A Refined Technique for the Automated Determination of Friction Losses in the Toothing of Multithreaded Transmissions with Differential Mechanisms and Planetary Gears

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ABSTRACT. Multithreaded transmissions, which using differential mechanisms for separating and summing up power flows, have been widely used in the transmissions of transport vehicles and special drives. The quality of such transmissions and the magnitude of frictional losses in the toothing significantly depend on the adopted kinematic scheme and the ratio of powers passing through the corresponding links of the differential mechanisms. With the existing automated analysis and synthesis of such transmissions, a matrix approach has recently been widely used. The proposed work provides a refined technique for the automated determination of friction losses in the toothing of multithreaded transmissions which containing differential mechanisms and planetary gears. It makes it possible to more accurately determine friction losses in the toothing and at the same time preserves the linear structure of the system of equations, which permit to continue using the matrix approach to analyze and synthesize such transmissions. Examples are given for the formation of equations systems describing the force interaction between the elements of the scheme for all variants of the links commutation of planetary rows operating as three-link differential mechanisms and planetary gears with one stopped link. The work is based on the analysis of existing methods for determining friction losses in the toothing and personal practical experience in the design and study of multithreaded stepped and stepless transmissions. The received results allow to apply the offered technique in the modern program complexes focused on the automated analysis and synthesis of multithread transmissions with use of the matrix approach. This makes it possible to significantly reduce the time to develop new multithreaded transmissions and increase their technical characteristics.

Introduction. In modern transmissions of vehicles and other drives of machines, planetary gears are widely used as differential mechanisms. It is, first of all, the summation or separation of power flows in dual-threaded and multithreaded transmissions, as well as their use in planetary gearboxes.

A feature of differential transmissions is the essential dependence of the magnitude of the losses in the mechanism on the kinematic transmission scheme and the ratio of the powers passing through the corresponding links of the differential. Therefore, the study and refinement of the technique for determining losses on differential mechanisms in multithreaded transmissions is an actual task.

Analysis of recent achievements and publications.

In the modern technical literature, as a differential transmission, it is customary to consider an automotive inter-wheel differential:

– a differential is a gear train with three shafts that has the property that the angular velocity of one shaft is the average of the angular velocities of the others, or a fixed multiple of that average [Wikipedia] (access mode: https://en.wikipedia.org/wiki/Differential_(mechanical_device));

– a differential gear is a device that is fitted to the axle of a vehicle that allows the wheels to turn at different rates when going around a corner.

However, we will talk about the use of classical 2*k-h* planetary gears as differential mechanisms for the summation and separation of power flows in multithread transmissions.

The issue of determining the efficiency of planetary gears and differential mechanisms has always been of interest to engineers and scientists who create vehicle transmissions and technological drives. The foundations of the theory of power losses in these gears were laid in the writings of such scientists as R.H. MacMillan, J.H. Glover, P.W. Jensen [1–4]. In the USSR this problem were solving with by A.D. Vashets, K.I. Zablonsky, A.N. Ivanov, Yu.N. Kirdyashev, V.I. Krasnenkov, M.A. Kreines, M.K. Kristi, V.N. Kudryavtsev and others [5], [6], [7]. With the introduction of the matrix method in the analysis of planetary mechanisms and transmissions in general [8], [9], a new era in the automation of this process began. There was an opportunity to completely formalize and automate the process of determining the efficiency of planetary and differential transmissions. However, the proposed variants are either not very convenient for automated matrix analysis [10], [11], [12], [13], or are convenient, but they give a large error in special zones [14].

In modern publications on planetary gears, authors, as a rule, solve specific applied problems: consider the particular constructions of conventional and stepless transmissions consider the influence of lubricants and tooth geometry on the efficiency of planetary gears, and study the issues of increasing their load capacity [15–21].

The aim and problem statement. Therefore, it was decided to return to the problem of creating a universal algorithm that allows to correctly take into account the mechanical losses in arbitrary kinematic schemes of transmissions containing planetary transmissions and differential mechanisms. To ensure the possibility of using the matrix approach in analyzing and synthesizing planetary gears, this algorithm must be completed by compiling a basic system of equations, which contain all the necessary information for determining the losses in each of the branches of multithreaded gears with differential mechanisms.

Materials research. The classical matrix approach to the analysis of planetary transmissions assumes the decomposition of the kinematic scheme into elementary structural elements (planetary rows, stop brakes, locking clutches, etc.) and recording of a set of characteristic equations for each of them. To the resulting system of linear equations, the constraint equations that describe the connection scheme of the elements, and the equations of inclusion of the selected gear are added.

So, for example, for the classical 2*k-h* planetary row (differential mechanism), the kinematics description uses the Willys equation:

$$\omega_a - k\omega_b + \omega_h(k-1) = 0,$$

where ω_a – angular velocity of the sun gear;

ω_b – angular velocity of the epicyclic gear;

ω_h – angular velocity of the carrier (lever or arm);

k – internal (basic) gear ratio of planetary row, corresponding to the gear ratio from the sun gear to the epicyclic gear when the carrier is stopped, taking into account the direction of rotation.

To describe the force interaction without taking losses into account, use equations

$$\begin{cases} M_a^* + M_b^* + M_h^* = 0; \\ kM_a^* + M_b^* = 0, \end{cases}$$

where M_a^* – the torque input or output to the sun gear without taking losses into account;

M_b^* – the torque input or output to the epicyclic gear without taking losses into account;

M_h^* – the torque input or output to the carrier (lever or arm) without taking losses into account.

When determining the efficiency of a planetary transmission composed of planetary and differential mechanisms, it is necessary to carry out an analysis of the kinematics and the force analysis of the circuit without taking losses into account. As a result, at each gear on all links, we will have the values of angular velocities and torques, which will allow us to analyze the directions of power flows and determine the relative magnitudes of the powers transferred in relative and figurative motion.

Just as in the book [6], we divide and consider separately the modes of operation of the planetary row in the role of planetary or differential transmission. Planetary transmission is obtained by stopping one of the central links of the differential mechanism (planetary row).

Consider a planetary row as a separate element of the general structural scheme of a planetary or combined transmission. In this case, it is possible to distinguish three combinations of elements commutation for a planetary row operating in the planetary transmission mode (fig. 1), end six – for a planetary row operating in differential transmission mode (fig. 2).

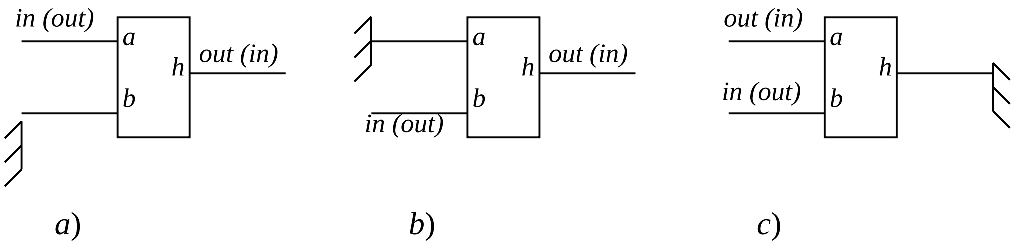

Fig. 1. Options of elements commutation for a planetary transmission: a) – the epicyclic gear is stopped; b) – the sun gear is stopped; c) – the carrier (lever or arm) is stopped

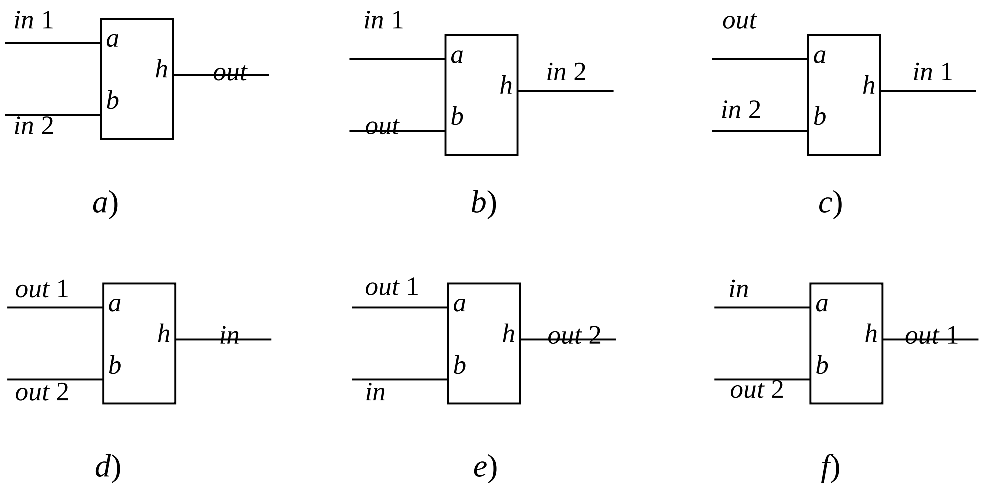

Fig. 2. Options of elements commutation for a differential transmission: with one output in the form: a) the carrier (lever or arm); b) epicyclic gear; c) sun gear; d)with one input in the form: d) the carrier (lever or arm); e) epicyclic gear; f) sun gear.

For example, for the scheme in Fig. 1(*a*) with the internal gear ratio k = –3 after the kinematic and preliminary force analysis in relative units, we will have the results given in Table. 1.

Table 1. The power characteristics of planetary transmission in Fig. 1 without losses.

	Input	Element *a*	Element *b*	Element *h*	Element *c* (satellite)	Output
ω	1	1	0	0,25	0,75	0,25
M^*	1	-1	-3	4	0	-4
N	1	-1	0	1	0	-1

In Table. 1 N is the power on the corresponding element. It should be noted that the power entering the element is negative, and the output from the element is positive.

According to the procedure described in [14], the torqueses on the elements, taking into account transmission losses, are determined as follows:

$$\begin{cases} M_a\left(1+sign\left(\omega_a M_a^*\right)\varphi\left(\psi_{a-c}^h+0{,}5\sum_1^{n-1}\psi_{c-d}^h\right)\right)+M_b\left(1+sign\left(\omega_b M_b^*\right)\varphi\left(\psi_{b-c}^h+0{,}5\sum_1^{n-1}\psi_{c-d}^h\right)\right)+M_h=0; \\ kM_a\left(1+sign\left(\omega_a M_a^*\right)\varphi\left(\psi_{a-c}^h+0{,}5\sum_1^{n-1}\psi_{c-d}^h\right)\right)+M_b\left(1+sign\left(\omega_b M_b^*\right)\varphi\left(\psi_{b-c}^h+0{,}5\sum_1^{n-1}\psi_{c-d}^h\right)\right)=0, \end{cases} \quad (1)$$

where M_a, M_b, M_h – torque on the links in the light of losses;

ω_c – relative angular velocity on the bearings of satellites;

ψ_{a-c}^h – coefficient of friction losses in the gearing "sun gear – satellite" with the carrier (lever or arm) stopped (for external gearing $\psi_{a-c}^h=0{,}02$);

ψ_{b-c}^h – coefficient of friction losses in the gearing "epicyclic gear – satellite" with the carrier (lever or arm) stopped (for internal gearing $\psi_{b-c}^h=0{,}01$);

ψ_{c-d}^h – coefficient of friction losses in the gearing "satellite 1 – satellite 2" with the carrier (lever or arm) stopped if the planetary row contains several successively meshed satellites (for external gearing $\psi_{a-c}^h=0{,}02$, in the absence of successively meshed satellites $\psi_{c-d}^h=0$);

φ – the fraction of power transmitted in relative rotation through the gearing, which is calculated from formula $\varphi=\dfrac{|\omega_c|}{|\omega_c|+|\omega_h|}$.

In this example $\varphi=\dfrac{|\omega_c|}{|\omega_c|+|\omega_h|}=\dfrac{0{,}75}{0{,}75+0{,}25}=0{,}75$, internal gear ratio k = –3. In accordance with Table. 1, assuming that rotation with angular velocity (+1) and torque (–1) is applied to the sun gear, system (1) can be written in the form:

$$\begin{cases} (-1)(1-0{,}75(0{,}02+0)) + M_b(1+0{,}75(0{,}01+0)) + M_h = 0; \\ (-3)(-1)(1-0{,}75(0{,}02+0)) + M_b(1+0{,}75(0{,}01+0)) = 0, \end{cases}$$

$$\begin{cases} -0{,}985 + 1{,}0075 M_b + M_h = 0; \\ (-3)(-0{,}985) + 1{,}0075 M_b = 0, \end{cases}$$

wherefrom $M_b = -2{,}933$ and $M_h = 0{,}985 + 1{,}0075 \cdot 2{,}933 = 3{,}94$.

Accordingly, the efficiency of the scheme will be $\eta = \frac{\hat{i}}{i} = \frac{-M_h / M_a}{\omega_a / \omega_h} = \frac{-3{,}94/(-1)}{1/0{,}25} = 0{,}985$. Here $\hat{i}$ – power ratio, i – kinematic ratio.

It should be noted that the technique [14] prescribes, with a power on the link equal to zero due to the zero speed of rotation, to take the power sign as positive.

The first drawback of this method is that in the presence of one stopped central gear and the passage of power by a single flow from the other central gear to the carrier (or vice versa), the overall efficiency does not take into account the share of losses at the stopped element. So, in fact, the efficiency obtained from the passage of 75% of power through the gearing of "the sun gear - the satellite", which is equal to 0,985, has become the overall transmission efficiency. This fact is confirmed by calculation using a method [6] repeatedly tested experimentally, which gives a smaller value of the efficiency:

$$\eta = 1 - \frac{|k|\left(\psi_{a-c}^{h} + \psi_{b-c}^{h}\right)}{|k|+1} = 1 - \frac{3 \cdot (0{,}02 + 0{,}01)}{4} = 0{,}9775.$$

The magnitude of the error that we get when using the technique [14] for the analysis of losses in planetary gears is shown in Fig. 3 and Fig. 4.

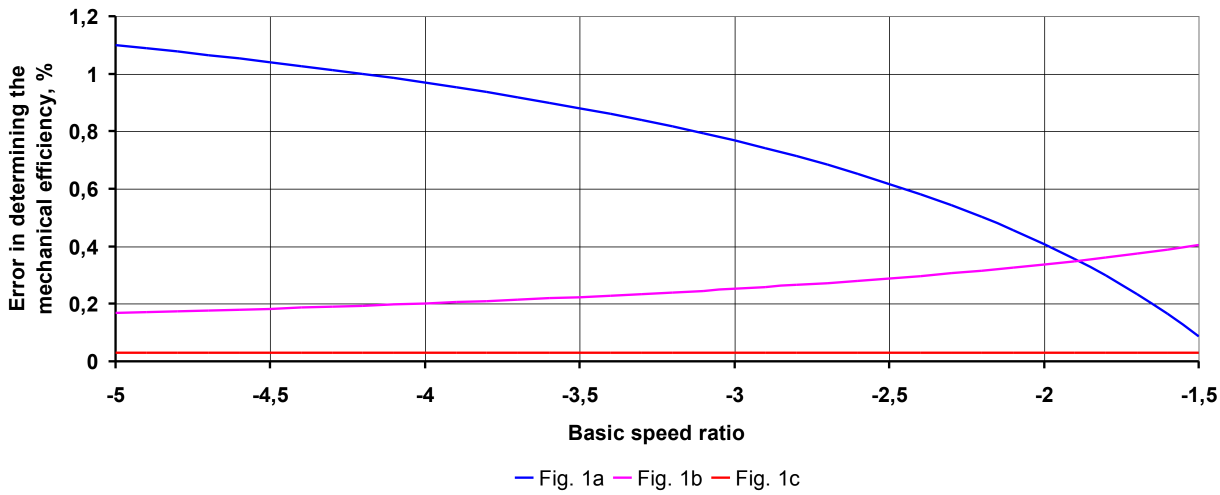

Fig. 3. Dependence of the magnitude of the error in determining the mechanical efficiency of the planetary row by the technique [14].

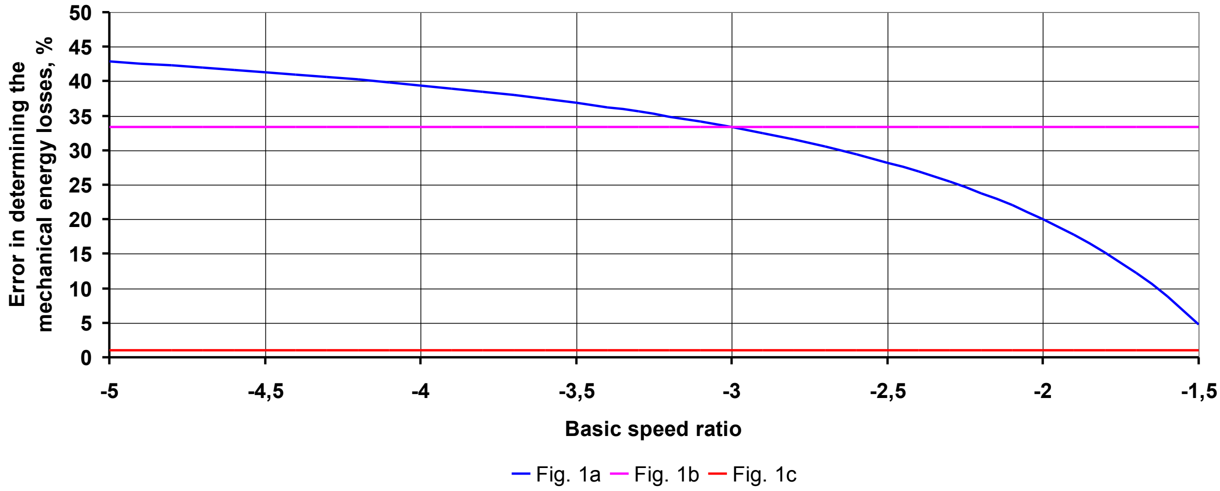

Fig. 4. Dependence of the magnitude of the error in determining the mechanical losses of the planetary row by the technique [14].

A similar picture arises when the planetary row works as a differential mechanism, when one of the three central elements is the mover, and the other two are driven. The power on the differential mechanism is divided into two flows, with one part of the power passing both gearing, and the second part – only one. The technique given in [14] works completely adequately only for the case of summation of two power fluxes, if both of them enter the differential mechanism through the sun gear and epicyclic gear, and not the carrier (lever or arm).

To eliminate the described inaccuracies in the methodology [14], while retaining the possibility of using it in the matrix approach to the analysis and synthesis of planetary gears, the following is proposed.

For the variant in Fig. 1(*a*), the system of equations describing the load balance can be written in the form:

$$\begin{cases} M_a \eta_\Sigma^{-sign\left(\omega_a M_a^*\right)} + \dfrac{M_b}{\eta_\Sigma} + M_h = 0; \\ kM_a \eta_\Sigma^{-sign\left(\omega_a M_a^*\right)} + \dfrac{M_b}{\eta_\Sigma} = 0, \end{cases} \qquad (2)$$

where η_Σ – the total efficiency at the passage of power by a single flow in accordance with [6]:

$$\eta_\Sigma = 1 - \frac{k}{1+k}\left(\psi_{a-c}^h + \psi_{b-c}^h\right).$$

For the variant in Fig. 1 (*b*) the system of equations describing the load balance can be written in the form:

$$\begin{cases} \dfrac{M_a}{\eta_\Sigma} + M_b \eta_\Sigma^{-sign\left(\omega_b M_b^*\right)} + M_h = 0; \\ \dfrac{kM_a}{\eta_\Sigma} + M_b \eta_\Sigma^{-sign\left(\omega_b M_b^*\right)} = 0, \end{cases} \qquad (3)$$

where η_Σ – the total efficiency at the passage of power by a single flow in accordance with [6]:

$$\eta_\Sigma = 1 - \frac{1}{1+k}\left(\psi_{a-c}^h + \psi_{b-c}^h\right).$$

For the variant in Fig. 1(*c*), the system of equations describing the load balance can be written in the form:

$$\begin{cases} M_a \eta_a^{-sign\left(\omega_a M_a^*\right)} + M_b \eta_b^{-sign\left(\omega_b M_b^*\right)} + M_h = 0; \\ kM_a \eta_a^{-sign\left(\omega_a M_a^*\right)} + M_b \eta_b^{-sign\left(\omega_b M_b^*\right)} = 0, \end{cases} \qquad (4)$$

where η_a – efficiency in gear “sun gear – satellite” $\eta_a = 1 - \psi_{a-c}^h$;

η_b – efficiency in gear “satellite – epicyclic gear” $\eta_b = 1 - \psi_{b-c}^h$.

For schemes with a differential mechanism the system of equations that describing the load balance will contain 4 equations. The first two equations of the system represent the balance of torque at the differential mechanism. And two additional equations allow you to distribute losses along the branches of the input (output), observing the overall balance of losses in accordance with [6]. It is assumed that the loss on the external toothing of the “sun gear – satellite” is twice as large as on the internal toothing of the “satellite – epicyclic gear”.

So for the variant in Fig. 2(*a*):

$$\begin{cases} M_a \eta_a + M_b \eta_b + M_h = 0; \\ kM_a \eta_a + M_b \eta_b = 0; \\ M_a^* \omega_a \eta_a + M_b^* \omega_b \eta_b = \left(M_a^* \omega_a + M_b^* \omega_b\right)\eta_\Sigma; \\ \dfrac{1-\eta_a}{1-\eta_b} = 2, \end{cases}$$

where the overall efficiency in accordance with [6] $\eta_\Sigma = 1 - \left|\dfrac{\omega_a - \omega_h}{(k-1)\omega_h}\right|\left(\psi_{a-c}^h + \psi_{b-c}^h\right)$, and the loss values at each input (output) η_a and η_b are determined from the last two equations on the basis of a preliminary calculation without taking losses into account before solving the general system of equations.

With the reverse power flow in the same scheme (Fig. 2(*d*)), the system will look like:

$$\begin{cases} \dfrac{M_a}{\eta_a} + \dfrac{M_b}{\eta_b} + M_h = 0; \\ \dfrac{kM_a}{\eta_a} + \dfrac{M_b}{\eta_b} = 0; \\ \dfrac{M_a^* \omega_a}{\eta_a} + \dfrac{M_b^* \omega_b}{\eta_b} = \dfrac{M_a^* \omega_a + M_b^* \omega_b}{\eta_\Sigma}; \\ \dfrac{1-\eta_a}{1-\eta_b} = 2, \end{cases}$$

which allows recording it for the schemes in Fig. 2(*a*) and Fig. 2(*d*) in the generalized form:

$$\begin{cases} M_a \eta_a^{-sign\left(\omega_a M_a^*\right)} + M_b \eta_b^{-sign\left(\omega_b M_b^*\right)} + M_h = 0; \\ kM_a \eta_a^{-sign\left(\omega_a M_a^*\right)} + M_b \eta_b^{-sign\left(\omega_b M_b^*\right)} = 0; \\ M_a^* \omega_a \eta_a^{-sign\left(\omega_a M_a^*\right)} + M_b^* \omega_b \eta_b^{-sign\left(\omega_b M_b^*\right)} = \left(M_a^* \omega_a + M_b^* \omega_b\right) \eta_\Sigma^{sign\left(\omega_h M_h^*\right)}; \\ \dfrac{1-\eta_a}{1-\eta_b} = 2. \end{cases} \quad (5)$$

Here it should be kept in mind that if the scheme really corresponds to Fig. 2(*a*) or Fig. 2(*d*), then for simplicity it is possible to equate

$$sign(\omega_a M_a^*) = sign(\omega_b M_b^*) = -sign(\omega_h M_h^*) . \quad (6)$$

Similarly for the variant in Fig. 2(*b*):

$$\begin{cases} M_a \eta_a + \dfrac{M_b}{\eta_b} + M_h = 0; \\ kM_a \eta_a + \dfrac{M_b}{\eta_b} = 0; \\ \left(M_a^* \omega_a \eta_a + M_h^* \omega_h\right) \eta_b = \left(M_a^* \omega_a + M_h^* \omega_h\right) \eta_\Sigma; \\ \dfrac{1-\eta_a}{1-\eta_h} = 2, \end{cases}$$

where the overall efficiency in accordance with [6] $\eta_\Sigma = 1 - \left|\dfrac{\omega_b - \omega_h}{\omega_b}\right| \left(\psi_{a-c}^h + \psi_{b-c}^h\right)$, and the loss values at each input (output) η_a and η_b are determined from the last two equations on the basis of a preliminary calculation without taking losses into account before solving the general system of equations.

With the reverse power flow in the same scheme (Fig. 2(*e*)), the system will look like

$$\begin{cases} \dfrac{M_a}{\eta_a} + M_b\eta_b + M_h = 0; \\ \dfrac{kM_a}{\eta_a} + M_b\eta_b = 0; \\ \left(\dfrac{M_a^*\omega_a}{\eta_a} + M_h^*\omega_h\right)\dfrac{1}{\eta_b} = \dfrac{M_a^*\omega_a + M_h^*\omega_h}{\eta_\Sigma}; \\ \dfrac{1-\eta_a}{1-\eta_h} = 2, \end{cases}$$

which allows recording it for the schemes in Fig. 2(*b*) and Fig. 2(*e*) in the generalized form:

$$\begin{cases} M_a\eta_a^{-sign\left(\omega_a M_a^*\right)} + M_b\eta_b^{-sign\left(\omega_b M_b^*\right)} + M_h = 0; \\ kM_a\eta_a^{-sign\left(\omega_a M_a^*\right)} + M_b\eta_b^{-sign\left(\omega_b M_b^*\right)} = 0; \\ \left(M_a^*\omega_a\eta_a^{-sign\left(\omega_a M_a^*\right)} + M_h^*\omega_h\right)\eta_b^{sign\left(\omega_b M_b^*\right)} = \left(M_a^*\omega_a + M_h^*\omega_h\right)\eta_\Sigma^{-sign\left(\omega_a M_a^*\right)}; \\ \dfrac{1-\eta_a}{1-\eta_h} = 2. \end{cases} \tag{7}$$

Similarly for the variant in Fig. 2(*c*)

$$\begin{cases} \dfrac{M_a}{\eta_a} + M_b\eta_b + M_h = 0; \\ \dfrac{kM_a}{\eta_a} + M_b\eta_b = 0; \\ \left(M_b^*\omega_b\eta_b + M_h^*\omega_h\right)\eta_a = \left(M_b^*\omega_b + M_h^*\omega_h\right)\eta_\Sigma; \\ \dfrac{1-\eta_a}{1-\eta_h} = 2, \end{cases}$$

where the overall efficiency in accordance with [6] $\eta_\Sigma = 1 - \left|\dfrac{\omega_a - \omega_h}{\omega_a}\right|\left(\psi_{a-c}^h + \psi_{b-c}^h\right)$, and the loss values at each input (output) η_a and η_b are determined from the last two equations on the basis of a preliminary calculation without taking losses into account before solving the general system of equations.

With the reverse power flow in the same scheme (Fig. 2(*f*)), the system will look like

$$\begin{cases} M_a\eta_a + \dfrac{M_b}{\eta_b} + M_h = 0; \\ kM_a\eta_a + \dfrac{M_b}{\eta_b} = 0; \\ \left(\dfrac{M_b^*\omega_b}{\eta_b} + M_h^*\omega_h\right)\dfrac{1}{\eta_a} = \dfrac{M_b^*\omega_b + M_h^*\omega_h}{\eta_\Sigma}; \\ \dfrac{1-\eta_a}{1-\eta_h} = 2, \end{cases}$$

which allows recording it for the schemes in Fig. 2(*c*) and Fig. 2(*f*) in the generalized form:

$$\begin{cases} M_a\eta_a^{-sign\left(\omega_a M_a^*\right)} + M_b\eta_b^{-sign\left(\omega_b M_b^*\right)} + M_h = 0; \\ kM_a\eta_a^{-sign\left(\omega_a M_a^*\right)} + M_b\eta_b^{-sign\left(\omega_b M_b^*\right)} = 0; \\ \left(M_b^*\omega_b\eta_b^{-sign\left(\omega_b M_b^*\right)} + M_h^*\omega_h\right)\eta_a^{sign\left(\omega_a M_a^*\right)} = \left(M_b^*\omega_b + M_h^*\omega_h\right)\eta_\Sigma^{sign\left(\omega_a M_a^*\right)}; \\ \dfrac{1-\eta_a}{1-\eta_h} = 2. \end{cases} \tag{8}$$

Summary. 1. Kinematics analysis, load distributions and loss determination in modern multithreaded transmissions containing differential mechanisms and planetary rows are performed sequentially on each step of gear or operating mode. At the same time, the same planetary rows on some transmissions can play the role of differential mechanisms, and on others – simple planetary gears with one stopped link. The classical algorithm for analysis of kinematics and distribution of loads without loss is universal and does not require the use of different formulas for one or the other case. When performing an accurate loss analysis in such schemes, the algorithms for analyzing simple planetary gears with one stopped link and differential mechanisms differ significantly from each other.

2. When analyzing losses in multithreaded transmissions, first, using the matrix analysis, calculate the kinematics and load distribution without taking into account the losses to determine the direction of the power flows and the preliminary values of the torque on the links along all branches of the circuit.

3. Further, for all planetary rows with one stopped link, equations (2), (3) or (4) are written into the system of equations, depending on the type of the stopped element.

4. For all differential mechanisms, the last two equations from systems (5), (7) or (8), depending on the direction of the power flows, are solved before composing a general linear system of equations. After this, the first two equations from the same systems are written in the general system. Such a sequential solution of the equations allows solving nonlinear equations for given differential mechanisms at the stage of preparation of a large general linear system of equations that describes the entire scheme. This makes it possible to solve a large linear system by a matrix method and to save the computation time. This is especially important in carrying out the structural-parametric synthesis of multithreaded transmissions, which is performed by multiple analysis of the generated circuits and sets of their parameters.

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