Ministry of Education and Science of Ukraine
National Technical University of Ukraine
“Igor Sikorsky Kyiv Polytechnic Institute”

LABORATORY PRACTICE

PHYSICAL FUNDAMENTALS OF MECHANICS

for foreign students of higher technical educational institutions

Faculty ______________________________

Group ______________________________

Name ______________________________

Kyiv 2018

Reviewers: *Voronov Sergiy, Dr. Sc. (Engineering Sciences), Professor*

Responsibleeditor: *Kotovskyi Vitalii, Dr. Sc. (Engineering Sciences), Professor*

Approval stamp is provided by the Methodical Council of the Igor Sikorsky Kyiv Polytechnic Institute (protocol №7 from 29.03.2018 y.)
on the submission of the Academic Councilof Faculty of Physics and Mathematicsof the Igor Sikorsky Kyiv Polytechnic Institute(protocol № 1 from 27.02.2018 y.)

Moiseenko Volodimir
Pugach Olha
Uzhva Valerii
Gareeva Faina
Pugach Alisa
Kulieznova Svitlana
ShtofelOlha

LABORATORY PRACTICE

PHYSICAL FUNDAMENTALS OF MECHANICS

LABORATORYPRACTICE.PHYSICALFUNDAMENTALSOFMECHANICS: роб. зошит для студентів технічних спеціальностей/*V. Moiseenko, O. Pugach,V. Uzhva, F. Gareeva, A. Pugach, S. Kulieznova, O. Shtofel*; *IgorSikorskyKyivPolytechnicInstitute*. – Kyiv : IgorSikorskyKyivPolytechnicInstitute, 2018. – 22 p.

This publication provides 4 laboratory works from the section "Mechanics" in accordance with the working curriculum of the credit module "Physics 1" of the discipline "Physics". The purpose of the educational publication is to provide foreign students with materials for laboratory work in physics that correspond to the module's curriculum for all technical specialties. The tasks of the work are to make laboratory works more accessible and understandable to foreign students.

Laboratory № 1-1.Examination of measured data processing theory in the laboratory of physics using mathematical pendulum as an example

Objectives: acquire the skills of building histograms, learn to process direct measurement data
Tools and equipment: mathematical pendulum, electronic stop watch

1.1. Theoretical basis

In physics a rigid object which swings freely about some motionless point or axis under the gravity force is considered as physical pendulum. It is customary to distinguish mathematical and physical pendulums. A mathematical pendulum is an idealized system, which consists of an imponderable and inextensible cord with the length ℓ, from which a material point with the mass *m*, which oscillates about the suspension point 0, is suspended (fig. 1.1). A small heavy ball suspended by a long tenuous poorly extensible cord can be pretty concrete approximation to the mathematical pendulum.

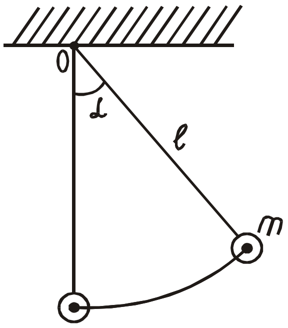

Fig.1.1

As the angles of deviation from equilibrium are small and the value of friction is so small, that it could be ignored, a physical pendulum performs harmonic oscillations. Their period is determined by the length of the pendulum ℓ and by free fall acceleration *g*:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

The oscillation period of a pendulum can be calculated by using the appropriate formula, or experimentally measured with the help of stopwatch. Such measurement of a period that is done with the help of stopwatch is called direct measurement. After the experiment we obtain the mean $\langle T \rangle$, but not the true value of *T*. We should evaluate the degree of approximation of the mean value to the true value of period *T.* For this experiment to be done successfully it is important to read (have a look at a small) short supplement "Error and measured at physical laboratory data processing theories""The theory of error and processing of measurement data at the physical laboratory".

1.2. Facility description and measurement method

The facility is a heavy ball, suspended by a poorly extensible cord, the length of which is much bigger than that of the ball. The time should be measured with electronic stop watch accurate to0,001 s. After measuring the time Δt_i of five full-wave oscillations the value of the period should be calculated using the following formula:

$$T_i = \frac{\Delta t_i}{5} \qquad (1.1)$$

The pendulum should be deviated through small angle (about 4^o) so that the oscillations can be considered as harmonic (such oscillations that occur by the sine or cosine law).

1.3. The order of work procedure

1. Set the pendulum in oscillative motion. Using stopwatch measure the time of five oscillations, write it down accurate to 0,001 s in table 1.1.
Repeat these measurements 50 times.

2. Accomplish another set of 50 measurements, write down the results in table 1.2, which issimilar to table 1.1, but is meant for 100 measurements.

3. Write down data about the stopwatch: δ (value of scale division) =…………

Table1.1

Experiment number n	Time of five oscillations Δt_i, s	Period $T_i = \frac{\Delta t_i}{5}$, s	$\Delta T_i = T_i - \langle T \rangle$, s	ΔT_i^2, s^2
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$$\sum_{i=1}^{n} T_i = \text{.............................};\quad \sum_{i=1}^{n} \Delta T_i^2 = \text{...........................};\quad \langle T \rangle = \frac{\sum_{i=1}^{n} T_i}{n} = \text{........................} \qquad (1.2)$$

Table 1.2 (for n=100)

Experiment number n	Time of five oscillations Δt_i, s	Period $T_i = \frac{\Delta t_i}{5}$, s	$\Delta T_i = T_i - \langle T \rangle$, s	ΔT_i^2, s^2
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100 !				

$\sum_{i=1}^{n} T_i =$ ....; $\sum_{i=1}^{n} \Delta T_i^2 =$ ....; $\langle T \rangle = \frac{\sum_{i=1}^{n} T_i}{n} =$ .... (1.2)

1.4.Measured data processing

1. Calculate accurate to 0,001 s using formula (1.1) the oscillation period $\langle T \rangle$ for 50 measurements. Write down the calculation results in table 1.1.

2. Accomplish such calculations for the second set of 50 measurements and write down the results in table 1.2.

3. Add to the data already available in table 1.2 values of periods T_i from the first set of measurements, as a result, composing series of 100 values of T_i.

4. Calculate the sampling mean value of oscillations period $\langle T \rangle$ for series of n = 50 (table 1.1) and series of n = 100 (table 1.2) using formula (1.2) .

5. Calculate the deviation ΔT_i of each value of period $T_i$from mean value $\langle T \rangle$:

$$\Delta T_i = T_i - \langle T \rangle \qquad (1.3)$$

For both sets: n = 50 and n = 100. Write down the results in table 1.1 and 1.2. Take into account positive and negative value ΔT_i. Make all of the calculations accurate to 0,001 s. The deviation range ΔT_i from -0,10 s to +0,10 s with equal intervals widths 0,01 s is given in tables 1.3 and 1.4.

6.Calculate the number Δn_i of values ΔT_i, that enter each of the intervals. Write them down in the second row in tables 1.3 and 1.4.

7. Calculate the fraction of values $\Delta n_i / n$, that enter each of the intervals (1,2,3, ...20.), by dividing Δn_i table 1.3 by n = 50, and Δn_i table 1.4 by n = 100. Write down the results in table 1.5.

8. Construct a histogram for series of 50 and 100 measurements. Set on the vertical axis of histogram the value $\Delta n_i / n$, and on horizontal axis – set the value of random deviation ΔT_i through the time interval. An example of a histogram for n = 50 is depicted on fig.1.2.

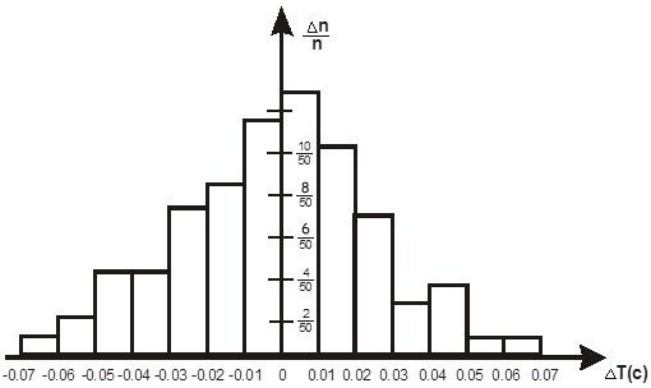

Fig.1.2

9. Calculate the standard error of the mean $S_{\langle T \rangle}$ for n = 50 and n = 100 using formula:

$$S_{\langle T \rangle} = \sqrt{\frac{\sum_{i=1}^{n} \left(T_i - \langle T \rangle\right)^2}{n(n-1)}} = \sqrt{\frac{\sum_{i=1}^{n} \Delta T_i^2}{n(n-1)}}. \qquad (1.4)$$

10. Calculate the summary standard deviation, caused by systematic errors, using formula:

$$\sigma_{\langle T \rangle \Sigma} = \frac{\sigma_{\langle \Delta T \rangle}}{m} = \frac{\delta}{m\sqrt{12}}, \qquad (1.5)$$

where δ – value of scale division of stopwatch, m = 5 (number of oscillations).

11. Check if the three sigma rule executed and write down the final. Use the data from tables 1.1 and 1.2 to accomplish the calculations.

Table 1.3 (for n = 50)

Deviation interval ΔT_i	$-0,10 \le \Delta T_i < -0,09$	$-0,09 \le \Delta T_i < -0,08$	$-0,08 \le \Delta T_i < -0,07$	$-0,07 \le \Delta T_i < -0,06$	$-0,06 \le \Delta T_i < -0,05$	$-0,05 \le \Delta T_i < -0,04$	$-0,04 \le \Delta T_i < -0,03$
№	1	2	3	4	5	6	7
Δn_i							

ΔT_i	-0,03≤ΔT_i<-0,02	-0,02≤ΔT_i<-0,01	-0,01≤ΔT_i<-0	0 ≤ΔT_i< 0,01	0,01 ≤ΔT_i< 0,02	0,02 ≤ΔT_i< 0,03	0,03 ≤ΔT_i< 0,04
№	8	9	10	11	12	13	14
Δn_i							
ΔT_i	0,04≤ΔT_i< 0,05	0,05 ≤ΔT_i< 0,06	0,06 ≤ΔT_i< 0,07	0,07 ≤ΔT_i< 0,08	0,08 ≤ΔT_i< 0,09	0,09 ≤ΔT_i< 0,10	
№	15	16	17	18	19	20	
Δn_i							

Table 1.4 (for *n* = 100)

Deviation interval ΔT_i	-0,10≤ΔT_i<-0,09	-0,09≤ΔT_i<-0,08	-0,08≤ΔT_i<-0,07	-0,07≤ΔT_i<-0,06	-0,06≤ΔT_i<-0,05	-0,05≤ΔT_i<-0,04	-0,04≤ΔT_i<-0,03
№	1	2	3	4	5	6	7
Δn_i							
ΔT_i	-0,03≤ΔT_i<-0,02	-0,02≤ΔT_i<-0,01	-0,01≤ΔT_i<-0	0 ≤ΔT_i< 0,01	0,01 ≤ΔT_i< 0,02	0,02 ≤ΔT_i< 0,03	0,03 ≤ΔT_i< 0,04
№	8	9	10	11	12	13	14
Δn_i							
ΔT_i	0,04≤ΔT_i< 0,05	0,05 ≤ΔT_i< 0,06	0,06 ≤ΔT_i< 0,07	0,07 ≤ΔT_i< 0,08	0,08 ≤ΔT_i< 0,09	0,09 ≤ΔT_i< 0,10	
№	15	16	17	18	19	20	
Δn_i							

Table 1.5

Deviation intervals ΔT_i by the number №	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$\Delta n_i / n$, *n*=50																				
$\Delta n_i / n$, *n*=100																				

Questions to answer

1. What is mathematical pendulum?
2. Which oscillations are called harmonic?
3. Name the measurement types and point out the measurement errors classification.
4. How to construct a histogram?
5. Enlist the means of getting the results of direct measurements.
6. Derive formulas for $\sigma_{\langle g\rangle\sum}$ and $S_{\langle g\rangle}$.
7. The principle of universal gravitation. Ways of getting formula ‘g’ in this work.

Laboratory № 1-2. Study of physical pendulum

Objectives: study of oscillative motion laws on the example of physical pendulum, determination of gravitational acceleration

Tools and equipment: physical pendulum (homogeneous steel rod), ruler, stop-watch.

2.1.Theoretical basis

A physical pendulum is any rigid object which swings freely about some horizontal axis under the gravity force. In this experiment physical pendulum is a homogeneous pencil rod with the length*L*. The scale is given on the rod (applied)and the prism (the edge of the prism is the axis of oscillation of pendulum) is fixed on it (a pencil rod). By moving the prism along the pencil rod you can change the distance between suspension point of a pendulum *O* and its center of mass *C*(fig.2.1).

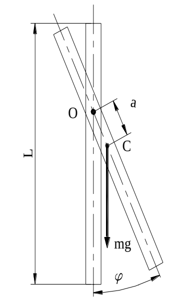

Fig. 2.1. Pendulummotionscheme

Let's assume that the moments of friction and resistance are small. In this case pendulum motion is only determined by the gravitational moment.

$$M = -mga \cdot \sin\varphi,$$

where a – distance OC from suspension point of a pendulum to the center of mass, φ – angle of deviation from the equilibrium of pendulum. By using the basic rotational equation of motion of a rigid body we will obtain:

$$I\ddot{\varphi} = -mga \cdot \sin\varphi, \quad (2.1)$$

where I – moment of inertia about the axis O; $\ddot{\varphi} = d^2\varphi / dt^2$ – angular acceleration. For small deviations from the equilibrium $\sin\varphi \approx \varphi$ we will introduce the following notation:

$$\omega_0^2 = mga / I.$$

Considering the notation above, the equation (2.1) will be as follows:

$$\ddot{\varphi} + \omega_0^2\varphi = 0. \quad (2.2)$$

The solution of this equation is well-known: it is the harmonic motion with a frequency $\omega_0 = \sqrt{mga/I}$. The solution of equation (2.2) takes the form:

$$\varphi = \varphi_0\cos(\omega_0 t + \alpha), \quad (2.3)$$

so φ is the function, φ_0 – oscillatory amplitude, α – input phase angle. It is easy to verify this information by substituting the former suggested solution in (2.1).

The oscillatory amplitude φ_0 and input phase angle α both depend on the way the oscillations excitation of pendulum is done, i.e. they are governed by so called initial conditions of the problem: initial angular deviation φ (t = 0) and initial angular velocity $\dot{\varphi}$:

$$d\varphi/dt\ (t=0) = \dot{\varphi}\ (t = 0).$$

The oscillation period T, which is connected to the frequency ω_0 through the following formula $T = 2\pi / \omega_0$, is determined by physical properties of a pendulum and the acceleration of the earth gravity g:

$$T = 2\pi\sqrt{\frac{I}{mga}}. \quad (2.4)$$

Let us denote the moment of inertia of pendulum by I_0, which is about the axis that is parallel to the axis of oscillation and passes through the center of mass C. According to the Steiner's theorem:

$$I = I_0 + ma^2, \quad (2.5)$$

(a - distances from center of mass to pivot of a pendulum),

therefore $T = 2\pi\sqrt{\frac{I_0}{mga} + \frac{a}{g}}$. (2.6)

Formula (2.6) depicts the relation between the oscillation period T and distance from the center of mass to pivot of pendulum a. This function is continuous on

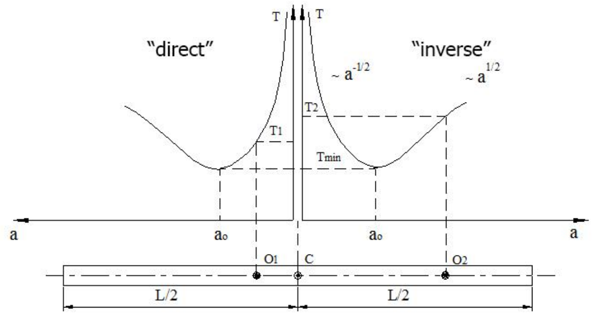

Fig.2.2. Relation between the oscillation period T and distance from center of mass to

the interval (0, ∞) and aims for the perpetuityat the end point of interval.

Let us analyse the relation $T(a)$ for very big ($a \to \infty$) and small ($a \to 0$) values of$a$. It is distinct, that at a → ∞ $T(a) \approx 2\pi\sqrt{a/g}$, so$T \sim a^{1/2}$. As before, at small values of "a" $T(a) \approx 2\pi\sqrt{I_0 / mga}$.or$T(a) \sim a^{-1/2}$. In such a situation, it is claimed thatat$a \to \infty$ period$T(a) \to \infty$ as$a^{1/2}$;at$a \to 0$ period also aims for the perpetuity, but at this time as $a^{-1/2}$. Consequently, it should reach some minimum value$a \in (0, \infty)$. (Use the optimum analysis method for the expression under the radical in formula (2.6) to show that minimum value of period $T_{min}$is achievedat $a_0 = \sqrt{I_0 / m}$.

Formula (2.6) also governs the relation$T(a)$ for both "simple" and "inverse" pendulum. All these deliberations provide the way for simple graphing of function$T(a)$, as shown on fig. 2.2.

Axes of function T on fig.2.2 must be taken ascoincident axes. While suspending pendulum at pointsO_1 i $O_2$the corresponding periods will be T_1andT_2. We take as our example of pendulum – pencil rod, but all of the obtained results can be applied to any other physical pendulum.

For homogenous pencil rod the moment of inertia$I_0 = mL^2/12$. In this case formula (2.6) can be rewritten as:

$$T^2 a = (4\pi^2/g)\, a^2 + \pi^2 L^2/3g. \quad (2.7)$$

By this means we can simplify the experimental validation of empirical relation $T(a)$, tracing it to a simple line functionwith variablesT^2a anda^2.

The function graph$T^2a\ (a^2)$is written as a line with slope ratio:

$$k = 4\pi^2/g. \quad (2.8)$$

The line has T^2a-shift in magnitude:

$$b = \pi^2 L^2/3g, \quad (2.9)$$

as shown on fig.2.3.

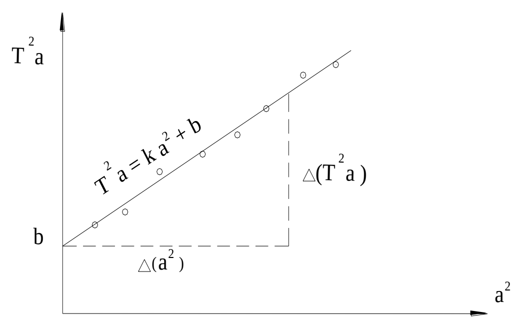

Fig.2.3. Experimental validation of theoretical relation $T(a)$.

If the obtained points fit into the line (taking into account themeasure of inaccuracy of the experiment), it means that the empirical relation (2.6) is correct. In this case we can draw the nearest equivalent to all of the values of(T^2a; a^2) line through the experimental points, which would allow us to determinate the slope ratio

$$k = (\Delta T^2 a) / (\Delta a^2)$$

and the acceleration of earth gravity$g$using formula (2.8). On fig.2.3 there are experimental values of (T^2a; a^2) with the points. By substituting the pivot point of pendulum$O$and measuring the according values of$a$and$T$, we can find the experimental relation between T^2a and a^2.

2.2.The order of work procedure

1. Study the construction of physical pendulum. Determine the center-of-gravity position of pendulum, by equilibrating it on a convenient prop.

2. Fixthe prism at the extreme left scale division value that means the maximum distance from the center of mass; with the help of dividing rule measure off the obtained length a.

3. Set the pendulum in oscillative motion in such a way thatoscillatory amplitude will not exceed 10˚ ($\sin\varphi \approx \varphi$). Measure off the time *t* of 10 full-wave oscillations at least three times and, having the obtained data as the basis,calculate the mean value of the oscillation period $\langle T \rangle$.

4. Determine the mean value of the oscillation period$\langle T \rangle$ for each value of *a* by displacing the prismover a distance of2-3 divisions of the scale, according to item 2.The empirical relation*T* (*a*) should contain not less than 14 points.

5. Calculate the values of T^2a and a^2, basing on the obtained data. Write down all of the experimental data in the table 2.1.

6. Plot *T* against *a* on a graph-paper; EvaluateT_{min} and the corresponding value $a = a_0$, which should be compared to the theoretical valueof a_0 for physical pendulum:

$$a_0 = \sqrt{I_0 / m} = L/\sqrt{12} = 0,29 \cdot L.$$

7. Plot experimental points (T^2a, a^2) on a graph-paper; draw a line, which would be proximate to all of the points. Make a conclusion whether the theoretical relation *T*(*a*) is valid.

8. Find the value of slope ratio *k*and parameter point*b*.

9. On the basis of formula (2.8) calculate the value of acceleration of the earth gravity*g*and compare it to the table value of *g*. Find the value of pendulum length $L_{exp.}$ using the value of parameter *b* and compare it to the measured value of pendulum length $L_{measured}$. Write down all of the dimensions in the table 2.2.

Table 2.1

	a (m)	*T* (s) = *t* / 10	$\langle T \rangle$ (s)	a^2 (m^2)	$\langle T \rangle^2 a$ (s^2.m)
1					
2					
3					
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Table 2.2

T_{min} (s) =……………………………………	k (s²/ m) =………………………………………
a_0 (m) =………………………………………..	b (m·s²) =………………………………………..
g_{table}= 9.8 m / s² $g_{exp.}$=…………………………	$L(m)_{exp.}$=………………… $L(m)_{measured}$=………………………
Measureofinaccuracyε = { $\lvert g_{exp.} - g_{table} \rvert / g_{table}$ }· 100% =..	

Questions to answer

1. Derive an equation of motion of physical pendulum and write down its solution for small equilibrium deviations (harmonic oscillations).
2. Verify that relation (2.3) is a solution for differential equation (2.2) using direct substitution.
3. Derive formulas that connect the oscillatory amplitude of pendulum with input phase angle under initial conditions.
4. Formulate and establish the Steiner's theorem.
5. Derive the relation between the oscillation period T and distance from the center of mass to pivot of a pendulum "a". Analyze the behavior of function T(a) as $a \to 0$ and $a \to \infty$. Depict, that T_{min} is achieved as $a_0 = \sqrt{I_0 / m}$.
6. How to obtain an experimental validation of theoretical relation $T(a)$?
7. How is the value of acceleration of earth gravity measured in this work?

Laboratory № 1-3. Study of dynamics of rotational motion by applying Oberbeck's pendulum

Objectives: experimental validation of the fundamental rotational equation of motion of a rigid body; determination inertia moment of the system

Tools and equipment: Oberbeck's pendulum, plumb bob, electronic stop watch, caliper gauge, dividing rule.

3.1. Theory(Theoretical basis)

The relation derived from the postulates of classical mechanics (Newton's laws) is the equation of motion of a rigid body rotating about a fixed axis:

$$I\beta = M_{\Sigma}, \qquad (3.1)$$

where$I$– is the moment of inertia with respect to the axis of rotation, β – angular acceleration, M_{Σ} – algebraic sum of moments of external forces with respect to the axis of rotation. Hence, the experimental validation of equation (3.1) is the validation of fundamental principles of classical mechanics.

Fig. 3.1.is a schematic representation of the experimental set-up (Oberbeck's pendulum). It consists of four pencil rods that are fixed on a bush attheright angle. Two sheaves with two different radii r_1 and r_2 are forced on that bush. Four plumb bobs with equal mass m_0 can be shifted along the rods and fixed at different distances L, providing a way to change the inertia

moment of the system. The entire construction can rotate clear about the horizontal axis. A thread with fixed plumb bob with mass *m* on its end is twirled on one of the sheaves; as a result, pendulum begins to rotate. The plumb bob is being under the action of gravity *mg* and spring (tensile) force of the thread *T*, as shown on fig.3.1.

From Newton's second law:

$$m\vec{a} = m\vec{g} + \vec{T}, \qquad (3.2)$$

where $\vec{a}$ – is plumb bob acceleration.

If we project relation (3.2) onto the vector of acceleration direction, we obtain the equation of plumb bob motion:

$$ma = mg - T. \qquad (3.3)$$

Due to Newton's second law, the opposite force *T* acts on the sheave and its moment about the axis of rotation is:

$$M = T \cdot r. \qquad (3.4)$$

The equation of pendulum motion can be simplified. The only thing that has to be done is to balance out the pendulum, in other words, try to get an indifferent equilibrium position of the pendulum in a free state (think about a pragmatic approach to accomplish and verify it). In the state of equilibriumthe pendulum mass centercoincides with the point O, which is located on a rotational axis, and gravitational moment about this axis is equal to zero. In such a case the pendulum motion is determined by the moment of tensile force of the thread *M* and moment of friction M_{fr}, which makes it possible to write down the fundamental equation of rotational motion (3.1) as:

$$I\beta = M - M_{fr}. \qquad (3.5)$$

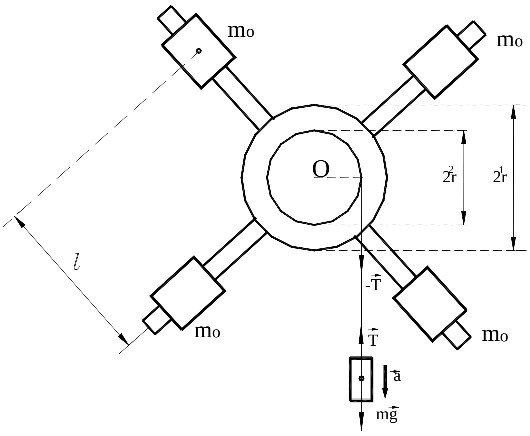

Fig.3.1. ConstructionofOberbeck'spendulum

If we solve the system of equations (3.3), (3.4), (3.5) and use the well-known relation between angular and linear acceleration

$$\beta = a / r, \qquad (3.6)$$

we obtain:

$$a = (mg\, r - M_{fr}) \cdot r / I. \qquad (3.7)$$

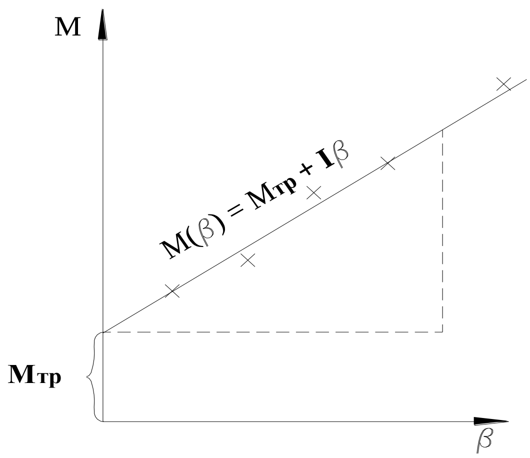

Fig.3.2. Thedeviation of experimental points $(M_i, \beta_i)$from the line$M(\beta) = M_{fr} + I\beta$is in the range of observational error value of the experiment

The moment of friction forces during the rotation can be taken as constant. In this case the motion of plumb bob is with constant acceleration (a = const).

Measuring the time *t*, which it takes for the plumb bob to come down from the state of rest at a length h, enables us to observe the angular acceleration of pendulum. As long as $h = a\, t^2 / 2$, for β the following exists:

$$\beta = 2\, h / r\, t^2 \qquad (3.8)$$

The moment of tensile force of the thread about the rotational axis can be derived from (3.3) and (3.4):

$$M = m\,(g - a) \cdot r \qquad (3.9)$$

Let us note that β and *M* can be calculated with the help of equations (3.8) and

(3.9) that are obtained independently of the fundamental equation (3.5). Let us rewrite the equation (3.5) in an opportune for the check-up form:

$$M = M_{тр} + I\beta. \qquad (3.10)$$

The equation above shows that functional relation M (β) is a line with slope ratio k that is numerically equal to the moment of inertia of the system:

$$I = k = \Delta M / \Delta\beta, \qquad (3.11)$$

and the M-intercept point is equal to the moment of friction forces M_{fr} which is depicted on fig.3.2.

If we have a number of plumb bobs, we can change mass m in a broad range and, due to this, also change values of M and β, in other words we can examine the experimental dependence: M (β). If the obtained points $(M_i,\ \beta_i)$ fit into a line (taking into account theobservational error value of the experiment) it means that the empirical relation (3.10) and the fundamental rotational equation of motion (3.1) are correct. While examining the dependence we can draw the nearest equivalent to all of the values of (M_i, β_i) line. Such a line must be situated in the range of observational error value of the experiment. Using graphical relationship M (β) we can determine the value of moment of friction forces and calculate the value of moment of inertia of the system $I$with the help of formula (3.11), fig 3.2.

3.2 The order of work procedure

1. Study (Get to know) the Oberbeck's pendulum construction. Check if it rotates clearly on the axis. Make sure that the external screw, which is fixing the bush, doesn't tighten during the rotation. Otherwise, you will not obtain correlation with the theorem, as long as pendulum motion will be affected by additional forces and their moment; this will lead to the loss of simplicity of the equation.

2. Set the plumb bobs $m_0$at certain distance from the rotational axis L in such a way that the pendulum will be in an indifferent equilibrium state. It is preferable (to be wished) that in the first experiment the value of this length would be maximum $L = L_{max}$. To make sure, that the pendulum is in balance, try to put it into a rotational motion and then let it be terminated. If the pendulum is balanced, it easily terminates at every new position and avoids oscillative motion around the equilibrium point.

3. Twirl the pendulum thread on the sheave with bigger radius ($r = r_1$) and fix the plumb bob with the mass m_1 at its end. Using formula (3.9) calculate the moment of tensile force of the thread M_1. As long as $a \ll g$, for calculation of M_1 value the approximate formula can be used:

$$M_1 \approx m_1 g r_1 . \qquad (3.12)$$

4. During the rotation of pendulum note the time t it takes for the plumb bob with the mass m_1 to accomplish height h (h = 1m). Repeat the time measuring three times and calculate the mean value $< t >$.

5. Using formula (3.8) calculate the angular acceleration β_1 which corresponds with the tensile force M_1. Plug $< t >$into the formula instead of$t$. Write down in table 3.1 data from items 3, 4, 5.

6. Repeat the experiment for several different values of mass m, each time adding one plumb bob out of given or combining them. Calculate the corresponding values of β_iiM_i, i = 1, 2, 3,… Write down all of the obtained data in the table 1.

7. Accomplish the same set of experiments for the sheave with smaller radius ($r = r_2$) and for the same value of $L = L_{max}$. Write down the obtained data in table 3.1.

8. Take the readings of value of inertia moment I of the system by setting the plumb bob m_0 at minimum distance from the rotational axis $L = L_{min}$.Repeat the experiments described in items 3 – 6. Write down the obtained data in table 3.2.

9. Plot M against β on a graph-paper for four sets of measurements. For each of the set determine the values of M_{fr} and J. Compare the results. Find the mean value for M_{fr} and also mean values of I_{min} and I_{max}.

10. To evaluate the observational error values use formulas, provided by the data processing theory (lab. work 1-1):

$$(\sigma_\beta / \beta)^2 = (\sigma_h / h)^2 + (\sigma_r / r)^2 + 4\,(\sigma_t / t)^2, \qquad (3.13)$$

$$S_{<\beta>} / \beta = 2\,(S_{<t>} / t), \qquad (3.14)$$

$$(\sigma_M / M)^2 = (\sigma_m / m)^2 + (\sigma_g / g)^2 + (\sigma_r / r)^2, \qquad (3.15)$$

where $S_{<\beta>}$ and $S_{<t>}$ – are sample standard deviations for corresponding mean values; σ_β, σ_h,... σ_r– systematic inaccuracies β, h, …, r.

11. Plot the following values on one of the experimental graphs

$$\sigma_{<\beta>} = \sqrt{S^2_{<\beta>} + \sigma^2_\beta} \text{ i} \sigma_M ,$$

which characterize the observational errors in such a way, as shown on fig.3.2. Make conclusions whether the equation (3.10) is valid in the range of observational error values.

3.3 Calculation of observational error values

It is required to calculate systematic inaccuracies for certain measurements, random mean inaccuracies (standard deviations) and relative measurement errors for β and M :

σ_t=........... σ_m =............ σ_g=............. σ_β............. σ_h = ...

σ_r =............σ_M =.............$\sigma_{<\beta>\Sigma}$ =............ $S_{<t>}$ =......................

$S_{<\beta>}$=.........................σ_M / M =........................σ_β / β=.....................

For measure of inaccuracies determination use resourse book «Data processing theory

3.4. Inertia moment $<I_{max}>$,$<I_{min}>$ and moment of friction force $< M_{fr}>$ determination

1) From table 3.1 by two values of I_{max} determine the mean value of inertia moment $<I_{max}>$ and same for moment of friction force $< M_{fr}>$:

$<I_{max}>$= ………………; $< M_{fr}>$ =………………….

2) From table 3.2 be two values of I_{min} determine the mean value of inertia moment $<I_{min}>$ and same for moment of friction force $< M_{тр}>$:

$<I_{min}>$= ………………; $< M_{fr}>$ =………………….

Table 3.1

$L = L_{max}$										
	$r = r_1 =$ (m)					$r = r_2 =$ (m)				
	m(kg) x10³	M_i N· m	$t_{1,2,3}$	$<t_i>$ (s)	β_i rad/s²	m(kg) x10³	M_i N· m	$t_{1,2,3}$	$<t_i>$ (s)	β_i rad/s²

2										
3										
4										
5										
6										
	M_{fr} = ______________(N· m) I_{max}= ______________(kg· m²)					M_{fr} = ______________(N· m) I_{max} = ______________(kg· m²)				

Table 3.2

$L = L_{min}$										
	$r = r_1$ = (m)					$r = r_2$ = (m)				
	m(kg) x10³	M_i N· m	$t_{1,2,3}$	$\langle t_i \rangle$ (s)	β_i rad/s²	m(kg) x10³	M_i N· m	$t_{1,2,3}$	$\langle t_i \rangle$ (s)	β_i rad/s²
2										
3										
4										
5										
6										
	M_{fr} = ______________(N· m) I_{min}= ______________(kg· m²)					M_{fr} = ______________(N· m) I_{min} = ______________(kg· m²)				

Questions to answer

1. Moment of force and angular momentum of system of material points relative to certain origin (point O). Relation among them – the equation of moments for system of material points.
2. Law of conservation of angular momentum for system of material points.
3. Moment of force and angular momentum relative to certain axis. The equation of momentsin this axis.
4. Moment of rigid body inertia with the respect to fixed axis. Steiner's theorem. Fundamental rotational equation of motion of a rigid body about a fixed axis.
5. How is inertia moment of pendulum measured in this experiment? What does it depend on?
6. How to determine the moment of friction force with Mvsβcurve?
7. How to evaluate theobservationalerror values?

Laboratory № 1-4. Examination of acceleration of the earth gravity with the help of the reversiblependulum

Objectives: study of acceleration of the earth gravity, examination of the reversible pendulum
Tools and equipment: reversible pendulum, electric stopwatch, dividing rule.

4.1.Theoretical basis

A physical pendulum is simply a rigid object which swings freely about some horizontal axis under the gravity force (fig.4.1). The point of intersection of vertical plane (which passes through the center of mass of the pendulum point C) and horizontal axis O is called the suspension point. The deviation from the equilibrium of pendulum is characterized by angle φ.

Let's assume that moments of friction and resistance are small. In this case pendulum motion is only determined by the gravitational moment.

$$M = -mga \cdot \sin\varphi,$$

where a – distance OC from suspension point of a pendulum to the center of mass, φ – angle of deviation from equilibrium of pendulum. By using the basic rotational equation of motion of a rigid bodywe will obtain:

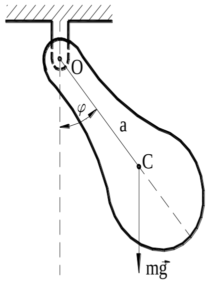

Fig.4.1.Pendulum motion scheme.

$$I\ddot{\varphi} = -mga \cdot \sin\varphi, \tag{4.1}$$

where I – moment of inertia about the axis O; $\ddot{\varphi} = d^2\varphi / dt^2$ – angular acceleration. For small deviations from the equilibrium sinφ ≈ φ. Let us introduce the following notation:

$$\omega_0^2 = mga / I.$$

Considering the notation above, the equation (4.1) will be as follows:

$$\ddot{\varphi} + \omega_0^2\varphi = 0. \tag{4.2}$$

The solution of this equation is well-known – it's the harmonic motion with the frequency $\omega_0 = \sqrt{mga/I}$. The solution of equation (4.2) takes the form:

$$\varphi = \varphi_0 \cos(\omega_0 t + \alpha), \tag{4.3}$$

so φ – is the function, φ_0 – oscillatory amplitude, α – input phase angle.

The oscillation period of physical pendulum: $T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{mga}}$ (4.4)

Let us denote the moment of inertia of pendulum by I_0, which is about the axis that is parallel to the axis of oscillation and passes through the center of mass C. According to the Steiner's theorem:

$$I = I_0 + ma^2, \tag{4.5}$$

therefore

$$T = 2\pi\sqrt{\frac{I_0 + ma^2}{mga}} . \tag{4.6}$$

Formula (4.6), which depicts the relation between the oscillation period T and distance from center of mass to pivot of a pendulum a, can be transformed to the formula below:

$$T(a) = 2\pi\sqrt{\frac{I_0}{mga} + \frac{a}{g}} \ ,$$

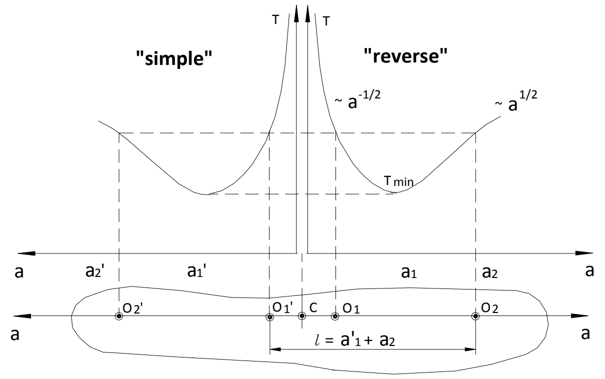

Fig.4.2. The relation between the oscillation period of physical pendulum T and the distance betweenpivot point and center of mass "a"

which provides a way to analyze the relation $T(a)$ for very big ($a \to \infty$) and small ($a \to 0$) values of a. It is distinct, that at $a \to \infty$ $T(a) \approx 2\pi\sqrt{a/g}$, so $T \sim a^{1/2}$. As before, at small values of "a" $T(a) \approx 2\pi\sqrt{I_0 / mga}$ or $T(a) \sim a^{-1/2}$. In such a situation, it is claimed that at $a \to \infty$ period $T(a) \to \infty$ as $a^{1/2}$; at $a \to 0$ period also aims for the perpetuity, but at this time as $a^{-1/2}$. Function (4.6) is continuous on the interval $(0, \infty)$ and aims for the perpetuity at the end point of interval. Consequently, it should reach some minimum value T_{min} at a □ $(0, \infty)$. Formula (4.6) also governs the relation $T(a)$ for both "simple" and "inverse" pendulum. All these deliberations provide the way for simple graphing of function $T(a)$, as shown on fig. 4.2.

Formula (4.6) provides a way for experimental observation of the acceleration of the earth gravity. Genuinely, while suspending the pendulum at different distances a_1 and $a_2$from the center of mass, the corresponding oscillation period values T_1 and T_2 can be measured. Using relation (4.6), a system of equations can be obtained:

$$T_1^2 = 4\pi^2 [(J_0 + ma_1^2) / mga_1] ,$$

$$T_2^2 = 4\pi^2 [(J_0 + ma_2^2) / mga_2] .$$

If we eliminate J_0 from the equations, we will obtain:

$$g = 4\pi^2 [(a_1^2 - a_2^2) / (a_1T_1^2 - a_2T_2^2) \qquad (4.7)$$

Period axes T should be must be taken as coincident: $a_1 = a_1'$;$a_2 = a_2'$. One and the same value of period T (under condition $T > T_{min}$) can be achieved while suspending the pendulum at points O_1, O_2, O'_1, O'_2.

However, formula (4.7) can be simplified. Let's assume that we have found positions of points $O_2$and O'_1, which are situated on different sides of the center of mass (fig.4.2). In such a case $T_1 = T_2 = T$, and formula (4.7) takes a simpler form:

$$g = 4\pi^2 \ell / T^2, \qquad (4.8)$$

where $\ell = a'_1 + a_2$.

All of the variables, that enter formula (4.8), can be easily measured to high precision. The hardest part is to determine the suspension points, at which periods of "simple" and "reverse" pendulums are almost coincident (from here comes the name – reversible pendulum).

There are lots of different constructions of reversible pendulum, one of which is depicted on fig.4.3. Two edge fulcrums ($F_1$and F_2) and two plumb bobs ($B_1$and B_2) are fixed on a steel rod, by moving them you can change period over wide range. Scales are put on the surface of the rod and they determine the position of moving elements of the construction. Their influence on the periods $T_1$and $T_2$are depicted on fig.4.4. As we can see, the movement of F_2 influences period T_2 more, than that of F_1 influences T_1. As this takes place, the position of the center of mass hardly changes, as long as the edge fulcrums are quite light. However, even small movement of plumb bob B_2 in the arrow direction result in serious center-of-mass C movement. This means that the distance a_2 elongates, and a_1 undergoes shortening by as many times. Both periods undergo shortening, however, for T_2 the process is going on at a far quicker rate, thereby the periods can be equalized. Let's examine the case, when at ordinary moment $T_1>T_2$.

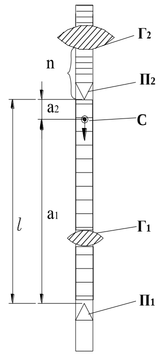

Fig.4.3 Construction of reversible pendulum.

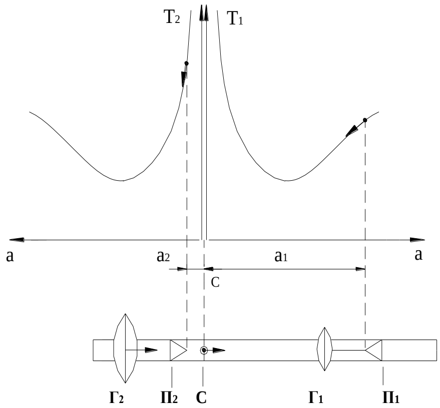

Fig.4.4. The influence of movement of reversible pendulum on period values T_1 and T_2; C – center of mass; $F_1$and F_2– movable edge fulcrums; $B_1$and B_2– movable plumb bobs

Which plumb bobs should be moved and at which direction so that the periods are equalized? Is that possible? Does the movement of plumb bobs change both periodsT_1 and T_2 in the same direction, or in different ones?

The periods $T_1$and T_2can be equalized in such a way that the difference between them will be in range of random scatter of repeat observations, which makes it possible to consider a number of values of $T_1$and T_2 as a common set T and calculate the corresponding mean value $<T>$ and standard error of the mean $S_{\langle T\rangle}$. Substantially, we are considering T_1–T_2as random inaccuracy.

Let’s examine the random measurement error impact on measure of inaccuracy g. For this purpose formula (4.7) instead of (4.8), which doesn’t take into account differences between periods, should be used. The appropriate calculation leads to the following result:

$$S_{<g>} = \sqrt{\left(\frac{\partial g}{\partial T_1} S_{<T>}\right)^2 + \left(\frac{\partial g}{\partial T_2} S_{<T>}\right)^2} = \frac{8\pi^2 l\sqrt{a_1^2 + a_2^2}}{|a_1 - a_2| \cdot T^3} S_{<T>},$$

where $S_{<g>}$.– standard error of the mean value of g. The equation for relative error calculation looks quite simple:

$$\frac{S_{<g>}}{g} = \frac{2\sqrt{a_1^2 + a_2^2}}{|a_1 - a_2|} \frac{S_{<T>}}{T}. \qquad (4.9)$$

Similarly, the systematic error can be calculated:

$$\frac{\sigma_g}{g} = \sqrt{\left(\frac{\sigma_l}{l}\right)^2 + \frac{4(a_1^2 + a_2^2)}{(a_1 - a_2)^2}\left(\frac{\sigma_T}{T}\right)^2 + 4\left(\frac{\sigma_\pi}{\pi}\right)^2}, \qquad (4.10)$$

where σ_l , σ_T ,σ_π – systematic errors of ℓ , T and π. These equations depict that the relative error g increases without limit if the subtraction $a_1 - a_2$tends to zero, and hence if $T \to T_{min}$ (fig.4. 2). That is why the experiment must be planned in such a way, that $a_1$and $a_2$are markedly different. It is simple to depict, that as the difference between $a_1$and $a_2$is big, the oscillations decay ascends, which leads to loss of accuracy while measuring the period T. The satisfactory result can be achieved if you choose $3 > a_1 / a_2 > 1.5$.

4.2. The order of work procedure

1. Study (Get to know) the construction of reversible pendulum. Set the plumb bob B_2as close as possible to the edge fulcrum F_2.
2. Set the pendulum in oscillative motion on one of the edge fulcrums in such a way that the oscillatory amplitude does not to exceed 10º. The period should be measured by the time of 10 oscillations. In the process it is not necessary to measure the period value itself each time, the time of 10 oscillations $t_1$and $t_2$that are done on edge fulcrums $F_1$and F_2respectively, can be measured instead.
3. While moving the plumb bob B_2 along the divisions, scaled on the steel rod, with 1– 2 interval, measure the values of $t_1$and t_2 no fewer than 3 times each; determine the corresponding mean values $<t_1>$and $<t_2>$ to each new position of B_2. Write down the results in table 1. Plot the mean values $<t_1>$i $<t_2>$ against n, where n is scale mark on a graph-paper. The intersecting pointwill determine the optimum position of plumb bob B, at which the values of periods$T_1$and

Table 4.2

	On F_1		On F_2	
n	t_1, s	$<t_1>$, s	t_2, s	$<t_2>$, s

T_2 will be nearest to one another. Write n_0 for intersecting point.

4. Place the pendulum on edge fulcrum F_2, and fix the plumb bob B_2 at position n_0. Set the pendulum in oscillative motion with deviation within the confines of the angle 10° and measure the time *t* of 50 oscillations. Take measurements three times.

5. Suspend the pendulum on the edge fulcrum F_1, but don't change the plumb bob position. Repeat time measurements for 50 oscillations (3 sets of measurements) (item 4). Write down the results of items 4, 5 in table 4.2.

6. For each of 6 sets of measurements determine the value of oscillations period *T*. Calculate the mean value of period $\langle T \rangle$.

7. Measure the parametric variable ℓ – distance between edge fulcrums $F_1$and F_2.

8. Using formula (4.8) calculate the free fall acceleration value $\langle g \rangle$, by substituting for *T* its mean value $\langle T \rangle$.

9. With the help of equations (4.9) and (4.10), evaluate the $\langle g \rangle$ determination inaccuracy.

Calculation parameters:

ℓ (m) =…........................; a_1 (m) =…...............................;a_2 (m) =…..............................;

σ_l (m) =…..........................; σ_T(s) =…..................................; σ_π =…....................................;

Error calculation:

$$S_{\langle T \rangle} = \sqrt{\frac{\sum_{i=1}^{6}(T_i - \langle T \rangle)^2}{6 \cdot 5}} = \ldots\ldots\ldots\ldots\ldots;\quad (S_{\langle g \rangle}/g)\cdot 100\% = \ldots\ldots\ldots\ldots\ldots;$$

$(\sigma_{\langle g \rangle}/g)\cdot 100\%$ =………………………

The net result:

$\langle g \rangle$ =.………………… ;$S_{\langle g \rangle}$ = ……………….; $\sigma_{\langle g \rangle}$=………………… Table 4.1

	Timeof 50oscillations , s	Period *T*, s	$T_i - \langle T \rangle$, s	$(T_i - \langle T \rangle)^2$, s^2
n	Edge fulcrum F_1			
1				
2				
3				
	Edge fulcrum F_2			
1				
2				
3				
$\langle T \rangle = (\sum_{i=1}^{6} T_i)/6$ =………………………………		$\sum_{i=1}^{6}(T_i - \langle T \rangle)^2$ =……………………………		

Questions to answer

1. Moment of force and impulsive moment of system of material points with the respect to certain origin (point O). Relation among them – the equation of moments for the system of material points.
2. Law of conservation of angular momentum for system of material points.
3. Moment of force and angular momentum relative to certain axis. The equation of momentsin this axis.
4. Moment of rigid body inertia with the respect to fixed axis. Steiner's theorem.Fundamental rotational equation of motion of a rigid body about a fixed axis.

5. The equation of motion of physical pendulum. Solution to this equation for small deviations from equilibrium state – harmonic oscillations.
6. The relation between the oscillation period T and distance from center of mass to pivot of a pendulum " a "
7. The method used to measure the acceleration of earth gravity g with the help of reversible pendulum.
8. How to plan the experiment, so that the uncertainty of g measurement would be the smallest?
9. Give answers to the questions, suggested in the text.

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