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«Investigation of the rotational motion of a solid body and determination of the velocity of the bullet with the help of a torsion ballistic pendulum»

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METHODOLOGICAL INSTRUCTIONS FOR LABORATORY WORK NO.7(1)

≪Investigation of the rotational motion of a solid body and determination of the velocity of the bullet with the help of a torsion ballistic pendulum≫

Compilers:	Iakunina Natalia Oleksandrivna, Ph.D., Assoc. Prof. Danylevych Oleksandr Gennadiiovych, Ph.D., Assoc. Prof. Yurchenko Irina Oleksandrivna, Ph.D., Assoc. Prof. Rebenchuk Tetyana Lvivna, Sen.Lect. Fedotov Viacheslav Vitalievich, Sen.Lect. Klymuk Olena Serhiivna, Ph.D., Sen.Lect.
Responsible editor	Linchevskyi I.V., D. Phys. Math., Prof.
Reviewers:	Dzhezherya Yu. I., D. Phys. Math., Prof.

The tutorial outlines a methodology for carrying out laboratory work devoted to the study of physical laws that describe the rotational motion of a solid. The proposed method is based on an experimental study of the motion of a solid body with the help of a torsion ballistic pendulum.

The publication gives a brief description of the theoretical data, which are needed to describe the phenomenon that is being studied in laboratory work. The theoretical calculations that are necessary for the interpretation of the corresponding experimental data and confirmation of the known theoretical laws are given. Also the step-by-step instruction for conducting experimental measurements, and corresponding calculations of physical quantities is presented.

For the students of higher educational institutions with technical profile.

Investigation of the rotational motion of a solid body and determination of the velocity of the bullet with the help of a torsion ballistic pendulum

The aim of the work:

to investigate the application of the laws of rotational motion and to determine the velocity of the bullet.

Brief theoretical information and method of measurement.

Direct measurement of the flight velocity of a bullet is too complicated because of its large value ($10^2 - 10^3$m/s, depending on the type of weapon). For this purpose, indirect methods are used, for example, the measurement of the period of oscillations of a torsion ballistic pendulum caused by an inelastic impact of the bullet on its target. The ballistic pendulum has a significant (in comparison with the bullet) mass and, consequently, exhibits greater inertia properties.

The moment of inertia characterizes inertial properties of solids. An absolutely solid body can be considered as a system of material points with unchanged distances between them during the motion. The moment of inertia of the system of material points relative to the Z axis is the quantity I_Z, which is equal to the sum of products of masses m_i of all material points of the system, and the squares of their distances r_i to this axis:

$$I_Z = \sum_{i=1}^{n} m_i r_i^2,$$

or in the case of a solid body

$$I_Z = \int r^2 dm.$$

It is necessary to apply an external force at least to one point of the body to set a solid body with a fixed axis into a rotating motion. The direction of this force should not cross through the axis of rotation and should not be parallel to it. That is calling, to act on the body with torque of a force.

The vector product of the radius vector $\overrightarrow{r}$ drawn from the point O to the point A of application of force $\overrightarrow{F}$, with this force is called the torque of a force $\overrightarrow{F}$ relative to a stationary point O: $\overrightarrow{M} = \left[\overrightarrow{r}\overrightarrow{F}\right]$. The vector $\overrightarrow{M}$ is directed perpendicular to the plane of the vectors $\overrightarrow{r}$ and $\overrightarrow{F}$ (Fig. 7.1). Its absolute value is equal $M = Fr\sin\alpha = Fl$, where α is the angle between $\overrightarrow{r}$ and $\overrightarrow{F}$ and $l = r\sin\alpha$ is called the lever arm (moment arm) of the force $\overrightarrow{F}$, which is equal to the length of the perpendicular, traced from the point O on the line of action of the force. The torque of a force F_Z relative to the axis OZ is the projection on this axis of the moment of force relative to any point selected on this axis.

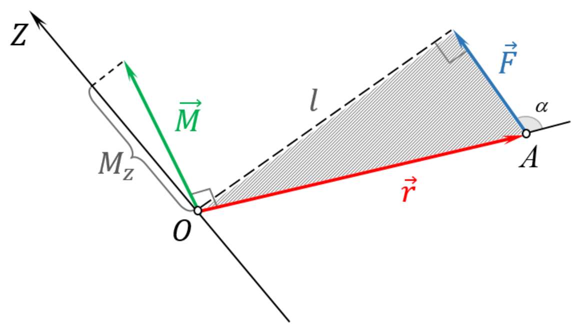

Fig 7.1. *Torque of a force*

The concept of moment of momentum (angular momentum) is introduced for a mechanical system similarly. The angular momentum of a material point relative to a stationary point O is the vector product of the radius vector of the material point , with the momentum of this material point $\overrightarrow{p}_i = m_i\overrightarrow{V}_i$:

$$\overrightarrow{L}_i = [\overrightarrow{r}_i\overrightarrow{p}_i] = \left[\overrightarrow{r}_i m_i \overrightarrow{V}_i\right].$$

The angular momentum of the system of material points relative to the point O is determined as the sum of the vectors of the angular momentum of all material points of the system relative to the same point. The momentum of a mechanical system relative to the OZ axis is a projection L_Z on this axis of the vector of angular momentum of the system relative to any point that is selected on the axis.

The basic law of the dynamics of the rotational motion is fulfilled for the system of points: the velocity of changing of the angular momentum of the system is equal to the total torque M_{Zext} of all external forces acting on the system relative to this axis (equation of moments):

$$\frac{d}{dt}L_Z = \sum M_{Zext}. \tag{7.1}$$

The angular momentum of the system relative to the OZ axis can be defined as the product of the moment of inertia I_Z of the solid body with its angular velocity ω: $L_Z = I_Z\omega$. If the total torque of external forces is zero: $\sum M_{Zext} = 0$ then (7.1) takes the form of the angular momentum conservation law:

$$L_Z = \text{const}$$

or

$$I_Z\omega = \text{const}. \tag{7.2}$$

Let's consider this law in relation to a torsion ballistic pendulum, in which the flying bullet hits horizontally.

The torsion ballistic pendulum is a cross-section, symmetrical about the OZ axis, with two masses m_0 which can move along the horizontal part of the cross-section and they are used to change the moment of inertia of the pendulum (Fig. 7.2.). The fixed load-attachment of mass m_1 is required for balancing the target 2. Cross-section pendulum is attached to supporting holder 4 with the help of elastic wire 3. The pendulum can make oscillations around the vertical axis OZ. After the bullet hits in the target 2, the pendulum deviates to some angle φ, which can be determined by a circular scale 1.

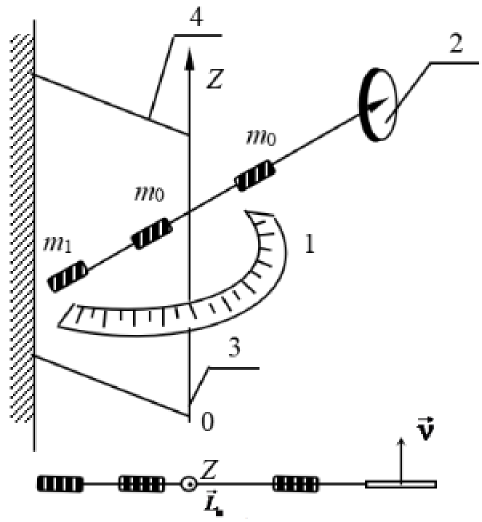

Fig 7.2. *Torsion ballistic pendulum*

A bullet of mass m is flying horizontally, and a torsion ballistic pendulum represent a mechanical system for which $\sum M_{Zext} = 0$, and therefore the law of conservation of angular momentum is fulfilled

$$\vec{L}_B + \vec{L}_P = \vec{L'}_B + \vec{L'}_P, \tag{7.3}$$

$\vec{L}_B$ and $\vec{L}_P$ — are vectors of angular momentum of the bullet and of the pendulum respectively, before the shot; $\vec{L'}_B$ and $\vec{L'}_P$ — are vectors of angular momentum of the bullet and of the pendulum after the shot.

Since the direction of the vectors of angular momentum coincides with the axis of rotation OZ, the sum of their vectors can be replaced by the algebraic sum of their projections:

$$L_{BZ} + L_{PZ} = L'_{BZ} + L'_{PZ} \tag{7.4}$$

where $L_{BZ}, L_{PZ}, L'_{BZ}, L'_{PZ}$ — are the projections of vectors of angular momentum of the bullet and of the pendulum.

Projection of angular momentum of a flying bullet on the axis OZ is:

$$L_{BZ} = mvR, \tag{7.5}$$

where m is the mass of the bullet; v — is the velocity of the bullet; R — is the distance from the point where the bullet hits the target 2 to the pendulum axis.

Since the pendulum does not move until the hit, the projection of the angular momentum of the pendulum on the OZ axis:

$$L_{PZ} = 0. \tag{7.6}$$

After the hit, when the bullet stuck in the target (absolutely inelastic impact), the pendulum with a bullet will start to rotate relative to OZ axis with some initial angular velocity ω_0, therefore

$$L'_{BZ} + L'_{PZ} = I\omega_0, \tag{7.7}$$

where I — is total moment of inertia of the pendulum and stucked bullet.

The moment of inertia of the bullet is small in comparison with the moment of inertia of the pendulum I_P and can be neglected:

$$I \approx I_P. \tag{7.8}$$

Substituting the values of angular momentums of the bullet and the pendulum in equation (7.4), we can find the velocity of the bullet

$$v = \frac{I_P \omega_0}{mR}. \tag{7.9}$$

In this expression values of the initial angular velocity and moment of inertia of the pendulum are unknown. It is difficult to measure them directly. Therefore, they should be expressed through easily measurable quantities. In practice the value of ω_0 is determined by the initial angle of twisting φ_0 and the period of oscillations T.

In the case of small deformations in the twisting wire a braking moment of elastic forces M_Z arises, which, by analogy with Hooke's law, can be considered to be proportional to the angle of rotation of the wire φ:

$$M_Z = -D\varphi, \tag{7.10}$$

where D — is the torsion modulus of wire. After substituting (7.10) into the basic equation of the dynamics of the rotational motion of a solid with a fixed axis of rotation, we can obtain:

$$I_Z \beta_Z = M_Z,$$

$$I_P \frac{d^2\varphi}{dt^2} = -D\varphi,$$

$$\frac{d^2\varphi}{dt^2} + \frac{D\varphi}{I_P} = 0.$$

This equation for small angles of deflection ($\varphi \ll 2\pi$) has a solution $\varphi(t) = \varphi_0 \cos\left(\sqrt{\frac{D}{I_M}} \cdot t + \alpha\right)$ which implies that the pendulum performs a harmonic oscillations with cyclic frequency $\sqrt{\frac{D}{I_M}}$ and angular amplitude φ_0.

The potential energy of the twisted wire, which is equal to $E_P = \frac{D\varphi_0^2}{2}$ becomes kinetic one $E_K = \frac{I_P \omega_0^2}{2}$ during untwisting. In the absence of friction in accordance with the law of conservation of mechanical energy we can write:

$$\frac{D\varphi_0^2}{2} = \frac{I_P \omega_0^2}{2},$$

whence follows

$$\omega_0 = \sqrt{\frac{D}{I_P}} \cdot \varphi_0.$$

The cyclic frequency of oscillations $\sqrt{\frac{D}{I_P}}$ is associated with the period of oscillations T by ratio

$$\sqrt{\frac{D}{I_M}} = \frac{2\pi}{T}. \tag{7.11}$$

So,

$$\omega_0 = \frac{2\pi}{T} \cdot \varphi_0. \tag{7.12}$$

The value of I_P can be calculated basing on the property of the additivity of the moment of inertia, or it can be expressed in the same way as ω_0 through the period of oscillations T. In the second case we can use the relation (7.11):

$$I_P = \frac{DT^2}{4\pi^2}. \tag{7.13}$$

To find the torsion modulus D of the wire, one can proceed from the property of the additivity of the moment of inertia. Let's write down its value for two symmetrical positions of masses m_0 relatively to the axis: when they are shifted together and when they are extended as far as possible. At this case, the distances from the centers of masses to the axis of rotation are equal R_1 and R_2, respectively. Consequently, the moment of inertia of the pendulum consists of moments of inertia I_{m_0} of two masses

m_0, of moments of inertia of the target I_{tar} and of the counterweight which are equal in magnitude, as well as of the moment of inertia of the rod I_{rod}:

$$I_P = 2I_{m_0} + 2I_{tar} + I_{rod}.$$

Taking into account (7.13) and considering masses m_0 as point masses symmetrically located at distances R_1 and R_2 from the axis of rotation, we can write for the first and second experiments, respectively:

$$\frac{DT_1^2}{4\pi^2} = 2m_0R_1^2 + 2I_{tar} + I_{rod},$$

$$\frac{DT_2^2}{4\pi^2} = 2m_0R_2^2 + 2I_{tar} + I_{rod},$$

where T_1 and T_2 — are the periods of oscillations of the pendulum with the shifted and extended masses, respectively. Subtracting the first equation from the second one and performing elementary transformations, we can obtain the expression for the torsion modulus of the wire:

$$D = 8\pi^2 m_0 \frac{R_2^2 - R_1^2}{T_2^2 - T_1^2}. \tag{7.14}$$

Substituting (7.14) into (7.13), we obtain an expression for the moment of inertia of the pendulum through easily measurable quantities:

$$I_P = 2m_0 \frac{R_2^2 - R_1^2}{T_2^2 - T_1^2} T^2. \tag{7.15}$$

The final formula for determining the velocity of a bullet with the account (7.12) and (7.15) takes the form:

$$v = \frac{4\pi m_0 \varphi_0}{mR} T \frac{R_2^2 - R_1^2}{T_2^2 - T_1^2}. \tag{7.16}$$

In this expression T and φ_0 are determined from the experiments for which the velocity of the bullet is calculated twice: with the shifted together masses (then $T = T_1$, $\varphi_0 = \varphi_{01}$) and with extended masses (then $T = T_2$, $\varphi_0 = \varphi_{02}$) and R is the distance from the point where the bullet hits the target to the pendulum axis.

The measurement procedure.

Task 1. Determination of the moment of inertia of the torsion ballistic pendulum.

1. Fix the masses m_0 as close to each other as possible and measure the distance R_1 from the axis of rotation to one of the masses. Install the pendulum at the equilibrium position ($\varphi_0 = 0$).

2. Deflect the pendulum at an angle $\varphi_0 \approx 20$.

3. Release the pendulum and measure the time t_i of $n = 5$ oscillations of the pendulum. Calculate the period of oscillation using formula $T_{1i} = \frac{t_i}{n}$. Enter the results of measurements in table 7.1.

4. Repeat the experiment according to steps 2 and 3 another two times and calculate average period of oscillation for shifted together masses $\langle T_1 \rangle$.

5. Then fix the masses m_0 as far from each other as possible and measure the distance R_2 from the axis of rotation to one of the masses. Install the pendulum at the equilibrium position ($\varphi_0 = 0$).

6. Repeat the series of experiments according to steps 2-4 three times and calculate average period of oscillation for extended masses $\langle T_2 \rangle$.

7. Calculate the torsion modulus of the wire D using (14) and the moment of inertia of the pendulum I_P using (15).

8. The results of the measurements and calculations must be entered in the table. 7.1.

Mass $m_0 =$ _________ . _________ kg

Table. 7.1

<table>
<tr><th>Masses</th><th>t, s</th><th>T,s</th><th>D</th><th>I_P</th></tr>
<tr><td rowspan="4">Shifted
together
$R_1 =$. m</td><td>.</td><td></td><td rowspan="8"></td><td rowspan="4"></td></tr>
<tr><td>.</td><td></td></tr>
<tr><td>.</td><td></td></tr>
<tr><td></td><td>$\langle T_1 \rangle =$</td></tr>
<tr><td rowspan="4">Extended

$R_2 =$. m</td><td>.</td><td></td><td rowspan="4"></td></tr>
<tr><td>.</td><td></td></tr>
<tr><td>.</td><td></td></tr>
<tr><td></td><td>$\langle T_2 \rangle =$</td></tr>
</table>

9. Make the conclusions from the obtained experimental results.

Task 2. Determination of the moment of inertia of the torsion ballistic pendulum.

1. Fix the masses m_0 as close to each other as possible and measure the distance R_1 from the axis of rotation to one of the masses. Install the pendulum at the equilibrium position ($\varphi_0 = 0$).
2. Prepare the spring pistol for the shot.
3. Shoot from the pistol and measure the maximum angle of deflection of the pendulum φ_{01}. Convert value of the angle from degrees into radians.
4. Measure the time t_i of $n = 5$ oscillations of the pendulum. Calculate the period of oscillation using formula $T_{1i} = \frac{t_i}{n}$. Enter the results of measurements in table 7.2.
5. Repeat the experiment according to steps 2-4 another two times and calculate average period of oscillation for shifted together masses $\langle T_1 \rangle$.
6. Measure the distance R from the pendulum axis to the point where the bullet hits the target.
7. Then fix the masses m_0 as far from each other as possible and measure the distance R_2 from the axis of rotation to one of the masses. Install the pendulum at the equilibrium position ($\varphi_0 = 0$).
8. Repeat the series of experiments according to steps 2-5 three times and calculate average period of oscillation for extended masses $\langle T_2 \rangle$.
9. Calculate the torsion modulus of the wire D using (14) and the moment of inertia of the pendulum I_P using (15).
10. Calculate the velocity v of the bullet using (16) for both positions of the masses and determine the average value of the velocity $\langle v \rangle$.
11. Determine the deviation $\Delta v = |\langle v \rangle - v|$ and estimate the measurement error $E = \frac{\Delta v}{v} \dot{1}00\%$.

12. The results of the measurements and calculations must be entered in the table. 7.2.

Bullet mass $m =$ ________·________ kg
Distance $R =$ ________·________ m

Table. 7.2

Masses	t_i,s	T_{1i},s	φ_0,°	φ_0, rad	D	I_P	v,m/s	$\langle v \rangle$,m/s	Δv, m/s	E,%
Shifted together	·		·	·						
	·		·	·						
	·		·	·						
$R_1 =$ · $m.$		$\langle T_1 \rangle =$	$\langle \varphi_{01} \rangle =$	$\langle \varphi_{01} \rangle =$						
Extended	·		·	·						
	·		·	·						
	·		·	·						
$R_2 =$ · $m.$		$\langle T_2 \rangle =$	$\langle \varphi_{02} \rangle =$	$\langle \varphi_{02} \rangle =$						

13. Make the conclusions from the obtained experimental results.

Control questions.

1. Formulate the basic law of the dynamics of the rotational motion of a solid. Derive the law of dynamics of solid body in the case of rotation around the stationary axis.
2. Formulate the law of conservation of angular momentum.
3. Give the determinations of the period of oscillation, how can it be measured experimentally?
4. Derive the expression for calculating the bullet velocity. Will the period of oscillations of the pendulum change if you increase the mass of its rod, or if you increase the mass of the bullet?
5. What is the technique of measuring of the velocity of the bullet?
6. Is the law of conservation of mechanical energy is fulfilled when the bullet is stuck in the target?
7. What should be considered as the most important parameters for measurement, to minimize the error of measurement of the velocity?
8. In what kind of energy will the kinetic energy of the pendulum transfer if the angle of pendulum's deviation will be maximal?

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