, \quad Q(t_0) = \mathscr{E}(t_0) \tag{36}$$

where

$$\begin{aligned} \tilde{\mho}_{111i}(t_k) &= -P_i(t_k) + \sum_{\ell=1}^{q} \widetilde{\Gamma_\ell}(t_k)\epsilon_\ell(t_k), \\ \Psi_{13i}(t_k) &= \begin{bmatrix} \tilde{N}_i(t_k)\pi_1 & \tilde{N}_i(t_k)\pi_2 & \cdots & \tilde{N}_i(t_k)\pi_q \end{bmatrix}, \\ \Psi_{33i}(t_k) &= \mathrm{diag}\{-\varrho_1(t_k)I, -\varrho_2(t_k)I, \cdots, -\varrho_q(t_k)I\}, \\ \varrho_\ell(t_k) &= \left(\mathrm{tr}\left[\widetilde{\Gamma_\ell}(t_k)Q(t_k)\right]\right)^{-1} \quad (\ell = 1, 2, \cdots, q). \end{aligned}$$

P r o o f. Under the given initial condition, we need to prove that inequalities (32) and (33) guarantee (18), and the inequality (34) is equivalent to (30). To this end, we first denote

$$\Pi_\ell(t_k) = \pi_\ell(t_k)\pi_\ell^T(t_k) = \begin{bmatrix} \pi_{1\ell}(t_k) \\ \pi_{2\ell}(t_k) \\ \pi_{3\ell}(t_k) \\ \pi_{4\ell}(t_k) \end{bmatrix} \begin{bmatrix} \pi_{1\ell}(t_k) \\ \pi_{2\ell}(t_k) \\ \pi_{3\ell}(t_k) \\ \pi_{4\ell}(t_k) \end{bmatrix}^T$$

where $\pi_\ell(t_k) = \begin{bmatrix} \pi_{1\ell}^T(t_k) & \pi_{2\ell}^T(t_k) & \pi_{3\ell}^T(t_k) & \pi_{4\ell}^T(t_k) \end{bmatrix}^T$ $(\ell = 1, 2, \cdots, q)$ are column vectors of appropriate dimensions.

Using the Schur Complement Lemma, the inequality (32) is equivalent to

$$-\epsilon_\ell(t_k) + \pi_\ell^T(t_k)\tilde{N}_i^T(t_k)\bar{P}_i(t_{k+1})\tilde{N}_i(t_k)\pi_\ell(t_k) < 0, \tag{37}$$

which, by the property of matrix trace, can be rewritten as

$$\mathrm{tr}\left[\pi_\ell^T(t_k)\tilde{N}_i^T(t_k)\bar{P}_i(t_{k+1})\tilde{N}_i(t_k)\pi_\ell(t_k)\right] < \epsilon_\ell(t_k). \tag{38}$$

Likewise, employing the Schur Complement Lemma to the inequality (34) again, we have

$$Q(t_{k+1}) \geq \mathscr{F}(Q(t_k)) \tag{39}$$

Then it can be easily verified that the inequality (34) is equivalent to (30). As such, according to Theorem 3.2 and Theorem 3.3, the H_∞ index defined in (16) is satisfied, and, at the same time, the covariance of system (15)is achieved. The proof is now complete. □

Remark 3.5. Similar to Theorem 3.2, the following inequality

$$\begin{bmatrix} -\epsilon_\ell (t_k) & \pi_\ell^T (t_k) \tilde{N}_i^T (t_k) Z_i (t_k) \\ * & \bar{P}_i (t_{k+1}) - Z_i (t_k) - Z_i^T (t_k) \end{bmatrix} < 0 \tag{40}$$

is equivalent to (32).

3.3. Finite horizon filter design

In this subsection, we shall give the sufficient condition for the existence of the desired control strategy that is capable of ensuring both expected multiple objectives.

Theorem 3.6. Let the disturbance attenuation level $\gamma > 0$, the positive definite weighted matrices $U > 0, S > 0$ and a sequence of variance upper bounds $\{\Theta (t_k)\}_{0 \le k \le N}$ be given. For the augmented system (1)-(3), the performance criteria (10) and (11) are guaranteed if there exists families of positive scalars $\{\lambda (t_k)\}_{0 \le k \le N-1}$, $\{\epsilon_\ell (t_k)\}_{0 \le k \le N}$ and positive definite matrices $\{P_{1i} (t_k), P_{2i} (t_k), P_{3i} (t_k), P_{4i} (t_k)\}_{1 \le k \le N}$ and $\{Q_{1i} (t_k),$ $Q_{2i} (t_k), Q_{3i} (t_k), Q_{4i} (t_k)\}_{1 \le k \le N}$, families of matrices $\{G_i (t_k), H_i (t_k), K_i (t_k), X_i (t_k),$ $Y_i (t_k)\}_{0 \le k \le N-1}$, real-valued matrices $\{Z_{1i} (t_k), Z_{2i} (t_k), Z_{3i} (t_k), Z_{4i} (t_k)\}_{0 \le k \le N}$ satisfying the following recursive matrix inequalities

$$\begin{bmatrix} -\epsilon_\ell (t_k) & * \\ \Omega_{21i}^1 (t_k) & \Omega_{ci} (t_k) \end{bmatrix} < 0, \tag{41}$$

$$\begin{bmatrix} \Omega_{11i}^2 (t_k) & * & * & * \\ 0 & -\gamma^2 U & * & * \\ \Omega_{31i}^2 (t_k) & \Omega_{32i}^2 (t_k) & \Omega_{ci} (t_k) & * \\ \Omega_{41i}^2 (t_k) & 0 & 0 & -I \end{bmatrix} < 0, \tag{42}$$

$$\begin{bmatrix} \Omega_{11i}^3 (t_k) & \Omega_{12i}^3 (t_k) \\ * & \Omega_{22i}^3 (t_k) \end{bmatrix} < 0, \tag{43}$$

$$Q_{1i} (t_{k+1}) + Q_{4i} (t_{k+1}) - \Theta (t_{k+1}) < 0 \tag{44}$$

with the relationship

$$X_i (t_k) = Z_{4i} (t_k) H_i (t_k), Y_i (t_k) = Z_{4i} (t_k) G_i (t_k)$$

and the initial condition

$$\begin{cases} P_{1i} (t_0) \le \gamma^2 S \\ P_{2i} (t_0) = P_{3i} (t_0) = P_{4i} (t_0) = Q_{2i} (t_0) = Q_{3i} (t_0) = 0 \\ Q_{1i} (t_0) + Q_{4i} (t_0) = \Upsilon (t_0) < \Theta (t_0) \end{cases}$$

where

$$\begin{aligned} \Omega_{ci} (t_k) = \mathrm{diag}\big\{ & \bar{P}_{1i} (t_{k+1}) - Z_{1i} (t_k) - Z_{1i}^T (t_k), \\ & \cdots, \bar{P}_{4i} (t_{k+1}) - Z_{4i} (t_k) - Z_{4i}^T (t_k) \big\}, \end{aligned}$$

$$
\begin{aligned}
\Omega_{11i}^{2}(t_k) &= \mathrm{diag}\left\{-P_{1i}(t_k)+\sum_{\ell=1}^{q}\Gamma_\ell(t_k)\,\epsilon_\ell(t_k), P_{2i}(t_k), P_{3i}(t_k), P_{4i}(t_k)\right\},\\
\Omega_{41i}^{2}(t_k) &= \begin{bmatrix} L(t_k) & 0 & 0 & -K_i(t_k)\end{bmatrix},\\
\Omega_{11i}^{3}(t_k) &= \mathrm{diag}\left\{-Q_{1i}(t_{k+1}), -Q_{2i}(t_{k+1}), -Q_{3i}(t_{k+1}), -Q_{4i}(t_{k+1})\right\},\\
\Omega_{31i}^{2}(t_k) &= \begin{bmatrix}
Z_{1i}(t_k)A(t_k) & 0 & 0 & 0\\
Z_{2i}(t_k)C(t_k) & 0 & 0 & 0\\
Z_{3i}(t_k)\Xi(\sigma(t_k))C(t_k) & Z_{3i}(t_k)\psi_1\left(\bar\sigma\left(t_k^{d-1}\right)\right) & Z_{3i}(t_k)\psi_2\left(\bar\sigma\left(t_k^{d-1}\right)\right) & 0\\
X_i(t_k)\Xi(\sigma(t_k))C(t_k) & X_i(t_k)\psi_1\left(\bar\sigma\left(t_k^{d-1}\right)\right) & X_i(t_k)\psi_2\left(\bar\sigma\left(t_k^{d-1}\right)\right) & Y_i(t_k)
\end{bmatrix},\\
\Phi(t_k) &= \Omega_{312}(t_k),\\
\Omega_{12i}^{3}(t_k) &= \begin{bmatrix}\Phi_{1i}(t_k) & \Phi_{2i}(t_k) & \Phi_{23i}(t_k)\end{bmatrix},\\
\Phi_{1i}(t_k) &= \begin{bmatrix}
A(t_k)Q_{1i}(t_k) & 0 & 0 & 0\\
C(t_k)Q_{1i}(t_k) & 0 & \psi_2\left(\bar\sigma\left(t_k^{d-1}\right)\right)Q_{3i}(t_k) & 0\\
\Phi_{31i}^{1}(t_k) & \Phi_{32i}^{1}(t_k) & \Phi_{43i}^{1}(t_k) & G_i(t_k)Q_{4i}(t_k)\\
\Phi_{41i}^{1}(t_k) & \Phi_{42i}^{1}(t_k) & Z_{3i}(t_k)\psi_2\left(\bar\sigma\left(t_k^{d-1}\right)\right) & 0\\
 & & X_i(t_k)\psi_2\left(\bar\sigma\left(t_k^{d-1}\right)\right) & Y_i(t_k)
\end{bmatrix},\\
\Phi_{31i}^{1}(t_k) &= \Xi(\sigma(t_k))C(t_k)Q_{1i}(t_k),\\
\Phi_{41i}^{1}(t_k) &= H_i(t_k)\Xi(\sigma(t_k))C(t_k)Q_{1i}(t_k),\\
\Phi_{32i}^{1}(t_k) &= \psi_1\left(\bar\sigma\left(t_k^{d-1}\right)\right)Q_{2i}(t_k),\\
\Phi_{42i}^{1}(t_k) &= H_i(t_k)\psi_1\left(\bar\sigma\left(t_k^{d-1}\right)\right)Q_{2i}(t_k),\\
\Phi_{43i}^{1}(t_k) &= H_i(t_k)\psi_2\left(\bar\sigma\left(t_k^{d-1}\right)\right)Q_{3i}(t_k),\\
\Phi_{2i}(t_k) &= \begin{bmatrix}\Phi_{21i}(t_k) & \Phi_{22i}(t_k) & \cdots & \Phi_{2qi}(t_k)\end{bmatrix},\\
\Phi_{21i}(t_k) &= \begin{bmatrix}\pi_{11}(t_k)\\ \pi_{21}(t_k)\\ \bar E(t_k)\pi_{31}(t_k)\\ H_i(t_k)\bar E(t_k)\pi_{41}(t_k)\end{bmatrix},\\
\Phi_{22i}(t_k) &= \begin{bmatrix}\pi_{12}(t_k)\\ \pi_{22}(t_k)\\ \bar E(t_k)\pi_{32}(t_k)\\ H_i(t_k)\bar E(t_k)\pi_{42}(t_k)\end{bmatrix},\\
&\cdots
\end{aligned}
$$

$$\Phi_{2qi}(t_k) = \begin{bmatrix} \pi_{1q}(t_k) \\ \pi_{2q}(t_k) \\ \bar{E}(t_k)\,\pi_{3q}(t_k) \\ H_i(t_k)\,\bar{E}(t_k)\,\pi_{4q}(t_k) \end{bmatrix},$$

$$\Phi_{3i}(t_k) = \begin{bmatrix} lB(t_k) & 0 \\ D(t_k) & 0 \\ \Xi(\sigma(t_k))\,D(t_k) & \psi_3\left(\bar{\sigma}\left(t_k^{d-1}\right)\right) \\ H_i(t_k)\,\Xi(\sigma(t_k))\,D(t_k) & H_i(t_k)\,\psi_3\left(\bar{\sigma}\left(t_k^{d-1}\right)\right) \end{bmatrix},$$

$$\begin{aligned} \Gamma(t_k) =& \mathit{\Omega}_{322}(t_k) \\ \mathit{\Omega}^3_{22i}(t_k) =& \mathrm{diag}\{\Gamma_{1i}(t_k), \Gamma_{2i}(t_k), \Gamma_{3i}(t_k)\}, \\ \Gamma_{1i}(t_k) =& \mathrm{diag}\{-Q_{1i}(t_k), -Q_{2i}(t_k), -Q_{3i}(t_k), -Q_{4i}(t_k)\}, \\ \Gamma_{2i}(t_k) =& \mathrm{diag}\{-\bar{\varrho}_1(t_k)I, -\bar{\varrho}_2(t_k)I, \cdots, -\bar{\varrho}_\ell(t_k)I\}, \\ \Gamma_{3i}(t_k) =& -R^{-1}(t_k), \quad \bar{\varrho}_\ell(t_k) = \left(\mathrm{tr}\left[\Gamma_\ell((t_k))\,Q_{1i}(t_k)\right]\right)^{-1}. \end{aligned}$$

Proof. First, $P_i(t_k)$, $Q_i(t_k)$ and $Z_i(t_k)$ in Theorem 3.4 are selected as follows:

$$\begin{aligned} P_i(t_k) =& \mathrm{diag}\{P_{1i}(t_k), P_{2i}(t_k), P_{3i}(t_k), P_{4i}(t_k)\}, \\ Q_i(t_k) =& \mathrm{diag}\{Q_{1i}(t_k), Q_{2i}(t_k), Q_{3i}(t_k), Q_{4i}(t_k)\}, \\ Z_i(t_k) =& \mathrm{diag}\{Z_{1i}(t_k), Z_{2i}(t_k), Z_{3i}(t_k), Z_{4i}(t_k)\}. \end{aligned}$$

Then, in light of (36), (37) and (42), we can obtain (41), (42) and (43).

Based on the definitions of (17) and (29), we have

$$\bar{I}\mathscr{E}(t_k)\,\bar{I}^T = \mathit{\Upsilon}(t_k)$$

where $\bar{I} = \begin{bmatrix} I & 0 & 0 & -I \end{bmatrix}$.

Noting $\bar{I}Q(t_k)\,\bar{I}^T = Q_{1i}(t_k) + Q_{4i}(t_k)$, we can arrive at the estimation error covariances index defined in (17) from (44). Now the proof is complete. □

4. NUMERICAL SIMULATIONS

In this section,a numerical example is provided to demonstrate the effectiveness of developed approach. Considered the discrete time-varying system with following parameters:

$$\begin{aligned} x(t_{k+1}) &= \begin{bmatrix} 0.6 & -0.5 \\ 0.1 & -0.8\cos(t_k) \end{bmatrix} x(t_k) + \begin{bmatrix} 0.05 \\ -0.02 \end{bmatrix} \omega(t_k) + f(t_k), \\ y(t_k) &= \begin{bmatrix} 0.3 & 0.15 \\ 0.05 & 0.4 - 0.15\cos(t_k) \end{bmatrix} x(t_k) + \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix} \omega(t_k) + g(t_k), \\ z(t_k) &= \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 + 0.2\sin(0.5t_k) \end{bmatrix} x(t_k) \\ y_i(\bar{t}_k) &= y_i(t_k) + \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix} \nu(\bar{t}_k), \; i = \sigma(\bar{t}_k) \end{aligned}$$

where

$$f(t_k) = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix} \times (0.3\sin(x_1(t_k))x_1(t_k)\omega(t_k) + 0.4\sin(x_2(t_k))x_2(t_k)\omega(t_k)),$$
$$g(t_k) = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix} \times (0.3\sin(x_1(t_k))x_1(t_k)\omega(t_k) + 0.4\sin(x_2(t_k))x_2(t_k)\omega(t_k)).$$

In addition, the covariance of noises are $\mathrm{Cov}\{\omega(t_k)\} = 0.4$ and $\mathrm{Cov}\{\nu(t_k)\} = 0.1$, the sampling parameters are $h = 1$ and $h_c = \frac{h}{b}$ $(b = 1, 2, 3, 4)$, and other parameters are selected as $\gamma = 0.9$, $q = 1$, $S = I$, $U = I$, and

$$\Theta(t_k) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad \Lambda(T_k) = \begin{bmatrix} 0.2 & 0.8 \\ 0.4 & 0.6 \end{bmatrix}.$$

Tables 4 − 4 show the selection lists of sensor nodes at different times based on random protocols in the case of different rate ratios. r represents the node selection after a moment in which the b moments are mapped to the original system in the high-speed network, $S_{t_k^0}$,$S_{t_k^1}$,...,$S_{t_k^b}$ indicate the node selection corresponding to b times in the high-speed network. Tables 4 − 4 show a list of estimator gains required by the system at different times in different b cases. Figures 1 − 4 show the trajectory of the signal $z_1(t_k)$ and its estimated $\hat{z}_1(t_k)$ at different times. It can be seen that as b increases, the estimated effect of the system improves. Figure 5 and Figure 6 depict the variance effects of the two components of the state $x(t_k)$ and its estimated $\hat{x}(t_k)$ error for different b cases, which are clearly smaller than the given performance index, and with the increase of b, the error variance of each component is improved.

t_k	0	1	2	3	4	···	9	10	11	12	···	16	17	18	19	20
r	2	2	1	2	1	···	1	1	2	2	···	1	2	1	1	1
$S_{t_k^0}$	2	2	1	2	1	···	1	1	2	2	···	1	2	1	1	1

Tab. 1. Selections of nodes at different times when $b = 1$.

t_k	0	1	2	3	4	···	9	10	11	12	···	16	17	18	19	20
r	2	4	1	4	2	···	1	3	2	2	···	4	3	3	3	1
$S_{t_k^0}$	2	2	1	2	2	···	1	1	2	2	···	2	1	1	1	1
$S_{t_k^1}$	1	2	1	2	1	···	1	2	1	1	···	2	2	2	2	1

Tab. 2. Selections of nodes at different times when $b = 2$.

t_k	0	1	2	3	4	$\cdots$	9	10	11	12	$\cdots$	16	17	18	19	20
r	6	6	2	8	2	$\cdots$	4	2	3	5	$\cdots$	8	3	3	8	1
$S_{t_k^0}$	2	2	2	2	2	$\cdots$	2	2	1	1	$\cdots$	2	1	1	2	1
$S_{t_k^1}$	1	1	1	2	1	$\cdots$	2	1	2	1	$\cdots$	2	2	2	2	1
$S_{t_k^2}$	2	2	1	2	1	$\cdots$	1	1	1	2	$\cdots$	2	1	1	2	1

Tab. 3. Selections of nodes at different times when $b = 3$.

t_k	0	1	2	3	4	$\cdots$	9	10	11	12	$\cdots$	16	17	18	19	20
r	6	16	5	12	4	$\cdots$	10	4	14	16	$\cdots$	6	15	3	5	9
$S_{t_k^0}$	2	2	1	2	2	$\cdots$	2	2	2	2	$\cdots$	2	1	1	1	1
$S_{t_k^1}$	1	2	1	2	2	$\cdots$	1	2	1	2	$\cdots$	1	2	2	1	1
$S_{t_k^3}$	2	2	2	1	1	$\cdots$	1	1	2	2	$\cdots$	2	2	1	2	1
$S_{t_k^4}$	1	2	1	2	1	$\cdots$	2	1	2	2	$\cdots$	1	2	1	1	2

Tab. 4. Selections of nodes at different times when $b = 4$.

t_k	0	1	$\cdots$	20
G	$\begin{bmatrix} 0.2311 & 0.4860 \\ 0.6068 & 0.8913 \end{bmatrix}$	$\begin{bmatrix} 0.1763 & 0.9355 \\ 0.4057 & 0.9169 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} 0.6252 & 0.3759 \\ 0.7334 & 0.0099 \end{bmatrix}$
H	$\begin{bmatrix} 0.7621 & 0.0185 \\ 0.4565 & 0.8214 \end{bmatrix}$	$\begin{bmatrix} 0.4103 & 0.0579 \\ 0.8936 & 0.3529 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} 0.4199 & 0.7939 \\ 0.7537 & 0.9200 \end{bmatrix}$
K	$\begin{bmatrix} 0.4447 & 0.7919 \\ 0.6154 & 0.9218 \end{bmatrix}$	$\begin{bmatrix} 0.8132 & 0.1389 \\ 0.0099 & 0.2028 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} 0.8447 & 0.6208 \\ 0.3678 & 0.7313 \end{bmatrix}$

Tab. 5. Estimator gains at different times when $b = 1$.

t_k	0	1	$\cdots$	20
G	$\begin{bmatrix} 0.6552 & 0.3716 \\ 0.8376 & 0.4253 \end{bmatrix}$	$\begin{bmatrix} 0.8066 & 0.4850 \\ 0.7036 & 0.1146 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} 0.6234 & 0.6773 \\ 0.6859 & 0.8768 \end{bmatrix}$
H	$\begin{bmatrix} 0.5947 & 0.7165 \\ 0.5657 & 0.5113 \end{bmatrix}$	$\begin{bmatrix} 0.6649 & 0.1400 \\ 0.3654 & 0.5668 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} 0.0129 & 0.7791 \\ 0.3104 & 0.3073 \end{bmatrix}$
K	$\begin{bmatrix} 0.7764 & 0.1859 \\ 0.4893 & 0.7006 \end{bmatrix}$	$\begin{bmatrix} 0.8230 & 0.9994 \\ 0.6739 & 0.9616 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} 0.9267 & 0.0743 \\ 0.6787 & 0.0707 \end{bmatrix}$

Tab. 6. Estimator gains at different times when $b = 2$.

t_k	0		1		...	20	
G	0.9866	0.9477	0.7540	0.8835	...	0.9222	0.7675
	0.5029	0.8280	0.6632	0.2722		0.0133	0.9473
H	0.9176	0.8121	0.4194	0.0356	...	0.8133	0.1990
	0.1131	0.9083	0.2130	0.0812		0.9238	0.6743
K	0.1564	0.7627	0.8506	0.4662	...	0.9271	0.5945
	0.1221	0.7218	0.3402	0.9138		0.3438	0.6155

Tab. 7. Estimator gains at different times when $b = 3$.

t_k	0		1		...	20	
G	0.9442	0.2584	0.8159	0.3099	...	0.7843	0.0310
	0.8386	0.0429	0.9302	0.2688		0.3879	0.5855
H	0.0059	0.7439	0.5365	0.2110	...	0.5586	0.0874
	0.5744	0.8068	0.1633	0.2168		0.2007	0.9332
K	0.6376	0.1443	0.6518	0.2293	...	0.2594	0.0492
	0.2513	0.6516	0.0528	0.6674		0.2042	0.6062

Tab. 8. Estimator gains at different times when $b = 4$.

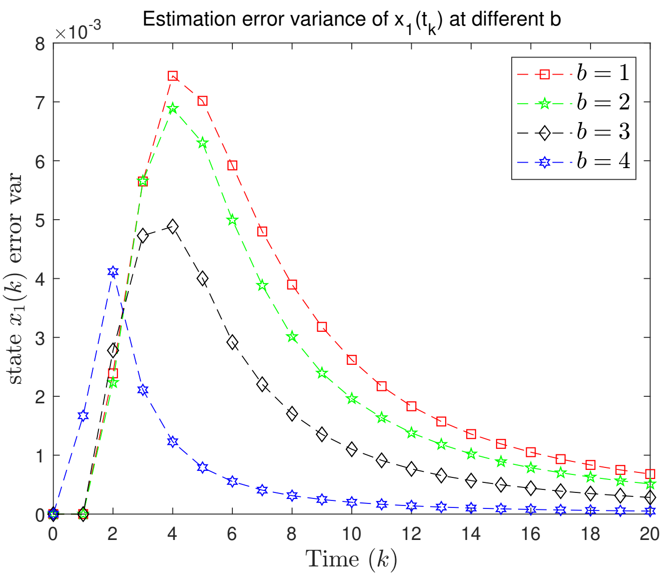

Fig. 1. Estimation error variance of $x_1(t_k)$.

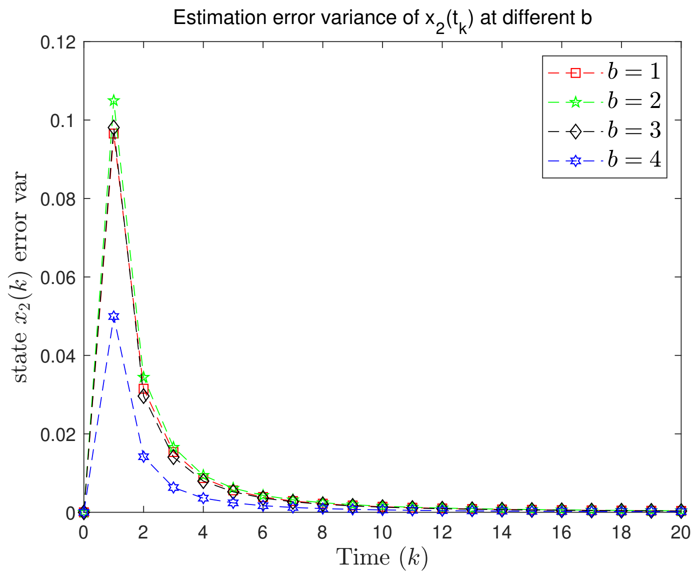

Fig. 2. Estimation error variance of $x_2(t_k)$.

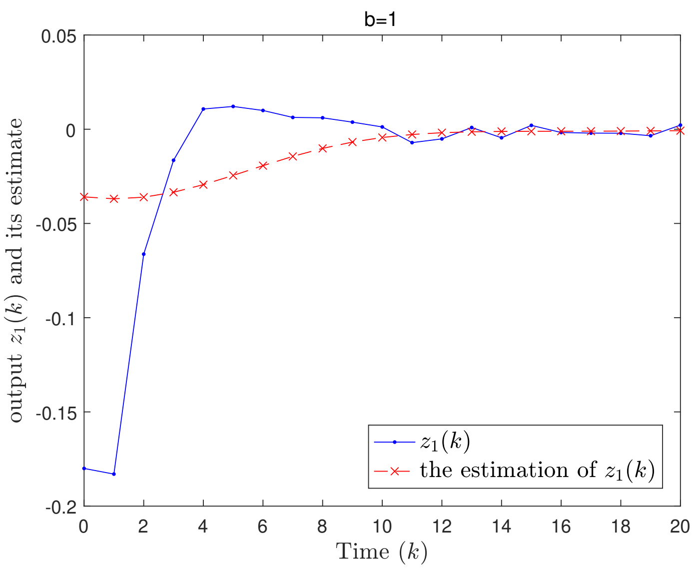

Fig. 3. $z_1(t_k)$ and $\hat{z}_1(t_k)$ under b=1.

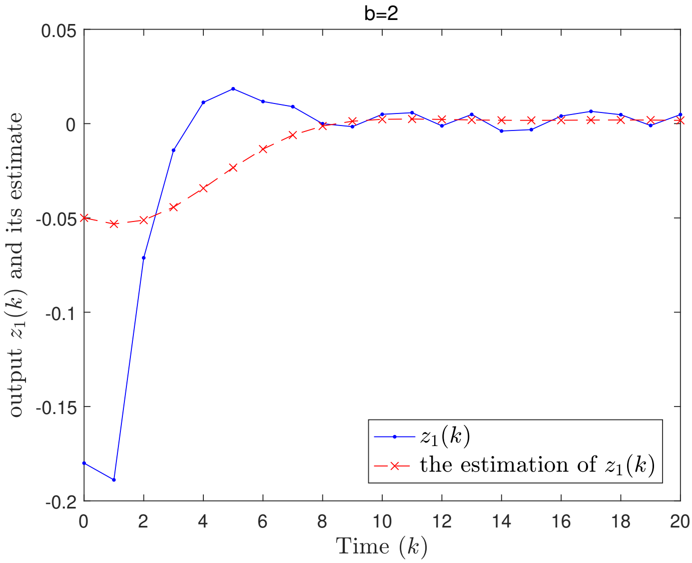

Fig. 4. $z_1(t_k)$ and $\hat{z}_1(t_k)$ under b=2.

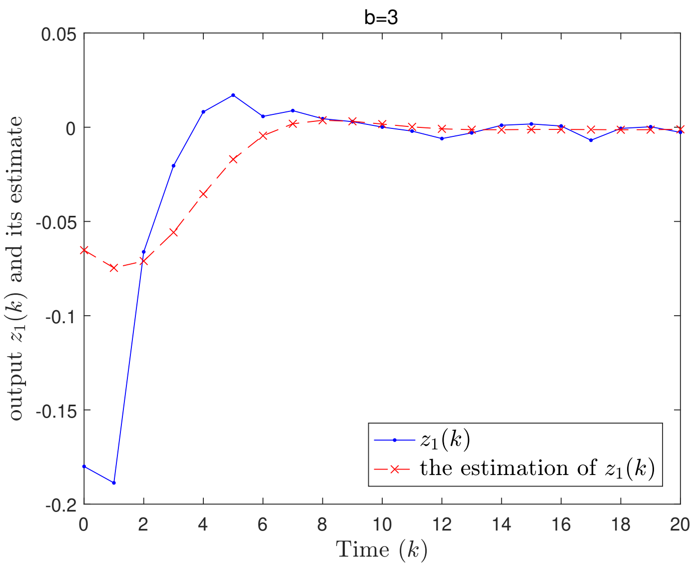

Fig. 5. $z_1(t_k)$ and $\hat{z}_1(t_k)$.

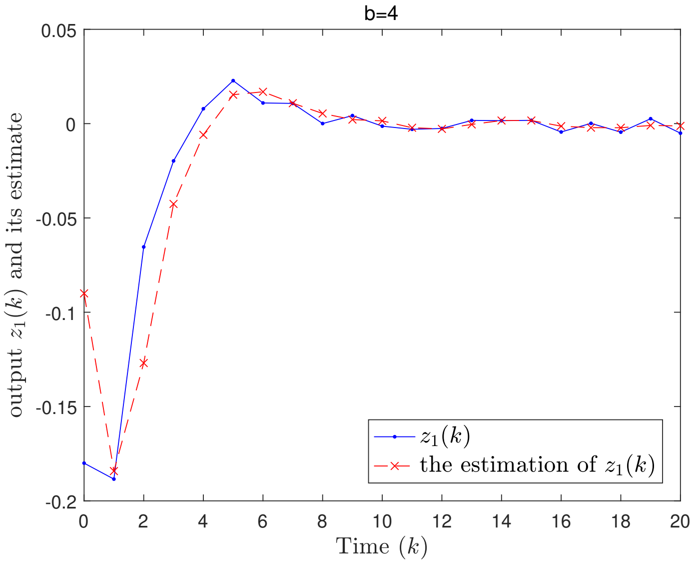

Fig. 6. $z_1(t_k)$ and $\hat{z}_1(t_k)$.

5. CONCLUSIONS

In this paper, we have investigated a finite-time H_∞ finite-horizon filtering problem for a class of nonlinear systems with time-varying multi-rate networks based on random protocols. First, the rules based on random protocol transmission are modeled as a random handover model. Then, take into account the H_∞ performance and variance constraint two performance indicators, the sufficient conditions for the system performance index are given by the relaxation method. The filter solutions satisfying the above criteria are obtained by using linear matrix inequalities (LMIs). Finally, the correctness and effectiveness of the proposed filter scheme are verified by numerical simulation.

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REFERENCES

[1] M. Brunot, A. Janot, P. Young, and F. Carrillo: An instrumental variable method for robot identification based on time variable parameter estimation. Kybernetika *54* (2019), 1, 202–220. DOI:10.14736/kyb-2018-1-0202

[2] W. Chen, D.-R. Ding, X.-H. Ge, Q.-L. Han, and G. Wei: H_∞ containment control of multi-agent systems under event-triggered communication scheduling: The finite-horizon case. IEEE Trans. Cybernet. *50* (2020), 4, 1372–1382. DOI:10.1109/tcyb.2018.2885567

[3] W. Chen, D.-R. Ding, H.-L. Dong, and G.-L. Wei: Distributed resilient filtering for power systems subject to denial-of-service attacks. IEEE Trans. Systems Man Cybernet.: Systems *49* (2019), 8, 1688–1697. DOI:10.1109/tsmc.2019.2905253

[4] A. Cetinkaya, H. Ishii, and T. Hayakawa: Analysis of Stochastic Switched Systems With Application to Networked Control Under Jamming Attacks IEEE Trans. Automat. Control *64* (2019), 5, 2013–2028. DOI:10.1109/tac.2018.2832466

[5] F. Deng, H.-L. Yang, and L.-J. Wang: Adaptive unscented Kalman filter based estimation and filtering for dynamic positioning with model uncertainties. Int. J. Control Automat. Systems *17* (2019), 3, 667–678. DOI:10.1007/s12555-018-9503-4

[6] D.-R. Ding, Z.-D. Wang, Q.-L. Han, and G.-L. Wei: Neural-network-based output-feedback control under Round–Robin scheduling protocols. IEEE Trans. Cybernet. *49* (2019), 6, 2372–2384. DOI:10.1109/tcyb.2018.2827037

[7] D.-R. Ding, Q.-L. Han, Z.-R. Wang, and X.-H. Ge: A survey on model-based distributed control and filtering for industrial cyber-physical systems. IEEE Trans. Industr. Inform. *15* (2019), 5, 2483–2499. DOI:10.1109/tii.2019.2905295

[8] H.-L. Dong, Z.-D. Wang, B. Shen, and D.-R. Ding: Variance-constrained H_∞ control for a class of nonlinear stochastic discrete time-varying systems: The event-triggered design. Automatica *72* (2016), 28–36. DOI:10.1016/j.automatica.2016.05.012

[9] Z.-L. Du and X.-M. Li: Strong tracking Tobit Kalman filter with model uncertainties. Int. J. Control Automat. Systems *17* (2019), 2, 345–355. DOI:10.1007/s12555-017-0655-4

[10] X.-H. Ge and Q.-L. Han: Consensus of multiagent systems subject to partially accessible and overlapping Markovian network topologies. IEEE Trans. Cybernet. *47* (2017), 8, 1807–1819. DOI:10.1109/tcyb.2016.2570860

[11] X.-H. Ge, Q.-L. Han, and Z.-D. Wang: A threshold-parameter-dependent approach to designing distributed event-triggered H_∞ consensus filters over sensor networks. IEEE Trans. Cybernet. *49* (2019), 4, 1148–1159. DOI:10.1109/tcyb.2017.2789296

[12] X.-H. Ge, Q.-L. Han, and Z.-D. Wang: A dynamic event-triggered transmission scheme for distributed set-membership estimation over wireless sensor networks. IEEE Trans. Cybernet. *49* (2019), 1, 171–183. DOI:10.1109/tcyb.2017.2769722

[13] E. Fridman, U. Shaked, and L. Xie: Robust H_∞ filtering of linear systems with time-varying delay. IEEE Trans. Automat. Control *48* (2003), 1, 159–165. DOI:10.1109/tac.2002.806674

[14] Y. He, G.-P. Liu, D. Rees, and M. Wu: H_∞ Filtering for discrete-time systems with time-varying delay. Signal Process. *89* (2009), 3, 275–282. DOI:10.1016/j.sigpro.2008.08.008

[15] C. Hu, W. Qin, Z. Li, B. He, and G. Liu: Consensus-based state estimation for multi-agent systems with constraint information. Kybernetika *3* (2017), 53, 545–561. DOI:10.14736/kyb-2017-3-0545

[16] Y.-S. Hung, and F.-W. Yang: Robust H_∞ filtering with error variance constraints for uncertain discrete time-varying systems with uncertainty. Automatica *39* (2003), 7, 1185–1194. DOI:10.1016/s0005-1098(03)00117-1

[17] Y. Liang, T.-W. Chen, and Q. Pan: Multi-rate optimal state estimation. Int. J. Control *82* (2009), 11, 2059–2076. DOI:10.1080/00207170902906132

[18] X.-G. Li and X.-J. Zhu: Stability analysis of neutral systems with distributed delays. Automatica *44* (2008), 8, 2197–2201. DOI:10.1016/j.automatica.2007.12.009

[19] H. Li and Y. Shi: Robust H_∞ filtering for nonlinear stochastic systems with uncertainties and Markov delays. Automatica *48* (2012), 1, 159–166.

[20] K. Liu, E. Fridman, and L. Hetel: Stability and $L_2 - gain$ analysis of networked control systems under round-robin scheduling: a time-delay approach. Systems Control Lett. *61* (2012), 5, 666–675. DOI:10.1016/j.sysconle.2012.03.002

[21] S. Liu, Z.-D. Wang, L.-C. Wang, and G.-L. Wei: On quantized H_∞ filtering for multi-rate systems under stochastic communication protocols: The finite-horizon case. Inform. Sci. *459* (2018), 211–223. DOI:10.1016/j.ins.2018.02.050

[22] Q. Qi, H. Zhang, and Z. Wu: Stabilization control for linear continuous-time mean-field systems IEEE Trans. Automat. Control *64* (2019), 8, 3461–3468. DOI:10.1109/tac.2018.2881141

[23] M. Tabbara and D. Nesic: Input-output stability of networked control systems with stochastic protocols and channels. IEEE Trans. Automat. Control *53* (2008), 5, 1160–1175. DOI:10.1109/tac.2008.923691

[24] B. Shen, Z.-D. Wang, and T.-W. Huang: Stabilization for sampled-data systems under noisy sampling interval. Automatica *63* (2016), 162–166. DOI:10.1016/j.automatica.2015.10.005

[25] A. Subramanian and A.-H. Sayed: Multiobjective filter design for uncertain stochastic time-delay systems. IEEE Trans. Automat. Control *49* (2004), 1, 149–154. DOI:10.1109/tac.2003.821422

[26] H. Tan, B. Shen, Y. Liu, A. Alsaedi, and B. Ahmad: Event-triggered multi-rate fusion estimation for uncertain system with stochastic nonlinearities and colored measurement noises. Inform. Fusion *36*, (2017), 313–320. DOI:10.1016/j.inffus.2016.12.003

[27] Z.-D. Wang, D.-W.C. Ho, and X. Liu: Variance-constrained filtering for uncertain stochastic systems with missing measurements. IEEE Trans. Automat. Control *48* (2003), 7, 560–567.

[28] Y. Xu, H. Su, Y.-J. Pan, Z. Wu, and W. Xu: Stability analysis of networked control systems with round-robin scheduling and packet dropouts. J. Franklin Inst. *350* (2013), 8, 2013–2027. DOI:10.1016/j.jfranklin.2013.05.024

[29] Y.-I. Yaz and E.-E. Yaz: On LMI formulations of some problems arising in nonlinear stochastic system analysis. IEEE Trans. Automat. Control *44* (1999), 4, 813–816. DOI:10.1109/9.754824

[30] W.-A. Zhang, G Feng, and L. Yu: Multi-rate distributed fusion estimation for sensor networks with packet losses. Automatica *48* (2012), 9, 2016–2028. DOI:10.1016/j.automatica.2012.06.027

[31] X.-M. Zhang, and Q.-L. Han: A decentralized event-triggered dissipative control scheme for systems with multiple sensors to sample the system outputs. IEEE Trans. Cybernet. *46* (2016), 12, 2745–2757. DOI:10.1109/tcyb.2015.2487420

[32] X.-M. Zhang, Q.-L. Han, X.-H. Ge, D.-R. Ding, L. Ding, D. Yue, and C. Peng: Networked control systems: a survey of trends and techniques. IEEE/CAA J. Automat. Sinica, 1–17. DOI:10.1109/jas.2019.1911651

[33] Y. Zhang, Z.-D. Wang, and L.-F. Ma: Variance-constrained state estimation for networked multi-rate systems with measurement quantization and probabilistic sensor failures. Int. J. Robust Nonlinear Control *26* (2016), 16, 3507–3523. DOI:10.1002/rnc.3520

[34] S. Zheng: Stability analysis of uncertain complex-variable delayed nonlinear systems via intermittent control with multiple switched periods. Kybernetika *54*, (2018), 5, 937–957. DOI:10.14736/kyb-2018-5-0937

[35] Z. Zuo, Q.-L. Han, B. Ning, X.-H. Ge, and X.-M. Zhang: An overview of recent advances in fixed-time cooperative control of multi-agent systems. IEEE Trans. Industr. Inform. *14* (2018), 6, 2322–2334. DOI:10.1109/tii.2018.2817248

[36] D.-R. Ding, Z.-D. Wang, and Q.-L. Han: A set-membership approach to event-triggered filtering for general nonlinear systems over sensor networks. IEEE Trans. Automat. Control, 1–1. DOI:10.1109/tac.2019.2934389

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