Gauge-invariant spectral description of the $U(1)$ Higgs model from local composite operators

D. Dudal,[a,b] **D.M. van Egmond,**[c] **M.S. Guimarães,**[c] **O. Holanda,**[c] **L.F. Palhares,**[c] **G. Peruzzo**[c] **and S.P. Sorella**[c]

[a]*KU Leuven Campus Kortrijk — Kulak, Department of Physics,*
Etienne Sabbelaan 53 bus 7657, 8500 Kortrijk, Belgium

[b]*Ghent University, Department of Physics and Astronomy,*
Krijgslaan 281-S9, 9000 Gent, Belgium

[c]*Universidade do Estado do Rio de Janeiro, Instituto de Física — Departamento de Física Teórica,*
Rua São Francisco Xavier 524, 20550-013, Maracanã, Rio de Janeiro, Brasil

E-mail: david.dudal@kuleuven.be, duifjemaria@gmail.com,
msguimaraes@uerj.br, ozorio.neto@uerj.br, leticia.palhares@uerj.br,
gperuzzofisica@gmail.com, silvio.sorella@gmail.com

ABSTRACT: The spectral properties of a set of local gauge-invariant composite operators are investigated in the $U(1)$ Higgs model quantized in the 't Hooft R_ξ gauge. These operators enable us to give a gauge-invariant description of the spectrum of the theory, thereby surpassing certain incommodities when using the standard elementary fields. The corresponding two-point correlation functions are evaluated at one-loop order and their spectral functions are obtained explicitly. As expected, the above mentioned correlation functions are independent from the gauge parameter ξ, while exhibiting positive spectral densities as well as gauge-invariant pole masses corresponding to the massive photon and Higgs physical excitations.

Contents

1 Introduction

An essential aspect of gauge theories is that all physical observable quantities have to be gauge-invariant [1, 2]. However, in practice, the explicit calculations of the S-matrix elements and corresponding cross sections are done by employing the non-gauge-invariant elementary fields such as the W bosons and the Higgs field of the electroweak theory, giving results in quite accurate agreement with experimental ones.

The success of making use of the non-gauge-invariant elementary fields can be traced back to the so called Nielsen identities [3–7] which follow from the Slavnov-Taylor identities encoding the BRST symmetry of quantized gauge theories. The Nielsen identities ensure that the pole masses of both transverse gauge bosons and Higgs field propagators do not depend on the gauge parameters entering the gauge fixing condition, a pivotal property shared by the S-matrix elements. Nevertheless, as one can easily figure out, the use of the non-gauge-invariant fields has its own limitations which show up in several ways. For example, the analysis of the spectral properties of the elementary two-point correlation functions in terms of the Källén-Lehmann (KL) representation is often plagued by an undesired dependence of the spectral densities on the gauge parameters and/or the densities attaining negative values, obscuring their physical interpretation. Indeed, from e.g. non-perturbative lattice QCD studies, it is well known that not only direct particle-spectrum related properties are hiding in the spectral functions, but at finite temperature also information on transport properties in the quark-gluon plasma etc., see for instance [8–12]. The spectral functions considered are those of gauge-invariant operators. Moreover, it is also known that in certain classes of gauges, the Nielsen identities can suffer from fatal infrared singularities [3, 13, 14], obscuring what happens with e.g. the pole mass or effective potential governing the Higgs vacuum expectation value in such gauges.

A formulation of the properties of the observable excitations in terms of gauge-invariant variables is thus very welcome. Such an endeavour has been addressed by several authors[1] [17–20], who have been able to construct, out of the elementary fields, a set of local gauge-invariant composite operators which can effectively implement a gauge-invariant framework by using the tools of quantum gauge field theories: renormalizability, locality, Lorentz covariance and BRST exact symmetry.

The aim of this work, which generalizes a previous one [21] devoted to the study of the analytic properties of the propagators of the non gauge-invariant elementary fields, is that of discussing the features of two local gauge-invariant operators within the framework of the U(1) Abelian Higgs model, whose action is specified by

$$S_0 = \int d^4x \left\{ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + (D_\mu \varphi)^\dagger D_\mu \varphi + \frac{\lambda}{2} \left(\varphi^\dagger \varphi - \frac{v^2}{2} \right)^2 \right\}, \tag{1.1}$$

where the photon field-strength tensor and the covariant derivative are respectively given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$
$$D_\mu \varphi = \partial_\mu \varphi + ie A_\mu \varphi \tag{1.2}$$

and the scalar field may be decomposed to account for the Higgs mechanism as

$$\varphi = \frac{1}{\sqrt{2}}((v + h) + i\rho), \tag{1.3}$$

with h and ρ denoting, respectively, the Higgs and the Goldstone fields, while v is the classical minimum of the Higgs potential of eq. (1.1), responsible for the photon mass generation

---

[1]See [15, 16] for a general review.

in this Higgs model. The action S_0 is left invariant by the following gauge transformations

$$\delta A_\mu = -\partial_\mu \omega, \qquad \delta\varphi = ie\omega\varphi, \qquad \delta\varphi^\dagger = -ie\omega\varphi^\dagger,$$
$$\delta h = -e\omega\rho, \qquad \delta\rho = e\omega(v+h) \,, \tag{1.4}$$

where ω is the gauge parameter.

Following [17, 18], we shall consider the two local composite operators $O(x)$ and $V_\mu(x)$ invariant under (1.4), given by

$$O(x) = 1/2(h^2(x) + 2vh(x) + \rho^2(x)) = \varphi^\dagger(x)\varphi(x) - \frac{v^2}{2} \,,$$
$$V_\mu(x) = -i\varphi^\dagger(x)(D_\mu\varphi)(x) \,. \tag{1.5}$$

The relevance of these operators can be understood by using the expansion (1.3) and retaining the first order terms. For the two-point correlator of the scalar operator one finds (cf. eq. (2.21) for the full expression):

$$\langle O(x)O(y)\rangle \sim v^2 \langle h(x)h(y)\rangle_{\text{tree level}} + \mathcal{O}(\hbar) + \langle \mathcal{O}\left(h^3; h\rho^2; \rho^4\right)\rangle \,, \tag{1.6}$$

while the contributions to the vector operator at lowest order in the fields read

$$V_\mu(x) \sim \frac{ev^2}{2}A_\mu(x) + \text{total derivative} + \text{higher orders} \,. \tag{1.7}$$

We see therefore that the gauge-invariant operator $O(x)$ is related to the Higgs excitation, while $V_\mu(x)$ is associated with the photon.

In the sequel, we shall compute the BRST invariant two-point correlation functions

$$\langle O(x)O(y)\rangle \,, \qquad \langle V_\mu(x)V_\nu(y)\rangle \,, \tag{1.8}$$

at one-loop order in the 't Hooft R_ξ-gauge and discuss the differences with respect to the corresponding one-loop elementary propagators $\langle h(x)h(y)\rangle$ and $\langle A_\mu(x)A_\nu(y)\rangle$ already evaluated in [21] .

As expected, both correlation functions of eq. (1.8) turn out to be independent from the gauge parameter ξ. Moreover, we shall show that the one-loop pole masses of $\langle V_\mu(x)V_\nu(y)\rangle^T$ and $\langle O(x)O(y)\rangle$ are exactly the same as those of the elementary propagators $\langle A_\mu(x)A_\nu(y)\rangle^T$ and $\langle h(x)h(y)\rangle$, where $\langle A_\mu(x)A_\nu(y)\rangle^T$ stands for the transverse component of $\langle A_\mu(x)A_\nu(y)\rangle$, i.e.

$$\langle A_\mu(x)A_\nu(y)\rangle^T = \left(\delta_{\mu\rho} - \frac{\partial_\mu\partial_\rho}{\partial^2}\right)\langle A_\rho(x)A_\nu(y)\rangle \,. \tag{1.9}$$

This important feature makes apparent that the operators $V_\mu(x)$ and $O(x)$ give a gauge-invariant picture for the photon and Higgs modes. In addition, the correlation functions $\langle V_\mu(x)V_\nu(y)\rangle^T$ and $\langle O(x)O(y)\rangle$ exhibit a spectral KL representation with positive spectral densities, allowing for a physical interpretation in terms of observable particles. This property is in sharp contrast with the one-loop spectral density of the elementary non-gauge-invariant Higgs propagator $\langle h(x)h(y)\rangle$, which displays an explicit dependence on the gauge

parameter ξ [21]. Moreover, the longitudinal part of the correlator $\langle V_\mu(x)V_\nu(y)\rangle$ —which is independently gauge-invariant— is shown to exhibit the pole mass of the Higgs excitation. This last feature reinforces the consistency of the present description of the physical degrees of freedom of the theory, since the only physically expected elementary excitations are indeed the Higgs and the photon ones. Let us also underline that, to our knowledge, this is the first explicit one-loop calculation of the gauge-invariant correlators (1.8) and of their analytical properties in Higgs-like models.

This work is organized as follows. In section 2.1, we give a short review of the U(1) Abelian Higgs model and of its quantization in the R_ξ-gauge. Then, we compute at one-loop order the two-point functions: for the elementary fields in 2.2 and for the composite operators in 2.3. We pay attention on how to partially resum contributions to the connected propagator. This is of particular relevance when considering a composite operator propagator, where the standard connection of the $1PI$ self-energy being the inverse connected propagator is lost. In section 3.1, we present an overview of the techniques employed in [21] to obtain the spectral function up to first order in $\hbar$. In section 3.2 we review, for the benefit of the reader, the results for the spectral functions of the propagators of the elementary fields [21]. In section 3.3, we provide the detailed analysis of the computation of the gauge-invariant correlators (1.8) and compare them with the corresponding correlators of the elementary fields. We connect the subtracted spectral KL representations with the contact terms that can be added to the action in presence of composite operators. The unitary limit, in which the gauge parameter ξ tends to infinity, is investigated in section 4, where we also make the connection with the gauge-invariant spectral densities. Section 5 collects our conclusion and outlook. The final appendices contain the derivation of the Feynman rules and of the one-loop diagrams contributing to (1.8).

2 The gauge-invariant operators $V_\mu(x)$ and $O(x)$ in the U(1) Higgs model

In this section, we will follow the steps outlined in [15, 16, 22] to obtain the two-point functions for the composite gauge-invariant operators $(V_\mu(x), O(x))$ in the Abelian Higgs model. In 2.1 we shall lay out some of the essential properties of the Abelian Higgs model quantized in the R_ξ-gauge. The cancellation of the gauge parameter ξ will help us to verify the explicit gauge independence of the correlation functions (1.8). In 2.2 we will shortly review the expressions of the two-point functions of the elementary fields, obtained in [21]. In 2.3 we shall compute the two-point function of the two composite gauge-invariant operators $(V_\mu(x), O(x))$.

2.1 The Abelian Higgs model: some essentials

We start from the U(1) Abelian Higgs classical action as given in eq. (1.1). The parameter v, corresponding to the minimum of the classical potential present in the starting action, gives the vacuum expectation value (vev) of the scalar field to zeroth order in $\hbar$, $\langle \varphi \rangle_0 = v$. As usual, the Higgs mechanism [23–26] is implemented by expressing the scalar field as an

expansion around its vev, namely

$$\varphi = \frac{1}{\sqrt{2}}((v+h) + i\rho), \tag{2.1}$$

where the real part h is identified as the Higgs field and ρ is the (unphysical) Goldstone boson, with $\langle \rho \rangle = 0$. Here we choose to expand around the classical value of the vev,[2] so that $\langle h \rangle$ is zero at the classical level, but receives loop corrections.[3] The action (1.1) now becomes

$$S_0 = \int d^4x \left\{ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} \partial_\mu h \partial_\mu h + \frac{1}{2} \partial_\mu \rho \partial_\mu \rho - e\, \rho\, \partial_\mu h\, A_\mu + e\, (h+v) A_\mu \partial_\mu \rho \right.$$
$$\left. + \frac{1}{2} e^2 A_\mu [(h+v)^2 + \rho^2] A_\mu + \frac{1}{8} \lambda (h^2 + 2hv + \rho^2)^2 \right\} \tag{2.2}$$

and we notice that both the gauge field and the Higgs field have acquired the following masses

$$m^2 = e^2 v^2, \; m_h^2 = \lambda v^2. \tag{2.3}$$

With this parametrization, the Higgs coupling λ and the parameter v can be fixed in terms of m, m_h and e, whose values will be suitably chosen later on in the text.

2.1.1 Gauge fixing

Quantization of the theory (2.2) requires a proper gauge fixing. We shall employ the gauge fixing term

$$S_{gf} = \int d^4x \left\{ \frac{1}{2\xi} \left(\partial_\mu A_\mu + \xi m \rho \right)^2 \right\}, \tag{2.4}$$

known as the 't Hooft or R_ξ-gauge, which has the pleasant property of cancelling the mixed term $\int d^4x (ev\, A_\mu \partial_\mu \rho)$ in the expression (2.2). Of course, (2.4) breaks the gauge invariance of the action. As is well known, the latter is replaced by the BRST invariance. In fact, introducing the FP ghost fields $\bar{c}, c$ as well as the auxiliary field b, for the BRST transformations we have

$$\begin{array}{cccc} sA_\mu = -\partial_\mu c, & sc = 0, & s\varphi = iec\varphi, & s\varphi^\dagger = -iec\varphi^\dagger, \\ sh = -ec\rho, & s\rho = ec(v+h), & s\bar{c} = ib, & sb = 0. \end{array} \tag{2.5}$$

Importantly, the operator s is nilpotent, i.e. $s^2 = 0$, allowing to work with the so-called BRST cohomology [27], a useful concept to prove unitarity and renormalizability of the

---

[2]In principle, a non-perturbative gauge-invariant setup implies that $\langle \varphi \rangle = 0$, so that this expansion with $\langle \varphi \rangle \neq 0$ is only well-defined in the gauge fixed framework that will be described in the next subsection. As is well-known the gauge fixed description of the Higgs mechanism is a successful approach to perturbative calculations in the continuum as the one pursued in the current work. For a more thorough discussion on gauge invariance and the Higgs mechanism, the reader is referred to [19, 20].

[3]There is of course an equivalent procedure of fixing $\langle h \rangle$ to zero at all orders, by expanding φ around the full vev: $\varphi = \frac{1}{\sqrt{2}}((\langle \varphi \rangle + h) + i\rho)$. See [21] for details.

Abelian Higgs model [28–30], see also [31]. We can now introduce the gauge fixing in a BRST invariant way via

$$
\begin{aligned}
\mathcal{S}_{gf} &= s \int d^d x \left\{ -i\frac{\xi}{2}\bar{c}b + \bar{c}(\partial_\mu A_\mu + \xi m\rho) \right\}, \\
&= \int d^d x \left\{ \frac{\xi}{2}b^2 + ib\partial_\mu A_\mu + ib\xi m\rho + \bar{c}\partial^2 c - \xi m^2 \bar{c}c - \xi m e\bar{c}hc \right\}.
\end{aligned}
\tag{2.6}
$$

Notice that the ghosts $(\bar{c}, c)$ get a gauge parameter-dependent mass, while interacting directly with the Higgs field.

The total gauge fixed BRST-invariant action then becomes

$$
\begin{aligned}
S = S_0 + S_{gf} = \int d^4 x \bigg\{ &\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}\partial_\mu h \partial_\mu h + \frac{1}{2}\partial_\mu \rho \partial_\mu \rho - e\rho \partial_\mu h A_\mu + e h A_\mu \partial_\mu \rho + \frac{1}{2}m^2 A_\mu A_\mu \\
&+ \frac{1}{2}e^2 A_\mu [h^2 + 2vh + \rho^2]A_\mu + \frac{1}{8}\lambda(h^2+\rho^2)(h^2+\rho^2+4hv) + \frac{1}{2}m_h^2 h^2 + m A_\mu \partial_\mu \rho + \frac{\xi}{2}b^2 + ib\partial_\mu A_\mu \\
&+ ib\xi m\rho + \bar{c}(\partial^2)c - m^2\xi\bar{c}c - m\xi e\bar{c}ch \bigg\},
\end{aligned}
\tag{2.7}
$$

with

$$
sS = 0 .
\tag{2.8}
$$

In appendix A we collect the propagators and vertices corresponding to the action (2.7) of the Abelian Higgs model in the R_ξ gauge.

Let us end this section by pointing out that the two local operators $(V_\mu(x), O(x))$ belong to the cohomology of the BRST operator [27], i.e.

$$
\begin{aligned}
sV_\mu(x) &= 0 , & V_\mu(x) &\neq s\Delta_\mu(x) \\
sO(x) &= 0 , & O(x) &\neq s\Delta(x) ,
\end{aligned}
\tag{2.9}
$$

for any local quantities $(\Delta_\mu(x), \Delta(x))$.

2.2 One-loop propagators for the elementary fields

In [21], we studied the spectral properties of the one-loop propagators for the photon field $A_\mu(x)$ and the Higgs field $h(x)$ and evaluated them for $d = 4$ through dimensional regularization in the $\overline{\text{MS}}$-scheme.

For the photon field, the transverse part of the connected propagator $G_{\mu\nu}^{AA}(p^2)$ up to order $\hbar$ is given in momentum space by

$$
\langle A_\mu(p) A_\nu(-p) \rangle = \frac{1}{p^2 + m^2} + \frac{1}{(p^2+m^2)^2}\Pi_{AA}(p^2) + \mathcal{O}(\hbar^2)
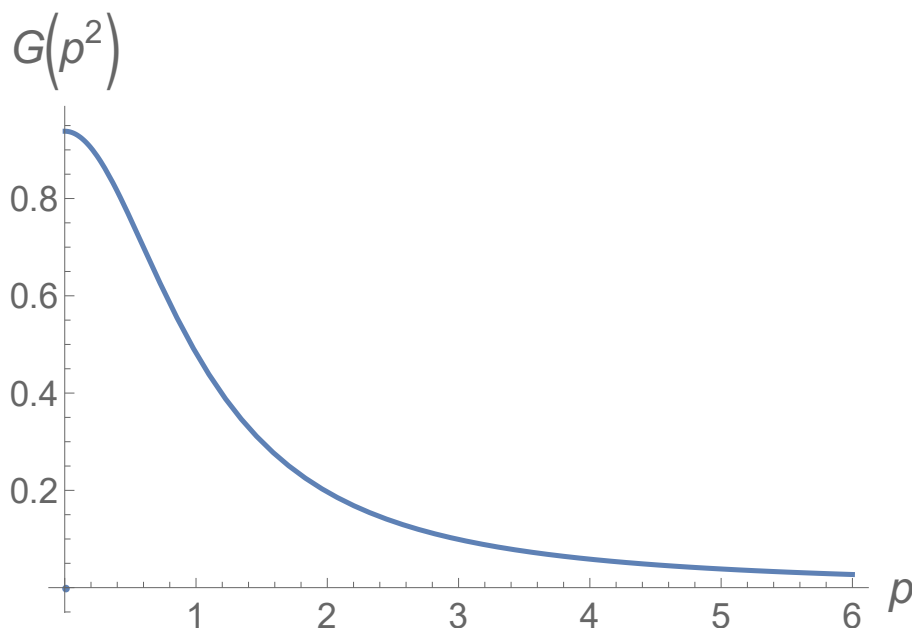\tag{2.10}
$$

Figure 1. Resummed photon propagator, with all quantities given in units of appropriate powers of the energy scale μ, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

with the self-energy given by

$$
\Pi_{AA}(p^2) = 2\frac{e^2}{(4\pi)^2} \int_0^1 dx \left\{ p^2 x(1-x) + m^2 x \right.
$$
$$
+ m_h^2(1-x)(1 - \ln\frac{p^2 x(1-x) + m^2 x + m_h^2(1-x)}{\mu^2}) + m_h^2\left(1 - \ln\frac{m_h^2}{\mu^2}\right)
$$
$$
\left. + \frac{m^4}{m_h^2}\left(1 - 3\ln\frac{m^2}{\mu^2}\right) + 2m^2 \ln\frac{p^2 x(1-x) + m^2 x + m_h^2(1-x)}{\mu^2} \right\}, \tag{2.11}
$$

and we can resum all one-loop self-energy insertions into the connected propagator via

$$
G_{AA}^T(p^2) = \frac{1}{p^2 + m^2 - \Pi(p^2)}, \tag{2.12}
$$

shown in figure 1.

For the Higgs field, we find

$$
\langle h(p)h(-p)\rangle = \frac{1}{p^2 + m_h^2} + \frac{1}{(p^2 + m_h^2)^2}\Pi_{hh}(p^2) + \mathcal{O}(\hbar^2) \tag{2.13}
$$

with

$$\Pi_{hh}(p^2) = \frac{1}{(4\pi)^2} \int_0^1 dx \Bigg\{ e^2 \Bigg[p^2 \left(1 - \ln\frac{m^2}{\mu^2} - 2\ln\frac{p^2 x(1-x) + m^2}{\mu^2} \right) \tag{2.14}$$
$$- \frac{p^4}{2m^2} \ln\frac{p^2 x(1-x) + m^2}{\mu^2} - 6m^2 \left(1 - \ln\frac{m^2}{\mu^2} + \ln\frac{p^2 x(1-x) + m^2}{\mu^2} \right) \Bigg]$$
$$+ \lambda \Bigg[\frac{1}{2} m_h^2 \left(-6 + 6\ln\frac{m_h^2}{\mu^2} - 9\ln\frac{p^2 x(1-x) + m_h^2}{\mu^2} \right) \Bigg]$$
$$- \Bigg[\xi(e^2 p^2 + \lambda m^2) \left(1 - \ln\frac{\xi m^2}{\mu^2} \right) - \left(e^2 \frac{p^4}{2m^2} - \lambda \frac{m_h^2}{2} \right) \ln\frac{p^2 x(1-x) + \xi m^2}{\mu^2} \Bigg] \Bigg\}.$$

Before trying to resum the self-energy insertions again, we notice that this resummation is tacitly assuming that the second term in (2.13) is much smaller than the first term. Then, we see that eq. (2.13) contains terms of the order of $\frac{p^4}{(p^2+m_h^2)^2} \ln\frac{p^2 x(1-x)+m_h^2}{\mu^2}$, which cannot be resummed for big values of p. We therefore use the identity

$$p^4 = (p^2 + m_h^2)^2 - m_h^4 - 2p^2 m_h^2, \tag{2.15}$$

to rewrite

$$\frac{p^4}{(p^2 + m_h^2)^2} \ln\frac{p^2 x(1-x) + m_h^2}{\mu^2} = \ln\frac{p^2 x(1-x) + m_h^2}{\mu^2} - \underline{\frac{(m^4 + 2p^2 m^2)}{(p^2 + m^2)^2} \ln\frac{p^2 x(1-x) + m_h^2}{\mu^2}}. \tag{2.16}$$

The underlined term in (2.16) can be safely resummed. We thence rewrite

$$\frac{\Pi_{hh}(p^2)}{(p^2 + m_h^2)^2} = \frac{\hat{\Pi}_{hh}(p^2)}{(p^2 + m_h^2)^2} + C_{hh}(p^2), \tag{2.17}$$

with

$$\hat{\Pi}_{hh}(p^2) = \frac{1}{(4\pi)^2} \int_0^1 dx \Bigg\{ e^2 \Bigg[p^2 \left(1 - \ln\frac{m^2}{\mu^2} - 2\ln\frac{p^2 x(1-x) + m^2}{\mu^2} \right)$$
$$+ \frac{(m_h^4 + 2p^2 m_h^2)}{2m^2} \ln\frac{p^2 x(1-x) + m_h^2}{\mu^2} - 6m^2 \left(1 - \ln\frac{m^2}{\mu^2} + \ln\frac{p^2 x(1-x) + m^2}{\mu^2} \right) \Bigg]$$
$$+ \lambda \Bigg[\frac{1}{2} m_h^2 \left(-6 + 6\ln\frac{m_h^2}{\mu^2} - 9\ln\frac{p^2 x(1-x) + m_h^2}{\mu^2} \right) \Bigg] \tag{2.18}$$
$$- \Bigg[\xi(e^2 p^2 + \lambda m^2) \left(1 - \ln\frac{\xi m^2}{\mu^2} \right) + \left(e^2 \frac{(m_h^4 + 2p^2 m_h^2)}{2m^2} + \lambda \frac{m_h^2}{2} \right) \ln\frac{p^2 x(1-x) + \xi m^2}{\mu^2} \Bigg] \Bigg\}.$$

and

$$C_{hh}(p^2) = -\frac{e^2}{2m^2(4\pi)^2} \int_0^1 dx \Bigg\{ \ln\left(\frac{p^2 x(1-x) + m_h^2}{\mu^2} \right) - \ln\left(\frac{p^2 x(1-x) + \xi m^2}{\mu^2} \right) \Bigg\} \tag{2.19}$$

and the reliable resummed approximation becomes

$$G_{hh}(p^2) = \frac{1}{p^2 + m^2 - \hat{\Pi}(p^2)} + C_{hh}(p^2), \tag{2.20}$$
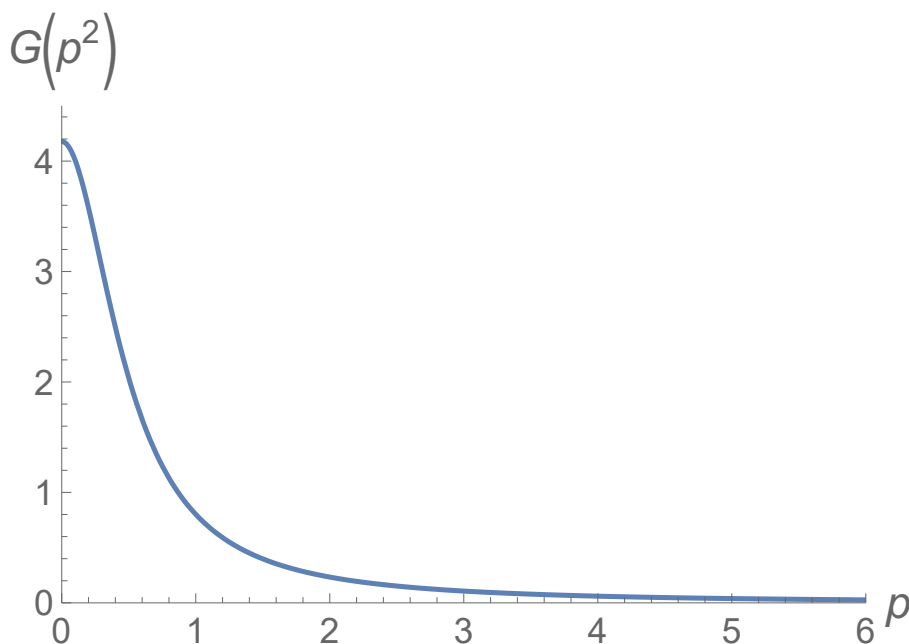
which is shown in figure 2.

Figure 2. Resummed Higgs propagator, with all quantities given in units of appropriate powers of the energy scale μ, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

For completeness, let us mention here that the integrals over the Feynman parameter x that appear in the propagators can be done analytically, see appendix B. Since the transverse component A_μ^T of the Abelian gauge field is gauge-invariant, it turns out that the transverse photon propagator is independent from the gauge parameter ξ, while the Higgs propagator does depend on ξ, in agreement with the Nielsen identities analyzed in [32].

2.3 The correlation functions $\langle O(x)O(y)\rangle$ and $\langle V_\mu(x)V_\nu(y)\rangle$ at one-loop order

We are now ready to study the two-point correlation functions of the local gauge-invariant operators $(V_\mu(x), O(x))$. For the correlator of the scalar composite operator we get:

$$\langle O(x)O(y)\rangle = v^2 \langle h(x)h(y)\rangle + v \langle h(x)\rho(y)^2\rangle + v \langle h(x)h(y)^2\rangle +$$
$$+ \frac{1}{4}\Big(\langle h(x)^2\rho(y)^2\rangle + \langle h(x)^2 h(y)^2\rangle + \langle \rho(x)^2\rho(y)^2\rangle \Big). \tag{2.21}$$

Individually, the terms in the expansion (2.21) are not gauge-invariant, but their sum is. We can now analyze the connected diagrams for each term, up to one-loop order, through the action (2.2). We calculated the one-loop diagrams in appendix C. Looking at the diagrams in figure 15, we can see that the correlation function $\langle O(p)O(-p)\rangle$ will have the following structure

$$\langle O(p)O(-p)\rangle^{1-\text{loop}} = \frac{A_{\text{fin}}(p^2) + \delta A_{\text{div}}(p^2)}{(p^2 + m_h^2)^2} + \frac{B_{\text{fin}}(p^2) + \delta B_{\text{div}}(p^2)}{(p^2 + m_h^2)}$$
$$+ C_{\text{fin}}(p^2) + \delta C_{\text{div}}(p^2), \tag{2.22}$$

where $(A_{\text{fin}}, B_{\text{fin}}, C_{\text{fin}})$ stand for the finite parts and $(\delta A_{\text{div}}, \delta B_{\text{div}}, \delta C_{\text{div}})$ for the purely divergent terms, i.e. the one-loop pole terms in $\frac{1}{\epsilon}$ obtained by means of the dimensional regularization ($d = 4 - \epsilon$), namely

$$
\delta A_{\text{div}}(p^2) \overset{\epsilon \to 0}{=} \frac{v^2}{8\pi^2\epsilon}\Big(2v^2\lambda^2 - e^2(p^2(-3+\xi) + v^2\lambda\xi)\Big),
$$
$$
\delta B_{\text{div}}(p^2) \overset{\epsilon \to 0}{=} \frac{v^2(6e^4 - \lambda^2 + e^2\lambda\xi)}{8\pi^2\epsilon\lambda},
$$
$$
\delta C_{\text{div}}(p^2) \overset{\epsilon \to 0}{=} \frac{1}{8\pi^2\epsilon}, \tag{2.23}
$$

while

$$
\begin{aligned}
A_{\text{fin}}(p^2) &= \frac{v^2}{(4\pi)^2} \int_0^1 dx \bigg\{ e^2 \bigg[p^2 \left(1 - \ln\frac{m^2}{\mu^2} - 2\ln\frac{p^2x(1-x)+m^2}{\mu^2} \right) \\
&\quad - \frac{p^4}{2m^2}\ln\frac{p^2x(1-x)+m^2}{\mu^2} - 6m^2\left(1 - \ln\frac{m^2}{\mu^2} + \ln\frac{p^2x(1-x)+m^2}{\mu^2} \right) \bigg] \\
&\quad + \lambda\bigg[\frac{1}{2}m_h^2\left(-6 + 6\ln\frac{m_h^2}{\mu^2} - 9\ln\frac{p^2x(1-x)+m_h^2}{\mu^2} \right) \bigg] \\
&\quad - \bigg[\xi(e^2p^2 + \lambda m^2)\left(1 - \ln\frac{\xi m^2}{\mu^2} \right) - \left(e^2\frac{p^4}{2m^2} - \lambda\frac{m_h^2}{2} \right)\ln\frac{p^2x(1-x)+\xi m^2}{\mu^2} \bigg] \bigg\}, \\
B_{\text{fin}}(p^2) &= \frac{1}{(4\pi)^2 m_h^2} \int_0^1 dx \bigg\{ -m^2\xi m_h^2 \ln\left(\frac{m^2\xi}{\mu^2} \right) + m^2\xi m_h^2 + m_h^4\left(3\ln\left(\frac{m_h^2+p^2(1-x)x}{\mu^2} \right) \right. \\
&\quad + \left. \ln\left(\frac{m^2\xi+p^2(1-x)x}{\mu^2} \right) \right) - 3m_h^4\ln\left(\frac{m_h^2}{\mu^2} \right) + 3m_h^4 + 2m^4 - 6m^4\ln\left(\frac{m^2}{\mu^2} \right) \bigg\}, \\
C_{\text{fin}}(p^2) &= -\frac{1}{2(4\pi)^2} \int_0^1 dx \bigg\{ \ln\left(\frac{m_h^2+p^2(1-x)x}{\mu^2} \right) + \ln\left(\frac{m^2\xi+p^2(1-x)x}{\mu^2} \right) \bigg\}. \tag{2.24}
\end{aligned}
$$

The divergent terms $(\delta A_{\text{div}}, \delta B_{\text{div}}, \delta C_{\text{div}})$ can be eliminated by means of the standard counterterms as well as by suitable counterterms in the external source part of the action S_J accounting for the introduction of the composite operator $O(x)$, see [27, 33–36] for a general account on this topic, i.e.

$$
S_J = S + \int d^4x \bigg[(1 + \delta Z_{\text{div}}^0) J(x) O(x) + (1 + \delta Z_{\text{div}}) \frac{(J(x))^2}{2} \bigg], \tag{2.25}
$$

where $J(x)$ is a BRST invariant dimension two source needed to define the generator $Z^c(J)$ of the connected Green function $\langle O(x)O(y)\rangle$:

$$
\langle O(x)O(y)\rangle = \left.\frac{\delta^2 Z^c(J)}{\delta J(x)\delta J(y)}\right|_{J=0}. \tag{2.26}
$$

It is worth emphasizing here that we have the freedom of introducing a pure BRST invariant contact term in the external source $J(x)$:

$$
\int d^4x \frac{\alpha}{2} J^2(x), \tag{2.27}
$$

which can be arbitrarily added to the action (2.25). Including such a term in (2.25) will have the effect of adding a dimensionless constant to $G_{OO} = \langle O(p)O(-p)\rangle$, i.e.

$$G_{OO}(p^2) \rightarrow G_{OO}(p^2) + \alpha. \tag{2.28}$$

In particular, α can be chosen to be equal to $-G_{OO}(0)$, implying then that the modified Green's function

$$G_{OO}(p^2) - G_{OO}(0) \tag{2.29}$$

will obey a once substracted KL representaion, see section 3 for more details on this.

Inserting the unity

$$1 = (p^2 + m_h^2)/(p^2 + m_h^2) = ((p^2 + m_h^2)/(p^2 + m_h^2))^2, \tag{2.30}$$

into the finite part of $\langle O(p)O(-p)\rangle$, we write

$$\langle O(p)O(-p)\rangle_{\text{fin}} = \frac{v^2}{p^2 + m_h^2} + \frac{\hbar v^2}{(p^2 + m_h^2)^2}\Pi(p^2) + \mathcal{O}(\hbar^2) \tag{2.31}$$

where

$$\Pi_{OO}(p^2) = \frac{1}{v^2}\Big((A_{\text{fin}}(p^2)) + (p^2 + m_h^2)(B_{\text{fin}}(p^2)) + (C_{\text{fin}}(p^2))(p^2 + m_h^2)^2\Big),$$

$$= \frac{1}{32\pi^2 v^2 m_h^2}\int_0^1 dx\bigg\{-8m_h^2 m^4 - 2m^2 p^2(m_h^2 + 6m^2)\ln\left(\frac{m^2}{\mu^2}\right) +$$

$$+ m_h^2\bigg[-(p^2 - 2m_h^2)^2\ln\left(\frac{m_h^2 + p^2(1-x)x}{\mu^2}\right)$$

$$-(12m^4 + 4m^2 p^2 + p^4)\ln\left(\frac{m^2 + p^2(1-x)x}{\mu^2}\right)\bigg] +$$

$$+ 2p^2(3m_h^4 + m_h^2 m^2 + 2m^4) - 6m_h^4 p^2 \ln\left(\frac{m_h^2}{\mu^2}\right)\bigg\}. \tag{2.32}$$

Since (2.32) contains terms of the order of $\frac{p^4}{p^2+m^2}\ln(p^2)$, we follow the steps (2.15)–(2.17) to find the resummed propagator in the one-loop approximation

$$G_{OO}(p^2) = \frac{v^2}{p^2 + m_h^2 - \hat{\Pi}_{OO}(p^2)} + C_{OO}(p^2) \tag{2.33}$$

with

$$\hat{\Pi}_{OO}(p^2) = \frac{1}{32\pi^2 v^2 m_h^2}\int_0^1 dx\bigg\{-8m_h^2 m^4 - 2m^2 p^2(m_h^2 + 6m^2)\ln\left(\frac{m^2}{\mu^2}\right) +$$

$$+ m_h^2\bigg[3(m_h^4 + 2m_h^2 p^2)\ln\left(\frac{m_h^2 + p^2(1-x)x}{\mu^2}\right)$$

$$-(12m^4 + 4m^2 p^2 - m_h^4 - 2p^2 m_h^2)\ln\left(\frac{m^2 + p^2(1-x)x}{\mu^2}\right)\bigg]$$

$$+ 2p^2(3m_h^4 + m_h^2 m^2 + 2m^4) - 6m_h^4 p^2 \ln\left(\frac{m_h^2}{\mu^2}\right)\bigg\}, \tag{2.34}$$

and

$$C_{OO}(p^2) = -\frac{1}{32\pi^2} \int_0^1 dx \left\{ \ln\left(\frac{m_h^2 + p^2 x(1-x)}{\mu^2}\right) + \ln\left(\frac{m^2 + p^2 x(1-x)}{\mu^2}\right) \right\}. \quad (2.35)$$

It is crucial to stress here that, if we had also resummed $C_{OO}(p^2)$ into the inverse propagator, we would have encountered an (unphysical) tachyon into the composite operator propagator, as at some point the exploding large p^2-behaviour in $C_{OO}(p^2)$ would completely wash out the other "UV tamed" contributions.

The propagator is depicted in figure 3 and we notice that the Green function $G_{OO}(p^2)$ becomes negative for large enough values of the momentum p. As one realizes from expression (2.33), this feature is due to the growing in the UV region of the logarithms contained in the term $C_{OO}(p^2)$, see eq. (2.35). It is worth mentioning that this behaviour is also present when the parameter v is completely removed from the theory. In fact, setting $v = 0$, the action S_0 in eq. (1.1), reduces to that of massless scalar QED, namely

$$S_0|_{v=0} = \int d^4x \left(\frac{F_{\mu\nu}^2}{4} + (D_\mu\varphi)^\dagger(D_\mu\varphi) + \frac{\lambda}{2}(\varphi^\dagger\varphi)^2 \right), \quad (2.36)$$

with

$$\varphi|_{v=0} = \frac{1}{\sqrt{2}}(h + i\rho). \quad (2.37)$$

Of course, when $v = 0$, the operators $O = \varphi^\dagger\varphi$ and $V_\mu = -i\varphi^\dagger D_\mu\varphi$ are still gauge-invariant. Though, from eqs. (2.34)–(2.35), computing $\langle O(p)O(-p)\rangle_{v=0}$, one immediately gets

$$\langle O(p)O(-p)\rangle_{v=0} = C_{OO}(p^2)|_{v=0} = -\frac{1}{16\pi^2} \int_0^1 dx \ln\frac{p^2 x(1-x)}{\mu^2}. \quad (2.38)$$

This equation precisely shows that the term $C_{OO}(p^2)$, and thus the negative behaviour for large enough values of p, is what one usually obtains in a theory for which $v = 0$, making evident that the presence of $C_{OO}(p^2)$ is not peculiarity of the U(1) Higgs model, on the contrary. However, in addition to the term $C_{OO}(p^2)$ and unlike massless scalar QED, the correlation function $\langle O(p)O(-p)\rangle$ of the U(1) Higgs model exhibits the term $\frac{v^2}{p^2+m_h^2-\hat{\Pi}_{OO}}$, which will play a pivotal role. Indeed, as we shall see later on, this term, originating from the expansion of φ around the minimum of the Higgs potential, $\varphi = \frac{1}{\sqrt{2}}(v + h + i\rho)$, will enable us to devise a gauge-invariant description of the elementary excitations of the model.

Let us end the analysis of the correlation function $G_{OO}(p^2)$ by displaying the behaviour of its first derivative, $\frac{\partial G_{OO}(p^2)}{\partial p^2}$, as well as of the once subtracted correlator $G_{OO}(p^2) - G_{OO}(0)$, see figure 4. The first derivative, as expected, is negative while, unlike $G_{OO}(p^2)$, it decays to zero for $p^2 \to \infty$. The quantity $\frac{\partial G_{OO}(p^2)}{\partial p^2}$ will be helpful when discussing the spectral representation corresponding to $\langle O(p)O(-p)\rangle$.

Then, for the vectorial composite operator $V_\mu(x)$, we first observe that

$$\begin{aligned}
V_\mu(x) &= -i\varphi^\dagger(x)(D_\mu\varphi)(x) \\
&= e\varphi^\dagger(x)A_\mu(x)\varphi(x) - \frac{1}{2}i\varphi^\dagger(x)\partial_\mu\varphi(x) + \frac{1}{2}i\varphi(x)\partial_\mu\varphi^\dagger(x) - i\partial_\mu O(x), \quad (2.39)
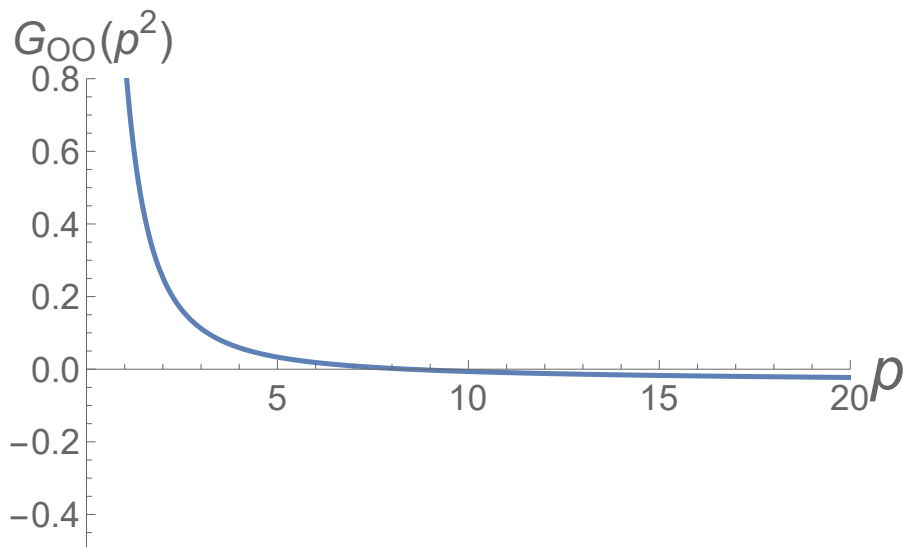\end{aligned}$$

Figure 3. Resummed propagator for the scalar composite operator. All quantities are given in units of appropriate powers of the energy scale μ, with the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.
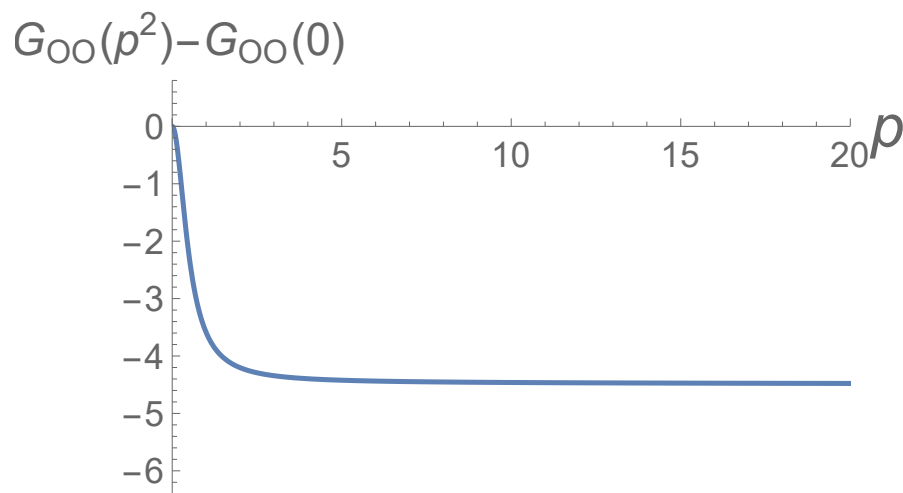
Figure 4. The resummed propagator with a single subtraction. All quantities are given in units of appropriate powers of the energy scale μ, with the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

and since we know that the last term is gauge-invariant, the first three terms together must also be. We can thus define a new gauge-invariant operator

$$V'_\mu(x) = e\varphi^\dagger(x)A_\mu(x)\varphi(x) - \frac{1}{2}i\varphi^\dagger(x)\partial_\mu\varphi(x) + \frac{1}{2}i\varphi(x)\partial_\mu\varphi^\dagger(x), \quad (2.40)$$

expanding the scalar field $\varphi(x)$ we find

$$V'_\mu(x) = \frac{1}{2}\Big(e(v + h(x))^2 A_\mu(x) + e\rho^2(x)A_\mu(x) + (v + h(x))\partial_\mu\rho(x) - \rho(x)\partial_\mu h(x)\Big) \quad (2.41)$$

Figure 5. The first derivative of the resummed propagator. All quantities are given in units of appropriate powers of the energy scale μ, with the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.
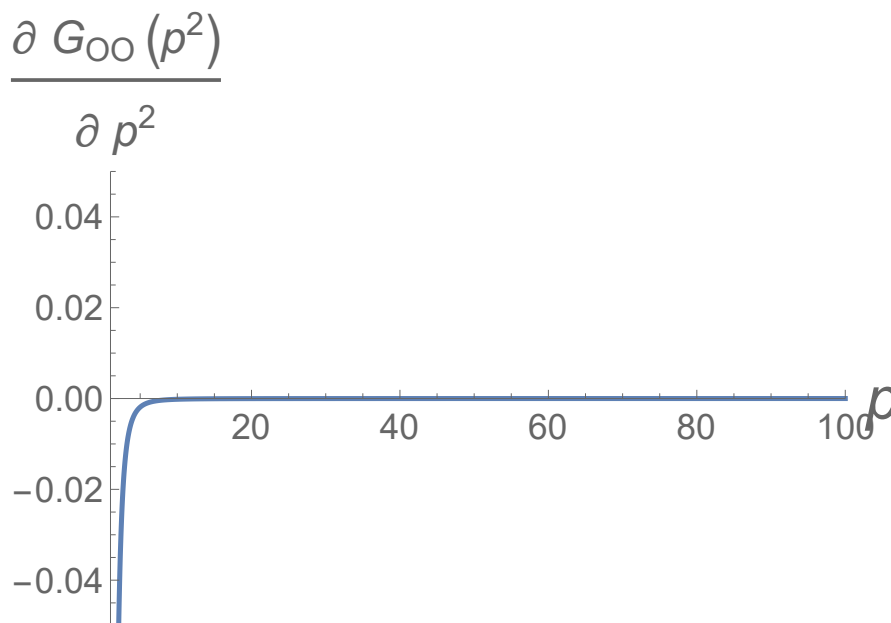
so that

$$
\begin{aligned}
\langle V'_\mu(x) V'_\nu(y) \rangle \overset{\varphi \to \frac{1}{\sqrt{2}}(v+h+i\rho)}{=} -\frac{1}{4}\Big\{ &- e^2 v^4 \langle A_\mu(x) A_\nu(y) \rangle - 4e^2 v^3 \langle h(x) A_\mu(x) A_\nu(y) \rangle \\
&- 2e^2 v^2 \langle h(x)^2 A_\mu(x) A_\nu(y) \rangle - 4e^2 v^2 \langle h(x) A_\mu(x) h(y) A_\nu(y) \rangle \\
&- 2e^2 v^2 \langle \rho(x)^2 A_\mu(x) A_\nu(y) \rangle - 2ev^2 \partial_\mu^x \langle h(x)\rho(x) A_v(y) \rangle \\
&+ 4ev^2 \langle \partial_\mu^x h(x) \rho(x) A_\nu(y) \rangle - 2ev^3 \partial_\mu^x \langle \rho(x) A_\nu(y) \rangle \\
&- 4ev^2 \partial_\mu^x \langle \rho(x) h(y) A_v(y) \rangle - 2v \partial_\mu^x \partial_\nu^y \langle h(x)\rho(x)\rho(y) \rangle \\
&+ 4v \partial_\nu^y \langle \partial_\mu^x h(x) \rho(x) \rho(y) \rangle - \partial_\mu^x \partial_\nu^y \langle h(x)\rho(x)h(y)\rho(y) \rangle \\
&+ 4 \langle \partial_\mu^x h(x)\rho(x) h(y) \partial_\nu^y \rho(y) \rangle - v^2 \partial_\mu^x \partial_\nu^y \langle \rho(x)\rho(y) \rangle \Big\} \\
&+ O(\hbar^2),
\end{aligned}
\tag{2.42}
$$

where we have discarded the terms that do not have one-loop contributions. In momentum space, we can split the two-point function into transverse and longitudinal parts in the usual way:

$$
\langle V'_\mu(p) V'_\nu(-p) \rangle = \langle V'(p) V'(-p) \rangle^T \mathcal{P}_{\mu\nu} + \langle V'(p) V'(-p) \rangle^L \mathcal{L}_{\mu\nu},
\tag{2.43}
$$

where we have introduced the transverse and longitudinal projectors, given respectively by

$$
\mathcal{P}_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \qquad \mathcal{L}_{\mu\nu}(p) = \frac{p_\mu p_\nu}{p^2}.
\tag{2.44}
$$

At tree-level, we find in momentum space

$$\langle V'_\mu(p)V'_\nu(-p)\rangle_{\text{tree}} = -\frac{1}{4}\left(-e^2v^4\langle A_\mu(p)A_\nu(-p)\rangle - v^2 p_\mu p_\nu\langle\rho(p)\rho(-p)\rangle\right)$$

$$= \frac{1}{4}\left(e^2v^4\frac{1}{p^2+m^2}\mathcal{P}_{\mu\nu} + e^2v^4\frac{\xi}{p^2+\xi m^2}\mathcal{L}_{\mu\nu} + v^2\frac{p^2}{p^2+\xi m^2}\mathcal{L}_{\mu\nu}\right)$$

$$= \frac{e^2v^4}{4}\frac{1}{p^2+m^2}\mathcal{P}_{\mu\nu} + v^2\mathcal{L}_{\mu\nu}. \tag{2.45}$$

We can now analyze the connected diagrams for each term, up to one-loop order, through the action (2.2). We calculated the one-loop diagrams in appendix D. Let us start with the transverse part. Looking at the diagrams in figure 16, we can see that the one-loop correlation function will have the following structure

$$\langle V'(p)V'(-p)\rangle^{T,1-\text{loop}} = \frac{A^V_{\text{fin}}(p^2) + \delta A^V_{\text{div}}(p^2)}{(p^2+m^2)^2} + \frac{B^V_{\text{fin}}(p^2) + \delta B^V_{\text{div}}(p^2)}{(p^2+m^2)}$$

$$+ C^V_{\text{fin}}(p^2) + \delta C^V_{\text{div}}(p^2) \tag{2.46}$$

where $(A^V_{\text{fin}}, B^V_{\text{fin}}, C^V_{\text{fin}})$ stand for the finite parts and $(\delta A^V_{\text{div}}, \delta B^V_{\text{div}}, \delta C^V_{\text{div}})$ for the purely divergent terms, i.e. the one-loop pole terms in $\frac{1}{\epsilon}$ obtained by means of the dimensional regularization, namely

$$\delta A^V_{\text{div}} \overset{\epsilon\to 0}{=} \frac{e^4v^4}{2(4\pi)^2\epsilon}\left(\frac{1}{3}p^2 - 6\left(\frac{e^2}{\lambda} - \frac{1}{2}\right)e^2v^2 + 3\lambda v^2\right),$$

$$\delta B^V_{\text{div}} \overset{\epsilon\to 0}{=} \frac{v^2}{(4\pi)^2\epsilon}\left(6\frac{e^6v^2}{\lambda} - 3e^4v^2 - \frac{e^2p^2}{3} + 3e^2\lambda v^2\right),$$

$$\delta C^V_{\text{div}} \overset{\epsilon\to 0}{=} \frac{1}{6(4\pi)^2\epsilon}(9e^2v^2 - p^2 - 3\lambda v^2) \tag{2.47}$$

and

$$A^V_{\text{fin}} = \frac{e^4v^4}{2(4\pi)^2}\int_0^1 dx\left\{p^2x(1-x) + m^2x\right.$$

$$+ m_h^2(1-x)(1-\ln\frac{p^2x(1-x)+m^2x+m_h^2(1-x)}{\mu^2}) + m_h^2\left(1-\ln\frac{m_h^2}{\mu^2}\right)$$

$$\left.+ \frac{m^4}{m_h^2}\left(1-3\ln\frac{m^2}{\mu^2}\right) + 2m^2\ln\frac{p^2x(1-x)+m^2x+m_h^2(1-x)}{\mu^2}\right\},$$

$$B^V_{\text{fin}} = \frac{m^2}{18m_h^2p^2(4\pi)^2}\int_0^1 dx\left\{3m_h^4\left(m_h^2-m^2-7p^2\right)\ln\left(\frac{m_h^2}{\mu^2}\right),\right.$$

$$- 3m_h^2\left(2p^2\left(m_h^2-5m^2\right)+\left(m_h^2-m^2\right)^2+p^4\right)\ln\left(\frac{xm_h^2+m^2(1-x)+p^2(1-x)x}{\mu^2}\right)$$

$$- 3\left(m_h^3-m^2m_h\right)^2 + 9p^2\left(m^2m_h^2+3m_h^4+2m^4\right)+2p^4m_h^2$$

$$\left.- 3m^2\left(m_h^2\left(p^2-m^2\right)+m_h^4+18m^2p^2\right)\ln\left(\frac{m^2}{\mu^2}\right)\right\}$$

$$C^V_{\text{fin}} = \frac{1}{36(4\pi)^2p^2}\int_0^1 dx\left\{3m^2\left(m_h^2-m^2+p^2\right)\ln\left(\frac{m^2}{\mu^2}\right)\right.$$

$$\left.+ 3m_h^2\left(-m_h^2+m^2+p^2\right)\ln\left(\frac{m_h^2}{\mu^2}\right)+6m_h^2\left(p^2-m^2\right)-5p^2\left(3m_h^2-9m^2+p^2\right)\right.$$

$$+3\left(2m_h^2\left(p^2-m^2\right)+m_h^4+m^4-10m^2p^2+p^4\right)\ln\left(\frac{p^2x(1-x)+xm^2+(1-x)m_h^2}{\mu^2}\right)$$

$$+3m_h^4+3m^4-54m^2p^2+3p^4\bigg\}. \tag{2.48}$$

The divergent terms $(\delta A_{\mathrm{div}}^V, \delta B_{\mathrm{div}}^V, \delta C_{\mathrm{div}}^V)$ can again be eliminated by means of the standard counterterms as well as by suitable counterterms in the external source part of the action S_J^V accounting for the introduction of the composite operator $V_\mu'(x)$, i.e.

$$S_J^V = S + \int d^4x\left[(1+\delta Z_{\mathrm{div}}^{V,0})J_\mu(x)V_\mu(x) + (1+\delta Z_{\mathrm{div}}^V)\frac{J_\mu(x)J_\mu(x)}{2}\right], \tag{2.49}$$

where $J_\mu(x)$ is a BRST invariant dimension one source needed to define the generator $Z^c(J)$ of the connected Green function $\langle V_\mu'(x)V_\nu'(y)\rangle$:

$$\langle V_\mu'(x)V_\nu'(y)\rangle = \left.\frac{\delta^2 Z^c(J)}{\delta J_\mu(x)\delta J_\nu(y)}\right|_{J=0}, \tag{2.50}$$

and like in the scalar case, we have the freedom of introducing BRST invariant pure contact terms in the external source $J_\mu(x)$:

$$\int d^4x\,\frac{1}{2}(\beta v^2\,J_\mu(x)J_\mu(x) + \gamma J_\mu(x)\partial^2 J_\mu(x) + \sigma(\partial_\mu J_\mu(x))^2), \tag{2.51}$$

which can be arbitrarily added to the action eq. (2.49). Including such terms in (2.49) will have the effect of adding a first order polynomial in p^2 to $G_{VV}^T(p^2) = \langle V'(p)V'(-p)\rangle^T$, i.e.

$$G_{VV}^T(p^2) \to G_{VV}^T(p^2) + \beta v^2 + \gamma p^2, \tag{2.52}$$

where we notice that the last term in (2.51) does not contribute to the transversal part of the propagator. In particular, β and γ can be chosen so that (2.52) becomes

$$G_{VV}^T(p^2) - G_{VV}^T(0) - p^2\left.\frac{\partial G_{VV}^T(p^2)}{\partial p^2}\right|_{p=0}. \tag{2.53}$$

Eventually, we have a Green's function that obeys a twice substracted KL representation, see section 3. Following similar steps as for the scalar composite field, (2.25)–(2.30), we find

$$\langle V'(p)V'(-p)\rangle^T = \frac{e^2v^4}{4}\frac{1}{p^2+m^2} + \frac{\hbar e^2v^4}{4}\frac{\Pi_{VV}^T(p^2)}{(p^2+m^2)^2} + \mathcal{O}(\hbar^2), \tag{2.54}$$

with

$$\Pi_{VV}^T(p^2) = -\frac{1}{9(4\pi)^2 e^2 v^4 m_h^2} \int_0^1 dx \Big\{ -18m^4(m_h^4 + m^4) + 9m_h^2 p^4(m_h^2 + m^2)$$
$$-3m_h^2 p^2 \big[2p^2(m_h^2 - 5m^2) + (m_h^2 - m^2)^2 + p^4 \big]$$
$$\times \ln\left(\frac{m_h^2(1-x) + m^2 x + p^2(1-x)x}{\mu^2} \right) + 2m_h^2 p^6$$
$$+3m_h^4 \ln\left(\frac{m_h^2}{\mu^2} \right) \big[p^2(m_h^2 + 11m^2) + 6m^4 - p^4 \big]$$
$$+3m^2 \big[-m_h^2 p^4 + p^2(-m_h^4 + m_h^2 m^2 + 36m^4) + 18m^6 \big] \ln\left(\frac{m^2}{\mu^2} \right)$$
$$-3p^2(m_h^6 + 10m_h^4 m^2 + m_h^2 m^4 + 12m^6) \Big\}, \tag{2.55}$$

and following the steps (2.15)–(2.17), we find

$$G_{VV}^T = \frac{e^2 v^4}{4} \left(\frac{1}{p^2 + m^2 - \hat{\Pi}_{VV}^T(p^2)} \right) + C_{VV}(p^2) \tag{2.56}$$

with

$$\hat{\Pi}_{VV}^T(p^2) = -\frac{1}{9(4\pi)^2 e^2 v^4 m_h^2} \int_0^1 dx \Big\{ -18m^4(m_h^4 + m^4) + 9m_h^2 p^4(m_h^2 + m^2)$$
$$-3m_h^2 \big[2(-2m^2 p^2 - m^4)(m_h^2 - 5m^2) + p^2(m_h^2 - m^2)^2 - 2m^2 p^2 - m^4 \big]$$
$$\times \ln\left(\frac{m_h^2(1-x) + m^2 x + p^2(1-x)x}{\mu^2} \right)$$
$$+2m_h^2 p^6 + 3m_h^4 \ln\left(\frac{m_h^2}{\mu^2} \right) \big[p^2(m_h^2 + 11m^2) + 6m^4 - p^4 \big]$$
$$+3m^2 \big[-m_h^2 p^4 + p^2(-m_h^4 + m_h^2 m^2 + 36m^4) + 18m^6 \big] \ln\left(\frac{m^2}{\mu^2} \right)$$
$$-3p^2(m_h^6 + 10m_h^4 m^2 + m_h^2 m^4 + 12m^6) \Big\}, \tag{2.57}$$

and

$$C_{VV}(p^2) = \frac{1}{12(4\pi)^2} \int_0^1 dx \Big\{ (2m_h^2 + p^2) \ln\left(\frac{m_h^2(1-x) + m^2 x + p^2(1-x)x}{\mu^2} \right) \Big\}. \tag{2.58}$$

The resummed propagator (2.56) is depicted in figure 6, as well as the subtracted version (2.53) in figure 7 and the second derivative in figure 8, which will be important for the spectral analysis in section 3.

For the longitudinal part of the propagator (see appendix D for details), we find the divergent part

$$\langle V'(p)V'(-p) \rangle_{\text{div}}^L \overset{\epsilon \to 0}{=} -\frac{v^2 \left(3e^4 + \lambda^2 \right)}{(4\pi)^2 \lambda \epsilon} \tag{2.59}$$

Figure 6. Resummed propagator for the vector composite operator. All quantities are given in units of appropriate powers of the energy scale μ, with the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.
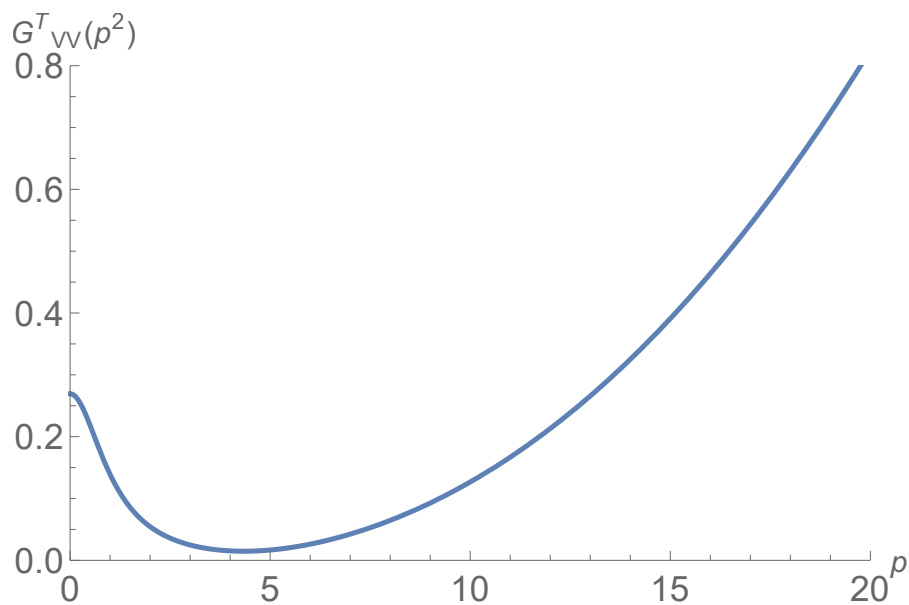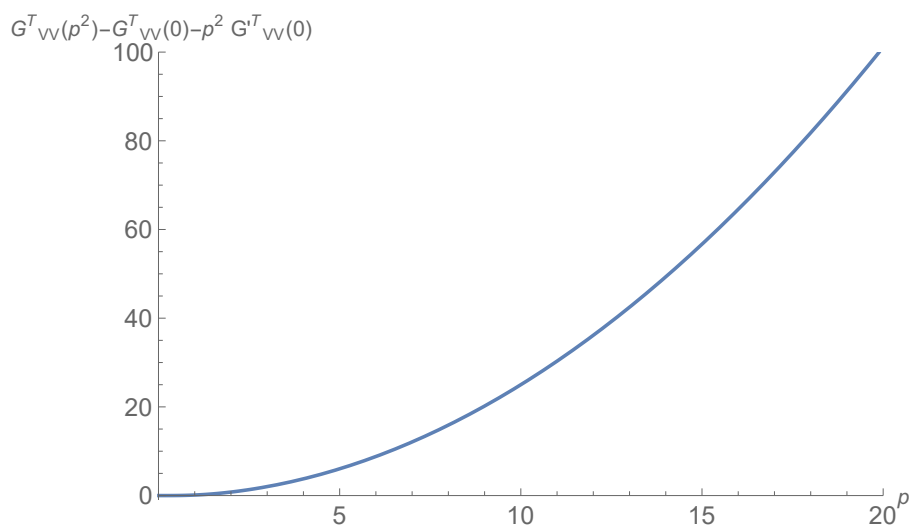

Figure 7. Resummed propagator with a double subtraction for the vector composite operator. All quantities are given in units of appropriate powers of the energy scale μ, with the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.
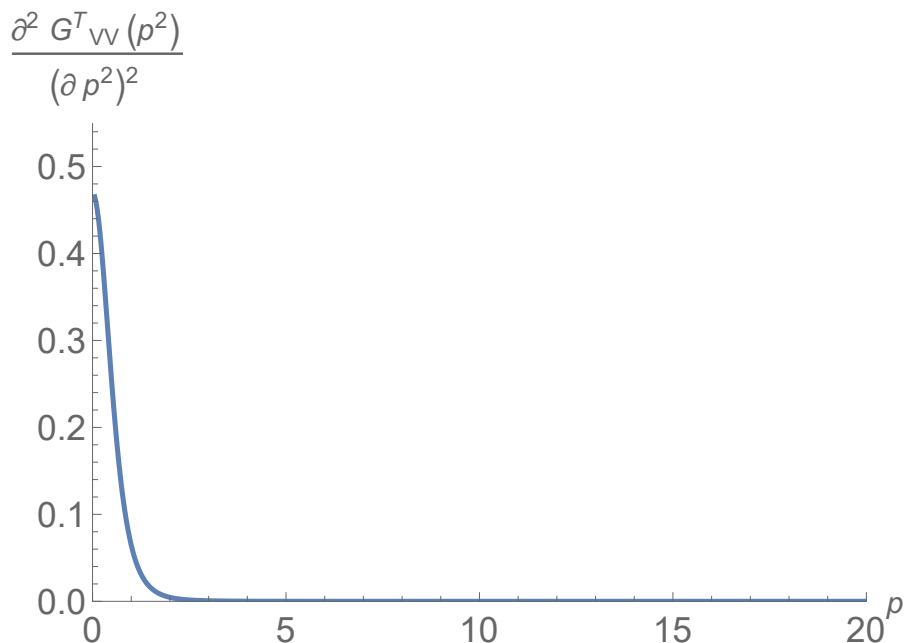
Figure 8. Second derivative of the vector propagator. All quantities given in units of appropriate powers of the energy scale μ, with the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

and the total finite correction up to first order in $\hbar$ is given by

$$\langle V'(p)V'(-p)\rangle^L_{\text{fin}} = v^2 - \left(\frac{m_h^4 - m_h^4 \ln\left(\frac{m_h^2}{\mu^2}\right) + m^4 - 3m^4 \ln\left(\frac{m^2}{\mu^2}\right)}{32\pi^2 m_h^2} \right). \tag{2.60}$$

From this expression one sees that, as in the case of the tree level, the one-loop correction to the longitudinal part of the correlator $\langle V'_\mu(x)V'_\nu(y)\rangle^L_{\text{fin}}$ remains independent from the momentum p^2. As such, it is not associated to any physical mode.

3 Spectral properties of the gauge-invariant local operators $(V_\mu(x), O(x))$

In this section, we will study the spectral properties associated with the correlation functions obtained in the last section. In 3.1, we will shortly review the techniques employed in [21] to obtain the pole mass, residue and spectral density up to first order in $\hbar$. In 3.2.1, we analyze the spectral properties of the elementary propagators. In 3.3, the spectral properties of the composite operators $(V_\mu(x), O(x))$ are discussed.

3.1 Obtaining the spectral function

For elementary fields we obtain the spectral density function by comparing the (Euclidean) KL spectral representation for the propagator of a generic field $\widetilde{O}(p)$

$$\langle \widetilde{O}(p)\widetilde{O}(-p)\rangle = G(p^2) = \int_0^\infty dt \frac{\rho(t)}{t + p^2}, \tag{3.1}$$

where $\rho(t)$ is the spectral density function and $G(p^2)$ stands for the resummed propagator

$$G(p^2) = \frac{1}{p^2 + m^2 - \Pi(p^2)}. \tag{3.2}$$

For higher-dimensional operators, the resummed propagator acquires an overall (dimensionful) factor identical to the one appearing in its tree level result, as we have seen in section 2.3. We also note that in the case of higher dimensional operators, the spectral representation, eq. (3.1), might require appropriate subtraction terms in order to ensure a convergent spectral integral. A standard way to cure this problem is to subtract from $G(p^2)$ the first few (divergent) terms of its Taylor expansion at $p = 0$ [37], making the integral more and more convergent. These subtraction terms are directly related to the renormalization of the composite operators, and one can see that the modified Green's functions for the composite scalar field (2.29) and for the composite vector field (2.53) are in fact subtractions of the Taylor series to first and second order, respectively. In our theory we can make use of the subtracted equations at $p = 0$ because all fields are massive in the R_ξ-gauge, so there are no divergences at zero momentum. Also, we stress that the spectral function $\rho(t)$ is not affected by the subtraction procedure as polynomials do not display discontinuities in the complex p^2-plane, whilst the spectral function is proportional to the jump across the branch cut. Moreover, we can see that these subtractions do not have an influence either on the (second) derivative of the propagator. For the scalar composite operator

$$\frac{\partial(G_{OO}(p^2) - G_{OO}(0))}{\partial p^2} = \frac{\partial G_{OO}(p^2)}{\partial p^2} = -\int_0^\infty dt \frac{\rho(t)}{(t+p^2)^2}, \tag{3.3}$$

which means that for a positive spectral function, the first derivative of $G_{OO}(p^2)$ ought to be strictly negative, as is indeed confirmed from figure 5. For the vector composite operator

$$\frac{\partial^2(G_{VV}(p^2) - G_{VV}(0) - p^2 G'_{VV}(0))}{(\partial p^2)^2} = \frac{\partial^2 G_{VV}(p^2)}{(\partial p^2)^2} = 2\int_0^\infty dt \frac{\rho(t)}{(t+p^2)^3}, \tag{3.4}$$

which should be strictly positive for a positive spectral function, consistent with figure 8.

We can also obtain the spectral function directly in the following way. The pole mass for any massless or massive field excitation is obtained by calculating the pole of the resummed propagator, that is, by solving

$$p^2 + m^2 - \Pi(p^2) = 0 \tag{3.5}$$

and its solution defines the pole mass $p^2 = -m^2_{\text{pole}}$. As consistency requires us to work up to a fixed order in perturbation theory, we should solve eq. (3.5) for the pole mass in an iterative fashion. Therefore, to first order in $\hbar$, we find

$$m^2_{\text{pole}} = m^2 - \Pi^{1-\text{loop}}(-m^2) + \mathcal{O}(\hbar^2), \tag{3.6}$$

where $\Pi^{1-\mathrm{loop}}$ is the first order, or one-loop, correction to the propagator. Now, we write eq. (3.2) in a slightly different way, namely

$$
\begin{aligned}
G(p^2) &= \frac{1}{p^2 + m^2 - \Pi(p^2)} \\
&= \frac{1}{p^2 + m^2 - \Pi^{1-\mathrm{loop}}(-m^2) - (\Pi(p^2) - \Pi^{1-\mathrm{loop}}(-m^2))} \\
&= \frac{1}{p^2 + m^2_{\mathrm{pole}} - \widetilde{\Pi}(p^2)},
\end{aligned}
\tag{3.7}
$$

where we defined $\widetilde{\Pi}(p^2) = \Pi(p^2) - \Pi^{1-\mathrm{loop}}(-m^2)$. At one-loop, expanding $\widetilde{\Pi}(p^2)$ around $p^2 = -m^2_{\mathrm{pole}} = -m^2 + \mathcal{O}(\hbar)$ gives the residue

$$
\begin{aligned}
Z &= \lim_{p^2 \to -m^2_{\mathrm{pole}}} (p^2 + m^2_{\mathrm{pole}}) G(p^2) \\
&= \frac{1}{1 - \partial_{p^2}\Pi(p^2)|_{p^2=-m^2}} \\
&= 1 + \partial_{p^2}\Pi(p^2)|_{p^2=-m^2} + \mathcal{O}(\hbar^2).
\end{aligned}
\tag{3.8}
$$

We now write (3.7) to first order in $\hbar$ as

$$
\begin{aligned}
G(p^2) &= \frac{Z}{(p^2 + m^2_{\mathrm{pole}} - \widetilde{\Pi}(p^2))Z} \\
&= \frac{Z}{p^2 + m^2_{\mathrm{pole}} - \widetilde{\Pi}(p^2) + (p^2 + m^2_{\mathrm{pole}})\frac{\partial \widetilde{\Pi}(p^2)}{\partial p^2}|_{p^2=-m^2}} \\
&= \frac{Z}{p^2 + m^2_{\mathrm{pole}}} + Z\left(\frac{\widetilde{\Pi}(p^2) - (p^2 + m^2_{\mathrm{pole}})\frac{\partial \widetilde{\Pi}(p^2)}{\partial p^2}|_{p^2=-m^2}}{(p^2 + m^2_{\mathrm{pole}})^2}\right),
\end{aligned}
\tag{3.9}
$$

where in the last line we used a first-order Taylor expansion so that the propagator has an isolated pole at $p^2 = -m^2_{\mathrm{pole}}$. In (3.1) we can isolate this pole in the same way, by defining the spectral density function as $\rho(t) = Z\delta(t - m^2_{\mathrm{pole}}) + \widetilde{\rho}(t)$, giving

$$
G(p^2) = \frac{Z}{p^2 + m^2_{\mathrm{pole}}} + \int_0^\infty dt \frac{\widetilde{\rho}(t)}{t + p^2}
\tag{3.10}
$$

and we identify the second term in each of the representations (3.9) and (3.10) as the *reduced propagator*

$$
\widetilde{G}(p^2) \equiv G(p^2) - \frac{Z}{p^2 + m^2_{\mathrm{pole}}},
\tag{3.11}
$$

so that

$$
\widetilde{G}(p^2) = \int_0^\infty dt \frac{\widetilde{\rho}(t)}{t + p^2} = Z\left(\frac{\widetilde{\Pi}(p^2) - (p^2 + m^2_{\mathrm{pole}})\frac{\partial \widetilde{\Pi}(p^2)}{\partial p^2}|_{p^2=-m^2}}{(p^2 + m^2_{\mathrm{pole}})^2}\right).
\tag{3.12}
$$

Finally, using Cauchy's integral theorem in complex analysis, we can find the spectral density $\widetilde{\rho}(t)$ as a function of $\widetilde{G}(p^2)$, giving

$$
\widetilde{\rho}(t) = \frac{1}{2\pi i}\lim_{\epsilon \to 0^+}\left(\widetilde{G}(-t - i\epsilon) - \widetilde{G}(-t + i\epsilon)\right).
\tag{3.13}
$$

Although we restricted our analysis to first order in $\hbar$ in this paper, it should not come as a surprise the foregoing methodology can be adapted order per order in $\hbar$.

3.2 Spectral properties of the elementary fields

We first discuss the spectral properties of the elementary fields: the transverse photon field $A_\mu^T(x)$ and the scalar Higgs field $h(x)$. For illustrational purposes, for the rest of this section and the next, we shall write all quantities as a function of the renormalization scale μ and choose the parameters $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$, so that $m = 1\,\mu$ and $m_h = \frac{1}{\sqrt{5}}\,\mu$. For this choice of parameters, all one-loop corrections computed are within 20% of the tree-level results, indicating that our perturbative approximation is under control.

3.2.1 The transverse photon field

Since in the Abelian case the transverse component of the gauge field $A_\mu^T(x)$ is explicitly gauge-invariant, the corresponding propagator (2.12) is independent from the gauge parameter ξ, and so are its pole mass, residue and spectral function. Following the steps from section 3.1, we find the first-order pole mass of the transverse photon to be

$$m_{\text{pole}}^2 = 1.05417\,\mu^2 \tag{3.14}$$

and the first-order residue

$$Z = 0.984983. \tag{3.15}$$

These values are small corrections of the tree-level ones, $m^2 = \mu^2$ and $Z_{\text{tree}} = 1$, so that the one-loop approximation appears to be consistent.

The spectral function is given in figure 9. We can distinguish a two-particle state threshold at $t = (m + m_h)^2 = 2.09\,\mu^2$, and the spectral density function is positive, adequately describing the physical photon excitation.

3.2.2 The Higgs field

For the Higgs fields, following the steps from section 3.1, we find the pole mass to first order in $\hbar$ to be

$$m_{h,\text{pole}}^2 = 0.237987\,\mu^2 = 1.1899\,m_h^2, \tag{3.16}$$

for all values of the parameter ξ. This means that while the Higgs propagator (2.14) is itself gauge dependent, the pole mass is gauge independent. This is in full agreement with the Nielsen identities of the Abelian U(1) Higgs model studied in [32]. For the residue, we distinguish three regions:

- $\xi < \frac{1}{20} = \frac{\lambda}{4e^2}$: for these values $m_h > 2\sqrt{\xi}m$, which means the Higgs particle is unstable and can decay into two Goldstone modes. Of course, this process is physically impossible because the Goldstone boson itself is not physical. It therefore clearly demonstrates the unphysical nature of the propagator $\langle h(x)h(y)\rangle$. For these values of ξ, the pole mass is a real number located on the (unphysical) branch cut created by the two-particle Goldstone state. This means that we cannot even properly define the derivative of the one-loop correction to obtain the corresponding residue (3.8).
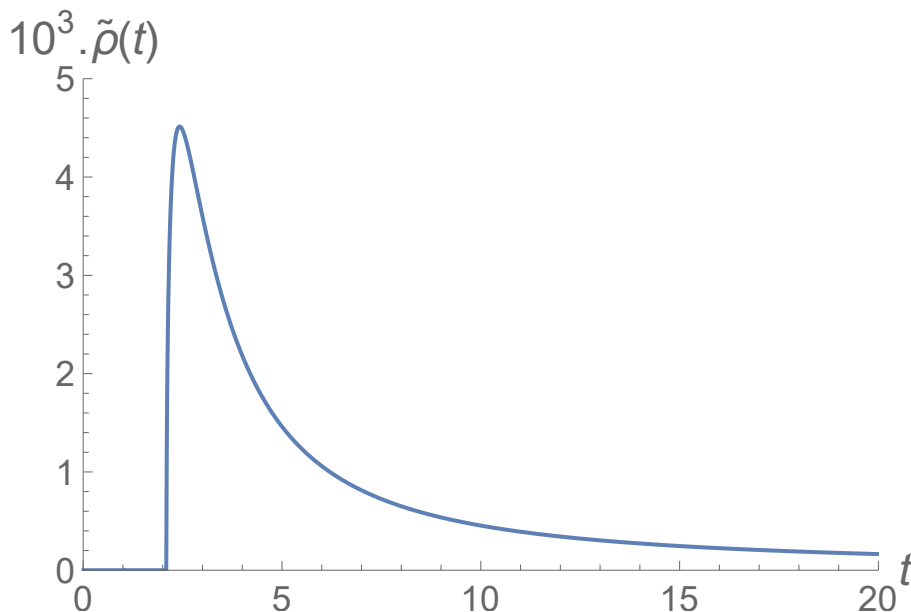
Figure 9. Spectral function for the transverse part of the reduced photon propagator $\langle A(p)A(-p)\rangle^T$, with all quantities given in units of appropriate powers of the energy scale μ, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

- $\xi \leq 3$: for these values we find $Z > 1$.

- $\xi > 3$: for these values we find $Z < 1$.

In figure 10, we display the spectral density functions for three values of ξ: $2, 3, 5$. For small t, their behaviour is the same, with a two-particle Higgs state at $t = (m_h + m_h)^2 = 0.8\,\mu^2$, and a two-particle state for the photon field, starting at $t = (m + m)^2 = 4\mu^2$. Then, we see that there is a negative contribution, different for each case, at $t = (\sqrt{\xi}m + \sqrt{\xi}m)^2$. This corresponds to the threshold for creation of two (unphysical) Goldstone bosons. This negative contribution eventually overcomes the other ones, leading to a negative regime in the spectral function, independently of the value of ξ. This feature is consistent with the large-momentum behaviour of the Higgs propagator (2.14), for a detailed discussion see [21]. As one lowers the value of the gauge parameter ξ, this unphysical threshold is shifted towards lower t's and may occur for momentum values lower than the physical two-particle states of two Higgs particles or two photons. As discussed above, for $\xi < \frac{\lambda}{4e^2}$ even the one-particle delta peak becomes located within the unphysical Goldstone production region and the standard interpretation of the spectral properties is completely lost. It is therefore clear that this correlation function does not display the desired spectral properties to describe the Higgs mode in this theory, indicating the necessity of resorting to another operator as we shall do in what follows.
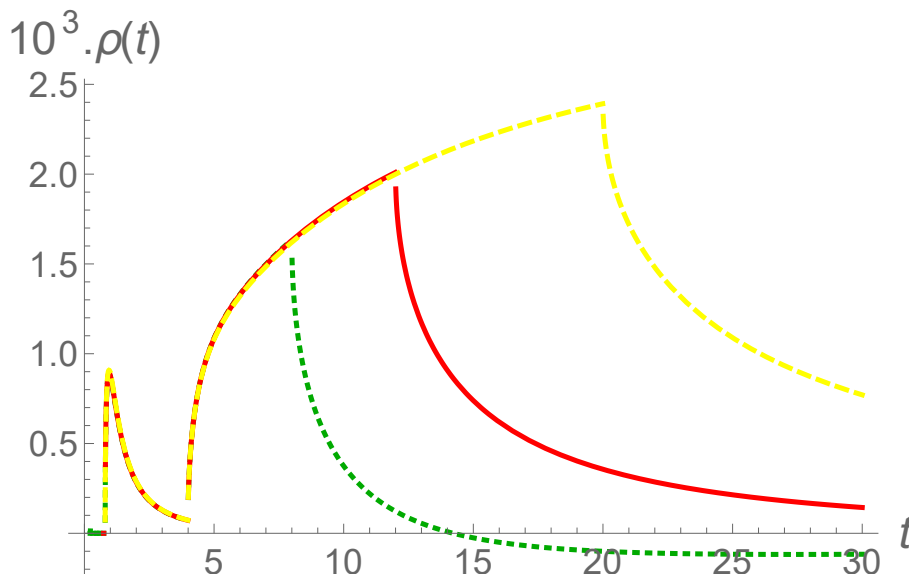
Figure 10. Spectral function for the reduced Higgs propagator $\langle h(p)h(-p)\rangle$, for gauge parameters $\xi = 2$ (Green, dotted), $\xi = 3$ (Yellow, dashed), $\xi = 5$ (Red, Solid), with all quantities given in units of appropriate powers of the energy scale μ, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

3.3 Spectral properties of the gauge-invariant composite operators $V_\mu(x)$ and $O(x)$

3.3.1 The scalar composite operator $O(x)$

For the scalar composite operator $O(x)$ with two-point function given by expression (2.34), we find the first-order pole mass for our set of parameter values to be

$$m_{h,\text{pole}}^2 = 0.213472\,\mu^2, \tag{3.17}$$

which is exactly equal to the pole mass of the elementary Higgs field correlator. Following the steps from 3.1, we find the first-order residue to correct the tree-level result $Z_{\text{tree}} = v^2$ by $\sim 7\%$:

$$Z = v^2(1 + \partial_{p^2}\Pi_{OO}(p^2)_{p^2=-m_h^2}) = 1.06577v^2\,, \tag{3.18}$$

while the first-order spectral function is shown in figure 11. Similarly as for the spectral function of the Higgs field in figure 10, one finds a two-particle threshold for Higgs pair production at $t = (m_h + m_h)^2 = 0.8\,\mu^2$, and a two-photon state starting at $t = (m + m)^2 = 4\,\mu^2$. The difference is that for this gauge-invariant correlation function we no longer have the unphysical Goldstone two-particle state. Due to the absence of this negative contribution, the spectral function is always positive. Therefore, this quantity is suitable for describing a physical Higgs excitation spectrum as opposed to the elementary propagator $\langle hh\rangle$.

Finally, it is interesting to note that below the unphysical threshold the elementary correlator displays the same qualitative spectral properties as this gauge-invariant approach. This means that spectral description of the physical Higgs mode could in principle be successfully encoded in the elementary propagator in the unitary gauge, in which $\xi \to \infty$
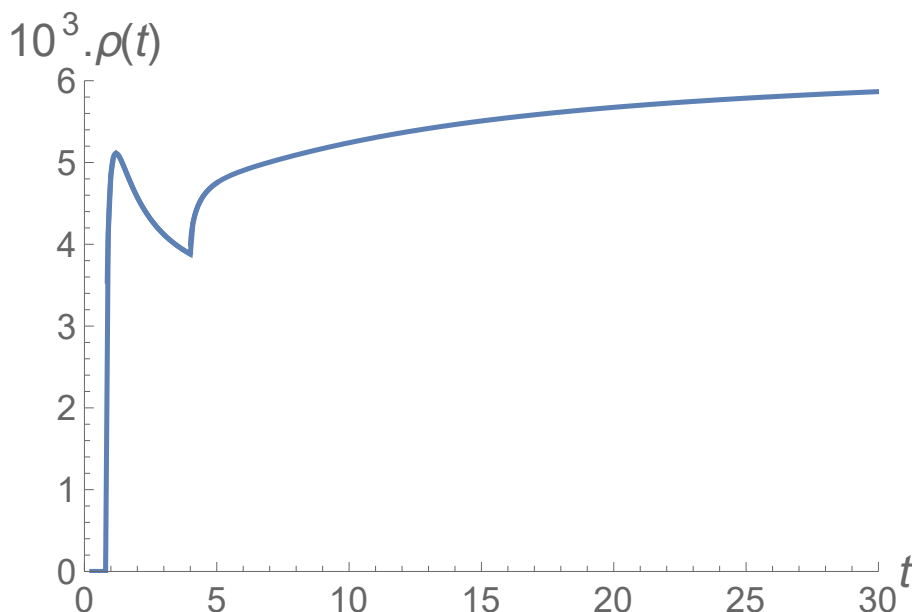
Figure 11. Spectral function for the reduced propagator of the scalar composite operator, $\langle O(p)O(-p)\rangle$, with all quantities given in units of appropriate powers of the energy scale μ, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

and the Goldstone bosons are infinitely heavy. We shall make an explicit comparison in section 4.

3.3.2 The vector composite operator $V_\mu(x)$

For the transverse vector composite operator $V_\mu^T(x)$, with our set of parameters we find the first-order pole mass

$$m_{\text{pole}}^2 = 1.05417\mu^2, \tag{3.19}$$

which is —as expected from the Nielsen identities— exactly the same as the pole mass of the transverse photon field correlator (3.14). Furthermore, we find the first-order residue

$$Z = \frac{e^2 v^4}{4}(1 + \partial_{p^2}\Pi_{VV}^T(p^2)_{p^2=-m^2}) = 1.09332\frac{e^2 v^4}{4}, \tag{3.20}$$

and the first order spectral density for the reduced propagator is displayed in figure 12. Like the photon spectral density in figure 9, we find a photon-Higgs two-particle state at $t = (m_h + m)^2 = 2.09\,\mu^2$, and the spectral density is again positive for all values of t.

4 Unitary gauge limit

It is well-known [1] that for the Higgs model, the unitary gauge represents the "most physical" gauge, as it decouples the unphysical fields, i.e. the ghost field and the Goldstone field. The unitary gauge can be formally obtained from the R_ξ-gauges by taking $\xi \to \infty$.
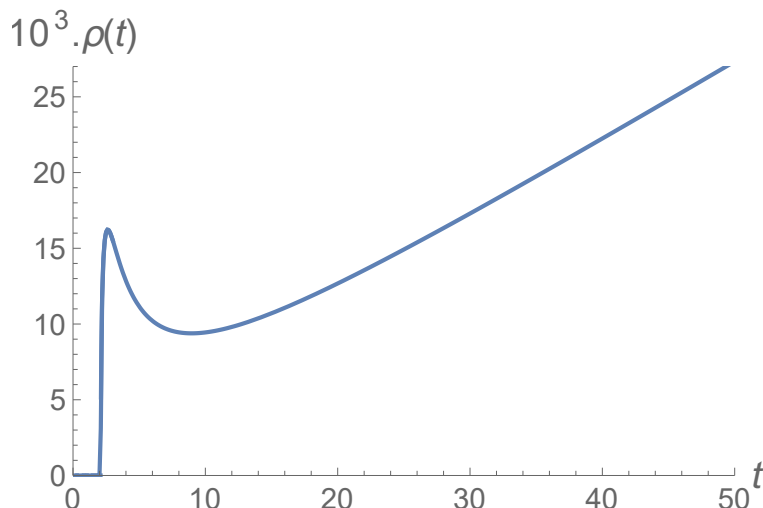
Figure 12. Spectral function for reduced transverse propagator of the vector composite operator $\langle V(p)V(-p)\rangle^T$, with all quantities given in units of appropriate powers of the energy scale μ, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

However, this gauge is non-renormalizable, as one can see by looking at this limit for the tree-level propagator of the photon field

$$\langle A_\mu(p)A_\nu(-p)\rangle_{\text{tree}} \overset{\xi\to\infty}{=} \frac{1}{p^2 + m^2}\mathcal{P}_{\mu\nu} + \frac{1}{m^2}\mathcal{L}_{\mu\nu}. \tag{4.1}$$

Nonetheless, we can approximate the unitary gauge by taking larger and larger values of ξ. This is especially interesting when looking at the spectral function of the elementary Higgs field, which is ξ-dependent. In figure 13 one finds the spectral function for $\xi = 1000$ for small and large ranges of t. In figure 14 we show the spectral function of the scalar composite field $O(x)$ for the same ranges of t. As one can see, the pictures are qualitatively very similar. This means that when approximating the unitary gauge, the spectral function of the gauge dependent, elementary field $h(x)$ approximates that of its composite, gauge-invariant counterpart, thereby clearly showing the physical nature of this gauge. It is intuitively clear why this happens: all unphysical threshold effects related to ghost and Goldstone modes are pushed to higher and higher energy scale as the gauge parameter ξ grows.

5 Conclusion and outlook

In the present work, following the local gauge-invariant setup of [17–20], we have evaluated at one-loop order the two-point correlation functions $\langle V'_\mu(x)V'_\nu(y)\rangle$, $\langle O(x)O(y)\rangle$ of the two local gauge-invariant operators $V'_\mu(x) = -i\varphi^\dagger(x)D_\mu\varphi(x)+i\partial_\mu O(x)$ and $O(x) = \varphi^\dagger(x)\varphi(x) - \frac{v^2}{2}$ in the U(1) Abelian Higgs model quantized in the R_ξ gauge.

Our results can be summarized as follows:

- both $\langle V'_\mu(x)V'_\nu(y)\rangle$ and $\langle O(x)O(y)\rangle$ do not depend on the gauge parameter ξ, as expected;
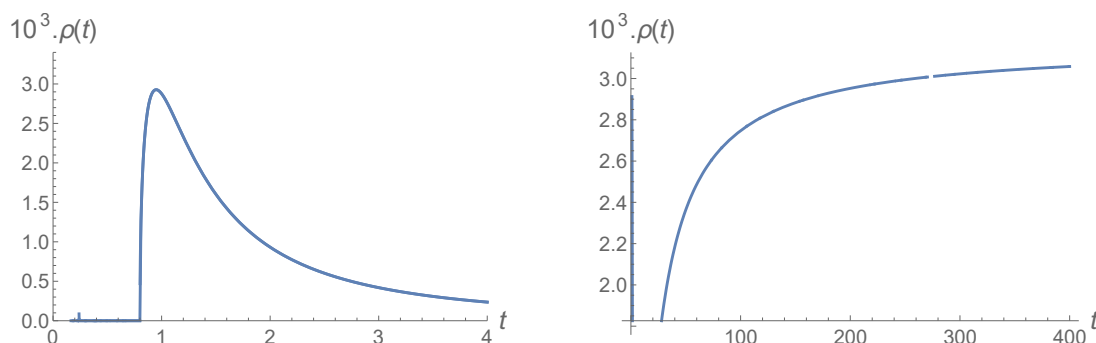
Figure 13. Spectral function for the reduced elementary propagator $\langle h(p)h(-p)\rangle$ for small values of t (left) and large values of t (right), with all quantities given in units of appropriate powers of the energy scale μ, for $\xi = 1000$ the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.
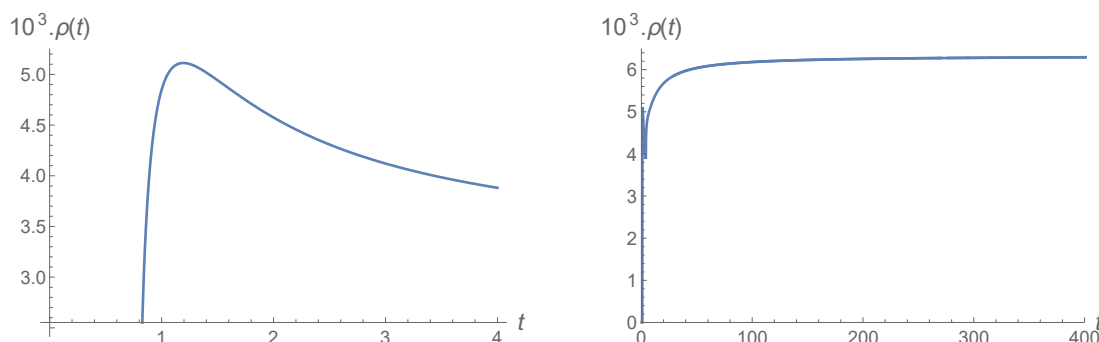

Figure 14. Spectral function for the reduced composite propagator $\langle O(p)O(-p)\rangle$ for small values of t (left) and large values of t (right), with all quantities given in units of appropriate powers of the energy scale μ, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

- the pole masses of $\langle V'_\mu(x)V'_\nu(y)\rangle^T$ and $\langle O(x)O(y)\rangle$ are exactly the same as those of the correlation functions of the elementary fields $\langle A_\mu(x)A_\nu(y)\rangle^T$ and $\langle h(x)h(y)\rangle$, respectively, where $\langle\cdots\rangle^T$ stands for the transverse component of the corresponding propagator;

- the Källén-Lehmann spectral densities of the correlation functions $\langle V'_\mu(x)V'_\nu(y)\rangle$ and $\langle O(x)O(y)\rangle$ turn out to be always positive, in contrast to the one associated with the (gauge dependent) elementary Higgs field.

These important features give us a fully gauge-invariant picture in order to describe the spectrum of elementary excitations of the model, i.e. the massive photon and the Higgs mode.

It is worth underlining that the local gauge-invariant operators $V'_\mu(x)$ and $O(x)$ have their generalization to the non-Abelian case [17–20].[4] This might enable us to extend

---

[4]See also the recent work [38] for a discussion on higher dimensional gauge-invariant operators for different gauge groups and representations of the Higgs fields.

the present work to the case of asymptotically free Higgs-Yang-Mills theories such as, for example, the SU(2) theory with a single Higgs field in the fundamental representation, see for example [39]. This model might be of particular interest since non-perturbative effects can be introduced in order to achieve a better understanding of SU(2) Higgs-Yang-Mills theory and the fate of the excitations in the infrared region. Indeed, as there is not even an strict order parameter discriminating between confinement- or Higgs-like behaviour in such theory, it should intuitively be possible to interpolate from one behaviour to the other without encountering sharp phase boundaries, a feature potentially encoded in the gauge-invariant correlation functions.

More specifically, one may for example introduce the Gribov-Zwanziger horizon term, in its BRST-invariant formulation encoded in the so called Refined Zwanziger-Gribov action (cf. [40–43] and refs. therein) implementing the restriction to the Gribov region Ω [44] in order to take into account the existence of the Gribov copies plaguing the non-Abelian Faddeev-Popov quantization procedure. As a consequence, the gauge-invariant pole masses of the non-Abelian generalization of the correlation functions $\langle V_\mu(x)V_\nu(y)\rangle$ and $\langle O(x)O(y)\rangle$ will now show an explicit dependence on the (BRST invariant) Gribov mass parameter as well as on the dimension-two condensates present in the Refined-Gribov-Zwanziger action [40–43]. Thus, extending the framework already outlined in [45], the aforementioned pole masses and further spectral properties could be employed as gauge-invariant probing quantities in order to extract non-perturbative information about the behaviour of the excitations of Higgs-Yang-Mills theories in the light of the Fradkin-Shenker [46, 47] results.

Another most interesting extension of our methodology would be to the Glashow-Weinberg-Salam electroweak theory, to have a genuinely gauge-invariant description of at least the $W^\pm, Z^0$- and Higgs boson sector of the theory, including their spectral functions bearing information on both pole mass and decay channels [48–51].

We hope to report soon on these interesting and relevant issues.

Acknowledgments

A Propagators and vertices of the Abelian Higgs model in the R_ξ gauge

A.1 Field propagators

The quadratic part of the action (2.7) in the bosonic sector is given by

$$S_{\text{bos}}^{\text{quad}} = \frac{1}{2}\int d^4x \Big\{ A_\mu(-\partial_{\mu\nu}(\partial^2 - m^2) + \partial_\mu\partial_\nu)A_\nu - \rho\partial^2\rho - h(\partial^2 - m_h^2)h + \bar{c}(\partial^2 - m^2\xi)c$$
$$+ 2ib\partial_\mu A_\mu + \xi b^2 + 2im\xi b\rho + 2mA_\mu\partial_\mu\rho \Big\}. \tag{A.1}$$

Putting this in a matrix form yields

$$S_{bos}^{quad} = \frac{1}{2} \int d^4x \, \Psi_\mu^T O_{\mu\nu} \Psi_\nu, \tag{A.2}$$

where

$$\Psi_\mu^T = \begin{pmatrix} A_\mu \ b \ \rho \ h \end{pmatrix}, \quad \Psi_\nu = \begin{pmatrix} A_\nu \\ b \\ \rho \\ h \end{pmatrix},$$

and

$$O_{\mu\nu} = \begin{pmatrix} (-\partial_{\mu\nu}(\partial^2 - m^2) + \partial_\mu\partial_\nu) & -i\partial_\mu & m\partial_\mu & 0 \\ i\partial_\nu & \xi & im\xi & 0 \\ -m\partial_\nu & im\xi & -\partial^2 & 0 \\ 0 & 0 & 0 & -(\partial^2 - m_h^2) \end{pmatrix}.$$

The tree-level field propagators can be read off from the inverse of $\mathcal{O}$, leading to the following expressions in momentum space:

$$\langle A_\mu(p) A_\nu(-p) \rangle = \frac{1}{p^2 + m^2} \mathcal{P}_{\mu\nu} + \frac{\xi}{p^2 + \xi m^2} \mathcal{L}_{\mu\nu},$$

$$\langle \rho(p)\rho(-p) \rangle = \frac{1}{p^2 + \xi m^2},$$

$$\langle h(p)h(-p) \rangle = \frac{1}{p^2 + m_h^2},$$

$$\langle A_\mu(p)b(-p) \rangle = \frac{p_\mu}{p^2 + \xi m^2},$$

$$\langle b(p)\rho(-k) \rangle = \frac{-im}{p^2 + \xi m^2}, \tag{A.3}$$

where $\mathcal{P}_{\mu\nu} = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$ and $\mathcal{L}_{\mu\nu} = \frac{p_\mu p_\nu}{p^2}$ are the transversal and longitudinal projectors, respectively. The ghost propagator is

$$\langle \bar{c}(p)c(-p) \rangle = \frac{1}{p^2 + \xi m^2}. \tag{A.4}$$

A.2 Vertices

From the action (2.7), we find the following vertices

$$\Gamma_{A_\mu\rho h}(-p_1, -p_2, -p_3) = ie(p_{\mu,3} - p_{\mu,2})\delta(p_1 + p_2 + p_3),$$

$$\Gamma_{A_\mu A_\nu h}(-p_1, -p_2, -p_3) = -2e^2 v \delta_{\mu\nu}\delta(p_1 + p_2 + p_3),$$

$$\Gamma_{A_\mu A_\nu hh}(-p_1, -p_2, -p_3, -p_4) = -2e^2 \delta_{\mu\nu}\delta(p_1 + p_2 + p_3 + p_4),$$

$$\Gamma_{A_\mu A_\nu \rho\rho}(-p_1, -p_2, -p_3, -p_4) = -2e^2 \delta_{\mu\nu}\delta(p_1 + p_2 + p_3 + p_4),$$

$$\Gamma_{hhhh}(-p_1, -p_2, -p_3, -p_4) = -3\lambda \, \delta(p_1 + p_2 + p_3 + p_4),$$

$$\Gamma_{hh\rho\rho}(-p_1, -p_2, -p_3, -p_4) = -\lambda \, \delta(p_1 + p_2 + p_3 + p_4),$$

$$\Gamma_{\rho\rho\rho\rho}(-p_1, -p_2, -p_3, -p_4) = -3\lambda \, \delta(p_1 + p_2 + p_3 + p_4),$$

$$\Gamma_{hhh}(-p_1, -p_2, -p_3) = -3\lambda v \, \delta(p_1 + p_2 + p_3),$$

$$\Gamma_{h\rho\rho}(-p_1, -p_2, -p_3) = -\lambda v \, \delta(p_1 + p_2 + p_3),$$

$$\Gamma_{\bar{c}hc}(-p_1, -p_2, -p_3) = -m\xi e \, \delta(p_1 + p_2 + p_3). \tag{A.5}$$

Figure 15. One-loop contributions to the propagator $\langle O(p)O(-p)\rangle$. Wavy lines represent the photon field, dashed lines the Higgs field, solid lines the Goldstone boson and double lines the ghost field. The • indicates the insertion of a composite operator.

B Basic Feynman integral

$$
\int_0^1 dx \, \ln\left[\frac{p^2 x\,(1-x)+xm_1^2+(1-x)\,m_2^2}{\mu^2}\right]
$$
$$
= \frac{1}{2p^2}\Bigg\{ (m_1^2 - m_2^2)\ln\left(\frac{m_2^2}{m_1^2}\right) + p^2\ln\left(\frac{m_1^2 m_2^2}{\mu^4}\right) - 4p^2
$$
$$
- 2i\sqrt{\left(m_1^2 - m_2^2\right)^2 + p^4 + 2p^2\left(m_1^2 + m_2^2\right)}
$$
$$
\times \Bigg[\tan^{-1}\left(\frac{-m_1^2 + m_2^2 - p^2}{\sqrt{-m_1^4 + 2m_1^2\left(m_2^2 - p^2\right) - \left(m_2^2 + p^2\right)^2}}\right)
$$
$$
- \tan^{-1}\left(\frac{-m_1^2 + m_2^2 + p^2}{\sqrt{-m_1^4 + 2m_1^2\left(m_2^2 - p^2\right) - \left(m_2^2 + p^2\right)^2}}\right) \Bigg] \Bigg\} \qquad \text{(B.1)}
$$

C   Contributions to $\langle O(p)O(-p)\rangle$

We consider each term in the two-point function $\langle O(p)O(-p)\rangle$, given by eq. (2.21). We will use the following definitions:

$$
\eta(m_1, m_2, p^2) \equiv \frac{1}{(4\pi)^{d/2}}\Gamma\left(2 - \frac{d}{2}\right)\int_0^1 dx \left(p^2 x(1-x) + xm_1 + (1-x)m_2\right)^{d/2-2}
$$
$$
\chi(m_1) \equiv \frac{1}{(4\pi)^{d/2}}\Gamma\left(1 - \frac{d}{2}\right)m_1^{d/2-1}. \tag{C.1}
$$

The first term is the one-loop correction to the Higgs propagator $\langle h(p)h(-p)\rangle$ known from [21], shown in frame $\boxed{1}$ in figure 15, which gives

$$
v\langle h(p)\,\rho(-p)^2\rangle = -\frac{m_h^2}{p^2 + m_h^2}\eta\left(\xi m^2, \xi m^2, p^2\right) \tag{C.2}
$$

The second term, the one-loop correction shown in frame $\boxed{2}$ of figure 15, gives

$$
v\langle h(p)\,\rho(-p)^2\rangle = -\frac{m_h^2}{p^2 + m_h^2}\eta\left(\xi m^2, \xi m^2, p^2\right) \tag{C.3}
$$

The third term, the one-loop correction shown in frame $\boxed{3}$ of figure 15, gives

$$
v\langle h(p)h(-p)^2\rangle = \frac{1}{p^2 + m_h^2}\left\{-3m_h^2\eta\left(m_h^2, m_h^2, p^2\right) - 3\chi\left(m_h^2\right) - \frac{2(d-1)m^2}{m_h^2}\chi\left(m^2\right) - \chi\left(\xi m^2\right)\right\} \tag{C.4}
$$

The fourth term has no one-loop contributions. The fifth term, the one-loop correction shown in frame $\boxed{4}$ of figure 15, gives

$$
\frac{1}{4}\langle h(p)^2\,h(-p)^2\rangle = \frac{1}{2}\eta\left(m_h^2, m_h^2, p^2\right). \tag{C.5}
$$

The sixth term, the one-loop correction shown in frame $\boxed{5}$ of figure 15, gives

$$
\frac{1}{4}\langle \rho(p)^2\,\rho(-p)^2\rangle = \frac{1}{2}\eta\left(\xi m^2, \xi m^2, p^2\right) \tag{C.6}
$$

Using the identity (2.30) we are able to write the whole one-loop correlation function $\langle O(-p)O(p)\rangle$, up to the order $\hbar$, as

$$
\langle O(p)O(-p)\rangle = \frac{v^2}{p^2 + m_h^2} + \frac{1}{(p^2 + m_h^2)^2}\left\{\frac{1}{2}\left[4(d-1)m^4 + 4m^2p^2 + p^4\right]\eta\left(m^2, m^2, p^2\right)\right. \tag{C.7}
$$
$$
\left. + \frac{1}{2}\left(p^2 - 2m_h^2\right)^2\eta\left(m_h^2, m_h^2, p^2\right) - p^2\left[2(d-1)\frac{m^2}{m_h^2} + 1\right]\chi\left(m^2\right) - 3p^2\chi\left(m_h^2\right)\right\}
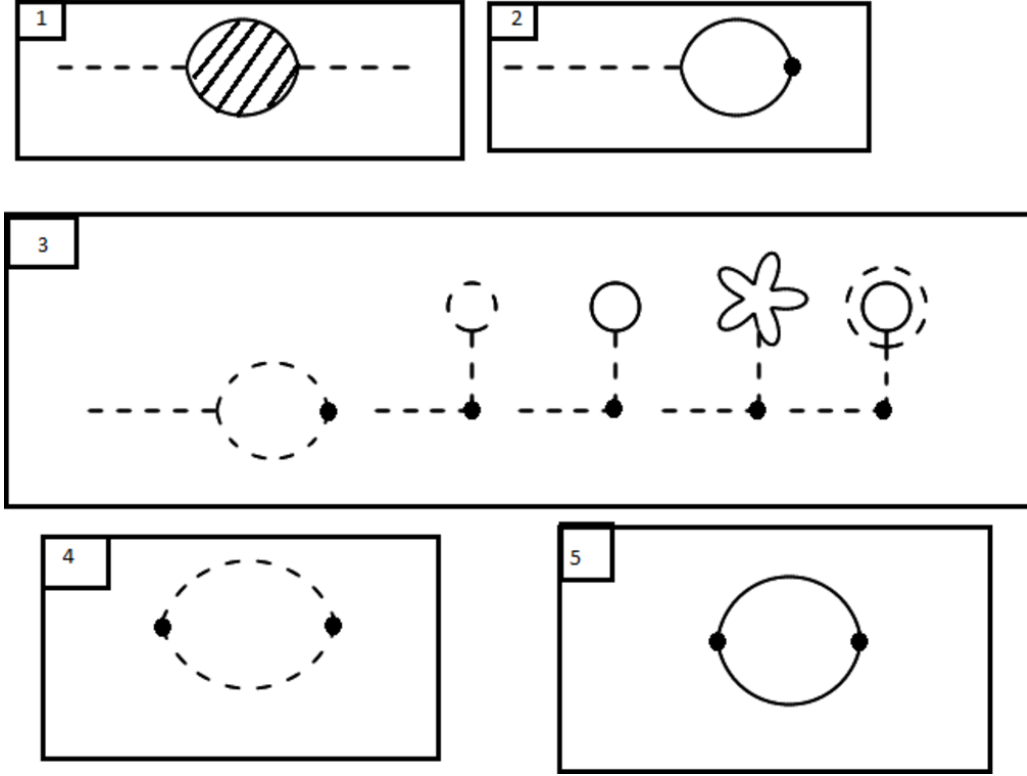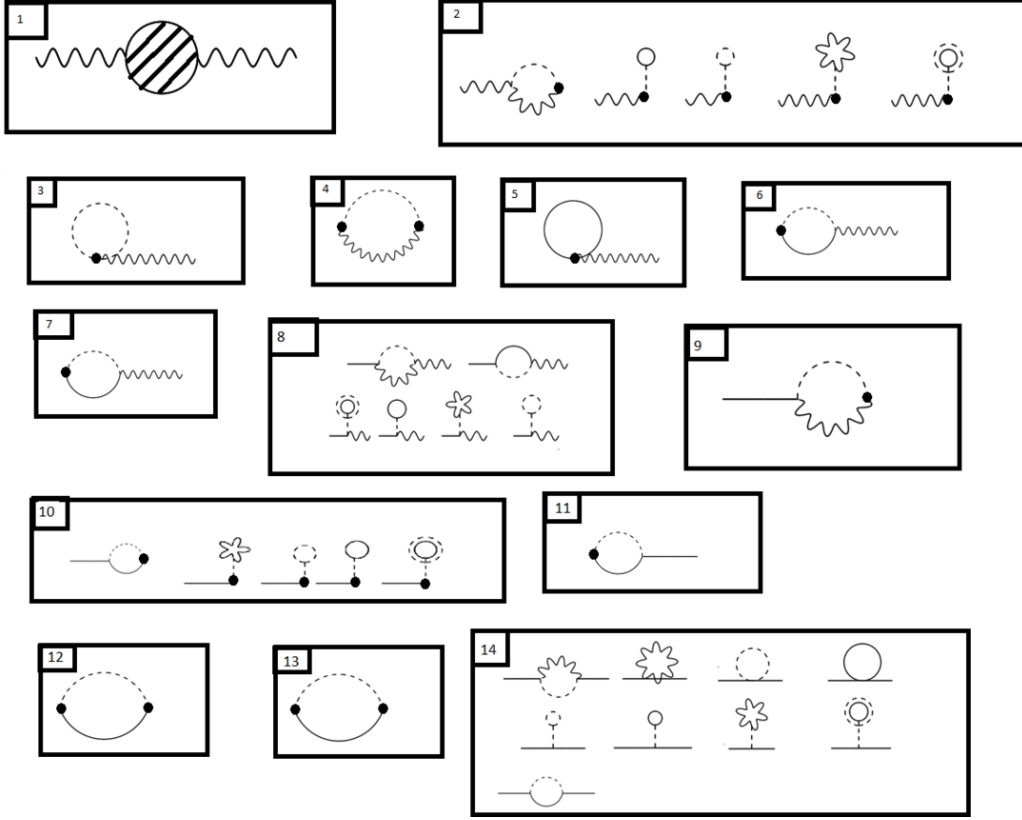$$

Figure 16. One-loop contributions for the propagator $\langle V_\mu(p)V_\nu(-p)\rangle$. Wavy lines represent the photon field, dashed lines the Higgs field, solid lines the Goldstone boson and double lines the ghost field. The $\bullet$ indicates the insertion of a composite operator.

D Contributions to $\langle V_\mu(x)V_\nu(y)\rangle$

We consider each term in the two-point function $\langle V_\mu(p)V_\nu(-p)\rangle$, given by eq. (2.42). The first term is the one-loop correction to the photon propagator $\langle A_\mu(p)A_\nu(-p)\rangle$ known from [21], shown in frame $\boxed{1}$ in figure 16, which gives

$$
\frac{e^2 v^4}{4}\langle A_\mu(p)A_\nu(-p)\rangle_{1-\text{loop}}
$$
$$
= \frac{e^2 v^4}{4}\mathcal{P}_{\mu\nu}(p)\left\{ \frac{1}{(p^2+m^2)^2}\left[-\frac{m^2\left((m_h^2-m^2+p^2)^2-4(d-2)m^2p^2\right)}{(d-1)v^2p^2}\eta\left(m^2,m_h^2,p^2\right)\right.\right.
$$
$$
+\frac{m^2\left(2(d-1)^2m^2p^2+m_h^2(p^2-m^2)+m_h^4\right)}{(d-1)v^2p^2m_h^2}\chi\left(m^2\right)
$$
$$
\left.\left.+\frac{m^2\left((2d-1)p^2-m_h^2+m^2\right)}{(d-1)v^2p^2}\chi\left(m_h^2\right)\right]\right\}
$$

$$+\frac{e^2v^4}{4}\mathcal{L}_{\mu\nu}(p)\left\{\frac{\xi^2}{(p^2+\xi m^2)^2}\left[\frac{m^2\left(-2m_h^2\left(m^2-p^2\right)+m_h^4+\left(m^2+p^2\right)^2\right)}{v^2p^2}\eta\left(m^2,m_h^2,p^2\right)\right.\right.$$

$$-\frac{m^2\left(2m_h^2-2\xi m^2+p^2\right)}{v^2}\eta\left(m_h^2,\xi m^2,p^2\right)$$

$$+\frac{m^2\left(2\left(d-1\right)m^2p^2+m_h^2\left(m^2-p^2\right)-m_h^4\right)}{v^2p^2m_h^2}\chi\left(m^2\right)$$

$$\left.\left.-\frac{m^2\left(-m_h^2+m^2-3p^2\right)}{v^2p^2}\chi\left(m_h^2\right)+2\frac{m^2}{v^2}\chi\left(\xi m^2\right)\right]\right\}\quad\text{(D.1)}$$

The second term, the one-loop correction shown in frame $\boxed{2}$ of figure 16, gives

$$e^2v^3\langle(hA_\mu)(p)A_\nu(-p)\rangle$$

$$=\frac{e^2v^3}{p^2+m^2}\mathcal{P}_{\mu\nu}(p)\left\{-\frac{e\left[2evm_h^4\left(p^2-\xi m^2\right)+evm_h^2\left(p^2+\xi m^2\right)^2+evm_h^6\right]}{2\left(d-1\right)m^2m_h^2p^2}\eta\left(m_h^2,\xi m^2,p^2\right)\right.$$

$$-\frac{e\left[evm_h^2\left(-2\left(3-2d\right)m^2p^2-m^4-p^4\right)+2evm_h^4\left(m^2-p^2\right)-evm_h^6\right]}{2\left(d-1\right)m^2m_h^2p^2}\eta\left(m^2,m_h^2,p^2\right)$$

$$-\frac{e\left[(d-2)evm_h^2p^2+\xi evm^2m_h^2-evm_h^4\right]}{2\left(d-1\right)m^2m_h^2p^2}\chi\left(\xi m^2\right)$$

$$-\frac{e\left[3devm_h^2p^2-\xi evm^2m_h^2+evm^2m_h^2-3evm_h^2p^2\right]}{2\left(d-1\right)m^2m_h^2p^2}\chi\left(m_h^2\right)$$

$$\left.-\frac{e\left[2\left(d-1\right)^2evm^2p^2-evm^2m_h^2+evm_h^2p^2+evm_h^4\right]}{2\left(d-1\right)m^2m_h^2p^2}\chi\left(m^2\right)\right\}$$

$$+\frac{e^2v^3\xi}{p^2+\xi m^2}\frac{1}{2vp^2}\mathcal{L}_{\mu\nu}(p)\left\{\left(m_h^2-\xi m^2+p^2\right)^2\eta\left(m_h^2,\xi m^2,p^2\right)\right.$$

$$-\left[-2m_h^2\left(m^2-p^2\right)+m_h^4+\left(p^2+m^2\right)^2\right]\eta\left(m^2,m_h^2,p^2\right)$$

$$+\frac{\left[-2\left(d-1\right)m^2p^2+m_h^2\left(p^2-m^2\right)+m_h^4\right]}{m_h^2}\chi\left(m^2\right)$$

$$\left.-\left(m_h^2-\xi m^2+2p^2\right)\chi\left(\xi m^2\right)-\left[m^2\left(\xi-1\right)+3p^2\right]\chi\left(m_h^2\right)\right\}\quad\text{(D.2)}$$

The third term, the one-loop correction shown in frame $\boxed{3}$ of figure 16, gives

$$\frac{e^2v^2}{2}\langle\left(h^2A_\mu\right)(p)A_\nu(-p)\rangle=\frac{e^2v^2}{2}\left(\frac{1}{p^2+m^2}\mathcal{P}_{\mu\nu}(p)+\frac{\xi}{p^2+\xi m^2}\mathcal{L}_{\mu\nu}(p)\right)\chi\left(m_h^2\right)\quad\text{(D.3)}$$

The fourth term, the one-loop correction shown in frame $\boxed{4}$ of figure 16, gives

$$
\begin{aligned}
e^2 v^2 \langle (hA_\mu)(p)(hA_\nu)(-p) \rangle = {} & e^2 v^2 \mathcal{P}_{\mu\nu}(p) \left\{ \frac{\left(m_h^2 - \xi m^2 + p^2\right)^2 + 4\xi m^2 p^2}{4 m^2 p^2 (d-1)} \eta\left(m_h^2, \xi m^2, p^2\right) \right. \\
& + \frac{4(d-2) m^2 p^2 - \left(m_h^2 - m^2 + p^2\right)^2}{4(d-1) m^2 p^2} \eta\left(m^2, m_h^2, p^2\right) \\
& - \frac{\left(m_h^2 - \xi m^2 + p^2\right)}{4(d-1) m^2 p^2} \chi\left(\xi m^2\right) + \frac{\left(m_h^2 - m^2 + p^2\right)}{4(d-1) m^2 p^2} \chi\left(m^2\right) \\
& \left. - \frac{(\xi - 1)}{4(d-1) p^2} \chi\left(m_h^2\right) \right\} \\
& + \frac{e^2 v^2}{4 m^2 p^2} \mathcal{L}_{\mu\nu}(p) \left\{ -\left(m_h^2 - \xi m^2 + p^2\right)^2 \eta\left(m_h^2, \xi m^2, p^2\right) \right. \\
& + \left[\left(m_h^2 - m^2 + p^2\right)^2 + 4 m^2 p^2\right] \eta\left(m^2, m_h^2, p^2\right) \\
& + m^2 (\xi - 1) \chi\left(m_h^2\right) + \left(m_h^2 - \xi m^2 + p^2\right) \chi\left(\xi m^2\right) \\
& \left. + \left(-m_h^2 + m^2 - p^2\right) \chi\left(m^2\right) \right\}
\end{aligned}
\tag{D.4}
$$

The fifth term, the one-loop correction shown in frame $\boxed{5}$ of figure 16, gives

$$
\frac{e^2 v^2}{2} \langle (\rho^2 A_\mu)(p) A_\nu(-p) \rangle = \frac{e^2 v^2}{2} \left(\frac{1}{p^2 + m^2} \mathcal{P}_{\mu\nu}(p) + \frac{\xi}{p^2 + \xi m^2} \mathcal{L}_{\mu\nu}(p) \right) \chi\left(\xi m^2\right)
\tag{D.5}
$$

The sixth term, the one-loop correction shown in frame $\boxed{6}$ of figure 16, gives

$$
\begin{aligned}
& -\frac{iev^2}{2} p_\mu \langle (h\rho)(p) A_\nu(-p) \rangle \\
& = -\frac{iev^2}{2} \frac{\xi}{p^2 + \xi m^2} \mathcal{L}_{\mu\nu}(p) \left[ie\left(\xi m^2 - m_h^2\right) \eta\left(m_h^2, \xi m^2, p^2\right) - ie\chi\left(m_h^2\right) + ie\chi\left(\xi m^2\right) \right]
\end{aligned}
\tag{D.6}
$$

The seventh term, the one-loop correction shown in frame $\boxed{7}$ of figure 16, gives

$$
\begin{aligned}
& ev^2 \langle (\rho\partial_\mu h)(p) A_\nu(-p) \rangle \\
& = \frac{ev^2}{2(d-1) p^2} \frac{1}{p^2 + m^2} \mathcal{P}_{\mu\nu}(p) \left\{ -e\left[\left(-m_h^2 + \xi m^2 + p^2\right)^2 + 4 m_h^2 p^2\right] \eta\left(m_h^2, \xi m^2, p^2\right) \right. \\
& \left. + e\left(m_h^2 - \xi m^2 + p^2\right) \chi\left(\xi m^2\right) + e\left(-m_h^2 + \xi m^2 + p^2\right) \chi\left(m_h^2\right) \right\} \\
& + \frac{ev^2}{2 p^2} \frac{\xi}{p^2 + \xi m^2} \mathcal{L}_{\mu\nu}(p) \left\{ e\left(m_h^2 - \xi m^2\right) \chi\left(m_h^2\right) + e\left(-m_h^2 + \xi m^2 + 2 p^2\right) \chi\left(\xi m^2\right) \right. \\
& \left. + e\left[-3 p^2\left(m_h^2 - \xi m^2 + p^2\right) + \left(m_h^2 - \xi m^2 + p^2\right)^2 + 2 p^4\right] \eta\left(\xi m^2, m_h^2, p^2\right) \right\}
\end{aligned}
\tag{D.7}
$$

The eighth term, the one-loop correction shown in frame $\boxed{8}$ of figure 16, gives

$$-\frac{iev^3}{2}p_\mu\langle\rho\left(p\right)A_\nu\left(-p\right)\rangle$$

$$= -\frac{iev^3}{2}\frac{\xi}{(p^2+\xi m^2)^2}\mathcal{L}_{\mu\nu}\left(p\right)\left\{\frac{ie^3v\left[\left(m_h^2-m^2+p^2\right)^2+4m^2p^2\right]}{m^2}\eta\left(m^2,m_h^2,p^2\right)\right.$$

$$+\frac{\left[ie^3vm_h^2\left(m_h^2-\xi m^2\right)-ie^3v\left(p^2+m_h^2\right)\left(m_h^2-\xi m^2+p^2\right)\right]}{m^2}\eta\left(m_h^2,\xi m^2,p^2\right)$$

$$+\left[\frac{i\left(d-1\right)e^3vp^2}{m_h^2}-\frac{ie^3v\left(m_h^2-m^2+p^2\right)}{m^2}\right]\chi\left(m^2\right)$$

$$\left.+\left[\frac{2ie^3vm_h^2+3ie^3vp^2-2ie^3vm^2}{2m^2}\right]\chi\left(m_h^2\right)+\frac{3ie^3vp^2}{2m^2}\chi\left(\xi m^2\right)\right\}\tag{D.8}$$

The ninth term, the one-loop correction shown in frame $\boxed{9}$ of figure 16, gives

$$-iev^3p_\mu\langle\rho(p)(hA_\nu)(-p)\rangle = \frac{1}{2(p^2+\xi m^2)}\mathcal{L}_{\mu\nu}(p)\left\{\left(p^2+m_h^2\right)\left(m_h^2-\xi m^2+p^2\right)\eta\left(\xi m^2,m_h^2,p^2\right)\right.$$

$$-\left[\left(m_h^2-m^2+p^2\right)^2+4m^2p^2\right]\eta\left(m^2,m_h^2,p^2\right)\tag{D.9}$$

$$\left.-\left(p^2+m_h^2\right)\chi\left(\xi m^2\right)+m^2\chi\left(m_h^2\right)+\left(p^2+m_h^2-m^2\right)\chi\left(m^2\right)\right\}$$

The tenth term, the one-loop correction shown in frame $\boxed{10}$ of figure 16, gives

$$-\frac{v}{2}p_\mu p_\nu\langle(h\rho)(p)\rho(-p)\rangle = \frac{vp^2}{2}\frac{1}{p^2+\xi m^2}\mathcal{L}_{\mu\nu}(p)\left[\frac{e^2vm_h^2}{m^2}\eta\left(\xi m^2,m_h^2,p^2\right)\right.\tag{D.10}$$

$$\left.+\frac{(d-1)e^2v}{m_h^2}\chi\left(m^2\right)+\frac{3e^2v}{2m^2}\chi\left(m_h^2\right)+\frac{e^2v}{2m^2}\chi\left(\xi m^2\right)\right]$$

The eleventh term, the one-loop correction shown in frame $\boxed{11}$ of figure 16, gives

$$ivp_\nu\langle(\rho\partial_\mu h)(p)\rho(-p)\rangle = \frac{m_h^2}{2}\frac{1}{p^2+\xi m^2}\mathcal{L}_{\mu\nu}(p)\left[\left(m_h^2-\xi m^2-p^2\right)\eta\left(\xi m^2,m_h^2,p^2\right)\right.$$

$$\left.+\chi\left(m_h^2\right)-\chi\left(\xi m^2\right)\right]\tag{D.11}$$

The twelfth term, the one-loop correction shown in frame $\boxed{12}$ of figure 16, gives

$$\frac{1}{4}p_\mu p_\nu\langle(hp)(p)(hp)(-p)\rangle = \frac{p^2}{4}\eta\left(\xi m^2,m_h^2,p^2\right)\mathcal{L}_{\mu\nu}(p)\tag{D.12}$$

The thirteenth term, the one-loop correction shown in frame $\boxed{13}$ of figure 16, gives

$$-\langle(\rho\partial_\mu h)(p)(h\partial_\nu\rho)(-p)\rangle$$

$$= -\frac{1}{4(d-1)p^2}\mathcal{P}_{\mu\nu}(p)\left\{\left[\left(-m_h^2+\xi m^2+p^2\right)^2+4m_h^2p^2\right]\eta\left(m_h^2,\xi m^2,p^2\right)\right.$$

$$\left.+\left(-m_h^2+\xi m^2-p^2\right)\chi\left(\xi m^2\right)-\left(-m_h^2+\xi m^2+p^2\right)\chi\left(m_h^2\right)\right\}$$

$$-\frac{1}{4p^2}\mathcal{L}_{\mu\nu}(p)\left\{-\left(m_h^2-\xi m^2-p^2\right)\left(m_h^2-\xi m^2+p^2\right)\eta\left(\xi m^2,m_h^2,p^2\right)\right.$$

$$+\left(m_h^2-\xi m^2-p^2\right)\chi\left(\xi m^2\right)$$

$$\left.-\left(m_h^2-\xi m^2+p^2\right)\chi\left(m_h^2\right)\right\}$$

The fourteenth term, the one-loop correction shown in frame $\boxed{14}$ of figure 16, gives

$$
\frac{v^2}{4} p_\mu p_\nu \langle \rho(p) \, \rho(-p) \rangle_{1-\text{loop}} = \frac{1}{(p^2 + \xi m^2)^2} \frac{p^2}{4} \Big\{ \left[m_h^4 - (p^2 + m_h^2)^2 \right] \eta \left(\xi m^2, m_h^2, p^2 \right)
$$
$$
+ \left[(m_h^2 - m^2 + p^2)^2 + 4m^2 p^2 \right] \eta \left(m^2, m_h^2, p^2 \right) + p^2 \chi \left(\xi m^2 \right)
$$
$$
+ (m_h^2 - m^2) \chi (m_h^2) - (m_h^2 - m^2 + p^2) \chi (m^2) \Big\} \mathcal{L}_{\mu\nu}(p)
$$

Open Access. This article is distributed under the terms of the Creative Commons Attribution License (CC-BY 4.0), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

[1] M.E. Peskin and D.V. Schroeder, *An introduction to quantum field theory*, Addison-Wesley, Reading, MA, U.S.A. (1995).

[2] G. 't Hooft et al., *Recent developments in gauge theories. Proceedings, Nato Advanced Study Institute, Cargese, France,* 26 *August–8 September* 1979, *NATO Sci. Ser.* **B 59** (1980) 1 [INSPIRE].

[3] N.K. Nielsen, *On the gauge dependence of spontaneous symmetry breaking in gauge theories*, *Nucl. Phys.* **B 101** (1975) 173 [INSPIRE].

[4] O. Piguet and K. Sibold, *Gauge independence in ordinary Yang-Mills theories*, *Nucl. Phys.* **B 253** (1985) 517 [INSPIRE].

[5] P. Gambino, P.A. Grassi and F. Madricardo, *Fermion mixing renormalization and gauge invariance*, *Phys. Lett.* **B 454** (1999) 98 [hep-ph/9811470] [INSPIRE].

[6] P. Gambino and P.A. Grassi, *The Nielsen identities of the SM and the definition of mass*, *Phys. Rev.* **D 62** (2000) 076002 [hep-ph/9907254] [INSPIRE].

[7] P.A. Grassi, B.A. Kniehl and A. Sirlin, *Width and partial widths of unstable particles*, *Phys. Rev. Lett.* **86** (2001) 389 [hep-th/0005149] [INSPIRE].

[8] M. Asakawa, T. Hatsuda and Y. Nakahara, *Maximum entropy analysis of the spectral functions in lattice QCD*, *Prog. Part. Nucl. Phys.* **46** (2001) 459 [hep-lat/0011040] [INSPIRE].

[9] M. Asakawa and T. Hatsuda, *J/ψ and η_c in the deconfined plasma from lattice QCD*, *Phys. Rev. Lett.* **92** (2004) 012001 [hep-lat/0308034] [INSPIRE].

[10] G. Aarts, C. Allton, M.B. Oktay, M. Peardon and J.-I. Skullerud, *Charmonium at high temperature in two-flavor QCD*, *Phys. Rev.* **D 76** (2007) 094513 [arXiv:0705.2198] [INSPIRE].

[11] H.B. Meyer, *Transport properties of the quark-gluon plasma: a lattice QCD perspective*, *Eur. Phys. J.* **A 47** (2011) 86 [arXiv:1104.3708] [INSPIRE].

[12] A. Rothkopf, *Heavy quarkonium in extreme conditions*, arXiv:1912.02253 [INSPIRE].

[13] I.J.R. Aitchison and C.M. Fraser, *Gauge invariance and the effective potential*, *Annals Phys.* **156** (1984) 1 [INSPIRE].

[14] A. Andreassen, W. Frost and M.D. Schwartz, *Consistent use of effective potentials*, *Phys. Rev.* **D 91** (2015) 016009 [arXiv:1408.0287] [INSPIRE].

[15] A. Maas, *Brout-Englert-Higgs physics: from foundations to phenomenology*, *Prog. Part. Nucl. Phys.* **106** (2019) 132 [`arXiv:1712.04721`] [ɪɴSPIRE].

[16] A. Maas, R. Sondenheimer and P. Törek, *On the observable spectrum of theories with a Brout-Englert-Higgs effect*, *Annals Phys.* **402** (2019) 18 [`arXiv:1709.07477`] [ɪɴSPIRE].

[17] G. 't Hooft, *Why do we need local gauge invariance in theories with vector particles? An introduction*, in *Recent developments in gauge theories*, Springer, Boston, MA, U.S.A. (1980), pg. 101.

[18] G. 't Hooft, A. Jaffe, G. Mack, P. Mitter and R. Stora, *Nonperturbative quantum field theory*, Springer Science & Business Media **185**, Springer, Boston, MA, U.S.A. (2012).

[19] J. Fröhlich, G. Morchio and F. Strocchi, *Higgs phenomenon without a symmetry breaking order parameter*, *Phys. Lett.* **B 97** (1980) 249 [ɪɴSPIRE].

[20] J. Fröhlich, G. Morchio and F. Strocchi, *Higgs phenomenon without symmetry breaking order parameter*, *Nucl. Phys.* **B 190** (1981) 553 [ɪɴSPIRE].

[21] D. Dudal et al., *Some remarks on the spectral functions of the Abelian Higgs model*, *Phys. Rev.* **D 100** (2019) 065009 [`arXiv:1905.10422`] [ɪɴSPIRE].

[22] P. Törek and A. Maas, *Testing gauge-invariant perturbation theory*, `PoS(LATTICE2016)203` (2016) [`arXiv:1610.04188`] [ɪɴSPIRE].

[23] P.W. Higgs, *Broken symmetries and the masses of gauge bosons*, *Phys. Rev. Lett.* **13** (1964) 508 [ɪɴSPIRE].

[24] P.W. Higgs, *Broken symmetries, massless particles and gauge fields*, *Phys. Lett.* **12** (1964) 132 [ɪɴSPIRE].

[25] F. Englert and R. Brout, *Broken symmetry and the mass of gauge vector mesons*, *Phys. Rev. Lett.* **13** (1964) 321 [ɪɴSPIRE].

[26] G.S. Guralnik, C.R. Hagen and T.W.B. Kibble, *Global conservation laws and massless particles*, *Phys. Rev. Lett.* **13** (1964) 585 [ɪɴSPIRE].

[27] O. Piguet and S.P. Sorella, *Algebraic renormalization: perturbative renormalization, symmetries and anomalies*, *Lect. Notes Phys. Monogr.* **28** (1995) 1.

[28] C. Becchi, A. Rouet and R. Stora, *Renormalization of the Abelian Higgs-Kibble model*, *Commun. Math. Phys.* **42** (1975) 127 [ɪɴSPIRE].

[29] C. Becchi, A. Rouet and R. Stora, *The Abelian Higgs-Kibble model. Unitarity of the S operator*, *Phys. Lett.* **B 52** (1974) 344 [ɪɴSPIRE].

[30] T. Kugo and I. Ojima, *Local covariant operator formalism of non-Abelian gauge theories and quark confinement problem*, *Prog. Theor. Phys. Suppl.* **66** (1979) 1 [ɪɴSPIRE].

[31] E. Kraus and K. Sibold, *Rigid invariance as derived from BRS invariance: the Abelian Higgs model*, *Z. Phys.* **C 68** (1995) 331 [`hep-th/9503140`] [ɪɴSPIRE].

[32] R. Haussling and E. Kraus, *Gauge parameter dependence and gauge invariance in the Abelian Higgs model*, *Z. Phys.* **C 75** (1997) 739 [`hep-th/9608160`] [ɪɴSPIRE].

[33] H. Kluberg-Stern and J.B. Zuber, *Ward identities and some clues to the renormalization of gauge invariant operators*, *Phys. Rev.* **D 12** (1975) 467 [ɪɴSPIRE].

[34] C. Itzykson and J.-B. Zuber, *Quantum field theory*, Courier Corporation, U.S.A. (1980).

[35] S.D. Joglekar and B.W. Lee, *General theory of renormalization of gauge invariant operators*, *Annals Phys.* **97** (1976) 160 [ɪɴSPIRE].

[36] D. Dudal, S.P. Sorella, N. Vandersickel and H. Verschelde, *A purely algebraic construction of a gauge and renormalization group invariant scalar glueball operator*, *Eur. Phys. J.* **C 64** (2009) 147 [arXiv:0812.2401] [ɪɴSPIRE].

[37] P. Colangelo and A. Khodjamirian, *QCD sum rules, a modern perspective*, in *At the frontier of particle physics: handbook of QCD*, World Scientific, Singapore (2001), pg. 1495 [hep-ph/0010175] [ɪɴSPIRE].

[38] R. Sondenheimer, *Analytical relations for the bound state spectrum of gauge theories with a Brout-Englert-Higgs mechanism*, arXiv:1912.08680 [ɪɴSPIRE].

[39] K.-I. Kondo, *Gauge-independent Brout-Englert-Higgs mechanism and Yang-Mills theory with a gauge-invariant gluon mass term*, *Eur. Phys. J.* **C 78** (2018) 577 [arXiv:1804.03279] [ɪɴSPIRE].

[40] N. Vandersickel and D. Zwanziger, *The Gribov problem and QCD dynamics*, *Phys. Rept.* **520** (2012) 175 [arXiv:1202.1491] [ɪɴSPIRE].

[41] D. Dudal, J.A. Gracey, S.P. Sorella, N. Vandersickel and H. Verschelde, *A refinement of the Gribov-Zwanziger approach in the Landau gauge: infrared propagators in harmony with the lattice results*, *Phys. Rev.* **D 78** (2008) 065047 [arXiv:0806.4348] [ɪɴSPIRE].

[42] M.A.L. Capri et al., *Exact nilpotent nonperturbative BRST symmetry for the Gribov-Zwanziger action in the linear covariant gauge*, *Phys. Rev.* **D 92** (2015) 045039 [arXiv:1506.06995] [ɪɴSPIRE].

[43] M.A.L. Capri et al., *The universal character of Zwanziger's horizon function in Euclidean Yang-Mills theories*, *Phys. Lett.* **B 781** (2018) 48 [arXiv:1802.04582] [ɪɴSPIRE].

[44] V.N. Gribov, *Quantization of non-Abelian gauge theories*, *Nucl. Phys.* **B 139** (1978) 1 [ɪɴSPIRE].

[45] M.A.L. Capri et al., *Semiclassical analysis of the phases of 4d* SU(2) *Higgs gauge systems with cutoff at the Gribov horizon*, *Phys. Rev.* **D 88** (2013) 085022 [arXiv:1212.1003] [ɪɴSPIRE].

[46] E.H. Fradkin and S.H. Shenker, *Phase diagrams of lattice gauge theories with Higgs fields*, *Phys. Rev.* **D 19** (1979) 3682 [ɪɴSPIRE].

[47] W. Caudy and J. Greensite, *On the ambiguity of spontaneously broken gauge symmetry*, *Phys. Rev.* **D 78** (2008) 025018 [arXiv:0712.0999] [ɪɴSPIRE].

[48] F. Jegerlehner, M. Yu. Kalmykov and O. Veretin, *MS versus pole masses of gauge bosons: electroweak bosonic two loop corrections*, *Nucl. Phys.* **B 641** (2002) 285 [hep-ph/0105304] [ɪɴSPIRE].

[49] F. Jegerlehner, M. Yu. Kalmykov and O. Veretin, *MS-bar versus pole masses of gauge bosons. 2. Two loop electroweak fermion corrections*, *Nucl. Phys.* **B 658** (2003) 49 [hep-ph/0212319] [ɪɴSPIRE].

[50] S.P. Martin, *Pole mass of the W boson at two-loop order in the pure $\bar{MS}$ scheme*, *Phys. Rev.* **D 91** (2015) 114003 [arXiv:1503.03782] [ɪɴSPIRE].

[51] S.P. Martin, *Z-boson pole mass at two-loop order in the pure $\overline{MS}$ scheme*, *Phys. Rev.* **D 92** (2015) 014026 [arXiv:1505.04833] [ɪɴSPIRE].