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## Analysis of the Interdependence between the Economic Growth and the Development of the Railway Sector

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**Abstract:**

**Purpose:** The authors directly formulated the ‘research hypothesis’ as the interdependence between economic growth and the volume of rail freight. It is a very valid question of primary importance not only to economic historians, but also to development economists.

**Approach/Methodology/Design:** Econometric modelling was used to set out the factors of Germany’s economic growth between 1872 and 1913, with a particular focus on the importance of railways. While analysing many determinants of German economic growth, the authors also took into account the *t* time variable, which made possible the practical application of the theorem by Frisch-Waugh with the generalization of Stone.

**Findings:** Considering these results, it can be confirmed that the railways, being a symbol of the era, and also a leading sector of the German economy, however, played an important role in shaping the modern economy and multiplying Germany’s social well-being. It can be considered as one of the causes of economic growth. On the other hand, the high economic growth rate of the German Reich was an important factor determining the development of transport. The obtained results are the basis for the construction of vector-autoregressive models (VAR) including the error correction model (ECM) and long-term relationship research.

**Practical Implications:** A systematic, quantitative inquiry on the impact of railway expansion on economic growth would most likely contribute not only to better understanding of economic history but could also shed light on the foundations of the recent monetary theory.

**Originality/Value:** Considering the importance of transport for the economy, it is particularly important to examine whether the development of transport had an impact on the level of economic growth, and whether economic growth led to the development of the transport industry, and perhaps was bidirectional relationships.

**Keywords:** Frisch–Waugh–Stone’s theorem, cliometrics, economic growth, development of rail transport.

**JEL classification:** A12, C10, N13, O47.

**Paper Type:** Research study.

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## **1. Introduction**

Although the main methodological assumptions about the New Economic History (NEH) paradigm were made in the 1950s and 1960s, it can be safely argued that they are still rarely used. As added by Diebolt and Hauper (2017), the rapid increase in the interest in the methods of NEH occurred in the 1990s, when in 1993 R. Fogel and D. North, both NEH pioneers, were honoured with the Nobel Prize of the Bank of Sweden in the field of economics. The committee recognized the contribution of these scientists both to the development of economic history and to demonstrating the effectiveness of the use of quantitative methods in the process of seeking the correctness of economic life, and moreover – the verification of many common and doubtful theories from the past (see Diebolt and Wallusch, 2015). One of such theories that, according to the authors, required verification using econometric methods, is to look for factors of economic growth in the German Reich during the period of building the capitalist order, with particular emphasis on the role of railways. The authors assumed the research hypothesis as follows:

*H1: the expansion of railways, manifested by an increase in the volume of rail freight (expressed in tonne-kilometres (tkm)) stimulated Germany's economic growth, while the high rate of German economic growth stimulated the expansion of the rail sector, which resulted in an increase in the volume of rail freight (tkm).*

One can talk about the occurrence of feedback between the railways impact on the economic growth and the economic growth on the development of railways system. To verify the formulated research hypothesis, the authors used the model of interdependent equations, including the time variable  $t$ .

The main purpose of the article, in addition to using the latest, corrected time series of the level of economic growth of the German Reich (1872-1913) was to determine the correlation between economic growth and railway deployment using the Frisch-Waugh theorem with Stone's generalization, as well as to search for other factors of economic growth.

## **2. The Pace of Railway Development in German States and the German Reich: The Level of Economic Growth**

Politically broken Germany was still an agricultural country in the mid-19th century, economically backward, after unification in 1871, long before the outbreak of World War I, it was already included in the world economic powers, with the growing importance of the industrial sector. As a result of the technological breakthrough in 1850-1873, a process of rapid change took place in the economies of German countries (take-off), which was an impulse for the development of capitalist relations (Borchardt, 1978). The effect of the growing industry was the increase in production capacity and the need to use new, efficient transport. Up to the time of rail transport, land transport was poorly developed, based mainly on inland waterway transport and

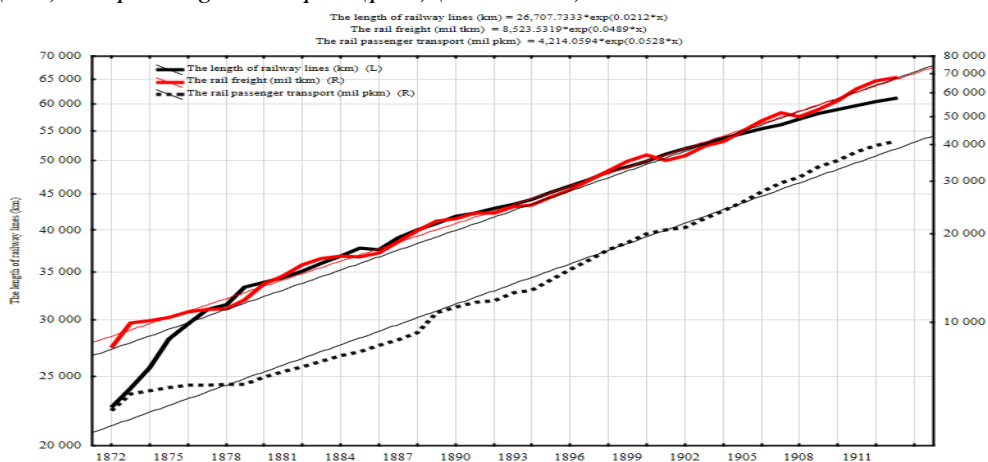
inefficient transport with rather local road significance. The construction of iron roads gave economies entering the capitalist order phase a clear revival, greater involvement of the private sector, manifested in an increase in investment expenditure, including those for the development of railways. For example, Hornung (2015) argues that railways have had a significant causal impact on urban population growth in the years 1838-1871. Railways were widely recognized as the primary growth sector of the then economies (Rostow, 1971; Fremdling, 1985; Ziegler, 1996; Myszczyzyn, 2013).

Although the first railway line was built in England, from the mid-1830s the railway has been gradually developing in particular regions of Germany. After commissioning of the first section of Nuremberg-Fürth (1835/36) (Bavaria) railway line with a length of 6 km, in 1845 – 2162 km of iron roads were already built in Germany – in comparison to 1840 an increase of 1644 km was recorded, which meant an average annual increase of almost 330 km. In 1855–1856, of 8288 km iron roads, 4264 km were private property. In the period of 1870–1913, railways were still an important factor for economic growth. The length of the iron roads on the globe increased over five times. The greatest progress in this area was made in the United States, where the length of railway lines increased in the years 1870–1913 from about 85,000 km to almost 411,000 km. Germany came second (an increase from around 43,000 to almost 64,000 km) (Möthes, 1950).

To sum up the period of rapid railway expansion: from the beginning of the expansion of rail transport in the 1830s to 1913, almost 1 million km of iron roads were built in the world; almost half of them were in the United States, another 30% in Europe. The expansion of the railway, seen as a huge and expensive investment, enabled the opening of new lands often distant from the centres for development, led to the development of exchange relations, increased the scope of internal migration and urbanization, contributed to the change in the economy of the location of industry, as well as increased international specialization (Jahrbuch für Volkswirtschaft und Statistik, 1857; Maddison, 2000). United Germany as one of the economically more developed countries reached the maximum pace of railway construction in the 1880s and 1890s, and the dynamic development of railway lines ended just before the outbreak of World War I.

The pace of railway expansion expressed in the length of railway lines in Germany against the background of freight (tkm) and passenger transport (pkm) is illustrated in the figure below (see Figure 1). In the period analysed by the authors (1872-1913), the length of German rail routes increased from 22,500 km to almost 61,200 km, rail freight reached an average annual growth rate of 4.89% (from 8.2 billion tkm to 67.7 billion tkm), while passenger transport by 5.28% (from 5 billion pkm to 41.2 billion pkm).

**Figure 1.** Length of railway lines in Germany against the background of freight (tkm) and passenger transport (pkm) (1872-1913).



**Source:** Authors' calculations based on *Statistisches Jahrbuch für das Deutsche Reich, 1885–1913*, Fremdling, 1985.

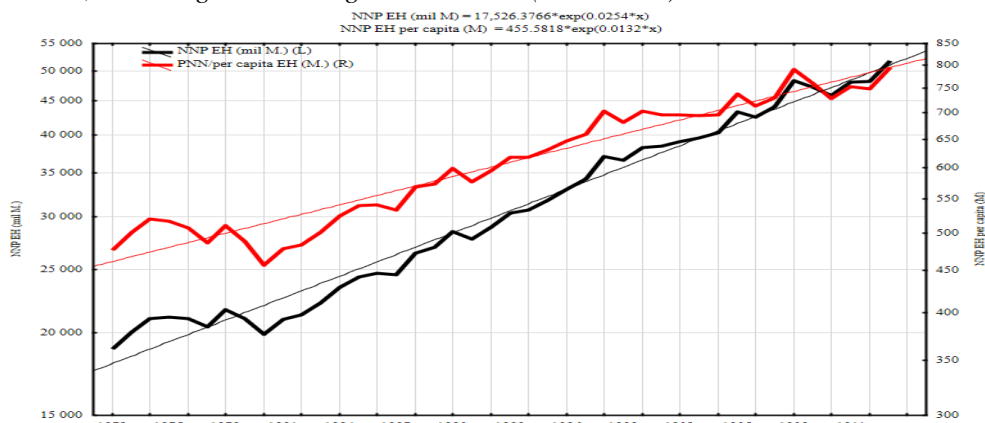
Iron railways can undoubtedly be considered one of the most important technical innovations in the 19th century. It should be added that the social savings from railways for Germany estimated by Myszczyzyn (2019) in accordance with the assumptions of Fogel (1964) were relatively low (in 1895 about 5% of Gross Domestic Product (GDP), in 1911 about 2.3% of GDP), which could mean their relatively small impact on economic growth (Mielcarek, 2010). Hence, the assessment of the impact of railway development on the economic growth of Germany using econometric modelling plays a special role.

A much more difficult issue is the analysis of available data on the level and dynamics of economic growth of the German Empire (1872-1913). National accounts published by Kaiserliches Statistisches Amt have only been available since 1891. In addition, global product accounts (NNP – Net National Product) are only available using the revenue aggregation method. Until recently, it was widely based on Germany's economic growth data in the 1960s by Hoffmann and Müller (1959), Hoffmann (1965) which were also used by Maddison (2006). At that time, German scientists had been reporting the need to revise Germany's economic growth data since the 1970s (Holtfrerich, 1973; Fremdling, 1988; Burhop and Wolff, 2005).

Hence, an important element of the authors research is the use of the latest estimates of the level of economic growth of the German Reich (1850-1913) developed by Burhop and Wolff (2005). They proposed four data series illustrating the NNP level of the German Reich at market prices. Despite the removal of deficiencies, the differences between the series (expenditure method, income method and production method) still remained, therefore they proposed a compromise NNP estimation for 1851-1913, using the weighted average of the four corrected original series.

For further analysis, the authors used the expenditure method (NNP EH), which is an example of a stationary series (Burhop and Wolff, 2005; Myszczyzyn and Mickiewicz, 2019). They also used the expenditure method (NNP EH), which is an example of a stationary series (Burhop and Wolff, 2005). In the analysed years 1872-1913, the average annual economic growth rate of NNP EH was almost 2.54% (an increase from 18,920 billion M. to 51,540 billion M.), while NNP per capita increased by 1.32% (from 476.68 M in 1872 to 792.22 M in 1913). The level of economic growth of NNP EH and NNP EH per capita (with exponential trend lines) is illustrated in the figure below (Figure 2).

**Figure 2.** The level of NNP and NNP per capita of the German Reich – expenditure method, including determining the trend line (1872-1913).



**Source:** Authors' calculations based on Burhop and Wolff, 2005.

### 3. Applied Methods

As indicated by Hozer and Zawadzki (1990), focusing mainly on studies of regularities in the field of interdependence of phenomena severely limits the utilitarian nature of econometrics. The phenomenon of strong correlation of explanatory variables is frequent, especially in the case of data in the form of time series. Thus, the authors based on the conclusions from the Frisch-Waugh-Stone theorem put forward the thesis about the need to include a time variable in the set of variables explaining the causal-descriptive econometric model and presented several reasons for using it. In addition, they proved that the omission of the time variable  $t$  generally leads to receiving weighted assessments of model parameters.

The Frisch-Waugh-Stone theorem indicates that the variable  $t$  introduced into the equation eliminates the linear trend from all processes used in the equation (Kufel, 2002). According to the above theorem, estimation of the parameters of the linear regression equation with the least squares method (OLS) based on deviations from linear trends gives the same results that are obtained after entering the time variable  $t$

into the equation (Frisch and Waugh, 1933; Stone, 1960; Hozer and Zawadzki, 1990; Hozer, 2005).

Building their own econometric model, the authors referred, among others, to the econometric model used by Clemens and Williamson (2002), also implemented by Mata and Love (2008), in which data on the length of railway lines was used as a symbol of physical capital. The econometric model used by the authors in its original form contained a dozen or so variables describing the state of the railway infrastructure, human capital, production potential, including: employment in agriculture, industrial production value, export and import value, banking sector<sup>i</sup>, as well as the time variable  $t$ , in accordance with the symbolic notation (Hozer and Zawadzki, 1990):

$$Y_t = \sum_{i=1}^k \alpha_i X_{it} + \beta_1 t + \beta_0 + U_t \quad (1)$$

where:

$Y_t$  – explained variable,  $X_{it}$ , ( $i=1, \dots, k$ ) – explanatory variables (non-random),  $U_t$  – random component,  $\beta$  – model parameters, wherein  $\alpha_0 = \beta_0$ .

Using theoretical foundations, it is possible to apply:

- least squares estimation of trend function parameters;
- based on the deviations from trends, estimation of parameters  $\alpha_i$  ( $i=1, \dots, k$ ) of the model.

Assuming that the variables  $Y_t$  and  $X_{it}$  of equation 1) there are linear trends:

$$Y_t = \delta_0 + \delta_1 t + Y_t^* \quad (2)$$

and

$$X_{it} = \delta_{i0} + \delta_{i1} t + X_{it}^* \quad (i= 1, \dots, k) \quad (3)$$

where:

$Y_t$  – explained variable,  $X_{it}$ , ( $i = 1, \dots, k$ ) – explanatory variables (non-random),  $Y_t^*$  and  $X_{it}^*$  – deviations from trends.

Therefore, substituting equations (2) and (3) to equation (1), we get:

$$\delta_0 + \delta_1 t + Y_t^* = \sum_{i=1}^k (\delta_{i0} + \delta_{i1} t + X_{it}^*) \alpha_i + \beta_1 t + \beta_0 + U_t \quad (4)$$

The Frisch-Waugh-Stone theorem further shows that the estimation of parameters  $\alpha_i$  ( $i = 1, \dots, k$ ) of the model 4) with the OLS is equivalent to the estimation of the parameters  $\alpha_i$  ( $i = 1, \dots, k$ ) of the following model:

$$Y_t^* = \sum_{i=1}^k \alpha_i X_{it}^* + U_t \quad (5)$$

After transformations, the 4) formula takes the form:

$$\delta_0 + \delta_1 t + Y_t^* = \sum_{i=1}^k \alpha_i \delta_{i0} + \sum_{i=1}^k \alpha_i \delta_{i1} t + \sum_{i=1}^k \alpha_i X_{it}^* + \beta_1 t + \beta_0 + U_t \quad (6)$$

By subtracting the 5) equation from the 6) equation, we get:

$$\delta_0 + \delta_1 t + Y_t^* = \sum_{i=1}^k \alpha_i \delta_{i0} + \sum_{i=1}^k \alpha_i \delta_{i1} t + \sum_{i=1}^k \alpha_i X_{it}^* + \beta_1 t + \beta_0 + U_t \quad (7)$$

The following relations occur between the structural parameters  $\alpha_i$  and trend parameters:

$$\beta_1 = \delta_1 - \sum_{i=1}^k \alpha_i \delta_{i1} \quad (8)$$

$$\beta_0 = \delta_0 - \sum_{i=1}^k \alpha_i \delta_{i0} \quad (9)$$

Thus, the Frisch-Waugh-Stone theorem shows that the trend parameters  $\beta_0$  and  $\beta_1$  in (1) after taking into account (2) and (3) are the differences between the trend parameters of the explanatory variable  $\delta_0$  and  $\delta_1$  and the sum of the corresponding weighted parameters of the trends of the explanatory variables, while the weights are the parameters  $\alpha_i$ .

Using the above, estimation of parameters  $\alpha_i$  ( $i = 1, \dots, k$ ) of the model based on deviations from trends was used:

$$Y_t^* = \sum_{i=1}^k \alpha_i X_{it}^* + U_t \quad (10)$$

The inclusion of the time variable  $t$  in the construction of the cause-effect model had strong theoretical and practical justification. As indicated by Hozer and Zawadzki (1990) the need to include a time variable is evidenced by the following premises:

- incomplete set of explanatory variables, and the variable  $t$  is a symptom of unobserved variables;
- load of  $X_{it}$  parameter evaluations resulting from omitting the variable  $t$ ;
- load of constant term;
- reduction of the intensity of the phenomenon of catalysis and negative consequences associated with the phenomenon of collinearity;
- positive impact on the coincidence of the model;
- $t$  variable should appear in the final econometric model instead of delayed endogenous and exogenous variables (Hozer and Zawadzki, 1990).

It is worth mentioning that Hozer (2005) considered that the Frisch-Waugh-Stone theorem seems to be of fundamental importance in modelling economic phenomena. However, he added that it is rarely explored in the economic literature. This further strengthens and justifies the use of the above theorem for the author's own calculations. The authors built a two-equation model in which he assumed that:

- the NNP EH of the German Reich was the explained variable (1872–1913);
- the freight (million tkm) was the explained variable in the second equation.

In order to estimate the constructed two-equation model with interdependent equations, the double least squares method (2SLS) was used. The method can be used for both uniquely and ambiguously identifiable models.

The structural model for the two-equation model is presented below (taking into account the deviations from trends calculated by the authors, in accordance with the assumed Frisch-Waugh-Stone theorem):

$$Y_t^* = \sum_{i=1}^k \alpha_i X_{it}^* + U_t \tag{11}$$

$$R_t^* = \sum_{i=1}^k \alpha_i X_{it}^* + U_t \tag{12}$$

where:

$Y_t^*$  – explained variable: NNP EH (Mark (M.) in prices from 1913), deviations from the trend,  $R_t^*$  – explained variable, rail freight (tkm), deviations from the trend,  $X_{it}^*$  ( $i=1, \dots, k$ ) – explanatory variables (deviations from the trend),  $U_t$  – random component.

An important step in building interdependent models was to check traceability. This assumption was met in the analysed equations – the equations are ambiguously identifiable (Goldberger, 1972; Nowak, 2006).



The two-equation model of interdependent equations for the estimated 2SLS of the explained variables: NNP EH, expected value ( $\hat{Y}_T^*$ ) and rail freight, expected value (million tkm) ( $\hat{R}_T^*$ ) took the following form:

$$\hat{Y}_T^* = \alpha_{10} + \alpha_{11}R_T^* + \alpha_{12}E_G^* + \alpha_{13}C_B^* + \alpha_{14}A_E^* \quad (13)$$

$$\hat{R}_T^* = \alpha_{20} + \alpha_{21}Y_T^* + \alpha_{22}W_R^* + \alpha_{23}E_X^*$$

where:

$R_T^*$  – rail freight (million tkm),  $E_G^*$  – level of emigration (thousand people),  $C_B^*$  – level of capital and reserves of issuing banks (million M.),  $A_E^*$  – employment in agriculture (thousand people),  $W_R^*$  – employment in railways (thousand people),  $E_X^*$  – export (million M.),  $\alpha_{mi}$  – estimated parameters of the equation, where  $m$  = the next number of the equation ( $i = 1, \dots, k$ ) – the next parameter at the explanatory variable.

There are two stages in estimating the model:

- in the first stage, using the OLS, the theoretical values for NNP EH (M.) and rail freight (tkm);
- in the second stage, the calculated theoretical variables for rail freight (tkm) and other exogenous variables were used to estimate NNP EH from the first equation. Then the theoretical values of NNP EH and other explanatory variables were used to estimate the explained variable rail freight (tkm).

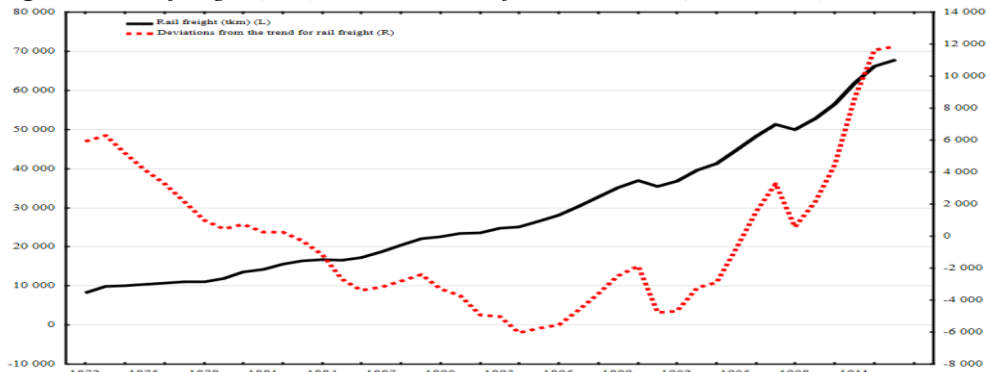
At the same time, the authors assumed that in the case of autocorrelation of a random component or collinearity, an additional generalized OLS Cochrane-Orcutt method will be used.

#### 4. Results

After specifying the model specification and selection of explained variables based on own economic knowledge and general statistical analysis, the variables selected in the first stage were evaluated using descriptive statistics of explanatory variables. After verification, according to the conclusions resulting from the Frisch-Waugh-Stone theorem, the authors calculated linear trends for all explained and explanatory variables and deviations from these trends.

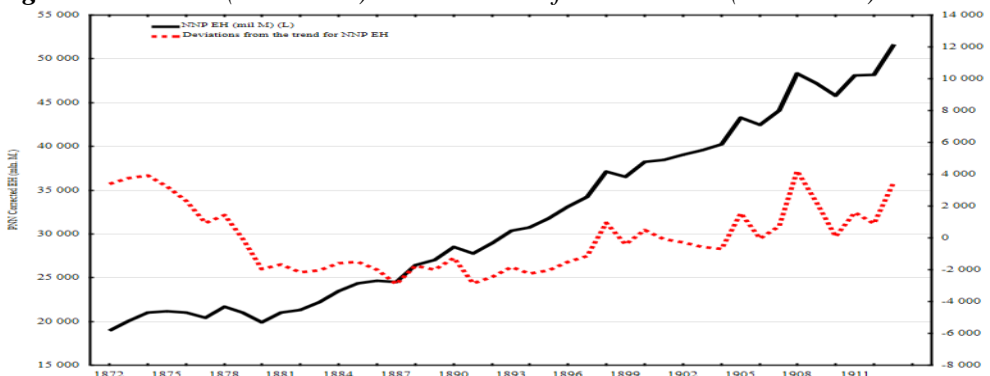
The time series of the variable: rail freight (million tkm) including the determination of deviations from the trend and the level of economic growth in Germany expressed as NNP EH (expenditure method) including the estimation of deviations from the trend is illustrated below. Figure 3 and Figure 4 illustrate the time series of the analysed variables.

**Figure 3.** Rail freight (tkm) and deviations from the trend (1872-1913).



**Source:** Authors' calculations based on *Statistisches Jahrbuch für das Deutsche Reich, 1885–1913*, Fremdling, 1985.

**Figure 4.** NNP EH (million M.) and deviations from the trend (1872-1913).



**Source:** Authors' calculations based on *Burhop and Wolff, 2005*.

The results of estimation of the explained variable (deviation) of NNP EH with the 2SLS method are presented in the Table 1.

**Table 1.** Results of the 2SLS estimation of the parameters of the linear model describing the explained variable NNP EH

Explanatory variable	Parameter of the variable	Standard error $S_E$	Statistics $t$ (37)	Value $p$
$\hat{\alpha}_{10}$ (free word)	44.3050	149.620	0.2961	0.7688
$\hat{\alpha}_{11}$ (rail freight, million tkm) – theoretical values)	0.2516	0.0565	4.452	7.55e-05
$\hat{\alpha}_{12}$ (emigration, thousand)	-16.445	3.4833	-4.721	3.33e-05
$\hat{\alpha}_{13}$ (capital and reserves of issuing banks, million M)	22.146	8.8257	2.509	0.0166

$\hat{\alpha}_{14}$ (employment agriculture, thousand people)	-2.9751	0.9233	-3.222	0.0027
Determination coefficient $R^2$ (Cor. $R^2$ )	0.8031 (0.7818)			
Standard estimation error	946.782			
Statistics $F$ $p$ – value	F(4.37) = 37.7314 $p < 1.39e-12$			
Durbin-Watson test: DW test statistics Autocorrelation of residuals	1.8795 0.0539			
Lagrange multipliers test:				
LM statistics Critical value $\chi^2$	0.1329 3.8415			
Ljung-Box statistics Critical value $\chi^2$	0.1312 3.8415			

**Source:** Authors' calculations.

The results of 2SLS estimation for the NNP EH variable showed that the adopted independent variables allow to explain 80.31% of the total variance of the variable ( $R^2=0.8031$ ), which turned out to be statistically significant (statistics  $F=37.7314$ ;  $p<1.39e-12$ ). Low values of variance inflation factors (VIF) for the independent variables appearing in the model confirmed that there was no collinearity phenomenon. The results of the White test for residue heteroscedasticity ( $H_0$ : heteroscedasticity of residuals does not occur) show that there are no grounds for rejection of  $H_0$ , because:  $nR^2 < \chi^2_*$  ( $11.1908 < 21.0261$ ), so there is homoscedasticity of variance of the random component.

The Durbin-Watson coefficient (DW=1.8795) means that the residuals of the model were characterized by the lack of autocorrelation. In addition, the Lagrange multipliers test was calculated, which only confirmed the DW test result. Assessments of structural parameters (assuming *ceteris paribus*) lead to the following conclusions:

- the increase in rail freight by 1 million tkm led to an increase in NNP EH by 251,6 thousand M. This means a positive impact of the expansion of the railway sector expressed in the increase of rail transport potential on the economic growth of Germany. This would mean that with an average annual increase in rail freight (698 million tkm), this factor would account for 27.5% of economic growth (NNP EH);
- the increase in the level of emigration by 1 thousand people impacted the decrease of NNP EH by nearly 16.445 million M.;
- the increase in the value of capital and reserves of issuing banks by 1 million M. caused by the increase of NNP EH by 22.146 million M.;
- the increase in the number of people employed in agriculture by 1 thousand people affected the decrease of NNP EH by almost 2.98 million M.

Of the four significant variables explaining the growth of two (rail freight and capital and reserves of issuing banks), it has a positive impact on the growth of NNP EH. The increase in emigration and employment in agriculture resulted in a decrease in the value of NNP EH.

The results of the 2SLS estimation for the variable: rail freight showed that the adopted independent variables allow to explain 96.44% of the total variance of the variable, which turned out to be statistically significant (statistics  $F=343.5218$ ;  $p<1.39e-27$ ). Low values of variance inflation factors (VIF) for independent variables appearing in the model confirm that there was no collinearity phenomenon. The results of the White test for residue heteroscedasticity ( $H_0$ : heteroscedasticity does not occur) show that we reject the  $H_0$  hypothesis because:  $nR^2 > \chi^2_*$  ( $33.732>16.919$ ), so there is heteroscedasticity of variance of the random component (Table 2).

**Table 2.** Results of the 2SLS estimation of the parameters of the linear model describing the explained variable rail freight (tkm)

Explanatory variable	Parameter of the variable	Standard error $S_E$	Statistics $t$ (37)	Value $p$
$\hat{\alpha}_{20}$ (free word)	54.1102	138.137	0.3917	0.6975
$\hat{\alpha}_{21}$ (NNP EH – theoretical values)	0.2851	0.1272	2.242	0.0309
$\hat{\alpha}_{23}$ (export. million M)	4.0790	0.2425	16.82	3.37e-019
Determination coefficient $R^2$ (Cor. $R^2$ )	0.9644 (0.9616)			
Standard estimation error	893.3648			
Statistics $F$	F(4,37) = 343.5218			
$p$ – value	$p < 1.45e-27$			
Durbin-Watson test: DW test statistics	1.3537			
Autocorrelation of residuals	0.26159			
Lagrange multipliers test:				
LM statistics	2.7865			
Critical value $\chi^2$	3.8415			
Ljung-Box statistics	1.819			
Critical value $\chi^2$	3.8415			

**Source:** Authors' calculations.

The study of the autocorrelation of model residuals showed that the Durbin-Watson coefficient (DW=1.3537), which meant that the model residuals had a possible positive autocorrelation, hence the Lagrange multiplier test was additionally calculated, the results of which confirm that there are no autocorrelation of residuals.

In view of residual heteroscedasticity, the author estimated the model using the generalized least squares method (random component heteroscedasticity correction method). The results are illustrated in Table 3.

Lower standard errors were obtained for exogenous variables, higher determination coefficient  $R^2=97.09$ . The values of parameters with exogenous variables have also changed. It should be added that even if random errors do not have a normal distribution, the assumptions of the Gauss-Markov theorem are still met. So the best estimator, the OLS obtained, the  $\beta$  parameter vector is the best linear and unbiased estimator. Also, the OLS estimator of the variance-covariance matrix is unbiased.

**Table 3.** Results of estimation of the parameters of the linear model calculated by the generalized least squares method (correction of heteroscedasticity of the random component) describing the explained variable: rail freight (million tkm)

Explanatory variable	Parameter of the variable	Standard error $S_E$	Statistics $t$ (37)	Value $p$
$\hat{\alpha}_{20}$ (free word)	54.9653	139.1	0.3952	0.6949
$\hat{\alpha}_{21}$ (NNP EH – theoretical values)	0.2629	0.0795	3.307	0.0021
$\hat{\alpha}_{22}$ (employment level in railways, thou. persons)	11.0243	3.443	3.202	0.0028
$\hat{\alpha}_{23}$ (export million M)	4.1143	0.1738	23.66	2.41e-024
Determination coefficient $R^2$ (Cor. $R^2$ )	0.9709 (0.9686)			
Standard estimation error	2.1897			
Statistics $F$ $p$ – value	$F(4,38)= 422.4712$ $p < 3.24e-29$			
Durbin-Watson test: DW test statistics Autocorrelation of residuals	1.367 0.246			

**Source:** Authors' calculations.

Assessments of structural parameters (assuming *ceteris paribus*) lead to the following conclusions:

- the increase in the NNP EH global product by 1 million M led to an increase in rail freight by 0.2629 million tkm. This means a positive impact of economic growth on the possibilities of expanding the railway sector expressed in the increase in the potential of rail freight (tkm). This would mean that with an average annual growth of NNP EH (by 756 million M.), this factor would account for almost 29% of the increase in the potential of rail freight (tkm);
- the increase of the employment level in railways by 1 thousand people contributed to the increase in rail freight (tkm) by almost 11.024 million tkm. This would mean that with an average annual increase in employment

- in railways (by 10.6 thousand people), this factor would account for almost 15.5% increase in the potential of rail freight (tkm);
- the increase in the value of exports of the German Reich by 1 million M. caused an increase in rail freight (tkm) by 4.1143 million tkm.

Of the three important explanatory variables, the increase in all (including the NNP EH global product) had a positive impact on the increase in rail reloading capacity.

## **5. Concluding Remarks**

United Germany relatively quickly became the economic power of the world, although in the mid-19th century nothing pointed to this. The expansion of industry, progress in agriculture, rapid population growth, changes in the education and higher education system, the development of international exchange with increasingly effective transport determined the systematic economic growth.

The expansion and widespread use of rail transport has revolutionized Germany's economic system. Unlike England, the expansion of railways in German countries took place at a similar time as the industrialization process. For this reason, the railway is considered to be the leading sector of the German industrialization process. The creation and development of the railway forced the involvement of German states, but more importantly unprecedented activity of the private sector. Rail quickly proved to be not only a faster means of transport, but also more convenient, reliable and, what is more important, profitable. The railway sector invested huge share capital estimated in 1909 at the level of almost 15 billion M. The forward and backward effects determined the increase in demand for many raw materials and materials affecting the development of the machinery industry, while the increase in the railway sector ensured a permanent decrease in transport tariffs.

Hence, there was a clear need to use econometric modelling to look for growth factors in Germany. The use of a two-equation model of interdependent equations confirmed the hypothesis that the high rate of economic growth in Germany, expressed as NNP EH, had a positive effect on the growth of the variable rail freight (tkm). In turn, the increase in the possibilities of increased transport of goods expressed in tonne-kilometres (tkm) had a positive effect on Germany's economic growth.

The test results confirm that there was feedback for these two variables. As the authors stated, the increase in rail potential manifested by an increase in rail transport (tkm) had a positive effect on Germany's economic growth, and this scale amounted to almost 27.5% of the total growth of NNP EH, while NNP EH determined 29% of the total increase in rail freight (tkm). It has been proven that the increase in NNP EH by 1 million M increased the possibility of rail transshipment by 285.1 thousand tkm (and after using the generalized method by 262.9 thousand tkm).

At the same time, the authors pointed to other factors influencing the economic growth of the German Reich, such as: the size of banks' capital and reserves, the level of emigration and the level of employment in agriculture. The econometric modelling used by the authors of the work, including the practical use of the conclusions drawn from the Frisch-Waugh-Stone theorem and social savings account, showed the effectiveness of these methods, while enabling the extension of the authors' previous research, together with the included monographs (Myszczyzyn, 2013; 2019). At the same time, the researchers plans to continue research on the factors of economic growth in Germany in the future using quantitative methods and use cointegration models as an alternative to a structural approach to multi-equation modelling (Myszczyzyn and Mickiewicz, 2019).

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## Notes

<sup>i</sup>Some explanatory variables proved to be strongly correlated with other explanatory variables and weakly correlated with explanatory variables, and they were omitted in the model.