Visnyk NTUU KPI Seriia – Radiotekhnika Radioaparatobuduvannia, 2019, Iss. 76, pp. 5–14

УДК 621.3.01

Investigation of energy processes in circuits of oscillatory charge of supercapacitors

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Introduction. At this stage of the continuous development of combined power supply systems there is a problem of improving the methods and electrical devices aimed at the accumulation of energy and its dynamic transmission to consumers. In modern modes of operation of electric vehicles (EV), energy storage devices must withstand millions of cycles of charge/discharge without degradation of electrical characteristics. Supercapacitors (SP) can process at least one million cycles and can be used in combined power supply systems of the EV and various electrical and electromechanical objects that are stochastically in need of high pulse power.

Problem statement. In combined power supplies from the SC and accumulator battery (AB) combine high performance with the specific power of the SC with high energy specific AB, which can easily provide high power at the beginning of the movement of the EV or at a sharp change in the speed of movement, while providing the required energy storage with AB in long motion. Using combined systems with SC and AB can significantly increase the life of AB and work with low losses in circuits oscillatory charge SC. The purpose of this work is to develop the theory of energy processes in the circle of the oscillatory charge SC from AB, which is based on the consideration of the dependence of the capacity of the SC on the voltage on their terminals and the purposeful change of the initial voltages of their charge, which improves the energy efficiency of the combined power supply systems.

Results. In this work, a study of the energy characteristics in the circuits of the oscillatory charge of supercapacitors from a storage battery, which is considered as a real source of electromotive force (EMF), has been carried out. A comparison of the power characteristics of circuits of oscillatory charge SC with different values of the quality factor of the charging circuit is carried out. The approximated solution of the nonlinear nonuniform differential equation of the second order for an oscillatory process of charge SC from AB, in which the capacitance is a linear function of the voltage at its terminals is obtained, which makes it possible to determine the dependence of energy losses in charge circles on the parameters of their elements.

Conclusions. The conditions for increasing the energy transfer coefficient from AB to SC in the circuits of the oscillatory charge are analyzed. The features of the influence of the initial voltages, capacities and Q-factor of the charging circuit on the energy transfer coefficient from AB to SC are determined. The regularities of increasing the energy transfer coefficient and reducing the power losses in the circuits of the oscillatory charge from the SC from AB with the increase of the effective Q-factor of the charge circuit and the initial voltages on the terminals of such a SC are established.

Key words: energy processes; charge; supercapacitor; internal resistance; battery; power losses

DOI: 10.20535/RADAP.2019.76.5-14

Introduction

At the stage of the steady development of combined power supply systems there is a problem of improving the methods and electrical devices aimed at the accumulation of energy and its dynamic transmission to consumers [1–4]. In modern modes of operation of electric vehicles (EV), energy storage devices must withstand millions of cycles of charge/discharge without degradation of electrical characteristics. Supercapacitors (equivalent denomination – ionistors, ultracapacitors, non-linear capacitors, or double-layer electrochemical capacitors) can process at least one million cycles and can be used in combined power supply systems of the EV and various electrical and electromechanical objects that are stochastically in need of high pulse power [2, 4–10, 15, 16].

Supercapacitor (SC) can provide currents and power tens of times larger compared to new lithium ion batteries and withstand a thousand times more cycles of charge/discharge without destroying [1, 2, 8, 11]. Specific power of industrial designs of SC is $9 \cdot 10^4$ W/kg, which is 22 times more than for lithium-ion accumulator battery (AB) [2, 11]. The duration of the charge processes of the SC is from 1 to 30 seconds, which is almost 1000 times less than in AB. In modern samples of SC, the specific energy is almost 7 times lower than in industrial lithium ion batteries [2]. According to these features, in combined power supply systems AB is used for long-term power modes, and SC – for providing pulsed modes with high capacities.

An analytical review of the work related to the study of power characteristics in electric circuits of combined power supplies from the SC and AB confirmed that the studies in most cases were conducted without an analysis of the energy transfer coefficient from AB to SC. There is also no analysis of electrical energy losses in the circuits of the oscillatory charge of the SC from AB in combined systems under different initial conditions under voltage on the terminals of the SC [1–9]. Such approaches for a long time have prevented the carrying out of studies on increasing the energy performance of combined power supplies from the SC and AB.

1 Formulation of the problem

In combined power supplies from the SC and AB combine high performance with the specific power of the SC with high energy specific AB, which can easily provide high power at the beginning of the movement of the EV or at a sharp change in the speed of movement, while providing the required energy storage with AB in long motion [3,9]. Using combined systems with SC and AB can significantly increase the life of AB and work with low losses in circuits oscillatory charge SC.

The purpose of this work is to develop the theory of energy processes in the circle of the oscillatory charge SC from AB, which is based on the consideration of the dependence of the capacity of the SC on the voltage on their terminals and the purposeful change of the initial voltages of their charge, which improves the energy efficiency of the combined power supply systems.

2 Energy processes in the circuits of oscillatory charge of supercapacitors

Let us investigate the oscillatory charge of the SC from the lithium ion battery, respectively, we will consider only the charging circuit of the combined power supply represented by the equivalent scheme in Fig. 1.

According to the equivalent scheme of the combined power supply EM (Fig. 1), the SC is charged from the lithium ion battery due to the active resistance of the charging circuit $R_\Sigma = R_{AB} + R_1 + R_W$, inductance coil L and switch. In this scheme, as a constant voltage source is used a lithium-ion battery with a nominal voltage $U_n = 2,3\,\mathrm{V}$ and an internal resistance $R_{AB} = 0,012\,\mathrm{ohm}$. SC is represented as the equivalent circuit with the parallel branches with different time constants $\tau = RC$ [5], resistance of the wires $R_w = 0,01\,\mathrm{ohm}$. This scheme with three branches with sufficient accuracy reflects the energy processes in the SC with the duration transients up to 30 minutes. The first branch is represented by the capacity, the value of which depends on the voltage. This branch consists of elements C_1 and R_1, the values of which do not change and the element $C_v\,(U_1)$, whose value depends on the applied voltage to the SC. The branch has such a small constant time that its capacities are recharged in a few seconds. The second branch with unchanged parameters C_2 and R_2 is used to display transient processes that take minutes. The third branch has the biggest time constant and reflects transient processes lasting more than 10 minutes, and it is assumed that the parameters C_3 and R_3 are unchanged, that is, they are not voltage dependent on the terminals of the SC. To take into account the self-discharge of the SC in the equivalent circuit, a resistor R_4 is used [5,11].

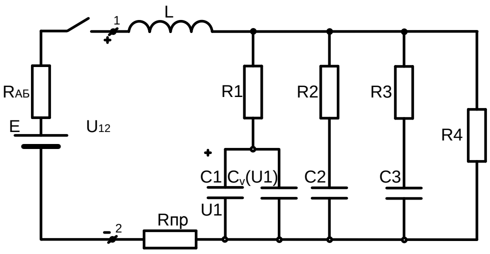

Fig. 1. Equivalent circuit for a combined power source with AB and SC

The dose of energy, that is selected from lithium ion battery at a charge of SC from zero initial conditions $U_{SC}\,(t=0) = 0, i\,(t=0) = 0$ for the final voltage $U_{SC}\,(t=\infty) = U_{AB}, i\,(t=\infty) = 0$, can be found by expression:

$$W_{AB} = \int\limits_0^\infty U_{AB} \cdot i(t) \cdot dt = U_{AB} \int\limits_0^{U_{AB}} C(U) \cdot dU, \quad (1)$$

where U_{AB} – the voltage of the battery.

The change of the dose of electric energy entering the SC during the charge from zero initial conditions $U_{SC}\,(t=0) = 0, i\,(t=0) = 0$ for the final voltage $U_{SC}\,(t=\infty) = U_{AB},\ i\,(t=\infty) = 0$(and, accordingly, change the value of charge Q on each of the plates from 0 до Q_{fin}) can be found from the expression:

$$\Delta W_{SC} = \int\limits_0^\infty U\,(t) \cdot i\,(t) \cdot dt = \int\limits_0^{Q_{fin}} U \cdot dQ =$$

$$= \int\limits_0^{U_{AB}} U \cdot (C_1 + 2kU) \cdot dU. \quad (2)$$

In the work [12], when analyzing the energy processes of the charge of a linear capacitor, variants of increasing the energy characteristics were proposed, using non-zero initial conditions under stress on terminals of linear capacitors, but the energy processes in the circuits of the charge SC, with non-zero stress conditions, were not considered.

To study the expedient operating modes of a combined power supply system with SC and AB, it is necessary to analyze the energy characteristics in the process of oscillatory charge of SC from lithium-ion AB at non-zero initial conditions under voltage at the terminals of the SC, in the range $-U_{AB} < U_{0SC} < +U_{AB}$.

In the study, the initial and final conditions for the current in the charge circuit were identical: $i\left(t=0\right)=i\left(t=\infty\right)=0$. The dose of energy entering the SC is analyzed; the dose of energy selected from AB; the energy of losses in the circuit of the charge SC and the coefficient of energy transfer from AB in the charge from the source of the constant voltage. The points in the range are studied $-U_n, -0,9\cdot U_n, ..., +U_n$. The total capacity of the SC was represented by the sum of the constant capacity $C_1 = const$ and the capacity $C_v\left(U\right) = k\cdot|U|$, which is linearly dependent on the voltage value U [3–7,10,11,13]:

$$C\left(U\right)=C_1+k\cdot\left|U\right|. \tag{3}$$

For the oscillatory charge of the SC from AB, the parameters of the electric circuit are chosen such that the condition for the Q-factor is satisfied. Considering the expression (3), we have:

$$Q_f=\frac{1}{R_{\sum}}\cdot\sqrt{\frac{L}{C\left(U\right)}}>0,5. \tag{4}$$

Taking into account the expressions (3)-(4), the Q-factor of the charging circuit is a function of the voltage on the terminals of the SC $Q_f\left(U\right)$. In the study, two different values of the inductance $L_1 = 1,697$ H and $L_2 = 42,438$ H (estimated values) and the total resistance of the charging circuit $R_\Sigma = 0,0245$ ohm were used. At the nominal voltage on the SC $|U_n|$ the Q-factor of the oscillating circle of charge (Fig. 1) is $Q_{f1}\left(|U_n|\right)=2$ and $Q_{f2}\left(|U_n|\right)=10$. The dependence of the Q-factor of the charge circuit from the voltage is taken into account in the study of combined power supply systems.

3 The solution of nonlinear nonhomogeneous differential equation

In fig. 2 is shown the voltage dependence on the terminals of the SC (a) and the current in the charge circuit (b) (at Q-factor $Q_{f1}\left(|U_n|\right)=2$) at a charge from zero initial conditions $U_{SC}\left(t=0\right)=0, i\left(t=0\right)=0$ to the moment, when the switch is closed $i\left(t\right)=0$ and the voltage on the terminals will be equal $U_{SC}\left(t_f\right)=U_f$.

The expression for the current in the circuit of charge SC can be written in the form [13]:

$$i\left(t\right)=\left(C_1+2k\left|U_{SC}\left(t\right)\right|\right)\cdot\left(\frac{dU_{SC}\left(t\right)}{dt}\right). \tag{5}$$

Given the second Kirchoff law for the scheme of the electric circuit of the oscillatory charge of the SC can be written:

$$E=U_{R\Sigma}\left(t\right)+U_L\left(t\right)+U_{SC}\left(t\right), \tag{6}$$

where $U_{R\Sigma}\left(t\right)=U_{R_{AB}}\left(t\right)+U_{R_1}\left(t\right)+U_{R_w}\left(t\right)$ – voltage drop on the resistive elements of the charging circuit.

The expressions for $U_{R\Sigma}\left(t\right)$ and $U_L\left(t\right)$ can be given:

$$\begin{aligned}U_{R_\Sigma}\left(t\right)&=R_\Sigma\cdot i\left(t\right)=\\&=R_\Sigma\cdot\left(\left(C_1+2k\left|U_{SC}\left(t\right)\right|\right)\cdot\left(\frac{dU_{SC}\left(t\right)}{dt}\right)\right),\end{aligned} \tag{7}$$

$$U_L\left(t\right)=L\frac{di\left(t\right)}{dt}. \tag{8}$$

The derivative of time for expression (5) has the form:

$$\begin{aligned}\frac{di\left(t\right)}{dt}&=\frac{d}{dt}\left(\left(C_1+2k\left|U_{SC}\left(t\right)\right|\right)\cdot\left(\frac{dU_{SC}\left(t\right)}{dt}\right)\right)=\\&=\left(C_1+2k\left|U_{SC}\left(t\right)\right|\right)^{/}\cdot\left(\frac{dU_{SC}\left(t\right)}{dt}\right)+\\&+\left(C_1+2k\left|U_{SC}\left(t\right)\right|\right)\cdot\left(\frac{dU_{SC}\left(t\right)}{dt}\right)^{/}=\\&=2k\left(\frac{dU_{SC}\left(t\right)}{dt}\right)^2\cdot\left(\frac{\left|U_{SC}\left(t\right)\right|}{U_{SC}\left(t\right)}\right)+\\&+\left(C_1+2k\left|U_{SC}\left(t\right)\right|\right)\cdot\left(\frac{d^2U_{SC}\left(t\right)}{dt^2}\right).\end{aligned} \tag{9}$$

The expression on Kirchhoff's second law, taking into account the expressions (5)-(9), will have the form:

$$\begin{aligned}&2kL\cdot\left(\frac{dU_{SC}\left(t\right)}{dt}\right)^2\cdot\left(\frac{\left|U_{SC}\left(t\right)\right|}{U_{SC}\left(t\right)}\right)+\\&+L\cdot\left(C_1+2k\left|U_{SC}\left(t\right)\right|\right)\cdot\left(\frac{d^2U_{SC}\left(t\right)}{dt^2}\right)+\\&+R_\Sigma\cdot\left(\left(C_1+2k\left|U_{SC}\left(t\right)\right|\right)\cdot\left(\frac{dU_{SC}\left(t\right)}{dt}\right)\right)+\\&+U_{SC}\left(t\right)=E.\end{aligned} \tag{10}$$

The following replacements should be introduced to simplify the appearance of the non-linear nonhomogeneous differential equation of the second order (10):

$$\begin{aligned}U_{SC}\left(t\right)&=U,\\\left|U_{SC}\left(t\right)\right|&=\left|U\right|,\\\frac{dU_{SC}\left(t\right)}{dt}&=U^{/},\\\frac{d^2U_{SC}\left(t\right)}{dt^2}&=U^{//}.\end{aligned} \tag{11}$$

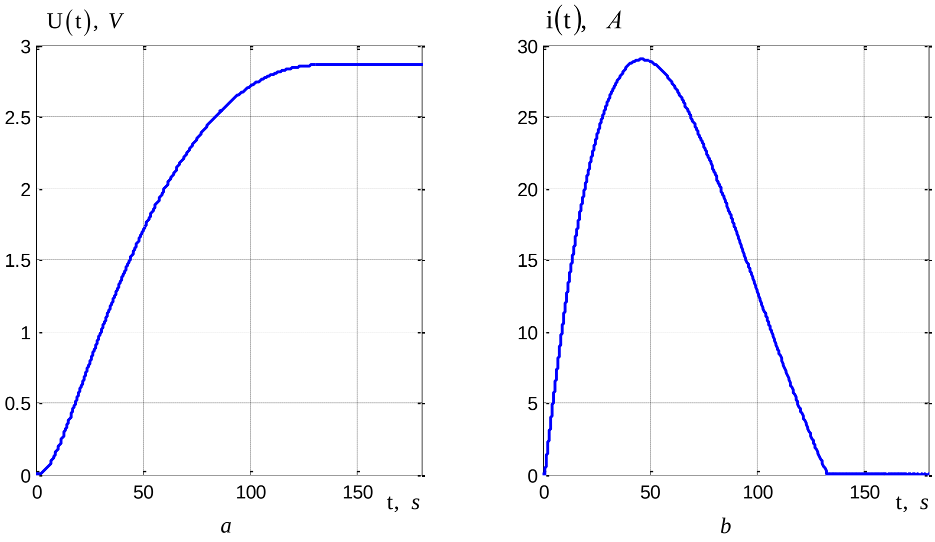

Fig. 2. The dependence of the voltage on the terminals of the SC (a) and the current in the charge circuit (b) at a charge from zero initial conditions to the moment of closing the switch

The nonlinear nonhomogeneous differential equation of the second order (10), after the use of substitution (11), will have the form:

$$L \cdot (C_1 + 2k\,|U|) \cdot U^{//} + 2kL \cdot \left(U^{/}\right)^2 \cdot \left(\frac{|U|}{U}\right) + \\ + R_{\Sigma} \cdot (C_1 + 2k\,|U|) \cdot U^{/} + U = E. \quad (12)$$

This nonlinear nonhomogeneous differential equation of second order can not be solved by direct methods. It is necessary to get the approximate value of the roots, using a numerical method in the package of applications MATLAB. Let's take the parameters of the equivalent circuit (Fig. 1) such that Q-factor is $Q_{f1}(U_n) = 2$. The oscillatory charge of SC occurs from zero initial conditions $U_{SC}(t=0) = 0, i(t=0) = 0$ to the moment when the switch is closed $i(t) = 0$ and the voltage on the terminals will be equal $U_{SC}(t_f) = U_f$.

4 Approximation of the voltage dependence on the supercapacitor terminals

The approximation of the voltage dependence on the terminals of the SC $U_{SC}(t)$, in the process of oscillatory charge $(Q_{f1}(U_f) = 2)$, was implemented with the Curve Fitting Tool (MATLAB) application, which allows describing the approximated function by the given data vectors $U_{SC}(t)$ and t from the MATLAB workspace. An exponential function is selected as an approximated function in the Curve Fitting Tool application. The dependence of the voltage on the terminals of the SC in the process of oscillatory charge over a time interval from $t = 0$ to $t = 180$ s is depicted in Fig. 2a. The voltage on the terminals of the SC, in the workspace of the application package MATLAB, is given by the data vector (with the dimension of 1x4699 points) [14].

The solution of the nonlinear nonhomogeneous differential equation of the second order (12) will be written as the sum of two exponents:

$$\mathrm{U}_{SC}(t) = a \cdot e^{bt} + c \cdot e^{dt}, \quad (13)$$

where a, b, c, d – the stable of integration, which are determined from the initial conditions.

For these initial conditions, the coefficients are defined with an accuracy of 95 %:

$$\begin{aligned} &a = -2.21e+04 \;\; (-1.648e+11, 1.648e+11)\,; \\ &b = -0.2949 \;\; (-336.9, 336.3)\,; \\ &c = 2.21e+04 \;\; (-1.648e+11, 1.648e+11)\,; \\ &d = -0.2949 \;\; (-336.9, 336.3)\,. \end{aligned}$$

The statistics of the data approximation in Curve Fitting Tool's application for the voltage dependence function on the SC terminals, depending on the time $U_{SC}(t)$:

- RMSE: 0.071 – the standard error. A value close to 0 indicates that the approximation can be used successfully because the standard deviation of the sample mean value is within the normal range.

- R-square: 0.997 – the correlation area between the initial values and the approximate values. A value close to 1 indicates that the variance is negligible [14].

- Adjusted R-square: 0.997 – this is the number of degrees of freedom of the approximated correlation area. A value close to 1 indicates a good approximation.

The current in the charging circuit is determined in accordance with the expressions (5) and (13):

$$i\,(t) = \left(C_1 + 2k\left(a\cdot e^{bt} + c\cdot e^{dt}\right)\right)\cdot$$
$$\cdot\left(a\cdot b\cdot e^{bt} + c\cdot d\cdot e^{dt}\right). \qquad (14)$$

The approximate solution of the nonlinear nonhomogeneous differential equation of the second order (12) in the form (13) satisfies the requirements for approximation. The root of the mean-square error is close to 0; accordingly, the solution of this nonlinear differential equation in the form (13) can be successfully used to analyze the energy processes in the circuits of the oscillatory charge SC from the source of a constant EMF.

The correlation area between the initial values and the approximate values and the number of degrees of freedom, of the approximated correlation area, which indicate a good approximation.

5 The analysis of the energy characteristics of the oscillatory charge of a supercapacitor

The dose of energy that is selected at the oscillatory charge of the SC from the source of the electromotive force is determined by the expression

$$W_{AB} = \int\limits_{t_i}^{t_f} U_{AB}\cdot i(t)\cdot dt, \qquad (15)$$

where U_{AB} – the voltage of the source of the electromotive force, in this case – the battery.

After substituting in this expression the formula for the current in the charging circuit (14), we obtain:

$$W_{AB} = \int\limits_{t_i}^{t_f} U_{AB}\left(C_1 + 2k\left(ae^{bt} + ce^{dt}\right)\right)$$
$$\cdot\left(abe^{bt} + cde^{dt}\right)dt. \qquad (16)$$

The dose of energy that enters the SC, with oscillatory charge from the initial voltage U_i to the final voltage U_f, is determined by the expression

$$\Delta W_{SC} = \frac{C_1\left(U_f^2 - U_i^2\right)}{2} + \frac{2k\left(U_f^3 - U_i^3\right)}{3} =$$
$$= \left(U_f - U_i\right)\cdot$$
$$\cdot\left[\frac{C_1\left(U_f + U_i\right)}{2} + \frac{2k\left(U_f^2 + U_fU_i + U_i^2\right)}{3}\right]. \qquad (17)$$

The energy transfer coefficient η_{SC} is determined by the ratio of the energy received in the SC to the energy selected from AB for the entire time of the oscillatory charge:

$$\eta_{SC} = \frac{\left(W_{SC}(t_f) - W_{SC}(t_i)\right)}{\left(W_{AB}(t_i) - W_{AB}(t_f)\right)} =$$
$$= \frac{\left(U_f - U_i\right)\cdot\left[\frac{C_1(U_f+U_i)}{2} + \frac{2k\left(U_f^2+U_fU_i+U_i^2\right)}{3}\right]}{\int\limits_{t_i}^{t_f} U_{AB}\cdot i\,(t)\,dt}, \qquad (18)$$

respectively $W_{SC}(t_i)$, $W_{SC}(t_f)$ – the energies that were accumulated in the SC accordingly before switching and after the end of transient process of the oscillatory charge from the AB; $\Delta W_{AB} = W_{AB}(t_i) - W_{AB}(t_f)$ – the energy given by AB during the oscillatory charge.

The energy of losses in the circuit of oscillatory charge of the SC from AB can be determined from the expressions (15) – (17). This is energy, which is the difference between the energy given by AB and the energy received SC during the oscillatory charge:

$$W_{losses} = \left(W_{AB}(t_i) - W_{AB}(t_f)\right) -$$
$$- \left(W_{SC}(t_f) - W_{SC}(t_i)\right) \qquad (19)$$

.

The dose of energy $W^{/}_{SC}$, that enters to the SC during the oscillatory charge from the initial voltage U_i to the final voltage U_f, is reduced to the value of W_{0AB}, that is to the quantity of the dose of energy selected from AB with an aperiodic charge of a fully discharged SC ($U_{0SC}\,(t = 0) = 0$), is determined by the expression:

$$W^{/}_{SC} = \frac{\left(U_f - U_i\right)}{\int\limits_{0}^{t_f} U_{AB}\cdot i(t)\cdot dt}\cdot$$
$$\cdot\left[\frac{C_1\left(U_f + U_i\right)}{2} + \frac{2k\left(U_f^2 + U_fU_i + U_i^2\right)}{3}\right]. \qquad (20)$$

The expression for the energy dose $W^{/}_{AB}$, which is taken from the AB during the charge from the initial voltage U_i to the final voltage U_f, has reduced to the value of W_{0AB}, can be written as

$$W^{/}_{AB} = \frac{\int\limits_{t_i}^{t_f} U_{AB}i(t)dt}{\int\limits_{0}^{t_f} U_{AB}i(t)dt} = \frac{\int\limits_{t_i}^{t_f} i(t)dt}{\int\limits_{0}^{t_f} i(t)dt}. \qquad (21)$$

The coefficient of energy transfer from AB η_{SC}, with oscillatory charge of the SC from the initial voltage U_i to the voltage U_f, for the given values of energy doses $W^{/}_{SC}$ and $W^{/}_{AB}$, has the form

$$\eta_{SC} = \frac{W^{/}_{SC}}{W^{/}_{AB}}. \qquad (22)$$

The energy of losses $W^{/}_{losses}$ in the circuit of the oscillatory charge SC from the initial voltage U_i to the final voltage U_f, has reduced to the value W_{0AB}, can be found from expressions (20)-(22)

$$W^{/}_{losses} = (W_{AB}(t_i) - W_{AB}(t_f))^{/} - \\ - (W_{SC}(t_f) - W_{SC}(t_i))^{/} = W^{/}_{AB} - W^{/}_{SC}. \quad (23)$$

6 Processing and analysis of the results

Accordingly in tab. 1, the values of the energy characteristics in the circuit of the oscillatory charge of the SC from the initial voltage U_i to the final voltage U_f, are given at Q-factor of charging circuit $Q_{f1}(U_n) = 2$ ($L = 1.697$ H):

- dose of energy W_{SC}, which enters to the SC during the oscillatory charge;
- dose of energy W_{AB}, which is selected from the AB during the oscillatory charge;
- energy of losses W_{losses} in the circuit of oscillatory charge SC;
- coefficient of energy transfer from AB η_{SC}, under given conditions. In tabl. 2 shows similar energy characteristics, when the charge from the SC from AB in a oscillatory charging circuit with $Q_{f2}(U_n) = 10$ ($L = 42.438$ H).

Numerical simulation in the MATLAB application package completely corresponds to mathematical dependencies (1), (23).

It is necessary to analyze the energy characteristics in the circuits of the oscillatory charge of the SC with variable initial voltages U_{0SC}: the dose of energy entering the SC; the dose of energy selected from AB; energy of losses in the circuit; the coefficient of energy transfer from AB.

From tabl. 1 it is evident that, when the Q-factor of the oscillatory charge circuit of SC is $Q_{f1}(U_n) = 2$, the coefficient of energy transfer from AB changes in the range from 27,13 %, when $U_{0SC} = -U_n$, to values exceeding 90,0 %, when $U_{0SC} > 0,7 \cdot U_n$.

At initial voltage on terminals of the SC $U_{0SC} = = 0,9 \cdot U_n$, the coefficient of energy transfer under the charge the SC (at Q-factor $Q_{f1}(U_n) = 2$) is increased by 1,27 times, when compared with the coefficient of energy transfer at zero initial conditions of voltage; and when changing the initial voltage of the SC from $U_{0SC} = -0,9 \cdot U_n$ to $U_{0SC} = 0,9 \cdot U_n$, the coefficient of energy transfer increases by 2,8 times.

The dose of energy W_{SC}, that entering to the SC at the initial voltage on the terminals $U_{0SC} = 0,5 \cdot U_n$, is 2912,52 J, which is 1,4 times less than with the voltage $U_{0SC} = 0$V, but the energy losses in the circuit of oscillatory charge of the SC, under given conditions, will be smaller in 2,61 times. At initial voltage on the terminals of the SC $U_{0SC} = 0,7 \cdot U_n$, the dose of energy W_{SC}, that entering to the SC, will be less than 2,032 times than the dose of energy, entering the SC with a oscillatory charge from zero initial conditions, but the energy of losses will be less in 6,35 times.

With Q-factor $Q_{f1}(U_n) = 2$, the dose of energy selected from AB, when charged from the initial voltage SC $U_{0SC} = -0,5 \cdot U_n$ to the nominal voltage on the terminals of the SC, 1.33 times more than when charged from the voltage $U_{0SC} = 0$ V. With a oscillatory charge of the SC from the voltage $U_{0SC} = 0,7 \cdot U_n$ to the voltage U_n, the dose of energy selected from AB will be less than 2,43 times, in comparison with the energy dose at a charge from zero initial conditions.

With the Q-factor $Q_{f2}(U_n) = 10$ ($L = 42,438$ H) of the oscillating charge circuit of the SC, the process of charge of the SC occurs at the coefficient of energy transfer from AB $\eta_{SC} > 57$ % (Table 2) and the higher the value of the initial voltage on the SC U_{0SC}, the higher the coefficient of energy transfer from AB.

At the charge of the SC from the initial voltage at the terminals $U_{0SC} = 0,9 \cdot U$ the coefficient of energy transfer from AB increases by 5,56 %, compared with U_n the coefficient of energy transfer from AB at zero initial conditions of voltage (Table 2). When changing the initial voltage on the terminals of the SC from $U_{0SC} = -0,8 \cdot U_n$ to $U_{0SC} = 0,8 \cdot U_n$, the coefficient of energy transfer changes 1,4 times and makes 98.23 %.

Accordingly, the dose of energy W_{SC}, accumulated during the charge in the SC from the initial voltage at the terminals $U_{0SC} = 0,5 \cdot U_n$ to the nominal voltage U_n, will be 1,55 times less than the dose of energy entering the SC during the oscillatory charge from zero initial voltage on the terminals of the SC; and the energy of losses W_{losses} in the circuit of the oscillatory charge of the SC, while, will be less than 2,59 times. At the initial voltage on the terminals of the SC $U_{0SC} = 0,9 \cdot U_n$, the dose of energy accumulated in the SC during the oscillatory charge will be 6,6 times less, than the dose of energy at the initial voltage of $U_{0SC} = 0$ V; under these conditions, the energy of losses W_{losses}, in the circuit of the oscillatory charge SC from the initial voltage on the terminals of the SC $U_{0SC} = 0,9 \cdot U_n$, will be less than 49 times for the energy of losses at oscillatory charge from zero initial conditions.

The dose of energy W_{AB}, which is selected from the AB during the oscillatory charge at Q-factor of the charge circuit $Q_{f2}(U_n) = 10$ ($L = 42,438$ H), varies nonlinearly from 12178.00 J with the initial voltage at the terminals $U_{0SC} = -0,9 \cdot U_n$ up to 1064,82 J, at initial voltage at the terminals $U_{0SC} = 0,9 \cdot U_n$. At the initial voltage at the terminals of the SC $U_{0SC} = 0,5 \cdot U_n$, the dose of energy selected from AB will be 1,59 times less, and at initial voltage $U_{0SC} = 0,9 \cdot U_n$- in

Table 1 Experimental data $Q_{f1}\left(U_n\right) = 2$

η_{SC}	W_{losses},J	W_{AB},J	U_{SCmax},V	W_{SC},J	U_{0SC},V	U_{0SC}/U_{AB}
27.13	7051.18	9677.18	3.05	2626.00	$-2,30$	$-1,0$
34.55	5963.24	9111.24	3.05	3148.00	$-2,07$	$-0,9$
41.91	4975.33	8565.33	3.05	3590.00	$-1,84$	$-0,8$
48.72	4127.39	8049.59	3.04	3922.2	$-1,61$	$-0,7$
54.91	3410.20	7563.10	3.02	4152.90	$-1,38$	$-0,6$
60.38	2814.39	7104.09	3.01	4289.70	$-1,15$	$-0,5$
65.12	2329.00	6677.10	2.98	4348.10	$-0,92$	$-0,4$
69.07	1943.40	6282.50	2.95	4339.10	$-0,69$	$-0,3$
72.19	1647.97	5926.07	2.92	4278.10	$-0,46$	$-0,2$
74.53	1429.92	5614.24	2.89	4184.32	$-0,23$	$-0,1$
76.05	1282.41	5355.17	2.86	4072.76	0	0
77.45	1148.28	5091.78	2.83	3943.49	$0,23$	$0,1$
79.12	994.27	4762.74	2.79	3768.47	$0,46$	$0,2$
81.05	828.17	4370.43	2.75	3542.25	$0,69$	$0,3$
83.20	657.89	3916.85	2.70	3258.96	$0,92$	$0,4$
85.56	491.37	3403.88	2.64	2912.52	$1,15$	$0,5$
88.12	336.68	2833.01	2.58	2496.34	$1,38$	$0,6$
90.84	201.95	2205.70	2.52	2003.73	$1,61$	$0,7$
93.74	95.42	1523.11	2.45	1427.69	$1,84$	$0,8$
96.78	25.31	786.36	2.37	761.05	$2,07$	$0,9$
100.00	0	0	2.30	0	$2,30$	$1,0$

Table 2 Experimental data $Q_{f2}\left(U_n\right) = 10$

η_{SC}	W_{losses},J	W_{AB},J	U_{SCmax},V	W_{SC},J	U_{0SC},V	U_{0SC}/U_{AB}
57.17	5462.00	12754.00	3.89	7292.000	$-2,30$	$-1,0$
63.93	4393.00	12178.00	3.89	7785.00	$-2,07$	$-0,9$
70.24	3445.00	11575.00	3.87	8130.00	$-1,84$	$-0,8$
75.77	2659.80	10978.00	3.84	8318.00	$-1,61$	$-0,7$
80.51	2025.10	10389.00	3.81	8363.90	$-1,38$	$-0,6$
84.48	1523.29	9814.99	3.76	8291.700	$-1,15$	$-0,5$
87.68	1140.87	9261.97	3.71	8121.10	$-0,92$	$-0,4$
90.17	859.23	8737.33	3.65	7878.10	$-0,69$	$-0,3$
91.92	666.93	8253.03	3.59	7586.10	$-0,46$	$-0,2$
93.02	545.33	7818.65	3.53	7273.32	$-0,23$	$-0,1$
93.51	483.19	7448.41	3.48	6965.22	0	0
93.87	433.08	7068.21	3.42	6635.13	$0,23$	$0,1$
94.31	375.32	6596.47	3.35	6221.15	$0,46$	$0,2$
94.81	313.13	6037.37	3.26	5724.24	$0,69$	$0,3$
95.39	248.93	5396.42	3.16	5147.49	$0,92$	$0,4$
96.02	186.24	4675.76	3.05	4489.52	$1,15$	$0,5$
96.70	127.84	3879.05	2.93	3751.21	$1,38$	$0,6$
97.44	79.91	3009.67	2.79	2932.76	$1,61$	$0,7$
98.23	36.56	2070.62	2.64	2034.06	$1,84$	$0,8$
99.07	9.86	1064.82	2.48	1054.96	$2,07$	$0,9$
100.00	0	0	2.30	0	$2,30$	$1,0$

6,99 times less than the dose of energy, which is selected from AB, when charged from zero initial conditions by voltage.

A generalized analysis of functional dependencies (Table 1, 2) with oscillatory charge of the SC from AB (with the Q-factors of the charging circuit $Q_{f1}(U_n) = 2$ and $Q_{f2}(U_n) = 10$), confirms that when changing the initial voltage on the SC within $-U_n \le U_{0SC} \le U_n$, the dose of energy W_{AB}, which is selected from AB during the oscillatory charge, and the energy of losses W_{losses} in the circuit of the oscillatory charge of the SC, are nonlinearly reduced from the maximum values at the initial voltage at the terminals $U_{0SC} = -U_n$ to the minimum values at the voltage U_{0SC} close to U_n. Under these conditions, the dose of energy W_{SC}, which enters to the SC during the oscillatory charge with the Q-factor of the charging circuit $Q_{f1}(U_n) = 2$, will be the maximum $W_{SC} = 4348,10$ J at the initial voltage at the terminals of the SC $U_{0SC} = -0,4 \cdot U_n$, and at Q-factor $Q_{f2}(U_n) = 10$ the maximum dose of energy $W_{SC} = 8363,90$ J, at initial voltage $U_{0SC} = -0,6 \cdot U_n$; after reaching these values, the dose of energy W_{SC} entering the SC will be nonlinearly reduced with values $U_{0SC} \to U_n$.

The coefficient of energy transfer from AB with oscillatory charge SC η_{SC}, with the Q-factors of the charge circuit $Q_{f1}(U_n) = 2$ and at $Q_{f2}(U_n) = 10$, increases nonlinearly from the minimum values at $U_{0SC} = -U_n$ to maximum at $U_{0SC} \to U_n$.

Moreover, the larger the Q-factor of the oscillatory charging circuit $Q_f(U_n)$, the higher is the dose of the energy W_{AB}, which is selected from AB during the charge of the SC, and the dose of the energy W_{SC} that enters the SC with a higher the coefficient of energy transfer η_{SC}. The energy of losses in the circuit of the oscillatory charge of the SC W_{losses} of a greater value of the Q-factor of the circuit will be less. Thus, with the oscillatory charge of the SC from a source of constant EMF (in this case, lithium ion AB), with a high value of Q-factor of the charge circuit $Q_f(U_n)$, negative values of the initial voltage on the terminals of the SC can be used and this will be advantageous from the energy point of view.

For example, with Q-factor of charging circuit $Q_{f2}(U_n) = 10$, you can accumulate large amounts of energy in the SC, with the coefficient of energy transfer $\eta_{SC} > 57$ % and the larger the value of the initial voltage U_{0SC} on the terminals of the SC, the greater will be η_{SC}. To increase the coefficient of energy transfer η_{SC}, at small values of the Q-factors of the charge circuit $Q_f(U_n)$, it is necessary to increase the positive initial U_{0SC} voltage on the terminals of the SC.

When choosing positive initial voltages on the terminals of the SC, it should be kept in mind that this reduces the dose of energy W'_{SC} entering to the SC during the oscillatory charge from the voltage U_{0SC} to U_n.

Conclusions

1. With the oscillatory charge of the SC from AB, the coefficient of energy transfer from AB varies in a nonlinear range from the minimum values, at the initial voltage at the terminals $U_{0SC} = -U_n$, to the maximum, at the initial voltage at U_{0SC} terminals close to U_n. It is determined that the higher the Q-factor of the charge circuit of the SC $Q_f(U_n)$, the higher will be the value of the coefficient of energy transfer from AB η_{SC}.

2. The dose of energy W'_{SC} entering into the SC during the time of one oscillatory charge at the Q-factor of the charging circuit $Q_{f1}(U_n) = 2$ will be the maximum $W'_{SCmax} = 1,163$, at initial voltage SC $U_{0SC} = -0,4 \cdot U_n$; and for Q-factor $Q_{f2}(U_n) = 10$, the maximum value of the energy dose is $W'_{SCmax} = 2,236$, with initial voltage $U_{0SC} = -0,6 \cdot U_n$; after reaching these values, the dose of energy W'_{SC} entering the SC will decrease nonlinearly at $U_{0SC} \to U_n$ values. According to the analysis, with the oscillatory charge of the SC from AB, with a high value of Q-factor of the charge circuit $Q_f(U_n)$, negative values of the initial voltage on the terminals of the SC can be used and this will be advantageous from the energy point of view.

3. The analysis of the functional dependences in the oscillatory charge of the SC from AB, confirms that when changing the initial voltage on the SC within the limits $-U_n \le U_{0SC} \le U_n$, the energy of losses in the circuit of the oscillatory charge SC is nonlinearly reduced from the maximum values at the initial voltage at the terminals $U_{0SC} = -U_n$ to the minimum values, at a voltage U_{0SC} close to U_n. The amount of the energy of losses W'_{losses} is reduced to the value of W_{AB}, with an oscillatory charge of the SC from zero initial voltage conditions, with a higher value of Q-factor of the charging circuit $Q_{f2}(U_n) = 10$, it will be less by 62,39 % (for W'_{losses} at $Q_{f1}(U_n) = 2$). In the transition to the region of positive values of the initial voltages on the terminals of the SC $U_{0SC} > 0$, the energy of losses W'_{losses} significantly decreases and at Q-factor $Q_{f2}(U_n) = 10$ ($U_{0SC} = 0,7 \cdot U_n$) energy losses make up 2.6 % of the energy, which gives AB.

4. The energy doses that are selected from AB W'_{AB} are given to the value W_{0AB}, with a Q-factor of charging circuit $Q_{f2}(U_n) = 10$ and the initial voltage of the SC $U_{0SC} = -0,7 \cdot U_n$ will be 1,36 times greater, and at a voltage $U_{0SC} = 0$ - in 1,39 times more, than the value of the dose of energy W'_{AB} at Q-factor $Q_{f1}(U_n) = 2$.

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Дослідження енергетичних процесів в колах коливального заряду суперконденсаторів

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Вступ. В даний час при сталому розвитку комбінованих джерел живлення виникає проблема поліпшення електричних пристроїв призначених для зберігання енергії і її динамічної передачі споживачам. При сучасних режимах експлуатації електромобілів (ЕМ), пристрої для накопичення енергії повинні працювати мільйони циклів заряд/розряд без погіршення енергетичних характеристик. Суперконденсатори (в літературі відомі, як іоністори, ультраконденсатори, нелінійні конденсатори або двошарові електрохімічні конденсатори) можуть працювати не менше мільйона циклів і можуть успішно використовуватись в комбінованих джерелах живлення ЕМ або в різноманітних електромеханічних об'єктах, які стохастично потребують велику імпульсну потужність.

Постановка задачі. В комбінованих системах електроживлення з суперконденсаторами (СК) та акумуляторними батареями (АБ) поєднуються висока питома потужність СК з високою питомою енергією АБ. Такий підхід дозволяє легко забезепечити необхідну потужність на початку руху електротранспорту або при різкому збільшенні швидкості руху, при цьому забезпечуючи необхідний запас енергії з АБ, при тривалому русі. Застосування комбінованих систем з СК та АБ може значно підвищити термін служби АБ та дозволяє працювати з низькими втратами енергії в колі коливального заряду СК. Метою цієї роботи є вдосконалення теорії енергетичних процесів в електричних колах коливального заряду СК від джерел постійної ЕРС (АБ), яка базується на врахуванні залежності ємності СК від прикладеної до його клем напруги та врахуванні змінних початкових умов по напрузі при заряді СК, що поліпшує енергоефективність комбінованих систем електроживлення.

Результати. В даній роботі досліджено енергетичні характеристики в колах коливального заряду СК від реального джерела постійної ЕРС (АБ). Проведено порівняння енергетичних характеристик кіл коливального заряду СК при різних добротностях зарядного контуру. Знайдено наближене рішення для нелінійного неоднорідного диференціального рівняння другого порядку при коливальному заряді СК від АБ, при врахуванні, що ємність є лінійною функцією від напруги на клемах СК. Дане рішення дає можливість визначати залежність енергії втрат в колах коливального заряду нелінійного конденсатора від електротехнічних параметрів елементів.

Висновки. Проаналізовані умови, за яких зростає коефіцієнт передачі енергії від АБ до СК, в колах коливального заряду СК. Визначено вплив початкових умов по напрузі на клемах СК, добротності зарядного контуру, ємності СК на коефіцієнт передачі енергії від АБ до СК в процесі заряду. Досліджено, що при підвищенні добротності зарядного контуру та збільшенні початкових умов по напрузі на клемах СК, можна збільшити коефіцієнт передачі енергії від АБ до СК та зменшити енергію втрат при коливальному заряді СК від АБ.

Ключові слова: енергетичні процеси; заряд; суперконденсатор; внутрішній опір; акумуляторна батарея; втрати електроенергії

Исследование энергетических процессов в цепях колебательного заряда суперконденсаторов

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В работе проведено исследование энергетических характеристик в цепях колебательного заряда суперконденсаторов от аккумуляторной батареи, которая рассматривается как реальный источник ЭДС. Проведено сравнение энергетических характеристик цепей колебательного заряда суперконденсатора (СК) при различных значениях добротности зарядного контура. Получено аппроксимированые решения нелинейного неоднородного дифференциального уравнения второго порядка для колебательного процесса заряда СК от аккумуляторной батареи (АБ), у которого емкость является линейной функцией от напряжения на его клеммах, что дает возможность определять зависимости энергетических потерь в цепях заряда от параметров их элементов. Проанализированы условия увеличения коэффициента передачи энергии от АБ в СК в цепях колебательного заряда. Определены особенности влияния начальных напряжений, емкостей и добротностей зарядного контура на коэффициент передачи энергии от АБ в СК.

Ключевые слова: энергетические процессы; заряд; суперконденсатор; внутреннее сопротивление; аккумуляторная батарея; потери электроэнергии